

# Package ‘VGAM’

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**Title** Vector Generalized Linear and Additive Models

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**Depends** R (>= 3.0.0), methods, splines, stats, stats4

**Suggests** VGAMdata, MASS

**Description** Vector generalized linear and additive models, and associated models (Reduced-Rank VGLMs, Quadratic RR-VGLMs, Reduced-Rank VGAMs). This package fits many models and distribution by maximum likelihood estimation (MLE) or penalized MLE. Also fits constrained ordination models in ecology.

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**URL** <http://www.stat.auckland.ac.nz/~yee/VGAM>

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## Description

**VGAM** provides functions for fitting vector generalized linear and additive models (VGLMs and VGAMs), and associated models (Reduced-rank VGLMs, Quadratic RR-VGLMs, Reduced-rank VGAMs). This package fits many models and distributions by maximum likelihood estimation (MLE) or penalized MLE. Also fits constrained ordination models in ecology such as constrained quadratic ordination (CQO).

## Details

This package centers on the *iteratively reweighted least squares* (IRLS) algorithm. Other key words include Fisher scoring, additive models, penalized likelihood, reduced-rank regression and constrained ordination. The central modelling functions are [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), [cao](#). For detailed control of fitting, each of these has its own control function, e.g., [vglm.control](#). The package uses S4 (see [methods-package](#)). A companion package called **VGAMdata** contains some larger data sets which were shifted from **VGAM**.

The classes of GLMs and GAMs are special cases of VGLMs and VGAMs. The VGLM/VGAM framework is intended to be very general so that it encompasses as many distributions and models as possible. VGLMs are limited only by the assumption that the regression coefficients enter through a set of linear predictors. The VGLM class is very large and encompasses a wide range of multivariate response types and models, e.g., it includes univariate and multivariate distributions, categorical data analysis, time series, survival analysis, generalized estimating equations, extreme values, correlated binary data, quantile and expectile regression, bioassay data and nonlinear least-squares problems.

VGAMs are to VGLMs what GAMs are to GLMs. Vector smoothing (see [vsmooth.spline](#)) allows several additive predictors to be estimated as a sum of smooth functions of the covariates.

For a complete list of this package, use `library(help = "VGAM")`. New **VGAM** family functions are continually being written and added to the package. A monograph about VGLM and VGAMs etc. is currently in the making.

## Warning

This package is undergoing continual development and improvement. Until my monograph comes out and this package is released as version 1.0-0 the user should treat everything subject to change. This includes the family function names, argument names, many of the internals, the use of link functions, and slot names. Some future pain can be minimized by using good programming techniques, e.g., using extractor/accessor functions such as `coef()`, `weights()`, `vcov()`, `predict()`. Nevertheless, please expect changes in all aspects of the package. See the NEWS file for a list of changes from version to version.

**Author(s)**

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- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.
- (Oldish) documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee/VGAM> contains some further information and examples.

**See Also**

[vglm](#), [vgam](#), [rrvglm](#), [cqo](#), [TypicalVGAMfamilyFunction](#), [CommonVGAMffArguments](#), [Links](#).

**Examples**

```
# Example 1; proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo))
depvar(fit1) # Better than using fit1@y; dependent variable (response)
weights(fit1, type = "prior") # Number of observations
coef(fit1, matrix = TRUE) # p.179, in McCullagh and Nelder (1989)
constraints(fit1) # Constraint matrices
summary(fit1)

# Example 2; zero-inflated Poisson model
zdata <- data.frame(x2 = runif(nn <- 2000))
zdata <- transform(zdata, pstr0 = logit(-0.5 + 1*x2, inverse = TRUE),
                  lambda = loge( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y = rzipois(nn, lambda, pstr0 = pstr0))
with(zdata, table(y))
fit2 <- vglm(y ~ x2, zipoisson, zdata, trace = TRUE)
```

```

coef(fit2, matrix = TRUE) # These should agree with the above values

# Example 3; fit a two species GAM simultaneously
fit3 <- vgam(cbind(agaaus, kniexc) ~ s(altitude, df = c(2, 3)),
             binomialff(mv = TRUE), hunua)
coef(fit3, matrix = TRUE) # Not really interpretable
## Not run: plot(fit3, se = TRUE, overlay = TRUE, lcol = 3:4, scol = 3:4)

ooo <- with(hunua, order(altitude))
with(hunua, matplot(altitude[ooo], fitted(fit3)[ooo, ], type = "l",
                    lwd = 2, col = 3:4,
                    xlab = "Altitude (m)", ylab = "Probability of presence", las = 1,
                    main = "Two plant species' response curves", ylim = c(0, 0.8)))
with(hunua, rug(altitude))
## End(Not run)

# Example 4; LMS quantile regression
fit4 <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1),
             data = bmi.nz, trace = TRUE)
head(predict(fit4))
head(fitted(fit4))
head(bmi.nz) # Person 1 is near the lower quartile among people his age
head(cdf(fit4))

## Not run: par(mfrow = c(1, 1), bty = "l", mar = c(5,4,4,3)+0.1, xpd = TRUE)
qtplot(fit4, percentiles = c(5,50,90,99), main = "Quantiles", las = 1,
        xlim = c(15, 90), ylab = "BMI", lwd = 2, lcol = 4) # Quantile plot

ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2) # Density plot
aa <- deplot(fit4, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
             main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)")
aa
aa <- deplot(fit4, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit4, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue",
             Attach = TRUE)
aa$post$deplot # Contains density function values

## End(Not run)

# Example 5; GEV distribution for extremes
(fit5 <- vglm(maxtemp ~ 1, egev, data = oxtemp, trace = TRUE))
head(fitted(fit5))
coef(fit5, matrix = TRUE)
Coef(fit5)
vcov(fit5)
vcov(fit5, untransform = TRUE)
sqrt(diag(vcov(fit5))) # Approximate standard errors
## Not run: rlplot(fit5)

```

AA.Aa.aa

*The AA-Aa-aa Blood Group System***Description**

Estimates the parameter of the AA-Aa-aa blood group system.

**Usage**

```
AA.Aa.aa(link = "logit", init.pA = NULL)
```

**Arguments**

link	Link function applied to pA. See <a href="#">Links</a> for more choices.
init.pA	Optional initial value for pA.

**Details**

This one parameter model involves a probability called pA. The probability of getting a count in the first column of the input (an AA) is  $pA \cdot pA$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The input can be a 3-column matrix of counts, where the columns are AA, Ab and aa (in order). Alternatively, the input can be a 3-column matrix of proportions (so each row adds to 1) and the weights argument is used to specify the total number of counts for each row.

**Author(s)**

T. W. Yee

**References**

Weir, B. S. (1996) *Genetic Data Analysis II: Methods for Discrete Population Genetic Data*, Sunderland, MA: Sinauer Associates, Inc.

**See Also**

[AB.Ab.aB.ab](#), [AB.Ab.aB.ab2](#), [AB0](#), [G1G2G3](#), [MNSs](#).

**Examples**

```

y <- cbind(53, 95, 38)
fit <- vglm(y ~ 1, AA.Aa.aa(link = "probit"), trace = TRUE)
rbind(y, sum(y) * fitted(fit))
Coef(fit) # Estimated pA
summary(fit)

```

---

AB.Ab.aB.ab

*The AB-Ab-aB-ab Blood Group System*


---

**Description**

Estimates the parameter of the AB-Ab-aB-ab blood group system.

**Usage**

```
AB.Ab.aB.ab(link = "logit", init.p = NULL)
```

**Arguments**

link	Link function applied to p. See <a href="#">Links</a> for more choices.
init.p	Optional initial value for p.

**Details**

This one parameter model involves a probability called p.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The input can be a 4-column matrix of counts, where the columns are AB, Ab, aB and ab (in order). Alternatively, the input can be a 4-column matrix of proportions (so each row adds to 1) and the weights argument is used to specify the total number of counts for each row.

**Author(s)**

T. W. Yee

**References**

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

See Also

[AA.Aa.aa](#), [AB.Ab.aB.ab2](#), [ABO](#), [G1G2G3](#), [MNSs](#).

Examples

```
ymat <- cbind(AB=1997, Ab=906, aB=904, ab=32) # Data from Fisher (1925)
fit <- vglm(ymat ~ 1, AB.Ab.aB.ab(link = "identity", init.p = 0.9), trace = TRUE)
fit <- vglm(ymat ~ 1, AB.Ab.aB.ab, trace = TRUE)
rbind(ymat, sum(ymat)*fitted(fit))
Coef(fit) # Estimated p
p <- sqrt(4*(fitted(fit)[, 4]))
p*p
summary(fit)
```

---

AB.Ab.aB.ab2	<i>The AB-Ab-aB-ab2 Blood Group System</i>
--------------	--

---

Description

Estimates the parameter of the the AB-Ab-aB-ab2 blood group system.

Usage

```
AB.Ab.aB.ab2(link = "logit", init.p = NULL)
```

Arguments

- link                      Link function applied to p. See [Links](#) for more choices.
- init.p                    Optional initial value for p.

Details

This one parameter model involves a probability called p.

Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

Warning

There may be a bug in the deriv and weight slot of the family function.

Note

The input can be a 4-column matrix of counts. Alternatively, the input can be a 4-column matrix of proportions (so each row adds to 1) and the weights argument is used to specify the total number of counts for each row.



**Author(s)**

T. W. Yee

**References**

Elandt-Johnson, R. C. (1971) *Probability Models and Statistical Methods in Genetics*, New York: Wiley.

**See Also**

[AA.Aa.aa](#), [AB.Ab.aB.ab](#), [ABO](#), [G1G2G3](#), [MNSs](#).

**Examples**

```
ymat <- cbind(68, 11, 13, 21) # See Elandt-Johnson, pp.430,427
fit <- vglm(ymat ~ 1, AB.Ab.aB.ab2(link = cloglog), trace = TRUE, crit = "coef")
Coef(fit) # Estimated p
rbind(ymat, sum(ymat) * fitted(fit))
sqrt(diag(vcov(fit))) # Estimated variance is approx 0.0021
```

---

ABO

---

*The ABO Blood Group System*


---

**Description**

Estimates the two independent parameters of the the ABO blood group system.

**Usage**

```
ABO(link = "logit", ipA = NULL, ipO = NULL)
```

**Arguments**

link	Link function applied to pA and pB. See <a href="#">Links</a> for more choices.
ipA, ipO	Optional initial value for pA and pO. A NULL value means values are computed internally.

**Details**

The parameters pA and pB are probabilities, so that  $pO=1-pA-pB$  is the third probability. The probabilities pA and pB correspond to A and B respectively, so that pO is the probability for O. It is easier to make use of initial values for pO than for pB. In documentation elsewhere I sometimes use  $pA=p$ ,  $pB=q$ ,  $pO=r$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The input can be a 4-column matrix of counts, where the columns are A, B, AB, O (in order). Alternatively, the input can be a 4-column matrix of proportions (so each row adds to 1) and the weights argument is used to specify the total number of counts for each row.

**Author(s)**

T. W. Yee

**References**

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

**See Also**

[AA.Aa.aa](#), [AB.Ab.aB.ab](#), [AB.Ab.aB.ab2](#), [G1G2G3](#), [MNSs](#).

**Examples**

```
ymat <- cbind(A = 725, B = 258, AB = 72, O = 1073) # Order matters, not the name
fit <- vglm(ymat ~ 1, ABO(link = identity), trace = TRUE, cri = "coef")
coef(fit, matrix = TRUE)
Coef(fit) # Estimated pA and pB
rbind(ymat, sum(ymat) * fitted(fit))
sqrt(diag(vcov(fit)))
```

---

acat	<i>Ordinal Regression with Adjacent Categories Probabilities</i>
------	--

---

**Description**

Fits an adjacent categories regression model to an ordered (preferably) factor response.

**Usage**

```
acat(link = "log", parallel = FALSE, reverse = FALSE,
      zero = NULL, whitespace = FALSE)
```

**Arguments**

link	Link function applied to the ratios of the adjacent categories probabilities. See <a href="#">Links</a> for more choices.
parallel	A logical, or formula specifying which terms have equal/unequal coefficients.
reverse	Logical. By default, the linear/additive predictors used are $\eta_j = \log(P[Y = j + 1]/P[Y = j])$ for $j = 1, \dots, M$ . If reverse is TRUE then $\eta_j = \log(P[Y = j]/P[Y = j + 1])$ will be used.

zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ .
whitespace	See <a href="#">CommonVGAMffArguments</a> for information.

## Details

In this help file the response  $Y$  is assumed to be a factor with ordered values  $1, 2, \dots, M + 1$ , so that  $M$  is the number of linear/additive predictors  $\eta_j$ .

By default, the log link is used because the ratio of two probabilities is positive.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see [ordered](#).

## Note

The response should be either a matrix of counts (with row sums that are all positive), or an ordered factor. In both cases, the  $y$  slot returned by [vglm](#)/[vgam](#)/[rrvglm](#) is the matrix of counts.

For a nominal (unordered) factor response, the multinomial logit model ([multinomial](#)) is more appropriate.

Here is an example of the usage of the `parallel` argument. If there are covariates  $x_1$ ,  $x_2$  and  $x_3$ , then `parallel = TRUE ~ x1 + x2 -1` and `parallel = FALSE ~ x3` are equivalent. This would constrain the regression coefficients for  $x_1$  and  $x_2$  to be equal; those of the intercepts and  $x_3$  would be different.

## Author(s)

Thomas W. Yee

## References

- Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.
- Simonoff, J. S. (2003) *Analyzing Categorical Data*, New York: Springer-Verlag.
- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[cumulative](#), [cratio](#), [sratio](#), [multinomial](#), [pneumo](#).

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, acat, pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
model.matrix(fit)
```

AICv1m

*Akaike's Information Criterion*

## Description

Calculates the Akaike information criterion for a fitted model object for which a log-likelihood value has been obtained.

## Usage

```
AICv1m(object, ..., corrected = FALSE, k = 2)
AICvgam(object, ..., k = 2)
AICrrvglm(object, ..., k = 2)
AICqrrvglm(object, ..., k = 2)
AICcao(object, ..., k = 2)
```

## Arguments

object	Some <b>VGAM</b> object, for example, having class <code>vglmff-class</code> .
...	Other possible arguments fed into <code>logLik</code> in order to compute the log-likelihood.
corrected	Logical, perform the finite sample correction?
k	Numeric, the penalty per parameter to be used; the default is the classical AIC.

## Details

The following formula is used for VGLMs:  $-2\log\text{-likelihood} + kn_{par}$ , where  $n_{par}$  represents the number of parameters in the fitted model, and  $k = 2$  for the usual AIC. One could assign  $k = \log(n)$  ( $n$  the number of observations) for the so-called BIC or SBC (Schwarz's Bayesian criterion). This is the function `AICv1m()`.

This code relies on the log-likelihood being defined, and computed, for the object. When comparing fitted objects, the smaller the AIC, the better the fit. The log-likelihood and hence the AIC is only defined up to an additive constant.

Any estimated scale parameter (in GLM parlance) is used as one parameter.

For VGAMs and CAO the nonlinear effective degrees of freedom for each smoothed component is used. This formula is heuristic. These are the functions `AICvgam()` and `AICcao()`.

The finite sample correction is usually recommended when the sample size is small or when the number of parameters is large. When the sample size is large their difference tends to be negligible. The correction is described in Hurvich and Tsai (1989), and is based on a (univariate) linear model with normally distributed errors.

**Value**

Returns a numeric value with the corresponding AIC (or BIC, or ..., depending on k).

**Warning**

This code has not been double-checked. The general applicability of AIC for the VGLM/VGAM classes has not been developed fully. In particular, AIC should not be run on some **VGAM** family functions because of violation of certain regularity conditions, etc.

**Note**

AIC has not been defined for QRR-VGLMs yet.

Using AIC to compare [posbinomial](#) models with, e.g., [posbernoulli.tb](#) models, requires `posbinomial(omit.constant = FALSE)`. See [posbinomial](#) for an example. A warning is given if it suspects a wrong `omit.constant` value was used.

Where defined, `AICc(...)` is the same as `AIC(..., corrected = TRUE)`.

**Author(s)**

T. W. Yee.

**References**

Hurvich, C. M. and Tsai, C.-L. (1989) Regression and time series model selection in small samples, *Biometrika*, **76**, 297–307.

**See Also**

VGLMs are described in [vg1m-class](#); VGAMs are described in [vgam-class](#); RR-VGLMs are described in [rrvg1m-class](#); [AIC](#), [BICv1m](#).

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = TRUE, reverse = TRUE), pneumo))
coef(fit1, matrix = TRUE)
AIC(fit1)
AICc(fit1) # Quick way
AIC(fit1, corrected = TRUE) # Slow way
(fit2 <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = FALSE, reverse = TRUE), pneumo))
coef(fit2, matrix = TRUE)
AIC(fit2)
AICc(fit2)
AIC(fit2, corrected = TRUE)
```

---

alaplace

*Asymmetric Laplace Distribution Family Functions*


---

## Description

Maximum likelihood estimation of the 1, 2 and 3-parameter asymmetric Laplace distributions (ALDs). The 1-parameter ALD may be used for quantile regression.

## Usage

```
alaplace1(tau = NULL, llocation = "identity",
           ilocation = NULL, kappa = sqrt(tau/(1 - tau)), Scale.arg = 1,
           shrinkage.init = 0.95, parallelLocation = FALSE, digt = 4,
           dfmu.init = 3, intparloc = FALSE, imethod = 1)
```

```
alaplace2(tau = NULL, llocation = "identity", lscale = "log",
           ilocation = NULL, iscale = NULL, kappa = sqrt(tau/(1 - tau)),
           shrinkage.init = 0.95,
           parallelLocation = FALSE, digt = 4, eq.scale = TRUE,
           dfmu.init = 3, intparloc = FALSE,
           imethod = 1, zero = -2)
```

```
alaplace3(llocation = "identity", lscale = "log", lkappa = "log",
           ilocation = NULL, iscale = NULL, ikappa = 1,
           imethod = 1, zero = 2:3)
```

## Arguments

**tau, kappa**      Numeric vectors with  $0 < \tau < 1$  and  $\kappa > 0$ . Most users will only specify tau since the estimated location parameter corresponds to the  $\tau$ th regression quantile, which is easier to understand. See below for details.

**llocation, lscale, lkappa**      Character. Parameter link functions for location parameter  $\xi$ , scale parameter  $\sigma$ , asymmetry parameter  $\kappa$ . See [Links](#) for more choices. For example, the argument llocation can help handle count data by restricting the quantiles to be positive (use llocation = "log"). However, llocation is best left alone since the theory only works properly with the identity link.

**ilocation, iscale, ikappa**      Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.

**parallelLocation, intparloc**      Logical. Should the quantiles be parallel on the transformed scale (argument llocation)? Assigning this argument to TRUE circumvents the seriously embarrassing quantile crossing problem. The argument intparloc applies to intercept term; the argument parallelLocation applies to other terms.

eq.scale	Logical. Should the scale parameters be equal? It is advised to keep eq.scale = TRUE unchanged because it does not make sense to have different values for each tau value.
imethod	Initialization method. Either the value 1, 2, 3 or 4.
dfmu.init	Degrees of freedom for the cubic smoothing spline fit applied to get an initial estimate of the location parameter. See <a href="#">vsmooth.spline</a> . Used only when imethod = 3.
shrinkage.init	How much shrinkage is used when initializing $\xi$ . The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used only when imethod = 4. See <a href="#">CommonVGAMffArguments</a> for more information.
Scale.arg	The value of the scale parameter $\sigma$ . This argument may be used to compute quantiles at different $\tau$ values from an existing fitted <code>alaplace2()</code> model (practical only if it has a single value). If the model has <code>parallelLocation = TRUE</code> then only the intercept need be estimated; use an offset. See below for an example.
digit	Passed into <a href="#">Round</a> as the <code>digits</code> argument for the tau values; used cosmetically for labelling.
zero	See <a href="#">CommonVGAMffArguments</a> for more information. Where possible, the default is to model all the $\sigma$ and $\kappa$ as an intercept-only term.

## Details

These **VGAM** family functions implement one variant of asymmetric Laplace distributions (ALDs) suitable for quantile regression. Kotz et al. (2001) call it *the* ALD. Its density function is

$$f(y; \xi, \sigma, \kappa) = \frac{\sqrt{2}}{\sigma} \frac{\kappa}{1 + \kappa^2} \exp \left( -\frac{\sqrt{2}}{\sigma \kappa} |y - \xi| \right)$$

for  $y \leq \xi$ , and

$$f(y; \xi, \sigma, \kappa) = \frac{\sqrt{2}}{\sigma} \frac{\kappa}{1 + \kappa^2} \exp \left( -\frac{\sqrt{2} \kappa}{\sigma} |y - \xi| \right)$$

for  $y > \xi$ . Here, the ranges are for all real  $y$  and  $\xi$ , positive  $\sigma$  and positive  $\kappa$ . The special case  $\kappa = 1$  corresponds to the (symmetric) Laplace distribution of Kotz et al. (2001). The mean is  $\xi + \sigma(1/\kappa - \kappa)/\sqrt{2}$  and the variance is  $\sigma^2(1 + \kappa^4)/(2\kappa^2)$ . The enumeration of the linear/additive predictors used for `alaplace2()` is the first location parameter followed by the first scale parameter, then the second location parameter followed by the second scale parameter, etc. For `alaplace3()`, only a vector response is handled and the last (third) linear/additive predictor is for the asymmetry parameter.

It is known that the maximum likelihood estimate of the location parameter  $\xi$  corresponds to the regression quantile estimate of the classical quantile regression approach of Koenker and Bassett (1978). An important property of the ALD is that  $P(Y \leq \xi) = \tau$  where  $\tau = \kappa^2/(1 + \kappa^2)$  so that  $\kappa = \sqrt{\tau/(1 - \tau)}$ . Thus `alaplace1()` might be used as an alternative to `rq` in the **quantreg** package.

Both `alaplace1()` and `alaplace2()` can handle multiple responses, and the number of linear/additive predictors is dictated by the length of tau or kappa. The function `alaplace2()` can also handle a matrix response with a single-valued tau or kappa.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

In the extra slot of the fitted object are some list components which are useful, e.g., the sample proportion of values which are less than the fitted quantile curves.

**Warning**

The MLE regularity conditions do not hold for this distribution so that misleading inferences may result, e.g., in the summary and vcov of the object.

Care is needed with tau values which are too small, e.g., for count data with `llocation = "loge"` and if the sample proportion of zeros is greater than tau.

**Note**

These **VGAM** family functions use Fisher scoring. Convergence may be slow and half-stepping is usual (although one can use `trace = TRUE` to see which is the best model and then use `maxit` to choose that model) due to the regularity conditions not holding.

For large data sets it is a very good idea to keep the length of tau/kappa low to avoid large memory requirements. Then for `parallelLoc = FALSE` one can repeatedly fit a model with `alaplace1()` with one  $\tau$  at a time; and for `parallelLoc = TRUE` one can refit a model with `alaplace1()` with one  $\tau$  at a time but using offsets and an intercept-only model.

A second method for solving the noncrossing quantile problem is illustrated below in Example 3. This is called the *accumulative quantile method* (AQM) and details are in Yee (2012). It does not make the strong parallelism assumption.

The functions `alaplace2()` and [laplace](#) differ slightly in terms of the parameterizations.

**Author(s)**

Thomas W. Yee

**References**

- Koenker, R. and Bassett, G. (1978) Regression quantiles. *Econometrica*, **46**, 33–50.
- Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001) *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*, Boston: Birkhauser.
- Yee, T. W. (2012) Quantile regression for counts and proportions. In preparation.

**See Also**

[ralap](#), [laplace](#), [lms.bcn](#), [amlnormal](#), [koenker](#).



## Examples

```
## Not run:
# Example 1: quantile regression with smoothing splines
adata <- data.frame(x = sort(runif(n <- 500)))
mymu <- function(x) exp(-2 + 6*sin(2*x-0.2) / (x+0.5)^2)
adata <- transform(adata, y = rpois(n, lambda = mymu(x)))
mytau <- c(0.25, 0.75); mydof <- 4

fit <- vgam(y ~ s(x, df = mydof),
  alaplace1(tau = mytau, llocation = "loge",
    parallelLoc = FALSE), adata, trace = TRUE)
fitp <- vgam(y ~ s(x, df = mydof), data = adata, trace = TRUE,
  alaplace1(tau = mytau, llocation = "loge", parallelLoc = TRUE))

par(las = 1); mylwd = 1.5
with(adata, plot(x, jitter(y, factor = 0.5), col = "orange",
  main = "Example 1; green: parallelLoc = TRUE",
  ylab = "y", pch = "o", cex = 0.75))
with(adata, matlines(x, fitted(fit), col = "blue",
  lty = "solid", lwd = mylwd))
with(adata, matlines(x, fitted(fitp), col = "green",
  lty = "solid", lwd = mylwd))
finexgrid <- seq(0, 1, len = 1001)
for (ii in 1:length(mytau))
  lines(finexgrid, qpois(p = mytau[ii], lambda = mymu(finexgrid)),
    col = "blue", lwd = mylwd)
fit@extra # Contains useful information

# Example 2: regression quantile at a new tau value from an existing fit
# Nb. regression splines are used here since it is easier.
fitp2 <- vglm(y ~ bs(x, df = mydof),
  family = alaplace1(tau = mytau, llocation = "loge",
    parallelLoc = TRUE),
  adata, trace = TRUE)

newtau <- 0.5 # Want to refit the model with this tau value
fitp3 <- vglm(y ~ 1 + offset(predict(fitp2)[,1]),
  family = alaplace1(tau = newtau, llocation = "loge"),
  adata)
with(adata, plot(x, jitter(y, factor = 0.5), col = "orange",
  pch = "o", cex = 0.75, ylab = "y",
  main = "Example 2; parallelLoc = TRUE"))
with(adata, matlines(x, fitted(fitp2), col = "blue",
  lty = 1, lwd = mylwd))
with(adata, matlines(x, fitted(fitp3), col = "black",
  lty = 1, lwd = mylwd))

# Example 3: noncrossing regression quantiles using a trick: obtain
# successive solutions which are added to previous solutions; use a log
```

```

# link to ensure an increasing quantiles at any value of x.

mytau <- seq(0.2, 0.9, by = 0.1)
answer <- matrix(0, nrow(adata), length(mytau)) # Stores the quantiles
adata <- transform(adata, offsety = y*0)
usetau <- mytau
for (ii in 1:length(mytau)) {
  # cat("\n\nii = ", ii, "\n")
  adata <- transform(adata, usey = y-offsety)
  iloc <- ifelse(ii == 1, with(adata, median(y)), 1.0) # Well-chosen!
  mydf <- ifelse(ii == 1, 5, 3) # Maybe less smoothing will help
  lloc <- ifelse(ii == 1, "identity", "loge") # 2nd value must be "loge"
  fit3 <- vglm(usey ~ ns(x, df = mydf), data = adata, trace = TRUE,
               alaplace1(tau = usetau[ii], lloc = lloc, iloc = iloc))
  answer[,ii] <- (if(ii == 1) 0 else answer[,ii-1]) + fitted(fit3)
  adata <- transform(adata, offsety = answer[,ii])
}

# Plot the results.
with(adata, plot(x, y, col = "blue",
  main = paste("Noncrossing and nonparallel; tau = ",
    paste(mytau, collapse = ", "))))
with(adata, matlines(x, answer, col = "orange", lty = 1))

# Zoom in near the origin.
with(adata, plot(x, y, col = "blue", xlim = c(0, 0.2), ylim = 0:1,
  main = paste("Noncrossing and nonparallel; tau = ",
    paste(mytau, collapse = ", "))))
with(adata, matlines(x, answer, col = "orange", lty = 1))

## End(Not run)

```

---

alaplaceUC

*The Laplace Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the 3-parameter asymmetric Laplace distribution with location parameter location, scale parameter scale, and asymmetry parameter kappa.

## Usage

```

dalap(x, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)),
      log = FALSE)
palap(q, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)))
qalap(p, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)))
ralap(n, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)))

```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
location	the location parameter $\xi$ .
scale	the scale parameter $\sigma$ . Must consist of positive values.
tau	the quantile parameter $\tau$ . Must consist of values in $(0, 1)$ . This argument is used to specify kappa and is ignored if kappa is assigned.
kappa	the asymmetry parameter $\kappa$ . Must consist of positive values.
log	if TRUE, probabilities p are given as <code>log(p)</code> .

**Details**

There are many variants of asymmetric Laplace distributions (ALDs) and this one is known as *the* ALD by Kotz et al. (2001). See [alaplace3](#), the **VGAM** family function for estimating the three parameters by maximum likelihood estimation, for formulae and details.

**Value**

`dalap` gives the density, `palap` gives the distribution function, `qalap` gives the quantile function, and `ralap` generates random deviates.

**Author(s)**

T. W. Yee

**References**

Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001) *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*, Boston: Birkhauser.

**See Also**

[alaplace3](#).

**Examples**

```
x <- seq(-5, 5, by = 0.01)
loc <- 0; sigma <- 1.5; kappa <- 2
## Not run: plot(x, dalap(x, loc, sigma, kappa = kappa), type = "l", col = "blue",
  main = "Blue is density, orange is cumulative distribution function",
  ylim = c(0, 1), sub = "Purple are 5, 10, ..., 95 percentiles",
  las = 1, ylab = "", cex.main = 0.5)
abline(h = 0, col = "blue", lty = 2)
lines(qalap(seq(0.05, 0.95, by = 0.05), loc, sigma, kappa = kappa),
  dalap(qalap(seq(0.05, 0.95, by = 0.05), loc, sigma, kappa = kappa),
```

```

      loc, sigma, kappa = kappa), col = "purple", lty = 3, type = "h")
lines(x, palap(x, loc, sigma, kappa = kappa), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)

pp <- seq(0.05, 0.95, by = 0.05) # Test two functions
max(abs(palap(qalap(pp, loc, sigma, kappa = kappa),
             loc, sigma, kappa = kappa) - pp)) # Should be 0

```

Amh

*Ali-Mikhail-Haq Bivariate Distribution***Description**

Density, distribution function, and random generation for the (one parameter) bivariate Ali-Mikhail-Haq distribution.

**Usage**

```

damh(x1, x2, alpha, log = FALSE)
pamh(q1, q2, alpha)
ramh(n, alpha)

```

**Arguments**

`x1`, `x2`, `q1`, `q2` vector of quantiles.  
`n` number of observations. Same as [runif](#)  
`alpha` the association parameter.  
`log` Logical. If TRUE then the logarithm is returned.

**Details**

See [amh](#), the **VGAM** family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

**Value**

`damh` gives the density, `pamh` gives the distribution function, and `ramh` generates random deviates (a two-column matrix).

**Author(s)**

T. W. Yee and C. S. Chee

**See Also**

[amh](#).

### Examples

```
x <- seq(0, 1, len = (N <- 101)); alpha <- 0.7
ox <- expand.grid(x, x)
zedd <- damh(ox[, 1], ox[, 2], alpha = alpha)
## Not run:
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pamh(ox[, 1], ox[, 2], alpha = alpha)
contour(x, x, matrix(zedd, N, N), col = "blue")

plot(r <- ramh(n = 1000, alpha = alpha), col = "blue")
par(mfrow = c(1, 2))
hist(r[, 1]) # Should be uniform
hist(r[, 2]) # Should be uniform

## End(Not run)
```

amh

*Ali-Mikhail-Haq Distribution Family Function*

### Description

Estimate the association parameter of Ali-Mikhail-Haq's bivariate distribution by maximum likelihood estimation.

### Usage

```
amh(lalpha = "rhobit", ialpha = NULL, imethod = 1, nsimEIM = 250)
```

### Arguments

lalpha	Link function applied to the association parameter $\alpha$ , which is real and $-1 < \alpha < 1$ . See <a href="#">Links</a> for more choices.
ialpha	Numeric. Optional initial value for $\alpha$ . By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument imethod.
imethod	An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ialpha.
nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information.

### Details

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = y_1 y_2 / (1 - \alpha(1 - y_1)(1 - y_2))$$

for  $-1 < \alpha < 1$ . The support of the function is the unit square. The marginal distributions are the standard uniform distributions. When  $\alpha = 0$  the random variables are independent.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 0.5. This is because each marginal distribution corresponds to a standard uniform distribution.

**Author(s)**

T. W. Yee and C. S. Chee

**References**

Balakrishnan, N. and Lai, C.-D. (2009) *Continuous Bivariate Distributions*, 2nd ed. New York: Springer.

**See Also**

[ramh](#), [fgm](#), [bigumbell](#).

**Examples**

```
ymat <- ramh(1000, alpha = rhobit(2, inverse = TRUE))
fit <- vglm(ymat ~ 1, amh, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

---

amlbinomial

---

*Binomial Logistic Regression by Asymmetric Maximum Likelihood Estimation*


---

**Description**

Binomial quantile regression estimated by maximizing an asymmetric likelihood function.

**Usage**

```
amlbinomial(w.aml = 1, parallel = FALSE, digw = 4, link = "logit")
```

## Arguments

w.aml	Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the “w-regression plane”). The default value of unity results in the ordinary maximum likelihood (MLE) solution.
parallel	If w.aml has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be TRUE should force the quantile curves to not cross (although they may not cross anyway). See <a href="#">CommonVGAMffArguments</a> for more information.
digw	Passed into <a href="#">Round</a> as the digits argument for the w.aml values; used cosmetically for labelling.
link	See <a href="#">binomialff</a> .

## Details

The general methodology behind this **VGAM** family function is given in Efron (1992) and full details can be obtained there.

This model is essentially a logistic regression model (see [binomialff](#)) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by *w.aml* for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviance-type values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

## Value

An object of class “vglmff” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

If w.aml has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the w.aml values. See Equation (1.6) of Efron (1992).

## Note

On fitting, the extra slot has list components “w.aml” and “percentile”. The latter is the percent of observations below the “w-regression plane”, which is the fitted values. Also, the individual deviance values corresponding to each element of the argument w.aml is stored in the extra slot.

For amlbinomial objects, methods functions for the generic functions `qtpplot` and `cdf` have not been written yet.

See [amlpoisson](#) about comments on the jargon, e.g., *expectiles* etc.

In this documentation the word *quantile* can often be interchangeably replaced by *expectile* (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1992) Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. *Journal of the American Statistical Association*, **87**, 98–107.

## See Also

[amlpoisson](#), [amlexponential](#), [amlnormal](#), [alaplace1](#), [denorm](#).

## Examples

```
# Example: binomial data with lots of trials per observation
set.seed(1234)
sizevec <- rep(100, length = (nn <- 200))
mydat <- data.frame(x = sort(runif(nn)))
mydat <- transform(mydat, prob = logit(-0 + 2.5*x + x^2, inverse = TRUE))
mydat <- transform(mydat, y = rbinom(nn, size = sizevec, prob = prob))
(fit <- vgam(cbind(y, sizevec - y) ~ s(x, df = 3),
             amlbinomial(w = c(0.01, 0.2, 1, 5, 60)),
             mydat, trace = TRUE))

fit@extra

## Not run:
par(mfrow = c(1,2))
# Quantile plot
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
  paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
        "percentile-expectile curves")))
with(mydat, matlines(x, 100 * fitted(fit), lwd = 2, col = "blue", lty = 1))

# Compare the fitted expectiles with the quantiles
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
  paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
        "percentile curves are red")))
with(mydat, matlines(x, 100 * fitted(fit), lwd = 2, col = "blue", lty = 1))

for (ii in fit@extra$percentile)
  with(mydat, matlines(x, 100 *
    qbinom(p = ii/100, size = sizevec, prob = prob) / sizevec,
    col = "red", lwd = 2, lty = 1))

## End(Not run)
```

---

amlexponential

---

*Exponential Regression by Asymmetric Maximum Likelihood Estimation*


---

## Description

Exponential expectile regression estimated by maximizing an asymmetric likelihood function.



**Usage**

```
amlexponential(w.aml = 1, parallel = FALSE, imethod = 1, digw = 4,
               link = "loge")
```

**Arguments**

<code>w.aml</code>	Numeric, a vector of positive constants controlling the expectiles. The larger the value the larger the fitted expectile value (the proportion of points below the “w-regression plane”). The default value of unity results in the ordinary maximum likelihood (MLE) solution.
<code>parallel</code>	If <code>w.aml</code> has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be <code>TRUE</code> should force the quantile curves to not cross (although they may not cross anyway). See <a href="#">CommonVGAMffArguments</a> for more information.
<code>imethod</code>	Integer, either 1 or 2 or 3. Initialization method. Choose another value if convergence fails.
<code>digw</code>	Passed into <a href="#">Round</a> as the <code>digits</code> argument for the <code>w.aml</code> values; used cosmetically for labelling.
<code>link</code>	See <a href="#">exponential</a> and the warning below.

**Details**

The general methodology behind this **VGAM** family function is given in Efron (1992) and full details can be obtained there.

This model is essentially an exponential regression model (see [exponential](#)) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by *w.aml* for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviance-type values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

**Value**

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

Note that the `link` argument of [exponential](#) and [amlexponential](#) are currently different: one is the rate parameter and the other is the mean (expectile) parameter.

If `w.aml` has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the `w.aml` values. See Equation (1.6) of Efron (1992).

**Note**

On fitting, the extra slot has list components “`w.aml`” and “`percentile`”. The latter is the percent of observations below the “w-regression plane”, which is the fitted values. Also, the individual deviance values corresponding to each element of the argument `w.aml` is stored in the extra slot.

For amlexponential objects, methods functions for the generic functions `qtplot` and `cdf` have not been written yet.

See [amlpoisson](#) about comments on the jargon, e.g., *expectiles* etc.

In this documentation the word *quantile* can often be interchangeably replaced by *expectile* (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1992) Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. *Journal of the American Statistical Association*, **87**, 98–107.

## See Also

[exponential](#), [amlbinomial](#), [amlpoisson](#), [amlnormal](#), [alaplace1](#), [lms.bcg](#), [deexp](#).

## Examples

```
nn <- 2000
mydat <- data.frame(x = seq(0, 1, length = nn))
mydat <- transform(mydat, mu = loge(-0 + 1.5*x + 0.2*x^2, inverse = TRUE))
mydat <- transform(mydat, mu = loge(0 - sin(8*x), inverse = TRUE))
mydat <- transform(mydat, y = rexp(nn, rate = 1/mu))
(fit <- vgam(y ~ s(x,df = 5), amlexponential(w = c(0.001, 0.1, 0.5, 5, 60)),
            mydat, trace = TRUE))
fit@extra

## Not run: # These plots are against the sqrt scale (to increase clarity)
par(mfrow = c(1,2))
# Quantile plot
with(mydat, plot(x, sqrt(y), col = "blue", las = 1, main =
  paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
    "percentile-expectile curves")))
with(mydat, matlines(x, sqrt(fitted(fit)), lwd = 2, col = "blue", lty = 1))

# Compare the fitted expectiles with the quantiles
with(mydat, plot(x, sqrt(y), col = "blue", las = 1, main =
  paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
    "percentile curves are orange")))
with(mydat, matlines(x, sqrt(fitted(fit)), lwd = 2, col = "blue", lty = 1))

for (ii in fit@extra$percentile)
  with(mydat, matlines(x, sqrt(qexp(p = ii/100, rate = 1/mu)), col = "orange"))
## End(Not run)
```

amlnormal

*Asymmetric Least Squares Quantile Regression***Description**

Asymmetric least squares, a special case of maximizing an asymmetric likelihood function of a normal distribution. This allows for expectile/quantile regression using asymmetric least squares error loss.

**Usage**

```
amlnormal(w.aml = 1, parallel = FALSE, lexpectile = "identity",
          iexpectile = NULL, imethod = 1, digw = 4)
```

**Arguments**

w.aml	Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the “w-regression plane”). The default value of unity results in the ordinary least squares (OLS) solution.
parallel	If w.aml has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be TRUE should force the quantile curves to not cross (although they may not cross anyway). See <a href="#">CommonVGAMffArguments</a> for more information.
lexpectile, iexpectile	See <a href="#">CommonVGAMffArguments</a> for more information.
imethod	Integer, either 1 or 2 or 3. Initialization method. Choose another value if convergence fails.
digw	Passed into <a href="#">Round</a> as the digits argument for the w.aml values; used cosmetically for labelling.

**Details**

This is an implementation of Efron (1991) and full details can be obtained there. Equation numbers below refer to that article. The model is essentially a linear model (see [lm](#)), however, the asymmetric squared error loss function for a residual  $r$  is  $r^2$  if  $r \leq 0$  and  $wr^2$  if  $r > 0$ . The solution is the set of regression coefficients that minimize the sum of these over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

**Value**

An object of class “vglmff” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

On fitting, the extra slot has list components "w.aml" and "percentile". The latter is the percent of observations below the "w-regression plane", which is the fitted values.

One difficulty is finding the w.aml value giving a specified percentile. One solution is to fit the model within a root finding function such as [uniroot](#); see the example below.

For amlnormal objects, methods functions for the generic functions `qtplot` and `cdf` have not been written yet.

See the note in [amlpoisson](#) on the jargon, including *expectiles* and *regression quantiles*.

The deviance slot computes the total asymmetric squared error loss (2.5). If w.aml has more than one value then the value returned by the slot is the sum taken over all the w.aml values.

This **VGAM** family function could well be renamed `amlnormal()` instead, given the other function names [amlpoisson](#), [amlbinomial](#), etc.

In this documentation the word *quantile* can often be interchangeably replaced by *expectile* (things are informal here).

**Author(s)**

Thomas W. Yee

**References**

Efron, B. (1991) Regression percentiles using asymmetric squared error loss. *Statistica Sinica*, **1**, 93–125.

**See Also**

[amlpoisson](#), [amlbinomial](#), [amlexponential](#), [bmi.nz](#), [alaplace1](#), [denorm](#), [lms.bcn](#) and similar variants are alternative methods for quantile regression.

**Examples**

```
## Not run:
# Example 1
ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo,] # Sort by age
(fit <- vglm(BMI ~ bs(age), fam = amlnormal(w.aml = 0.1), bmi.nz))
fit@extra # Gives the w value and the percentile
coef(fit, matrix = TRUE)

# Quantile plot
with(bmi.nz, plot(age, BMI, col = "blue", main =
  paste(round(fit@extra$percentile, digits = 1),
    "expectile-percentile curve")))
with(bmi.nz, lines(age, c(fitted(fit)), col = "black"))

# Example 2
# Find the w values that give the 25, 50 and 75 percentiles
findw <- function(w, percentile = 50) {
```

```

fit2 <- vglm(BMI ~ bs(age), fam = amlnormal(w = w), data = bmi.nz)
fit2@extra$percentile - percentile
}
# Quantile plot
with(bmi.nz, plot(age, BMI, col = "blue", las = 1, main =
  "25, 50 and 75 expectile-percentile curves"))
for (myp in c(25, 50, 75)) {
# Note: uniroot() can only find one root at a time
bestw <- uniroot(f = findw, interval = c(1/10^4, 10^4), percentile = myp)
fit2 <- vglm(BMI ~ bs(age), fam = amlnormal(w = bestw$root), data = bmi.nz)
with(bmi.nz, lines(age, c(fitted(fit2)), col = "red"))
}

# Example 3; this is Example 1 but with smoothing splines and
# a vector w and a parallelism assumption.
ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo,] # Sort by age
fit3 <- vgam(BMI ~ s(age, df = 4), bmi.nz, trace = TRUE,
  fam = amlnormal(w = c(0.1, 1, 10), parallel = TRUE))
fit3@extra # The w values, percentiles and weighted deviances

# The linear components of the fit; not for human consumption:
coef(fit3, matrix = TRUE)

# Quantile plot
with(bmi.nz, plot(age, BMI, col="blue", main =
  paste(paste(round(fit3@extra$percentile, digits = 1), collapse = ", "),
    "expectile-percentile curves")))
with(bmi.nz, matlines(age, fitted(fit3), col = 1:fit3@extra$M, lwd = 2))
with(bmi.nz, lines(age, c(fitted(fit )), col = "black")) # For comparison

## End(Not run)

```

amlpoisson

*Poisson Regression by Asymmetric Maximum Likelihood Estimation***Description**

Poisson quantile regression estimated by maximizing an asymmetric likelihood function.

**Usage**

```
amlpoisson(w.aml = 1, parallel = FALSE, imethod = 1, digw = 4,
  link = "loge")
```

**Arguments**

**w.aml** Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the “w-regression plane”). The default value of unity results in the ordinary maximum likelihood (MLE) solution.

parallel	If <code>w.aml</code> has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be <code>TRUE</code> should force the quantile curves to not cross (although they may not cross anyway). See <a href="#">CommonVGAMffArguments</a> for more information.
imethod	Integer, either 1 or 2 or 3. Initialization method. Choose another value if convergence fails.
digw	Passed into <a href="#">Round</a> as the <code>digits</code> argument for the <code>w.aml</code> values; used cosmetically for labelling.
link	See <a href="#">poissonff</a> .

### Details

This method was proposed by Efron (1992) and full details can be obtained there.

The model is essentially a Poisson regression model (see [poissonff](#)) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by `w.aml` for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviance-type values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

### Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

### Warning

If `w.aml` has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the `w.aml` values. See Equation (1.6) of Efron (1992).

### Note

On fitting, the extra slot has list components `"w.aml"` and `"percentile"`. The latter is the percent of observations below the "w-regression plane", which is the fitted values. Also, the individual deviance values corresponding to each element of the argument `w.aml` is stored in the extra slot.

For `amlpoisson` objects, methods functions for the generic functions `qtplot` and `cdf` have not been written yet.

About the jargon, Newey and Powell (1987) used the name *expectiles* for regression surfaces obtained by asymmetric least squares. This was deliberate so as to distinguish them from the original *regression quantiles* of Koenker and Bassett (1978). Efron (1991) and Efron (1992) use the general name *regression percentile* to apply to all forms of asymmetric fitting. Although the asymmetric maximum likelihood method very nearly gives regression percentiles in the strictest sense for the normal and Poisson cases, the phrase *quantile regression* is used loosely in this **VGAM** documentation.

In this documentation the word *quantile* can often be interchangeably replaced by *expectile* (things are informal here).

**Author(s)**

Thomas W. Yee

**References**

- Efron, B. (1991) Regression percentiles using asymmetric squared error loss. *Statistica Sinica*, **1**, 93–125.
- Efron, B. (1992) Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. *Journal of the American Statistical Association*, **87**, 98–107.
- Koenker, R. and Bassett, G. (1978) Regression quantiles. *Econometrica*, **46**, 33–50.
- Newey, W. K. and Powell, J. L. (1987) Asymmetric least squares estimation and testing. *Econometrica*, **55**, 819–847.

**See Also**

[amlnormal](#), [amlbinomial](#), [alaplacel](#).

**Examples**

```
set.seed(1234)
mydat <- data.frame(x = sort(runif(nn <- 200)))
mydat <- transform(mydat, y = rpois(nn, exp(0 - sin(8*x))))
(fit <- vgam(y ~ s(x), fam = amlpoisson(w.aml = c(0.02, 0.2, 1, 5, 50)),
  mydat, trace = TRUE))
fit@extra

## Not run:
# Quantile plot
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
  paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
    "percentile-expectile curves")))
with(mydat, matlines(x, fitted(fit), lwd = 2))
## End(Not run)
```

---

auuc

Auckland University Undergraduate Counts Data

---

**Description**

Undergraduate student enrolments at the University of Auckland in 1990.

**Usage**

```
data(auuc)
```

**Format**

A data frame with 4 observations on the following 5 variables.

**Commerce** a numeric vector of counts.

**Arts** a numeric vector of counts.

**SciEng** a numeric vector of counts.

**Law** a numeric vector of counts.

**Medicine** a numeric vector of counts.

**Details**

Each student is cross-classified by their colleges (Science and Engineering have been combined) and the socio-economic status (SES) of their fathers (1 = highest, down to 4 = lowest).

**Source**

Dr Tony Morrison.

**References**

Wild, C. J. and Seber, G. A. F. (2000) *Chance Encounters: A First Course in Data Analysis and Inference*, New York: Wiley.

**Examples**

```
auuc
## Not run:
round(fitted(grc(auuc)))
round(fitted(grc(auuc, Rank = 2)))

## End(Not run)
```

---

aux.posbernoulli.t	<i>Auxiliary Function for the Positive Bernoulli Family Function with Time Effects</i>
--------------------	--

---

**Description**

Returns behavioural effects indicator variables from a capture history matrix.

**Usage**

```
aux.posbernoulli.t(y, check.y = FALSE, rename = TRUE, name = "bei")
```



**Arguments**

<code>y</code>	Capture history matrix. Rows are animals, columns are sampling occasions, and values should be 0s and 1s only.
<code>check.y</code>	Logical, if TRUE then some basic checking is performed.
<code>rename, name</code>	If <code>rename = TRUE</code> then the behavioural effects indicator are named using the value of <code>name</code> as the prefix. If FALSE then use the same column names as <code>y</code> .

**Details**

This function can help fit certain capture–recapture models (commonly known as  $M_{tb}$  or  $M_{tbh}$  (no prefix  $h$  means it is an intercept-only model) in the literature). See [posbernoulli.t](#) for details.

**Value**

A list with the following components.

**cap.hist1** A matrix the same dimension as `y`. In any particular row there are 0s up to the first capture. Then there are 1s thereafter.

**cap1** A vector specifying which time occasion the animal was first captured.

**y0i** Number of noncaptures before the first capture.

**yr0i** Number of noncaptures after the first capture.

**yr1i** Number of recaptures after the first capture.

**See Also**

[posbernoulli.t](#), [deermice](#).

**Examples**

```
# Fit a M_tbh model to the deermice data:
(pdata <- aux.posbernoulli.t(with(deermice, cbind(y1, y2, y3, y4, y5, y6))))

deermice <- data.frame(deermice,
                      bei = 0, # Add this
                      pdata$cap.hist1) # Incorporate these
head(deermice) # Augmented with behavioural effect indicator variables
tail(deermice)
```

---

backPain*Data on Back Pain Prognosis, from Anderson (1984)*

---

### Description

Data from a study of patients suffering from back pain. Prognostic variables were recorded at presentation and progress was categorised three weeks after treatment.

### Usage

```
data(backPain)
```

### Format

A data frame with 101 observations on the following 4 variables.

**x1** length of previous attack.

**x2** pain change.

**x3** lordosis.

**pain** an ordered factor describing the progress of each patient with levels worse < same < slight.improvement < moderate.improvement < marked.improvement < complete.relief.

### Source

<http://ideas.repec.org/c/boc/bocode/s419001.html>

The data set and this help file was copied from **gnm** so that a vignette in **VGAM** could be run; the analysis is described in Yee (2010).

### References

Anderson, J. A. (1984) Regression and Ordered Categorical Variables. *J. R. Statist. Soc. B*, **46**(1), 1-30.

Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.

### Examples

```
summary(backPain)
```

---

beggs

*Bacon and Eggs Data*

---

### Description

Purchasing of bacon and eggs.

### Usage

```
data(beggs)
```

### Format

Data frame of a two way table.

**b0, b1, b2, b3, b4** The b refers to bacon. The number of times bacon was purchased was 0, 1, 2, 3, or 4.

**e0, e1, e2, e3, e4** The e refers to eggs. The number of times eggs was purchased was 0, 1, 2, 3, or 4.

### Details

The data is from Information Resources, Inc., a consumer panel based in a large US city [see Bell and Lattin (1998) for further details]. Starting in June 1991, the purchases in the bacon and fresh eggs product categories for a sample of 548 households over four consecutive store trips was tracked. Only those grocery shopping trips with a total basket value of at least five dollars was considered. For each household, the total number of bacon purchases in their four eligible shopping trips and the total number of egg purchases (usually a package of eggs) for the same trips, were counted.

### Source

Bell, D. R. and Lattin, J. M. (1998) Shopping Behavior and Consumer Preference for Store Price Format: Why 'Large Basket' Shoppers Prefer EDLP. *Marketing Science*, **17**, 66–88.

### References

Danaher, P. J. and Hardie, B. G. S. (2005) Bacon with Your Eggs? Applications of a New Bivariate Beta-Binomial Distribution. *American Statistician*, **59**(4), 282–286.

### See Also

[rrvglm](#), [rcim](#), [grc](#).

### Examples

```
beggs
colSums(beggs)
rowSums(beggs)
```

Benford

*Benford's Distribution***Description**

Density, distribution function, quantile function, and random generation for Benford's distribution.

**Usage**

```
dbenf(x, ndigits = 1, log = FALSE)
pbenf(q, ndigits = 1, log.p = FALSE)
qbenf(p, ndigits = 1)
rbenf(n, ndigits = 1)
```

**Arguments**

<code>x, q</code>	Vector of quantiles. See <code>ndigits</code> .
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. A single positive integer. Else if <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>ndigits</code>	Number of leading digits, either 1 or 2. If 1 then the support of the distribution is $\{1, \dots, 9\}$ , else $\{10, \dots, 99\}$ .
<code>log, log.p</code>	Logical. If <code>log.p = TRUE</code> then all probabilities <code>p</code> are given as $\log(p)$ .

**Details**

Benford's Law (aka *the significant-digit law*) is the empirical observation that in many naturally occurring tables of numerical data, the leading significant (nonzero) digit is not uniformly distributed in  $\{1, 2, \dots, 9\}$ . Instead, the leading significant digit ( $= D$ , say) obeys the law

$$P(D = d) = \log_{10} \left( 1 + \frac{1}{d} \right)$$

for  $d = 1, \dots, 9$ . This means the probability the first significant digit is 1 is approximately 0.301, etc.

Benford's Law was apparently first discovered in 1881 by astronomer/mathematician S. Newcombe. It started by the observation that the pages of a book of logarithms were dirtiest at the beginning and progressively cleaner throughout. In 1938, a General Electric physicist called F. Benford re-discovered the law on this same observation. Over several years he collected data from different sources as different as atomic weights, baseball statistics, numerical data from *Reader's Digest*, and drainage areas of rivers.

Applications of Benford's Law has been as diverse as to the area of fraud detection in accounting and the design computers.

**Value**

`dbenf` gives the density, `pbenf` gives the distribution function, and `qbenf` gives the quantile function, and `rbenf` generates random deviates.

**Author(s)**

T. W. Yee

**References**

Benford, F. (1938) The Law of Anomalous Numbers. *Proceedings of the American Philosophical Society*, **78**, 551–572.

Newcomb, S. (1881) Note on the Frequency of Use of the Different Digits in Natural Numbers. *American Journal of Mathematics*, **4**, 39–40.

**Examples**

