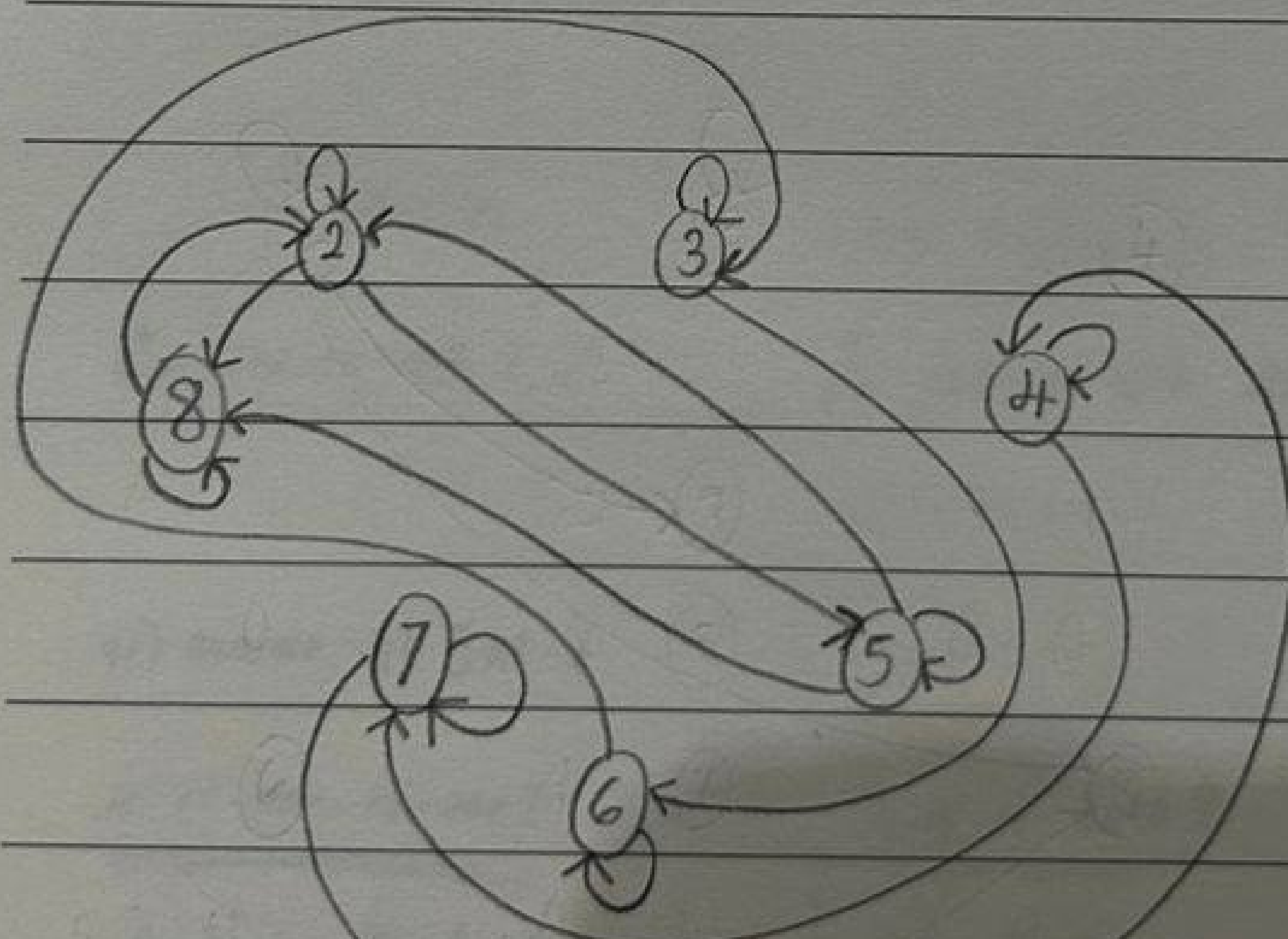
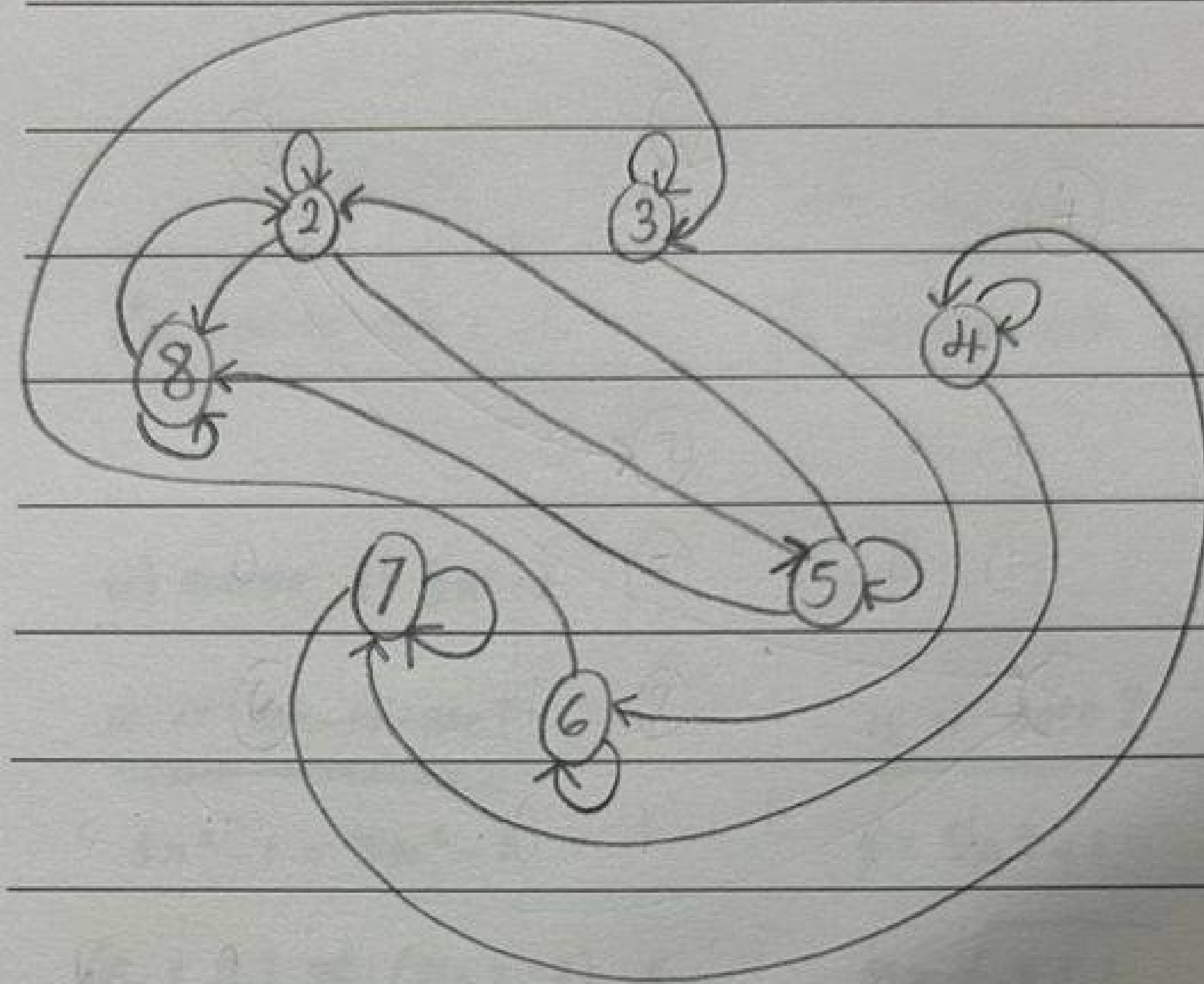
① $A = \{2, 3, 4, 5, 6, 7, 8\}$

