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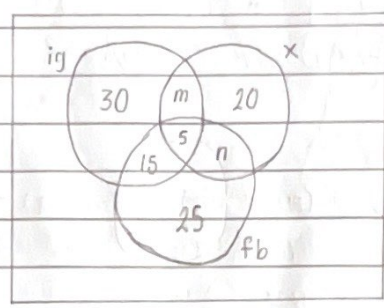
by nurul adriana , a23cs0258

ASSIGNMENT 1

– DISCRETE STRUCTURE –

① total = 150 , ig = instagram , fb = facebook , x = twitter

a] i-



ii- ig, $55 = 30 + 15 + 5 + m$

$m = 5$

fb, $65 = 25 + 15 + 5 + n$

$n = 20$

total users $= 30 + 25 + 20 + 5 + 15 + 20 + 5$

$= 120$

didn't use $= 150 - 120 = 30$

iii- have exact 2 $= 15 + m + n$

$= 15 + 20 + 5$

$= 40$

iv- $(ig \cup x) \cap \neg fb$

= total users − fb users

$= 120 - 65$

$= 55$

b] given $A = \{ n \in N \mid n \text{ odd},\ 1 < n < 10 \}$

$B = \{ n \in N \mid n \text{ is prime},\ 1 < n < 10 \}$

$C = \{ n \in N \mid n \text{ divisible by } 3,\ 1 < n < 10 \}$

$N = \{ \text{natural numbers} \}$

$\therefore A = \{3, 5, 7, 9\}$; $B = \{2, 3, 5, 7\}$; $C = \{3, 6, 9\}$

i- $|A| = 4$, $|B| = 4$; $|C| = 3$

ii- $\emptyset, \{3\}, \{5\}, \{7\}, \{9\}, \{3,5\}, \{3,7\}, \{3,9\}, \{5,7\}, \{5,9\}, \{7,9\},$
$\{3,5,7\}, \{3,7,9\}, \{3,5,9\}, \{5,7,9\}$

iii- $C \times B = \{3,2\}, \{3,3\}, \{3,5\}, \{3,7\}, \{6,2\}, \{6,3\}, \{6,5\}, \{6,7\}$
$\{9,2\}, \{9,3\}, \{9,5\}, \{9,7\}$

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② a] verify $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$,

using truth table ;

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg p \wedge q$	$\neg(p \vee q) \vee (\neg p \wedge q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

$\therefore \neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$ (proven).

using logic property law ;

$\neg(p \vee q) \vee (\neg p \wedge q) \equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q)$ → de morgan laws

$\equiv \neg p \wedge (\neg q \vee q)$ → distributive laws

$\equiv \neg p \wedge T$ → negation laws

$\equiv \neg p$ → identity laws

$\therefore \neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$ (proven).

b] i- $p(x) \rightarrow (r(x) \wedge q(x))$

ii- $\neg(r(x) \vee q(x)) \rightarrow \neg p(x)$

iii- $\neg p(x) \leftrightarrow (r(x) \vee q(x))$

c] given $\forall x (x^2 + 2x - 3 = 0)$, write negation;

$\neg \forall x \ (x^2 + 2x - 3 = 0)$

$\exists x \ \neg (x^2 + 2x - 3 = 0)$ → proves this

could also ↓ written as

$\exists x \ (x^2 + 2x - 3 \neq 0)$

assume $x = 2$,

$(2)^2 + 2(2) - 3 = 0$

$5 \neq 0$

$\therefore$ the statement is true.

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d] use predicates, quantifier & logical connective
[domain of discourse is all students]

i- there is a student at your school who can speak Russian but doesn't know C++.

$P(x)$ = x is a student at the school
$Q(x)$ = x can speak Russian
$R(x)$ = x doesn't know C++

$\exists x\,(S(x) \rightarrow (Q(x) \wedge R(x)))$

ii- every student at your school either can speak Russian or knows C++.

$\forall x\,(Q(x) \vee \neg R(x))$

iii- no student at your school can speak Russian or knows C++.

$\forall x\, \neg(Q(x) \vee \neg R(x))$ or $\neg(\exists x\,(Q(x) \vee \neg R(x)))$

(3) domain of discourse is all integers, using indirect proof;

if $a^2 - 3b$ is even, then a is even and b is even.

$P(a)$: $a^2 - 3b$, $Q(a)$: a is even, $R(b)$: b is even.

$\forall a\, P(a) \rightarrow [Q(a) \wedge R(b)]$

contradiction

$\neg[Q(a) \wedge R(b)] = \neg P(a)$

assume $\neg P(a)$ is true, show $\neg[Q(a) \wedge R(b)]$ is true too.

$a = 2k+1$, $b = 2k+1$ } odd definition

subs into $P(a)$,

$= (2k+1)^2 - 3(2k+1)$

$= 4k^2 + 4k + 1 - 6k - 3$

$= 4k^2 - 2k - 2$

$= 2\underbrace{(2k^2 - k - 1)}$

↳ assume $2k^2 - k - 1 = t$

$= 2t \longrightarrow$ even definition

it is shown that $P(a)$ is an even number, $\therefore \neg P(a)$ is true.