```
dbenf(x <- c(0:10, NA, NaN, -Inf, Inf))
pbenf(x)

## Not run:
xx = 1:9; # par(mfrow=c(2,1))
barplot(dbenf(xx), col = "lightblue", las = 1, xlab = "Leading digit",
        ylab = "Probability", names.arg = as.character(xx),
        main = paste("Benford's distribution", sep = ""))

hist(rbenf(n = 1000), border = "blue", prob = TRUE,
     main = "1000 random variates from Benford's distribution",
     xlab = "Leading digit", sub="Red is the true probability",
     breaks = 0:9 + 0.5, ylim = c(0, 0.35), xlim = c(0, 10.0))
lines(xx, dbenf(xx), col = "red", type = "h")
points(xx, dbenf(xx), col = "red")

## End(Not run)
```

---

Benini

---

*The Benini Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the Benini distribution with parameter shape.

**Usage**

```
dbenini(x, shape, y0, log = FALSE)
pbenini(q, shape, y0)
qbenini(p, shape, y0)
rbenini(n, shape, y0)
```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Same as <code>runif</code> .
<code>shape</code>	the shape parameter $b$ .
<code>y0</code>	the scale parameter $y_0$ .
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

See [benini](#), the **VGAM** family function for estimating the parameter  $b$  by maximum likelihood estimation, for the formula of the probability density function and other details.

## Value

`dbenini` gives the density, `pbenini` gives the distribution function, `qbenini` gives the quantile function, and `rbenini` generates random deviates.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

[benini](#).

## Examples

```
## Not run:
y0 <- 1; shape <- exp(1)
xx <- seq(0.0, 4, len = 101)
plot(xx, dbenini(xx, y0 = y0, shape = shape), type = "l", col = "blue",
      main = "Blue is density, orange is cumulative distribution function",
      sub = "Purple lines are the 10,20,...,90 percentiles", ylim = 0:1,
      las = 1, ylab = "", xlab = "x")
abline(h = 0, col = "blue", lty = 2)
lines(xx, pbenini(xx, y0 = y0, shape = shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qbenini(probs, y0 = y0, shape = shape)
lines(Q, dbenini(Q, y0 = y0, shape = shape),
      col = "purple", lty = 3, type = "h")
pbenini(Q, y0 = y0, shape = shape) - probs # Should be all zero

## End(Not run)
```

benini

*Benini Distribution Family Function***Description**

Estimating the 1-parameter Benini distribution by maximum likelihood estimation.

**Usage**

```
benini(y0 = stop("argument 'y0' must be specified"), lshape = "log",
       ishape = NULL, imethod = 1, zero = NULL)
```

**Arguments**

<code>y0</code>	Positive scale parameter.
<code>lshape</code>	Parameter link function and extra argument of the parameter $b$ , which is the shape parameter. See <a href="#">Links</a> for more choices. A log link is the default because $b$ is positive.
<code>ishape</code>	Optional initial value for the shape parameter. The default is to compute the value internally.
<code>imethod, zero</code>	Details at <a href="#">CommonVGAMffArguments</a> .

**Details**

The Benini distribution has a probability density function that can be written

$$f(y) = 2b \exp(-b[(\log(y/y_0))^2]) \log(y/y_0)/y$$

for  $0 < y_0 < y$ , and  $b > 0$ . The cumulative distribution function for  $Y$  is

$$F(y) = 1 - \exp(-b[(\log(y/y_0))^2]).$$

Here, Newton-Raphson and Fisher scoring coincide. The median of  $Y$  is now returned as the fitted values. This **VGAM** family function can handle a multiple responses, which is inputted as a matrix.

On fitting, the extra slot has a component called `y0` which contains the value of the `y0` argument.

**Value**

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

Yet to do: the 2-parameter Benini distribution estimates  $y_0$  as well, and the 3-parameter Benini distribution estimates another shape parameter  $a$  too.

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[Benini](#).

**Examples**

```
y0 <- 1; nn <- 3000
bdata <- data.frame(y = rbenini(nn, y0 = y0, shape = exp(2)))
fit <- vglm(y ~ 1, benini(y0 = y0), bdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
fit@extra$y0
c(head(fitted(fit), 1), with(bdata, median(y))) # Should be equal
```

beta.ab

*The Two-parameter Beta Distribution Family Function***Description**

Estimation of the shape parameters of the two-parameter beta distribution.

**Usage**

```
beta.ab(lshape1 = "loge", lshape2 = "loge",
        i1 = NULL, i2 = NULL, trim = 0.05,
        A = 0, B = 1, parallel = FALSE, zero = NULL)
```

**Arguments**

lshape1, lshape2, i1, i2	Details at <a href="#">CommonVGAMffArguments</a> . See <a href="#">Links</a> for more choices.
trim	An argument which is fed into <code>mean()</code> ; it is the fraction (0 to 0.5) of observations to be trimmed from each end of the response <code>y</code> before the mean is computed. This is used when computing initial values, and guards against outliers.
A, B	Lower and upper limits of the distribution. The defaults correspond to the <i>standard beta distribution</i> where the response lies between 0 and 1.
parallel, zero	See <a href="#">CommonVGAMffArguments</a> for more information.



## Details

The two-parameter beta distribution is given by  $f(y) =$

$$(y - A)^{shape1-1} \times (B - y)^{shape2-1} / [Beta(shape1, shape2) \times (B - A)^{shape1+shape2-1}]$$

for  $A < y < B$ , and  $Beta(.,.)$  is the beta function (see [beta](#)). The shape parameters are positive, and here, the limits  $A$  and  $B$  are known. The mean of  $Y$  is  $E(Y) = A + (B - A) \times shape1 / (shape1 + shape2)$ , and these are the fitted values of the object.

For the standard beta distribution the variance of  $Y$  is  $shape1 \times shape2 / [(1 + shape1 + shape2) \times (shape1 + shape2)^2]$ . If  $\sigma^2 = 1 / (1 + shape1 + shape2)$  then the variance of  $Y$  can be written  $\sigma^2 \mu(1 - \mu)$  where  $\mu = shape1 / (shape1 + shape2)$  is the mean of  $Y$ .

Another parameterization of the beta distribution involving the mean and a precision parameter is implemented in [betaff](#).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Note

The response must have values in the interval  $(A, B)$ . **VGAM** 0.7-4 and prior called this function [betaff](#).

## Author(s)

Thomas W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) Chapter 25 of: *Continuous Univariate Distributions*, 2nd edition, Volume 2, New York: Wiley.

Gupta, A. K. and Nadarajah, S. (2004) *Handbook of Beta Distribution and Its Applications*, New York: Marcel Dekker.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[betaff](#), [Beta](#), [genbetaII](#), [betaII](#), [betabinomial.ab](#), [betageometric](#), [betaprime](#), [rbetageom](#), [rbetanorm](#), [kumar](#).

## Examples

```
bdata <- data.frame(y = rbeta(n = 1000, shape1 = exp(0), shape2 = exp(1)))
fit <- vglm(y ~ 1, beta.ab(lshape1 = "identity", lshape2 = "identity"),
           data = bdata, trace = TRUE, crit = "coef")
fit <- vglm(y ~ 1, beta.ab, bdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
```

```

Coef(fit) # Useful for intercept-only models

bdata <- transform(bdata, Y = 5 + 8 * y) # From 5 to 13, not 0 to 1
fit <- vglm(Y ~ 1, beta.ab(A = 5, B = 13), bdata, trace = TRUE)
Coef(fit)
c(meanY = with(bdata, mean(Y)), head(fitted(fit),2))

```

Betabinom

*The Beta-Binomial Distribution*

## Description

Density, distribution function, and random generation for the beta-binomial distribution.

## Usage

```

dbetabinom(x, size, prob, rho = 0, log = FALSE)
pbetabinom(q, size, prob, rho, log.p = FALSE)
rbetabinom(n, size, prob, rho = 0)
dbetabinom.ab(x, size, shape1, shape2, log = FALSE, .dontuse.prob = NULL)
pbetabinom.ab(q, size, shape1, shape2, log.p = FALSE)
rbetabinom.ab(n, size, shape1, shape2, .dontuse.prob = NULL)

```

## Arguments

x, q	vector of quantiles.
size	number of trials.
n	number of observations. Same as <a href="#">runif</a> .
prob	the probability of success $\mu$ . Must be in the unit closed interval $[0, 1]$ .
rho	the correlation parameter $\rho$ . Usually must be in the unit open interval $(0, 1)$ , however, the value 0 is sometimes supported (if so then it corresponds to the usual binomial distribution).
shape1, shape2	the two (positive) shape parameters of the standard beta distribution. They are called a and b in <a href="#">beta</a> respectively.
log, log.p	Logical. If TRUE then all probabilities p are given as $\log(p)$ .
.dontuse.prob	An argument that should be ignored and unused.

## Details

The beta-binomial distribution is a binomial distribution whose probability of success is not a constant but it is generated from a beta distribution with parameters shape1 and shape2. Note that the mean of this beta distribution is  $\mu = \text{shape1}/(\text{shape1}+\text{shape2})$ , which therefore is the mean or the probability of success.

See [betabinomial](#) and [betabinomial.ab](#), the **VGAM** family functions for estimating the parameters, for the formula of the probability density function and other details.

**Value**

dbetabinom and dbetabinom.ab give the density, pbetabinom and pbetabinom.ab give the distribution function, and

rbetabinom and rbetabinom.ab generate random deviates.

**Note**

pbetabinom and pbetabinom.ab can be particularly slow. The functions here ending in .ab are called from those functions which don't. The simple transformations  $\mu = \alpha/(\alpha + \beta)$  and  $\rho = 1/(1 + \alpha + \beta)$  are used, where  $\alpha$  and  $\beta$  are the two shape parameters.

**Author(s)**

T. W. Yee

**See Also**

[betabinomial](#), [betabinomial.ab](#).

**Examples**

```
set.seed(1); rbetabinom(10, 100, prob = 0.5)
set.seed(1);      rbinom(10, 100, prob = 0.5) # The same since rho = 0

## Not run: N <- 9; xx <- 0:N; s1 <- 2; s2 <- 3
dy <- dbetabinom.ab(xx, size = N, shape1 = s1, shape2 = s2)
barplot(rbind(dy, dbinom(xx, size = N, prob = s1 / (s1+s2))),
        beside = TRUE, col = c("blue", "green"), las = 1,
        main = paste("Beta-binomial (size=", N, ", shape1=", s1,
                      ", shape2=", s2, ") (blue) vs\n",
                      " Binomial(size=", N, ", prob=", s1/(s1+s2), ") (green)", sep = ""),
        names.arg = as.character(xx), cex.main = 0.8)
sum(dy * xx) # Check expected values are equal
sum(dbinom(xx, size = N, prob = s1 / (s1+s2)) * xx)
cumsum(dy) - pbetabinom.ab(xx, N, shape1 = s1, shape2 = s2) # Should be all 0

y <- rbetabinom.ab(n = 10000, size = N, shape1 = s1, shape2 = s2)
ty <- table(y)
barplot(rbind(dy, ty / sum(ty)),
        beside = TRUE, col = c("blue", "orange"), las = 1,
        main = paste("Beta-binomial (size=", N, ", shape1=", s1,
                      ", shape2=", s2, ") (blue) vs\n",
                      " Random generated beta-binomial(size=", N, ", prob=", s1/(s1+s2),
                      ") (orange)", sep = ""), cex.main = 0.8,
        names.arg = as.character(xx))
## End(Not run)
```

betabinomial

*Beta-binomial Distribution Family Function***Description**

Fits a beta-binomial distribution by maximum likelihood estimation. The two parameters here are the mean and correlation coefficient.

**Usage**

```
betabinomial(lmu = "logit", lrho = "logit",
             irho = NULL, imethod = 1, shrinkage.init = 0.95,
             nsimEIM = NULL, zero = 2)
```

**Arguments**

lmu, lrho	Link functions applied to the two parameters. See <a href="#">Links</a> for more choices. The defaults ensure the parameters remain in $(0, 1)$ , however, see the warning below.
irho	Optional initial value for the correlation parameter. If given, it must be in $(0, 1)$ , and is recycled to the necessary length. Assign this argument a value if a convergence failure occurs. Having <code>irho = NULL</code> means an initial value is obtained internally, though this can give unsatisfactory results.
imethod	An integer with value 1 or 2 or $\dots$ , which specifies the initialization method for $\mu$ . If failure to converge occurs try the another value and/or else specify a value for <code>irho</code> .
zero	An integer specifying which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value should be either 1 or 2. The default is to have a single correlation parameter. To model both parameters as functions of the covariates assign <code>zero = NULL</code> . See <a href="#">CommonVGAMffArguments</a> for more information.
shrinkage.init, nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information. The argument <code>shrinkage.init</code> is used only if <code>imethod = 2</code> . Using the argument <code>nsimEIM</code> may offer large advantages for large values of $N$ and/or large data sets.

**Details**

There are several parameterizations of the beta-binomial distribution. This family function directly models the mean and correlation parameter, i.e., the probability of success. The model can be written  $T|P = p \sim \text{Binomial}(N, p)$  where  $P$  has a beta distribution with shape parameters  $\alpha$  and  $\beta$ . Here,  $N$  is the number of trials (e.g., litter size),  $T = NY$  is the number of successes, and  $p$  is the probability of a success (e.g., a malformation). That is,  $Y$  is the *proportion* of successes. Like [binomialff](#), the fitted values are the estimated probability of success (i.e.,  $E[Y]$  and not  $E[T]$ ) and the prior weights  $N$  are attached separately on the object in a slot.

The probability function is

$$P(T = t) = \binom{N}{t} \frac{B(\alpha + t, \beta + N - t)}{B(\alpha, \beta)}$$

where  $t = 0, 1, \dots, N$ , and  $B$  is the [beta](#) function with shape parameters  $\alpha$  and  $\beta$ . Recall  $Y = T/N$  is the real response being modelled.

The default model is  $\eta_1 = \text{logit}(\mu)$  and  $\eta_2 = \text{logit}(\rho)$  because both parameters lie between 0 and 1. The mean (of  $Y$ ) is  $p = \mu = \alpha/(\alpha + \beta)$  and the variance (of  $Y$ ) is  $\mu(1 - \mu)(1 + (N - 1)\rho)/N$ . Here, the correlation  $\rho$  is given by  $1/(1 + \alpha + \beta)$  and is the correlation between the  $N$  individuals within a litter. A *litter effect* is typically reflected by a positive value of  $\rho$ . It is known as the *over-dispersion parameter*.

This family function uses Fisher scoring. Elements of the second-order expected derivatives with respect to  $\alpha$  and  $\beta$  are computed numerically, which may fail for large  $\alpha$ ,  $\beta$ ,  $N$  or else take a long time.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#).

Suppose `fit` is a fitted beta-binomial model. Then `fit@y` contains the sample proportions  $y$ , `fitted(fit)` returns estimates of  $E(Y)$ , and `weights(fit, type="prior")` returns the number of trials  $N$ .

### Warning

If the estimated rho parameter is close to zero then it pays to try `lrho = "rhobit"`. One day this may become the default link function.

This family function is prone to numerical difficulties due to the expected information matrices not being positive-definite or ill-conditioned over some regions of the parameter space. If problems occur try setting `lrho` to some numerical value, `nsimEIM = 100`, say, or else use `etastart` argument of [vglm](#), etc.

### Note

This function processes the input in the same way as [binomialff](#). But it does not handle the case  $N = 1$  very well because there are two parameters to estimate, not one, for each row of the input. Cases where  $N = 1$  can be omitted via the subset argument of [vglm](#).

The *extended* beta-binomial distribution of Prentice (1986) is currently not implemented in the **VGAM** package as it has range-restrictions for the correlation parameter that are currently too difficult to handle in this package. However, try `lrho = "rhobit"`.

### Author(s)

T. W. Yee

## References

Moore, D. F. and Tsatis, A. (1991) Robust estimation of the variance in moment methods for extra-binomial and extra-Poisson variation. *Biometrics*, **47**, 383–401.

Prentice, R. L. (1986) Binary regression using an extended beta-binomial distribution, with discussion of correlation induced by covariate measurement errors. *Journal of the American Statistical Association*, **81**, 321–327.

## See Also

[betabinomial.ab](#), [Betabinom](#), [binomialff](#), [betaff](#), [dirmultinomial](#), [lirat](#).

## Examples

```
# Example 1
bdata <- data.frame(N = 10, mu = 0.5, rho = 0.8)
bdata <- transform(bdata,
  y = rbetabinom(n = 100, size = N, prob = mu, rho = rho))
fit <- vglm(cbind(y, N-y) ~ 1, betabinomial, bdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(cbind(depvar(fit), weights(fit, type = "prior")))
```

```
# Example 2
fit <- vglm(cbind(R, N-R) ~ 1, betabinomial, lirat,
  trace = TRUE, subset = N > 1)
coef(fit, matrix = TRUE)
Coef(fit)
t(fitted(fit))
t(depvar(fit))
t(weights(fit, type = "prior"))
```

```
# Example 3, which is more complicated
lirat <- transform(lirat, fgrp = factor(grp))
summary(lirat) # Only 5 litters in group 3
fit2 <- vglm(cbind(R, N-R) ~ fgrp + hb, betabinomial(zero = 2),
  data = lirat, trace = TRUE, subset = N > 1)
coef(fit2, matrix = TRUE)
## Not run: with(lirat, plot(hb[N > 1], fit2@misc$rho,
  xlab = "Hemoglobin", ylab = "Estimated rho",
  pch = as.character(grp[N > 1]), col = grp[N > 1]))
## End(Not run)
## Not run: # cf. Figure 3 of Moore and Tsatis (1991)
with(lirat, plot(hb, R / N, pch = as.character(grp), col = grp, las = 1,
  xlab = "Hemoglobin level", ylab = "Proportion Dead",
  main = "Fitted values (lines)"))
smalldf <- with(lirat, lirat[N > 1, ])
for (gp in 1:4) {
  xx <- with(smalldf, hb[grp == gp])
  yy <- with(smalldf, fitted(fit2)[grp == gp])
```

```

ooo <- order(xx)
lines(xx[ooo], yy[ooo], col = gp)
}
## End(Not run)

```

betabinomial.ab

*Beta-binomial Distribution Family Function*

### Description

Fits a beta-binomial distribution by maximum likelihood estimation. The two parameters here are the shape parameters of the underlying beta distribution.

### Usage

```
betabinomial.ab(lshape12 = "loge", i1 = 1, i2 = NULL,
               imethod = 1, shrinkage.init = 0.95, nsimEIM = NULL,
               zero = NULL)
```

### Arguments

<code>lshape12</code>	Link function applied to both (positive) shape parameters of the beta distribution. See <a href="#">Links</a> for more choices.
<code>i1, i2</code>	Initial value for the shape parameters. The first must be positive, and is recycled to the necessary length. The second is optional. If a failure to converge occurs, try assigning a different value to <code>i1</code> and/or using <code>i2</code> .
<code>zero</code>	An integer specifying which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value should be either 1 or 2. The default is to model both shape parameters as functions of the covariates. If a failure to converge occurs, try <code>zero = 2</code> .
<code>shrinkage.init, nsimEIM, imethod</code>	See <a href="#">CommonVGAMffArguments</a> for more information. The argument <code>shrinkage.init</code> is used only if <code>imethod = 2</code> . Using the argument <code>nsimEIM</code> may offer large advantages for large values of $N$ and/or large data sets.

### Details

There are several parameterizations of the beta-binomial distribution. This family function directly models the two shape parameters of the associated beta distribution rather than the probability of success (however, see **Note** below). The model can be written  $T|P = p \sim \text{Binomial}(N, p)$  where  $P$  has a beta distribution with shape parameters  $\alpha$  and  $\beta$ . Here,  $N$  is the number of trials (e.g., litter size),  $T = NY$  is the number of successes, and  $p$  is the probability of a success (e.g., a malformation). That is,  $Y$  is the *proportion* of successes. Like [binomialff](#), the fitted values are the estimated probability of success (i.e.,  $E[Y]$  and not  $E[T]$ ) and the prior weights  $N$  are attached separately on the object in a slot.

The probability function is

$$P(T = t) = \binom{N}{t} \frac{B(\alpha + t, \beta + N - t)}{B(\alpha, \beta)}$$

where  $t = 0, 1, \dots, N$ , and  $B$  is the beta function with shape parameters  $\alpha$  and  $\beta$ . Recall  $Y = T/N$  is the real response being modelled.

The default model is  $\eta_1 = \log(\alpha)$  and  $\eta_2 = \log(\beta)$  because both parameters are positive. The mean (of  $Y$ ) is  $p = \mu = \alpha/(\alpha + \beta)$  and the variance (of  $Y$ ) is  $\mu(1 - \mu)(1 + (N - 1)\rho)/N$ . Here, the correlation  $\rho$  is given by  $1/(1 + \alpha + \beta)$  and is the correlation between the  $N$  individuals within a litter. A *litter effect* is typically reflected by a positive value of  $\rho$ . It is known as the *over-dispersion parameter*.

This family function uses Fisher scoring. The two diagonal elements of the second-order expected derivatives with respect to  $\alpha$  and  $\beta$  are computed numerically, which may fail for large  $\alpha$ ,  $\beta$ ,  $N$  or else take a long time.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#).

Suppose `fit` is a fitted beta-binomial model. Then `fit@y` (better: `depvar(fit)`) contains the sample proportions  $y$ , `fitted(fit)` returns estimates of  $E(Y)$ , and `weights(fit, type = "prior")` returns the number of trials  $N$ .

### Warning

This family function is prone to numerical difficulties due to the expected information matrices not being positive-definite or ill-conditioned over some regions of the parameter space. If problems occur try setting `i1` to be some other positive value, using `i2` and/or setting `zero = 2`.

This family function may be renamed in the future. See the warnings in [betabinomial](#).

### Note

This function processes the input in the same way as [binomialff](#). But it does not handle the case  $N = 1$  very well because there are two parameters to estimate, not one, for each row of the input. Cases where  $N = 1$  can be omitted via the subset argument of [vglm](#).

Although the two linear/additive predictors given above are in terms of  $\alpha$  and  $\beta$ , basic algebra shows that the default amounts to fitting a logit link to the probability of success; subtracting the second linear/additive predictor from the first gives that logistic regression linear/additive predictor. That is,  $\text{logit}(p) = \eta_1 - \eta_2$ . This is illustrated in one of the examples below.

The *extended* beta-binomial distribution of Prentice (1986) is currently not implemented in the **VGAM** package as it has range-restrictions for the correlation parameter that are currently too difficult to handle in this package.

### Author(s)

T. W. Yee



## References

Moore, D. F. and Tsatis, A. (1991) Robust estimation of the variance in moment methods for extra-binomial and extra-Poisson variation. *Biometrics*, **47**, 383–401.

Prentice, R. L. (1986) Binary regression using an extended beta-binomial distribution, with discussion of correlation induced by covariate measurement errors. *Journal of the American Statistical Association*, **81**, 321–327.

## See Also

[betabinomial](#), [Betabinom](#), [binomialff](#), [betaff](#), [dirmultinomial](#), [lirat](#).

## Examples

```
# Example 1
N <- 10; s1 <- exp(1); s2 <- exp(2)
y <- rbetabinom.ab(n = 100, size = N, shape1 = s1, shape2 = s2)
fit <- vglm(cbind(y, N-y) ~ 1, betabinomial.ab, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fit@misc$rho) # The correlation parameter
head(cbind(depvar(fit), weights(fit, type = "prior"))))

# Example 2
fit <- vglm(cbind(R, N-R) ~ 1, betabinomial.ab, data = lirat,
            trace = TRUE, subset = N > 1)
coef(fit, matrix = TRUE)
Coef(fit)
fit@misc$rho # The correlation parameter
t(fitted(fit))
t(depvar(fit))
t(weights(fit, type = "prior"))
# A "loge" link for the 2 shape parameters is a logistic regression:
all.equal(c(fitted(fit)),
          as.vector(logit(predict(fit)[, 1] -
                              predict(fit)[, 2], inverse = TRUE))))

# Example 3, which is more complicated
lirat <- transform(lirat, fgrp = factor(grp))
summary(lirat) # Only 5 litters in group 3
fit2 <- vglm(cbind(R, N-R) ~ fgrp + hb, betabinomial.ab(zero = 2),
            data = lirat, trace = TRUE, subset = N > 1)
coef(fit2, matrix = TRUE)
coef(fit2, matrix = TRUE)[, 1] -
coef(fit2, matrix = TRUE)[, 2] # logit(p)
## Not run: with(lirat, plot(hb[N > 1], fit2@misc$rho,
                           xlab = "Hemoglobin", ylab = "Estimated rho",
                           pch = as.character(grp[N > 1]), col = grp[N > 1]))
## End(Not run)
## Not run: # cf. Figure 3 of Moore and Tsatis (1991)
```

```

with(lirat, plot(hb, R / N, pch = as.character(grp), col = grp, las = 1,
  xlab = "Hemoglobin level", ylab = "Proportion Dead",
  main = "Fitted values (lines)"))

smalldf <- with(lirat, lirat[N > 1, ])
for (gp in 1:4) {
  xx <- with(smalldf, hb[grp == gp])
  yy <- with(smalldf, fitted(fit2)[grp == gp])
  ooo <- order(xx)
  lines(xx[ooo], yy[ooo], col = gp)
}
## End(Not run)

```

betaff

*The Two-parameter Beta Distribution Family Function*

## Description

Estimation of the mean and precision parameters of the beta distribution.

## Usage

```

betaff(A = 0, B = 1, lmu = "logit", lphi = "loge",
  imu = NULL, iphi = NULL, imethod = 1, zero = NULL)

```

## Arguments

A, B	Lower and upper limits of the distribution. The defaults correspond to the <i>standard beta distribution</i> where the response lies between 0 and 1.
lmu, lphi	Link function for the mean and precision parameters. The values <i>A</i> and <i>B</i> are extracted from the min and max arguments of <a href="#">elogit</a> . Consequently, only <a href="#">elogit</a> is allowed.
imu, iphi	Optional initial value for the mean and precision parameters respectively. A NULL value means a value is obtained in the initialize slot.
imethod, zero	See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The two-parameter beta distribution can be written  $f(y) =$

$$(y - A)^{\mu_1 \phi - 1} \times (B - y)^{(1 - \mu_1) \phi - 1} / [\text{beta}(\mu_1 \phi, (1 - \mu_1) \phi) \times (B - A)^{\phi - 1}]$$

for  $A < y < B$ , and  $\text{beta}(\cdot, \cdot)$  is the beta function (see [beta](#)). The parameter  $\mu_1$  satisfies  $\mu_1 = (\mu - A) / (B - A)$  where  $\mu$  is the mean of  $Y$ . That is,  $\mu_1$  is the mean of of a standard beta distribution:  $E(Y) = A + (B - A) \times \mu_1$ , and these are the fitted values of the object. Also,  $\phi$  is positive and  $A < \mu < B$ . Here, the limits *A* and *B* are *known*.

Another parameterization of the beta distribution involving the raw shape parameters is implemented in [beta.ab](#).

For general  $A$  and  $B$ , the variance of  $Y$  is  $(B - A)^2 \times \mu_1 \times (1 - \mu_1)/(1 + \phi)$ . Then  $\phi$  can be interpreted as a *precision* parameter in the sense that, for fixed  $\mu$ , the larger the value of  $\phi$ , the smaller the variance of  $Y$ . Also,  $\mu_1 = \text{shape1}/(\text{shape1} + \text{shape2})$  and  $\phi = \text{shape1} + \text{shape2}$ . Fisher scoring is implemented.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

The response must have values in the interval  $(A, B)$ . The user currently needs to manually choose `lmu` to match the input of arguments  $A$  and  $B$ , e.g., with [elogit](#); see the example below.

### Author(s)

Thomas W. Yee

### References

Ferrari, S. L. P. and Francisco C.-N. (2004) Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, **31**, 799–815.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

### See Also

[beta.ab](#), [Beta](#), [genbetaII](#), [betaII](#), [betabinomial.ab](#), [betageometric](#), [betaprime](#), [rbetageom](#), [rbetanorm](#), [kumar](#), [elogit](#).

### Examples

```
bdata <- data.frame(y = rbeta(nn <- 1000, shape1 = exp(0), shape2 = exp(1)))
fit1 <- vglm(y ~ 1, betaff, bdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1) # Useful for intercept-only models

# General A and B, and with a covariate
bdata <- transform(bdata, x2 = runif(nn))
bdata <- transform(bdata, mu = logit(0.5 - x2, inverse = TRUE),
  prec = exp(3.0 + x2)) # prec == phi
bdata <- transform(bdata, shape2 = prec * (1 - mu),
  shape1 = mu * prec)
bdata <- transform(bdata,
  y = rbeta(nn, shape1 = shape1, shape2 = shape2))
bdata <- transform(bdata, Y = 5 + 8 * y) # From 5 to 13, not 0 to 1
fit <- vglm(Y ~ x2, data = bdata, trace = TRUE,
  betaff(A = 5, B = 13, lmu = elogit(min = 5, max = 13)))
coef(fit, matrix = TRUE)
```

**Description**

Density, distribution function, and random generation for the beta-geometric distribution.

**Usage**

```
dbtageom(x, shape1, shape2, log = FALSE)
pbetageom(q, shape1, shape2, log.p = FALSE)
rbetageom(n, shape1, shape2)
```

**Arguments**

x, q	vector of quantiles.
n	number of observations. Same as <a href="#">runif</a> .
shape1, shape2	the two (positive) shape parameters of the standard beta distribution. They are called a and b in <a href="#">beta</a> respectively.
log, log.p	Logical. If TRUE then all probabilities p are given as log(p).

**Details**

The beta-geometric distribution is a geometric distribution whose probability of success is not a constant but it is generated from a beta distribution with parameters shape1 and shape2. Note that the mean of this beta distribution is  $\text{shape1}/(\text{shape1}+\text{shape2})$ , which therefore is the mean of the probability of success.

**Value**

dbtageom gives the density, pbetageom gives the distribution function, and rbetageom generates random deviates.

**Note**

pbetageom can be particularly slow.

**Author(s)**

T. W. Yee

**See Also**

[geometric](#), [betaff](#), [Beta](#).

## Examples

```
## Not run:
shape1 <- 1; shape2 <- 2; y <- 0:30
proby <- dbetageom(y, shape1, shape2, log = FALSE)
plot(y, proby, type = "h", col = "blue", ylab = "P[Y=y]", main = paste(
  "Y ~ Beta-geometric(shape1=", shape1,", shape2=", shape2, ")", sep = ""))
sum(proby)

## End(Not run)
```

betageometric

*Beta-geometric Distribution Family Function*

## Description

Maximum likelihood estimation for the beta-geometric distribution.

## Usage

```
betageometric(lprob = "logit", lshape = "loge",
  iprob = NULL,   ishape = 0.1,
  moreSummation = c(2, 100), tolerance = 1.0e-10, zero = NULL)
```

## Arguments

lprob, lshape	Parameter link functions applied to the parameters $p$ and $\phi$ (called prob and shape below). The former lies in the unit interval and the latter is positive. See <a href="#">Links</a> for more choices.
iprob, ishape	Numeric. Initial values for the two parameters. A NULL means a value is computed internally.
moreSummation	Integer, of length 2. When computing the expected information matrix a series summation from 0 to $\text{moreSummation}[1] \cdot \max(y) + \text{moreSummation}[2]$ is made, in which the upper limit is an approximation to infinity. Here, $y$ is the response.
tolerance	Positive numeric. When all terms are less than this then the series is deemed to have converged.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. If used, the value must be from the set $\{1,2\}$ .

## Details

A random variable  $Y$  has a 2-parameter beta-geometric distribution if  $P(Y = y) = p(1 - p)^y$  for  $y = 0, 1, 2, \dots$  where  $p$  are generated from a standard beta distribution with shape parameters  $\text{shape1}$  and  $\text{shape2}$ . The parameterization here is to focus on the parameters  $p$  and  $\phi = 1/(\text{shape1} + \text{shape2})$ , where  $\phi$  is shape. The default link functions for these ensure that the appropriate range of the parameters is maintained. The mean of  $Y$  is  $E(Y) = \text{shape2}/(\text{shape1} - 1) = (1 - p)/(p - \phi)$ .

The geometric distribution is a special case of the beta-geometric distribution with  $\phi = 0$  (see [geometric](#)). However, fitting data from a geometric distribution may result in numerical problems because the estimate of  $\log(\phi)$  will 'converge' to  $-\text{Inf}$ .

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

The first iteration may be very slow; if practical, it is best for the weights argument of [vglm](#) etc. to be used rather than inputting a very long vector as the response, i.e., `vglm(y ~ 1, ..., weights = wts)` is to be preferred over `vglm(rep(y, wts) ~ 1, ...)`. If convergence problems occur try inputting some values of argument `ishape`.

If an intercept-only model is fitted then the `misc` slot of the fitted object has list components `shape1` and `shape2`.

### Author(s)

T. W. Yee

### References

Paul, S. R. (2005) Testing goodness of fit of the geometric distribution: an application to human fecundability data. *Journal of Modern Applied Statistical Methods*, **4**, 425–433.

### See Also

[geometric](#), [betaff](#), [rbetageom](#).

### Examples

```
bdata <- data.frame(y = 0:11, wts = c(227,123,72,42,21,31,11,14,6,4,7,28))
fitb <- vglm(y ~ 1, betageometric, bdata, weight = wts, trace = TRUE)
fitg <- vglm(y ~ 1,      geometric, bdata, weight = wts, trace = TRUE)
coef(fitb, matrix = TRUE)
Coef(fitb)
sqrt(diag(vcov(fitb, untransform = TRUE)))
fitb@misc$shape1
fitb@misc$shape2
# Very strong evidence of a beta-geometric:
pchisq(2 * (logLik(fitb) - logLik(fitg)), df = 1, lower.tail = FALSE)
```

betaII

*Beta Distribution of the Second Kind***Description**

Maximum likelihood estimation of the 3-parameter beta II distribution.

**Usage**

```
betaII(lscale = "loge", lshape2.p = "loge", lshape3.q = "loge",
       iscale = NULL, ishape2.p = 2, ishape3.q = 2, zero = NULL)
```

**Arguments**

`lscale`, `lshape2.p`, `lshape3.q`  
 Parameter link functions applied to the (positive) parameters `scale`, `p` and `q`.  
 See [Links](#) for more choices.

`iscale`, `ishape2.p`, `ishape3.q`  
 Optional initial values for `scale`, `p` and `q`.

`zero`  
 An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2,3} which correspond to `scale`, `p`, `q`, respectively.

**Details**

The 3-parameter beta II is the 4-parameter *generalized* beta II distribution with shape parameter  $a = 1$ . It is also known as the Pearson VI distribution. Other distributions which are special cases of the 3-parameter beta II include the Lomax ( $p = 1$ ) and inverse Lomax ( $q = 1$ ). More details can be found in Kleiber and Kotz (2003).

The beta II distribution has density

$$f(y) = y^{p-1} / [b^p B(p, q) \{1 + y/b\}^{p+q}]$$

for  $b > 0$ ,  $p > 0$ ,  $q > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter `scale`, and the others are shape parameters. The mean is

$$E(Y) = b \Gamma(p+1) \Gamma(q-1) / (\Gamma(p) \Gamma(q))$$

provided  $q > 1$ ; these are returned as the fitted values.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

See the note in [genbetaII](#).

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[betaff](#), [genbetaII](#), [dagum](#), [sinmad](#), [fisk](#), [invlmax](#), [lomax](#), [paralogistic](#), [invparalogistic](#).

**Examples**

```
bdata <- data.frame(y = rsinmad(2000, shape1.a = 1, exp(2), exp(1))) # Not genuine data!
fit <- vglm(y ~ 1, betaII, bdata, trace = TRUE)
fit <- vglm(y ~ 1, betaII(ishape2.p = 0.7, ishape3.q = 0.7),
            bdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

Betanorm

*The Beta-Normal Distribution***Description**

Density, distribution function, quantile function and random generation for the univariate beta-normal distribution.

**Usage**

```
dbetanorm(x, shape1, shape2, mean = 0, sd = 1, log = FALSE)
pbetanorm(q, shape1, shape2, mean = 0, sd = 1,
          lower.tail = TRUE, log.p = FALSE)
qbetanorm(p, shape1, shape2, mean = 0, sd = 1)
rbetanorm(n, shape1, shape2, mean = 0, sd = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Same as <a href="#">runif</a> .
shape1, shape2	the two (positive) shape parameters of the standard beta distribution. They are called a and b respectively in <a href="#">beta</a> .
mean, sd	the mean and standard deviation of the univariate normal distribution ( <a href="#">Normal</a> ).
log, log.p	Logical. If TRUE then all probabilities p are given as log(p).
lower.tail	Logical. If TRUE then the upper tail is returned, i.e., one minus the usual answer.



**Details**

The function `betauninormal`, the **VGAM** family function for estimating the parameters, has not yet been written.

**Value**

`dbetanorm` gives the density, `pbetanorm` gives the distribution function, `qbetanorm` gives the quantile function, and `rbetanorm` generates random deviates.

**Author(s)**

T. W. Yee

**References**

pp.146–152 of Gupta, A. K. and Nadarajah, S. (2004) *Handbook of Beta Distribution and Its Applications*, New York: Marcel Dekker.

**Examples**

```
## Not run:
shape1 <- 0.1; shape2 <- 4; m <- 1
x <- seq(-10, 2, len = 501)
plot(x, dbetanorm(x, shape1, shape2, m = m), type = "l", ylim = 0:1, las = 1,
      ylab = paste("betanorm(", shape1, ", ", shape2, ", m=", m, ", sd=1)", sep = ""),
      main = "Blue is density, orange is cumulative distribution function",
      sub = "Purple lines are the 10,20,...,90 percentiles", col = "blue")
lines(x, pbetanorm(x, shape1, shape2, m = m), col = "orange")
abline(h = 0)
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qbetanorm(probs, shape1, shape2, m = m)
lines(Q, dbetanorm(Q, shape1, shape2, m = m), col = "purple", lty = 3, type = "h")
lines(Q, pbetanorm(Q, shape1, shape2, m = m), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
pbetanorm(Q, shape1, shape2, m = m) - probs # Should be all 0

## End(Not run)
```

---

betaprime

*The Beta-Prime Distribution*


---

**Description**

Estimation of the two shape parameters of the beta-prime distribution by maximum likelihood estimation.

**Usage**

```
betaprime(link = "log", i1 = 2, i2 = NULL, zero = NULL)
```

## Arguments

link	Parameter link function applied to the two (positive) shape parameters. See <a href="#">Links</a> for more choices.
i1, i2	Initial values for the first and second shape parameters. A NULL value means it is obtained in the initialize slot. Note that i2 is obtained using i1.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The value must be from the set {1,2} corresponding respectively to shape1 and shape2 respectively. If zero=NULL then both parameters are modelled with the explanatory variables.

## Details

The beta-prime distribution is given by

$$f(y) = y^{shape1-1}(1+y)^{-shape1-shape2}/B(shape1, shape2)$$

for  $y > 0$ . The shape parameters are positive, and here,  $B$  is the beta function. The mean of  $Y$  is  $shape1/(shape2 - 1)$  provided  $shape2 > 1$ ; these are returned as the fitted values.

If  $Y$  has a  $Beta(shape1, shape2)$  distribution then  $Y/(1-Y)$  and  $(1-Y)/Y$  have a  $Betaprime(shape1, shape2)$  and  $Betaprime(shape2, shape1)$  distribution respectively. Also, if  $Y_1$  has a  $gamma(shape1)$  distribution and  $Y_2$  has a  $gamma(shape2)$  distribution then  $Y_1/Y_2$  has a  $Betaprime(shape1, shape2)$  distribution.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Note

The response must have positive values only.

The beta-prime distribution is also known as the *beta distribution of the second kind* or the *inverted beta distribution*.

## Author(s)

Thomas W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) Chapter 25 of: *Continuous Univariate Distributions*, 2nd edition, Volume 2, New York: Wiley.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[betaff](#), [Beta](#).

### Examples

```
nn <- 1000
bdata <- data.frame(shape1 = exp(1), shape2 = exp(3))
bdata <- transform(bdata, yb = rbeta(nn, shape1, shape2))
bdata <- transform(bdata, y1 = (1-yb) / yb,
                  y2 = yb / (1-yb),
                  y3 = rgamma(nn, exp(3)) / rgamma(nn, exp(2)))

fit1 <- vglm(y1 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit1, matrix = TRUE)

fit2 <- vglm(y2 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit2, matrix = TRUE)

fit3 <- vglm(y3 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit3, matrix = TRUE)

# Compare the fitted values
with(bdata, mean(y3))
head(fitted(fit3))
Coef(fit3) # Useful for intercept-only models
```

---

Biclaytoncop

*Clayton Copula (Bivariate) Distribution*


---

### Description

Density and random generation for the (one parameter) bivariate Clayton copula distribution.

### Usage

```
dbiclaytoncop(x1, x2, alpha = 0, log = FALSE)
rbiclaytoncop(n, alpha = 0)
```

### Arguments

x1, x2	vector of quantiles. The x1 and x2 should both be in the interval (0, 1).
n	number of observations. Same as <a href="#">rnorm</a> .
alpha	the association parameter. Should be in the interval $[0, \infty)$ . The default corresponds to independence.
log	Logical. If TRUE then the logarithm is returned.

### Details

See [biclaytoncop](#), the **VGAM** family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

**Value**

dbiclaytoncop gives the density at point (x1,x2), rbiclaytoncop generates random deviates (a two-column matrix).

**Note**

dbiclaytoncop() does not yet handle  $x_1 = 0$  and/or  $x_2 = 0$ .

**Author(s)**

R. Feyter and T. W. Yee

**References**

Clayton, D. (1982) A model for association in bivariate survival data. *Journal of the Royal Statistical Society, Series B, Methodological*, **44**, 414–422.

**See Also**

[binormalcop](#), [binormal](#).

**Examples**

```
## Not run:  edge <- 0.01 # A small positive value
N <- 101; x <- seq(edge, 1.0 - edge, len = N); Rho <- 0.7
ox <- expand.grid(x, x)
zedd <- dbiclaytoncop(ox[, 1], ox[, 2], alpha = Rho, log = TRUE)
par(mfrow = c(1, 2))
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5, las = 1)
plot(rbiclaytoncop(1000, 2), col = "blue", las = 1)

## End(Not run)
```

---

biclaytoncop

*Clayton Copula (Bivariate) Family Function*

---

**Description**

Estimate the correlation parameter of the (bivariate) Clayton copula distribution by maximum likelihood estimation.

**Usage**

```
biclaytoncop(lalpha = "loge", ialpha = NULL, imethod = 1,
             parallel = FALSE, zero = NULL)
```

**Arguments**

lalpha, ialpha, imethod

Details at [CommonVGAMffArguments](#). See [Links](#) for more link function choices.

parallel, zero Details at [CommonVGAMffArguments](#). If `parallel = TRUE` then the constraint is also applied to the intercept.

**Details**

The cumulative distribution function is

$$P(u_1, u_2; \alpha) = (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha}$$

for  $0 \leq \alpha$ . The support of the function is the interior of the unit square; however, values of 0 and/or 1 are not allowed (currently). The marginal distributions are the standard uniform distributions. When  $\alpha = 0$  the random variables are independent.

This **VGAM** family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.5. This is because each marginal distribution corresponds to a standard uniform distribution.

This **VGAM** family function is fragile; each response must be in the interior of the unit square.

**Author(s)**

R. Feyter and T. W. Yee

**References**

Clayton, D. (1982) A model for association in bivariate survival data. *Journal of the Royal Statistical Society, Series B, Methodological*, **44**, 414–422.

Stober, J. and Schepsmeier, U. (2013) Derivatives and Fisher information of bivariate copulas. *Statistical Papers*.

**See Also**

[rbiclaytoncop](#), [dbiclaytoncop](#), [kendall.tau](#).

## Examples

```
ymat <- rbiclaytoncop(n = (nn <- 1000), alpha = exp(2))
bdata <- data.frame(y1 = ymat[, 1],
                   y2 = ymat[, 2],
                   y3 = ymat[, 1],
                   y4 = ymat[, 2],
                   x2 = runif(nn))

summary(bdata)
## Not run: plot(ymat, col = "blue")
fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the first
            biclaytoncop, data = bdata,
            trace = TRUE, crit = "coef") # Sometimes a good idea

coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
summary(fit1)

# Another example; alpha is a function of x2
bdata <- transform(bdata, alpha = exp(-0.5 + x2))
ymat <- rbiclaytoncop(n = nn, alpha = with(bdata, alpha))
bdata <- transform(bdata, y5 = ymat[, 1],
                   y6 = ymat[, 2])
fit2 <- vgam(cbind(y5, y6) ~ s(x2), data = bdata,
            biclaytoncop(lalpha = "loge"), trace = TRUE)
## Not run: plot(fit2, lcol = "blue", scol = "orange", se = TRUE, las = 1)
```

---

BICv1m

*Bayesian Information Criterion*


---

## Description

Calculates the Bayesian information criterion (BIC) for a fitted model object for which a log-likelihood value has been obtained.

## Usage

```
BICv1m(object, ..., k = log(nobs(object)))
```

## Arguments

object, ...	Same as <a href="#">AICv1m</a> .
k	Numeric, the penalty per parameter to be used; the default is $\log(n)$ where $n$ is the number of observations).

## Details

The so-called BIC or SBC (Schwarz's Bayesian criterion) can be computed by calling [AICv1m](#) with a different  $k$  argument. See [AICv1m](#) for information and caveats.

**Value**

Returns a numeric value with the corresponding BIC, or ..., depending on k.

**Warning**

Like [AICvlm](#), this code has not been double-checked. The general applicability of BIC for the VGLM/VGAM classes has not been developed fully. In particular, BIC should not be run on some **VGAM** family functions because of violation of certain regularity conditions, etc.

Many **VGAM** family functions such as [cumulative](#) can have the number of observations absorbed into the prior weights argument (e.g., weights in [vglm](#)), either before or after fitting. Almost all **VGAM** family functions can have the number of observations defined by the weights argument, e.g., as an observed frequency. BIC simply uses the number of rows of the model matrix, say, as defining n, hence the user must be very careful of this possible error. Use at your own risk!!

**Note**

BIC, AIC and other ICs can have many additive constants added to them. The important thing are the differences since the minimum value corresponds to the best model. Preliminary testing shows absolute differences with some **VGAM** family functions such as [gaussianff](#), however, they should agree with non-normal families.

BIC has not been defined for QRR-VGLMs yet.

**Author(s)**

T. W. Yee.

**See Also**

[AICvlm](#), VGLMs are described in [vglm-class](#); VGAMs are described in [vgam-class](#); RR-VGLMs are described in [rrvglm-class](#); [BIC](#), [AIC](#).

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = TRUE, reverse = TRUE), pneumo))
coef(fit1, matrix = TRUE)
BIC(fit1)
(fit2 <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = FALSE, reverse = TRUE), pneumo))
coef(fit2, matrix = TRUE)
BIC(fit2)

# These do not agree in absolute terms:
gdata <- data.frame(x2 = sort(runif(n <- 40)))
gdata <- transform(gdata, y1 = 1 + 2*x2 + rnorm(n, sd = 0.1))
fit.v <- vglm(y1 ~ x2, gaussianff, data = gdata)
fit.g <- glm(y1 ~ x2, gaussian, data = gdata)
fit.l <- lm(y1 ~ x2, data = gdata)
c(BIC(fit.l), BIC(fit.g), BIC(fit.v))
```

```
c(AIC(fit.l), AIC(fit.g), AIC(fit.v))
c(AIC(fit.l) - AIC(fit.v),
  AIC(fit.g) - AIC(fit.v))
c(logLik(fit.l), logLik(fit.g), logLik(fit.v))
```

bifrankcop

*Frank's Bivariate Distribution Family Function***Description**

Estimate the association parameter of Frank's bivariate distribution by maximum likelihood estimation.

**Usage**

```
bifrankcop(lapar = "loge", iapar = 2, nsimEIM = 250)
```

**Arguments**

lapar	Link function applied to the (positive) association parameter $\alpha$ . See <a href="#">Links</a> for more choices.
iapar	Numeric. Initial value for $\alpha$ . If a convergence failure occurs try assigning a different value.
nsimEIM	See <a href="#">CommonVGAMffArguments</a> .

**Details**

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = H_\alpha(y_1, y_2) = \log_\alpha[1 + (\alpha^{y_1} - 1)(\alpha^{y_2} - 1)/(\alpha - 1)]$$

for  $\alpha \neq 1$ . Note the logarithm here is to base  $\alpha$ . The support of the function is the unit square.

When  $0 < \alpha < 1$  the probability density function  $h_\alpha(y_1, y_2)$  is symmetric with respect to the lines  $y_2 = y_1$  and  $y_2 = 1 - y_1$ . When  $\alpha > 1$  then  $h_\alpha(y_1, y_2) = h_{1/\alpha}(1 - y_1, y_2)$ .

If  $\alpha = 1$  then  $H(y_1, y_2) = y_1 y_2$ , i.e., uniform on the unit square. As  $\alpha$  approaches 0 then  $H(y_1, y_2) = \min(y_1, y_2)$ . As  $\alpha$  approaches infinity then  $H(y_1, y_2) = \max(0, y_1 + y_2 - 1)$ .

The default is to use Fisher scoring implemented using [rbifrankcop](#). For intercept-only models an alternative is to set `nsimEIM=NULL` so that a variant of Newton-Raphson is used.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).



**Note**

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to a half. This is because the marginal distributions correspond to a standard uniform distribution.

**Author(s)**

T. W. Yee

**References**

Genest, C. (1987) Frank's family of bivariate distributions. *Biometrika*, **74**, 549–555.

**See Also**

[rbifrankcop](#), [fgm](#).

**Examples**

```
## Not run:
ymat <- rbifrankcop(n = 2000, alpha = exp(4))
plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = bifrankcop, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
head(fitted(fit))
summary(fit)

## End(Not run)
```

---

bigamma.mckay

---

*Bivariate Gamma: McKay's Distribution*


---

**Description**

Estimate the three parameters of McKay's bivariate gamma distribution by maximum likelihood estimation.

**Usage**

```
bigamma.mckay(lscale = "log", lshape1 = "log", lshape2 = "log",
              iscale = NULL, ishape1 = NULL, ishape2 = NULL,
              imethod = 1, zero = 1)
```

### Arguments

lscale, lshape1, lshape2

Link functions applied to the (positive) parameters  $a$ ,  $p$  and  $q$  respectively. See [Links](#) for more choices.

iscale, ishape1, ishape2

Optional initial values for  $a$ ,  $p$  and  $q$  respectively. The default is to compute them internally.

imethod, zero See [CommonVGAMffArguments](#).

### Details

One of the earliest forms of the bivariate gamma distribution has a joint probability density function given by

$$f(y_1, y_2; a, p, q) = (1/a)^{p+q} y_1^{p-1} (y_2 - y_1)^{q-1} \exp(-y_2/a) / [\Gamma(p)\Gamma(q)]$$

for  $a > 0$ ,  $p > 0$ ,  $q > 0$  and  $0 < y_1 < y_2$  (Mckay, 1934). Here,  $\Gamma$  is the gamma function, as in [gamma](#). By default, the linear/additive predictors are  $\eta_1 = \log(a)$ ,  $\eta_2 = \log(p)$ ,  $\eta_3 = \log(q)$ .

The marginal distributions are gamma, with shape parameters  $p$  and  $p + q$  respectively, but they have a common scale parameter  $a$ . Pearson's product-moment correlation coefficient of  $y_1$  and  $y_2$  is  $\sqrt{p/(p+q)}$ . This distribution is also known as the bivariate Pearson type III distribution. Also,  $Y_2 - y_1$ , conditional on  $Y_1 = y_1$ , has a gamma distribution with shape parameter  $q$ .

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

### Note

The response must be a two column matrix where the first column is  $y_1$  and the second  $y_2$ . It is necessary that each element of the vectors  $y_1$  and  $y_2 - y_1$  be positive. Currently, the fitted value is a matrix with two columns; the first column has values  $ap$  for the marginal mean of  $y_1$ , while the second column has values  $a(p+q)$  for the marginal mean of  $y_2$  (all evaluated at the final iteration).

### Author(s)

T. W. Yee

### References

- McKay, A. T. (1934) Sampling from batches. *Journal of the Royal Statistical Society—Supplement*, **1**, 207–216.
- Kotz, S. and Balakrishnan, N. and Johnson, N. L. (2000) *Continuous Multivariate Distributions Volume 1: Models and Applications*, 2nd edition, New York: Wiley.
- Balakrishnan, N. and Lai, C.-D. (2009) *Continuous Bivariate Distributions*, 2nd edition. New York: Springer.