draw diagraph if $x - y = 3n$, $n \in \mathbb{Z}$ } must be reflexive, symmetric and transitive only

$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5),$
$(5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$



2 3 4 5 6 7 8



$$
\begin{array}{c|ccccccc}
 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
2 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
5 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
6 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
8 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}
$$

2. $A = \{1, 2, 3\}$ $\quad B = \{9, 8, 7\}$

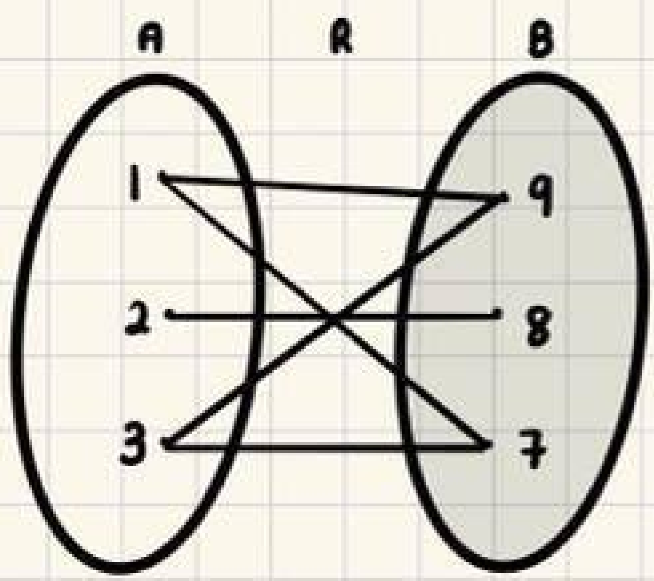
$R : A \to B \quad (a,b) \in A \times B \quad aRb \leftrightarrow a+b$ (even number)

