Question 1

a. How many $\overset{n?}{\text{students}}$ in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on scale from 0 to 100 points. (5 marks)

b. What is the minimum number of students required in a Structure Discrete class So that at least six students will receive the same letter grade (A, B, C, D or F). (5 marks)

a. pigeonhole, k = Score grade = 100

pigeon, n :

$$\text{at least 2 Students} = \frac{n}{m}$$

$$\text{Let } 1.01 = \frac{n}{100}$$

$$n = 101$$
$$\approx 102$$

$\therefore$ 102 students in a class

b. pigeonhole, k = grade = $\{A, B, C, D, F\}$
$= 5$

pigeon, n :

$$\text{at least 6 students} = \frac{n}{m}$$

$$\text{Let } 5.01 = \frac{n}{5}$$

$$n = 25.05$$
$$\approx 26$$

$\therefore$ minimum number of students required in a Discrete Structure class is 26 students.

Question 2

The following table gives information on mobile phone sold by a certain store :

	Percentage of customer purchasing	Of those who purchase, percentage who purchase extended warranty
Brand 1	70	20
Brand 2	30	40

A purchaser is randomly selected from among all those bought a mobile phone from same Store. Determine the probability that :

a. customer purchased Brand 1

$P(A)$ = customer purchased Brand 1.

$P(A) = 0.7$

b. $P(B)$ = customer purchased Brand 2.

$P(B) = 0.3$

c. $P(W)$ = Customer purchased extended warranty.

$P(W|A) = 0.2$

d. $P(A \cap W) = P(A) \cdot P(W|A)$

$= 0.7 \cdot 0.2$

$= 0.14$

e. $P(B \cap W)$

$P(W|B) = 0.4$

$P(B \cap W) = P(B) \cdot P(W|B)$

$= 0.3 \cdot 0.4$

$= 0.12$

f. $P(W) = P(W|A) \cdot P(A) + P(W|B) \cdot P(B)$

$= (0.2)(0.7) + (0.4)(0.3)$

$= 0.26$

g. $P(A|W) = \dfrac{P(A \cap W)}{P(W)}$

$= \dfrac{0.14}{0.26}$

$= 0.5385$

Question 3

Question 3

Explain the given keyword using your own word and represent your understanding by drawing the graph.

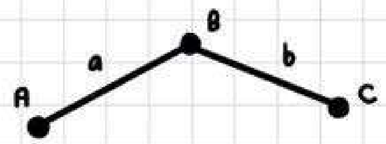
a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. Loop
g. Parallel Edges

(7 Marks)

a) vertices → a point that can connect edges.

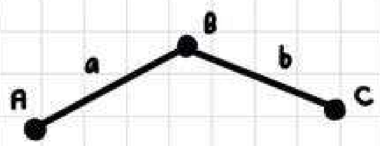
∴ point A and point B are vertices.

b) Edges → lines between two or more points



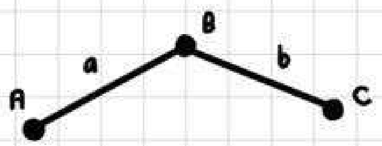
∴ line a and b are edges.

c) adjacent vertices → two vertices that are connect by an edge



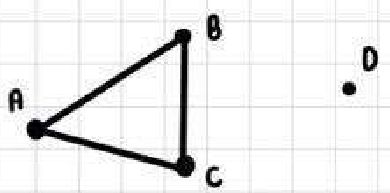
∴ point A and point B are adjacent to each other while point B and point C are adjacent to each other.

d) Incident edge → One of two vertices is connect by edges



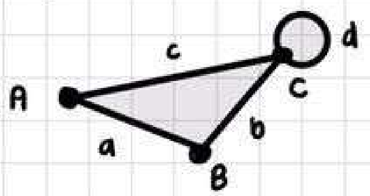
∴ point A and point B are incident to edge a.

e) isolated vertex → vertex that not connected with any edge



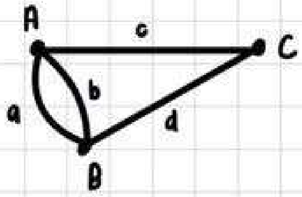
∴ D is isolated vertex.

f) loop ⟶ edge with one endpoint



∴ edge d is a loop.

g) parallel edge ⟶ edges that share one vertex



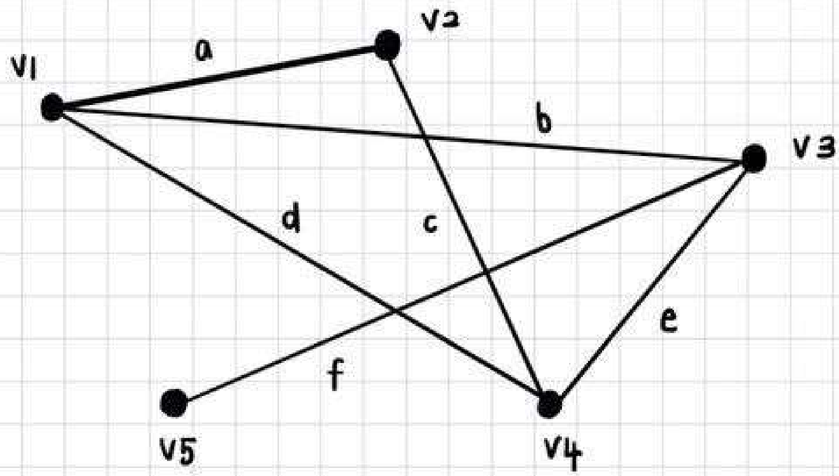
∴ edge a and edge b are parallel edges.