**See Also**[gamma2](#).**Examples**

```

shape1 <- exp(1); shape2 <- exp(2); scalepar <- exp(3)
mdata <- data.frame(y1 = rgamma(nn <- 1000, shape = shape1, scale = scalepar))
mdata <- transform(mdata, zedd = rgamma(nn, shape = shape2, scale = scalepar))
mdata <- transform(mdata, y2 = y1 + zedd) # Z is defined as Y2-y1|Y1=y1
fit <- vglm(cbind(y1, y2) ~ 1, bigamma.mckay, mdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)

colMeans(depvar(fit)) # Check moments
head(fitted(fit), 1)

```

bigumbellI

*Gumbel's Type I Bivariate Distribution Family Function***Description**

Estimate the association parameter of Gumbel's Type I bivariate distribution by maximum likelihood estimation.

**Usage**

```
bigumbellI(lapar = "identity", iapar = NULL, imethod = 1)
```

**Arguments**

lapar	Link function applied to the association parameter $\alpha$ . See <a href="#">Links</a> for more choices.
iapar	Numeric. Optional initial value for $\alpha$ . By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument imethod.
imethod	An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ia.

**Details**

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = e^{-y_1 - y_2 + \alpha y_1 y_2} + 1 - e^{-y_1} - e^{-y_2}$$

for real  $\alpha$ . The support of the function is for  $y_1 > 0$  and  $y_2 > 0$ . The marginal distributions are an exponential distribution with unit mean.

A variant of Newton-Raphson is used, which only seems to work for an intercept model. It is a very good idea to set trace=TRUE.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 1. This is because each marginal distribution corresponds to a exponential distribution with unit mean.

This **VGAM** family function should be used with caution.

**Author(s)**

T. W. Yee

**References**

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[morgenstern](#).

**Examples**

```
nn <- 1000
gdata <- data.frame(y1 = rexp(nn), y2 = rexp(nn))
## Not run: with(gdata, plot(cbind(y1, y2)))
fit <- vglm(cbind(y1, y2) ~ 1, fam = bigumbell, gdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

---

bilogis4

*Bivariate Logistic Distribution*

---

**Description**

Density, distribution function, quantile function and random generation for the 4-parameter bivariate logistic distribution.

**Usage**

```
dbilogis4(x1, x2, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1, log = FALSE)
pbilogis4(q1, q2, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1)
rbilogis4(n, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1)
```

**Arguments**

`x1, x2, q1, q2` vector of quantiles.  
`n` number of observations. Same as `rlogis`.  
`loc1, loc2` the location parameters  $l_1$  and  $l_2$ .  
`scale1, scale2` the scale parameters  $s_1$  and  $s_2$ .  
`log` Logical. If `log = TRUE` then the logarithm of the density is returned.

**Details**

See `bilogis4`, the **VGAM** family function for estimating the four parameters by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

**Value**

`dbilogis4` gives the density, `pbilogis4` gives the distribution function, and `rbilogis4` generates random deviates (a two-column matrix).

**Author(s)**

T. W. Yee

**References**

Gumbel, E. J. (1961) Bivariate logistic distributions. *Journal of the American Statistical Association*, **56**, 335–349.

**See Also**

`bilogistic4`.

**Examples**

```
## Not run: par(mfrow = c(1, 3))
ymat <- rbilogis4(n = 2000, loc1 = 5, loc2 = 7, scale2 = exp(1))
myxlim <- c(-2, 15); myylim <- c(-10, 30)
plot(ymat, xlim = myxlim, ylim = myylim)

N <- 100
x1 <- seq(myxlim[1], myxlim[2], len = N)
x2 <- seq(myylim[1], myylim[2], len = N)
ox <- expand.grid(x1, x2)
z <- dbilogis4(ox[,1], ox[,2], loc1 = 5, loc2 = 7, scale2 = exp(1))
contour(x1, x2, matrix(z, N, N), main = "density")
z <- pbilogis4(ox[,1], ox[,2], loc1 = 5, loc2 = 7, scale2 = exp(1))
contour(x1, x2, matrix(z, N, N), main = "cdf")
## End(Not run)
```

bilogistic4

*Bivariate Logistic Distribution Family Function***Description**

Estimates the four parameters of the bivariate logistic distribution by maximum likelihood estimation.

**Usage**

```
bilogistic4(llocation = "identity", lscale = "loge",
            iloc1 = NULL, iscale1 = NULL, iloc2 = NULL, iscale2 = NULL,
            imethod = 1, zero = NULL)
```

**Arguments**

llocation	Link function applied to both location parameters $l_1$ and $l_2$ . See <a href="#">Links</a> for more choices.
lscale	Parameter link function applied to both (positive) scale parameters $s_1$ and $s_2$ . See <a href="#">Links</a> for more choices.
iloc1, iloc2	Initial values for the location parameters. By default, initial values are chosen internally using imethod. Assigning values here will override the argument imethod.
iscale1, iscale2	Initial values for the scale parameters. By default, initial values are chosen internally using imethod. Assigning values here will override the argument imethod.
imethod	An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose values from the set {1,2,3,4}.

**Details**

The four-parameter bivariate logistic distribution has a density that can be written as

$$f(y_1, y_2; l_1, s_1, l_2, s_2) = 2 \frac{\exp[-(y_1 - l_1)/s_1 - (y_2 - l_2)/s_2]}{s_1 s_2 (1 + \exp[-(y_1 - l_1)/s_1] + \exp[-(y_2 - l_2)/s_2])^3}$$

where  $s_1 > 0$   $s_2 > 0$  are the scale parameters, and  $l_1$  and  $l_2$  are the location parameters. Each of the two responses are unbounded, i.e.,  $-\infty < y_j < \infty$ . The mean of  $Y_1$  is  $l_1$  etc. The fitted values are returned in a 2-column matrix. The cumulative distribution function is

$$F(y_1, y_2; l_1, s_1, l_2, s_2) = (1 + \exp[-(y_1 - l_1)/s_1] + \exp[-(y_2 - l_2)/s_2])^{-1}$$

The marginal distribution of  $Y_1$  is

$$P(Y_1 \leq y_1) = F(y_1; l_1, s_1) = (1 + \exp[-(y_1 - l_1)/s_1])^{-1}.$$

By default,  $\eta_1 = l_1$ ,  $\eta_2 = \log(s_1)$ ,  $\eta_3 = l_2$ ,  $\eta_4 = \log(s_2)$  are the linear/additive predictors.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

### Note

This family function uses the BFGS quasi-Newton update formula for the working weight matrices. Consequently the estimated variance-covariance matrix may be inaccurate or simply wrong! The standard errors must be therefore treated with caution; these are computed in functions such as [vcov\(\)](#) and [summary\(\)](#).

### Author(s)

T. W. Yee

### References

- Gumbel, E. J. (1961) Bivariate logistic distributions. *Journal of the American Statistical Association*, **56**, 335–349.
- Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

### See Also

[logistic](#), [rbilogis4](#).

### Examples

```
yamat <- rbilogis4(n <- 1000, loc1 = 5, loc2 = 7, scale2 = exp(1))
## Not run: plot(yamat)
fit <- vglm(yamat ~ 1, fam = biogistic4, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
vcov(fit)
head(weights(fit, type = "work"))
summary(fit)
```

## Description

Density and random generation for a bivariate binary regression model using an odds ratio as the measure of dependency.

## Usage

```
rbinom2.or(n, mu1,
           mu2 = if(exchangeable) mu1 else stop("'mu2' not specified"),
           oratio = 1, exchangeable = FALSE, tol = 0.001, twoCols = TRUE,
           colnames = if(twoCols) c("y1", "y2") else c("00", "01", "10", "11"),
           ErrorCheck = TRUE)
dbinom2.or(mu1,
           mu2 = if(exchangeable) mu1 else stop("'mu2' not specified"),
           oratio = 1, exchangeable = FALSE, tol = 0.001,
           colnames = c("00", "01", "10", "11"), ErrorCheck = TRUE)
```

## Arguments

n	number of observations. Must be a single positive integer. The arguments mu1, mu2, oratio are recycled to length n.
mu1, mu2	The marginal probabilities. Only mu1 is needed if exchangeable = TRUE. Values should be between 0 and 1.
oratio	Odds ratio. Must be numeric and positive. The default value of unity means the responses are statistically independent.
exchangeable	Logical. If TRUE, the two marginal probabilities are constrained to be equal.
twoCols	Logical. If TRUE, then a $n \times 2$ matrix of 1s and 0s is returned. If FALSE, then a $n \times 4$ matrix of 1s and 0s is returned.
colnames	The dimnames argument of <a href="#">matrix</a> is assigned <code>list(NULL, colnames)</code> .
tol	Tolerance for testing independence. Should be some small positive numerical value.
ErrorCheck	Logical. Do some error checking of the input parameters?

## Details

The function `rbinom2.or` generates data coming from a bivariate binary response model. The data might be fitted with the **VGAM** family function [binom2.or](#).

The function `dbinom2.or` does not really compute the density (because that does not make sense here) but rather returns the four joint probabilities.



**Value**

The function `rbinom2.or` returns either a 2 or 4 column matrix of 1s and 0s, depending on the argument `twoCols`.

The function `dbinom2.or` returns a 4 column matrix of joint probabilities; each row adds up to unity.

**Author(s)**

T. W. Yee

**See Also**

[binom2.or](#).

**Examples**

```
nn <- 2000 # Example 1
ymat <- rbinom2.or(n = nn, mu1 = 0.8, oratio = exp(2), exch = TRUE)
(mytab <- table(ymat[, 1], ymat[, 2], dnn = c("Y1", "Y2")))
(myor <- mytab["0", "0"] * mytab["1", "1"] / (mytab["1", "0"] * mytab["0", "1"]))
fit <- vglm(ymat ~ 1, binom2.or(exch = TRUE))
coef(fit, matrix = TRUE)

bdata <- data.frame(x2 = sort(runif(nn))) # Example 2
bdata <- transform(bdata, mu1 = logit(-2 + 4*x2, inverse = TRUE),
                  mu2 = logit(-1 + 3*x2, inverse = TRUE))
dmat <- with(bdata, dbinom2.or(mu1 = mu1, mu2 = mu2, oratio = exp(2)))
ymat <- with(bdata, rbinom2.or(n = nn, mu1 = mu1, mu2 = mu2, oratio = exp(2)))
fit2 <- vglm(ymat ~ x2, binom2.or, data = bdata)
coef(fit2, matrix = TRUE)
## Not run:
matplot(with(bdata, x2), dmat, lty = 1:4, col = 1:4, type = "l",
        main = "Joint probabilities", ylim = 0:1, lwd = 2,
        ylab = "Probabilities", xlab = "x2", las = 1)
legend(x = 0, y = 0.5, lty = 1:4, col = 1:4, lwd = 2,
       legend = c("1 = (y1=0, y2=0)", "2 = (y1=0, y2=1)",
                  "3 = (y1=1, y2=0)", "4 = (y1=1, y2=1)"))

## End(Not run)
```

**Description**

Fits a Palmgren (bivariate odds-ratio model, or bivariate logistic regression) model to two binary responses. Actually, a bivariate logistic/probit/cloglog/cauchit model can be fitted. The odds ratio is used as a measure of dependency.

## Usage

```
binom2.or(lmu = "logit", lmu1 = lmu, lmu2 = lmu, loratio = "loge",
          imu1 = NULL, imu2 = NULL, ioratio = NULL, zero = 3,
          exchangeable = FALSE, tol = 0.001, more.robust = FALSE)
```

## Arguments

<code>lmu</code>	Link function applied to the two marginal probabilities. See <a href="#">Links</a> for more choices. See the note below.
<code>lmu1, lmu2</code>	Link function applied to the first and second of the two marginal probabilities.
<code>loratio</code>	Link function applied to the odds ratio. See <a href="#">Links</a> for more choices.
<code>imu1, imu2, ioratio</code>	Optional initial values for the marginal probabilities and odds ratio. See <a href="#">CommonVGAMffArguments</a> for more details. In general good initial values are often required so use these arguments if convergence failure occurs.
<code>zero</code>	Which linear/additive predictor is modelled as an intercept only? A NULL means none.
<code>exchangeable</code>	Logical. If TRUE, the two marginal probabilities are constrained to be equal.
<code>tol</code>	Tolerance for testing independence. Should be some small positive numerical value.
<code>more.robust</code>	Logical. If TRUE then some measures are taken to compute the derivatives and working weights more robustly, i.e., in an attempt to avoid numerical problems. Currently this feature is not debugged if set TRUE.

## Details

Also known informally as the *Palmgren model*, the bivariate logistic model is a full-likelihood based model defined as two logistic regressions plus  $\log(\text{oratio}) = \eta_3$  where  $\eta_3$  is the third linear/additive predictor relating the odds ratio to explanatory variables. Explicitly, the default model is

$$\text{logit}[P(Y_j = 1)] = \eta_j, \quad j = 1, 2$$

for the marginals, and

$$\log[P(Y_{00} = 1)P(Y_{11} = 1)/(P(Y_{01} = 1)P(Y_{10} = 1))] = \eta_3,$$

specifies the dependency between the two responses. Here, the responses equal 1 for a success and a 0 for a failure, and the odds ratio is often written  $\psi = p_{00}p_{11}/(p_{10}p_{01})$ . The model is fitted by maximum likelihood estimation since the full likelihood is specified. The two binary responses are independent if and only if the odds ratio is unity, or equivalently, the log odds ratio is zero. Fisher scoring is implemented.

The default models  $\eta_3$  as a single parameter only, i.e., an intercept-only model, but this can be circumvented by setting `zero = NULL` in order to model the odds ratio as a function of all the explanatory variables. The function `binom2.or()` can handle other probability link functions such as [probit](#), [cloglog](#) and [cauchit](#) links as well, so is quite general. In fact, the two marginal probabilities can each have a different link function. A similar model is the *bivariate probit model*

([binom2.rho](#)), which is based on a standard bivariate normal distribution, but the bivariate probit model is less interpretable and flexible.

The exchangeable argument should be used when the error structure is exchangeable, e.g., with eyes or ears data.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

When fitted, the `fitted.values` slot of the object contains the four joint probabilities, labelled as  $(Y_1, Y_2) = (0,0), (0,1), (1,0), (1,1)$ , respectively. These estimated probabilities should be extracted with the `fitted` generic function.

## Note

At present we call [binom2.or](#) families a *bivariate odds-ratio model*. The response should be either a 4-column matrix of counts (whose columns correspond to  $(Y_1, Y_2) = (0,0), (0,1), (1,0), (1,1)$  respectively), or a two-column matrix where each column has two distinct values, or a factor with four levels. The function [rbinom2.or](#) may be used to generate such data. Successful convergence requires at least one case of each of the four possible outcomes.

By default, a constant odds ratio is fitted because `zero = 3`. Set `zero = NULL` if you want the odds ratio to be modelled as a function of the explanatory variables; however, numerical problems are more likely to occur.

The argument `lmu`, which is actually redundant, is used for convenience and for upward compatibility: specifying `lmu` only means the link function will be applied to `lmu1` and `lmu2`. Users who want a different link function for each of the two marginal probabilities should use the `lmu1` and `lmu2` arguments, and the argument `lmu` is then ignored. It doesn't make sense to specify `exchangeable = TRUE` and have different link functions for the two marginal probabilities.

Regarding Yee and Dirnbock (2009), the `xij` (see [vglm.control](#)) argument enables environmental variables with different values at the two time points to be entered into an exchangeable [binom2.or](#) model. See the author's webpage for sample code.

## Author(s)

Thomas W. Yee

## References

- McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.
- le Cessie, S. and van Houwelingen, J. C. (1994) Logistic regression for correlated binary data. *Applied Statistics*, **43**, 95–108.
- Palmgren, J. (1989) *Regression Models for Bivariate Binary Responses*. Technical Report no. 101, Department of Biostatistics, University of Washington, Seattle.
- Yee, T. W. and Dirnbock, T. (2009) Models for analysing species' presence/absence data at two time points. *Journal of Theoretical Biology*, **259**(4), 684–694.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

### See Also

[rbinom2.or](#), [binom2.rho](#), [loglinb2](#), [zipebcom](#), [coalminers](#), [binomialff](#), [logit](#), [probit](#), [cloglog](#), [cauchit](#).

### Examples

```
# Fit the model in Table 6.7 in McCullagh and Nelder (1989)
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW, nBW, BnW, BW) ~ Age, binom2.or(zero = NULL), coalminers)
fitted(fit)
summary(fit)
coef(fit, matrix = TRUE)
c(weights(fit, type = "prior")) * fitted(fit) # Table 6.8

## Not run: with(coalminers, matplot(Age, fitted(fit), type = "l", las = 1,
                                   xlab = "(age - 42) / 5", lwd = 2))
with(coalminers, matpoints(Age, depvar(fit), col=1:4))
legend(x = -4, y = 0.5, lty = 1:4, col = 1:4, lwd = 2,
       legend = c("1 = (Breathlessness=0, Wheeze=0)",
                  "2 = (Breathlessness=0, Wheeze=1)",
                  "3 = (Breathlessness=1, Wheeze=0)",
                  "4 = (Breathlessness=1, Wheeze=1)"))

## End(Not run)

# Another model: pet ownership
## Not run: require(VGAMdata)
# More homogeneous:
petdata <- subset(xs.nz, ethnic == "0" & age < 70 & sex == "M")
petdata <- na.omit(petdata[, c("cat", "dog", "age")])
summary(petdata)
with(petdata, table(cat, dog)) # Can compute the odds ratio

fit <- vgam(cbind((1-cat) * (1-dog), (1-cat) * dog,
                 cat * (1-dog), cat * dog) ~ s(age, df = 5),
            binom2.or(zero = 3), data = petdata, trace = TRUE)
colSums(depvar(fit))
coef(fit, matrix = TRUE)

## End(Not run)

## Not run: # Plot the estimated probabilities
ooo <- order(with(petdata, age))
matplot(with(petdata, age)[ooo], fitted(fit)[ooo, ], type = "l",
        xlab = "Age", ylab = "Probability", main = "Pet ownership",
        ylim = c(0, max(fitted(fit))), las = 1, lwd = 1.5)
legend("topleft", col=1:4, lty = 1:4, leg = c("no cat or dog ",
        "dog only", "cat only", "cat and dog"), lwd = 1.5)

## End(Not run)
```

## Description

Density and random generation for a bivariate probit model. The correlation parameter rho is the measure of dependency.

## Usage

```
rbinom2.rho(n, mu1,
            mu2 = if(exchangeable) mu1 else stop("'mu2' not specified"),
            rho = 0, exchangeable = FALSE, twoCols = TRUE,
            colnames = if(twoCols) c("y1", "y2") else c("00", "01", "10", "11"),
            ErrorCheck = TRUE)
dbinom2.rho(mu1,
            mu2 = if(exchangeable) mu1 else stop("'mu2' not specified"),
            rho = 0, exchangeable = FALSE,
            colnames = c("00", "01", "10", "11"), ErrorCheck = TRUE)
```

## Arguments

n	number of observations. Must be a single positive integer. The arguments mu1, mu2, rho are recycled to length n.
mu1, mu2	The marginal probabilities. Only mu1 is needed if exchangeable = TRUE. Values should be between 0 and 1.
rho	The correlation parameter. Must be numeric and lie between $-1$ and $1$ . The default value of zero means the responses are uncorrelated.
exchangeable	Logical. If TRUE, the two marginal probabilities are constrained to be equal.
twoCols	Logical. If TRUE, then a $n \times 2$ matrix of 1s and 0s is returned. If FALSE, then a $n \times 4$ matrix of 1s and 0s is returned.
colnames	The dimnames argument of <code>matrix</code> is assigned <code>list(NULL, colnames)</code> .
ErrorCheck	Logical. Do some error checking of the input parameters?

## Details

The function `rbinom2.rho` generates data coming from a bivariate probit model. The data might be fitted with the **VGAM** family function `binom2.rho`.

The function `dbinom2.rho` does not really compute the density (because that does not make sense here) but rather returns the four joint probabilities.

**Value**

The function `rbinom2.rho` returns either a 2 or 4 column matrix of 1s and 0s, depending on the argument `twoCols`.

The function `dbinom2.rho` returns a 4 column matrix of joint probabilities; each row adds up to unity.

**Author(s)**

T. W. Yee

**See Also**

[binom2.rho](#).

**Examples**

```
(myrho <- rhobit(2, inverse = TRUE)) # Example 1
ymat <- rbinom2.rho(nn <- 2000, mu1 = 0.8, rho = myrho, exch = TRUE)
(mytab <- table(ymat[, 1], ymat[, 2], dnn = c("Y1", "Y2")))
fit <- vglm(ymat ~ 1, binom2.rho(exch = TRUE))
coef(fit, matrix = TRUE)

bdata <- data.frame(x2 = sort(runif(nn))) # Example 2
bdata <- transform(bdata, mu1 = probit(-2+4*x2, inverse = TRUE),
                  mu2 = probit(-1+3*x2, inverse = TRUE))
dmat <- with(bdata, dbinom2.rho(mu1, mu2, myrho))
ymat <- with(bdata, rbinom2.rho(nn, mu1, mu2, myrho))
fit2 <- vglm(ymat ~ x2, binom2.rho, bdata)
coef(fit2, matrix = TRUE)
## Not run: matplot(with(bdata, x2), dmat, lty = 1:4, col = 1:4,
                    type = "l", main = "Joint probabilities",
                    ylim = 0:1, lwd = 2, ylab = "Probability")
legend(x = 0.25, y = 0.9, lty = 1:4, col = 1:4, lwd = 2,
       legend = c("1 = (y1=0, y2=0)", "2 = (y1=0, y2=1)",
                  "3 = (y1=1, y2=0)", "4 = (y1=1, y2=1)"))
## End(Not run)
```

---

binom2.rho

*Bivariate Probit Model (Family Function)*

---

**Description**

Fits a bivariate probit model to two binary responses. The correlation parameter `rho` is the measure of dependency.

**Usage**

```
binom2.rho(lrho = "rhobit", lmu = "probit", imu1 = NULL, imu2 = NULL,
           irho = NULL, imethod = 1, zero = 3,
           exchangeable = FALSE, grho = seq(-0.95, 0.95, by = 0.05),
           nsimEIM = NULL)
binom2.Rho(rho = 0, imu1 = NULL, imu2 = NULL,
           exchangeable = FALSE, nsimEIM = NULL)
```

**Arguments**

lrho	Link function applied to the $\rho$ association parameter. See <a href="#">Links</a> for more choices.
lmu	Link function applied to the marginal probabilities. Should be left alone.
irho	Optional initial value for $\rho$ . If given, this should lie between $-1$ and $1$ . See below for more comments.
imu1, imu2	Optional initial values for the two marginal probabilities. May be a vector.
zero	Which linear/additive predictor is modelled as an intercept only? A NULL means none. Numerically, the $\rho$ parameter is easiest modelled as an intercept only, hence the default.
exchangeable	Logical. If TRUE, the two marginal probabilities are constrained to be equal.
imethod, nsimEIM, grho	See <a href="#">CommonVGAMffArguments</a> for more information. A value of at least 100 for nsimEIM is recommended; the larger the value the better.
rho	Numeric vector. Values are recycled to the needed length, and ought to be in range.

**Details**

The *bivariate probit model* was one of the earliest regression models to handle two binary responses jointly. It has a probit link for each of the two marginal probabilities, and models the association between the responses by the  $\rho$  parameter of a standard bivariate normal distribution (with zero means and unit variances). One can think of the joint probabilities being  $\Phi(\eta_1, \eta_2; \rho)$  where  $\Phi$  is the cumulative distribution function of a standard bivariate normal distribution.

Explicitly, the default model is

$$\text{probit}[P(Y_j = 1)] = \eta_j, \quad j = 1, 2$$

for the marginals, and

$$\text{rhobit}[\text{rho}] = \eta_3.$$

The joint probability  $P(Y_1 = 1, Y_2 = 1) = \Phi(\eta_1, \eta_2; \rho)$ , and from these the other three joint probabilities are easily computed. The model is fitted by maximum likelihood estimation since the full likelihood is specified. Fisher scoring is implemented.

The default models  $\eta_3$  as a single parameter only, i.e., an intercept-only model for rho, but this can be circumvented by setting `zero = NULL` in order to model rho as a function of all the explanatory variables.

The bivariate probit model should not be confused with a *bivariate logit model* with a probit link (see [binom2.or](#)). The latter uses the odds ratio to quantify the association. Actually, the bivariate

logit model is recommended over the bivariate probit model because the odds ratio is a more natural way of measuring the association between two binary responses.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

When fitted, the `fitted.values` slot of the object contains the four joint probabilities, labelled as  $(Y_1, Y_2) = (0,0), (0,1), (1,0), (1,1)$ , respectively.

## Note

See [binom2.or](#) about the form of input the response should have.

By default, a constant  $\rho$  is fitted because `zero = 3`. Set `zero = NULL` if you want the  $\rho$  parameter to be modelled as a function of the explanatory variables. The value  $\rho$  lies in the interval  $(-1, 1)$ , therefore a [rhobit](#) link is default.

Converge problems can occur. If so, assign `irho` a range of values and monitor convergence (e.g., set `trace = TRUE`). Else try `imethod`. Practical experience shows that local solutions can occur, and that `irho` needs to be quite close to the (global) solution. Also, `imu1` and `imu2` may be used.

This help file is mainly about `binom2.rho()`. `binom2.Rho()` fits a bivariate probit model with *known*  $\rho$ . The inputted  $\rho$  is saved in the `misc` slot of the fitted object, with `rho` as the component name.

In some econometrics applications (e.g., Freedman 2010, Freedman and Sekhon 2010) one response is used as an explanatory variable, e.g., a *recursive* binomial probit model. Such will not work here. Historically, the bivariate probit model was the first VGAM I ever wrote, based on Ashford and Sowden (1970). I don't think they ever thought of it either! Hence the criticisms raised go beyond the use of what was originally intended.

## Author(s)

Thomas W. Yee

## References

- Ashford, J. R. and Sowden, R. R. (1970) Multi-variate probit analysis. *Biometrics*, **26**, 535–546.
- Freedman, D. A. (2010) *Statistical Models and Causal Inference: a Dialogue with the Social Sciences*, Cambridge: Cambridge University Press.
- Freedman, D. A. and Sekhon, J. S. (2010) Endogeneity in probit response models. *Political Analysis*, **18**, 138–150.

## See Also

[rbinom2.rho](#), [rhobit](#), [pnorm2](#), [binom2.or](#), [loglinb2](#), [coalminers](#), [binomialff](#), [rhobit](#), [fisherz](#).



## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW, nBW, BnW, BW) ~ Age,
            binom2.rho, data = coalminers, trace = TRUE)
summary(fit)
coef(fit, matrix = TRUE)
```

---

binomialff

*Binomial Family Function*


---

## Description

Family function for fitting generalized linear models to binomial responses, where the dispersion parameter may be known or unknown.

## Usage

```
binomialff(link = "logit", dispersion = 1, mv = FALSE,
            onedpar = !mv, parallel = FALSE,
            zero = NULL, bred = FALSE, earg.link = FALSE)
```

## Arguments

link	Link function; see <a href="#">Links</a> and <a href="#">CommonVGAMffArguments</a> for more information.
dispersion	Dispersion parameter. By default, maximum likelihood is used to estimate the model because it is known. However, the user can specify <code>dispersion = 0</code> to have it estimated, or else specify a known positive value (or values if <code>mv</code> is TRUE).
mv	Multivariate response? If TRUE, then the response is interpreted as $M$ independent binary responses, where $M$ is the number of columns of the response matrix. In this case, the response matrix should have zero/one values only. If FALSE and the response is a (2-column) matrix, then the number of successes is given in the first column, and the second column is the number of failures.
onedpar	One dispersion parameter? If <code>mv</code> , then a separate dispersion parameter will be computed for each response (column), by default. Setting <code>onedpar = TRUE</code> will pool them so that there is only one dispersion parameter to be estimated.
parallel	A logical or formula. Used only if <code>mv</code> is TRUE. This argument allows for the parallelism assumption whereby the regression coefficients for a variable is constrained to be equal over the $M$ linear/additive predictors. If <code>parallel = TRUE</code> then the constraint is not applied to the intercepts.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ , where $M$ is the number of columns of the matrix response. See <a href="#">CommonVGAMffArguments</a> for more information.

earg.link	Details at <a href="#">CommonVGAMffArguments</a> .
bred	Details at <a href="#">CommonVGAMffArguments</a> . Setting <code>bred = TRUE</code> should work for multiple responses ( <code>mv = TRUE</code> ) and all <b>VGAM</b> link functions; it has been tested for <a href="#">logit</a> only (and it gives similar results to <b>brglm</b> but not identical), and further testing is required. One result from fitting bias reduced binary regression is that finite regression coefficients occur when the data is separable (see example below).

## Details

This function is largely to mimic [binomial](#), however there are some differences.

If the dispersion parameter is unknown, then the resulting estimate is not fully a maximum likelihood estimate (see pp.124–8 of McCullagh and Nelder, 1989).

A dispersion parameter that is less/greater than unity corresponds to under-/over-dispersion relative to the binomial model. Over-dispersion is more common in practice.

Setting `mv = TRUE` is necessary when fitting a Quadratic RR-VGLM (see [cqo](#)) because the response is a matrix of  $M$  columns (e.g., one column per species). Then there will be  $M$  dispersion parameters (one per column of the response matrix).

When used with [cqo](#) and [cao](#), it may be preferable to use the [cloglog](#) link.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), and [cao](#).

## Warning

With a multivariate response, assigning a known dispersion parameter for *each* response is not handled well yet. Currently, only a single known dispersion parameter is handled well.

See the above note regarding `bred`.

The maximum likelihood estimate will not exist if the data is *completely separable* or *quasi-completely separable*. See Chapter 10 of Altman et al. (2004) for more details, and **safeBinaryRegression**. Yet to do: add a `sepccheck = TRUE`, say, argument to detect this problem and give an appropriate warning.

## Note

If `mv` is `FALSE` (default) then the response can be of one of two formats: a factor (first level taken as failure), or a 2-column matrix (first column = successes) of counts. The argument `weights` in the modelling function can also be specified as any vector of positive values. In general, 1 means success and 0 means failure (to check, see the `y` slot of the fitted object). Note that a general vector of proportions of success is no longer accepted.

The notation  $M$  is used to denote the number of linear/additive predictors.

If `mv` is `TRUE`, then the matrix response can only be of one format: a matrix of 1's and 0's (1 = success).

The call `binomialff(dispersion = 0, ...)` is equivalent to `quasibinomialff(...)`. The latter was written so that R users of `quasibinomial()` would only need to add a “ff” to the end of the family function name.

Regardless of whether the dispersion parameter is to be estimated or not, its value can be seen from the output from the `summary()` of the object.

Fisher scoring is used. This can sometimes fail to converge by oscillating between successive iterations (Ridout, 1990). See the example below.

### Author(s)

Thomas W. Yee

### References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

Altman, M. and Gill, J. and McDonald, M. P. (2004) *Numerical Issues in Statistical Computing for the Social Scientist*, Hoboken, NJ, USA: Wiley-Interscience.

Ridout, M. S. (1990) Non-convergence of Fisher’s method of scoring—a simple example. *GLIM Newsletter*, 20(6).

### See Also

[quasibinomialff](#), [Links](#), [rrvglm](#), [cqo](#), [cao](#), [betabinomial](#), [posbinomial](#), [zibinomial](#), [double.expbinoimial](#), [matched.binomial](#), [seq2binomial](#), [amlbinomial](#), [simplex](#), [binomial](#), [safeBinaryRegression](#).

### Examples

```
quasibinomialff()
quasibinomialff(link = "probit")

shunua <- hunua[sort.list(with(hunua, altitude)), ] # Sort by altitude
fit <- vglm(agaas ~ poly(altitude, 2), binomialff(link = cloglog), shunua)
## Not run:
plot(agaas ~ jitter(altitude), shunua, col = "blue", ylab = "P(Agaas = 1)",
     main = "Presence/absence of Agathis australis", las = 1)
with(shunua, lines(altitude, fitted(fit), col = "orange", lwd = 2))
## End(Not run)

# Fit two species simultaneously
fit2 <- vgam(cbind(agaas, kniexc) ~ s(altitude), binomialff(mv = TRUE), shunua)
with(shunua, matplot(altitude, fitted(fit2), type = "l",
                    main = "Two species response curves", las = 1))

# Shows that Fisher scoring can sometime fail. See Ridout (1990).
ridout <- data.frame(v = c(1000, 100, 10), r = c(4, 3, 3), n = c(5, 5, 5))
(ridout <- transform(ridout, logv = log(v)))
# The iterations oscillates between two local solutions:
```

```

glm.fail <- glm(r / n ~ offset(logv) + 1, weight = n,
               binomial(link = 'cloglog'), ridout, trace = TRUE)
coef(glm.fail)
# vglm()'s half-stepping ensures the MLE of -5.4007 is obtained:
vglm.ok <- vglm(cbind(r, n-r) ~ offset(logv) + 1,
               binomialff(link = cloglog), ridout, trace = TRUE)
coef(vglm.ok)

# Separable data
set.seed(123)
threshold <- 0
bdata <- data.frame(x2 = sort(rnorm(nn <- 100)))
bdata <- transform(bdata, y1 = ifelse(x2 < threshold, 0, 1))
fit <- vglm(y1 ~ x2, binomialff(bred = TRUE),
            data = bdata, criter = "coef", trace = TRUE)
coef(fit, matrix = TRUE) # Finite!!
summary(fit)
## Not run: plot(depvar(fit) ~ x2, data = bdata, col = "blue", las = 1)
lines(fitted(fit) ~ x2, data = bdata, col = "orange")
abline(v = threshold, col = "gray", lty = "dashed")
## End(Not run)

```

Binorm

*Bivariate normal distribution cumulative distribution function*

## Description

Density, cumulative distribution function and random generation for the bivariate normal distribution.

## Usage

```

dbinorm(x1, x2, mean1 = 0, mean2 = 0, var1 = 1, var2 = 1, cov12 = 0, log = FALSE)
pbinorm(x1, x2, mean1 = 0, mean2 = 0, var1 = 1, var2 = 1, cov12 = 0)
rbinorm(n, mean1 = 0, mean2 = 0, var1 = 1, var2 = 1, cov12 = 0)
pnorm2(x1, x2, mean1 = 0, mean2 = 0, var1 = 1, var2 = 1, cov12 = 0)

```

## Arguments

**x1, x2** vector of quantiles.

**mean1, mean2, var1, var2, cov12** vector of means, variances and the covariance.

**n** number of observations. Same as [rnorm](#).

**log** Logical. If `log = TRUE` then the logarithm of the density is returned.

## Details

The default arguments correspond to the standard bivariate normal distribution with correlation parameter  $\rho = 0$ . That is, two independent standard normal distributions. Let `sd1` (say) be `sqrt(var1)` and written  $\sigma_1$ , etc. Then the general formula for the correlation coefficient is  $\rho = cov/(\sigma_1\sigma_2)$  where `cov` is argument `cov12`. Thus if arguments `var1` and `var2` are left alone then `cov12` can be inputted with  $\rho$ .

One can think of this function as an extension of [pnorm](#) to two dimensions, however note that the argument names have been changed for **VGAM** 0.9-1 onwards.

## Value

`dbinorm` gives the density, `pbinorm` gives the cumulative distribution function, `rbinorm` generates random deviates ( $n$  by 2 matrix).

## Warning

Being based on an approximation, the results of `pbinorm()` may be negative! Also, `pnorm2()` should be withdrawn soon; use `pbinorm()` instead because it is identical.

## Note

For `rbinorm()`, if the  $i$ th variance-covariance matrix is not positive-definite then the  $i$ th row is all NAs.

## References

`pbinorm()` is based on Donnelly (1973), the code was translated from FORTRAN to ratfor using `struct`, and then from ratfor to C manually. The function was originally called `bivnor`, and TWY only wrote a wrapper function.

Donnelly, T. G. (1973) Algorithm 462: Bivariate Normal Distribution. *Communications of the ACM*, **16**, 638.

## See Also

[pnorm](#), [binormal](#), [uninormal](#).

## Examples

```
yvec <- c(-5, -1.96, 0, 1.96, 5)
ymat <- expand.grid(yvec, yvec)
cbind(ymat, pbinorm(ymat[, 1], ymat[, 2]))

## Not run: rhovec <- seq(-0.95, 0.95, by = 0.01)
plot(rhovec, pbinorm(0, 0, cov12 = rhovec), type = "l", col = "blue", las = 1)
abline(v = 0, h = 0.25, col = "gray", lty = "dashed")
## End(Not run)
```

binormal

*Bivariate normal distribution family function***Description**

Maximum likelihood estimation of the five parameters of a bivariate normal distribution.

**Usage**

```
binormal(lmean1 = "identity", lmean2 = "identity",
        lsd1 = "log", lsd2 = "log",
        lrho = "rhobit",
        imean1 = NULL, imean2 = NULL,
        isd1 = NULL, isd2 = NULL,
        irho = NULL, imethod = 1,
        eq.mean = FALSE, eq.sd = FALSE,
        zero = 3:5)
```

**Arguments**

`lmean1`, `lmean2`, `lsd1`, `lsd2`, `lrho`

Link functions applied to the means, standard deviations and rho parameters. See [Links](#) for more choices. Being positive quantities, a log link is the default for the standard deviations.

`imean1`, `imean2`, `isd1`, `isd2`, `irho`, `imethod`, `zero`

See [CommonVGAMffArguments](#) for more information.

`eq.mean`, `eq.sd` Logical or formula. Constrains the means or the standard deviations to be equal. Only one of these arguments may be assigned a value.

**Details**

For the bivariate normal distribution, this fits a linear model (LM) to the means, and by default, the other parameters are intercept-only. The response should be a two-column matrix. The correlation parameter is rho, which lies between  $-1$  and  $1$  (thus the [rhobit](#) link is a reasonable choice). The fitted means are returned as the fitted values, which is in the form of a two-column matrix. Fisher scoring is implemented.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

This function may be renamed to `normal2()` or something like that at a later date.

**Note**

If both equal means and equal standard deviations are desired then use something like `constraints = list("(Intercept)"` and maybe `zero = NULL` etc.

**Author(s)**

T. W. Yee

**See Also**

[uninormal](#), [gaussianff](#), [pnorm2](#), [bistudentt](#).

**Examples**

```
set.seed(123); nn <- 1000
bdata <- data.frame(x2 = runif(nn), x3 = runif(nn))
bdata <- transform(bdata, y1 = rnorm(nn, 1 + 2 * x2),
                    y2 = rnorm(nn, 3 + 4 * x2))
fit1 <- vglm(cbind(y1, y2) ~ x2,
             binormal(eq.sd = TRUE), data = bdata, trace = TRUE)
coef(fit1, matrix = TRUE)
constraints(fit1)
summary(fit1)

# Estimated P(Y1 <= y1, Y2 <= y2) under the fitted model
var1 <- loge(2 * predict(fit1)[, "log(sd1)"], inverse = TRUE)
var2 <- loge(2 * predict(fit1)[, "log(sd2)"], inverse = TRUE)
cov12 <- rhobit(predict(fit1)[, "rhobit(rho)"], inverse = TRUE)
head(with(bdata, pnorm2(y1, y2,
                        mean1 = predict(fit1)[, "mean1"],
                        mean2 = predict(fit1)[, "mean2"],
                        var1 = var1, var2 = var2, cov12 = cov12)))
```

---

binormalcop

*Gaussian Copula (Bivariate) Family Function*


---

**Description**

Estimate the correlation parameter of the (bivariate) Gaussian copula distribution by maximum likelihood estimation.

**Usage**

```
binormalcop(lrho = "rhobit", irho = NULL, imethod = 1,
            parallel = FALSE, zero = NULL)
```

## Arguments

lrho, irho, imethod

Details at [CommonVGAMffArguments](#). See [Links](#) for more link function choices.

parallel, zero Details at [CommonVGAMffArguments](#). If `parallel = TRUE` then the constraint is applied to the intercept too.

## Details

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = \Phi_2(\Phi^{-1}(y_1), \Phi^{-1}(y_2); \rho)$$

for  $-1 < \rho < 1$ ,  $\Phi_2$  is the cumulative distribution function of a standard bivariate normal (see [pbinorm](#)), and  $\Phi$  is the cumulative distribution function of a standard univariate normal (see [pnorm](#)).

The support of the function is the interior of the unit square; however, values of 0 and/or 1 are not allowed. The marginal distributions are the standard uniform distributions. When  $\rho = 0$  the random variables are independent.

This **VGAM** family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.5. This is because each marginal distribution corresponds to a standard uniform distribution.

This **VGAM** family function is fragile; each response must be in the interior of the unit square. Setting `crit = "coef"` is sometimes a good idea because inaccuracies in [pbinorm](#) might mean unnecessary half-stepping will occur near the solution.

## Author(s)

T. W. Yee

## References

Schepsmeier, U. and Stober, J. (2013) Derivatives and Fisher information of bivariate copulas. *Statistical Papers*.

## See Also

[rbinormcop](#), [pnorm](#), [kendall.tau](#).



**Examples**

```

nn <- 1000
ymat <- rbinormcop(n = nn, rho = rhobit(-0.9, inverse = TRUE))
bdata <- data.frame(y1 = ymat[, 1],
                   y2 = ymat[, 2],
                   y3 = ymat[, 1],
                   y4 = ymat[, 2],
                   x2 = runif(nn))

summary(bdata)
## Not run: plot(ymat, col = "blue")
fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the first
             fam = binormalcop,
             crit = "coef", # Sometimes a good idea
             data = bdata, trace = TRUE)

coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
summary(fit1)

# Another example; rho is a linear function of x2
bdata <- transform(bdata, rho = -0.5 + x2)
ymat <- rbinormcop(n = nn, rho = with(bdata, rho))
bdata <- transform(bdata, y5 = ymat[, 1],
                  y6 = ymat[, 2])
fit2 <- vgam(cbind(y5, y6) ~ s(x2), data = bdata,
             binormalcop(lrho = "identity"), trace = TRUE)
## Not run: plot(fit2, lcol = "blue", scol = "orange", se = TRUE, las = 1)

```

Binormcop

*Gaussian Copula (Bivariate) Distribution***Description**

Density, distribution function, and random generation for the (one parameter) bivariate Gaussian copula distribution.

**Usage**

```

dbinormcop(x1, x2, rho = 0, log = FALSE)
pbinormcop(q1, q2, rho = 0)
rbinormcop(n, rho = 0)

```

**Arguments**

**x1, x2, q1, q2** vector of quantiles. The x1 and x2 should be in the interval (0, 1). Ditto for q1 and q2.

**n** number of observations. Same as [rnorm](#).

rho                    the correlation parameter. Should be in the interval  $(-1, 1)$ .  
 log                    Logical. If TRUE then the logarithm is returned.

### Details

See [binormalcop](#), the **VGAM** family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

### Value

dbinormcop gives the density, pbinormcop gives the distribution function, and rbinormcop generates random deviates (a two-column matrix).

### Note

Yettodo: allow x1 and/or x2 to have values 1, and to allow any values for x1 and/or x2 to be outside the unit square.

### Author(s)

T. W. Yee

### See Also

[binormalcop](#), [binormal](#).

### Examples

```
## Not run:  edge <- 0.01 # A small positive value
N <- 101; x <- seq(edge, 1.0 - edge, len = N); Rho <- 0.7
ox <- expand.grid(x, x)
zedd <- dbinormcop(ox[, 1], ox[, 2], rho = Rho, log = TRUE)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)
zedd <- pbinormcop(ox[, 1], ox[, 2], rho = Rho)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)

## End(Not run)
```

### Description

biplot is a generic function applied to RR-VGLMs and QRR-VGLMs etc. These apply to rank-1 and rank-2 models of these only. For RR-VGLMs these plot the second latent variable scores against the first latent variable scores.

**Methods**

**x** The object from which the latent variables are extracted and/or plotted.

**Note**

See [lvplot](#) which is very much related to biplots.

---

 Bisa

---

*The Birnbaum-Saunders Distribution*


---

**Description**

Density, distribution function, and random generation for the Birnbaum-Saunders distribution.

**Usage**

```
dbisa(x, shape, scale = 1, log = FALSE)
pbisa(q, shape, scale = 1)
qbisa(p, shape, scale = 1)
rbisa(n, shape, scale = 1)
```

**Arguments**

<b>x, q</b>	vector of quantiles.
<b>p</b>	vector of probabilities.
<b>n</b>	Same as in <a href="#">runif</a> .
<b>shape, scale</b>	the (positive) shape and scale parameters.
<b>log</b>	Logical. If TRUE then the logarithm of the density is returned.

**Details**

The Birnbaum-Saunders distribution is a distribution which is used in survival analysis. See [bisa](#), the **VGAM** family function for estimating the parameters, for more details.

**Value**

`dbisa` gives the density, `pbisa` gives the distribution function, and `qbisa` gives the quantile function, and `rbisa` generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[bisa](#).

## Examples

```
## Not run:
x <- seq(0, 6, len = 400)
plot(x, dbisa(x, shape = 1), type = "l", col = "blue",
      ylab = "Density", lwd = 2, ylim = c(0,1.3), lty = 3,
      main = "X ~ Birnbaum-Saunders(shape, scale = 1)")
lines(x, dbisa(x, shape = 2), col = "orange", lty = 2, lwd = 2)
lines(x, dbisa(x, shape = 0.5), col = "green", lty = 1, lwd = 2)
legend(x = 3, y = 0.9, legend = paste("shape = ", c(0.5, 1, 2)),
      col = c("green", "blue", "orange"), lty = 1:3, lwd = 2)

shape <- 1; x <- seq(0.0, 4, len = 401)
plot(x, dbisa(x, shape = shape), type = "l", col = "blue", las = 1, ylab = "",
      main = "Blue is density, orange is cumulative distribution function",
      sub = "Purple lines are the 10,20,...,90 percentiles", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pbisa(x, shape = shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qbisa(probs, shape = shape)
lines(Q, dbisa(Q, shape = shape), col = "purple", lty = 3, type = "h")
pbisa(Q, shape = shape) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
lines(Q, pbisa(Q, shape), col = "purple", lty = 3, type = "h")

## End(Not run)
```

---

bisa

*Birnbaum-Saunders Distribution Family Function*


---

## Description

Estimates the shape and scale parameters of the Birnbaum-Saunders distribution by maximum likelihood estimation.

## Usage

```
bisa(lshape = "loge", lscale = "loge",
     ishape = NULL, iscale = 1, imethod = 1, zero = NULL)
```

## Arguments

- |                |  |
|----------------|--|
| lscale, lshape | Parameter link functions applied to the shape and scale parameters ( $a$ and $b$ below). See <a href="#">Links</a> for more choices. A log link is the default for both because they are positive. |
| iscale, ishape | Initial values for $a$ and $b$ . A NULL means an initial value is chosen internally using imethod.   |
| imethod        | An integer with value 1 or 2 or 3 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ishape and/or iscale.                  |

**zero** An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose one value from the set  $\{1,2\}$ .

## Details

The (two-parameter) Birnbaum-Saunders distribution has a cumulative distribution function that can be written as

$$F(y; a, b) = \Phi[\xi(y/b)/a]$$

where  $\Phi(\cdot)$  is the cumulative distribution function of a standard normal (see [pnorm](#)),  $\xi(t) = \sqrt{t} - 1/\sqrt{t}$ ,  $y > 0$ ,  $a > 0$  is the shape parameter,  $b > 0$  is the scale parameter. The mean of  $Y$  (which is the fitted value) is  $b(1 + a^2/2)$ . and the variance is  $a^2b^2(1 + \frac{5}{4}a^2)$ . By default,  $\eta_1 = \log(a)$  and  $\eta_2 = \log(b)$  for this family function.

Note that  $a$  and  $b$  are orthogonal, i.e., the Fisher information matrix is diagonal. This family function implements Fisher scoring, and it is unnecessary to compute any integrals numerically.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Author(s)

T. W. Yee

## References

- Lemonte, A. J. and Cribari-Neto, F. and Vasconcellos, K. L. P. (2007) Improved statistical inference for the two-parameter Birnbaum-Saunders distribution. *Computational Statistics & Data Analysis*, **51**, 4656–4681.
- Birnbaum, Z. W. and Saunders, S. C. (1969) A new family of life distributions. *Journal of Applied Probability*, **6**, 319–327.
- Birnbaum, Z. W. and Saunders, S. C. (1969) Estimation for a family of life distributions with applications to fatigue. *Journal of Applied Probability*, **6**, 328–347.
- Engelhardt, M. and Bain, L. J. and Wright, F. T. (1981) Inferences on the parameters of the Birnbaum-Saunders fatigue life distribution based on maximum likelihood estimation. *Technometrics*, **23**, 251–256.
- Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) *Continuous Univariate Distributions*, 2nd edition, Volume 2, New York: Wiley.

## See Also

[pbisa](#), [inv.gaussianff](#).

**Examples**

```

bdata1 <- data.frame(x2 = runif(nn <- 1000))
bdata1 <- transform(bdata1, shape = exp(-0.5 + x2), scale = exp(1.5))
bdata1 <- transform(bdata1, y = rbisa(nn, shape, scale))
fit1 <- vglm(y ~ x2, bisa(zero = 2), bdata1, trace = TRUE)
coef(fit1, matrix = TRUE)

## Not run:
bdata2 <- data.frame(shape = exp(-0.5), scale = exp(0.5))
bdata2 <- transform(bdata2, y = rbisa(nn, shape, scale))
fit <- vglm(y ~ 1, bisa, bdata2, trace = TRUE)
with(bdata2, hist(y, prob = TRUE, ylim = c(0, 0.5), col = "lightblue"))
coef(fit, matrix = TRUE)
with(bdata2, mean(y))
head(fitted(fit))
x <- with(bdata2, seq(0, max(y), len = 200))
lines(dbisa(x, Coef(fit)[1], Coef(fit)[2]) ~ x, bdata2, col = "orange", lwd = 2)
## End(Not run)

```

Bistudentt

*Bivariate Student-t distribution cumulative distribution function***Description**

Density  
for the bivariate Student-t distribution distribution.

**Usage**

```
dbistudentt(x1, x2, df, rho = 0, log = FALSE)
```

**Arguments**

x1, x2	vector of quantiles.
df, rho	vector of degrees of freedom and correlation parameter. For df, a value Inf is currently not working.
log	Logical. If log = TRUE then the logarithm of the density is returned.

**Details**

One can think of this function as an extension of [dt](#) to two dimensions. See [bistudentt](#) for more information.

**Value**

dbistudentt gives the density.

## References

Schepsmeier, U. and Stober, J. (2013) Derivatives and Fisher information of bivariate copulas. *Statistical Papers*.

## See Also

[bistudentt](#), [dt](#).

## Examples

```
## Not run: N <- 101; x <- seq(-4, 4, len = N); Rho <- 0.7; mydf <- 10
ox <- expand.grid(x, x)
zedd <- dbistudentt(ox[, 1], ox[, 2], df = mydf, rho = Rho, log = TRUE)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)

## End(Not run)
```

---

bistudentt

*Bivariate Student-t Family Function*


---

## Description

Estimate the degrees of freedom and correlation parameters of the (bivariate) Student-t distribution by maximum likelihood estimation.

## Usage

```
bistudentt(ldf = "loglog", lrho = "rhobit",
            idf = NULL, irho = NULL, imethod = 1,
            parallel = FALSE, zero = -1)
```

## Arguments

ldf, lrho, idf, irho, imethod

Details at [CommonVGAMffArguments](#). See [Links](#) for more link function choices.

parallel, zero Details at [CommonVGAMffArguments](#).

## Details

The density function is

$$f(y_1, y_2; \nu, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} (1 + y_1^2 + y_2^2 - 2\rho y_1 y_2) / (\nu(1-\rho^2))^{(\nu+2)/2}$$

for  $-1 < \rho < 1$ , and real  $y_1$  and  $y_2$ .

This **VGAM** family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

The working weight matrices have not been fully checked.

**Note**

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.0.

**Author(s)**

T. W. Yee, with help from Thibault Vatter.

**References**

Schepsmeier, U. and Stober, J. (2013) Derivatives and Fisher information of bivariate copulas. *Statistical Papers*.

**See Also**

[dbistudentt](#), [binormal](#), [pt](#).

**Examples**

```
nn <- 1000
mydof <- loglog(1, inverse = TRUE)
ymat <- cbind(rt(nn, df = mydof), rt(nn, df = mydof))

bdata <- data.frame(y1 = ymat[, 1],
                   y2 = ymat[, 2],
                   y3 = ymat[, 1],
                   y4 = ymat[, 2],
                   x2 = runif(nn))

summary(bdata)
## Not run: plot(ymat, col = "blue")
fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the first
            fam = bistudentt,
            #      crit = "coef", # Sometimes a good idea
            data = bdata, trace = TRUE)

coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
summary(fit1)
```



---

bmi.nz*Body Mass Index of New Zealand Adults Data*

---

**Description**

The body mass indexes and ages from an approximate random sample of 700 New Zealand adults.

**Usage**

```
data(bmi.nz)
```

**Format**

A data frame with 700 observations on the following 2 variables.

**age** a numeric vector; their age (years).

**BMI** a numeric vector; their body mass indexes, which is their weight divided by the square of their height ( $\text{kg} / \text{m}^2$ ).

**Details**

They are a random sample from the Fletcher Challenge/Auckland Heart and Health survey conducted in the early 1990s.

There are some outliers in the data set.

A variable gender would be useful, and may be added later.

**Source**

Clinical Trials Research Unit, University of Auckland, New Zealand, <http://www.ctr.u.auckland.ac.nz>.