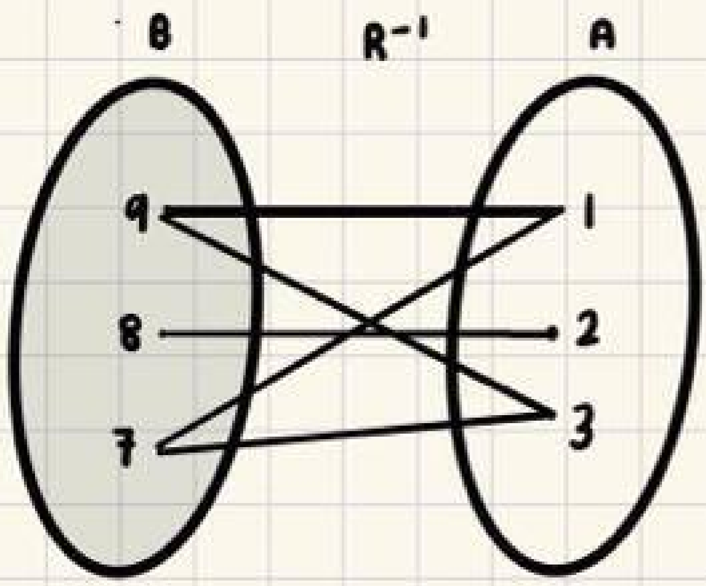
a) $R = \{(1,9)\ (1,7)\ (2,8)\ (3,9)\ (3,7)\}$

$R^{-1} = \{(9,1)\ (7,1)\ (8,2)\ (9,3)\ (7,3)\}$

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \qquad R^{-1} = \begin{array}{c} \\ 7 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b)

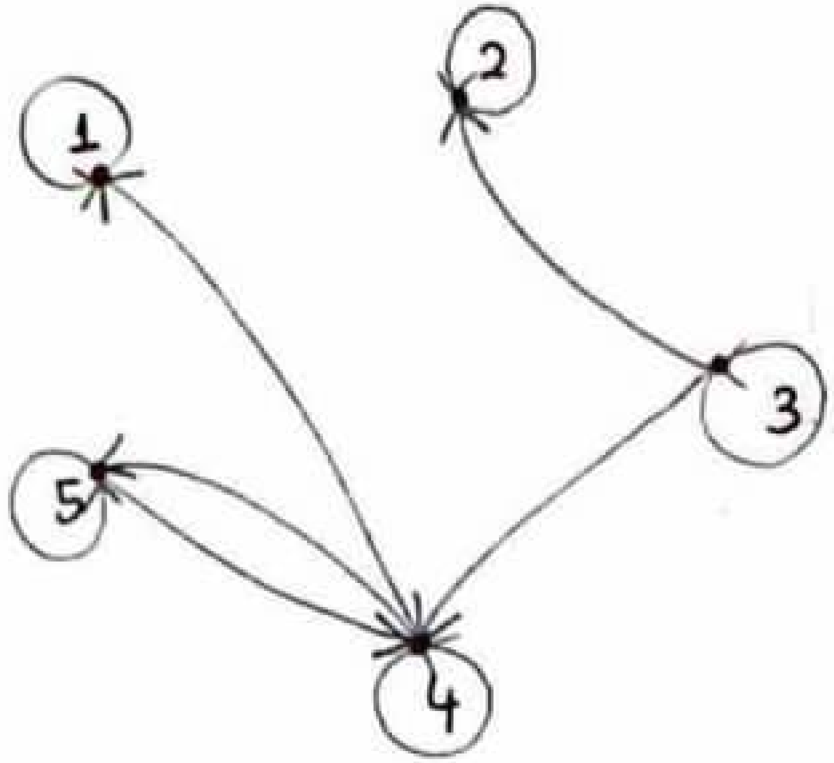




3. Let $A = \{1, 2, 3, 4, 5\}$, and let R be the relation on A that has the matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Construct the diagraph of R, and list in-degrees and out-degrees of all verticles.