Question 4

Let G = {V, E} be a graph. An undirected graph having V ={v1, v2, v3, v4, v5} and E = {a, b, c, d, e, f}. Where a = (v1,v2), b = (v1,v3), c = (v2,v4), d = (v1,v4), e = (v3,v4) and f = (v3,v5).

Find the degree of each vertex.

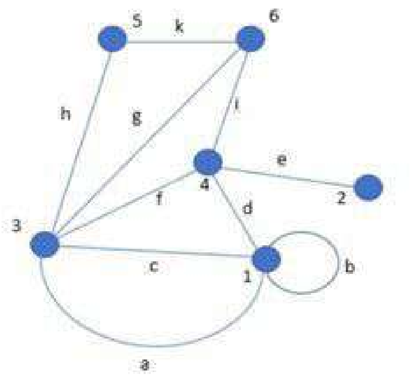
(5 Marks)



deg(v1) = 3 , deg(v2) = 2 , deg(v3) = 3 ,

deg(v4) = 3 , deg(v5) = 1

Question 5



Given the graph shown above, Find:

i. Incidence Matrix (6 Marks)
ii. Adjacency Matrix (6 Marks)

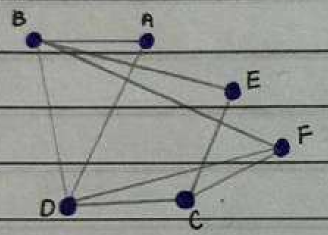
i) Incidence Matrix

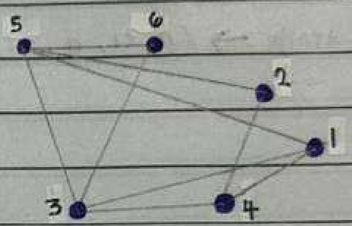
$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6
\end{array}
\begin{array}{c}
\begin{array}{cccccccccc} a & b & c & d & e & f & g & h & i & k \end{array} \\
\begin{bmatrix}
1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}
\end{array}
$$

ii) adjacency matrix

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
1 & 0 & 2 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
2 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

6 determine whether graph Y and Z ~~below~~ are isomorphic. If it is, find their ~~adjant~~ adjacency matrix.





$\rightarrow$ $f(B) = 5$, $f(A) = 6$, $f(D) = 3$, $f(E) = 2$, $f(F) = 1$, $f(C) = 4$.

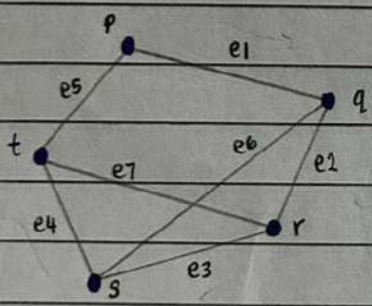
$\therefore$ graph Y and graph Z are proven to be isomorphic to each other.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array} \begin{array}{c} \begin{array}{cccccc} 6 & 5 & 4 & 3 & 2 & 1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

F $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ 1 $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

(7) consider an undirected graph with vertices $V = \{p, q, r, s, t\}$ and edges $E = \{e1, e2, e3, e4, e5, e6, e7\}$.

$e1 = \{p, q\}$, $e2 = \{q, r\}$, $e3 = \{r, s\}$, $e4 = \{s, t\}$, $e5 = \{t, p\}$, $e6 = \{q, s\}$, $e7 = \{r, t\}$



i) find paths from p to t.

→ (p, e5, t), (p, e1, q, e2, r, e3, s, e4, t), (p, e1, q, e6, s, e4, t), (p, e1, q, e2, r, e7, t), and (p, e1, q, e6, s, e3, r, e7, t).

ii) find trails from p to t.

→ (p, e5, t), (p, e1, q, e2, r, e3, s, e4, t), (p, e1, q, e6, s, e4, t), (p, e1, q, e2, r, e7, t), (p, e1, q, e6, s, e3, r, e7, t), (p, e5, t, e7, r, e2, q, e6, s, e4, t), (p, e5, t, e7, r, e3, s, e4, t).

No: .... Date: ....

iii) shortest → (p, e5, t)

longest → (p, e1, q, e2, r, e3, s, e4, t)

iv) shortest trails → (p, e5, t)

longest trails → (p, e5, t, e7, r, e2, q, e6, s, e4, t)