**References**

MacMahon, S., Norton, R., Jackson, R., Mackie, M. J., Cheng, A., Vander Hoorn, S., Milne, A., McCulloch, A. (1995) Fletcher Challenge-University of Auckland Heart & Health Study: design and baseline findings. *New Zealand Medical Journal*, **108**, 499–502.

**Examples**

```
## Not run: with(bmi.nz, plot(age, BMI, col = "blue"))
fit <- vgam(BMI ~ s(age, df = c(2, 4, 2)), lms.yjn, data = bmi.nz, trace = TRUE)
qtplot(fit, pcol = "blue", tcol = "brown", lcol = "brown")
## End(Not run)
```

borel.tanner

*Borel-Tanner Distribution Family Function***Description**

Estimates the parameter of a Borel-Tanner distribution by maximum likelihood estimation.

**Usage**

```
borel.tanner(Qsize = 1, link = "logit", imethod = 1)
```

**Arguments**

Qsize	A positive integer. It is called $Q$ below and is the initial queue size.
link	Link function for the parameter; see <a href="#">Links</a> for more choices and for general information.
imethod	See <a href="#">CommonVGAMffArguments</a> . Valid values are 1, 2, 3 or 4.

**Details**

The Borel-Tanner distribution (Tanner, 1953) describes the distribution of the total number of customers served before a queue vanishes given a single queue with random arrival times of customers (at a constant rate  $r$  per unit time, and each customer taking a constant time  $b$  to be served). Initially the queue has  $Q$  people and the first one starts to be served. The two parameters appear in the density only in the form of the product  $rb$ , therefore we use  $a = rb$ , say, to denote the single parameter to be estimated. The density function is

$$f(y; a) = \frac{Q!}{(y - Q)!} y^{y-Q-1} a^{y-Q} \exp(-ay)$$

where  $y = Q, Q + 1, Q + 2, \dots$ . The case  $Q = 1$  corresponds to the *Borel* distribution (Borel, 1942). For the  $Q = 1$  case it is necessary for  $0 < a < 1$  for the distribution to be proper. The Borel distribution is a basic Lagrangian distribution of the first kind. The Borel-Tanner distribution is an  $Q$ -fold convolution of the Borel distribution.

The mean is  $Q/(1 - a)$  (returned as the fitted values) and the variance is  $Qa/(1 - a)^3$ . The distribution has a very long tail unless  $a$  is small. Fisher scoring is implemented.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

T. W. Yee

## References

- Tanner, J. C. (1953) A problem of interference between two queues. *Biometrika*, **40**, 58–69.
- Borel, E. (1942) Sur l'emploi du theoreme de Bernoulli pour faciliter le calcul d'une infinite de coefficients. Application au probleme de l'attente a un guichet. *Comptes Rendus, Academie des Sciences, Paris, Series A*, **214**, 452–456.
- Page 328 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey: Wiley.
- Consul, P. C. and Famoye, F. (2006) *Lagrangian Probability Distributions*, Boston: Birkhauser.

## See Also

[rbort](#), [poissonff](#), [felix](#).

## Examples

```
bdata <- data.frame(y = rbort(n <- 200))
fit <- vglm(y ~ 1, borel.tanner, bdata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

Bort

*The Borel-Tanner Distribution*

---

## Description

Density  
and random generation for the Borel-Tanner distribution.

## Usage

```
dbort(x, Qsize = 1, a = 0.5, log = FALSE)
```

```
rbort(n, Qsize = 1, a = 0.5)
```

## Arguments

x	vector of quantiles.
n	number of observations. Must be a positive integer of length 1.
Qsize, a	See <a href="#">borel.tanner</a> .
log	Logical. If log=TRUE then the logarithm of the density is returned.

**Details**

See [borel.tanner](#), the **VGAM** family function for estimating the parameter, for the formula of the probability density function and other details.

**Value**

`dbort` gives the density,  
`rbort` generates random deviates.

**Warning**

Looping is used for [rbort](#), therefore values of `a` close to 1 will result in long (or infinite!) computational times. The default value of `a` is subjective.

**Author(s)**

T. W. Yee

**See Also**

[borel.tanner](#).

**Examples**

```
## Not run:
qsize = 1; a = 0.5
x = qsize:(qsize+10)
plot(x, dbort(x, qsize, a), type="h", las=1, col="blue",
      ylab=paste("fbort(qsize=", qsize, ", a=", a, ")"),
      main="Borel-Tanner density function")

## End(Not run)
```

---

 Brat

*Inputting Data to fit a Bradley Terry Model*


---

**Description**

Takes in a square matrix of counts and outputs them in a form that is accessible to the [brat](#) and [bratt](#) family functions.

**Usage**

```
Brat(mat, ties = 0 * mat, string = c(">", "=="), whitespace = FALSE)
```

**Arguments**

mat	Matrix of counts, which is considered $M$ by $M$ in dimension when there are ties, and $M + 1$ by $M + 1$ when there are no ties. The rows are winners and the columns are losers, e.g., the 2-1 element is now many times Competitor 2 has beaten Competitor 1. The matrices are best labelled with the competitors' names.
ties	Matrix of counts. This should be the same dimension as mat. By default, there are no ties. The matrix must be symmetric, and the diagonal should contain NAs.
string	Character. The matrices are labelled with the first value of the descriptor, e.g., "NZ > Oz" 'means' NZ beats Australia in rugby. Suggested alternatives include " beats " or " wins against ". The second value is used to handle ties.
whitespace	Logical. If TRUE then a white space is added before and after string; it generally enhances readability. See <a href="#">CommonVGAMffArguments</a> for some similar-type information.

**Details**

In the **VGAM** package it is necessary for each matrix to be represented as a single row of data by [brat](#) and [bratt](#). Hence the non-diagonal elements of the  $M + 1$  by  $M + 1$  matrix are concatenated into  $M(M + 1)$  values (no ties), while if there are ties, the non-diagonal elements of the  $M$  by  $M$  matrix are concatenated into  $M(M - 1)$  values.

**Value**

A matrix with 1 row and either  $M(M + 1)$  or  $M(M - 1)$  columns.

**Note**

This is a data preprocessing function for [brat](#) and [bratt](#).

Yet to do: merge InverseBrat into brat.

**Author(s)**

T. W. Yee

**References**

Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.

**See Also**

[brat](#), [bratt](#), [InverseBrat](#).

## Examples

```
journal <- c("Biometrika", "Comm Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284, 730, NA, 813, 276,
                498, 68, NA, 325, 221, 17, 142, NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)
Brat(mat) # Less readable
Brat(mat, whitespace = TRUE) # More readable
vglm(Brat(mat, whitespace = TRUE) ~ 1, brat, trace = TRUE)
```

brat

*Bradley Terry Model*

## Description

Fits a Bradley Terry model (intercept-only model) by maximum likelihood estimation.

## Usage

```
brat(refgp = "last", refvalue = 1, init.alpha = 1)
```

## Arguments

refgp	Integer whose value must be from the set $\{1, \dots, M + 1\}$ , where there are $M + 1$ competitors. The default value indicates the last competitor is used—but don't input a character string, in general.
refvalue	Numeric. A positive value for the reference group.
init.alpha	Initial values for the $\alpha$ s. These are recycled to the appropriate length.

## Details

The Bradley Terry model involves  $M + 1$  competitors who either win or lose against each other (no draws/ties allowed in this implementation—see [bratt](#) if there are ties). The probability that Competitor  $i$  beats Competitor  $j$  is  $\alpha_i / (\alpha_i + \alpha_j)$ , where all the  $\alpha$ s are positive. Loosely, the  $\alpha$ s can be thought of as the competitors' 'abilities'. For identifiability, one of the  $\alpha_i$  is set to a known value `refvalue`, e.g., 1. By default, this function chooses the last competitor to have this reference value. The data can be represented in the form of a  $M + 1$  by  $M + 1$  matrix of counts, where winners are the rows and losers are the columns. However, this is not the way the data should be inputted (see below).

Excluding the reference value/group, this function chooses  $\log(\alpha_j)$  as the  $M$  linear predictors. The log link ensures that the  $\alpha$ s are positive.

The Bradley Terry model can be fitted by logistic regression, but this approach is not taken here. The Bradley Terry model can be fitted with covariates, e.g., a home advantage variable, but unfortunately, this lies outside the VGLM theoretical framework and therefore cannot be handled with this code.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#).

**Warning**

Presently, the residuals are wrong, and the prior weights are not handled correctly. Ideally, the total number of counts should be the prior weights, after the response has been converted to proportions. This would make it similar to family functions such as [multinomial](#) and [binomialff](#).

**Note**

The function [Brat](#) is useful for coercing a  $M + 1$  by  $M + 1$  matrix of counts into a one-row matrix suitable for [brat](#). Diagonal elements are skipped, and the usual S order of `c(a.matrix)` of elements is used. There should be no missing values apart from the diagonal elements of the square matrix. The matrix should have winners as the rows, and losers as the columns. In general, the response should be a 1-row matrix with  $M(M + 1)$  columns.

Only an intercept model is recommended with [brat](#). It doesn't make sense really to include covariates because of the limited VGLM framework.

Notationally, note that the **VGAM** family function [brat](#) has  $M + 1$  contestants, while [bratt](#) has  $M$  contestants.

**Author(s)**

T. W. Yee

**References**

Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.

Stigler, S. (1994) Citation patterns in the journals of statistics and probability. *Statistical Science*, **9**, 94–108.

The **BradleyTerry2** package has more comprehensive capabilities than this function.

**See Also**

[bratt](#), [Brat](#), [multinomial](#), [binomialff](#).

**Examples**

```
# Citation statistics: being cited is a 'win'; citing is a 'loss'
journal <- c("Biometrika", "Comm.Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284,
                730, NA, 813, 276,
                498, 68,  NA, 325,
                221, 17, 142,  NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)
fit <- vglm(Brat(mat) ~ 1, brat(refgp = 1), trace = TRUE)
fit <- vglm(Brat(mat) ~ 1, brat(refgp = 1), trace = TRUE, crit = "coef")
summary(fit)
```

```

c(0, coef(fit)) # Log-abilities (in order of "journal")
c(1, Coef(fit)) # Abilities (in order of "journal")
fitted(fit)     # Probabilities of winning in awkward form
(check <- InverseBrat(fitted(fit))) # Probabilities of winning
check + t(check) # Should be 1's in the off-diagonals

```

bratt

*Bradley Terry Model With Ties*

## Description

Fits a Bradley Terry model with ties (intercept-only model) by maximum likelihood estimation.

## Usage

```
bratt(refgp = "last", refvalue = 1, init.alpha = 1, i0 = 0.01)
```

## Arguments

refgp	Integer whose value must be from the set $\{1, \dots, M\}$ , where there are $M$ competitors. The default value indicates the last competitor is used—but don't input a character string, in general.
refvalue	Numeric. A positive value for the reference group.
init.alpha	Initial values for the $\alpha$ s. These are recycled to the appropriate length.
i0	Initial value for $\alpha_0$ . If convergence fails, try another positive value.

## Details

There are several models that extend the ordinary Bradley Terry model to handle ties. This family function implements one of these models. It involves  $M$  competitors who either win or lose or tie against each other. (If there are no draws/ties then use [brat](#)). The probability that Competitor  $i$  beats Competitor  $j$  is  $\alpha_i/(\alpha_i + \alpha_j + \alpha_0)$ , where all the  $\alpha$ s are positive. The probability that Competitor  $i$  ties with Competitor  $j$  is  $\alpha_0/(\alpha_i + \alpha_j + \alpha_0)$ . Loosely, the  $\alpha$ s can be thought of as the competitors' 'abilities', and  $\alpha_0$  is an added parameter to model ties. For identifiability, one of the  $\alpha_i$  is set to a known value refvalue, e.g., 1. By default, this function chooses the last competitor to have this reference value. The data can be represented in the form of a  $M$  by  $M$  matrix of counts, where winners are the rows and losers are the columns. However, this is not the way the data should be inputted (see below).

Excluding the reference value/group, this function chooses  $\log(\alpha_j)$  as the first  $M - 1$  linear predictors. The log link ensures that the  $\alpha$ s are positive. The last linear predictor is  $\log(\alpha_0)$ .

The Bradley Terry model can be fitted with covariates, e.g., a home advantage variable, but unfortunately, this lies outside the VGLM theoretical framework and therefore cannot be handled with this code.



**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#).

**Note**

The function [Brat](#) is useful for coercing a  $M$  by  $M$  matrix of counts into a one-row matrix suitable for `bratt`. Diagonal elements are skipped, and the usual S order of `c(a.matrix)` of elements is used. There should be no missing values apart from the diagonal elements of the square matrix. The matrix should have winners as the rows, and losers as the columns. In general, the response should be a matrix with  $M(M - 1)$  columns.

Also, a symmetric matrix of ties should be passed into [Brat](#). The diagonal of this matrix should be all NAs.

Only an intercept model is recommended with `bratt`. It doesn't make sense really to include covariates because of the limited VGLM framework.

Notationally, note that the **VGAM** family function [brat](#) has  $M + 1$  contestants, while `bratt` has  $M$  contestants.

**Author(s)**

T. W. Yee

**References**

Torsney, B. (2004) Fitting Bradley Terry models using a multiplicative algorithm. In: Antoch, J. (ed.) *Proceedings in Computational Statistics COMPSTAT 2004*, Physica-Verlag: Heidelberg. Pages 513–526.

**See Also**

[brat](#), [Brat](#), [binomialff](#).

**Examples**

```
# citation statistics: being cited is a 'win'; citing is a 'loss'
journal <- c("Biometrika", "Comm.Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284,
                730, NA, 813, 276,
                498, 68,  NA, 325,
                221, 17, 142,  NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)

# Add some ties. This is fictional data.
ties <- 5 + 0 * mat
ties[2, 1] <- ties[1,2] <- 9

# Now fit the model
fit <- vglm(Brat(mat, ties) ~ 1, bratt(refgp = 1), trace = TRUE)
fit <- vglm(Brat(mat, ties) ~ 1, bratt(refgp = 1), trace = TRUE, crit = "coef")
```

```

summary(fit)
c(0, coef(fit)) # Log-abilities (in order of "journal"); last is log(alpha0)
c(1, Coef(fit)) # Abilities (in order of "journal"); last is alpha0

fit@misc$alpha # alpha_1,...,alpha_M
fit@misc$alpha0 # alpha_0

fitted(fit) # Probabilities of winning and tying, in awkward form
predict(fit)
(check <- InverseBrat(fitted(fit))) # Probabilities of winning
qprob <- attr(fitted(fit), "probtie") # Probabilities of a tie
qprobmatt <- InverseBrat(c(qprob), NCo = nrow(ties)) # Probabilities of a tie
check + t(check) + qprobmatt # Should be 1s in the off-diagonals

```

---

calibrate

---

*Model Calibrations*


---

## Description

calibrate is a generic function used to produce calibrations from various model fitting functions. The function invokes particular ‘methods’ which depend on the ‘class’ of the first argument.

## Usage

```
calibrate(object, ...)
```

## Arguments

object	An object for which a calibration is desired.
...	Additional arguments affecting the calibration produced. Usually the most important argument in ... is newdata which, for calibrate, contains new <i>response</i> data, <b>Y</b> , say.

## Details

Given a regression model with explanatory variables **X** and response **Y**, calibration involves estimating **X** from **Y** using the regression model. It can be loosely thought of as the opposite of [predict](#) (which takes an **X** and returns a **Y**.)

## Value

In general, given a new response **Y**, the explanatory variables **X** are returned. However, for constrained ordination models such as CQO and CAO models, it is usually not possible to return **X**, so the latent variables are returned instead (they are linear combinations of the **X**). See the specific calibrate methods functions to see what they return.

**Note**

This function was not called `predictx` because of the inability of constrained ordination models to return **X**; they can only return the latent variable values (site scores) instead.

**Author(s)**

T. W. Yee

**See Also**

[predict](#), [calibrate.qrrvglm](#).

**Examples**

```
## Not run:
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
set.seed(123)
p1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
  WaterCon + BareSand + FallTwig +
  CoveMoss + CoveHerb + ReflLux,
  family = poissonff, data = hspider, Rank = 1,
  df1.nl = c(Zoraspin = 2, 1.9),
  Bestof = 3, Crowlpositive = TRUE)

siteNos <- 1:2 # Calibrate these sites
cp1 <- calibrate(p1, new = data.frame(depvar(p1)[siteNos, ]), trace = TRUE)

# Graphically compare the actual site scores with their calibrated values
persp(p1, main = "Solid=actual, dashed=calibrated site scores",
  label = TRUE, col = "blue", las = 1)
# Actual site scores:
abline(v = latvar(p1)[siteNos], lty = 1, col = 1:length(siteNos))
abline(v = cp1, lty = 2, col = 1:length(siteNos)) # Calibrated values

## End(Not run)
```

**Description**

`calibrate` is a generic function applied to QRR-VGLMs and RR-VGAMs etc.

**Methods**

**object** The object from which the calibration is performed.

calibrate.qrrvglm

*Calibration for CQO and CAO models***Description**

Performs maximum likelihood calibration for constrained and unconstrained quadratic and additive ordination models (CQO and CAO models are better known as QRR-VGLMs and RR-VGAMs respectively).

**Usage**

```
calibrate.qrrvglm(object, newdata = NULL,
  type = c("latvar", "predictors", "response", "vcov", "all3or4"),
  initial.vals = NULL, ...)
```

**Arguments**

<code>object</code>	The fitted CQO/CAO model.
<code>newdata</code>	A data frame with new response data (usually new species data). The default is to use the original data used to fit the model; however, the calibration may take a long time to compute because the computations are expensive.
<code>type</code>	What type of result is to be returned. The first are the calibrated latent variables or site scores. This must be computed always. The "predictors" are the linear/quadratic or additive predictors evaluated at the calibrated latent variables or site scores. The "response" are the fitted means evaluated at the calibrated latent variables or site scores. The "vcov" are the estimated variance-covariance matrices of the calibrated latent variables or site scores. The "all3or4" is for all of them, i.e., all types. For CAO models, "vcov" is unavailable, so all 3 are returned. For CQO models, "vcov" is available, so all 4 are returned.
<code>initial.vals</code>	Initial values for the search. For rank-1 models, this should be a vector of length <code>nrow(newdata)</code> , and for rank 2 models this should be a two column matrix with the number of rows equalling the number of rows in <code>newdata</code> . The default is a grid defined by arguments in <a href="#">calibrate.qrrvglm.control</a> .
<code>...</code>	Arguments that are fed into <a href="#">calibrate.qrrvglm.control</a> .

**Details**

Given a fitted regression CQO/CAO model, maximum likelihood calibration is theoretically easy and elegant. However, the method assumes that all species are independent, which is not really true in practice. More details and references are given in Yee (2012).

The function [optim](#) is used to search for the maximum likelihood solution. Good initial values are needed, and [calibrate.qrrvglm.control](#) allows the user some control over the choice of these.

**Value**

The argument type determines what is returned. If type = "all3or4" then all the type values are returned in a list, with the following components. Each component has length nrow(newdata).

latvar	Calibrated latent variables or site scores.
predictors	linear/quadratic or additive predictors. For example, for Poisson families, this will be on a log scale, and for binomial families, this will be on a logit scale.
response	Fitted values of the response, evaluated at the calibrated latent variables or site scores.
vcov	Estimated variance-covariance matrix of the calibrated latent variables or site scores. Actually, these are stored in an array whose last dimension is nrow(newdata).

**Warning**

This function is computationally expensive. Setting trace = TRUE to get a running log is a good idea.

**Note**

Despite the name of this function, CAO models are handled as well.

**Author(s)**

T. W. Yee

**References**

Yee, T. W. (2012) On constrained and unconstrained quadratic ordination. *Manuscript in preparation*.

ter Braak, C. J. F. 1995. Calibration. In: *Data Analysis in Community and Landscape Ecology* by Jongman, R. H. G., ter Braak, C. J. F. and van Tongeren, O. F. R. (Eds.) Cambridge University Press, Cambridge.

**See Also**

[calibrate.qrrvglm.control](#), [calibrate](#), [cqo](#), [cao](#).

**Examples**

```
## Not run:
hspider[,1:6] <- scale(hspider[, 1:6]) # Standardize the environmental variables
set.seed(123)
p1 <- cqo(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
  WaterCon + BareSand + FallTwig +
  CoveMoss + CoveHerb + ReflLux,
  family = poissonff, data = hspider, Rank = 1,
  IToler = TRUE, Crow1positive = TRUE)

siteNos <- 3:4 # Calibrate these sites
```

```

cp1 <- calibrate(p1, new = data.frame(depvar(p1)[siteNos, ]), trace = TRUE)

## End(Not run)

## Not run:
# Graphically compare the actual site scores with their calibrated values
persp(p1, main = "Site scores: solid=actual, dashed=calibrated",
      label = TRUE, col = "blue", las = 1)
# Actual site scores:
abline(v = latvar(p1)[siteNos], lty = 1, col = 1:length(siteNos))
abline(v = cp1, lty = 2, col = 1:length(siteNos)) # Calibrated values

## End(Not run)

```

---

```
calibrate.qrrvglm.control
```

*Control function for CQO/CAO calibration*

---

## Description

Algorithmic constants and parameters for running `calibrate.qrrvglm` are set using this function.

## Usage

```
calibrate.qrrvglm.control(object, trace = FALSE, Method.optim = "BFGS",
  gridSize = if (Rank == 1) 9 else 5, varI.latvar = FALSE, ...)
```

## Arguments

<code>object</code>	The fitted CQO/CAO model. The user should ignore this argument.
<code>trace</code>	Logical indicating if output should be produced for each iteration. It is a good idea to set this argument to be TRUE since the computations are expensive.
<code>Method.optim</code>	Character. Fed into the method argument of <code>optim</code> .
<code>gridSize</code>	Numeric, recycled to length Rank. Controls the resolution of the grid used for initial values. For each latent variable, an equally spaced grid of length <code>gridSize</code> is cast from the smallest site score to the largest site score. Then the likelihood function is evaluated on the grid, and the best fit is chosen as the initial value. Thus increasing the value of <code>gridSize</code> increases the chance of obtaining the global solution, however, the computing time increases proportionately.
<code>varI.latvar</code>	Logical. For CQO objects only, this argument is fed into <code>Coef.qrrvglm</code> .
<code>...</code>	Avoids an error message for extraneous arguments.

## Details

Most CQO/CAO users will only need to make use of `trace` and `gridSize`. These arguments should be used inside their call to `calibrate.qrrvglm`, not this function directly.

**Value**

A list which with the following components.

trace	Numeric (even though the input can be logical).
gridSize	Positive integer.
varI.latvar	Logical.

**Note**

Despite the name of this function, CAO models are handled as well.

**Author(s)**

T. W. Yee

**References**

Yee, T. W. (2013) On constrained and unconstrained quadratic ordination. *Manuscript in preparation*.

**See Also**

[calibrate.qrrvglm](#), [Coef.qrrvglm](#).

**Examples**

```
## Not run: hspider[, 1:6] <- scale(hspider[, 1:6]) # Needed when ITol = TRUE
set.seed(123)
p1 <- cgo(cbind(Alopacce, Alopcune, Pardlugu, Pardnigr,
               Pardpull, Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig +
          CoveMoss + CoveHerb + ReflLux,
          family = poissonff, data = hspider, ITol = TRUE)
sort(p1@misc$deviance.Bestof) # A history of all the iterations

siteNos <- 3:4 # Calibrate these sites
cp1 <- calibrate(p1, new = data.frame(depvar(p1)[siteNos, ]), trace = TRUE)

## End(Not run)

## Not run:
# Graphically compare the actual site scores with their calibrated values
persp(p1, main = "Site scores: solid=actual, dashed=calibrated",
      label = TRUE, col = "blue", las = 1)
abline(v = latvar(p1)[siteNos], lty = 1,
       col = 1:length(siteNos)) # Actual site scores
abline(v = cp1, lty = 2, col = 1:length(siteNos)) # Calibrated values

## End(Not run)
```

cao

*Fitting Constrained Additive Ordination (CAO)***Description**

A constrained additive ordination (CAO) model is fitted using the *reduced-rank vector generalized additive model* (RR-VGAM) framework.

**Usage**

```
cao(formula, family, data = list(),
    weights = NULL, subset = NULL, na.action = na.fail,
    etastart = NULL, mustart = NULL, coefstart = NULL,
    control = cao.control(...), offset = NULL,
    method = "cao.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
    contrasts = NULL, constraints = NULL,
    extra = NULL, qr.arg = FALSE, smart = TRUE, ...)
```

**Arguments**

formula	a symbolic description of the model to be fit. The RHS of the formula is used to construct the latent variables, upon which the smooths are applied. All the variables in the formula are used for the construction of latent variables except for those specified by the argument <code>noRRR</code> , which is itself a formula. The LHS of the formula contains the response variables, which should be a matrix with each column being a response (species).
family	a function of class "vglmff" (see <a href="#">vglmff-class</a> ) describing what statistical model is to be fitted. This is called a “ <b>VGAM</b> family function”. See <a href="#">CommonVGAMffArguments</a> for general information about many types of arguments found in this type of function. See <a href="#">cgo</a> for a list of those presently implemented.
data	an optional data frame containing the variables in the model. By default the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>cao</code> is called.
weights	an optional vector or matrix of (prior) weights to be used in the fitting process. For <code>cao</code> , this argument currently should not be used.
subset	an optional logical vector specifying a subset of observations to be used in the fitting process.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the <code>na.action</code> setting of <a href="#">options</a> , and is <code>na.fail</code> if that is unset. The “factory-fresh” default is <code>na.omit</code> .
etastart	starting values for the linear predictors. It is a $M$ -column matrix. If $M = 1$ then it may be a vector. For <code>cao</code> , this argument currently should not be used.
mustart	starting values for the fitted values. It can be a vector or a matrix. Some family functions do not make use of this argument. For <code>cao</code> , this argument currently should not be used.



<code>coefstart</code>	starting values for the coefficient vector. For <code>cao</code> , this argument currently should not be used.
<code>control</code>	a list of parameters for controlling the fitting process. See <a href="#">cao.control</a> for details.
<code>offset</code>	a vector or $M$ -column matrix of offset values. These are <i>a priori</i> known and are added to the linear predictors during fitting. For <code>cao</code> , this argument currently should not be used.
<code>method</code>	the method to be used in fitting the model. The default (and presently only) method <code>cao.fit</code> uses iteratively reweighted least squares (IRLS) within FORTRAN code called from <a href="#">optim</a> .
<code>model</code>	a logical value indicating whether the <i>model frame</i> should be assigned in the <code>model</code> slot.
<code>x.arg, y.arg</code>	logical values indicating whether the model matrix and response vector/matrix used in the fitting process should be assigned in the <code>x</code> and <code>y</code> slots. Note the model matrix is the linear model (LM) matrix.
<code>contrasts</code>	an optional list. See the <code>contrasts.arg</code> of <a href="#">model.matrix.default</a> .
<code>constraints</code>	an optional list of constraint matrices. For <code>cao</code> , this argument currently should not be used. The components of the list must be named with the term it corresponds to (and it must match in character format). Each constraint matrix must have $M$ rows, and be of full-column rank. By default, constraint matrices are the $M$ by $M$ identity matrix unless arguments in the family function itself override these values. If <code>constraints</code> is used it must contain <i>all</i> the terms; an incomplete list is not accepted.
<code>extra</code>	an optional list with any extra information that might be needed by the family function. For <code>cao</code> , this argument currently should not be used.
<code>qr.arg</code>	For <code>cao</code> , this argument currently should not be used.
<code>smart</code>	logical value indicating whether smart prediction ( <a href="#">smartpred</a> ) will be used.
<code>...</code>	further arguments passed into <a href="#">cao.control</a> .

## Details

The arguments of `cao` are a mixture of those from [vgam](#) and [cqo](#), but with some extras in [cao.control](#). Currently, not all of the arguments work properly.

CAO can be loosely be thought of as the result of fitting generalized additive models (GAMs) to several responses (e.g., species) against a very small number of latent variables. Each latent variable is a linear combination of the explanatory variables; the coefficients **C** (called *C* below) are called *constrained coefficients* or *canonical coefficients*, and are interpreted as weights or loadings. The **C** are estimated by maximum likelihood estimation. It is often a good idea to apply [scale](#) to each explanatory variable first.

For each response (e.g., species), each latent variable is smoothed by a cubic smoothing spline, thus CAO is data-driven. If each smooth were a quadratic then CAO would simplify to *constrained quadratic ordination* (CQO; formerly called *canonical Gaussian ordination* or CGO). If each smooth were linear then CAO would simplify to *constrained linear ordination* (CLO). CLO can theoretically be fitted with `cao` by specifying `df1.nl=0`, however it is more efficient to use [rrvglm](#).

Currently, only Rank=1 is implemented, and only noRRR = ~1 models are handled.

With binomial data, the default formula is

$$\text{logit}(P[Y_s = 1]) = \eta_s = f_s(\nu), \quad s = 1, 2, \dots, S$$

where  $x_2$  is a vector of environmental variables, and  $\nu = C^T x_2$  is a  $R$ -vector of latent variables. The  $\eta_s$  is an additive predictor for species  $s$ , and it models the probabilities of presence as an additive model on the logit scale. The matrix  $C$  is estimated from the data, as well as the smooth functions  $f_s$ . The argument noRRR = ~ 1 specifies that the vector  $x_1$ , defined for RR-VGLMs and QRR-VGLMs, is simply a 1 for an intercept. Here, the intercept in the model is absorbed into the functions. A [cloglog](#) link may be preferable over a [logit](#) link.

With Poisson count data, the formula is

$$\log(E[Y_s]) = \eta_s = f_s(\nu)$$

which models the mean response as an additive models on the log scale.

The fitted latent variables (site scores) are scaled to have unit variance. The concept of a tolerance is undefined for CAO models, but the optima and maxima are defined. The generic functions [Max](#) and [Opt](#) should work for CAO objects, but note that if the maximum occurs at the boundary then [Max](#) will return a NA. Inference for CAO models is currently undeveloped.

### Value

An object of class "cao" (this may change to "rrvgam" in the future). Several generic functions can be applied to the object, e.g., [Coef](#), [concoef](#), [lvplot](#), [summary](#).

### Warning

CAO is very costly to compute. With version 0.7-8 it took 28 minutes on a fast machine. I hope to look at ways of speeding things up in the future.

Use [set.seed](#) just prior to calling [cao\(\)](#) to make your results reproducible. The reason for this is finding the optimal CAO model presents a difficult optimization problem, partly because the log-likelihood function contains many local solutions. To obtain the (global) solution the user is advised to try *many* initial values. This can be done by setting Bestof some appropriate value (see [cao.control](#)). Trying many initial values becomes progressively more important as the nonlinear degrees of freedom of the smooths increase.

Currently the dispersion parameter for a [gaussianff](#) CAO model is estimated slightly differently and may be slightly biased downwards (usually a little too small).

### Note

CAO models are computationally expensive, therefore setting `trace = TRUE` is a good idea, as well as running it on a simple random sample of the data set instead.

Sometimes the IRLS algorithm does not converge within the FORTRAN code. This results in warnings being issued. In particular, if an error code of 3 is issued, then this indicates the IRLS algorithm has not converged. One possible remedy is to increase or decrease the nonlinear degrees of freedom so that the curves become more or less flexible, respectively.

**Author(s)**

T. W. Yee

**References**

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[cao.control](#), [Coef.cao](#), [cqo](#), [latvar](#), [Opt](#), [Max](#), [persp.cao](#), [poissonff](#), [binomialff](#), [negbinomial](#), [gamma2](#), [gaussianff](#), [set.seed](#), [gam](#).

**Examples**

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
set.seed(149) # For reproducible results
ap1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull) ~
           WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
           family = poissonff, data = hspider, Rank = 1,
           df1.nl = c(Pardpull= 2.7, 2.5),
           Bestof = 7, Crow1positive = FALSE)
sort(ap1@misc$deviance.Bestof) # A history of all the iterations

Coef(ap1)
concoef(ap1)

par(mfrow = c(2, 2))
plot(ap1) # All the curves are unimodal; some quite symmetric

par(mfrow = c(1, 1), las = 1)
index <- 1:ncol(depvar(ap1))
lvplot(ap1, lcol = index, pcol = index, y = TRUE)

trplot(ap1, label = TRUE, col = index)
abline(a = 0, b = 1, lty = 2)

trplot(ap1, label = TRUE, col = "blue", log = "xy", which.sp = c(1, 3))
abline(a = 0, b = 1, lty = 2)

persp(ap1, col = index, lwd = 2, label = TRUE)
abline(v = Opt(ap1), lty = 2, col = index)
abline(h = Max(ap1), lty = 2, col = index)

## End(Not run)
```

cao.control

*Control Function for RR-VGAMs (CAO)*

## Description

Algorithmic constants and parameters for a constrained additive ordination (CAO), by fitting a *reduced-rank vector generalized additive model* (RR-VGAM), are set using this function. This is the control function for [cao](#).

## Usage

```
cao.control(Rank = 1, all.knots = FALSE, criterion = "deviance", Cinit = NULL,
            Crow1positive = TRUE, epsilon = 1.0e-05, Etamat.colmax = 10,
            GradientFunction = FALSE, iKvector = 0.1, iShape = 0.1,
            noRRR = ~ 1, Norrr = NA,
            SmallNo = 5.0e-13, Use.Init.Poisson.Q0 = TRUE,
            Bestof = if (length(Cinit)) 1 else 10, maxitl = 10,
            imethod = 1, bf.epsilon = 1.0e-7, bf.maxit = 10,
            Maxit.optim = 250, optim.maxit = 20, sd.sitescores = 1.0,
            sd.Cinit = 0.02, suppress.warnings = TRUE,
            trace = TRUE, df1.nl = 2.5, df2.nl = 2.5,
            spar1 = 0, spar2 = 0, ...)
```

## Arguments

Rank	The numerical rank $R$ of the model, i.e., the number of latent variables. Currently only Rank = 1 is implemented.
all.knots	Logical indicating if all distinct points of the smoothing variables are to be used as knots. Assigning the value FALSE means fewer knots are chosen when the number of distinct points is large, meaning less computational expense. See <a href="#">vgam.control</a> for details.
criterion	Convergence criterion. Currently, only one is supported: the deviance is minimized.
Cinit	Optional initial <b>C</b> matrix which may speed up convergence.
Crow1positive	Logical vector of length Rank (recycled if necessary): are the elements of the first row of <b>C</b> positive? For example, if Rank is 4, then specifying Crow1positive = c(FALSE, TRUE) will force <b>C</b> [1,1] and <b>C</b> [1,3] to be negative, and <b>C</b> [1,2] and <b>C</b> [1,4] to be positive.
epsilon	Positive numeric. Used to test for convergence for GLMs fitted in FORTRAN. Larger values mean a loosening of the convergence criterion.
Etamat.colmax	Positive integer, no smaller than Rank. Controls the amount of memory used by .Init.Poisson.Q0(). It is the maximum number of columns allowed for the pseudo-response and its weights. In general, the larger the value, the better the initial value. Used only if Use.Init.Poisson.Q0 = TRUE.

GradientFunction	Logical. Whether <code>optim</code> 's argument <code>gr</code> is used or not, i.e., to compute gradient values. Used only if <code>FastAlgorithm</code> is <code>TRUE</code> . Currently, this argument must be set to <code>FALSE</code> .
iKvector, iShape	See <a href="#">qrrvglm.control</a> .
noRRR	Formula giving terms that are <i>not</i> to be included in the reduced-rank regression (or formation of the latent variables). The default is to omit the intercept term from the latent variables. Currently, only <code>noRRR = ~ 1</code> is implemented.
Norrr	Defunct. Please use <code>noRRR</code> . Use of <code>Norrr</code> will become an error soon.
SmallNo	Positive numeric between <code>.Machine\$double.eps</code> and <code>0.0001</code> . Used to avoid under- or over-flow in the IRLS algorithm.
Use.Init.Poisson.Q0	Logical. If <code>TRUE</code> then the function <code>.Init.Poisson.Q0</code> is used to obtain initial values for the canonical coefficients <code>C</code> . If <code>FALSE</code> then random numbers are used instead.
Bestof	Integer. The best of <code>Bestof</code> models fitted is returned. This argument helps guard against local solutions by (hopefully) finding the global solution from many fits. The argument works only when the function generates its own initial value for <code>C</code> , i.e., when <code>C</code> are <i>not</i> passed in as initial values. The default is only a convenient minimal number and users are urged to increase this value.
maxitl	Positive integer. Maximum number of Newton-Raphson/Fisher-scoring/local-scoring iterations allowed.
imethod	See <a href="#">qrrvglm.control</a> .
bf.epsilon	Positive numeric. Tolerance used by the modified vector backfitting algorithm for testing convergence.
bf.maxit	Positive integer. Number of backfitting iterations allowed in the compiled code.
Maxit.optim	Positive integer. Number of iterations given to the function <code>optim</code> at each of the <code>optim.maxit</code> iterations.
optim.maxit	Positive integer. Number of times <code>optim</code> is invoked.
sd.sitescores	Numeric. Standard deviation of the initial values of the site scores, which are generated from a normal distribution. Used when <code>Use.Init.Poisson.Q0</code> is <code>FALSE</code> .
sd.Cinit	Standard deviation of the initial values for the elements of <code>C</code> . These are normally distributed with mean zero. This argument is used only if <code>Use.Init.Poisson.Q0 = FALSE</code> .
suppress.warnings	Logical. Suppress warnings?
trace	Logical indicating if output should be produced for each iteration. Having the value <code>TRUE</code> is a good idea for large data sets.
df1.nl, df2.nl	Numeric and non-negative, recycled to length <code>S</code> . Nonlinear degrees of freedom for smooths of the first and second latent variables. A value of 0 means the smooth is linear. Roughly, a value between 1.0 and 2.0 often has the approximate flexibility of a quadratic. The user should not assign too large a value to this argument, e.g., the value 4.0 is probably too high. The argument <code>df1.nl</code> is ignored if <code>spar1</code> is assigned a positive value or values. Ditto for <code>df2.nl</code> .

spar1, spar2	Numeric and non-negative, recycled to length $S$ . Smoothing parameters of the smooths of the first and second latent variables. The larger the value, the more smooth (less wiggly) the fitted curves. These arguments are an alternative to specifying <code>df1.nl</code> and <code>df2.nl</code> . A value 0 (the default) for <code>spar1</code> means that <code>df1.nl</code> is used. Ditto for <code>spar2</code> . The values are on a scaled version of the latent variables. See Green and Silverman (1994) for more information.
...	Ignored at present.

## Details

Many of these arguments are identical to `qrrvglm.control`. Here,  $R$  is the Rank,  $M$  is the number of additive predictors, and  $S$  is the number of responses (species). Thus  $M = S$  for binomial and Poisson responses, and  $M = 2S$  for the negative binomial and 2-parameter gamma distributions.

Allowing the smooths too much flexibility means the CAO optimization problem becomes more difficult to solve. This is because the number of local solutions increases as the nonlinearity of the smooths increases. In situations of high nonlinearity, many initial values should be used, so that `Bestof` should be assigned a larger value. In general, there should be a reasonable value of `df1.nl` somewhere between 0 and about 3 for most data sets.

## Value

A list with the components corresponding to its arguments, after some basic error checking.

## Note

The argument `df1.nl` can be inputted in the format `c(spp1 = 2, spp2 = 3, 2.5)`, say, meaning the default value is 2.5, but two species have alternative values.

If `spar1 = 0` and `df1.nl = 0` then this represents fitting linear functions (CLO). Currently, this is handled in the awkward manner of setting `df1.nl` to be a small positive value, so that the smooth is almost linear but not quite. A proper fix to this special case should be done in the short future.

## Author(s)

T. W. Yee

## References

- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.
- Green, P. J. and Silverman, B. W. (1994) *Nonparametric Regression and Generalized Linear Models: A Roughness Penalty Approach*, London: Chapman & Hall.

## See Also

[cao](#).

### Examples

```
## Not run:
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
set.seed(123)
ap1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
           WaterCon + BareSand + FallTwig +
           CoveMoss + CoveHerb + ReflLux,
           family = poissonff, data = hspider,
           df1.nl = c(Zoraspin = 2.3, 2.1),
           Bestof = 10, Crowlpositive = FALSE)
sort(ap1@misc$deviance.Bestof) # A history of all the iterations

Coef(ap1)

par(mfrow = c(2, 3)) # All or most of the curves are unimodal; some are
plot(ap1, lcol = "blue") # quite symmetric. Hence a CQO model should be ok

par(mfrow = c(1, 1), las = 1)
index <- 1:ncol(depvar(ap1)) # lvplot is jagged because only 28 sites
lvplot(ap1, lcol = index, pcol = index, y = TRUE)

trplot(ap1, label = TRUE, col = index)
abline(a = 0, b = 1, lty = 2)

persp(ap1, label = TRUE, col = 1:4)

## End(Not run)
```

---

Card

---

*Cardioid Distribution*


---

### Description

Density, distribution function, quantile function and random generation for the cardioid distribution.

### Usage

```
dcard(x, mu, rho, log = FALSE)
pcard(q, mu, rho)
qcard(p, mu, rho, tolerance = 1e-07, maxits = 500)
rcard(n, mu, rho, ...)
```

### Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Must be a single positive integer.
mu, rho	See <a href="#">cardioid</a> for more information.

tolerance, maxits, ...

The first two are control parameters for the algorithm used to solve for the roots of a nonlinear system of equations; tolerance controls for the accuracy and maxits is the maximum number of iterations. rcard calls qcard so the ... can be used to vary the two arguments.

log Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See [cardioid](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for the formula of the probability density function and other details.

## Value

dcard gives the density, pcard gives the distribution function, qcard gives the quantile function, and rcard generates random deviates.

## Note

Convergence problems might occur with rcard.

## Author(s)

Thomas W. Yee

## See Also

[cardioid](#).

## Examples

```
## Not run:
mu <- 4; rho <- 0.4; x <- seq(0, 2*pi, len = 501)
plot(x, dcard(x, mu, rho), type = "l", las = 1, ylim = c(0, 1), col = "blue",
     ylab = paste("[dp]card(mu=", mu, ", rho=", rho, ")"),
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pcard(x, mu, rho), col = "orange")

probs <- seq(0.1, 0.9, by = 0.1)
Q <- qcard(probs, mu, rho)
lines(Q, dcard(Q, mu, rho), col = "purple", lty = 3, type = "h")
lines(Q, pcard(Q, mu, rho), col = "purple", lty = 3, type = "h")
abline(h = c(0, probs, 1), v = c(0, 2*pi), col = "purple", lty = 3)
max(abs(pcard(Q, mu, rho) - probs)) # Should be 0

## End(Not run)
```



cardioid

*Cardioid Distribution Family Function***Description**

Estimates the two parameters of the cardioid distribution by maximum likelihood estimation.

**Usage**

```
cardioid(lmu = elogit(min = 0, max = 2*pi),
         lrho = elogit(min = -0.5, max = 0.5),
         imu = NULL, irho = 0.3, nsimEIM = 100, zero = NULL)
```

**Arguments**

lmu, lrho	Parameter link functions applied to the $\mu$ and $\rho$ parameters, respectively. See <a href="#">Links</a> for more choices.
imu, irho	Initial values. A NULL means an initial value is chosen internally. See <a href="#">CommonVGAMffArguments</a> for more information.
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> for more information.

**Details**

The two-parameter cardioid distribution has a density that can be written as

$$f(y; \mu, \rho) = \frac{1}{2\pi} (1 + 2\rho \cos(y - \mu))$$

where  $0 < y < 2\pi$ ,  $0 < \mu < 2\pi$ , and  $-0.5 < \rho < 0.5$  is the concentration parameter. The default link functions enforce the range constraints of the parameters.

For positive  $\rho$  the distribution is unimodal and symmetric about  $\mu$ . The mean of  $Y$  (which make up the fitted values) is  $\pi + (\rho/\pi)((2\pi - \mu) \sin(2\pi - \mu) + \cos(2\pi - \mu) - \mu \sin(\mu) - \cos(\mu))$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Warning**

Numerically, this distribution can be difficult to fit because of a log-likelihood having multiple maxima. The user is therefore encouraged to try different starting values, i.e., make use of `imu` and `irho`.

**Note**

Fisher scoring using simulation is used.

**Author(s)**

T. W. Yee

**References**

Jammalamadaka, S. R. and SenGupta, A. (2001) *Topics in Circular Statistics*, Singapore: World Scientific.

**See Also**

[rcard](#), [elogit](#), [vonmises](#).

**CircStats** and **circular** currently have a lot more R functions for circular data than the **VGAM** package.

**Examples**

```
## Not run:
cdata <- data.frame(y = rcard(n = 1000, mu = 4, rho = 0.45))
fit <- vglm(y ~ 1, cardioid, cdata, trace = TRUE)
coef(fit, matrix=TRUE)
Coef(fit)
c(with(cdata, mean(y)), head(fitted(fit), 1))
summary(fit)

## End(Not run)
```

---

cauchit

---

*Cauchit Link Function*


---

**Description**

Computes the cauchit (tangent) link transformation, including its inverse and the first two derivatives.

**Usage**

```
cauchit(theta, bvalue = .Machine$double.eps,
        inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

**Arguments**

theta            Numeric or character. See below for further details.  
bvalue           See [Links](#).  
inverse, deriv, short, tag    Details at [Links](#).

## Details

This link function is an alternative link function for parameters that lie in the unit interval. This type of link bears the same relation to the Cauchy distribution as the probit link bears to the Gaussian. One characteristic of this link function is that the tail is heavier relative to the other links (see examples below).

Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.

## Value

For `deriv = 0`, the tangent of theta, i.e.,  $\tan(\pi * (\text{theta} - 0.5))$  when `inverse = FALSE`, and if `inverse = TRUE` then  $0.5 + \text{atan}(\text{theta})/\pi$ .

For `deriv = 1`, then the function returns  $d\text{theta}/d\text{eta}$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

## Note

Numerical instability may occur when theta is close to 1 or 0. One way of overcoming this is to use `bvalue`.

As mentioned above, in terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the Cauchy distribution (see [cauchy1](#)).

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[logit](#), [probit](#), [cloglog](#), [loge](#), [cauchy](#), [cauchy1](#).

## Examples

```
p <- seq(0.01, 0.99, by=0.01)
cauchit(p)
max(abs(cauchit(cauchit(p), inverse = TRUE) - p)) # Should be 0

p <- c(seq(-0.02, 0.02, by=0.01), seq(0.97, 1.02, by = 0.01))
cauchit(p) # Has no NAs

## Not run:
par(mfrow = c(2, 2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
p <- seq(0.01, 0.99, by = 0.01)

for (d in 0:1) {
```

```

matplot(p, cbind(logit(p, deriv = d), probit(p, deriv = d)),
        type = "n", col = "purple", ylab = "transformation",
        las = 1, main = if (d == 0) "Some probability link functions"
        else "First derivative")
lines(p, logit(p, deriv = d), col = "limegreen")
lines(p, probit(p, deriv = d), col = "purple")
lines(p, cloglog(p, deriv = d), col = "chocolate")
lines(p, cauchit(p, deriv = d), col = "tan")
if (d == 0) {
  abline(v = 0.5, h = 0, lty = "dashed")
  legend(0, 4.5, c("logit", "probit", "cloglog", "cauchit"), lwd = mylwd,
        col = c("limegreen", "purple", "chocolate", "tan"))
} else
  abline(v = 0.5, lty = "dashed")
}

for (d in 0) {
  matplot(y, cbind( logit(y, deriv = d, inverse = TRUE),
                  probit(y, deriv = d, inverse = TRUE)),
          type = "n", col = "purple", xlab = "transformation", ylab = "p",
          main = if (d == 0) "Some inverse probability link functions"
          else "First derivative", las=1)
  lines(y, logit(y, deriv = d, inverse = TRUE), col = "limegreen")
  lines(y, probit(y, deriv = d, inverse = TRUE), col = "purple")
  lines(y, cloglog(y, deriv = d, inverse = TRUE), col = "chocolate")
  lines(y, cauchit(y, deriv = d, inverse = TRUE), col = "tan")
  if (d == 0) {
    abline(h = 0.5, v = 0, lty = "dashed")
    legend(-4, 1, c("logit", "probit", "cloglog", "cauchit"), lwd = mylwd,
          col = c("limegreen", "purple", "chocolate", "tan"))
  }
}
par(lwd = 1)

## End(Not run)

```

---

cauchy

*Cauchy Distribution Family Function*


---

## Description

Estimates either the location parameter or both the location and scale parameters of the Cauchy distribution by maximum likelihood estimation.

## Usage

```

cauchy(llocation = "identity", lscale = "loge",
       ilocation = NULL, iscale = NULL,
       iprobs = seq(0.2, 0.8, by = 0.2),
       imethod = 1, nsimEIM = NULL, zero = 2)

```

```
cauchy1(scale.arg = 1, llocation = "identity",
        ilocation = NULL, imethod = 1)
```

### Arguments

llocation, lscale	Parameter link functions for the location parameter $a$ and the scale parameter $b$ . See <a href="#">Links</a> for more choices.
ilocation, iscale	Optional initial value for $a$ and $b$ . By default, an initial value is chosen internally for each.
imethod	Integer, either 1 or 2 or 3. Initial method, three algorithms are implemented. The user should try all possible values to help avoid converging to a local solution. Also, choose the another value if convergence fails, or use <code>ilocation</code> and/or <code>iscale</code> .
iprobs	Probabilities used to find the respective sample quantiles; used to compute <code>iscale</code> .
zero, nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information.
scale.arg	Known (positive) scale parameter, called $b$ below.

### Details

The Cauchy distribution has density function

$$f(y; a, b) = \{\pi b[1 + ((y - a)/b)^2]\}^{-1}$$

where  $y$  and  $a$  are real and finite, and  $b > 0$ . The distribution is symmetric about  $a$  and has a heavy tail. Its median and mode are  $a$  but the mean does not exist. The fitted values are the estimates of  $a$ . Fisher scoring is the default but if `nsimEIM` is specified then Fisher scoring with simulation is used.

If the scale parameter is known (`cauchy1`) then there may be multiple local maximum likelihood solutions for the location parameter. However, if both location and scale parameters are to be estimated (`cauchy`) then there is a unique maximum likelihood solution provided  $n > 2$  and less than half the data are located at any one point.

### Value

An object of class "`vglmff`" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Warning

It is well-known that the Cauchy distribution may have local maxima in its likelihood function; make full use of `imethod`, `ilocation`, `iscale` etc.

### Note

Good initial values are needed. By default these **VGAM** family functions search for a starting value for  $a$  on a grid. It also pays to select a wide range of initial values via the `ilocation` and/or `iscale` and/or `imethod` arguments.

**Author(s)**

T. W. Yee

**References**

- Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.
- Barnett, V. D. (1966) Evaluation of the maximum-likelihood estimator where the likelihood equation has multiple roots. *Biometrika*, **53**, 151–165.
- Copas, J. B. (1975) On the unimodality of the likelihood for the Cauchy distribution. *Biometrika*, **62**, 701–704.
- Efron, B. and Hinkley, D. V. (1978) Assessing the accuracy of the maximum likelihood estimator: Observed versus expected Fisher information. *Biometrika*, **65**, 457–481.

**See Also**

[Cauchy](#), [cauchit](#), [studentt](#).

**Examples**

```
# Both location and scale parameters unknown
cdata1 <- data.frame(x = runif(nn <- 1000))
cdata1 <- transform(cdata1, loc = exp(1+0.5*x), scale = exp(1))
cdata1 <- transform(cdata1, y = rcauchy(nn, loc, scale))
fit <- vglm(y ~ x, cauchy(lloc = "loge"), cdata1, trace = TRUE)
coef(fit, matrix = TRUE)
head(fitted(fit)) # Location estimates
summary(fit)

# Location parameter unknown
set.seed(123)
cdata2 <- data.frame(x = runif(nn <- 500))
cdata2 <- transform(cdata2, loc = 1 + 0.5 * x, scale = 0.4)
cdata2 <- transform(cdata2, y = rcauchy(nn, loc, scale))
fit <- vglm(y ~ x, cauchy1(scale = 0.4), cdata2, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
```

---

cdf.lmscreg

*Cumulative Distribution Function for LMS Quantile Regression*

---

**Description**

Computes the cumulative distribution function (CDF) for observations, based on a LMS quantile regression.

**Usage**

```
cdf.lmscreg(object, newdata = NULL, ...)
```

**Arguments**

object	A <b>VGAM</b> quantile regression model, i.e., an object produced by modelling functions such as <code>vglm</code> and <code>vgam</code> with a family function beginning with "lms."
newdata	Data frame where the predictions are to be made. If missing, the original data is used.
...	Parameters which are passed into functions such as <code>cdf.lms.yjn</code> .

**Details**

The CDFs returned here are values lying in [0,1] giving the relative probabilities associated with the quantiles newdata. For example, a value near 0.75 means it is close to the upper quartile of the distribution.

**Value**

A vector of CDF values lying in [0,1].

**Note**

The data are treated like quantiles, and the percentiles are returned. The opposite is performed by `qplot.lmscreg`.

The CDF values of the model have been placed in `@post$cdf` when the model was fitted.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

`deplot.lmscreg`, `qplot.lmscreg`, `lms.bcn`, `lms.bcg`, `lms.yjn`.

**Examples**

```
fit <- vgam(BMI ~ s(age, df=c(4, 2)), lms.bcn(zero = 1), data = bmi.nz)
head(fit@post$cdf)
head(cdf(fit)) # Same
head(depvar(fit))
head(fitted(fit))

cdf(fit, data.frame(age = c(31.5, 39), BMI = c(28.4, 24)))
```

cennormal

*Censored Normal Distribution***Description**

Maximum likelihood estimation for the normal distribution with left and right censoring.

**Usage**