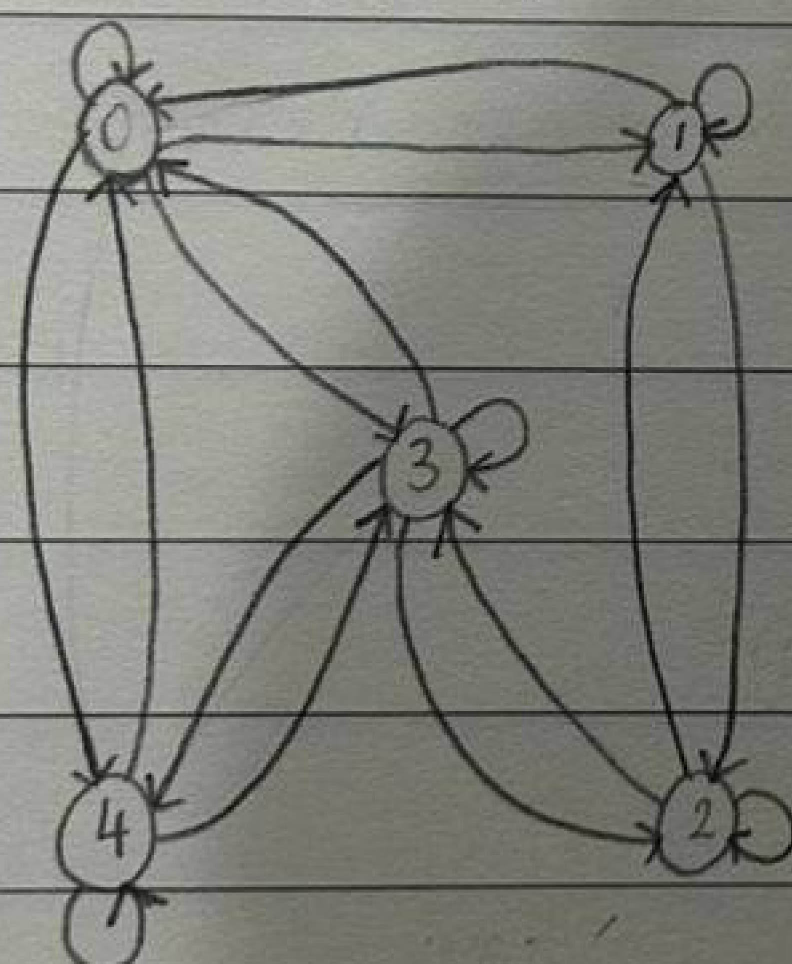


	1	2	3	4	5
In-degrees	2	2	1	3	2
Out-degrees	1	1	3	3	2

$x_1 = x_2$ $\therefore$ onto = ✓

$\therefore$ one-to-one = ✓

④ $\therefore$ R is reflexive, transitive, and symmetric //



$$\begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

5. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$R = \{(x, y) : 3x - y = 0\}$

$\boxed{R = \{(1,3)(2,6)(3,9)(4,12)\}}$

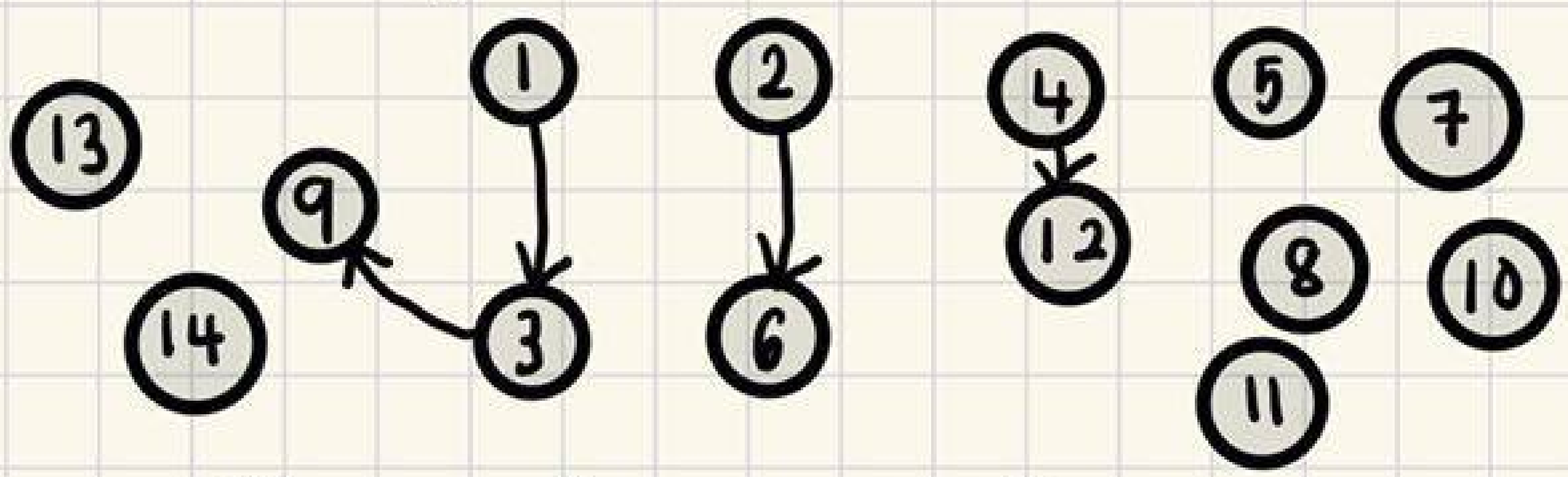
a) irreflexive $(x, y) R ; \forall x : y \in A$