```
cennormal(lmu = "identity", lsd = "log", imethod = 1, zero = 2)
```

**Arguments**

lmu, lsd	Parameter link functions applied to the mean and standard deviation parameters. See <a href="#">Links</a> for more choices. The standard deviation is a positive quantity, therefore a log link is the default.
imethod	Initialization method. Either 1 or 2, this specifies two methods for obtaining initial values for the parameters.
zero	An integer vector, containing the value 1 or 2. If so, the mean or standard deviation respectively are modelled as an intercept only. Setting zero = NULL means both linear/additive predictors are modelled as functions of the explanatory variables.

**Details**

This function is like [uninormal](#) but handles observations that are left-censored (so that the true value would be less than the observed value) else right-censored (so that the true value would be greater than the observed value). To indicate which type of censoring, input `extra = list(leftcensored = vec1, rightcensored = vec2)` where `vec1` and `vec2` are logical vectors the same length as the response. If the two components of this list are missing then the logical values are taken to be FALSE. The fitted object has these two components stored in the `extra` slot.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

This function is an alternative to [tobit](#) but cannot handle a matrix response and uses different working weights. If there are no censored observations then [uninormal](#) is recommended instead.

**Author(s)**

T. W. Yee



See Also

[tobit](#), [uninormal](#), [double.cennormal](#).

Examples

```
## Not run:
cdata <- data.frame(x2 = runif(nn <- 1000)) # ystar are true values
cdata <- transform(cdata, ystar = rnorm(nn, m = 100 + 15 * x2, sd = exp(3)))
with(cdata, hist(ystar))
cdata <- transform(cdata, L = runif(nn, 80, 90), # Lower censoring points
                  U = runif(nn, 130, 140)) # Upper censoring points
cdata <- transform(cdata, y = pmax(L, ystar)) # Left censored
cdata <- transform(cdata, y = pmin(U, y))    # Right censored
with(cdata, hist(y))
Extra <- list(leftcensored = with(cdata, ystar < L),
             rightcensored = with(cdata, ystar > U))
fit1 <- vglm(y ~ x2, cennormal, cdata, crit = "c", extra = Extra, trace = TRUE)
fit2 <- vglm(y ~ x2, tobit(Lower = with(cdata, L), Upper = with(cdata, U)),
            cdata, crit = "c", trace = TRUE)
coef(fit1, matrix = TRUE)
max(abs(coef(fit1, matrix = TRUE) - coef(fit2, matrix = TRUE))) # Should be 0
names(fit1@extra)

## End(Not run)
```

---

cenpoisson	<i>Censored Poisson Family Function</i>
------------	---

---

Description

Family function for a censored Poisson response.

Usage

```
cenpoisson(link = "log", imu = NULL)
```

Arguments

- link                      Link function applied to the mean; see [Links](#) for more choices.
- imu                        Optional initial value; see [CommonVGAMffArguments](#) for more information.

Details

Often a table of Poisson counts has an entry  $J+$  meaning  $\geq J$ . This family function is similar to [poissonff](#) but handles such censored data. The input requires [SurvS4](#). Only a univariate response is allowed. The Newton-Raphson algorithm is used.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

As the response is discrete, care is required with [Surv](#), especially with "interval" censored data because of the (start, end] format. See the examples below. The examples have  $y < L$  as left censored and  $y \geq U$  (formatted as U+) as right censored observations, therefore  $L \leq y < U$  is for uncensored and/or interval censored observations. Consequently the input must be tweaked to conform to the (start, end] format.

**Note**

The function [poissonff](#) should be used when there are no censored observations. Also, NAs are not permitted with [SurvS4](#), nor is type = "counting".

**Author(s)**

Thomas W. Yee

**References**

See [survival](#) for background.

**See Also**

[SurvS4](#), [poissonff](#), [Links](#).

**Examples**

```
# Example 1: right censored data
set.seed(123); U <- 20
cdata <- data.frame(y = rpois(N <- 100, exp(3)))
cdata <- transform(cdata, cy = pmin(U, y),
                  rcensored = (y >= U))
cdata <- transform(cdata, status = ifelse(rcensored, 0, 1))
with(cdata, table(cy))
with(cdata, table(rcensored))
with(cdata, table(ii <- print(SurvS4(cy, status)))) # Check; U+ means >= U
fit <- vglm(SurvS4(cy, status) ~ 1, cenpoisson, cdata, trace = TRUE)
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check; U+ means >= U

# Example 2: left censored data
L <- 15
cdata <- transform(cdata, cY = pmax(L, y),
                  lcensored = y < L) # Note y < L, not cY == L or y <= L
cdata <- transform(cdata, status = ifelse(lcensored, 0, 1))
with(cdata, table(cY))
```

```

with(cdata, table(lcensored))
with(cdata, table(ii <- print(SurvS4(cY, status, type = "left")))) # Check
fit <- vglm(SurvS4(cY, status, type = "left") ~ 1, cenpoisson, cdata, trace = TRUE)
coef(fit, matrix = TRUE)

# Example 3: interval censored data
cdata <- transform(cdata, Lvec = rep(L, len = N),
                  Uvec = rep(U, len = N))
cdata <- transform(cdata, icensored = Lvec <= y & y < Uvec) # Not lcensored or rcensored
with(cdata, table(icensored))
cdata <- transform(cdata, status = rep(3, N)) # 3 means interval censored
cdata <- transform(cdata, status = ifelse(rcensored, 0, status)) # 0 means right censored
cdata <- transform(cdata, status = ifelse(lcensored, 2, status)) # 2 means left censored
# Have to adjust Lvec and Uvec because of the (start, end] format:
cdata$Lvec[with(cdata, icensored)] <- cdata$Lvec[with(cdata, icensored)] - 1
cdata$Uvec[with(cdata, icensored)] <- cdata$Uvec[with(cdata, icensored)] - 1
cdata$Lvec[with(cdata, lcensored)] <- cdata$Lvec[with(cdata, lcensored)] # Unchanged
cdata$Lvec[with(cdata, rcensored)] <- cdata$Uvec[with(cdata, rcensored)] # Unchanged
with(cdata, table(ii <- print(SurvS4(Lvec, Uvec, status, type = "interval")))) # Check

fit <- vglm(SurvS4(Lvec, Uvec, status, type = "interval") ~ 1,
            cenpoisson, cdata, trace = TRUE)
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check

# Example 4: Add in some uncensored observations
index <- (1:N)[with(cdata, icensored)]
index <- head(index, 4)
cdata$status[index] <- 1 # actual or uncensored value
cdata$Lvec[index] <- cdata$y[index]
with(cdata, table(ii <- print(SurvS4(Lvec, Uvec, status,
                                   type = "interval")))) # Check

fit <- vglm(SurvS4(Lvec, Uvec, status, type = "interval") ~ 1,
            cenpoisson, cdata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check

```

---

cgo

*Redirects the user to cgo*


---

## Description

Redirects the user to the function [cgo](#).

## Usage

```
cgo(...)
```

**Arguments**

... Ignored.

**Details**

The former function `cgo` has been renamed `cqo` because CGO (for *canonical Gaussian ordination*) is a confusing and inaccurate name. CQO (for *constrained quadratic ordination*) is better. This new nomenclature described in Yee (2006).

**Value**

Nothing is returned; an error message is issued.

**Warning**

The code, therefore, in Yee (2004) will not run without changing the "g" to a "q".

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

**See Also**

`cqo`.

**Examples**

```
## Not run:  
cgo()  
  
## End(Not run)
```

cgumbel

*Censored Gumbel Distribution***Description**

Maximum likelihood estimation of the 2-parameter Gumbel distribution when there are censored observations. A matrix response is not allowed.

**Usage**

```
cgumbel(llocation = "identity", lscale = "log",
        iscale = NULL, mean = TRUE, percentiles = NULL, zero = 2)
```

**Arguments**

<code>llocation</code> , <code>lscale</code>	Character. Parameter link functions for the location and (positive) <i>scale</i> parameters. See <a href="#">Links</a> for more choices.
<code>iscale</code>	Numeric and positive. Initial value for <i>scale</i> . Recycled to the appropriate length. In general, a larger value is better than a smaller value. The default is to choose the value internally.
<code>mean</code>	Logical. Return the mean? If TRUE then the mean is returned, otherwise percentiles given by the percentiles argument.
<code>percentiles</code>	Numeric with values between 0 and 100. If mean=FALSE then the fitted values are percentiles which must be specified by this argument.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The value (possibly values) must be from the set {1,2} corresponding respectively to <i>location</i> and <i>scale</i> . If zero=NULL then all linear/additive predictors are modelled as a linear combination of the explanatory variables. The default is to fit the shape parameter as an intercept only.

**Details**

This **VGAM** family function is like [gumbel](#) but handles observations that are left-censored (so that the true value would be less than the observed value) else right-censored (so that the true value would be greater than the observed value). To indicate which type of censoring, input `extra = list(leftcensored = vec1, rightcensored = vec2)` where `vec1` and `vec2` are logical vectors the same length as the response. If the two components of this list are missing then the logical values are taken to be FALSE. The fitted object has these two components stored in the `extra` slot.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

Numerical problems may occur if the amount of censoring is excessive.

**Note**

See [gumbel](#) for details about the Gumbel distribution. The initial values are based on assuming all uncensored observations, therefore could be improved upon.

**Author(s)**

T. W. Yee

**References**

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

**See Also**

[gumbel](#), [egumbel](#), [rgumbel](#), [guplot](#), [gev](#), [venice](#).

**Examples**

```
# Example 1
ystar <- venice[["r1"]] # Use the first order statistic as the response
nn <- length(ystar)
L <- runif(nn, 100, 104) # Lower censoring points
U <- runif(nn, 130, 135) # Upper censoring points
y <- pmax(L, ystar) # Left censored
y <- pmin(U, y) # Right censored
extra <- list(leftcensored = ystar < L, rightcensored = ystar > U)
fit <- vglm(y ~ scale(year), data = venice, trace = TRUE, extra = extra,
            cgumbel(mean = FALSE, perc = c(5, 25, 50, 75, 95)))
coef(fit, matrix = TRUE)
head(fitted(fit))
fit@extra

# Example 2: simulated data
nn <- 1000
ystar <- rgumbel(nn, loc = 1, scale = exp(0.5)) # The uncensored data
L <- runif(nn, -1, 1) # Lower censoring points
U <- runif(nn, 2, 5) # Upper censoring points
y <- pmax(L, ystar) # Left censored
y <- pmin(U, y) # Right censored
## Not run: par(mfrow = c(1, 2)); hist(ystar); hist(y);
extra <- list(leftcensored = ystar < L, rightcensored = ystar > U)
fit <- vglm(y ~ 1, trace = TRUE, extra = extra, cgumbel)
coef(fit, matrix = TRUE)
```

chest.nz

*Chest Pain in NZ Adults Data***Description**

Presence/absence of chest pain in 10186 New Zealand adults.

**Usage**

```
data(chest.nz)
```

**Format**

A data frame with 73 rows and the following 5 variables.

**age** a numeric vector; age (years).

**nolnor** a numeric vector of counts; no pain on LHS or RHS.

**nolr** a numeric vector of counts; no pain on LHS but pain on RHS.

**lnor** a numeric vector of counts; no pain on RHS but pain on LHS.

**lr** a numeric vector of counts; pain on LHS and RHS of chest.

**Details**

Each adult was asked their age and whether they experienced any pain or discomfort in their chest over the last six months. If yes, they indicated whether it was on their LHS and/or RHS of their chest.

**Source**

MacMahon, S., Norton, R., Jackson, R., Mackie, M. J., Cheng, A., Vander Hoorn, S., Milne, A., McCulloch, A. (1995) Fletcher Challenge-University of Auckland Heart & Health Study: design and baseline findings. *New Zealand Medical Journal*, **108**, 499–502.

**Examples**

```
## Not run:
fit <- vgam(cbind(nolnor, nolr, lnor, lr) ~ s(age, c(4, 3)),
           binom2.or(exchan = TRUE, zero = NULL), data = chest.nz)
coef(fit, matrix = TRUE)

## End(Not run)
## Not run: plot(fit, which.cf = 2, se = TRUE)
```

chinese.nz

*Chinese Population in New Zealand 1867–2001 Data***Description**

The Chinese population in New Zealand from 1867 to 2001, along with the whole of the New Zealand population.

**Usage**

```
data(chinese.nz)
```

**Format**

A data frame with 27 observations on the following 4 variables.

year Year.

male Number of Chinese males.

female Number of Chinese females.

nz Total number in the New Zealand population.

**Details**

Historically, there was a large exodus of Chinese from the Guangdong region starting in the mid-1800s to the gold fields of South Island of New Zealand, California, and southern Australia, etc. Discrimination then meant that only men were allowed entry, to hinder permanent settlement. In the case of New Zealand, the government relaxed its immigration laws after WWII to allow wives of Chinese already in NZ to join them because China had been among the Allied powers. Gradual relaxation in the immigration and an influx during the 1980s meant the Chinese population became increasingly demographically normal over time.

The NZ total for the years 1867 and 1871 exclude the Maori population. Three modifications have been made to the female column to make the data internally consistent with the original table.

**References**

Page 6 of *Aliens At My Table: Asians as New Zealanders See Them* by M. Ip and N. Murphy, (2005). Penguin Books. Auckland, New Zealand.

**Examples**

```
## Not run: par(mfrow = c(1, 2))
plot(female / (male + female) ~ year, chinese.nz, type = "b",
     ylab = "Proportion", col = "blue", las = 1,
     cex = 0.015 * sqrt(male + female),
#    cex = 0.10 * sqrt((male + female)^1.5 / sqrt(female) / sqrt(male)),
     main = "Proportion of NZ Chinese that are female")
abline(h = 0.5, lty = "dashed", col = "gray")
```



```

fit1.cnz <- vglm(cbind(female, male) ~ year,          binomialff, chinese.nz)
fit2.cnz <- vglm(cbind(female, male) ~ poly(year, 2), binomialff, chinese.nz)
fit4.cnz <- vglm(cbind(female, male) ~ bs(year, 5), binomialff, chinese.nz)

lines(fitted(fit1.cnz) ~ year, chinese.nz, col = "purple", lty = 1)
lines(fitted(fit2.cnz) ~ year, chinese.nz, col = "green", lty = 2)
lines(fitted(fit4.cnz) ~ year, chinese.nz, col = "orange", lwd = 2, lty = 1)
legend("bottomright", col = c("purple", "green", "orange"),
      lty = c(1, 2, 1), leg = c("linear", "quadratic", "B-spline"))

plot(100*(male+female)/nz ~ year, chinese.nz, type = "b", ylab = "Percent",
     ylim = c(0, max(100*(male+female)/nz)), col = "blue", las = 1,
     main = "Percent of NZers that are Chinese")
abline(h = 0, lty = "dashed", col = "gray")
## End(Not run)

```

chisq

*Chi-squared Distribution***Description**

Maximum likelihood estimation of the degrees of freedom for a chi-squared distribution.

**Usage**

```
chisq(link = "log", zero = NULL)
```

**Arguments**

link, zero      See [CommonVGAMffArguments](#) for information.

**Details**

The degrees of freedom is treated as a parameter to be estimated, and as real (not integer). Being positive, a log link is used by default. Fisher scoring is used.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

Multiple responses are permitted. There may be convergence problems if the degrees of freedom is very large or close to zero.

**Author(s)**

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

[Chisquare.uninormal](#).

## Examples

```
cdata <- data.frame(x2 = runif(nn <- 1000))
cdata <- transform(cdata, y1 = rchisq(nn, df = exp(1 - 1 * x2)),
                    y2 = rchisq(nn, df = exp(2 - 2 * x2)))
fit <- vglm(cbind(y1, y2) ~ x2, chisq, cdata, trace = TRUE)
coef(fit, matrix = TRUE)
```

---

clo	<i>Redirects the user to rrvglm</i>
-----	-------------------------------------

---

## Description

Redirects the user to the function [rrvglm](#).

## Usage

```
clo(...)
```

## Arguments

... Ignored.

## Details

CLO stands for *constrained linear ordination*, and is fitted with a statistical class of models called *reduced-rank vector generalized linear models* (RR-VGLMs). It allows for generalized reduced-rank regression in that response types such as Poisson counts and presence/absence data can be handled.

Currently in the **VGAM** package, [rrvglm](#) is used to fit RR-VGLMs. However, the Author's opinion is that linear responses to a latent variable (composite environmental gradient) is not as common as unimodal responses, therefore [cqo](#) is often more appropriate.

The new CLO/CQO/CAO nomenclature described in Yee (2006).

## Value

Nothing is returned; an error message is issued.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

**See Also**

[rrvglm](#), [cqq](#).

**Examples**

```
## Not run:  
clo()  
  
## End(Not run)
```

---

cloglog

*Complementary Log-log Link Function*

---

**Description**

Computes the complementary log-log transformation, including its inverse and the first two derivatives.

**Usage**

```
cloglog(theta, bvalue = NULL, inverse = FALSE, deriv = 0,  
        short = TRUE, tag = FALSE)
```

**Arguments**

theta	Numeric or character. See below for further details.
bvalue	See <a href="#">Links</a> for general information about links.
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

**Details**

The complementary log-log link function is commonly used for parameters that lie in the unit interval. Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.

**Value**

For `deriv = 0`, the complimentary log-log of theta, i.e.,  $\log(-\log(1 - \text{theta}))$  when `inverse = FALSE`, and if `inverse = TRUE` then  $1 - \exp(-\exp(\text{theta}))$ .

For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

Numerical instability may occur when theta is close to 1 or 0. One way of overcoming this is to use `bvalue`.

Changing 1s to 0s and 0s to 1s in the response means that effectively a loglog link is fitted. That is, transform  $y$  by  $1 - y$ . That's why only one of `cloglog` and `loglog` is written.

With constrained ordination (e.g., `cqo` and `cao`) used with `binomialff`, a complementary log-log link function is preferred over the default `logit` link, for a good reason. See the example below.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the extreme value distribution.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [logit](#), [probit](#), [cauchit](#).

**Examples**

```
p <- seq(0.01, 0.99, by = 0.01)
cloglog(p)
max(abs(cloglog(cloglog(p), inverse = TRUE) - p)) # Should be 0

p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
cloglog(p) # Has NAs
cloglog(p, bvalue = .Machine$double.eps) # Has no NAs

## Not run:
p <- seq(0.01, 0.99, by = 0.01)
plot(p, logit(p), type = "l", col = "limegreen", ylab = "transformation",
     lwd = 2, las = 1, main = "Some probability link functions")
lines(p, probit(p), col = "purple", lwd = 2)
lines(p, cloglog(p), col = "chocolate", lwd = 2)
lines(p, cauchit(p), col = "tan", lwd = 2)
abline(v = 0.5, h = 0, lty = "dashed")
```

```

legend(0.1, 4, c("logit", "probit", "cloglog", "cauchit"),
      col = c("limegreen", "purple", "chocolate", "tan"), lwd = 2)

## End(Not run)

## Not run:
# This example shows that a cloglog link is preferred over the logit
n <- 500; p <- 5; S <- 3; Rank <- 1 # Species packing model:
mydata <- rcqo(n, p, S, eq.tol = TRUE, es.opt = TRUE, eq.max = TRUE,
  family = "binomial", hi.abundance = 5, seed = 123, Rank = Rank)
fitc <- cqo(attr(mydata, "formula"), ITol = TRUE, data = mydata,
  fam = binomialff(mv = TRUE, link = "cloglog"), Rank = Rank)
fitl <- cqo(attr(mydata, "formula"), ITol = TRUE, data = mydata,
  fam = binomialff(mv = TRUE, link = "logit"), Rank = Rank)

# Compare the fitted models (cols 1 and 3) with the truth (col 2)
cbind(concoef(fitc), attr(mydata, "ccoefficients"), concoef(fitl))

## End(Not run)

```

---

coalminers

*Breathlessness and Wheeze Amongst Coalminers Data*


---

## Description

Coalminers who are smokers without radiological pneumoconiosis, classified by age, breathlessness and wheeze.

## Usage

```
data(coalminers)
```

## Format

A data frame with 9 age groups with the following 5 columns.

**BW** Counts with breathlessness and wheeze.

**BnW** Counts with breathlessness but no wheeze.

**nBW** Counts with no breathlessness but wheeze.

**nBnW** Counts with neither breathlessness or wheeze.

**age** Age of the coal miners (actually, the midpoints of the 5-year category ranges).

## Details

The data were published in Ashford and Sowden (1970). A more recent analysis is McCullagh and Nelder (1989, Section 6.6).

**Source**

Ashford, J. R. and Sowden, R. R. (1970) Multi-variate probit analysis. *Biometrics*, **26**, 535–546.

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*. 2nd ed. London: Chapman & Hall.

**Examples**

```
str(coalminers)
```

---

Coef	<i>Computes Model Coefficients and Quantities</i>
------	---

---

**Description**

Coef is a generic function which computes model coefficients from objects returned by modelling functions. It is an auxiliary function to [coef](#) that enables extra capabilities for some specific models.

**Usage**

```
Coef(object, ...)
```

**Arguments**

object	An object for which the computation of other types of model coefficients or quantities is meaningful.
...	Other arguments fed into the specific methods function of the model.

**Details**

This function can often be useful for [vglm](#) objects with just an intercept term in the RHS of the formula, e.g.,  $y \sim 1$ . Then often this function will apply the inverse link functions to the parameters. See the example below.

For reduced-rank VGLMs, this function can return the **A**, **C** matrices, etc.

For quadratic and additive ordination models, this function can return ecological meaningful quantities such as tolerances, optima, maxima.

**Value**

The value returned depends specifically on the methods function invoked.

**Warning**

This function may not work for *all* **VGAM** family functions. You should check your results on some artificial data before applying it to models fitted to real data.

Author(s)

Thomas W. Yee

References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

See Also

[coef](#), [Coef.vlm](#), [Coef.rrvglm](#), [Coef.qrrvglm](#), [depvar](#).

Examples

```
nn <- 1000
bdata <- data.frame(y = rbeta(nn, shape1 = 1, shape2 = 3)) # Original scale
fit <- vglm(y ~ 1, beta.ab, data = bdata, trace = TRUE) # Intercept-only model
coef(fit, matrix = TRUE) # Both on a log scale
Coef(fit) # On the original scale
```

---

Coef.qrrvglm	Returns Important Matrices etc. of a QO Object
--------------	--

---

Description

This methods function returns important matrices etc. of a QO object.

Usage

```
Coef.qrrvglm(object, varI.latvar = FALSE, reference = NULL, ...)
```

Arguments

object	A CQO object. The former has class "qrrvglm".
varI.latvar	Logical indicating whether to scale the site scores (latent variables) to have variance-covariance matrix equal to the rank- <i>R</i> identity matrix. All models have uncorrelated site scores (latent variables), and this option stretches or shrinks the ordination axes if TRUE. See below for further details.
reference	Integer or character. Specifies the <i>reference species</i> . By default, the reference species is found by searching sequentially starting from the first species until a positive-definite tolerance matrix is found. Then this tolerance matrix is transformed to the identity matrix. Then the sites scores (latent variables) are made uncorrelated. See below for further details.
...	Currently unused.

## Details

If `ITolerances=TRUE` or `EqualTolerances=TRUE` (and its estimated tolerance matrix is positive-definite) then all species' tolerances are unity by transformation or by definition, and the spread of the site scores can be compared to them. Vice versa, if one wishes to compare the tolerances with the sites score variability then setting `varI.latvar=TRUE` is more appropriate.

For rank-2 QRR-VGLMs, one of the species can be chosen so that the angle of its major axis and minor axis is zero, i.e., parallel to the ordination axes. This means the effect on the latent vars is independent on that species, and that its tolerance matrix is diagonal. The argument `reference` allows one to choose which is the reference species, which must have a positive-definite tolerance matrix, i.e., is bell-shaped. If `reference` is not specified, then the code will try to choose some reference species starting from the first species. Although the `reference` argument could possibly be offered as an option when fitting the model, it is currently available after fitting the model, e.g., in the functions `Coef.qrrvglm` and `lvplot.qrrvglm`.

## Value

The **A**, **B1**, **C**, **T**, **D** matrices/arrays are returned, along with other slots.

The returned object has class "`Coef.qrrvglm`" (see `Coef.qrrvglm-class`).

## Note

Consider an equal-tolerances Poisson/binomial CQO model with  $\text{noRRR} = \sim 1$ . For  $R = 1$  it has about  $2S + p_2$  parameters. For  $R = 2$  it has about  $3S + 2p_2$  parameters. Here,  $S$  is the number of species, and  $p_2 = p - 1$  is the number of environmental variables making up the latent variable. For an unequal-tolerances Poisson/binomial CQO model with  $\text{noRRR} = \sim 1$ , it has about  $3S - 1 + p_2$  parameters for  $R = 1$ , and about  $6S - 3 + 2p_2$  parameters for  $R = 2$ . Since the total number of data points is  $nS$ , where  $n$  is the number of sites, it pays to divide the number of data points by the number of parameters to get some idea about how much information the parameters contain.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

## See Also

`cqo`, `Coef.qrrvglm-class`, `print.Coef.qrrvglm`, `lvplot.qrrvglm`.

## Examples

```
set.seed(123)
x2 <- rnorm(n <- 100)
x3 <- rnorm(n)
x4 <- rnorm(n)
```



```

latvar1 <- 0 + x3 - 2*x4
lambda1 <- exp(3 - 0.5 * ( latvar1-0)^2)
lambda2 <- exp(2 - 0.5 * ( latvar1-1)^2)
lambda3 <- exp(2 - 0.5 * ((latvar1+4)/2)^2) # Unequal tolerances
y1 <- rpois(n, lambda1)
y2 <- rpois(n, lambda2)
y3 <- rpois(n, lambda3)
set.seed(111)
# vvv p1 <- cqo(cbind(y1, y2, y3) ~ x2 + x3 + x4, poissonff, trace = FALSE)
## Not run: lvplot(p1, y = TRUE, lcol = 1:3, pch = 1:3, pcol = 1:3)

# vvv Coef(p1)
# vvv print(Coef(p1), digits=3)

```

---

Coef.qrrvglm-class	Class “Coef.qrrvglm”
--------------------	----------------------

---

## Description

The most pertinent matrices and other quantities pertaining to a QRR-VGLM (CQO model).

## Objects from the Class

Objects can be created by calls of the form `Coef(object, ...)` where `object` is an object of class “qrrvglm” (created by [cqo](#)).

In this document,  $R$  is the *rank*,  $M$  is the number of linear predictors and  $n$  is the number of observations.

## Slots

**A:** Of class “matrix”, **A**, which are the linear ‘coefficients’ of the matrix of latent variables. It is  $M$  by  $R$ .

**B1:** Of class “matrix”, **B1**. These correspond to terms of the argument `noRRR`.

**C:** Of class “matrix”, **C**, the canonical coefficients. It has  $R$  columns.

**Constrained:** Logical. Whether the model is a constrained ordination model.

**D:** Of class “array”, `D[, , j]` is an order-Rank matrix, for  $j = 1, \dots, M$ . Ideally, these are negative-definite in order to make the response curves/surfaces bell-shaped.

**Rank:** The rank (dimension, number of latent variables) of the RR-VGLM. Called  $R$ .

**latvar:**  $n$  by  $R$  matrix of latent variable values.

**latvar.order:** Of class “matrix”, the permutation returned when the function [order](#) is applied to each column of `latvar`. This enables each column of `latvar` to be easily sorted.

**Maximum:** Of class “numeric”, the  $M$  maximum fitted values. That is, the fitted values at the optima for `noRRR = ~ 1` models. If `noRRR` is not `~ 1` then these will be NAs.

**NOS:** Number of species.

**Optimum:** Of class "matrix", the values of the latent variables where the optima are. If the curves are not bell-shaped, then the value will be NA or NaN.

**Optimum.order:** Of class "matrix", the permutation returned when the function `order` is applied to each column of Optimum. This enables each row of Optimum to be easily sorted.

**bellshaped:** Vector of logicals: is each response curve/surface bell-shaped?

**dispersion:** Dispersion parameter(s).

**Dzero:** Vector of logicals, is each of the response curves linear in the latent variable(s)? It will be if and only if  $D[, j]$  equals **O**, for  $j = 1, \dots, M$ .

**Tolerance:** Object of class "array",  $Tolerance[, j]$  is an order-Rank matrix, for  $j = 1, \dots, M$ , being the matrix of tolerances (squared if on the diagonal). These are denoted by **T** in Yee (2004). Ideally, these are positive-definite in order to make the response curves/surfaces bell-shaped. The tolerance matrices satisfy  $T_s = -\frac{1}{2}D_s^{-1}$ .

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

## See Also

`Coef.qrrvglm`, `cqo`,  
`print.Coef.qrrvglm`.

## Examples

```
x2 <- rnorm(n <- 100)
x3 <- rnorm(n)
x4 <- rnorm(n)
latvar1 <- 0 + x3 - 2*x4
lambda1 <- exp(3 - 0.5 * ( latvar1-0)^2)
lambda2 <- exp(2 - 0.5 * ( latvar1-1)^2)
lambda3 <- exp(2 - 0.5 * ((latvar1+4)/2)^2)
y1 <- rpois(n, lambda1)
y2 <- rpois(n, lambda2)
y3 <- rpois(n, lambda3)
yy <- cbind(y1, y2, y3)
# vvv p1 <- cqo(yy ~ x2 + x3 + x4, fam = poissonff, trace = FALSE)
## Not run:
lvplot(p1, y = TRUE, lcol = 1:3, pch = 1:3, pcol = 1:3)

## End(Not run)
# vvv print(Coef(p1), digits = 3)
```

---

Coef.rrvglm	<i>Returns Important Matrices etc. of a RR-VGLM Object</i>
-------------	--

---

**Description**

This methods function returns important matrices etc. of a RR-VGLM object.

**Usage**

```
Coef.rrvglm(object, ...)
```

**Arguments**

object	An object of class "rrvglm".
...	Currently unused.

**Details**

The **A**, **B1**, **C** matrices are returned, along with other slots. See [rrvglm](#) for details about RR-VGLMs.

**Value**

An object of class "Coef.rrvglm" (see [Coef.rrvglm-class](#)).

**Note**

This function is an alternative to `coef.rrvglm`.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

**See Also**

[Coef.rrvglm-class](#), `print.Coef.rrvglm`, [rrvglm](#).

**Examples**

```
# Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3, multinomial, pneumo)
coef(fit, matrix = TRUE)
Coef(fit)
```

---

Coef.rrvglm-class	Class “Coef.rrvglm”
-------------------	---------------------

---

### Description

The most pertinent matrices and other quantities pertaining to a RR-VGLM.

### Objects from the Class

Objects can be created by calls of the form `Coef(object, ...)` where `object` is an object of class `rrvglm` (see [rrvglm-class](#)).

In this document,  $M$  is the number of linear predictors and  $n$  is the number of observations.

### Slots

**A:** Of class “matrix”, **A**.

**B1:** Of class “matrix”, **B1**.

**C:** Of class “matrix”, **C**.

**Rank:** The rank of the RR-VGLM.

**colx1.index:** Index of the columns of the “vlm”-type model matrix corresponding to the variables in **x1**. These correspond to **B1**.

**colx2.index:** Index of the columns of the “vlm”-type model matrix corresponding to the variables in **x2**. These correspond to the reduced-rank regression.

**Atilde:** Object of class “matrix”, the **A** matrix with the corner rows removed. Thus each of the elements have been estimated. This matrix is returned only if corner constraints were used.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

### See Also

[Coef.rrvglm](#), [rrvglm](#), [rrvglm-class](#), `print.Coef.rrvglm`.

### Examples

```
# Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3, multinomial, pneumo)
coef(fit, matrix = TRUE)
Coef(fit)
# print(Coef(fit), digits = 3)
```

---

Coef.vlm*Extract Model Coefficients for VLM Objects*

---

## Description

Amongst other things, this function applies inverse link functions to the parameters of intercept-only VGLMs.

## Usage

```
Coef.vlm(object, ...)
```

## Arguments

object	A fitted model.
...	Arguments which may be passed into <a href="#">coef</a> .

## Details

Most **VGAM** family functions apply a link function to the parameters, e.g., positive parameter are often have a log link, parameters between 0 and 1 have a logit link. This function can back-transform the parameter estimate to the original scale.

## Value

For intercept-only models (e.g., formula is  $y \sim 1$ ) the back-transformed parameter estimates can be returned.

## Warning

This function may not work for *all* **VGAM** family functions. You should check your results on some artificial data before applying it to models fitted to real data.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

## See Also

[Coef](#), [coef](#).

## Examples

```
set.seed(123); nn <- 1000
bdata <- data.frame(y = rbeta(nn, shape1 = 1, shape2 = 3))
fit <- vglm(y ~ 1, betaff, data = bdata, trace = TRUE) # intercept-only model
coef(fit, matrix = TRUE) # log scale
Coef(fit) # On the original scale
```

---

CommonVGAMffArguments    *Common VGAM family function Arguments*

---

## Description

Here is a description of some common and typical arguments found in many **VGAM** family functions, e.g., `lsigma`, `isigma`, `gsigma`, `nsimEI`, `parallel` and `zero`.

## Usage

```
TypicalVGAMfamilyFunction(lsigma = "loge",
                           isigma = NULL,
                           link.list = list("(Default)" = "identity",
                                             x2          = "loge",
                                             x3          = "logoff",
                                             x4          = "mlogit",
                                             x5          = "mlogit"),
                           earg.list = list("(Default)" = list(),
                                             x2          = list(),
                                             x3          = list(offset = -1),
                                             x4          = list(),
                                             x5          = list()),
                           gsigma = exp(-5:5),
                           parallel = TRUE,
                           shrinkage.init = 0.95,
                           nointercept = NULL, imethod = 1,
                           type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
                           probs.x = c(0.15, 0.85),
                           probs.y = c(0.25, 0.50, 0.75),
                           mv = FALSE, earg.link = FALSE,
                           whitespace = FALSE, bred = FALSE,
                           oim = FALSE, nsimEIM = 100, zero = NULL)
```

## Arguments

<code>lsigma</code>	Character. Link function applied to a parameter and not necessarily a mean. See <a href="#">Links</a> for a selection of choices. If there is only one parameter then this argument is often called link.
---------------------	---

link.list, earg.list

Some **VGAM** family functions (such as `normal.vcm`) implement models with potentially lots of parameter link functions. These two arguments allow many such links and extra arguments to be inputted more easily. One has something like `link.list = list("(Default)" = "identity", x2 = "loge", x3 = "logoff")` and `earg.list = list("(Default)" = list(), x2 = list(), x3 = "list(offset = -1)")`. Then any unnamed terms will have the default link with its corresponding extra argument. Note: the `mlogit` link is also possible, and if so, at least two instances of it are necessary. Then the last term is the baseline/reference group.

**isigma** Optional initial values can often be inputted using an argument beginning with "i". For example, "isigma" and "ilocation", or just "init" if there is one parameter. A value of NULL means a value is computed internally, i.e., a *self-starting* **VGAM** family function. If a failure to converge occurs make use of these types of arguments.

**gsigma** Grid-search initial values can be inputted using an argument beginning with "g", e.g., "gsigma", "gshape" and "gscale". If argument isigma is inputted then that has precedence over gsigma, etc.

If the grid search is 2-dimensional then it is advisable not to make the vectors too long as a nested for loop may be used. Ditto for 3-dimensions.

**parallel** A logical, or a simple formula specifying which terms have equal/unequal coefficients. The formula must be simple, i.e., additive with simple main effects terms. Interactions and nesting etc. are not handled. To handle complex formulas use the constraints argument (of `vglm` etc.); however, there is a lot more setting up involved and things will not be as convenient.

Here are some examples. 1. `parallel = TRUE ~ x2 + x5` means the parallelism assumption is only applied to  $X_2, X_5$  and the intercept. 2. `parallel = TRUE ~ -1` and `parallel = TRUE ~ 0` mean the parallelism assumption is applied to *no* variables at all. Similarly, `parallel = FALSE ~ -1` and `parallel = FALSE ~ 0` mean the parallelism assumption is applied to *all* the variables including the intercept. 3. `parallel = FALSE ~ x2 - 1` and `parallel = FALSE ~ x2 + 0` applies the parallelism constraint to all terms (including the intercept) except for  $X_2$ .

This argument is common in **VGAM** family functions for categorical responses, e.g., `cumulative`, `acat`, `cratio`, `sratio`. For the proportional odds model (`cumulative`) having parallel constraints applied to each explanatory variable (except for the intercepts) means the fitted probabilities do not become negative or greater than 1. However this parallelism or proportional-odds assumption ought to be checked.

**nsimEIM** Some **VGAM** family functions use simulation to obtain an approximate expected information matrix (EIM). For those that do, the `nsimEIM` argument specifies the number of random variates used per observation; the mean of `nsimEIM` random variates is taken. Thus `nsimEIM` controls the accuracy and a larger value may be necessary if the EIMs are not positive-definite. For intercept-only models ( $y \sim 1$ ) the value of `nsimEIM` can be smaller (since the common value used is also then taken as the mean over the observations), especially if the number of observations is large.

Some **VGAM** family functions provide two algorithms for estimating the EIM. If applicable, set `nsimEIM = NULL` to choose the other algorithm.

<code>imethod</code>	An integer with value 1 or 2 or 3 or ... which specifies the initialization method for some parameters or a specific parameter. If failure to converge occurs try the next higher value, and continue until success. For example, <code>imethod = 1</code> might be the method of moments, and <code>imethod = 2</code> might be another method. If no value of <code>imethod</code> works then it will be necessary to use arguments such as <code>isigma</code> . For many <b>VGAM</b> family functions it is advisable to try this argument with all possible values to safeguard against problems such as converging to a local solution. <b>VGAM</b> family functions with this argument usually correspond to a model or distribution that is relatively hard to fit successfully, therefore care is needed to ensure the global solution is obtained. So using all possible values that this argument supplies is a good idea.
<code>type.fitted</code>	Character. Type of fitted value returned by the <code>fitted()</code> methods function. The first choice is always the default. The available choices depends on what kind of family function it is. Using the first few letters of the chosen choice is okay. See <code>fittedvglm</code> for more details.
<code>probs.x, probs.y</code>	Numeric, with values in (0, 1). The probabilities that define quantiles with respect to some vector, usually an <code>x</code> or <code>y</code> of some sort. This is used to create two subsets of data corresponding to 'low' and 'high' values of <code>x</code> or <code>y</code> . Each value is separately fed into the <code>probs</code> argument of <code>quantile</code> . If the data set size is small then it may be necessary to increase/decrease slightly the first/second values respectively.
<code>whitespace</code>	Logical. Should white spaces ( " ") be used in the labelling of the linear/additive predictors? Setting TRUE usually results in more readability but it occupies more columns of the output.
<code>oim</code>	Logical. Should the observed information matrices (OIMs) be used for the working weights? In general, setting <code>oim = TRUE</code> means the Newton-Raphson algorithm, and <code>oim = FALSE</code> means Fisher-scoring. The latter uses the EIM, and is usually recommended. If <code>oim = TRUE</code> then <code>nsimEIM</code> is ignored.
<code>zero</code>	An integer specifying which linear/additive predictor is modelled as intercept-only. That is, the regression coefficients are set to zero for all covariates except for the intercept. If <code>zero</code> is specified then it may be a vector with values from the set $\{1, 2, \dots, M\}$ . The value <code>zero = NULL</code> means model <i>all</i> linear/additive predictors as functions of the explanatory variables. Here, $M$ is the number of linear/additive predictors. Technically, if <code>zero</code> contains the value $j$ then the $j$ th row of every constraint matrix (except for the intercept) consists of all 0 values. Some <b>VGAM</b> family functions allow the <code>zero</code> argument to accept negative values; if so then its absolute value is recycled over each (usual) response. For example, <code>zero = -2</code> for the two-parameter negative binomial distribution would mean, for each response, the second linear/additive predictor is modelled as intercepts-only. That is, for all the $k$ parameters in <code>negbinomial</code> (this <b>VGAM</b> family function can handle a matrix of responses).  Suppose <code>zero = zerovec</code> where <code>zerovec</code> is a vector of negative values. If $G$ is the usual $M$ value for a univariate response then the actual values for argument <code>zero</code> are all values in <code>c(abs(zerovec), G + abs(zerovec), 2*G + abs(zerovec), ...)</code> lying in the integer range 1 to $M$ . For example, setting <code>zero = -c(2, 3)</code> for a matrix response of 4 columns with <code>zinegbinomial</code> (which usually has $G =$



$M = 3$  for a univariate response) would be equivalent to `zero = c(2, 3, 5, 6, 8, 9, 11, 12)`. This example has  $M = 12$ . Note that if `zero` contains negative values then their absolute values should be elements from the set  $1:G$ .

Note: `zero` may have positive and negative values, for example, setting `zero = c(-2, 3)` in the above example would be equivalent to `zero = c(2, 3, 5, 8, 11)`.

<code>shrinkage.init</code>	Shrinkage factor $s$ used for obtaining initial values. Numeric, between 0 and 1. In general, the formula used is something like $s\mu + (1 - s)y$ where $\mu$ is a measure of central tendency such as a weighted mean or median, and $y$ is the response vector. For example, the initial values are slight perturbations of the mean towards the actual data. For many types of models this method seems to work well and is often reasonably robust to outliers in the response. Often this argument is only used if the argument <code>imethod</code> is assigned a certain value.
<code>nointercept</code>	An integer-valued vector specifying which linear/additive predictors have no intercepts. Any values must be from the set $\{1, 2, \dots, M\}$ . A value of <code>NULL</code> means no such constraints.
<code>mv</code>	Logical. Some <b>VGAM</b> family functions allow a multivariate or vector response. If so, then usually the response is a matrix with columns corresponding to the individual response variables. They are all fitted simultaneously. Arguments such as <code>parallel</code> may then be useful to allow for relationships between the regressions of each response variable. If <code>mv = TRUE</code> then sometimes the response is interpreted differently, e.g., <code>posbinomial</code> chooses the first column of a matrix response as success and combines the other columns as failure, but when <code>mv = TRUE</code> then each column of the response matrix is the number of successes and the <code>weights</code> argument is of the same dimension as the response and contains the number of trials.
<code>earg.link</code>	Sometimes the link argument can receive <code>earg</code> -type input, such as <code>quasibinomial</code> calling <code>binomial</code> . This argument should be generally ignored.
<code>bred</code>	Logical. Some <b>VGAM</b> family functions will allow bias-reduction based on the work by Kosmidis and Firth. Currently none are working yet!

## Details

Full details will be given in documentation yet to be written, at a later date!

## Value

An object of class `"vglmff"` (see `vglmff-class`). The object is used by modelling functions such as `vglm` and `vgam`.

## Warning

The `zero` argument is supplied for convenience but conflicts can arise with other arguments, e.g., the `constraints` argument of `vglm` and `vgam`. See Example 5 below for an example. If not sure, use, e.g., `constraints(fit)` and `coef(fit, matrix = TRUE)` to check the result of a fit.

The arguments `zero` and `nointercept` can be inputted with values that fail. For example, `multinomial(zero = 2, nointercept = 2)` means the second linear/additive predictor is identically zero, which will cause a failure.

Be careful about the use of other potentially contradictory constraints, e.g., `multinomial(zero = 2, parallel = TRUE ~ x)`. If in doubt, apply `constraints()` to the fitted object to check.

**VGAM** family functions with the `nsimEIM` may have inaccurate working weight matrices. If so, then the standard errors of the regression coefficients may be inaccurate. Thus output from `summary(fit)`, `vcov(fit)`, etc. may be misleading.

### Note

See [Links](#) regarding a major change in link functions, for version 0.9-0 and higher (released during the 2nd half of 2012).

### Author(s)

T. W. Yee

### References

Kosmidis, I. and Firth, D. (2009) Bias reduction in exponential family nonlinear models. *Biometrika*, **96**(4), 793–804.

### See Also

[Links](#), [vglmff-class](#), [normal.vcm](#), [mlogit](#).

### Examples

```
# Example 1
cumulative()
cumulative(link = "probit", reverse = TRUE, parallel = TRUE)

# Example 2
wdata <- data.frame(x2 = runif(nn <- 1000))
wdata <- transform(wdata,
  y = rweibull(nn, shape = 2 + exp(1 + x2), scale = exp(-0.5)))
fit <- vglm(y ~ x2, weibull(lshape = logoff(offset = -2), zero = 2), wdata)
coef(fit, mat = TRUE)

# Example 3; multivariate (multiple) response
## Not run:
ndata <- data.frame(x = runif(nn <- 500))
ndata <- transform(ndata,
  y1 = rnbino(n, mu = exp(3+x), size = exp(1)), # k is size
  y2 = rnbino(n, mu = exp(2-x), size = exp(0)))
fit <- vglm(cbind(y1, y2) ~ x, negbinomial(zero = -2), ndata)
coef(fit, matrix = TRUE)

## End(Not run)
# Example 4
## Not run:
# fit1 and fit2 are equivalent
fit1 <- vglm(ymatrix ~ x2 + x3 + x4 + x5,
```

```

      cumulative(parallel = FALSE ~ 1 + x3 + x5), mydataframe)
fit2 <- vglm(ymatrix ~ x2 + x3 + x4 + x5,
      cumulative(parallel = TRUE ~ x2 + x4), mydataframe)

## End(Not run)

# Example 5
gdata <- data.frame(x2 = rnorm(nn <- 200))
gdata <- transform(gdata,
      y1 = rnorm(nn, mean = 1 - 3*x2, sd = exp(1 + 0.2*x2)),
      y2 = rnorm(nn, mean = 1 - 3*x2, sd = exp(1)))
args(uninormal)
fit1 <- vglm(y1 ~ x2, uninormal, gdata)          # This is okay
fit2 <- vglm(y2 ~ x2, uninormal(zero = 2), gdata) # This is okay

# This creates potential conflict
clist <- list("(Intercept)" = diag(2), "x2" = diag(2))
fit3 <- vglm(y2 ~ x2, uninormal(zero = 2), gdata,
      constraints = clist) # Conflict!
coef(fit3, matrix = TRUE) # Shows that clist[["x2"]] was overwritten,
constraints(fit3) # i.e., 'zero' seems to override the 'constraints' arg

# Example 6 ('whitespace' argument)
pneumo <- transform(pneumo, let = log(exposure.time))
fit1 <- vglm(cbind(normal, mild, severe) ~ let,
      sratio(whitespace = FALSE, parallel = TRUE), pneumo)
fit2 <- vglm(cbind(normal, mild, severe) ~ let,
      sratio(whitespace = TRUE, parallel = TRUE), pneumo)
head(predict(fit1), 2) # No white spaces
head(predict(fit2), 2) # Uses white spaces

```

concoef

*Extract Model Constrained/Canonical Coefficients***Description**

concoef is a generic function which extracts the constrained (canonical) coefficients from objects returned by certain modelling functions.

**Usage**

```
concoef(object, ...)
```

**Arguments**

object	An object for which the extraction of canonical coefficients is meaningful.
...	Other arguments fed into the specific methods function of the model.

## Details

For constrained quadratic and ordination models, *canonical coefficients* are the elements of the **C** matrix used to form the latent variables. They are highly interpretable in ecology, and are looked at as weights or loadings.

They are also applicable for reduced-rank VGLMs.

## Value

The value returned depends specifically on the methods function invoked.

## Warning

[concoef](#) and [cccoef](#) are identical, but the latter will be deprecated soon.

For QO models, there is a direct inverse relationship between the scaling of the latent variables (site scores) and the tolerances. One normalization is for the latent variables to have unit variance. Another normalization is for all the species' tolerances to be unit (provided `EqualTolerances` is `TRUE`). These two normalizations cannot simultaneously hold in general. For rank  $R$  models with  $R > 1$  it becomes more complicated because the latent variables are also uncorrelated. An important argument when fitting quadratic ordination models is whether `EqualTolerances` is `TRUE` or `FALSE`. See Yee (2004) for details.

## Author(s)

Thomas W. Yee

## References

- Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.
- Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.
- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

## See Also

[concoef-method](#), `concoef.qrrvglm`, `concoef.cao`, [coef](#).

## Examples

```
## Not run: set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cqc(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
               Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          fam = quasipoissonff, data = hspider, Crowlpositive = FALSE)
concoef(p1)

## End(Not run)
```

---

concoef-methods	<i>Constrained (Canonical) Coefficients</i>
-----------------	---

---

### Description

concoef is a generic function used to return the constrained (canonical) coefficients of a constrained ordination model. The function invokes particular methods which depend on the class of the first argument.

### Methods

**object** The object from which the constrained coefficients are extracted.

---

constraints	<i>Constraint Matrices</i>
-------------	----------------------------

---

### Description

Extractor function for the *constraint matrices* of objects in the **VGAM** package.

### Usage

```
constraints(object, ...)
constraints.vlm(object, type = c("lm", "term"), all = TRUE, which,
               matrix.out = FALSE, colnames.arg = TRUE, ...)
```

### Arguments

object	Some <b>VGAM</b> object, for example, having class <code>vglmff-class</code> .
type	Character. Whether LM- or term-type constraints are to be returned. The number of such matrices returned is equal to <code>nvar(object, type = "lm")</code> and the number of terms, respectively.
all, which	If <code>all = FALSE</code> then which gives the integer index or a vector of logicals specifying the selection.
matrix.out	Logical. If TRUE then the constraint matrices are <code>cbind()</code> ed together. The result is usually more compact because the default is a list of constraint matrices.
colnames.arg	Logical. If TRUE then column names are assigned corresponding to the variables.
...	Other possible arguments such as <code>type</code> .

## Details

Constraint matrices describe the relationship of coefficients/component functions of a particular explanatory variable between the linear/additive predictors in VGLM/VGAM models. For example, they may be all different (constraint matrix is the identity matrix) or all the same (constraint matrix has one column and has unit values).

VGLMs and VGAMs have constraint matrices which are *known*. The class of RR-VGLMs have constraint matrices which are *unknown* and are to be estimated.

## Value

The extractor function `constraints()` returns a list comprising of constraint matrices—usually one for each column of the VLM model matrix, and in that order. The list is labelled with the variable names. Each constraint matrix has  $M$  rows, where  $M$  is the number of linear/additive predictors, and whose rank is equal to the number of columns. A model with no constraints at all has an order  $M$  identity matrix as each variable's constraint matrix.

For `vglm` and `vgam` objects, feeding in `type = "term"` constraint matrices back into the same model should work and give an identical model. The default are the "lm"-type constraint matrices; this is a list with one constraint matrix per column of the LM matrix. See the `constraints` argument of `vglm`, and the example below.

## Note

In all **VGAM** family functions `zero = NULL` means none of the linear/additive predictors are modelled as intercepts-only. Other arguments found in certain **VGAM** family functions which affect constraint matrices include `parallel` and `exchangeable`.

The `constraints` argument in `vglm` and `vgam` allows constraint matrices to be inputted. If so, then `constraints(fit, type = "lm")` can be fed into the `constraints` argument of the same object to get the same model.

The `xij` argument does not affect constraint matrices; rather, it allows each row of the constraint matrix to be multiplied by a specified vector.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

<http://www.stat.auckland.ac.nz/~yee> contains additional information.

**See Also**

[is.parallel](#), [is.zero](#). VGLMs are described in [vglm-class](#); RR-VGLMs are described in [rrvglm-class](#).

Arguments such as `zero` and `parallel` found in many **VGAM** family functions are a way of creating/modifying constraint matrices conveniently, e.g., see [zero](#). See [CommonVGAMffArguments](#) for more information.

**Examples**

```
# Fit the proportional odds model:
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ bs(let, 3),
             cumulative(parallel = TRUE, reverse = TRUE), pneumo))
coef(fit1, matrix = TRUE)
constraints(fit1) # Parallel assumption results in this
constraints(fit1, type = "term") # This is the same as the default ("vlm"-type)
is.parallel(fit1)

# An equivalent model to fit1 (needs the type "term" constraints):
clist.term <- constraints(fit1, type = "term") # The "term"-type constraints
(fit2 <- vglm(cbind(normal, mild, severe) ~ bs(let, 3),
             cumulative(reverse = TRUE), pneumo, constraints = clist.term))
abs(max(coef(fit1, matrix = TRUE) -
        coef(fit2, matrix = TRUE))) # Should be zero

# Fit a rank-1 stereotype (RR-multinomial logit) model:
data(car.all)
fit <- rrvglm(Country ~ Width + Height + HP, multinomial, car.all, Rank = 1)
constraints(fit) # All except the first are the estimated A matrix
```

---

corbet

---

*Corbet's Butterfly Data*


---

**Description**

About 3300 individual butterflies were caught in Malaya by naturalist Corbet trapping butterflies. They were classified to about 500 species.

**Usage**

```
data(corbet)
```

**Format**

A data frame with 24 observations on the following 2 variables.

`species` Number of species.

`ofreq` Observed frequency of individual butterflies of that species.

## Details

In the early 1940s Corbet spent two years trapping butterflies in Malaya. Of interest was the total number of species. Some species were so rare (e.g., 118 species had only one specimen) that it was thought likely that there were many unknown species.

## References

Fisher, R. A., Corbet, A. S. and Williams, C. B. (1943) The Relation Between the Number of Species and the Number of Individuals in a Random Sample of an Animal Population. *Journal of Animal Ecology*, **12**, 42–58.

## Examples