= element in R do not have loop at all.

b) assymmetric

= all edges are "one way street"

= no loop at all



$= \forall x, y \in A, (x, y) \in A$

c)

$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{matrix} &
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\boxed{R = \{(1,3)(2,6)(3,9)(4,12)\}}$

not transitive

$(1,3)$ and $(3,9) \in R$, but $(1,9) \notin R$

6. Suppose that the given is a relation matrix for R and S.

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetic, find

a. RS

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b. SR

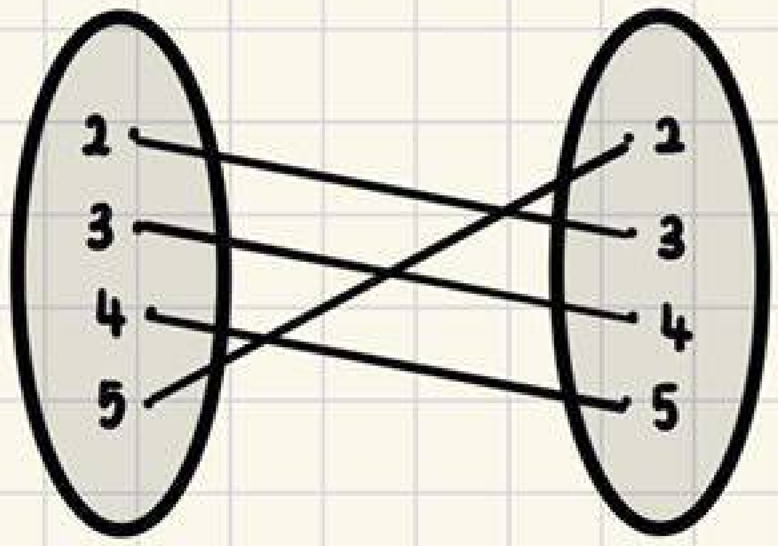
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c|ccccccc}
5 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
6 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
8 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}$$

7 the difference between a relation and a function is that a relation can have many outputs for a single input while a function has a single input for a single output.