```
summary(corbet)
```

---

cqo

*Fitting Constrained Quadratic Ordination (CQO)*


---

## Description

A *constrained quadratic ordination* (CQO; formerly called *canonical Gaussian ordination* or CGO) model is fitted using the *quadratic reduced-rank vector generalized linear model* (QRR-VGLM) framework.

## Usage

```
cqo(formula, family, data = list(), weights = NULL, subset = NULL,
     na.action = na.fail, etastart = NULL, mustart = NULL,
     coefstart = NULL, control = qrrvglm.control(...), offset = NULL,
     method = "cqo.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
     contrasts = NULL, constraints = NULL, extra = NULL,
     smart = TRUE, ...)
```

## Arguments

formula	a symbolic description of the model to be fit. The RHS of the formula is applied to each linear predictor. Different variables in each linear predictor can be chosen by specifying constraint matrices.
family	a function of class "vglmff" (see <a href="#">vglmff-class</a> ) describing what statistical model is to be fitted. This is called a “ <b>VGAM</b> family function”. See <a href="#">CommonVGAMffArguments</a> for general information about many types of arguments found in this type of function. Currently the following families are supported: <a href="#">poissonff</a> , <a href="#">binomialff</a> ( <a href="#">logit</a> and <a href="#">cloglog</a> links available), <a href="#">negbinomial</a> , <a href="#">gamma2</a> , <a href="#">gaussianff</a> . Sometimes special arguments are required for <code>cqo()</code> , e.g., <code>binomialff(mv = TRUE)</code> . Also, <a href="#">quasipoissonff</a> and <a href="#">quasibinomialff</a> may or may not work.



<code>data</code>	an optional data frame containing the variables in the model. By default the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>cgo</code> is called.
<code>weights</code>	an optional vector or matrix of (prior) weights to be used in the fitting process. Currently, this argument should not be used.
<code>subset</code>	an optional logical vector specifying a subset of observations to be used in the fitting process.
<code>na.action</code>	a function which indicates what should happen when the data contain NAs. The default is set by the <code>na.action</code> setting of <code>options</code> , and is <code>na.fail</code> if that is unset. The “factory-fresh” default is <code>na.omit</code> .
<code>etastart</code>	starting values for the linear predictors. It is a $M$ -column matrix. If $M = 1$ then it may be a vector. Currently, this argument probably should not be used.
<code>mustart</code>	starting values for the fitted values. It can be a vector or a matrix. Some family functions do not make use of this argument. Currently, this argument probably should not be used.
<code>coefstart</code>	starting values for the coefficient vector. Currently, this argument probably should not be used.
<code>control</code>	a list of parameters for controlling the fitting process. See <code>qrrvglm.control</code> for details.
<code>offset</code>	This argument must not be used.
<code>method</code>	the method to be used in fitting the model. The default (and presently only) method <code>cgo.fit</code> uses <i>iteratively reweighted least squares</i> (IRLS).
<code>model</code>	a logical value indicating whether the <i>model frame</i> should be assigned in the model slot.
<code>x.arg, y.arg</code>	logical values indicating whether the model matrix and response matrix used in the fitting process should be assigned in the x and y slots. Note the model matrix is the LM model matrix.
<code>contrasts</code>	an optional list. See the <code>contrasts.arg</code> of <code>model.matrix.default</code> .
<code>constraints</code>	an optional list of constraint matrices. The components of the list must be named with the term it corresponds to (and it must match in character format). Each constraint matrix must have $M$ rows, and be of full-column rank. By default, constraint matrices are the $M$ by $M$ identity matrix unless arguments in the family function itself override these values. If <code>constraints</code> is used it must contain <i>all</i> the terms; an incomplete list is not accepted. Constraint matrices for $x_2$ variables are taken as the identity matrix.
<code>extra</code>	an optional list with any extra information that might be needed by the family function.
<code>smart</code>	logical value indicating whether smart prediction ( <code>smartpred</code> ) will be used.
<code>...</code>	further arguments passed into <code>qrrvglm.control</code> .

## Details

QRR-VGLMs or *constrained quadratic ordination* (CQO) models are estimated here by maximum likelihood estimation. Optimal linear combinations of the environmental variables are computed,

called *latent variables* (these appear as `latvar` for  $R = 1$  else `latvar1`, `latvar2`, etc. in the output). Here,  $R$  is the *rank* or the number of ordination axes. Each species' response is then a regression of these latent variables using quadratic polynomials on a transformed scale (e.g., log for Poisson counts, logit for presence/absence responses). The solution is obtained iteratively in order to maximize the log-likelihood function, or equivalently, minimize the deviance.

The central formula (for Poisson and binomial species data) is given by

$$\eta = B_1^T x_1 + A\nu + \sum_{m=1}^M (\nu^T D_m \nu) e_m$$

where  $x_1$  is a vector (usually just a 1 for an intercept),  $x_2$  is a vector of environmental variables,  $\nu = C^T x_2$  is a  $R$ -vector of latent variables,  $e_m$  is a vector of 0s but with a 1 in the  $m$ th position. The  $\eta$  are a vector of linear/additive predictors, e.g., the  $m$ th element is  $\eta_m = \log(E[Y_m])$  for the  $m$ th species. The matrices  $B_1$ ,  $A$ ,  $C$  and  $D_m$  are estimated from the data, i.e., contain the regression coefficients. The tolerance matrices satisfy  $T_s = -\frac{1}{2} D_s^{-1}$ . Many important CQO details are directly related to arguments in `qrrvglm.control`, e.g., the argument `noRRR` specifies which variables comprise  $x_1$ .

Theoretically, the four most popular **VGAM** family functions to be used with `cqo` correspond to the Poisson, binomial, normal, and negative binomial distributions. The latter is a 2-parameter model. All of these are implemented, as well as the 2-parameter gamma. The Poisson is or should be catered for by `quasipoissonff` and `poissonff`, and the binomial by `quasibinomialff` and `binomialff`. Those beginning with "quasi" have dispersion parameters that are estimated for each species.

For initial values, the function `.Init.Poisson.QO` should work reasonably well if the data is Poisson with species having equal tolerances. It can be quite good on binary data too. Otherwise the `Cinit` argument in `qrrvglm.control` can be used.

It is possible to relax the quadratic form to an additive model. The result is a data-driven approach rather than a model-driven approach, so that CQO is extended to *constrained additive ordination* (CAO) when  $R = 1$ . See `cao` for more details.

In this documentation,  $M$  is the number of linear predictors,  $S$  is the number of responses (species). Then  $M = S$  for Poisson and binomial species data, and  $M = 2S$  for negative binomial and gamma distributed species data.

## Value

An object of class "qrrvglm". Note that the slot `misc` has a list component called `deviance.Bestof` which gives the history of deviances over all the iterations.

## Warning

Local solutions are not uncommon when fitting CQO models. To increase the chances of obtaining the global solution, increase the value of the argument `Bestof` in `qrrvglm.control`. For reproducibility of the results, it pays to set a different random number seed before calling `cqo` (the function `set.seed` does this). The function `cqo` chooses initial values for  $\mathbf{C}$  using `.Init.Poisson.QO()` if `Use.Init.Poisson.QO = TRUE`, else random numbers.

Unless `ITolerances = TRUE` or `EqualTolerances = FALSE`, CQO is computationally expensive with memory and time. It pays to keep the rank down to 1 or 2. If `EqualTolerances = TRUE` and `ITolerances = FALSE` then the cost grows quickly with the number of species and sites (in terms

of memory requirements and time). The data needs to conform quite closely to the statistical model, and the environmental range of the data should be wide in order for the quadratics to fit the data well (bell-shaped response surfaces). If not, RR-VGLMs will be more appropriate because the response is linear on the transformed scale (e.g., log or logit) and the ordination is called *constrained linear ordination* or CLO.

Like many regression models, CQO is sensitive to outliers (in the environmental and species data), sparse data, high leverage points, multicollinearity etc. For these reasons, it is necessary to examine the data carefully for these features and take corrective action (e.g., omitting certain species, sites, environmental variables from the analysis, transforming certain environmental variables, etc.). Any optimum lying outside the convex hull of the site scores should not be trusted. Fitting a CAO is recommended first, then upon transformations etc., possibly a CQO can be fitted.

For binary data, it is necessary to have ‘enough’ data. In general, the number of sites  $n$  ought to be much larger than the number of species  $S$ , e.g., at least 100 sites for two species. Compared to count (Poisson) data, numerical problems occur more frequently with presence/absence (binary) data. For example, if Rank = 1 and if the response data for each species is a string of all absences, then all presences, then all absences (when enumerated along the latent variable) then infinite parameter estimates will occur. In general, setting ITolerance = TRUE may help.

This function was formerly called cgo. It has been renamed to reinforce a new nomenclature described in Yee (2006).

#### Note

The input requires care, preparation and thought—a *lot more* than other ordination methods. Here is a partial **checklist**.

- (1) The number of species should be kept reasonably low, e.g., 12 max. Feeding in 100+ species wholesale is a recipe for failure. Choose a few species carefully. Using 10 well-chosen species is better than 100+ species thrown in willy-nilly.
- (2) Each species should be screened individually first, e.g., for presence/absence is the species totally absent or totally present at all sites? For presence/absence data `sort(colMeans(data))` can help avoid such species.
- (3) The number of explanatory variables should be kept low, e.g., 7 max.
- (4) Each explanatory variable should be screened individually first, e.g., is it heavily skewed or are there outliers? They should be plotted and then transformed where needed. They should not be too highly correlated with each other.
- (5) Each explanatory variable should be scaled, e.g., to mean 0 and unit variance. This is especially needed for ITolerance = TRUE.
- (6) Keep the rank low. Only if the data is very good should a rank-2 model be attempted. Usually a rank-1 model is all that is practically possible even after a lot of work. The rank-1 model should always be attempted first. Then might be clever and try use this for initial values for a rank-2 model.
- (7) If the number of sites is large then choose a random sample of them. For example, choose a maximum of 500 sites. This will reduce the memory and time expense of the computations.
- (8) Try ITolerance = TRUE or EqualTolerance = FALSE if the inputted data set is large, so as to reduce the computational expense. That’s because the default, ITolerance = FALSE and EqualTolerance = TRUE, is very memory hungry.

By default, a rank-1 equal-tolerances QRR-VGLM model is fitted (see `qrrvglm.control` for the default control parameters). If `Rank > 1` then the latent variables are always transformed so that they are uncorrelated. By default, the argument `trace` is `TRUE` meaning a running log is printed out while the computations are taking place. This is because the algorithm is computationally expensive, therefore users might think that their computers have frozen if `trace = FALSE`!

The argument `Bestof` in `qrrvglm.control` controls the number of models fitted (each uses different starting values) to the data. This argument is important because convergence may be to a *local* solution rather than the *global* solution. Using more starting values increases the chances of finding the global solution. Always plot an ordination diagram (use the generic function `lvplot`) and see if it looks sensible. Local solutions arise because the optimization problem is highly nonlinear, and this is particularly true for CAO.

Many of the arguments applicable to `cqo` are common to `vglm` and `rrvglm.control`. The most important arguments are `Rank`, `noRRR`, `Bestof`, `ITolerances`, `EqualTolerances`, `isd.latvar`, and `MUXfactor`.

When fitting a 2-parameter model such as the negative binomial or gamma, it pays to have `EqualTolerances = TRUE` and `ITolerances = FALSE`. This is because numerical problems can occur when fitting the model far away from the global solution when `ITolerances = TRUE`. Setting the two arguments as described will slow down the computation considerably, however it is numerically more stable.

In Example 1 below, an unequal-tolerances rank-1 QRR-VGLM is fitted to the hunting spiders dataset, and Example 2 is the equal-tolerances version. The latter is less likely to have convergence problems compared to the unequal-tolerances model. In Example 3 below, an equal-tolerances rank-2 QRR-VGLM is fitted to the hunting spiders dataset. The numerical difficulties encountered in fitting the rank-2 model suggests a rank-1 model is probably preferable. In Example 4 below, constrained binary quadratic ordination (in old nomenclature, constrained Gaussian logit ordination) is fitted to some simulated data coming from a species packing model. With multivariate binary responses, one must use `mv = TRUE` to indicate that the response (matrix) is multivariate. Otherwise, it is interpreted as a single binary response variable. In Example 5 below, the deviance residuals are plotted for each species. This is useful as a diagnostic plot. This is done by (re)regressing each species separately against the latent variable.

Sometime in the future, this function might handle input of the form `cqo(x, y)`, where `x` and `y` are matrices containing the environmental and species data respectively.

### Author(s)

Thomas W. Yee. Thanks to Alvin Sou for converting a lot of the original FORTRAN code into C.

### References

- Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.
- ter Braak, C. J. F. and Prentice, I. C. (1988) A theory of gradient analysis. *Advances in Ecological Research*, **18**, 271–317.
- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

### See Also

`qrrvglm.control`, `Coef.qrrvglm`, `predictqrrvglm`, `rcqo`, `cao`,

```
rrvglm,
poissonff, binomialff, negbinomial, gamma2, lvplot.qrrvglm, persp.qrrvglm,
trplot.qrrvglm,
vglm, set.seed, hspider.
```

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## Examples

```
## Not run:
# Example 1; Fit an unequal tolerances model to the hunting spiders data
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental variables
set.seed(1234) # For reproducibility of the results
p1ut <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
                 Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
                 Trocterr, Zoraspin) ~
            WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
            fam = poissonff, data = hspider, Crowlpositive = FALSE,
            EqualTol = FALSE)
sort(p1ut@misc$deviance.Bestof) # A history of all the iterations
if(deviance(p1ut) > 1177) warning("suboptimal fit obtained")

S <- ncol(depvar(p1ut)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
lvplot(p1ut, y = TRUE, lcol = clr, pch = 1:S, pcol = clr, las = 1) # Ordination diagram
legend("topright", leg = colnames(depvar(p1ut)), col = clr,
       pch = 1:S, merge = TRUE, bty = "n", lty = 1:S, lwd = 2)
(cp <- Coef(p1ut))

(a <- cp@latvar[cp@latvar.order]) # The ordered site scores along the gradient
# Names of the ordered sites along the gradient:
rownames(cp@latvar)[cp@latvar.order]
(aa <- (cp@Optimum)[,cp@Optimum.order]) # The ordered optima along the gradient
aa <- aa[!is.na(aa)] # Delete the species that is not unimodal
names(aa) # Names of the ordered optima along the gradient

trplot(p1ut, which.species = 1:3, log = "xy", type = "b", lty = 1, lwd = 2,
       col = c("blue", "red", "green"), label = TRUE) -> ii # Trajectory plot
legend(0.00005, 0.3, paste(ii$species[, 1], ii$species[, 2], sep = " and "),
       lwd = 2, lty = 1, col = c("blue", "red", "green"))
abline(a = 0, b = 1, lty = "dashed")

S <- ncol(depvar(p1ut)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
persp(p1ut, col = clr, label = TRUE, las = 1) # Perspective plot

# Example 2; Fit an equal tolerances model. Less numerically fraught.
set.seed(1234)
p1et <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
                 Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
                 Trocterr, Zoraspin) ~
```

```

      WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
      poissonff, data = hspider, Crowlpositive = FALSE)
sort(p1et@misc$deviance.Bestof) # A history of all the iterations
if (deviance(p1et) > 1586) warning("suboptimal fit obtained")
S <- ncol(depvar(p1et)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
persp(p1et, col = clr, label = TRUE, las = 1)

# Example 3: A rank-2 equal tolerances CQO model with Poisson data
# This example is numerically fraught... need IToler = TRUE too.
set.seed(555)
p2 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
  Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
  Trocterr, Zoraspin) ~
  WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
  poissonff, data = hspider, Crowlpositive = FALSE,
  IToler = TRUE, Rank = 2, Bestof = 3, isd.latvar = c(2.1, 0.9))
sort(p2@misc$deviance.Bestof) # A history of all the iterations
if (deviance(p2) > 1127) warning("suboptimal fit obtained")
lvplot(p2, ellips = FALSE, label = TRUE, xlim = c(-3,4),
  C = TRUE, Ccol = "brown", sites = TRUE, scol = "grey",
  pcol = "blue", pch = "+", chull = TRUE, ccol = "grey")

# Example 4: species packing model with presence/absence data
set.seed(2345)
n <- 200; p <- 5; S <- 5
mydata <- rcqo(n, p, S, fam = "binomial", hi.abundance = 4,
  eq.tol = TRUE, es.opt = TRUE, eq.max = TRUE)
myform <- attr(mydata, "formula")
set.seed(1234)
b1et <- cqo(myform, binomialff(mv = TRUE, link = "cloglog"), data = mydata)
sort(b1et@misc$deviance.Bestof) # A history of all the iterations
lvplot(b1et, y = TRUE, lcol = 1:S, pch = 1:S, pcol = 1:S, las = 1)
Coef(b1et)

# Compare the fitted model with the 'truth'
cbind(truth = attr(mydata, "concoefficients"), fitted = concoef(b1et))

# Example 5: Plot the deviance residuals for diagnostic purposes
set.seed(1234)
p1et <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
  Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
  Trocterr, Zoraspin) ~
  WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
  poissonff, data = hspider, EqualTol = TRUE, trace = FALSE)
sort(p1et@misc$deviance.Bestof) # A history of all the iterations
if (deviance(p1et) > 1586) warning("suboptimal fit obtained")
S <- ncol(depvar(p1et))
par(mfrow = c(3, 4))
for (ii in 1:S) {

```

```

tempdata <- data.frame(latvar1 = c(latvar(p1et)), sppCounts = depvar(p1et)[, ii])
tempdata <- transform(tempdata, myOffset = -0.5 * latvar1^2)

# For species ii, refit the model to get the deviance residuals
fit1 <- vglm(sppCounts ~ offset(myOffset) + latvar1, poissonff,
             data = tempdata, trace = FALSE)

# For checking: this should be 0
# print("max(abs(c(Coef(p1et)@B1[1,ii], Coef(p1et)@A[ii,1]) - coef(fit1)))")
# print( max(abs(c(Coef(p1et)@B1[1,ii], Coef(p1et)@A[ii,1]) - coef(fit1))) )

# Plot the deviance residuals
devresid <- resid(fit1, type = "deviance")
predvalues <- predict(fit1) + fit1@offset
ooo <- with(tempdata, order(latvar1))
with(tempdata, plot(latvar1, predvalues + devresid, col = "darkgreen",
                   xlab = "latvar1", ylab = "", main = colnames(depvar(p1et))[ii]))
with(tempdata, lines(latvar1[ooo], predvalues[ooo], col = "blue"))
}

## End(Not run)

```

---

crashes

---

*Crashes on New Zealand Roads in 2009*


---

## Description

A variety of reported crash data cross-classified by time (hour of the day) and day of the week, accumulated over 2009. These include fatalities and injuries (by car), trucks, motor cycles, bicycles and pedestrians. There are some alcohol-related data too.

## Usage

```

data(crashi)
data(crashf)
data(crashtr)
data(crashmc)
data(crashbc)
data(crashp)
data(alcoff)
data(alclevels)

```

## Format

Data frames with hourly times as rows and days of the week as columns. The `alclevels` dataset has hourly times and alcohol levels.

**Mon, Tue, Wed, Thu, Fri, Sat, Sun** Day of the week.

**0-30, 31-50, 51-80, 81-100, 101-120, 121-150, 151-200, 201-250, 251-300, 301-350, 350+** Blood alcohol level (milligrams alcohol per 100 millilitres of blood).

## Details

Each cell is the aggregate number of crashes reported at each hour-day combination, over the 2009 calendar year. The rownames of each data frame is the start time (hourly from midnight onwards) on a 24 hour clock, e.g., 21 means 9.00pm to 9.59pm.

For crashes, `chrashi` are the number of injuries by car, `crashf` are the number of fatalities by car (not included in `chrashi`), `crashtr` are the number of crashes involving trucks, `crashmc` are the number of crashes involving motorcyclists, `crashbc` are the number of crashes involving bicycles, and `crashp` are the number of crashes involving pedestrians. For alcohol-related offences, `alcoff` are the number of alcohol offenders from breath screening drivers, and `alclevels` are the blood alcohol levels of fatally injured drivers.

## Source

<http://www.transport.govt.nz/research/Pages/Motor-Vehicle-Crashes-in-New-Zealand-2009.aspx>. Thanks to Warwick Goold and Alfian F. Hadi for assistance.

## References

Motor Vehicles Crashes in New Zealand 2009; Statistical Statement Calendar Year 2009. Ministry of Transport, NZ Government; Yearly Report 2010. ISSN: 1176-3949

## See Also

[rrvglm](#), [rcim](#), [grc](#).

## Examples

```
## Not run: plot(unlist(alcoff), type = "l", frame.plot = TRUE,
  axes = FALSE, col = "blue", bty = "o",
  main = "Alcoholic offenders on NZ roads, aggregated over 2009",
  sub = "Vertical lines at midnight (purple) and noon (orange)",
  xlab = "Day/hour", ylab = "Number of offenders")
axis(1, at = 1 + (0:6) * 24 + 12, labels = colnames(alcoff))
axis(2, las = 1)
axis(3:4, labels = FALSE, tick = FALSE)
abline(v = sort(1 + c((0:7) * 24, (0:6) * 24 + 12)), lty = "dashed",
  col = c("purple", "orange"))
## End(Not run)

# Goodmans RC models
## Not run:
fitgrc1 <- grc(alcoff) # Rank-1 model
fitgrc2 <- grc(alcoff, Rank = 2, Corner = FALSE, Uncor = TRUE)
Coef(fitgrc2)

## End(Not run)
## Not run: biplot(fitgrc2, scaleA = 2.3, Ccol = "blue", Acol = "orange",
  Clabels = as.character(1:23), xlim = c(-1.3, 2.3),
  ylim = c(-1.2, 1))
## End(Not run)
```



cratio

*Ordinal Regression with Continuation Ratios***Description**

Fits a continuation ratio logit/probit/cloglog/cauchit/... regression model to an ordered (preferably) factor response.

**Usage**

```
cratio(link = "logit", parallel = FALSE, reverse = FALSE, zero = NULL,
       whitespace = FALSE)
```

**Arguments**

link	Link function applied to the $M$ continuation ratio probabilities. See <a href="#">Links</a> for more choices.
parallel	A logical, or formula specifying which terms have equal/unequal coefficients.
reverse	Logical. By default, the continuation ratios used are $\eta_j = \text{logit}(P[Y > j   Y \geq j])$ for $j = 1, \dots, M$ . If reverse is TRUE, then $\eta_j = \text{logit}(P[Y < j + 1   Y \leq j + 1])$ will be used.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ . The default value means none are modelled as intercept-only terms.
whitespace	See <a href="#">CommonVGAMffArguments</a> for information.

**Details**

In this help file the response  $Y$  is assumed to be a factor with ordered values  $1, 2, \dots, M + 1$ , so that  $M$  is the number of linear/additive predictors  $\eta_j$ .

There are a number of definitions for the *continuation ratio* in the literature. To make life easier, in the **VGAM** package, we use *continuation* ratios and *stopping* ratios (see [sratio](#)). Stopping ratios deal with quantities such as  $\text{logit}(P[Y=j | Y \geq j])$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Warning**

No check is made to verify that the response is ordinal if the response is a matrix; see [ordered](#).

**Note**

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the y slot returned by `vglm/vgam/rrvglm` is the matrix of counts.

For a nominal (unordered) factor response, the multinomial logit model ([multinomial](#)) is more appropriate.

Here is an example of the usage of the `parallel` argument. If there are covariates  $x_1$ ,  $x_2$  and  $x_3$ , then `parallel = TRUE ~ x1 + x2 -1` and `parallel = FALSE ~ x3` are equivalent. This would constrain the regression coefficients for  $x_1$  and  $x_2$  to be equal; those of the intercepts and  $x_3$  would be different.

**Author(s)**

Thomas W. Yee

**References**

- Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.
- Simonoff, J. S. (2003) *Analyzing Categorical Data*, New York: Springer-Verlag.
- McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.
- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.

**See Also**

[sratio](#), [acat](#), [cumulative](#), [multinomial](#), [pneumo](#), [logit](#), [probit](#), [cloglog](#), [cauchit](#).

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
             cratio(parallel = TRUE), pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
predict(fit)
predict(fit, untransform = TRUE)
```

---

cumulative

*Ordinal Regression with Cumulative Probabilities*

---

**Description**

Fits a cumulative link regression model to a (preferably ordered) factor response.

**Usage**

```
cumulative(link = "logit", parallel = FALSE, reverse = FALSE,
           mv = FALSE, whitespace = FALSE)
```

**Arguments**

link	Link function applied to the $J$ cumulative probabilities. See <a href="#">Links</a> for more choices, e.g., for the cumulative <a href="#">probit</a> / <a href="#">cloglog</a> / <a href="#">cauchit</a> /... models.
parallel	A logical or formula specifying which terms have equal/unequal coefficients. See below for more information about the parallelism assumption. The default results in what some people call the <i>generalized ordered logit model</i> to be fitted. If <code>parallel = TRUE</code> then it does not apply to the intercept.
reverse	Logical. By default, the cumulative probabilities used are $P(Y \leq 1)$ , $P(Y \leq 2)$ , ..., $P(Y \leq J)$ . If <code>reverse</code> is <code>TRUE</code> then $P(Y \geq 2)$ , $P(Y \geq 3)$ , ..., $P(Y \geq J + 1)$ are used. This should be set to <code>TRUE</code> for <code>link = golf, polf, nbolf</code> . For these links the cutpoints must be an increasing sequence; if <code>reverse = FALSE</code> for then the cutpoints must be an decreasing sequence.
mv	Logical. Multivariate response? If <code>TRUE</code> then the input should be a matrix with values $1, 2, \dots, L$ , where $L = J + 1$ is the number of levels. Each column of the matrix is a response, i.e., multivariate response. A suitable matrix can be obtained from <code>Cut</code> .
whitespace	See <a href="#">CommonVGAMffArguments</a> for information.

**Details**

In this help file the response  $Y$  is assumed to be a factor with ordered values  $1, 2, \dots, J + 1$ . Hence  $M$  is the number of linear/additive predictors  $\eta_j$ ; for `cumulative()` one has  $M = J$ .

This **VGAM** family function fits the class of *cumulative link models* to (hopefully) an ordinal response. By default, the *non-parallel* cumulative logit model is fitted, i.e.,

$$\eta_j = \text{logit}(P[Y \leq j])$$

where  $j = 1, 2, \dots, M$  and the  $\eta_j$  are not constrained to be parallel. This is also known as the *non-proportional odds model*. If the logit link is replaced by a complementary log-log link ([cloglog](#)) then this is known as the *proportional-hazards model*.

In almost all the literature, the constraint matrices associated with this family of models are known. For example, setting `parallel = TRUE` will make all constraint matrices (except for the intercept) equal to a vector of  $M$  1's. If the constraint matrices are equal, unknown and to be estimated, then this can be achieved by fitting the model as a reduced-rank vector generalized linear model (RR-VGLM; see [rrvglm](#)). Currently, reduced-rank vector generalized additive models (RR-VGAMs) have not been implemented here.

**Value**

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

No check is made to verify that the response is ordinal if the response is a matrix; see [ordered](#).

**Note**

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the `y` slot returned by `vglm/vgam/rrvglm` is the matrix of counts. The formula must contain an intercept term. Other **VGAM** family functions for an ordinal response include [acat](#), [cratio](#), [sratio](#). For a nominal (unordered) factor response, the multinomial logit model ([multinomial](#)) is more appropriate.

With the logit link, setting `parallel = TRUE` will fit a proportional odds model. Note that the `TRUE` here does not apply to the intercept term. In practice, the validity of the proportional odds assumption needs to be checked, e.g., by a likelihood ratio test (LRT). If acceptable on the data, then numerical problems are less likely to occur during the fitting, and there are less parameters. Numerical problems occur when the linear/additive predictors cross, which results in probabilities outside of (0, 1); setting `parallel = TRUE` will help avoid this problem.

Here is an example of the usage of the `parallel` argument. If there are covariates `x2`, `x3` and `x4`, then `parallel = TRUE ~ x2 + x3 -1` and `parallel = FALSE ~ x4` are equivalent. This would constrain the regression coefficients for `x2` and `x3` to be equal; those of the intercepts and `x4` would be different.

If the data is inputted in *long* format (not *wide* format, as in [pneumo](#) below) and the self-starting initial values are not good enough then try using `mustart`, `coefstart` and/or `etatstart`. See the example below.

To fit the proportional odds model one can use the **VGAM** family function [propodds](#). Note that `propodds(reverse)` is equivalent to `cumulative(parallel = TRUE, reverse = reverse)` (which is equivalent to `cumulative(parallel = TRUE, reverse = reverse, link = "logit")`). It is for convenience only. A call to `cumulative()` is preferred since it reminds the user that a parallelism assumption is made, as well as being a lot more flexible.

**Author(s)**

Thomas W. Yee

**References**

- Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.
- Agresti, A. (2010) *Analysis of Ordinal Categorical Data*, 2nd ed. New York: Wiley.
- Dobson, A. J. and Barnett, A. (2008) *An Introduction to Generalized Linear Models*, 3rd ed. Boca Raton: Chapman & Hall/CRC Press.
- McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.
- Simonoff, J. S. (2003) *Analyzing Categorical Data*, New York: Springer-Verlag.
- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

Further information and examples on categorical data analysis by the **VGAM** package can be found at <http://www.stat.auckland.ac.nz/~yee/VGAM/doc/categorical.pdf>.

### See Also

[propodds](#), [prplot](#), [margeff](#), [acat](#), [cratio](#), [sratio](#), [multinomial](#), [pneumo](#), [Links](#), [logit](#), [probit](#), [cloglog](#), [cauchit](#), [golf](#), [polf](#), [nbolf](#), [logistic1](#).

### Examples

```
# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = TRUE, reverse = TRUE), pneumo))
depvar(fit) # Sample proportions (good technique)
fit@y      # Sample proportions (bad technique)
weights(fit, type = "prior") # Number of observations
coef(fit, matrix = TRUE)
constraints(fit) # Constraint matrices
apply(fitted(fit), 1, which.max) # Classification
apply(predict(fit, newdata = pneumo, type = "response"), 1, which.max) # Classification

# Check that the model is linear in let -----
fit2 <- vgam(cbind(normal, mild, severe) ~ s(let, df = 2),
             cumulative(reverse = TRUE), pneumo)
## Not run: plot(fit2, se = TRUE, overlay = TRUE, lcol = 1:2, scol = 1:2)

# Check the proportional odds assumption with a LRT -----
(fit3 <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = FALSE, reverse = TRUE), pneumo))
pchisq(2 * (logLik(fit3) - logLik(fit)),
      df = length(coef(fit3)) - length(coef(fit)), lower.tail = FALSE)
lrtest(fit3, fit) # More elegant

# A factor() version of fit -----
# This is in long format (cf. wide format above)
Nobs <- round(depvar(fit) * c(weights(fit, type = "prior")))
sumNobs <- colSums(Nobs) # apply(Nobs, 2, sum)

pneumo.long <-
  data.frame(symptoms = ordered(rep(rep(colnames(Nobs), nrow(Nobs)),
                                   times = c(t(Nobs))),
            levels = colnames(Nobs)),
            let = rep(rep(with(pneumo, let), each = ncol(Nobs)),
                      times = c(t(Nobs))))
with(pneumo.long, table(let, symptoms)) # Check it; should be same as pneumo

(fit.long1 <- vglm(symptoms ~ let, data = pneumo.long, trace = TRUE,
                  cumulative(parallel = TRUE, reverse = TRUE)))
coef(fit.long1, matrix = TRUE) # Should be same as coef(fit, matrix = TRUE)
# Could try using mustart if fit.long1 failed to converge.
```

```

mymustart <- matrix(sumNobs / sum(sumNobs),
                    nrow(pneumo.long), ncol(Nobs), byrow = TRUE)
fit.long2 <- vglm(symptoms ~ let, mustart = mymustart,
                  cumulative(parallel = TRUE, reverse = TRUE),
                  data = pneumo.long, trace = TRUE)
coef(fit.long2, matrix = TRUE) # Should be same as coef(fit, matrix = TRUE)

```

Dagum

*The Dagum Distribution***Description**

Density, distribution function, quantile function and random generation for the Dagum distribution with shape parameters  $a$  and  $p$ , and scale parameter  $scale$ .

**Usage**

```

ddagum(x, shape1.a, scale = 1, shape2.p, log = FALSE)
pdagum(q, shape1.a, scale = 1, shape2.p)
qdagum(p, shape1.a, scale = 1, shape2.p)
rdagum(n, shape1.a, scale = 1, shape2.p)

```

**Arguments**

$x$ , $q$	vector of quantiles.
$p$	vector of probabilities.
$n$	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
$shape1.a$ , $shape2.p$	shape parameters.
$scale$	scale parameter.
$log$	Logical. If $log = TRUE$ then the logarithm of the density is returned.

**Details**

See [dagum](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

**Value**

`ddagum` gives the density, `pdagum` gives the distribution function, `qdagum` gives the quantile function, and `rdagum` generates random deviates.

**Note**

The Dagum distribution is a special case of the 4-parameter generalized beta II distribution.

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[dagum](#), [genbetaII](#).

**Examples**

```
probs <- seq(0.1, 0.9, by = 0.1)
shape1.a <- 1; shape2.p <- 2
# Should be 0:
max(abs(pdagum(qdagum(p = probs, shape1.a = shape1.a, shape2.p = shape2.p),
                shape1.a = shape1.a, shape2.p = shape2.p) - probs))

## Not run: par(mfrow = c(1, 2))
x <- seq(-0.01, 5, len = 401)
plot(x, dexp(x), type = "l", col = "black", ylab = "", las = 1, ylim = c(0, 1),
     main = "Black is standard exponential, others are ddagum(x, ...)")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 1), col = "orange")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 2), col = "blue")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 5), col = "green")
legend("topright", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
     legend = paste("shape1.a =", shape1.a, ", shape2.p =", c(1, 2, 5)))

plot(x, pexp(x), type = "l", col = "black", ylab = "", las = 1,
     main = "Black is standard exponential, others are pdagum(x, ...)")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 1), col = "orange")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 2), col = "blue")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 5), col = "green")
legend("bottomright", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
     legend = paste("shape1.a =", shape1.a, ", shape2.p =", c(1, 2, 5)))

## End(Not run)
```

---

dagum

Dagum Distribution Family Function

---

**Description**

Maximum likelihood estimation of the 3-parameter Dagum distribution.

**Usage**

```
dagum(lshape1.a = "loge", lscale = "loge", lshape2.p = "loge",
      ishape1.a = NULL, iscale = NULL, ishape2.p = 1, zero = NULL)
```

**Arguments**

`lshape1.a`, `lscale`, `lshape2.p`  
 Parameter link functions applied to the (positive) parameters `a`, `scale`, and `p`.  
 See [Links](#) for more choices.

`ishape1.a`, `iscale`, `ishape2.p`  
 Optional initial values for `a`, `scale`, and `p`.

`zero`  
 An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2,3} which correspond to `a`, `scale`, `p`, respectively.

**Details**

The 3-parameter Dagum distribution is the 4-parameter generalized beta II distribution with shape parameter  $q = 1$ . It is known under various other names, such as the Burr III, inverse Burr, beta-K, and 3-parameter kappa distribution. It can be considered a generalized log-logistic distribution. Some distributions which are special cases of the 3-parameter Dagum are the inverse Lomax ( $a = 1$ ), Fisk ( $p = 1$ ), and the inverse paralogistic ( $a = p$ ). More details can be found in Kleiber and Kotz (2003).

The Dagum distribution has a cumulative distribution function

$$F(y) = [1 + (y/b)^{-a}]^{-p}$$

which leads to a probability density function

$$f(y) = apy^{ap-1}/[b^{ap}\{1 + (y/b)^a\}^{p+1}]$$

for  $a > 0$ ,  $b > 0$ ,  $p > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter `scale`, and the others are shape parameters. The mean is

$$E(Y) = b \Gamma(p + 1/a) \Gamma(1 - 1/a) / \Gamma(p)$$

provided  $-ap < 1 < a$ ; these are returned as the fitted values.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

See the note in [genbetaII](#).

From Kleiber and Kotz (2003), the MLE is rather sensitive to isolated observations located sufficiently far from the majority of the data. Reliable estimation of the scale parameter require  $n > 7000$ , while estimates for  $a$  and  $p$  can be considered unbiased for  $n > 2000$  or 3000.



**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[Dagum](#), [genbetaII](#), [betaII](#), [sinmad](#), [fisk](#), [invlomax](#), [lomax](#), [paralogistic](#), [invparalogistic](#).

**Examples**

```
ddata <- data.frame(y = rdagum(n = 3000, exp(1), exp(2), exp(1)))
fit <- vglm(y ~ 1, dagum, ddata, trace = TRUE)
fit <- vglm(y ~ 1, dagum(ishape1.a = exp(1)), ddata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

deermice

---

*Captures of *Peromyscus maniculatus*, also known as deer mice.*


---

**Description**

Captures of *Peromyscus maniculatus* collected at East Stuart Gulch, Colorado, USA.

**Usage**

```
data(deermice)
```

**Format**

The format is a data frame.

**Details**

*Peromyscus maniculatus* is a rodent native to North America. The deer mouse is small in size, only about 8 to 10 cm long, not counting the length of the tail.

Originally, the columns of this data frame represent the sex (m or f), the ages (y: young, sa: semi-adult, a: adult), the weights in grams, and the capture histories of 38 individuals over 6 trapping occasions (1: captured, 0: not captured).

The data set was collected by V. Reid and distributed with the **CAPTURE** program of Otis et al. (1978).

deermice has 38 deermice whereas Perom had 36 deermice (Perom has been withdrawn.) In deermice the two semi-adults have been classified as adults. The sex variable has 1 for female, and 0 for male.

References

Huggins, R. M. (1991) Some practical aspects of a conditional likelihood approach to capture experiments. *Biometrics*, **47**, 725–732.

Otis, D. L. et al. (1978) Statistical inference from capture data on closed animal populations, *Wildlife Monographs*, **62**, 3–135.

See Also

[posbernoulli.b](#), [posbernoulli.t](#).

Examples

```
head(deermice)
## Not run:
fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + age,
             posbernoulli.t(parallel.t = TRUE), data = deermice, trace = TRUE)
coef(fit1)
coef(fit1, matrix = TRUE)

## End(Not run)
```

---

deplot.lmscreg	<i>Density Plot for LMS Quantile Regression</i>
----------------	---

---

Description

Plots a probability density function associated with a LMS quantile regression.

Usage

```
deplot.lmscreg(object, newdata = NULL, x0, y.arg, show.plot = TRUE, ...)
```

Arguments

object	A <b>VGAM</b> quantile regression model, i.e., an object produced by modelling functions such as <a href="#">vglm</a> and <a href="#">vgam</a> with a family function beginning with "lms.", e.g., <a href="#">lms.yjn</a> .
newdata	Optional data frame containing secondary variables such as sex. It should have a maximum of one row. The default is to use the original data.
x0	Numeric. The value of the primary variable at which to make the ‘slice’.
y.arg	Numerical vector. The values of the response variable at which to evaluate the density. This should be a grid that is fine enough to ensure the plotted curves are smooth.
show.plot	Logical. Plot it? If FALSE no plot will be done.
...	Graphical parameter that are passed into <a href="#">plotdeplot.lmscreg</a> .

## Details

This function calls, e.g., `deplot.lms.yjn` in order to compute the density function.

## Value

The original object but with a list placed in the slot `post`, called `@post$deplot`. The list has components

<code>newdata</code>	The argument <code>newdata</code> above, or a one-row data frame constructed out of the <code>x0</code> argument.
<code>y</code>	The argument <code>y.arg</code> above.
<code>density</code>	Vector of the density function values evaluated at <code>y.arg</code> .

## Note

`plotdeplot.lmscreg` actually does the plotting.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

## See Also

`plotdeplot.lmscreg`, `qtplot.lmscreg`, `lms.bcn`, `lms.bcg`, `lms.yjn`.

## Examples

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4, 2)), fam = lms.bcn(zero = 1), data = bmi.nz)
ygrid <- seq(15, 43, by = 0.25)
deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "green", llwd = 2,
       main = "BMI distribution at ages 20 (green), 40 (blue), 60 (red)")
deplot(fit, x0 = 40, y = ygrid, add = TRUE, col = "blue", llwd = 2)
deplot(fit, x0 = 60, y = ygrid, add = TRUE, col = "red", llwd = 2) -> a

names(a@post$deplot)
a@post$deplot$newdata
head(a@post$deplot$y)
head(a@post$deplot$density)

## End(Not run)
```

---

depvar

*Response variable extracted*


---

## Description

A generic function that extracts the response/dependent variable from objects.

## Usage

```
depvar(object, ...)
```

## Arguments

object	An object that has some response/dependent variable.
...	Other arguments fed into the specific methods function of the model. In particular, sometimes <code>type = c("lm", "lm2")</code> is available, in which case the first one is chosen if the user does not input a value. The latter value corresponds to argument <code>form2</code> , and sometimes a response for that is optional.

## Details

By default this function is preferred to calling `fit@y`, say.

## Value

The response/dependent variable, usually as a matrix or vector.

## Author(s)

Thomas W. Yee

## See Also

[model.matrix](#), [vglm](#).

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo))
fit@y      # Sample proportions (not recommended)
depvar(fit) # Better than using fit@y; dependent variable (response)
weights(fit, type = "prior") # Number of observations
```

---

df.residual	<i>Residual Degrees-of-Freedom</i>
-------------	------------------------------------

---

## Description

Returns the residual degrees-of-freedom extracted from a fitted VGLM object.

## Usage

```
df.residual_vlm(object, type = c("vlm", "lm"), ...)
```

## Arguments

object	an object for which the degrees-of-freedom are desired, e.g., a <a href="#">vglm</a> object.
type	the type of residual degrees-of-freedom wanted. In some applications the 'usual' LM-type value may be more appropriate. The default is the first choice.
...	additional optional arguments.

## Details

When a VGLM is fitted, a *large* (VLM) generalized least squares (GLS) fit is done at each IRLS iteration. To do this, an ordinary least squares (OLS) fit is performed by transforming the GLS using Cholesky factors. The number of rows is  $M$  times the 'ordinary' number of rows of the LM-type model:  $nM$ . Here,  $M$  is the number of linear/additive predictors. So the formula for the VLM-type residual degrees-of-freedom is  $nM - p^*$  where  $p^*$  is the number of columns of the 'big' VLM matrix. The formula for the LM-type residual degrees-of-freedom is  $n - p_j$  where  $p_j$  is the number of columns of the 'ordinary' LM matrix corresponding to the  $j$ th linear/additive predictor.

## Value

The value of the residual degrees-of-freedom extracted from the object. When `type = "vlm"` this is a single integer, and when `type = "lm"` this is a  $M$ -vector of integers.

## See Also

[vglm](#), [deviance](#), [lm](#).

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo))
head(model.matrix(fit, type = "vlm"))
head(model.matrix(fit, type = "lm"))

df.residual(fit, type = "vlm") # n * M - p_VLM
nobs(fit, type = "vlm") # n * M
nvar(fit, type = "vlm") # p_VLM
```

```
df.residual(fit, type = "lm") # n - p_LM(j); Useful in some situations
nobs(fit, type = "lm") # n
nvar(fit, type = "lm") # p_LM
nvar_vlm(fit, type = "lm") # p_LM(j) (<= p_LM elementwise)
```

---

dhuber

*Huber's least favourable distribution*


---

## Description

Density, distribution function, quantile function and random generation for Huber's least favourable distribution, see Huber and Ronchetti (2009).

## Usage

```
dhuber(x, k = 0.862, mu = 0, sigma = 1, log = FALSE)
edhuber(x, k = 0.862, mu = 0, sigma = 1, log = FALSE)
rhuber(n, k = 0.862, mu = 0, sigma = 1)
qhuber(p, k = 0.862, mu = 0, sigma = 1)
phuber(q, k = 0.862, mu = 0, sigma = 1)
```

## Arguments

x, q	numeric vector, vector of quantiles.
p	vector of probabilities.
n	number of random values to be generated. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
k	numeric. Borderline value of central Gaussian part of the distribution. This is known as the tuning constant, and should be positive. For example, <code>k = 0.862</code> refers to a 20% contamination neighborhood of the Gaussian distribution. If <code>k = 1.40</code> then this is 5% contamination.
mu	numeric. distribution mean.
sigma	numeric. Distribution scale ( <code>sigma = 1</code> defines the distribution in standard form, with standard Gaussian centre).
log	Logical. If <code>log = TRUE</code> then the logarithm of the result is returned.

## Details

Details are given in [huber2](#), the **VGAM** family function for estimating the parameters `mu` and `sigma`.

**Value**

dhuber gives out a vector of density values.

edhuber gives out a list with components val (density values) and eps (contamination proportion).

rhuber gives out a vector of random numbers generated by Huber's least favourable distribution.

phuber gives the distribution function, qhuber gives the quantile function.

**Author(s)**

Christian Hennig wrote [d,ed,r]huber() (from **smoothest**) and slight modifications were made by T. W. Yee to replace looping by vectorization and addition of the log argument. Arash Ardalan wrote [pq]huber(). This helpfile was adapted from **smoothest**.

**See Also**

[huber2](#).

**Examples**

```
set.seed(123456)
edhuber(1:5, k = 1.5)
rhuber(5)

## Not run: mu <- 3; xx <- seq(-2, 7, len = 100) # Plot CDF and PDF
plot(xx, dhuber(xx, mu = mu), type = "l", col = "blue", las = 1, ylab = "",
      main = "blue is density, red is cumulative distribution function",
      sub = "Purple lines are the 10,20,...,90 percentiles",
      ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(xx, phuber(xx, mu = mu), type = "l", col = "red")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qhuber(probs, mu = mu)
lines(Q, dhuber(Q, mu = mu), col = "purple", lty = 3, type = "h")
lines(Q, phuber(Q, mu = mu), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
phuber(Q, mu = mu) - probs # Should be all 0s

## End(Not run)
```

---

dirichlet

*Fitting a Dirichlet Distribution*


---

**Description**

Fits a Dirichlet distribution to a matrix of compositions.

**Usage**

```
dirichlet(link = "log", parallel = FALSE, zero = NULL)
```

## Arguments

`link` Link function applied to each of the  $M$  (positive) shape parameters  $\alpha_j$ . See [Links](#) for more choices. The default gives  $\eta_j = \log(\alpha_j)$ .

`parallel, zero` See [CommonVGAMffArguments](#) for more information.

## Details

In this help file the response is assumed to be a  $M$ -column matrix with positive values and whose rows each sum to unity. Such data can be thought of as compositional data. There are  $M$  linear/additive predictors  $\eta_j$ .

The Dirichlet distribution is commonly used to model compositional data, including applications in genetics. Suppose  $(Y_1, \dots, Y_M)^T$  is the response. Then it has a Dirichlet distribution if  $(Y_1, \dots, Y_{M-1})^T$  has density

$$\frac{\Gamma(\alpha_+)}{\prod_{j=1}^M \Gamma(\alpha_j)} \prod_{j=1}^M y_j^{\alpha_j-1}$$

where  $\alpha_+ = \alpha_1 + \dots + \alpha_M$ ,  $\alpha_j > 0$ , and the density is defined on the unit simplex

$$\Delta_M = \left\{ (y_1, \dots, y_M)^T : y_1 > 0, \dots, y_M > 0, \sum_{j=1}^M y_j = 1 \right\}.$$

One has  $E(Y_j) = \alpha_j/\alpha_+$ , which are returned as the fitted values. For this distribution Fisher scoring corresponds to Newton-Raphson.

The Dirichlet distribution can be motivated by considering the random variables  $(G_1, \dots, G_M)^T$  which are each independent and identically distributed as a gamma distribution with density  $f(g_j) = g_j^{\alpha_j-1} e^{-g_j} / \Gamma(\alpha_j)$ . Then the Dirichlet distribution arises when  $Y_j = G_j / (G_1 + \dots + G_M)$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

When fitted, the `fitted.values` slot of the object contains the  $M$ -column matrix of means.

## Note

The response should be a matrix of positive values whose rows each sum to unity. Similar to this is count data, where probably a multinomial logit model ([multinomial](#)) may be appropriate. Another similar distribution to the Dirichlet is the Dirichlet-multinomial (see [dirmultinomial](#)).

## Author(s)

Thomas W. Yee



References

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

See Also

[rdiric](#), [dirmultinomial](#), [multinomial](#), [simplex](#).

Examples

```
ydata <- data.frame(rdiric(n = 1000, shape = exp(c(-1, 1, 0))))
colnames(ydata) <- paste("y", 1:3, sep = "")
fit <- vglm(cbind(y1, y2, y3) ~ 1, dirichlet, ydata, trace = TRUE, crit = "coef")
Coef(fit)
coef(fit, matrix = TRUE)
head(fitted(fit))
```

---

dirmul.old	<i>Fitting a Dirichlet-Multinomial Distribution</i>
------------	---

---

Description

Fits a Dirichlet-multinomial distribution to a matrix of non-negative integers.

Usage

```
dirmul.old(link = "loge",
           init.alpha = 0.01, parallel = FALSE, zero = NULL)
```

Arguments

link	Link function applied to each of the $M$ (positive) shape parameters $\alpha_j$ for $j = 1, \dots, M$ . See <a href="#">Links</a> for more choices. Here, $M$ is the number of columns of the response matrix.
init.alpha	Numeric vector. Initial values for the alpha vector. Must be positive. Recycled to length $M$ .
parallel	A logical, or formula specifying which terms have equal/unequal coefficients.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ .

## Details

The Dirichlet-multinomial distribution, which is somewhat similar to a Dirichlet distribution, has probability function

$$P(Y_1 = y_1, \dots, Y_M = y_M) = \binom{2y_*}{y_1, \dots, y_M} \frac{\Gamma(\alpha_+)}{\Gamma(2y_* + \alpha_+)} \prod_{j=1}^M \frac{\Gamma(y_j + \alpha_j)}{\Gamma(\alpha_j)}$$

for  $\alpha_j > 0$ ,  $\alpha_+ = \alpha_1 + \dots + \alpha_M$ , and  $2y_* = y_1 + \dots + y_M$ . Here,  $\binom{a}{b}$  means “ $a$  choose  $b$ ” and refers to combinations (see [choose](#)). The (posterior) mean is

$$E(Y_j) = (y_j + \alpha_j) / (2y_* + \alpha_+)$$

for  $j = 1, \dots, M$ , and these are returned as the fitted values as a  $M$ -column matrix.

## Value

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Note

The response should be a matrix of non-negative values. Convergence seems to slow down if there are zero values. Currently, initial values can be improved upon.

This function is almost defunct and may be withdrawn soon. Use [dirmultinomial](#) instead.

## Author(s)

Thomas W. Yee

## References

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

Paul, S. R., Balasooriya, U. and Banerjee, T. (2005) Fisher information matrix of the Dirichlet-multinomial distribution. *Biometrical Journal*, **47**, 230–236.

Tvedebrink, T. (2010) Overdispersion in allelic counts and  $\theta$ -correction in forensic genetics. *Theoretical Population Biology*, **78**, 200–210.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[dirmultinomial](#), [dirichlet](#), [betabinomial.ab](#), [multinomial](#).

## Examples

```
# Data from p.50 of Lange (2002)
alleleCounts <- c(2, 84, 59, 41, 53, 131, 2, 0,
  0, 50, 137, 78, 54, 51, 0, 0,
  0, 80, 128, 26, 55, 95, 0, 0,
  0, 16, 40, 8, 68, 14, 7, 1)
dim(alleleCounts) <- c(8, 4)
alleleCounts <- data.frame(t(alleleCounts))
dimnames(alleleCounts) <- list(c("White", "Black", "Chicano", "Asian"),
  paste("Allele", 5:12, sep = ""))

set.seed(123) # @initialize uses random numbers
fit <- vglm(cbind(Allele5, Allele6, Allele7, Allele8, Allele9,
  Allele10, Allele11, Allele12) ~ 1, dirmul.old,
  trace = TRUE, crit = "c", data = alleleCounts)

(sfit <- summary(fit))
vcov(sfit)
round(eta2theta(coef(fit), fit@misc$link, fit@misc$earg), digits = 2) # not preferred
round(Coef(fit), digits = 2) # preferred
round(t(fitted(fit)), digits = 4) # 2nd row of Table 3.5 of Lange (2002)
coef(fit, matrix = TRUE)

pfit <- vglm(cbind(Allele5, Allele6, Allele7, Allele8, Allele9,
  Allele10, Allele11, Allele12) ~ 1,
  dirmul.old(parallel = TRUE), trace = TRUE,
  data = alleleCounts)
round(eta2theta(coef(pfit, matrix = TRUE), pfit@misc$link,
  pfit@misc$earg), digits = 2) # 'Right' answer
round(Coef(pfit), digits = 2) # 'Wrong' answer due to parallelism constraint
```

---

dirmultinomial

*Fitting a Dirichlet-Multinomial Distribution*


---

## Description

Fits a Dirichlet-multinomial distribution to a matrix response.

## Usage

```
dirmultinomial(lphi = "logit", iphi = 0.10, parallel = FALSE, zero = "M")
```

## Arguments

lphi	Link function applied to the $\phi$ parameter, which lies in the open unit interval $(0, 1)$ . See <a href="#">Links</a> for more choices.
iphi	Numeric. Initial value for $\phi$ . Must be in the open unit interval $(0, 1)$ . If a failure to converge occurs then try assigning this argument a different value.

parallel	A logical (formula not allowed here) indicating whether the probabilities $\pi_1, \dots, \pi_{M-1}$ are to be equal via equal coefficients. Note $\pi_M$ will generally be different from the other probabilities. Setting <code>parallel = TRUE</code> will only work if you also set <code>zero = NULL</code> because of interference between these arguments (with respect to the intercept term).
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ . If the character "M" then this means the numerical value $M$ , which corresponds to linear/additive predictor associated with $\phi$ . Setting <code>zero = NULL</code> means none of the values from the set $\{1, 2, \dots, M\}$ .

### Details

The Dirichlet-multinomial distribution arises from a multinomial distribution where the probability parameters are not constant but are generated from a multivariate distribution called the Dirichlet distribution. The Dirichlet-multinomial distribution has probability function

$$P(Y_1 = y_1, \dots, Y_M = y_M) = \binom{N_*}{y_1, \dots, y_M} \frac{\prod_{j=1}^M \prod_{r=1}^{y_j} (\pi_j(1 - \phi) + (r - 1)\phi)}{\prod_{r=1}^{N_*} (1 - \phi + (r - 1)\phi)}$$

where  $\phi$  is the *over-dispersion* parameter and  $N_* = y_1 + \dots + y_M$ . Here,  $\binom{a}{b}$  means “ $a$  choose  $b$ ” and refers to combinations (see [choose](#)). The above formula applies to each row of the matrix response. In this **VGAM** family function the first  $M - 1$  linear/additive predictors correspond to the first  $M - 1$  probabilities via

$$\eta_j = \log(P[Y = j]/P[Y = M]) = \log(\pi_j/\pi_M)$$

where  $\eta_j$  is the  $j$ th linear/additive predictor ( $\eta_M = 0$  by definition for  $P[Y = M]$  but not for  $\phi$ ) and  $j = 1, \dots, M - 1$ . The  $M$ th linear/additive predictor corresponds to `lphi` applied to  $\phi$ .

Note that  $E(Y_j) = N_*\pi_j$  but the probabilities (returned as the fitted values)  $\pi_j$  are bundled together as a  $M$ -column matrix. The quantities  $N_*$  are returned as the prior weights.

The beta-binomial distribution is a special case of the Dirichlet-multinomial distribution when  $M = 2$ ; see [betabinomial](#). It is easy to show that the first shape parameter of the beta distribution is  $shape1 = \pi(1/\phi - 1)$  and the second shape parameter is  $shape2 = (1 - \pi)(1/\phi - 1)$ . Also,  $\phi = 1/(1 + shape1 + shape2)$ , which is known as the *intra-cluster correlation* coefficient.

### Value

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

If the model is an intercept-only model then `@misc` (which is a list) has a component called `shape` which is a vector with the  $M$  values  $\pi_j(1/\phi - 1)$ .

### Warning

This **VGAM** family function is prone to numerical problems, especially when there are covariates.

**Note**

The response can be a matrix of non-negative integers, or else a matrix of sample proportions and the total number of counts in each row specified using the `weights` argument. This dual input option is similar to [multinomial](#).

To fit a ‘parallel’ model with the  $\phi$  parameter being an intercept-only you will need to use the `constraints` argument.

Currently, Fisher scoring is implemented. To compute the expected information matrix a for loop is used; this may be very slow when the counts are large. Additionally, convergence may be slower than usual due to round-off error when computing the expected information matrices.

**Author(s)**

Thomas W. Yee

**References**

Paul, S. R., Balasooriya, U. and Banerjee, T. (2005) Fisher information matrix of the Dirichlet-multinomial distribution. *Biometrical Journal*, **47**, 230–236.

Tvedebrink, T. (2010) Overdispersion in allelic counts and  $\theta$ -correction in forensic genetics. *Theoretical Population Biology*, **78**, 200–210.

**See Also**

[dirmul.old](#), [betabinomial](#), [betabinomial.ab](#), [dirichlet](#), [multinomial](#).

**Examples**

```
nn <- 10; M <- 5
ydata <- data.frame(round(matrix(runif(nn * M, max = 10), nn, M))) # Integer counts
colnames(ydata) <- paste("y", 1:M, sep = "")

fit <- vglm(cbind(y1, y2, y3, y4, y5) ~ 1, dirmultinomial, ydata, trace = TRUE)
head(fitted(fit))
depvar(fit) # Sample proportions
weights(fit, type = "prior", matrix = FALSE) # Total counts per row

ydata <- transform(ydata, x2 = runif(nn))
fit <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2, dirmultinomial, ydata, trace = TRUE)
## Not run: # This does not work:
Coef(fit)
## End(Not run)
coef(fit, matrix = TRUE)
(sfit <- summary(fit))
vcov(sfit)
```

---

`dlogF`*log F Distribution*

---

**Description**

Density, for the log F distribution.

**Usage**

```
dlogF(x, shape1, shape2, log = FALSE)
```

**Arguments**

`x`                      Vector of quantiles.  
`shape1, shape2`      Positive shape parameters.  
`log`                    if TRUE then the log density is returned, else the density.

**Details**

The details are given in [logF](#).

**Value**

`dlogF` gives the density.

**Author(s)**

T. W. Yee

**See Also**

[hypersecant](#).

**Examples**

```
## Not run: shape1 <- 1.5; shape2 <- 0.5; x <- seq(-5, 8, length = 1001)
plot(x, dlogF(x, shape1, shape2), type = "l",
     las = 1, col = "blue", ylab = "pdf",
     main = "log F density function")

## End(Not run)
```

---

double.cennormal	<i>Univariate Normal Distribution with Double Censoring</i>
------------------	---

---

## Description

Maximum likelihood estimation of the two parameters of a univariate normal distribution when there is double censoring.

## Usage

```
double.cennormal(r1 = 0, r2 = 0, lmu = "identity", lsd = "loge",
                 imu = NULL, isd = NULL, zero = 2)
```

## Arguments

r1, r2	Integers. Number of smallest and largest values censored, respectively.
lmu, lsd	Parameter link functions applied to the mean and standard deviation. See <a href="#">Links</a> for more choices.
imu, isd, zero	See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

This family function uses the Fisher information matrix given in Harter and Moore (1966). The matrix is not diagonal if either r1 or r2 are positive.

By default, the mean is the first linear/additive predictor and the log of the standard deviation is the second linear/additive predictor.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

This family function only handles a vector or one-column matrix response. The weights argument, if used, are interpreted as frequencies, therefore it must be a vector with positive integer values.

With no censoring at all (the default), it is better (and equivalent) to use [uninormal](#).

## Author(s)

T. W. Yee

## References

Harter, H. L. and Moore, A. H. (1966) Iterative maximum-likelihood estimation of the parameters of normal populations from singly and doubly censored samples. *Biometrika*, **53**, 205–213.

**See Also**

[uninormal](#), [cennormal](#), [tobit](#).

**Examples**

```
## Not run: # Repeat the simulations described in Harter and Moore (1966)
SIMS <- 100 # Number of simulations (change this to 1000)
mu.save <- sd.save <- rep(NA, len = SIMS)
r1 <- 0; r2 <- 4; nn <- 20
for (sim in 1:SIMS) {
  y <- sort(rnorm(nn))
  y <- y[(1+r1):(nn-r2)] # Delete r1 smallest and r2 largest
  fit <- vglm(y ~ 1, double.cennormal(r1 = r1, r2 = r2))
  mu.save[sim] <- predict(fit)[1, 1]
  sd.save[sim] <- exp(predict(fit)[1, 2]) # Assumes a log link and ~ 1
}
c(mean(mu.save), mean(sd.save)) # Should be c(0,1)
c(sd(mu.save), sd(sd.save))

## End(Not run)

# Data from Sarhan and Greenberg (1962); MLEs are mu = 9.2606, sd = 1.3754
strontium90 <- data.frame(y = c(8.2, 8.4, 9.1, 9.8, 9.9))
fit <- vglm(y ~ 1, double.cennormal(r1 = 2, r2 = 3, isd = 6), strontium90, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

---

double.expbinomial	<i>Double Exponential Binomial Distribution Family Function</i>
--------------------	---

---

**Description**

Fits a double exponential binomial distribution by maximum likelihood estimation. The two parameters here are the mean and dispersion parameter.

**Usage**

```
double.expbinomial(lmean = "logit", ldispersion = "logit",
  idispersion = 0.25, zero = 2)
```

**Arguments**

`lmean`, `ldispersion`

Link functions applied to the two parameters, called  $\mu$  and  $\theta$  respectively below. See [Links](#) for more choices. The defaults cause the parameters to be restricted to  $(0, 1)$ .

`idispersion`

Initial value for the dispersion parameter. If given, it must be in range, and is recycled to the necessary length. Use this argument if convergence failure occurs.



**zero** An integer specifying which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value should be either 1 or 2. The default is to have a single dispersion parameter value. To model both parameters as functions of the covariates assign `zero = NULL`.

## Details

This distribution provides a way for handling overdispersion in a binary response. The double exponential binomial distribution belongs to the family of double exponential distributions proposed by Efron (1986). Below, equation numbers refer to that original article. Briefly, the idea is that an ordinary one-parameter exponential family allows the addition of a second parameter  $\theta$  which varies the dispersion of the family without changing the mean. The extended family behaves like the original family with sample size changed from  $n$  to  $n\theta$ . The extended family is an exponential family in  $\mu$  when  $n$  and  $\theta$  are fixed, and an exponential family in  $\theta$  when  $n$  and  $\mu$  are fixed. Having  $0 < \theta < 1$  corresponds to overdispersion with respect to the binomial distribution. See Efron (1986) for full details.

This **VGAM** family function implements an *approximation* (2.10) to the exact density (2.4). It replaces the normalizing constant by unity since the true value nearly equals 1. The default model fitted is  $\eta_1 = \text{logit}(\mu)$  and  $\eta_2 = \text{logit}(\theta)$ . This restricts both parameters to lie between 0 and 1, although the dispersion parameter can be modelled over a larger parameter space by assigning the arguments `ldispersion` and `edispersion`.

Approximately, the mean (of  $Y$ ) is  $\mu$ . The *effective sample size* is the dispersion parameter multiplied by the original sample size, i.e.,  $n\theta$ . This family function uses Fisher scoring, and the two estimates are asymptotically independent because the expected information matrix is diagonal.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as `vglm`.

## Warning

Numerical difficulties can occur; if so, try using `idispersion`.

## Note

This function processes the input in the same way as `binomialff`, however multivariate responses are not allowed (`binomialff(mv = FALSE)`).

## Author(s)

T. W. Yee

## References

Efron, B. (1986) Double exponential families and their use in generalized linear regression. *Journal of the American Statistical Association*, **81**, 709–721.

**See Also**

[binomialff](#), [toxop](#), [CommonVGAMffArguments](#).

**Examples**

```
# This example mimics the example in Efron (1986).
# The results here differ slightly.

# Scale the variables
toxop <- transform(toxop,
  phat = positive / ssize,
  srainfall = scale(rainfall), # (6.1)
  sN = scale(ssize))          # (6.2)

# A fit similar (should be identical) to Section 6 of Efron (1986).
# But does not use poly(), and M = 1.25 here, as in (5.3)
cmlist <- list("(Intercept)" = diag(2),
  "I(srainfall)" = rbind(1,0),
  "I(srainfall^2)" = rbind(1,0),
  "I(srainfall^3)" = rbind(1,0),
  "I(sN)" = rbind(0,1),
  "I(sN^2)" = rbind(0,1))
fit <- vglm(cbind(phat, 1 - phat) * ssize ~
  I(srainfall) + I(srainfall^2) + I(srainfall^3) +
  I(sN) + I(sN^2),
  double.expbinomial(ldisp = elogit(min = 0, max = 1.25),
    idisp = 0.2, zero = NULL),
  toxop, trace = TRUE, constraints = cmlist)

# Now look at the results
coef(fit, matrix = TRUE)
head(fitted(fit))
summary(fit)
vcov(fit)
sqrt(diag(vcov(fit))) # Standard errors

# Effective sample size (not quite the last column of Table 1)
head(predict(fit))
Dispersion <- elogit(predict(fit)[,2], min = 0, max = 1.25, inverse = TRUE)
c(round(weights(fit, type = "prior") * Dispersion, digits = 1))

# Ordinary logistic regression (gives same results as (6.5))
ofit <- vglm(cbind(phat, 1 - phat) * ssize ~
  I(srainfall) + I(srainfall^2) + I(srainfall^3),
  binomialff, toxop, trace = TRUE)

# Same as fit but it uses poly(), and can be plotted (cf. Figure 1)
cmlist2 <- list("(Intercept)" = diag(2),
  "poly(srainfall, degree = 3)" = rbind(1, 0),
  "poly(sN, degree = 2)" = rbind(0, 1))
```

```

fit2 <- vglm(cbind(phat, 1 - phat) * ssize ~
             poly(srainfall, degree = 3) + poly(sN, degree = 2),
             double.expbinoomial(ldisp = elogit(min = 0, max = 1.25),
                                   idisp = 0.2, zero = NULL),
             toxop, trace = TRUE, constraints = cmlist2)
## Not run: par(mfrow = c(1, 2))
plotvgam(fit2, se = TRUE, lcol = "blue", scol = "orange") # Cf. Figure 1

# Cf. Figure 1(a)
par(mfrow = c(1,2))
ooo <- with(toxop, sort.list(rainfall))
with(toxop, plot(rainfall[ooo], fitted(fit2)[ooo], type = "l",
                 col = "blue", las = 1, ylim = c(0.3, 0.65)))
with(toxop, points(rainfall[ooo], fitted(ofit)[ooo], col = "orange",
                  type = "b", pch = 19))

# Cf. Figure 1(b)
ooo <- with(toxop, sort.list(ssize))
with(toxop, plot(ssize[ooo], Dispersion[ooo], type = "l", col = "blue",
                 las = 1, xlim = c(0, 100)))
## End(Not run)

```

enzyme

*Enzyme Data***Description**

Enzyme velocity and substrate concentration.

**Usage**

```
data(enzyme)
```

**Format**

A data frame with 12 observations on the following 2 variables.

**conc** a numeric explanatory vector; substrate concentration

**velocity** a numeric response vector; enzyme velocity

**Details**

Sorry, more details need to be included later.

**Source**

Sorry, more details need to be included later.

## References

Watts, D. G. (1981) An introduction to nonlinear least squares. In: L. Endrenyi (Ed.), *Kinetic Data Analysis: Design and Analysis of Enzyme and Pharmacokinetic Experiments*, pp.1–24. New York: Plenum Press.

## See Also

[micmen](#).

## Examples

```
## Not run:
fit <- vglm(velocity ~ 1, micmen, data = enzyme, trace = TRUE,
           form2 = ~ conc - 1, crit = "crit")
summary(fit)

## End(Not run)
```

---

erf

*Error Function*


---

## Description

Computes the error function based on the normal distribution.

## Usage

```
erf(x)
```

## Arguments

x                      Numeric.

## Details

$Er f(x)$  is defined as

$$Er f(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$$

so that it is closely related to [pnorm](#).

## Value

Returns the value of the function evaluated at x.

## Note

Some authors omit the term  $2/\sqrt{\pi}$  from the definition of  $Er f(x)$ . Although defined for complex arguments, this function only works for real arguments.

The *complementary error function*  $erfc(x)$  is defined as  $1 - erf(x)$ , and is implemented by `erfc`.

**Author(s)**

T. W. Yee

**References**

Abramowitz, M. and Stegun, I. A. (1972) *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover Publications Inc.

**See Also**

[pnorm](#).

**Examples**

```
## Not run:
curve(erf, -3, 3, col = "orange", ylab = "", las = 1)
curve(pnorm, -3, 3, add = TRUE, col = "blue", lty = "dotted", lwd = 2)
abline(v = 0, h = 0, lty = "dashed")
legend("topleft", c("erf(x)", "pnorm(x)"), col = c("orange", "blue"),
      lty = c("solid", "dotted"), lwd = 1:2)
## End(Not run)
```

---

erlang

*Erlang Distribution*

---

**Description**

Estimates the scale parameter of the Erlang distribution by maximum likelihood estimation.

**Usage**

```
erlang(shape.arg, link = "loge", imethod = 1, zero = NULL)
```

**Arguments**

shape.arg	The shape parameter. The user must specify a positive integer.
link	Link function applied to the (positive) <i>scale</i> parameter. See <a href="#">Links</a> for more choices.
imethod, zero	See <a href="#">CommonVGAMffArguments</a> for more details.

## Details

The Erlang distribution is a special case of the gamma distribution with *shape* that is a positive integer. If `shape.arg = 1` then it simplifies to the exponential distribution. As illustrated in the example below, the Erlang distribution is the distribution of the sum of `shape.arg` independent and identically distributed exponential random variates.

The probability density function of the Erlang distribution is given by

$$f(y) = \exp(-y/scale)y^{shape-1}scale^{-shape}/\Gamma(shape)$$

for known positive integer *shape*, unknown *scale* > 0 and *y* > 0. Here,  $\Gamma(shape)$  is the gamma function, as in [gamma](#). The mean of *Y* is  $\mu = shape \times scale$  and its variance is  $shape \times scale^2$ . The linear/additive predictor, by default, is  $\eta = \log(scale)$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

Multiple responses are permitted. The rate parameter found in [gamma2.ab](#) is 1/scale here—see also [rgamma](#).

## Author(s)

T. W. Yee

## References

Most standard texts on statistical distributions describe this distribution, e.g.,  
 Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

[gamma2.ab](#), [exponential](#).

## Examples

```
rate <- exp(2); myshape <- 3
edata <- data.frame(y = rep(0, nn <- 1000))
for (ii in 1:myshape)
  edata <- transform(edata, y = y + rexp(nn, rate = rate))
fit <- vglm(y ~ 1, erlang(shape = myshape), edata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit) # Answer = 1/rate
1/rate
summary(fit)
```

---

Expectiles-Exponential*Expectiles of the Exponential Distribution*

---

**Description**

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of an exponential distribution.

**Usage**

```
deexp(x, rate = 1, log = FALSE)
peexp(q, rate = 1, log = FALSE)
qeexp(p, rate = 1, Maxit.nr = 10, Tol.nr = 1.0e-6)
reexp(n, rate = 1)
```

**Arguments**

x, p, q            See [deunif](#).  
n, rate, log      See [rexp](#).  
Maxit.nr, Tol.nr  
                  See [deunif](#).

**Details**

General details are given in [deunif](#) including a note regarding the terminology used. Here, `exp` corresponds to the distribution of interest,  $F$ , and `eexp` corresponds to  $G$ . The addition of “e” is for the ‘other’ distribution associated with the parent distribution. Thus `deexp` is for  $g$ , `peexp` is for  $G$ , `qeexp` is for the inverse of  $G$ , `reexp` generates random variates from  $g$ .

For `qeexp` the Newton-Raphson algorithm is used to solve for  $y$  satisfying  $p = G(y)$ . Numerical problems may occur when values of  $p$  are very close to 0 or 1.

**Value**

`deexp(x)` gives the density function  $g(x)$ . `peexp(q)` gives the distribution function  $G(q)$ . `qeexp(p)` gives the expectile function: the value  $y$  such that  $G(y) = p$ . `reexp(n)` gives  $n$  random variates from  $G$ .

**Author(s)**

T. W. Yee

**See Also**

[deunif](#), [denorm](#), [dexp](#).

**Examples**

```

my_p <- 0.25; y <- rexp(nn <- 1000)
(myexp <- qexp(my_p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my_p

## Not run: par(mfrow = c(2,1))
yy <- seq(-0, 4, len = nn)
plot(yy, deexp(yy), col = "blue", ylim = 0:1, xlab = "y", ylab = "g(y)",
      type = "l", main = "g(y) for Exp(1); dotted green is f(y) = dexp(y)")
lines(yy, dexp(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'

plot(yy, peexp(yy), type = "l", col = "blue", ylim = 0:1,
      xlab = "y", ylab = "G(y)", main = "G(y) for Exp(1)")
abline(v = 1, h = 0.5, col = "red", lty = "dashed")
lines(yy, pexp(yy), col = "darkgreen", lty = "dotted", lwd = 2)
## End(Not run)

```

Expectiles-Koenker

*Expectiles/Quantiles of the Koenker Distribution***Description**

Density function, distribution function, and quantile/expectile function and random generation for the Koenker distribution.

**Usage**

```

dkoenker(x, location = 0, scale = 1, log = FALSE)
pkoenker(q, location = 0, scale = 1, log = FALSE)
qkoenker(p, location = 0, scale = 1)
rkoenker(n, location = 0, scale = 1)

```

**Arguments**

<code>x, q</code>	Vector of expectiles/quantiles. See the terminology note below.
<code>p</code>	Vector of probabilities. These should lie in $(0, 1)$ .
<code>n, log</code>	See <a href="#">runif</a> .
<code>location, scale</code>	Location and scale parameters. The latter should have positive values. Values of these vectors are recycled.

**Details**

A Student-t distribution with 2 degrees of freedom and a scale parameter of  $\sqrt{2}$  is equivalent to the standard Koenker distribution. Further details about this distribution are given in [koenker](#).



**Value**

`dkoenker(x)` gives the density function. `pkoenker(q)` gives the distribution function. `qkoenker(p)` gives the expectile and quantile function. `rkoenker(n)` gives  $n$  random variates.

**Author(s)**

T. W. Yee

**See Also**

[dt](#), [koenker](#).

**Examples**

```
my_p <- 0.25; y <- rkoenker(nn <- 5000)
(myexp <- qkoenker(my_p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my_p
# Equivalently:
I1 <- mean(y <= myexp) * mean( myexp - y[y <= myexp])
I2 <- mean(y > myexp) * mean(-myexp + y[y > myexp])
I1 / (I1 + I2) # Should be my_p
# Or:
I1 <- sum( myexp - y[y <= myexp])
I2 <- sum(-myexp + y[y > myexp])

# Non-standard Koenker distribution
myloc <- 1; myscale <- 2
yy <- rkoenker(nn, myloc, myscale)
(myexp <- qkoenker(my_p, myloc, myscale))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my_p
pkoenker(mean(yy), myloc, myscale) # Should be 0.5
abs(qkoenker(0.5, myloc, myscale) - mean(yy)) # Should be 0
abs(pkoenker(myexp, myloc, myscale) - my_p) # Should be 0
integrate(f = dkoenker, lower = -Inf, upper = Inf,
          locat = myloc, scale = myscale) # Should be 1

y <- seq(-7, 7, len = 201)
max(abs(dkoenker(y) - dt(y / sqrt(2), df = 2) / sqrt(2))) # Should be 0
## Not run: plot(y, dkoenker(y), type = "l", col = "blue", las = 1,
  ylim = c(0, 0.4), main = "Blue = Koenker; orange = N(0, 1)")
lines(y, dnorm(y), type = "l", col = "orange")
abline(h = 0, v = 0, lty = 2)
## End(Not run)
```

**Description**

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of a normal distribution.

**Usage**

```
denorm(x, mean = 0, sd = 1, log = FALSE)
penorm(q, mean = 0, sd = 1, log = FALSE)
qenorm(p, mean = 0, sd = 1, Maxit.nr = 10, Tol.nr = 1.0e-6)
renorm(n, mean = 0, sd = 1)
```

**Arguments**

`x`, `p`, `q`            See [deunif](#).  
`n`, `mean`, `sd`, `log`        See [rnorm](#).  
`Maxit.nr`, `Tol.nr`        See [deunif](#).

**Details**

General details are given in [deunif](#) including a note regarding the terminology used. Here, `norm` corresponds to the distribution of interest,  $F$ , and `enorm` corresponds to  $G$ . The addition of “e” is for the ‘other’ distribution associated with the parent distribution. Thus `denorm` is for  $g$ , `penorm` is for  $G$ , `qenorm` is for the inverse of  $G$ , `renorm` generates random variates from  $g$ .

For `qenorm` the Newton-Raphson algorithm is used to solve for  $y$  satisfying  $p = G(y)$ . Numerical problems may occur when values of  $p$  are very close to 0 or 1.

**Value**

`denorm(x)` gives the density function  $g(x)$ . `penorm(q)` gives the distribution function  $G(q)$ . `qenorm(p)` gives the expectile function: the value  $y$  such that  $G(y) = p$ . `renorm(n)` gives  $n$  random variates from  $G$ .

**Author(s)**

T. W. Yee

**See Also**

[deunif](#), [deexp](#), [dnorm](#), [amlnormal](#), [lms.bcn](#).

**Examples**

```
my_p <- 0.25; y <- rnorm(nn <- 1000)
(myexp <- qenorm(my_p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my_p

# Non-standard normal
```

```

mymean <- 1; mysd <- 2
yy <- rnorm(nn, mymean, mysd)
(myexp <- qenorm(my_p, mymean, mysd))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my_p
penorm(-Inf, mymean, mysd) # Should be 0
penorm( Inf, mymean, mysd) # Should be 1
penorm(mean(yy), mymean, mysd) # Should be 0.5
abs(qenorm(0.5, mymean, mysd) - mean(yy)) # Should be 0
abs(penorm(myexp, mymean, mysd) - my_p) # Should be 0
integrate(f = denorm, lower = -Inf, upper = Inf,
          mymean, mysd) # Should be 1

## Not run:
par(mfrow = c(2, 1))
yy <- seq(-3, 3, len = nn)
plot(yy, denorm(yy), type = "l", col="blue", xlab = "y", ylab = "g(y)",
     main = "g(y) for N(0,1); dotted green is f(y) = dnorm(y)")
lines(yy, dnorm(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'

plot(yy, penorm(yy), type = "l", col = "blue", ylim = 0:1,
     xlab = "y", ylab = "G(y)", main = "G(y) for N(0,1)")
abline(v = 0, h = 0.5, col = "red", lty = "dashed")
lines(yy, pnorm(yy), col = "darkgreen", lty = "dotted", lwd = 2)
## End(Not run)

```

Expectiles-Uniform

*Expectiles of the Uniform Distribution***Description**

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of a uniform distribution.

**Usage**

```

deunif(x, min = 0, max = 1, log = FALSE)
peunif(q, min = 0, max = 1, log = FALSE)
qeunif(p, min = 0, max = 1, Maxit.nr = 10, Tol.nr = 1.0e-6)
reunif(n, min = 0, max = 1)

```

**Arguments**

x, q	Vector of expectiles. See the terminology note below.
p	Vector of probabilities. These should lie in (0, 1).
n, min, max, log	See <a href="#">runif</a> .
Maxit.nr	Numeric. Maximum number of Newton-Raphson iterations allowed. A warning is issued if convergence is not obtained for all p values.
Tol.nr	Numeric. Small positive value specifying the tolerance or precision to which the expectiles are computed.

### Details

Jones (1994) elucidated on the property that the expectiles of a random variable  $X$  with distribution function  $F(x)$  correspond to the quantiles of a distribution  $G(x)$  where  $G$  is related by an explicit formula to  $F$ . In particular, let  $y$  be the  $p$ -expectile of  $F$ . Then  $y$  is the  $p$ -quantile of  $G$  where

$$p = G(y) = (P(y) - yF(y)) / (2[P(y) - yF(y)] + y - \mu),$$

and  $\mu$  is the mean of  $X$ . The derivative of  $G$  is

$$g(y) = (\mu F(y) - P(y)) / (2[P(y) - yF(y)] + y - \mu)^2.$$

Here,  $P(y)$  is the partial moment  $\int_{-\infty}^y xf(x) dx$  and  $0 < p < 1$ . The 0.5-expectile is the mean  $\mu$  and the 0.5-quantile is the median.

A note about the terminology used here. Recall in the *S* language there are the *dpqr*-type functions associated with a distribution, e.g., `dunif`, `punif`, `qunif`, `runif`, for the uniform distribution. Here, `unif` corresponds to  $F$  and `eunif` corresponds to  $G$ . The addition of “e” (for *expectile*) is for the ‘other’ distribution associated with the parent distribution. Thus `deunif` is for  $g$ , `peunif` is for  $G$ , `qeunif` is for the inverse of  $G$ , `reunif` generates random variates from  $g$ .

For `qeunif` the Newton-Raphson algorithm is used to solve for  $y$  satisfying  $p = G(y)$ . Numerical problems may occur when values of  $p$  are very close to 0 or 1.

### Value

`deunif(x)` gives the density function  $g(x)$ . `peunif(q)` gives the distribution function  $G(q)$ . `qeunif(p)` gives the expectile function: the expectile  $y$  such that  $G(y) = p$ . `reunif(n)` gives  $n$  random variates from  $G$ .

### Author(s)

T. W. Yee

### References

Jones, M. C. (1994) Expectiles and M-quantiles are quantiles. *Statistics and Probability Letters*, **20**, 149–153.

Yee, T. W. (2012) Vector generalized linear and additive quantile and expectile regression. *In preparation*.

### See Also

`deexp`, `denorm`, `dunif`, `dkoenker`.

### Examples

```
my_p <- 0.25; y <- runif(nn <- 1000)
(myexp <- qeunif(my_p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my_p
# Equivalently:
I1 <- mean(y <= myexp) * mean(myexp - y[y <= myexp])
```

```

I2 <- mean(y > myexp) * mean(-myexp + y[y > myexp])
I1 / (I1 + I2) # Should be my_p
# Or:
I1 <- sum( myexp - y[y <= myexp])
I2 <- sum(-myexp + y[y > myexp])

# Non-standard uniform
mymin <- 1; mymax <- 8
yy <- runif(nn, mymin, mymax)
(myexp <- qeunif(my_p, mymin, mymax))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my_p
peunif(mymin, mymin, mymax) # Should be 0
peunif(mymax, mymin, mymax) # Should be 1
peunif(mean(yy), mymin, mymax) # Should be 0.5
abs(qeunif(0.5, mymin, mymax) - mean(yy)) # Should be 0
abs(qeunif(0.5, mymin, mymax) - (mymin+mymax)/2) # Should be 0
abs(peunif(myexp, mymin, mymax) - my_p) # Should be 0
integrate(f = deunif, lower = mymin - 3, upper = mymax + 3,
          min = mymin, max = mymax) # Should be 1

## Not run:
par(mfrow = c(2,1))
yy <- seq(0.0, 1.0, len = nn)
plot(yy, deunif(yy), type = "l", col = "blue", ylim = c(0, 2),
      xlab = "y", ylab = "g(y)", main = "g(y) for Uniform(0,1)")
lines(yy, dunif(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'

plot(yy, peunif(yy), type = "l", col = "blue", ylim = 0:1,
      xlab = "y", ylab = "G(y)", main = "G(y) for Uniform(0,1)")
abline(a = 0.0, b = 1.0, col = "darkgreen", lty = "dotted", lwd = 2)
abline(v = 0.5, h = 0.5, col = "red", lty = "dashed")
## End(Not run)

```

---

expexp

---

*Exponentiated Exponential Distribution*


---

## Description

Estimates the two parameters of the exponentiated exponential distribution by maximum likelihood estimation.

## Usage

```

expexp(lshape = "log", lscale = "log",
       ishape = 1.1, iscale = NULL, tolerance = 1.0e-6, zero = NULL)

```

## Arguments

**lshape**, **lscale** Parameter link functions for the  $\alpha$  and  $\lambda$  parameters. See [Links](#) for more choices. The defaults ensure both parameters are positive.

ishape	Initial value for the $\alpha$ parameter. If convergence fails try setting a different value for this argument.
iscale	Initial value for the $\lambda$ parameter. By default, an initial value is chosen internally using ishape.
tolerance	Numeric. Small positive value for testing whether values are close enough to 1 and 2.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose one value from the set {1,2}.

### Details

The exponentiated exponential distribution is an alternative to the Weibull and the gamma distributions. The formula for the density is

$$f(y; \alpha, \lambda) = \alpha \lambda (1 - \exp(-\lambda y))^{\alpha-1} \exp(-\lambda y)$$

where  $y > 0$ ,  $\alpha > 0$  and  $\lambda > 0$ . The mean of  $Y$  is  $(\psi(\alpha + 1) - \psi(1))/\lambda$  (returned as the fitted values) where  $\psi$  is the digamma function. The variance of  $Y$  is  $(\psi'(1) - \psi'(\alpha + 1))/\lambda^2$  where  $\psi'$  is the trigamma function.

This distribution has been called the two-parameter generalized exponential distribution by Gupta and Kundu (2006). A special case of the exponentiated exponential distribution:  $\alpha = 1$  is the exponential distribution.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

### Warning

Practical experience shows that reasonably good initial values really helps. In particular, try setting different values for the ishape argument if numerical problems are encountered or failure to convergence occurs. Even if convergence occurs try perturbing the initial value to make sure the global solution is obtained and not a local solution. The algorithm may fail if the estimate of the shape parameter is too close to unity.

### Note

Fisher scoring is used, however, convergence is usually very slow. This is a good sign that there is a bug, but I have yet to check that the expected information is correct. Also, I have yet to implement Type-I right censored data using the results of Gupta and Kundu (2006).

Another algorithm for fitting this model is implemented in [expexp1](#).

### Author(s)

T. W. Yee

## References

Gupta, R. D. and Kundu, D. (2001) Exponentiated exponential family: an alternative to gamma and Weibull distributions, *Biometrical Journal*, **43**, 117–130.

Gupta, R. D. and Kundu, D. (2006) On the comparison of Fisher information of the Weibull and GE distributions, *Journal of Statistical Planning and Inference*, **136**, 3130–3144.

## See Also

[expexp1](#), [gamma2.ab](#), [weibull](#), [CommonVGAMffArguments](#).

## Examples

```
# A special case: exponential data
edata <- data.frame(y = rexp(n <- 1000))
fit <- vglm(y ~ 1, fam = expexp, edata, trace = TRUE, maxit = 99)
coef(fit, matrix = TRUE)
Coef(fit)

# Ball bearings data (number of million revolutions before failure)
bbearings <- c(17.88, 28.92, 33.00, 41.52, 42.12, 45.60,
48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64,
68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92,
128.04, 173.40)
fit <- vglm(bbearings ~ 1, fam = expexp(ishape = 0.05, ish = 5),
            trace = TRUE, maxit = 300)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(shape=5.2589, scale=0.0314)
logLik(fit) # Authors get -112.9763

# Failure times of the airconditioning system of an airplane
acplane <- c(23, 261, 87, 7, 120, 14, 62, 47,
225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14,
71, 11, 14, 11, 16, 90, 1, 16, 52, 95)
fit <- vglm(acplane ~ 1, fam = expexp(ishape = 0.8, isc = 0.15),
            trace = TRUE, maxit = 99)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(shape=0.8130, scale=0.0145)
logLik(fit) # Authors get log-lik -152.264
```

---

expexp1

---

*Exponentiated Exponential Distribution*


---

## Description

Estimates the two parameters of the exponentiated exponential distribution by maximizing a profile (concentrated) likelihood.

**Usage**

```
expexp1(lscale = "log", iscale = NULL, ishape = 1)
```

**Arguments**

lscale	Parameter link function for the (positive) $\lambda$ parameter. See <a href="#">Links</a> for more choices.
iscale	Initial value for the $\lambda$ parameter. By default, an initial value is chosen internally using ishape.
ishape	Initial value for the $\alpha$ parameter. If convergence fails try setting a different value for this argument.

**Details**

See [expexp](#) for details about the exponentiated exponential distribution. This family function uses a different algorithm for fitting the model. Given  $\lambda$ , the MLE of  $\alpha$  can easily be solved in terms of  $\lambda$ . This family function maximizes a profile (concentrated) likelihood with respect to  $\lambda$ . Newton-Raphson is used, which compares with Fisher scoring with [expexp](#).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

The standard errors produced by a summary of the model may be wrong.

**Note**

This family function works only for intercept-only models, i.e.,  $y \sim 1$  where  $y$  is the response.

The estimate of  $\alpha$  is attached to the `misc` slot of the object, which is a list and contains the component `shape`.

As Newton-Raphson is used, the working weights are sometimes negative, and some adjustment is made to these to make them positive.

Like [expexp](#), good initial values are needed. Convergence may be slow.

**Author(s)**

T. W. Yee

**References**

Gupta, R. D. and Kundu, D. (2001) Exponentiated exponential family: an alternative to gamma and Weibull distributions, *Biometrical Journal*, **43**, 117–130.



**See Also**

[expexp](#), [CommonVGAMffArguments](#).

**Examples**

```
# Ball bearings data (number of million revolutions before failure)
bbearings <- data.frame(y = c(17.88, 28.92, 33.00, 41.52, 42.12, 45.60,
48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64,
68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92,
128.04, 173.40))
fit <- vglm(y ~ 1, expexp1(ishape = 4), bbearings, trace = TRUE,
            maxit = 50, checkwz = FALSE)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(0.0314, 5.2589) with log-lik -112.9763
fit@misc$shape # Estimate of shape
logLik(fit)

# Failure times of the airconditioning system of an airplane
acplane <- data.frame(y = c(23, 261, 87, 7, 120, 14, 62, 47,
225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14,
71, 11, 14, 11, 16, 90, 1, 16, 52, 95))
fit <- vglm(y ~ 1, expexp1(ishape = 0.8), acplane, trace = TRUE,
            maxit = 50, checkwz = FALSE)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(0.0145, 0.8130) with log-lik -152.264
fit@misc$shape # Estimate of shape
logLik(fit)
```

---

expgeom

---

*The Exponential Geometric Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the exponential geometric distribution.

**Usage**

```
dexpgeom(x, scale = 1, shape, log = FALSE)
pexpgeom(q, scale = 1, shape)
qexpgeom(p, scale = 1, shape)
rexpgeom(n, scale = 1, shape)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.

n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
scale, shape	positive scale and shape parameters.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

### Details

See [expgeometric](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

### Value

`dexpgeom` gives the density, `pexpgeom` gives the distribution function, `qexpgeom` gives the quantile function, and `rexpgeom` generates random deviates.

### Note

We define scale as the reciprocal of the scale parameter used by Adamidis and Loukas (1998).

### Author(s)

J. G. Lauder and T. W. Yee

### See Also

[expgeometric](#), [exponential](#), [geometric](#).

### Examples

```
## Not run:
shape <- 0.5; scale <- 1; nn <- 501
x <- seq(-0.10, 3.0, len = nn)
plot(x, dexpgeom(x, scale, shape), type = "l", las = 1, ylim = c(0, 2),
     ylab = paste("[dp]expgeom(shape = ", shape, ", scale = ", scale, ")"),
     col = "blue", cex.main = 0.8,
     main = "Blue is density, red is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pexpgeom(x, scale, shape), col = "red")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qexpgeom(probs, scale, shape)
lines(Q, dexpgeom(Q, scale, shape), col = "purple", lty = 3, type = "h")
lines(Q, pexpgeom(Q, scale, shape), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pexpgeom(Q, scale, shape) - probs)) # Should be 0

## End(Not run)
```

expgeometric

*Exponential Geometric Distribution Family Function***Description**

Estimates the two parameters of the exponential geometric distribution by maximum likelihood estimation.

**Usage**

```
expgeometric(lscale = "loge", lshape = "logit",
             iscale = NULL,   ishape = NULL,
             tol12 = 1e-05, zero = 1, nsimEIM = 400)
```

**Arguments**

`lscale`, `lshape` Link function for the two parameters. See [Links](#) for more choices.  
`iscale`, `ishape` Numeric. Optional initial values for the scale and shape parameters.  
`tol12` Numeric. Tolerance for testing whether a parameter has value 1 or 2.  
`zero`, `nsimEIM` See [CommonVGAMffArguments](#).

**Details**

The exponential geometric distribution has density function

$$f(y; c = \text{scale}, s = \text{shape}) = (1/c)(1 - s)e^{-y/c}(1 - se^{-y/c})^{-2}$$

where  $y > 0$ ,  $c > 0$  and  $s \in (0, 1)$ . The mean,  $(c(s - 1)/s) \log(1 - s)$  is returned as the fitted values. Note the median is  $c \log(2 - s)$ . Simulated Fisher scoring is implemented.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

We define scale as the reciprocal of the scale parameter used by Adamidis and Loukas (1998).

**Author(s)**

J. G. Lauder and T. W. Yee

**References**

Adamidis, K., Loukas, S. (1998). A lifetime distribution with decreasing failure rate. *Statistics and Probability Letters*, **39**, 35–42.

**See Also**

[dexpgeom](#), [exponential](#), [geometric](#).

**Examples**

```
## Not run:
scale <- exp(2); shape = logit(-1, inverse = TRUE);
edata <- data.frame(y = rexpgeom(n = 2000, scale = scale, shape = shape))
fit <- vglm(y ~ 1, expgeometric, edata, trace = TRUE)
c(with(edata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

---

expint

---

*The Exponential Integral and Variants*


---

**Description**

Computes the exponential integral  $Ei(x)$  for real values, as well as  $\exp(-x) \times Ei(x)$  and  $E_1(x)$ .

**Usage**

```
expint(x)
expexpint(x)
expint.E1(x)
```

**Arguments**

x                      Numeric. Ideally a vector of positive reals.

**Details**

The exponential integral  $Ei(x)$  function is the integral of  $\exp(t)/t$  from 0 to  $x$ , for positive real  $x$ . The function  $E_1(x)$  is the integral of  $\exp(-t)/t$  from  $x$  to infinity, for positive real  $x$ .

**Value**

Function `expint(x)` returns  $Ei(x)$ , function `expexpint(x)` returns  $\exp(-x) \times Ei(x)$ , function `expint.E1(x)` returns  $E_1(x)$ .

**Note**

This function has not been tested thoroughly.

**Author(s)**

T. W. Yee has simply written a small wrapper function to call the above FORTRAN code.

**References**

<http://www.netlib.org/specfun/ei>.

**See Also**

[log](#), [exp](#).

**Examples**

```
## Not run:
par(mfrow = c(2, 2))
curve(expint, 0.01, 2, xlim = c(0, 2), ylim = c(-3, 5),
      las = 1, col = "orange")
abline(v = (-3):5, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")

curve(expexpint, 0.01, 2, xlim = c(0, 2), ylim = c(-3, 2),
      las = 1, col = "orange")
abline(v = (-3):2, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")

curve(expint.E1, 0.01, 2, xlim = c(0, 2), ylim = c(0, 5),
      las = 1, col = "orange")
abline(v = (-3):2, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")

## End(Not run)
```

---

explink

---

*Exponential Link Function*


---

**Description**

Computes the exponential transformation, including its inverse and the first two derivatives.

**Usage**

```
explink(theta, bvalue = NULL, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

**Arguments**

theta            Numeric or character. See below for further details.  
bvalue           See [cloglog](#).  
inverse, deriv, short, tag  
                 Details at [Links](#).

**Details**

The exponential link function is potentially suitable for parameters that are positive. Numerical values of theta close to negative or positive infinity may result in 0, Inf, -Inf, NA or NaN.

**Value**

For explink with `deriv = 0`, the exponential of theta, i.e., `exp(theta)` when `inverse = FALSE`. And if `inverse = TRUE` then `log(theta)`; if theta is not positive then it will return NaN.

For `deriv = 1`, then the function returns  $d\theta / d\eta$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

This function has particular use for computing quasi-variances when used with `rcim` and `uninormal`.

Numerical instability may occur when theta is close to negative or positive infinity. One way of overcoming this (one day) is to use `bvalue`.

**Author(s)**

Thomas W. Yee

**See Also**

[Links](#), [loge](#), [rcim](#), [Qvar](#), [uninormal](#).

**Examples**

```
theta <- rnorm(30)
explink(theta)
max(abs(explink(explink(theta), inverse = TRUE) - theta)) # Should be 0
```

---

explog

---

*The Exponential Logarithmic Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the exponential logarithmic distribution.

**Usage**

```
dexplog(x, scale = 1, shape, log = FALSE)
pexplog(q, scale = 1, shape)
qexplog(p, scale = 1, shape)
rexplog(n, scale = 1, shape)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
scale, shape	positive scale and shape parameters.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [explogff](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

**Value**

`dexplog` gives the density, `pexplog` gives the distribution function, `qexplog` gives the quantile function, and `rexlog` generates random deviates.

**Note**

We define scale as the reciprocal of the scale parameter used by Tahmasabi and Rezaei (2008).

**Author(s)**

J. G. Lauder and T. W. Yee

**See Also**

[explogff](#), [exponential](#).

**Examples**

```
## Not run:
shape <- 0.5; scale <- 2; nn <- 501
x <- seq(-0.50, 6.0, len = nn)
plot(x, dexplog(x, scale, shape), type = "l", las = 1, ylim = c(0, 1.1),
     ylab = paste("[dp]explog(shape = ", shape, ", scale = ", scale, ")"),
     col = "blue", cex.main = 0.8,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pexplog(x, scale, shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qexplog(probs, scale, shape = shape)
lines(Q, dexplog(Q, scale, shape = shape), col = "purple", lty = 3, type = "h")
lines(Q, pexplog(Q, scale, shape = shape), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pexplog(Q, scale, shape = shape) - probs)) # Should be 0

## End(Not run)
```

explogff

*Exponential Logarithmic Distribution Family Function***Description**

Estimates the two parameters of the exponential logarithmic distribution by maximum likelihood estimation.

**Usage**

```
explogff(lscale = "loge", lshape = "logit",
         iscale = NULL,  ishape = NULL,
         tol12 = 1e-05, zero = 1, nsimEIM = 400)
```

**Arguments**

lscale, lshape See [CommonVGAMffArguments](#) for information.  
 tol12 Numeric. Tolerance for testing whether a parameter has value 1 or 2.  
 iscale, ishape, zero, nsimEIM  
 See [CommonVGAMffArguments](#).

**Details**

The exponential logarithmic distribution has density function

$$f(y; c, s) = (1/(-\log p))(((1/c)(1-s)e^{-y/c})/(1 - (1-s)e^{-y/c}))$$

where  $y > 0$ , scale parameter  $c > 0$ , and shape parameter  $s \in (0, 1)$ . The mean,  $(-polylog(2, 1 - p)c)/\log(s)$  is *not* returned as the fitted values. Note the median is  $c \log(1 + \sqrt{s})$  and it is *currently* returned as the fitted values. Simulated Fisher scoring is implemented.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

We define scale as the reciprocal of the rate parameter used by Tahmasabi and Sadegh (2008).  
 Yet to do: find a polylog() function.

**Author(s)**

J. G. Lauder and T. W .Yee

**References**

Tahmasabi, R., Sadegh, R. (2008). A two-parameter lifetime distribution with decreasing failure rate. *Computational Statistics and Data Analysis*, **52**, 3889–3901.



See Also

[dexplog](#), [exponential](#),

Examples

```
## Not run:  scale <- exp(2); shape <- logit(-1, inverse = TRUE)
edata <- data.frame(y = rexplog(n = 2000, scale = scale, shape = shape))
fit <- vglm(y ~ 1, explogff, edata, trace = TRUE)
c(with(edata, median(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

---

exponential	<i>Exponential Distribution</i>
-------------	---------------------------------

---

Description

Maximum likelihood estimation for the exponential distribution.

Usage

```
exponential(link = "loge", location = 0, expected = TRUE,
            shrinkage.init = 0.95, zero = NULL)
```

Arguments

- link                   Parameter link function applied to the positive parameter *rate*. See [Links](#) for more choices.
- location               Numeric of length 1, the known location parameter, *A*, say.
- expected               Logical. If TRUE Fisher scoring is used, otherwise Newton-Raphson. The latter is usually faster.
- shrinkage.init, zero   See [CommonVGAMffArguments](#) for information.

Details

The family function assumes the response *Y* has density

$$f(y) = \lambda \exp(-\lambda(y - A))$$

for  $y > A$ , where *A* is the known location parameter. By default,  $A = 0$ . Then  $E(Y) = A + 1/\lambda$  and  $Var(Y) = 1/\lambda^2$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

Suppose  $A = 0$ . For a fixed time interval, the number of events is Poisson with mean  $\lambda$  if the time between events has a geometric distribution with mean  $\lambda^{-1}$ . The argument `rate` in `exponential` is the same as `rexp` etc. The argument `lambda` in `rpois` is somewhat the same as `rate` here.

**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[amlexponential](#), [laplace](#), [expgeometric](#), [explogff](#), [poissonff](#), [mix2exp](#), [freund61](#).

**Examples**

```
edata <- data.frame(x2 = runif(nn <- 100) - 0.5)
edata <- transform(edata, x3 = runif(nn) - 0.5)
edata <- transform(edata, eta = 0.2 - 0.7 * x2 + 1.9 * x3)
edata <- transform(edata, rate = exp(eta))
edata <- transform(edata, y = rexp(nn, rate = rate))
with(edata, stem(y))

fit.slow <- vglm(y ~ x2 + x3, exponential, edata, trace = TRUE, crit = "c")
fit.fast <- vglm(y ~ x2 + x3, exponential(exp = FALSE), edata,
               trace = TRUE, crit = "coef")
coef(fit.slow, mat = TRUE)
summary(fit.slow)
```

---

exppois

---

*The Exponential Poisson Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the exponential poisson distribution.

**Usage**

```
dexppois(x, lambda, betave = 1, log = FALSE)
pexppois(q, lambda, betave = 1)
qexppois(p, lambda, betave = 1)
rexppois(n, lambda, betave = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
lambda, betave	both positive parameters.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [exppoisson](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

**Value**

`dexppois` gives the density, `pexppois` gives the distribution function, `qexppois` gives the quantile function, and `rexppois` generates random deviates.

**Author(s)**

J. G. Lauder, jamesglaunder@gmail.com

**See Also**

[exppoisson](#).

**Examples**

```
## Not run:
lambda <- 2; betave <- 2; nn <- 201
x <- seq(-0.05, 1.05, len = nn)
plot(x, dexppois(x, lambda, betave), type = "l", las = 1, ylim = c(0, 5),
     ylab = paste("[dp]exppoisson(lambda = ", lambda, ", betave = ", betave, ")"),
     col = "blue", cex.main = 0.8,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pexppois(x, lambda, betave), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qexppois(probs, lambda, betave)
lines(Q, dexppois(Q, lambda, betave), col = "purple", lty = 3, type = "h")
lines(Q, pexppois(Q, lambda, betave), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
```

```
max(abs(pexppois(Q, lambda, betave) - probs)) # Should be 0
## End(Not run)
```

---

exppoisson

*Exponential Poisson Distribution Family Function*


---

## Description

Estimates the two parameters of the exponential Poisson distribution by maximum likelihood estimation.

## Usage

```
exppoisson(llambda = "loge", lbetave = "loge",
            ilambda = 1.1, ibetave = 2, zero = NULL)
```

## Arguments

llambda, lbetave

Link function for the two positive parameters. See [Links](#) for more choices.

ilambda, ibetave

Numeric. Initial values for the lambda and betave parameters. Currently this function is not intelligent enough to obtain better initial values.

zero

See [CommonVGAMffArguments](#).

## Details

The exponential Poisson distribution has density function

$$f(y; \lambda = \text{shape}, \beta = \text{scale}) = \frac{\lambda\beta}{1 - e^{-\lambda}} e^{-\lambda - \beta y + \lambda \exp(-\beta y)}$$

where  $y > 0$  and the parameters shape,  $\lambda$ , and scale,  $\beta$ , are positive. The distribution implies a population facing discrete hazard rates which are multiples of a base hazard. This **VGAM** family function requires the hypergeo package (to use their genhypergeo function).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

This **VGAM** family function does not work properly!

## Author(s)

J. G. Lauder, jamesglaunder@gmail.com

## References

Kus, C., (2007). A new lifetime distribution. *Computational Statistics and Data Analysis*, **51**, 4497–4509.

## See Also

[dexppois](#), [exponential](#), [poisson](#).

## Examples

```
## Not run:
lambda <- exp(1); betave <- exp(2)
rdata <- data.frame(y = rexppois(n = 1000, lambda, betave))
library(hypergeo)
fit <- vglm(y ~ 1, exppoisson, rdata, trace = TRUE)
c(with(rdata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

---

Felix

---

*The Felix Distribution*


---

## Description

Density

Felix distribution.

## Usage

```
dfelix(x, a = 0.25, log = FALSE)
```

## Arguments

x	vector of quantiles.
a	See <a href="#">felix</a> .
log	Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See [felix](#), the **VGAM** family function for estimating the parameter, for the formula of the probability density function and other details.

**Value**

dfelix gives the density.

**Warning**

The default value of a is subjective.

**Author(s)**

T. W. Yee

**See Also**

[felix.](#)

**Examples**

```
## Not run:
a <- 0.25; x <- 1:15
plot(x, dfelix(x, a), type = "h", las = 1, col = "blue",
      ylab = paste("dfelix(a=", a, ")"),
      main = "Felix density function")

## End(Not run)
```

---

felix

*Felix Distribution Family Function*


---

**Description**

Estimates the parameter of a Felix distribution by maximum likelihood estimation.

**Usage**

```
felix(link = elogit(min = 0, max = 0.5), imethod = 1)
```

**Arguments**

link	Link function for the parameter; see <a href="#">Links</a> for more choices and for general information.
imethod	See <a href="#">CommonVGAMffArguments</a> . Valid values are 1, 2, 3 or 4.

**Details**

The Felix distribution is an important basic Lagrangian distribution. The density function is

$$f(y; a) = \frac{1}{((y-1)/2)!} y^{(y-3)/2} a^{(y-1)/2} \exp(-ay)$$

where  $y = 1, 3, 5, \dots$  and  $0 < a < 0.5$ . The mean is  $1/(1 - 2a)$  (returned as the fitted values). Fisher scoring is implemented.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

T. W. Yee

**References**

Consul, P. C. and Famoye, F. (2006) *Lagrangian Probability Distributions*, Boston: Birkhauser.

**See Also**

[dfelix](#), [borel.tanner](#).