8. $A = \{ 2, 3, 4, 5)$

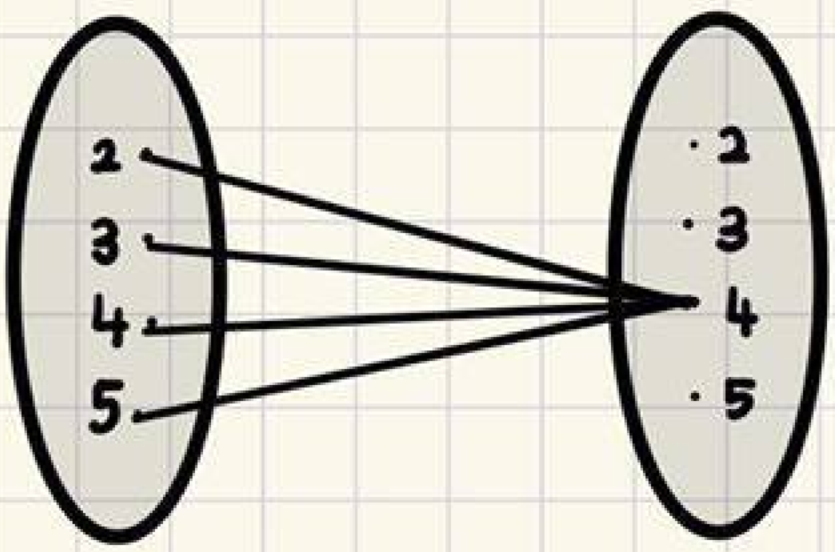
(i) $\{ (2,3) (3,4) (4,5) (5,2) \}$



- domain $R = \{2, 3, 4, 5\}$
- range $R = \{2, 3, 4, 5\}$
- one-to-one

$\therefore$ the set is function #

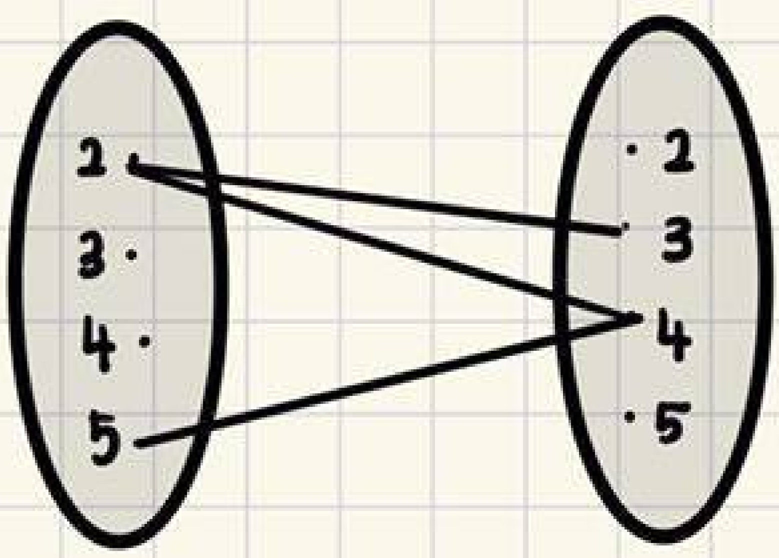
(ii) $\{ (2,4) (3,4) (5,4) (4,4) \}$



- domain $R = \{2, 3, 4, 5\}$
- range $R = \{4\}$
- all has arrow from domain

$\therefore$ the set is function

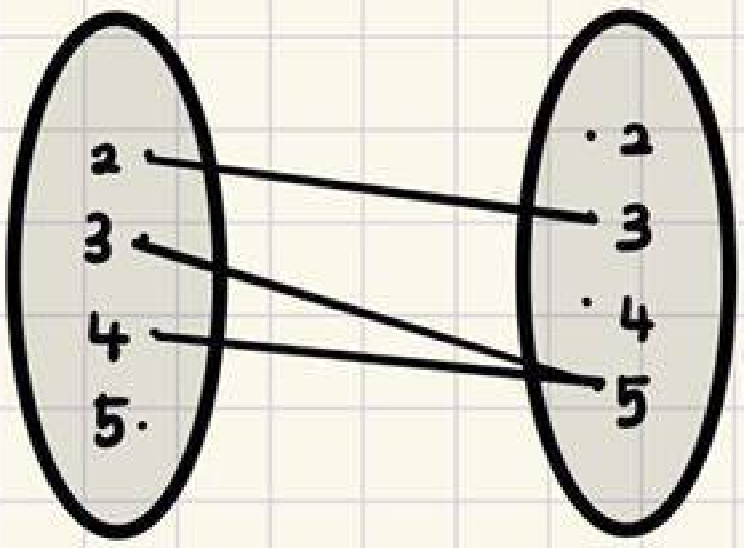
(iii) $\{ (2,3) (2,4) (5,4) \}$



- domain $R = \{2, 5\} \neq$ set A
- range $R = \{3, 4\}$
- no arrow from domain 3, 4

$\therefore$ the set is not function

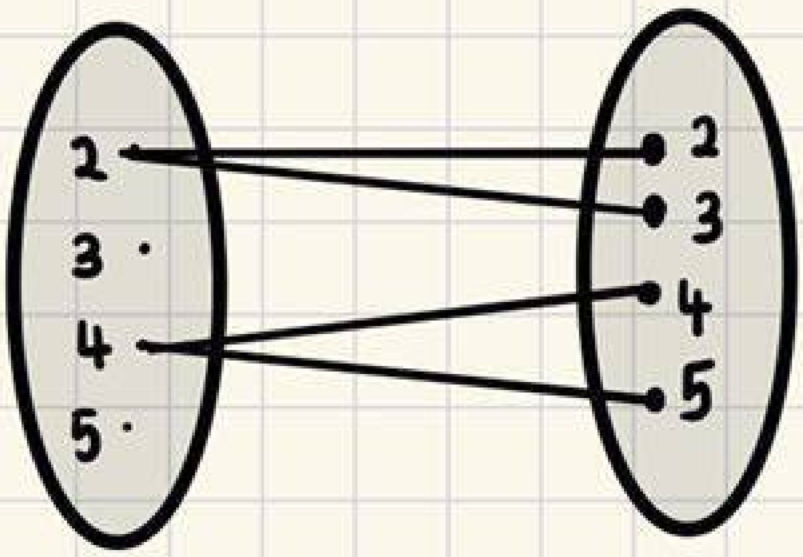
(iv) $\{(2,3)(3,5)(4,5)\}$



- domain R = $\{2,3,4\} \neq$ set A
- range R = $\{3,5\}$
- no arrow from domain 5

$\therefore$ the set is not function.

(v) $\{(2,2)(2,3)(4,4)(4,5)\}$



- domain R = $\{2,4\} \neq$ set A
- range R = $\{2,3,4,5\}$
- no arrow from domain 3,5

$\therefore$ the set is not function.

9. Given the relation of $R = \{(x,y) \mid y = x+5,\ x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$.

Depict this relationship using roster form. Write down the domain and the range.

$R = \{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Domain = 0, 1, 2, 3, 4, 5

Range = 5, 6, 7, 8, 9, 10

x	0	1	2	3	4	5
y	5	6	7	8	9	10

⑩ v) assume $y = 1 - 2x$,

is it one-to-one?

$1 - 2x_1 = 1 - 2x_2$

assume $x_1 = x_2 = 4$,

$1 - 2(4) = 1 - 2(4)$

$-7 = -7$

$x_1 = x_2$

$\therefore$ one-to-one = ✓

is it onto?

$y = 1 - 2x$

$x = \dfrac{y+1}{2}$

subs 3, $x = 2$

subs -3, $x = -1$

$\therefore$ domains with different range, onto = ✓

thus, $f(x) = 1 - 2x$ is a bijection.

vi) assume $y = 5x^2 - 1$

x^4 < not o-t-o, not onto

is it one-to-one?

$5x^2 - 1 = 5x^2 - 1$

is it onto?

$y = 5x^2 - 1$

thus $f(x) = 5x^2 - 1$

$x_1 = x_2$ subs -3, $x = -1$

∴ one-to-one = ✓ ∴ domains with different range, onto = ✓

vi) assume $y = 5x^2 - 1$

is it one-to-one?

$5x^2 - 1 = 5x^2 - 1$

$(x_1 = 2) \neq (x_2 = -2)$

$5(2)^2 - 1 = 5(-2)^2 - 1$

$19 = 19$

but $x_1 \neq x_2$

∴ different domains but same range, one-to-one = X

is it onto?

$y = 5x^2 - 1$

$x = \sqrt{\dfrac{y+1}{5}}$

subs 4, $x = 1$

subs -4, $x = -$

∴ no possible range for negative domains, onto = X

thus, $f(x) = 5x^2 - 1$ is not one-to-one nor onto.

x^4 < not o-t-o, not onto

vii) assume $y = x^4$

is it one-to-one?

$(x_1)^4 = (x_2)^4$

$(x_1 = 2) \neq (x_2 = -2)$

$16 = 16$

but $x_1 \neq x_2$

is it onto?

$y = x^4$

$x = \sqrt[4]{y}$

but same range, one-to-one = X

domains, onto = X

vii) assume $y = x^4$

is it one-to-one?

$(x_1)^4 = (x_2)^4$

$(x_1 = 2) \neq (x_2 = -2)$

$(2)^4 = (-2)^4$

$16 = 16$

but $x_1 \neq x_2$

$\therefore$ one-to-one = X

is it onto?

$y = x^4$

$x = \sqrt[4]{y}$

subs 16, $x = 2$

subs -16, $x = -$

$\therefore$ onto = X

thus $f(x) = 5x^2 - 1$ is not one-to-one nor onto function.

viii) assume $y = \frac{x-2}{x-3}$

is it one-to-one?

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

is it onto?

$y = \frac{x-2}{x-3}$

thus $f(x) = \frac{x-2}{x-3}$

viii) assume $y = \dfrac{x-2}{x-3}$

is it one-to-one?

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

assume $x_1 = x_2 = 4$,

$$\frac{(4)-2}{(4)-3} = \frac{(4)-2}{(4)-3}$$

$$2 = 2$$

$$x_1 = x_2$$

$\therefore$ one-to-one = ✓

is it onto?

$$y = \frac{x-2}{x-3}$$

$$x = \frac{3y-2}{y-1}$$

subs 2, $x = 4$

subs -2, $x = -8/-3$

$\therefore$ onto = ✓

thus $f(x) = \dfrac{x-2}{x-3}$ is a bijection.

(4) $\therefore$ R is reflexive, transitive, and symmetric

0 1 2 3 4

11. $fg(x)$ $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1$
$g(x) = x^2 - 1$

$$f[g(x)] = 3(x^2 - 1) - 1$$
$$= 3x^2 - 3 - 1$$
$$= 3x^2 - 4$$

$x = \{0, 1, 2, 3\}$

$$fg(0) = 3(0)^2 - 4 = -4$$

$$fg(2) = 3(2)^2 - 4 = 8$$

$$fg(1) = 3(1)^2 - 4 = -1$$

$$fg(3) = 3(3)^2 - 4 = 23$$

(x) $f(x) = x^2$
$g(x) = 5x - 6$

$$f[g(x)] = (5x - 6)^2$$
$$= (5x - 6)(5x - 6)$$
$$= 25x^2 - 30x - 30x + 36$$
$$= 25x^2 - 60x + 36$$

$x = \{0, 1, 2, 3\}$

$$fg(0) = 25(0)^2 - 60(0) + 36 = 36$$

$$fg(2) = 25(2)^2 - 60(2) + 36 = 16$$

$$fg(1) = 25(1)^2 - 60(1) + 36 = 1$$

$$fg(3) = 25(3)^2 - 60(3) + 36 = 81$$

(xi) $f(x) = x - 1$

$g(x) = x^3 + 1$

$$f[g(x)] = (x^3 + 1) - 1$$
$$= x^3 + 1 - 1$$
$$= x^3$$

$x = \{0, 1, 2, 3\}$

$$fg(0) = 0^3$$
$$= 0$$

$$fg(1) = 1^3$$
$$= 1$$

$$fg(2) = 2^3$$
$$= 8$$

$$fg(3) = 3^3$$
$$= 27$$

12. Solve the recurrence relation given;

i) $a_n = 6a_{n-1} - 9a_{n-2}$; initial condition $a_0 = 1$ and $a_1 = 6$

$$a_2 = 6a_1 - 9a_0 = 6(6) - 9(1)$$
$$= 27$$

$$a_3 = 6a_2 - 9a_1 = 6(27) - 9(6)$$
$$= 108$$

$$a_4 = 6a_3 - 9a_2 = 6(108) - 9(27)$$
$$= 405$$

$$a_5 = 6a_4 - 9a_3 = 6(405) - 9(108)$$
$$= 1458$$

new recurrence relations:

1, 6, 27, 108, 405, 1458, ...

ii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; initial condition $a_0 = 2, a_1 = 5$ and $a_2 = 15$

$$a_3 = 6a_2 - 11a_1 + 6a_0 = 6(15) - 11(5) + 6(2)$$
$$= 47$$

$$a_4 = 6a_3 - 11a_2 + 6a_1 = 6(47) - 11(15) + 6(5)$$
$$= 147$$

$$a_5 = 6a_4 - 11a_3 + 6a_2 = 6(147) - 11(47) + 6(15)$$
$$= 455$$

$$a_6 = 6a_5 - 11a_4 + 6a_3 = 6(455) - 11(147) + 6(47)$$
$$= 1395$$

new recurrence relations: 2, 5, 15, 47, 147, 455, 1395, ...

iii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; initial condition $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$

$$a_3 = -3a_2 - 3a_1 + a_0 = -3(-1) - 3(-2) + 1$$
$$= 10$$

$$a_4 = -3a_3 - 3a_2 + a_1 = -3(10) - 3(-1) + (-2)$$
$$= -29$$

$$a_5 = -3a_4 - 3a_3 + a_2 = -3(-29) - 3(10) + (-1)$$
$$= 56$$

$$a_6 = -3a_5 - 3a_4 + a_3 = -3(56) - 3(-29) + (10)$$
$$= -71$$

new recurrence relations : 1, -2, -1, 10, -29, 56, -71, ...

(13) given $a_{n+1} = 5a_n - 3$, $a_1 = k$

$k \neq 0$

i) find a_4 in terms of k,

$$a_{n+1} + 3 = 5a_n$$

$$a_n = \frac{a_{n+1} + 3}{5}$$

$$a_4 = \frac{a_5 + 3}{5} = \frac{5(a_1) + 3}{5} = \frac{5k + 3}{5}$$

ii) given $a_4 = 7$, find k

$$a_4 = \frac{5k + 3}{5} = 7$$

5

ii) given $a_4 = 7$, find k

$$a_4 = \frac{5k+3}{5} = 7$$

$$5k + 3 = 35$$

$$5k = 32$$

$$k = \frac{32}{5}$$ //