**Examples**

```
fdata <- data.frame(y = 2 * rpois(n = 200, 1) + 1) # Not real data!
fit <- vglm(y ~ 1, felix, fdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

fff

---

*F Distribution Family Function*


---

**Description**

Maximum likelihood estimation of the (2-parameter) F distribution.

**Usage**

```
fff(link = "log", idf1 = NULL, idf2 = NULL, nsimEIM = 100,
    imethod = 1, zero = NULL)
```

**Arguments**

link	Parameter link function for both parameters. See <a href="#">Links</a> for more choices. The default keeps the parameters positive.
idf1, idf2	Numeric and positive. Initial value for the parameters. The default is to choose each value internally.
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> for more information.
imethod	Initialization method. Either the value 1 or 2. If both fail try setting values for idf1 and idf2.

## Details

The F distribution is named after Fisher and has a density function that has two parameters, called `df1` and `df2` here. This function treats these degrees of freedom as *positive reals* rather than integers. The mean of the distribution is  $df2/(df2 - 2)$  provided  $df2 > 2$ , and its variance is  $2df2^2(df1 + df2 - 2)/(df1(df2 - 2)^2(df2 - 4))$  provided  $df2 > 4$ . The estimated mean is returned as the fitted values. Although the F distribution can be defined to accommodate a non-centrality parameter `ncp`, it is assumed zero here. Actually it shouldn't be too difficult to handle any known `ncp`; something to do in the short future.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

Numerical problems will occur when the estimates of the parameters are too low or too high.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

[FDist](#).

## Examples

```
## Not run:
fdata <- data.frame(x2 = runif(nn <- 2000))
fdata <- transform(fdata, df1 = exp(2+0.5*x2),
                  df2 = exp(2-0.5*x2))
fdata <- transform(fdata, y = rf(nn, df1, df2))
fit <- vglm(y ~ x2, fff, fdata, trace = TRUE)
coef(fit, matrix = TRUE)

## End(Not run)
```



**Description**

Density, distribution function, and random generation for the (one parameter) bivariate Farlie-Gumbel-Morgenstern's distribution.

**Usage**

```
dfgm(x1, x2, alpha, log = FALSE)
pfgm(q1, q2, alpha)
rfgm(n, alpha)
```

**Arguments**

`x1`, `x2`, `q1`, `q2` vector of quantiles.  
`n` number of observations. Must be a positive integer of length 1.  
`alpha` the association parameter.  
`log` Logical. If TRUE then the logarithm is returned.

**Details**

See [fgm](#), the **VGAM** family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

**Value**

`dfgm` gives the density, `pfgm` gives the distribution function, and `rfgm` generates random deviates (a two-column matrix).

**Author(s)**

T. W. Yee

**See Also**

[fgm](#).

**Examples**

```
## Not run: N <- 101; x <- seq(0.0, 1.0, len = N); alpha <- 0.7
ox <- expand.grid(x, x)
zedd <- dfgm(ox[, 1], ox[, 2], alpha = alpha)
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pfgm(ox[, 1], ox[, 2], alpha = alpha)
contour(x, x, matrix(zedd, N, N), col = "blue")
```

```

plot(r <- rfgm(n = 3000, alpha = alpha), col = "blue")
par(mfrow = c(1, 2))
hist(r[, 1]) # Should be uniform
hist(r[, 2]) # Should be uniform

## End(Not run)

```

fgm

*Farlie-Gumbel-Morgenstern's Bivariate Distribution Family Function*

### Description

Estimate the association parameter of Farlie-Gumbel-Morgenstern's bivariate distribution by maximum likelihood estimation.

### Usage

```
fgm(lapar = "rhobit", iapar = NULL, imethod = 1)
```

### Arguments

lapar, iapar, imethod

Details at [CommonVGAMffArguments](#). See [Links](#) for more link function choices.

### Details

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = y_1 y_2 (1 + \alpha(1 - y_1)(1 - y_2))$$

for  $-1 < \alpha < 1$ . The support of the function is the unit square. The marginal distributions are the standard uniform distributions. When  $\alpha = 0$  the random variables are independent.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

### Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 0.5. This is because each marginal distribution corresponds to a standard uniform distribution.

### Author(s)

T. W. Yee

References

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

Smith, M. D. (2007) Invariance theorems for Fisher information. *Communications in Statistics—Theory and Methods*, **36**(12), 2213–2222.

See Also

[rfgm](#), [bifrankcop](#), [morgenstern](#).

Examples

```
ymat <- rfgm(n = 1000, alpha = rhobit(3, inverse = TRUE))
## Not run: plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = fgm, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

---

fill	<i>Creates a Matrix of Appropriate Dimension</i>
------	--

---

Description

A support function for the argument `xij`, it generates a matrix of an appropriate dimension.

Usage

```
fill(x, values = 0, ncolx = ncol(x))
```

Arguments

- |        |  |
|--------|--|
| x      | A vector or matrix which is used to determine the dimension of the answer, in particular, the number of rows. After converting <code>x</code> to a matrix if necessary, the answer is a matrix of values <code>values</code> , of dimension <code>nrow(x)</code> by <code>ncolx</code> . |
| values | Numeric. The answer contains these values, which are recycled <i>columnwise</i> if necessary, i.e., as <code>matrix(values, ..., byrow=TRUE)</code> .  |
| ncolx  | The number of columns of the returned matrix. The default is the number of columns of <code>x</code> .   |

## Details

The `xij` argument for `vglm` allows the user to input variables specific to each linear/additive predictor. For example, consider the bivariate logit model where the first/second linear/additive predictor is the logistic regression of the first/second binary response respectively. The third linear/additive predictor is  $\log(\text{OR}) = \eta_3$ , where OR is the odds ratio. If one has ocular pressure as a covariate in this model then `xij` is required to handle the ocular pressure for each eye, since these will be different in general. [This contrasts with a variable such as age, the age of the person, which has a common value for both eyes.] In order to input these data into `vglm` one often finds that functions `fill`, `fill1`, etc. are useful.

All terms in the `xij` and `formula` arguments in `vglm` must appear in the `form2` argument too.

## Value

`matrix(values, nrow=nrow(x), ncol=ncol(x))`, i.e., a matrix consisting of values `values`, with the number of rows matching `x`, and the default number of columns is the number of columns of `x`.

## Note

The effect of the `xij` argument is after other arguments such as `exchangeable` and `zero`. Hence `xij` does not affect constraint matrices.

Additionally, there are currently 3 other identical `fill` functions, called `fill1`, `fill2` and `fill3`; if you need more then assign `fill4 = fill5 = fill1` etc. The reason for this is that if more than one `fill` function is needed then they must be unique. For example, if  $M = 4$  then `xij = op ~ lop + rop + fill(mop) + fill(mop)` would reduce to `xij = op ~ lop + rop + fill(mop)`, whereas `xij = op ~ lop + rop + fill1(mop) + fill2(mop)` would retain all  $M$  terms, which is needed.

In Examples 1 to 3 below, the `xij` argument illustrates covariates that are specific to a linear predictor. Here, `lop/rop` are the ocular pressures of the left/right eye in an artificial dataset, and `mop` is their mean. Variables `leye` and `reye` might be the presence/absence of a particular disease on the LHS/RHS eye respectively.

In Example 3, the `xij` argument illustrates fitting the (exchangeable) model where there is a common smooth function of the ocular pressure. One should use regression splines since `s` in `vgam` does not handle the `xij` argument. However, regression splines such as `bs` and `ns` need to have the same basis functions here for both functions, and Example 3 illustrates a trick involving a function `BS` to obtain this, e.g., same knots. Although regression splines create more than a single column per term in the model matrix, `fill(BS(lop, rop))` creates the required (same) number of columns.

## Author(s)

T. W. Yee

## References

More information can be found at <http://www.stat.auckland.ac.nz/~yee>.

## See Also

`vglm.control`, `vglm`, `multinomial`.

## Examples

```

fill(runif(5))
fill(runif(5), ncol = 3)
fill(runif(5), val = 1, ncol = 3)

# Generate eyes data for the examples below. Eyes are independent (OR=1).
nn <- 1000 # Number of people
eyesdat = data.frame(lop = round(runif(nn), 2),
                     rop = round(runif(nn), 2),
                     age = round(rnorm(nn, 40, 10)))
eyesdat <- transform(eyesdat,
  mop = (lop + rop) / 2,      # Mean ocular pressure
  op  = (lop + rop) / 2,      # Value unimportant unless plotting
#   op  = lop,                # Choose this if plotting
  eta1 = 0 - 2*lop + 0.04*age, # Linear predictor for left eye
  eta2 = 0 - 2*rop + 0.04*age) # Linear predictor for right eye
eyesdat <- transform(eyesdat,
  leye = rbinom(nn, size = 1, prob = logit(eta1, inverse = TRUE)),
  reye = rbinom(nn, size = 1, prob = logit(eta2, inverse = TRUE)))

# Example 1
# All effects are linear
fit1 <- vglm(cbind(leye, reye) ~ op + age,
             family = binom2.or(exchangeable=TRUE, zero=3),
             data=eyesdat, trace=TRUE,
             xij = list(op ~ lop + rop + fill(lop)),
             form2 = ~ op + lop + rop + fill(lop) + age)
head(model.matrix(fit1, type="lm")) # LM model matrix
head(model.matrix(fit1, type="vlm")) # Big VLM model matrix
coef(fit1)
coef(fit1, matrix = TRUE) # Unchanged with 'xij'
constraints(fit1)
max(abs(predict(fit1)-predict(fit1, new = eyesdat))) # Predicts correctly
summary(fit1)
## Not run: plotvgam(fit1, se = TRUE) # Wrong, e.g., because it plots against op, not lop.
# So set op=lop in the above for a correct plot.

## End(Not run)

# Example 2
# Model OR as a linear function of mop
fit2 <- vglm(cbind(leye, reye) ~ op + age, data = eyesdat, trace = TRUE,
             binom2.or(exchangeable = TRUE, zero = NULL),
             xij = list(op ~ lop + rop + mop),
             form2 = ~ op + lop + rop + mop + age)
head(model.matrix(fit2, type = "lm")) # LM model matrix
head(model.matrix(fit2, type = "vlm")) # Big VLM model matrix
coef(fit2)
coef(fit2, matrix = TRUE) # Unchanged with 'xij'
max(abs(predict(fit2) - predict(fit2, new = eyesdat))) # Predicts correctly

```

```

summary(fit2)
## Not run: plotvgam(fit2, se = TRUE) # Wrong because it plots against op, not lop.

# Example 3. This model uses regression splines on ocular pressure.
# It uses a trick to ensure common basis functions.
BS <- function(x, ...)
  bs(c(x,...), df = 3)[1:length(x), , drop = FALSE] # trick

fit3 <- vglm(cbind(leye,reye) ~ BS(lop,rop) + age,
  family = binom2.or(exchangeable = TRUE, zero = 3),
  data = eyesdat, trace = TRUE,
  xij = list(BS(lop,rop) ~ BS(lop,rop) +
    BS(rop,lop) +
    fill(BS(lop,rop))),
  form2 = ~ BS(lop,rop) + BS(rop,lop) + fill(BS(lop,rop)) +
    lop + rop + age)
head(model.matrix(fit3, type = "lm")) # LM model matrix
head(model.matrix(fit3, type = "vlm")) # Big VLM model matrix
coef(fit3)
coef(fit3, matrix = TRUE)
summary(fit3)
fit3@smart.prediction
max(abs(predict(fit3) - predict(fit3, new = eyesdat))) # Predicts correctly
predict(fit3, new = head(eyesdat)) # Note the 'scalar' OR, i.e., zero=3
max(abs(head(predict(fit3)) - predict(fit3, new = head(eyesdat)))) # Should be 0
## Not run:
plotvgam(fit3, se = TRUE, xlab = "lop") # Correct

## End(Not run)

```

finney44

*Toxicity trial for insects***Description**

A data frame of a toxicity trial.

**Usage**

```
data(finney44)
```

**Format**

A data frame with 6 observations on the following 3 variables.

pconc a numeric vector, percent concentration of pyrethrins.

hatched number of eggs that hatched.

unhatched number of eggs that did not hatch.

## Details

Finney (1944) describes a toxicity trial of five different concentrations of pyrethrins (percent) plus a control that were administered to eggs of *Ephesia kuhniella*. The natural mortality rate is large, and a common adjustment is to use Abbott's formula.

## References

Finney, D. J., 1944. The application of the probit method to toxicity test data adjusted for mortality in the controls. *Annals of Applied Biology*, **31**, 68–74.

Abbott, W. S. (1925). A method of computing the effectiveness of an insecticide. *Journal of Economic Entomology*, 18, 265–7.

## Examples

```
data(finney44)
transform(finney44, mortality = unhatched / (hatched + unhatched))
```

---

fisherz

---

*Fisher's Z Link Function*


---

## Description

Computes the Fisher Z transformation, including its inverse and the first two derivatives.

## Usage

```
fisherz(theta, bminvalue = NULL, bmaxvalue = NULL,
        inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

## Arguments

**theta**                Numeric or character. See below for further details.

**bminvalue, bmaxvalue**        Optional boundary values. Values of theta which are less than or equal to  $-1$  can be replaced by bminvalue before computing the link function value. Values of theta which are greater than or equal to  $1$  can be replaced by bmaxvalue before computing the link function value. See [Links](#).

**inverse, deriv, short, tag**        Details at [Links](#).

## Details

The fisherz link function is commonly used for parameters that lie between  $-1$  and  $1$ . Numerical values of theta close to  $-1$  or  $1$  or out of range result in Inf, -Inf, NA or NaN.

**Value**

For `deriv = 0`,  $0.5 * \log((1+\text{theta})/(1-\text{theta}))$  (same as `atanh(theta)`) when `inverse = FALSE`, and if `inverse = TRUE` then  $(\exp(2*\text{theta})-1)/(\exp(2*\text{theta})+1)$  (same as `tanh(theta)`).

For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of `theta` if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

Numerical instability may occur when `theta` is close to  $-1$  or  $1$ . One way of overcoming this is to use, e.g., `bminvalue`.

The link function `rhobit` is very similar to `fisherz`, e.g., just twice the value of `fisherz`. This link function may be renamed to `atanhlink` in the near future.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), `rhobit`, `atanh`, `logit`.

**Examples**

```
theta <- seq(-0.99, 0.99, by = 0.01)
y <- fisherz(theta)
## Not run: plot(theta, y, type = "l", las = 1, ylab = "",
  main = "fisherz(theta)", col = "blue")
abline(v = (-1):1, h = 0, lty = 2, col = "gray")
## End(Not run)

x <- c(seq(-1.02, -0.98, by = 0.01), seq(0.97, 1.02, by = 0.01))
fisherz(x) # Has NAs
fisherz(x, bminvalue = -1 + .Machine$double.eps,
  bmaxvalue = 1 - .Machine$double.eps) # Has no NAs
```



**Description**

Density, distribution function, quantile function and random generation for the Fisk distribution with shape parameter  $a$  and scale parameter  $scale$ .

**Usage**

```
dfisk(x, shape1.a, scale = 1, log = FALSE)
pfisk(q, shape1.a, scale = 1)
qfisk(p, shape1.a, scale = 1)
rfisk(n, shape1.a, scale = 1)
```

**Arguments**

$x$ , $q$	vector of quantiles.
$p$	vector of probabilities.
$n$	number of observations. If $\text{length}(n) > 1$ then the length is taken to be the number required.
$shape1.a$	shape parameter.
$scale$	scale parameter.
$log$	Logical. If $log = TRUE$ then the logarithm of the density is returned.

**Details**

See [fisk](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

**Value**

`dfisk` gives the density, `pfisk` gives the distribution function, `qfisk` gives the quantile function, and `rfisk` generates random deviates.

**Note**

The Fisk distribution is a special case of the 4-parameter generalized beta II distribution.

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[fisk](#), [genbetaII](#).

**Examples**

```
fdata <- data.frame(y = rfisk(n = 1000, 4, 6))
fit <- vglm(y ~ 1, fisk, data = fdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

---

fisk

*Fisk Distribution family function*


---

**Description**

Maximum likelihood estimation of the 2-parameter Fisk distribution.

**Usage**

```
fisk(lshape1.a = "loge", lscale = "loge",
     ishape1.a = NULL, iscale = NULL, zero = NULL)
```

**Arguments**

lshape1.a, lscale

Parameter link functions applied to the (positive) parameters a and scale. See [Links](#) for more choices.

ishape1.a, iscale

Optional initial values for a and scale.

zero

An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2} which correspond to a, scale, respectively.

**Details**

The 2-parameter Fisk (aka log-logistic) distribution is the 4-parameter generalized beta II distribution with shape parameter  $q = p = 1$ . It is also the 3-parameter Singh-Maddala distribution with shape parameter  $q = 1$ , as well as the Dagum distribution with  $p = 1$ . More details can be found in Kleiber and Kotz (2003).

The Fisk distribution has density

$$f(y) = ay^{a-1} / [b^a \{1 + (y/b)^a\}^2]$$

for  $a > 0$ ,  $b > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter scale, and  $a$  is a shape parameter. The cumulative distribution function is

$$F(y) = 1 - [1 + (y/b)^a]^{-1} = [1 + (y/b)^{-a}]^{-1}.$$

The mean is

$$E(Y) = b \Gamma(1 + 1/a) \Gamma(1 - 1/a)$$

provided  $a > 1$ ; these are returned as the fitted values.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

See the note in [genbetaII](#).

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ: Wiley-Interscience.

**See Also**

[Fisk](#), [genbetaII](#), [betaII](#), [dagum](#), [sinmad](#), [invlomax](#), [lomax](#), [paralogistic](#), [invparalogistic](#).

**Examples**

```
fdata <- data.frame(y = rfisk(n = 200, exp(1), exp(2)))
fit <- vglm(y ~ 1, fisk, fdata, trace = TRUE)
fit <- vglm(y ~ 1, fisk(ishape1.a = exp(1)), fdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

fittedvlm

*Fitted Values of a VLM object*


---

**Description**

Extractor function for the fitted values of a model object that inherits from a *vector linear model* (VLM), e.g., a model of class "vglm".

**Usage**

```
fittedvlm(object, matrix.arg = TRUE, type.fitted = NULL, ...)
```

### Arguments

<code>object</code>	a model object that inherits from a VLM.
<code>matrix.arg</code>	Logical. Return the answer as a matrix? If FALSE then it will be a vector.
<code>type.fitted</code>	Character. Some <b>VGAM</b> family functions have a <code>type.fitted</code> argument. If so then a different type of fitted value can be returned. It is recomputed from the model after convergence. Note: this is an experimental feature and not all <b>VGAM</b> family functions have this implemented yet.
<code>...</code>	Currently unused.

### Details

The “fitted values” usually corresponds to the mean response, however, because the **VGAM** package fits so many models, this sometimes refers to quantities such as quantiles. The mean may even not exist, e.g., for a Cauchy distribution.

Note that the fitted value is output from the `@linkinv` slot of the **VGAM** family function, where the `eta` argument is the  $n \times M$  matrix of linear predictors.

### Value

The fitted values evaluated at the final IRLS iteration.

### Note

This function is one of several extractor functions for the **VGAM** package. Others include `coef`, `deviance`, `weights` and `constraints` etc. This function is equivalent to the `methods` function for the generic function `fitted.values`.

If `fit` is a VLM or VGLM then `fitted(fit)` and `predict(fit, type = "response")` should be equivalent (see [predictvglm](#)). The latter has the advantage in that it handles a `newdata` argument so that the fitted values can be computed for a different data set.

### Author(s)

Thomas W. Yee

### References

Chambers, J. M. and T. J. Hastie (eds) (1992) *Statistical Models in S*. Wadsworth & Brooks/Cole.

### See Also

[fitted](#), [predictvglm](#), [vglmff-class](#).

### Examples

```
# Categorical regression example 1
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo))
fitted(fit1)
```

```
# LMS quantile regression example 2
fit2 <- vgam(BMI ~ s(age, df = c(4, 2)),
             lms.bcn(zero = 1), data = bmi.nz, trace = TRUE)
head(predict(fit2, type = "response")) # Equal to the the following two:
head(fitted(fit2))
predict(fit2, type = "response", newdata = head(bmi.nz))

# Zero-inflated example 3
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pstr0.3 = logit(-0.5, inverse = TRUE),
                  lambda.3 = loge(-0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(nn, lambda = lambda.3, pstr0 = pstr0.3))
fit3 <- vglm(y1 ~ x2, zipoisson (zero = NULL), data = zdata, crit = "coef")
head(fitted(fit3, type.fitted = "mean" ))      # E(Y), which is the default
head(fitted(fit3, type.fitted = "pobs0"))      # P(Y = 0)
head(fitted(fit3, type.fitted = "pstr0"))      # Prob of a structural 0
head(fitted(fit3, type.fitted = "onempstr0"))  # 1 - prob of a structural 0
```

Foldnorm

*The Folded-Normal Distribution*

## Description

Density, distribution function, quantile function and random generation for the (generalized) folded-normal distribution.

## Usage

```
dfoldnorm(x, mean = 0, sd = 1, a1 = 1, a2 = 1, log = FALSE)
pfoldnorm(q, mean = 0, sd = 1, a1 = 1, a2 = 1)
qfoldnorm(p, mean = 0, sd = 1, a1 = 1, a2 = 1, ...)
rfoldnorm(n, mean = 0, sd = 1, a1 = 1, a2 = 1)
```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Same as <a href="#">rnorm</a> .
<code>mean, sd</code>	see <a href="#">rnorm</a> .
<code>a1, a2</code>	see <a href="#">foldnormal</a> .
<code>log</code>	Logical. If TRUE then the log density is returned.
<code>...</code>	Arguments that can be passed into <a href="#">uniroot</a> .

## Details

See [foldnormal](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

**Value**

dfoldnorm gives the density, pfoldnorm gives the distribution function, qfoldnorm gives the quantile function, and rfoldnorm generates random deviates.

**Note**

qfoldnorm runs very slowly because it calls [uniroot](#) for each value of the argument p. The solution is consequently not exact; the `...` can be used to obtain a more accurate solution if necessary.

**Author(s)**

T. W. Yee

**See Also**

[foldnormal](#), [uniroot](#).

**Examples**

```
## Not run:
m <- 1.5; SD <- exp(0)
x <- seq(-1, 4, len = 501)
plot(x, dfoldnorm(x, m = m, sd = SD), type = "l", ylim = 0:1, las = 1,
      ylab = paste("foldnorm(m = ", m, ", sd = ", round(SD, digits = 3), ")"),
      main = "Blue is density, orange is cumulative distribution function",
      sub = "Purple lines are the 10,20,...,90 percentiles", col = "blue")
lines(x, pfoldnorm(x, m = m, sd = SD), col = "orange")
abline(h = 0)
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qfoldnorm(probs, m = m, sd = SD)
lines(Q, dfoldnorm(Q, m = m, sd = SD), col = "purple", lty = 3, type = "h")
lines(Q, pfoldnorm(Q, m = m, sd = SD), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pfoldnorm(Q, m = m, sd = SD) - probs)) # Should be 0

## End(Not run)
```

---

foldnormal

---

*Folded Normal Distribution Family Function*


---

**Description**

Fits a (generalized) folded (univariate) normal distribution.

**Usage**

```
foldnormal(lmean = "identity", lsd = "loge", imean = NULL, isd = NULL,
           a1 = 1, a2 = 1, nsimEIM = 500, imethod = 1, zero = NULL)
```

**Arguments**

lmean, lsd	Link functions for the mean and standard deviation parameters of the usual univariate normal distribution. They are $\mu$ and $\sigma$ respectively. See <a href="#">Links</a> for more choices.
imean, isd	Optional initial values for $\mu$ and $\sigma$ . A NULL means a value is computed internally. See <a href="#">CommonVGAMffArguments</a> .
a1, a2	Positive weights, called $a_1$ and $a_2$ below. Each must be of length 1.
nsimEIM, imethod, zero	See <a href="#">CommonVGAMffArguments</a> .

**Details**

If a random variable has an ordinary univariate normal distribution then the absolute value of that random variable has an ordinary *folded normal distribution*. That is, the sign has not been recorded; only the magnitude has been measured.

More generally, suppose  $X$  is normal with mean `mean` and standard deviation `sd`. Let  $Y = \max(a_1 X, -a_2 X)$  where  $a_1$  and  $a_2$  are positive weights. This means that  $Y = a_1 X$  for  $X > 0$ , and  $Y = a_2 X$  for  $X < 0$ . Then  $Y$  is said to have a *generalized folded normal distribution*. The ordinary folded normal distribution corresponds to the special case  $a_1 = a_2 = 1$ .

The probability density function of the ordinary folded normal distribution can be written `dnorm(y, mean, sd) + dnorm(y, for  $y \geq 0$ . By default, mean and log(sd) are the linear/additive predictors. Having mean=0 and sd=1 results in the half-normal distribution. The mean of an ordinary folded normal distribution is`

$$E(Y) = \sigma \sqrt{2/\pi} \exp(-\mu^2/(2\sigma^2)) + \mu[1 - 2\Phi(-\mu/\sigma)]$$

and these are returned as the fitted values. Here,  $\Phi()$  is the cumulative distribution function of a standard normal ([pnorm](#)).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

Under- or over-flow may occur if the data is ill-conditioned. It is recommended that several different initial values be used to help avoid local solutions.

**Note**

The response variable for this family function is the same as [uninormal](#) except positive values are required. Reasonably good initial values are needed. Fisher scoring using simulation is implemented.

See [CommonVGAMffArguments](#) for general information about many of these arguments.

**Author(s)**

Thomas W. Yee

## References

Lin, P. C. (2005) Application of the generalized folded-normal distribution to the process capability measures. *International Journal of Advanced Manufacturing Technology*, **26**, 825–830.

## See Also

[rfoldnorm](#), [uninormal](#), [dnorm](#), [skewnormal](#).

## Examples

```
## Not run: m <- 2; SD <- exp(1)
fdata <- data.frame(y = rfoldnorm(n <- 1000, m = m, sd = SD))
hist(with(fdata, y), prob = TRUE, main = paste("foldnormal(m = ", m,
  ", sd = ", round(SD, 2), ")"))
fit <- vglm(y ~ 1, fam = foldnormal, fdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
mygrid <- with(fdata, seq(min(y), max(y), len = 200)) # Add the fit to the histogram
lines(mygrid, dfoldnorm(mygrid, Cfit[1], Cfit[2]), col = "orange")

## End(Not run)
```

---

Frank

*Frank's Bivariate Distribution*

---

## Description

Density, distribution function, and random generation for the (one parameter) bivariate Frank distribution.

## Usage

```
dbifrankcop(x1, x2, alpha, log = FALSE)
pbifrankcop(q1, q2, alpha)
rbifrankcop(n, alpha)
```

## Arguments

`x1`, `x2`, `q1`, `q2` vector of quantiles.  
`n` number of observations. Must be a positive integer of length 1.  
`alpha` the positive association parameter  $\alpha$ .  
`log` Logical. If `log = TRUE` then the logarithm of the density is returned.

## Details

See [bifrankcop](#), the **VGAM** family functions for estimating the association parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.



**Value**

dbifrankcop gives the density, pbifrankcop gives the distribution function, and rbifrankcop generates random deviates (a two-column matrix).

**Author(s)**

T. W. Yee

**References**

Genest, C. (1987) Frank's family of bivariate distributions. *Biometrika*, **74**, 549–555.

**See Also**

[bifrankcop](#).

**Examples**

```
## Not run: N <- 100; alpha <- exp(2)
xx <- seq(-0.30, 1.30, len = N)
ox <- expand.grid(xx, xx)
zedd <- dbifrankcop(ox[, 1], ox[, 2], alpha = alpha)
contour(xx, xx, matrix(zedd, N, N))
zedd <- pbifrankcop(ox[, 1], ox[, 2], alpha = alpha)
contour(xx, xx, matrix(zedd, N, N))

alpha <- exp(4)
plot(rr <- rbifrankcop(n = 3000, alpha = alpha))
par(mfrow = c(1, 2))
hist(rr[, 1]) # Should be uniform
hist(rr[, 2]) # Should be uniform

## End(Not run)
```

**Description**

Density, distribution function, quantile function and random generation for the three parameter Frechet distribution.

**Usage**

```
dfrechet(x, location = 0, scale = 1, shape, log = FALSE)
pfrechet(q, location = 0, scale = 1, shape)
qfrechet(p, location = 0, scale = 1, shape)
rfrechet(n, location = 0, scale = 1, shape)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Passed into <a href="#">runif</a> .
<code>location, scale, shape</code>	the location parameter $a$ , scale parameter $b$ , and shape parameter $s$ .
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [frechet2](#), the **VGAM** family function for estimating the 2 parameters (without location parameter) by maximum likelihood estimation, for the formula of the probability density function and range restrictions on the parameters.

**Value**

`dfrechet` gives the density, `pfrechet` gives the distribution function, `qfrechet` gives the quantile function, and `rfrechet` generates random deviates.

**Author(s)**

T. W. Yee

**References**

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[frechet2](#).

**Examples**

```
## Not run: shape <- 5
x <- seq(-0.1, 3.5, length = 401)
plot(x, dfrechet(x, shape = shape), type = "l", ylab = "", las = 1,
     main = "Frechet density divided into 10 equal areas; orange = cdf")
abline(h = 0, col = "blue", lty = 2)
qq <- qfrechet(seq(0.1, 0.9, by = 0.1), shape = shape)
lines(qq, dfrechet(qq, shape = shape), col = "purple", lty = 3, type = "h")
lines(x, pfrechet(q = x, shape = shape), col = "orange")

## End(Not run)
```

---

 frechet

*Frechet Distribution Family Function*


---

## Description

Maximum likelihood estimation of the 2-parameter Frechet distribution.

## Usage

```
frechet2(location = 0, lscale = "loge", lshape = logoff(offset = -2),
         iscale = NULL, ishape = NULL, nsimEIM = 250, zero = NULL)
```

## Arguments

location            Numeric. Location parameter. It is called  $a$  below.  
 lscale, lshape    Link functions for the parameters; see [Links](#) for more choices.  
 iscale, ishape, zero, nsimEIM  
                     See [CommonVGAMffArguments](#) for information.

## Details

The (3-parameter) Frechet distribution has a density function that can be written

$$f(y) = \frac{sb}{(y-a)^2} [b/(y-a)]^{s-1} \exp[-(b/(y-a))^s]$$

for  $y > a$  and scale parameter  $b > 0$ . The positive shape parameter is  $s$ . The cumulative distribution function is

$$F(y) = \exp[-(b/(y-a))^s].$$

The mean of  $Y$  is  $a + b\Gamma(1 - 1/s)$  for  $s > 1$  (these are returned as the fitted values). The variance of  $Y$  is  $b^2[\Gamma(1 - 2/s) - \Gamma^2(1 - 1/s)]$  for  $s > 2$ .

Family `frechet2` has  $a$  known, and  $\log(b)$  and  $\log(s - 2)$  are the default linear/additive predictors. The working weights are estimated by simulated Fisher scoring.

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

Family function `frechet2` may fail for low values of the shape parameter, e.g., near 2 or lower.

**Author(s)**

T. W. Yee

**References**

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[rfrechets](#), [gev](#).

**Examples**

```
## Not run:
set.seed(123)
fdata <- data.frame(y1 = rfrechets(nn <- 1000, shape = 2 + exp(1)))
with(fdata, hist(y1))
fit2 <- vglm(y1 ~ 1, frechet2, fdata, trace = TRUE)
coef(fit2, matrix = TRUE)
Coef(fit2)
head(fitted(fit2))
with(fdata, mean(y1))
head(weights(fit2, type = "working"))
vcov(fit2)

## End(Not run)
```

---

freund61

---

*Freund's (1961) Bivariate Extension of the Exponential Distribution*


---

**Description**

Estimate the four parameters of the Freund (1961) bivariate extension of the exponential distribution by maximum likelihood estimation.

**Usage**

```
freund61(la = "loge", lap = "loge", lb = "loge", lbp = "loge",
         ia = NULL, iap = NULL, ib = NULL, ibp = NULL,
         independent = FALSE, zero = NULL)
```

**Arguments**

la, lap, lb, lbp

Link functions applied to the (positive) parameters  $\alpha$ ,  $\alpha'$ ,  $\beta$  and  $\beta'$ , respectively (the “p” stands for “prime”). See [Links](#) for more choices.

ia, iap, ib, ibp	Initial value for the four parameters respectively. The default is to estimate them all internally.
independent	Logical. If TRUE then the parameters are constrained to satisfy $\alpha = \alpha'$ and $\beta = \beta'$ , which implies that $y_1$ and $y_2$ are independent and each have an ordinary exponential distribution.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2,3,4\}$ . The default is none of them.

## Details

This model represents one type of bivariate extension of the exponential distribution that is applicable to certain problems, in particular, to two-component systems which can function if one of the components has failed. For example, engine failures in two-engine planes, paired organs such as peoples' eyes, ears and kidneys. Suppose  $y_1$  and  $y_2$  are random variables representing the lifetimes of two components  $A$  and  $B$  in a two component system. The dependence between  $y_1$  and  $y_2$  is essentially such that the failure of the  $B$  component changes the parameter of the exponential life distribution of the  $A$  component from  $\alpha$  to  $\alpha'$ , while the failure of the  $A$  component changes the parameter of the exponential life distribution of the  $B$  component from  $\beta$  to  $\beta'$ .

The joint probability density function is given by

$$f(y_1, y_2) = \alpha\beta' \exp(-\beta'y_2 - (\alpha + \beta - \beta')y_1)$$

for  $0 < y_1 < y_2$ , and

$$f(y_1, y_2) = \beta\alpha' \exp(-\alpha'y_1 - (\alpha + \beta - \alpha')y_2)$$

for  $0 < y_2 < y_1$ . Here, all four parameters are positive, as well as the responses  $y_1$  and  $y_2$ . Under this model, the probability that component  $A$  is the first to fail is  $\alpha/(\alpha + \beta)$ . The time to the first failure is distributed as an exponential distribution with rate  $\alpha + \beta$ . Furthermore, the distribution of the time from first failure to failure of the other component is a mixture of  $\text{Exponential}(\alpha')$  and  $\text{Exponential}(\beta')$  with proportions  $\beta/(\alpha + \beta)$  and  $\alpha/(\alpha + \beta)$  respectively.

The marginal distributions are, in general, not exponential. By default, the linear/additive predictors are  $\eta_1 = \log(\alpha)$ ,  $\eta_2 = \log(\alpha')$ ,  $\eta_3 = \log(\beta)$ ,  $\eta_4 = \log(\beta')$ .

A special case is when  $\alpha = \alpha'$  and  $\beta = \beta'$ , which means that  $y_1$  and  $y_2$  are independent, and both have an ordinary exponential distribution with means  $1/\alpha$  and  $1/\beta$  respectively.

Fisher scoring is used, and the initial values correspond to the MLEs of an intercept model. Consequently, convergence may take only one iteration.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

To estimate all four parameters, it is necessary to have some data where  $y_1 < y_2$  and  $y_2 < y_1$ .

The response must be a two-column matrix, with columns  $y_1$  and  $y_2$ . Currently, the fitted value is a matrix with two columns; the first column has values  $(\alpha' + \beta)/(\alpha'(\alpha + \beta))$  for the mean of  $y_1$ , while the second column has values  $(\beta' + \alpha)/(\beta'(\alpha + \beta))$  for the mean of  $y_2$ . The variance of  $y_1$  is

$$\frac{(\alpha')^2 + 2\alpha\beta + \beta^2}{(\alpha')^2(\alpha + \beta)^2},$$

the variance of  $y_2$  is

$$\frac{(\beta')^2 + 2\alpha\beta + \alpha^2}{(\beta')^2(\alpha + \beta)^2},$$

the covariance of  $y_1$  and  $y_2$  is

$$\frac{\alpha'\beta' - \alpha\beta}{\alpha'\beta'(\alpha + \beta)^2}.$$

**Author(s)**

T. W. Yee

**References**

Freund, J. E. (1961) A bivariate extension of the exponential distribution. *Journal of the American Statistical Association*, **56**, 971–977.

**See Also**

[exponential](#).

**Examples**

```
fdata <- data.frame(y1 = rexp(nn <- 1000, rate = exp(1)))
fdata <- transform(fdata, y2 = rexp(nn, rate = exp(2)))
fit1 <- vglm(cbind(y1, y2) ~ 1, fam = freund61, fdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)
vcov(fit1)
head(fitted(fit1))
summary(fit1)

# y1 and y2 are independent, so fit an independence model
fit2 <- vglm(cbind(y1, y2) ~ 1, freund61(indep = TRUE),
            data = fdata, trace = TRUE)
coef(fit2, matrix = TRUE)
constraints(fit2)
pchisq(2 * (logLik(fit1) - logLik(fit2)), # p-value
      df = df.residual(fit2) - df.residual(fit1), lower.tail = FALSE)
lrtest(fit1, fit2) # Better alternative
```

fsqrt

*Folded Square Root Link Function***Description**

Computes the folded square root transformation, including its inverse and the first two derivatives.

**Usage**

```
fsqrt(theta, min = 0, max = 1, mux = sqrt(2),
      inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

**Arguments**

theta                    Numeric or character. See below for further details.  
 min, max, mux        These are called  $L$ ,  $U$  and  $K$  below.  
 inverse, deriv, short, tag  
                          Details at [Links](#).

**Details**

The folded square root link function can be applied to parameters that lie between  $L$  and  $U$  inclusive. Numerical values of theta out of range result in NA or NaN.

**Value**

For fsqrt with deriv = 0:  $K(\sqrt{\theta - L} - \sqrt{U - \theta})$  or  $\text{mux} * (\text{sqrt}(\text{theta} - \text{min}) - \text{sqrt}(\text{max} - \text{theta}))$  when inverse = FALSE, and if inverse = TRUE then some more complicated function that returns a NA unless theta is between  $-\text{mux} * \text{sqrt}(\text{max} - \text{min})$  and  $\text{mux} * \text{sqrt}(\text{max} - \text{min})$ .

For deriv = 1, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if inverse = FALSE, else if inverse = TRUE then it returns the reciprocal.

**Note**

The default has, if theta is 0 or 1, the link function value is  $-\text{sqrt}(2)$  and  $+\text{sqrt}(2)$  respectively. These are finite values, therefore one cannot use this link function for general modelling of probabilities because of numerical problem, e.g., with [binomialff](#), [cumulative](#). See the example below.

**Author(s)**

Thomas W. Yee

**See Also**

[Links](#).

## Examples

```

p <- seq(0.01, 0.99, by = 0.01)
fsqrt(p)
max(abs(fsqrt(fsqrt(p), inverse = TRUE) - p)) # Should be 0

p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
fsqrt(p) # Has NAs

## Not run:
p <- seq(0.01, 0.99, by = 0.01)
par(mfrow = c(2, 2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
for (d in 0:1) {
  matplot(p, cbind(logit(p, deriv = d), fsqrt(p, deriv = d)),
    type = "n", col = "purple", ylab = "transformation", las = 1,
    main = if (d == 0) "Some probability link functions"
    else "First derivative")
  lines(p, logit(p, deriv = d), col = "limegreen")
  lines(p, probit(p, deriv = d), col = "purple")
  lines(p, cloglog(p, deriv = d), col = "chocolate")
  lines(p, fsqrt(p, deriv = d), col = "tan")
  if (d == 0) {
    abline(v = 0.5, h = 0, lty = "dashed")
    legend(0, 4.5, c("logit", "probit", "cloglog", "fsqrt"), lwd = 2,
      col = c("limegreen", "purple", "chocolate", "tan"))
  } else
    abline(v = 0.5, lty = "dashed")
}

for (d in 0) {
  matplot(y, cbind(logit(y, deriv = d, inverse = TRUE),
    fsqrt(y, deriv = d, inverse = TRUE)),
    type = "n", col = "purple", xlab = "transformation", ylab = "p",
    lwd = 2, las = 1,
    main = if (d == 0) "Some inverse probability link functions"
    else "First derivative")
  lines(y, logit(y, deriv = d, inverse = TRUE), col = "limegreen")
  lines(y, probit(y, deriv = d, inverse = TRUE), col = "purple")
  lines(y, cloglog(y, deriv = d, inverse = TRUE), col = "chocolate")
  lines(y, fsqrt(y, deriv = d, inverse = TRUE), col = "tan")
  if (d == 0) {
    abline(h = 0.5, v = 0, lty = "dashed")
    legend(-4, 1, c("logit", "probit", "cloglog", "fsqrt"), lwd = 2,
      col = c("limegreen", "purple", "chocolate", "tan"))
  }
}
par(lwd = 1)

## End(Not run)

# This is lucky to converge
fit.h <- vglm(agaas ~ bs(altitude), binomialff(link = fsqrt(mux = 5)),

```



```

      data = hunua, trace = TRUE)
## Not run:
plotvgam(fit.h, se = TRUE, lcol = "orange", scol = "orange",
        main = "Orange is Hunua, Blue is Waitakere")
## End(Not run)
head(predict(fit.h, hunua, type = "response"))

## Not run:
# The following fails.
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let,
           cumulative(link = fsqrt(mux = 10), par = TRUE, rev = TRUE),
           data = pneumo, trace = TRUE, maxit = 200)
## End(Not run)

```

G1G2G3

*The G1G2G3 Blood Group System***Description**

Estimates the three independent parameters of the the G1G2G3 blood group system.

**Usage**

```
G1G2G3(link = "logit", ip1 = NULL, ip2 = NULL, iF = NULL)
```

**Arguments**

`link`                      Link function applied to p1, p2 and f. See [Links](#) for more choices.  
`ip1, ip2, iF`            Optional initial value for p1, p2 and f.

**Details**

The parameters p1 and p2 are probabilities, so that  $p_3 = 1 - p_1 - p_2$  is the third probability. The parameter f is the third independent parameter.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The input can be a 6-column matrix of counts, with columns corresponding to G1G1, G1G2, G1G3, G2G2, G2G3, G3G3 (in order). Alternatively, the input can be a 6-column matrix of proportions (so each row adds to 1) and the `weights` argument is used to specify the total number of counts for each row.

Author(s)

T. W. Yee

References

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

See Also

[AA.Aa.aa](#), [AB.Ab.aB.ab](#), [AB.Ab.aB.ab2](#), [ABO](#), [MNSs](#).

Examples

```
ymat <- cbind(108, 196, 429, 143, 513, 559)
fit <- vglm(ymat ~ 1, G1G2G3(link = probit), trace = TRUE, crit = "coef")
fit <- vglm(ymat ~ 1, G1G2G3(link = logit, ip1 = 0.3, ip2 = 0.3, iF = 0.02),
            trace = TRUE, crit = "coef")
fit <- vglm(ymat ~ 1, G1G2G3(link = "identity"), trace = TRUE)
Coef(fit) # Estimated p1, p2 and f
rbind(ymat, sum(ymat)*fitted(fit))
sqrt(diag(vcov(fit)))
```

---

gamma1	<i>1-parameter Gamma Distribution</i>
--------	---------------------------------------

---

Description

Estimates the 1-parameter gamma distribution by maximum likelihood estimation.

Usage

```
gamma1(link = "loge", zero = NULL)
```

Arguments

- |      |   |
|------|---|
| link | Link function applied to the (positive) <i>shape</i> parameter. See <a href="#">Links</a> for more choices and general information. |
| zero | Details at <a href="#">CommonVGAMffArguments</a> .  |

Details

The density function is given by

$$f(y) = \exp(-y) \times y^{shape-1} / \Gamma(shape)$$

for  $shape > 0$  and  $y > 0$ . Here,  $\Gamma(shape)$  is the gamma function, as in [gamma](#). The mean of  $Y$  (returned as the fitted values) is  $\mu = shape$ , and the variance is  $\sigma^2 = shape$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

This **VGAM** family function can handle a multiple responses, which is inputted as a matrix.

The parameter *shape* matches with *shape* in [rgamma](#). The argument *rate* in [rgamma](#) is assumed 1 for this family function.

If *rate* is unknown use the family function [gamma2.ab](#) to estimate it too.

**Author(s)**

T. W. Yee

**References**

Most standard texts on statistical distributions describe the 1-parameter gamma distribution, e.g., Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[gamma2.ab](#) for the 2-parameter gamma distribution, [lgammaff](#), [lindley](#).

**Examples**

```
gdata <- data.frame(y = rgamma(n = 100, shape = exp(3)))
fit <- vglm(y ~ 1, gamma1, gdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

gamma2

2-parameter Gamma Distribution

---

**Description**

Estimates the 2-parameter gamma distribution by maximum likelihood estimation.

**Usage**

```
gamma2(lmu = "loge", lshape = "loge",
       imethod = 1, ishape = NULL,
       parallel = FALSE, deviance.arg = FALSE, zero = -2)
```

## Arguments

<code>lmu, lshape</code>	Link functions applied to the (positive) <i>mu</i> and <i>shape</i> parameters (called $\mu$ and $\lambda$ respectively). See <a href="#">Links</a> for more choices.
<code>ishape</code>	Optional initial value for <i>shape</i> . A NULL means a value is computed internally. If a failure to converge occurs, try using this argument. This argument is ignored if used within <code>cqo</code> ; see the <code>iShape</code> argument of <code>qrrvglm.control</code> instead.
<code>imethod</code>	An integer with value 1 or 2 which specifies the initialization method for the $\mu$ parameter. If failure to converge occurs try another value (and/or specify a value for <code>ishape</code> ).
<code>deviance.arg</code>	Logical. If TRUE, the deviance function is attached to the object. Under ordinary circumstances, it should be left alone because it really assumes the shape parameter is at the maximum likelihood estimate. Consequently, one cannot use that criterion to minimize within the IRLS algorithm. It should be set TRUE only when used with <code>cqo</code> under the fast algorithm.
<code>zero</code>	Integer valued vector, usually assigned $-2$ or $2$ if used at all. Specifies which of the two linear/additive predictors are modelled as an intercept only. By default, the shape parameter (after <code>lshape</code> is applied) is modelled as a single unknown number that is estimated. It can be modelled as a function of the explanatory variables by setting <code>zero = NULL</code> . A negative value means that the value is recycled, so setting $-2$ means all shape parameters are intercept only. See <a href="#">CommonVGAMffArguments</a> for more information.
<code>parallel</code>	Details at <a href="#">CommonVGAMffArguments</a> . If <code>parallel = TRUE</code> then the constraint is not applied to the intercept.

## Details

This distribution can model continuous skewed responses. The density function is given by

$$f(y; \mu, \lambda) = \frac{\exp(-\lambda y/\mu) \times (\lambda y/\mu)^{\lambda-1} \times \lambda}{\mu \times \Gamma(\lambda)}$$

for  $\mu > 0$ ,  $\lambda > 0$  and  $y > 0$ . Here,  $\Gamma(\cdot)$  is the gamma function, as in [gamma](#). The mean of  $Y$  is  $\mu = \mu$  (returned as the fitted values) with variance  $\sigma^2 = \mu^2/\lambda$ . If  $0 < \lambda < 1$  then the density has a pole at the origin and decreases monotonically as  $y$  increases. If  $\lambda = 1$  then this corresponds to the exponential distribution. If  $\lambda > 1$  then the density is zero at the origin and is unimodal with mode at  $y = \mu - \mu/\lambda$ ; this can be achieved with `lshape="loglog"`.

By default, the two linear/additive predictors are  $\eta_1 = \log(\mu)$  and  $\eta_2 = \log(\lambda)$ . This family function implements Fisher scoring and the working weight matrices are diagonal.

This **VGAM** family function handles *multivariate* responses, so that a matrix can be used as the response. The number of columns is the number of species, say, and `zero=-2` means that *all* species have a shape parameter equalling a (different) intercept only.

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as `vglm` and `vgam`.

**Note**

The response must be strictly positive. A moment estimator for the shape parameter may be implemented in the future.

If `mu` and `shape` are vectors, then `rgamma(n = n, shape = shape, scale = mu/shape)` will generate random gamma variates of this parameterization, etc.; see [GammaDist](#).

For [cqo](#) and [cao](#), taking the logarithm of the response means (approximately) a [gaussianff](#) family may be used on the transformed data.

**Author(s)**

T. W. Yee

**References**

The parameterization of this **VGAM** family function is the 2-parameter gamma distribution described in the monograph

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[gamma1](#) for the 1-parameter gamma distribution, [gamma2.ab](#) for another parameterization of the 2-parameter gamma distribution, [bigamma.mckay](#) for a bivariate gamma distribution, [expexp](#), [GammaDist](#), [golf](#), [CommonVGAMffArguments](#).

**Examples**

```
# Essentially a 1-parameter gamma
gdata <- data.frame(y = rgamma(n = 100, shape = exp(1)))
fit1 <- vglm(y ~ 1, gamma1, gdata)
fit2 <- vglm(y ~ 1, gamma2, gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)

# Essentially a 2-parameter gamma
gdata <- data.frame(y = rgamma(n = 500, rate = exp(1), shape = exp(2)))
fit2 <- vglm(y ~ 1, gamma2, gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)
summary(fit2)
```

gamma2.ab

2-parameter Gamma Distribution

## Description

Estimates the 2-parameter gamma distribution by maximum likelihood estimation.

## Usage

```
gamma2.ab(lrate = "log", lshape = "log",
          irate = NULL,  ishape = NULL, expected = TRUE, zero = 2)
```

## Arguments

lrate, lshape	Link functions applied to the (positive) <i>rate</i> and <i>shape</i> parameters. See <a href="#">Links</a> for more choices.
expected	Logical. Use Fisher scoring? The default is yes, otherwise Newton-Raphson is used.
irate, ishape	Optional initial values for <i>rate</i> and <i>shape</i> . A NULL means a value is computed internally. If a failure to converge occurs, try using these arguments.
zero	An integer specifying which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value should be either 1 or 2 or NULL. The default is to model <i>shape</i> as an intercept only. A value NULL means neither 1 or 2.

## Details

The density function is given by

$$f(y) = \exp(-rate \times y) \times y^{shape-1} \times rate^{shape} / \Gamma(shape)$$

for  $shape > 0$ ,  $rate > 0$  and  $y > 0$ . Here,  $\Gamma(shape)$  is the gamma function, as in [gamma](#). The mean of  $Y$  is  $\mu = shape/rate$  (returned as the fitted values) with variance  $\sigma^2 = \mu^2/shape = shape/rate^2$ . By default, the two linear/additive predictors are  $\eta_1 = \log(rate)$  and  $\eta_2 = \log(shape)$ .

The argument `expected` refers to the type of information matrix. The expected information matrix corresponds to Fisher scoring and is numerically better here. The observed information matrix corresponds to the Newton-Raphson algorithm and may be withdrawn from the family function in the future. If both algorithms work then the differences in the results are often not huge.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The parameters *rate* and *shape* match with the arguments *rate* and *shape* of [rgamma](#). Often,  $scale = 1/rate$  is used.

If  $rate = 1$  use the family function [gamma1](#) to estimate *shape*.

**Author(s)**

T. W. Yee

**References**

Most standard texts on statistical distributions describe the 2-parameter gamma distribution, e.g.,  
 Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[gamma1](#) for the 1-parameter gamma distribution, [gamma2](#) for another parameterization of the 2-parameter gamma distribution, [bigamma.mckay](#) for a bivariate gamma distribution, [expexp](#).

**Examples**

```
# Essentially a 1-parameter gamma
gdata <- data.frame(y1 = rgamma(n <- 100, shape = exp(1)))
fit1 <- vglm(y1 ~ 1, gamma1, gdata, trace = TRUE)
fit2 <- vglm(y1 ~ 1, gamma2.ab, gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)

# Essentially a 2-parameter gamma
gdata <- data.frame(y2 = rgamma(n = 500, rate = exp(1), shape = exp(2)))
fit2 <- vglm(y2 ~ 1, gamma2.ab, gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)
summary(fit2)
```

---

gammahyp

*Gamma Hyperbola Bivariate Distribution*


---

**Description**

Estimate the parameter of a gamma hyperbola bivariate distribution by maximum likelihood estimation.

**Usage**

```
gammahyp(ltheta = "log", itheta = NULL, expected = FALSE)
```

**Arguments**

ltheta	Link function applied to the (positive) parameter $\theta$ . See <a href="#">Links</a> for more choices.
itheta	Initial value for the parameter. The default is to estimate it internally.
expected	Logical. FALSE means the Newton-Raphson (using the observed information matrix) algorithm, otherwise the expected information matrix is used (Fisher scoring algorithm).

**Details**

The joint probability density function is given by

$$f(y_1, y_2) = \exp(-e^{-\theta}y_1/\theta - \theta y_2)$$

for  $\theta > 0$ ,  $y_1 > 0$ ,  $y_2 > 1$ . The random variables  $Y_1$  and  $Y_2$  are independent. The marginal distribution of  $Y_1$  is an exponential distribution with rate parameter  $\exp(-\theta)/\theta$ . The marginal distribution of  $Y_2$  is an exponential distribution that has been shifted to the right by 1 and with rate parameter  $\theta$ . The fitted values are stored in a two-column matrix with the marginal means, which are  $\theta \exp(\theta)$  and  $1 + 1/\theta$ .

The default algorithm is Newton-Raphson because Fisher scoring tends to be much slower for this distribution.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The response must be a two column matrix.

**Author(s)**

T. W. Yee

**References**

Reid, N. (2003) Asymptotics and the theory of inference. *Annals of Statistics*, **31**, 1695–1731.

**See Also**

[exponential](#).

**Examples**

```
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, theta = exp(-2 + x2))
gdata <- transform(gdata, y1 = rexp(nn, rate = exp(-theta)/theta),
  y2 = rexp(nn, rate = theta) + 1)
fit <- vglm(cbind(y1, y2) ~ x2, fam = gammahyp(expected = TRUE), gdata)
```



```
fit <- vglm(cbind(y1, y2) ~ x2, fam = gammahyp, gdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
summary(fit)
```

garma

*GARMA (Generalized Autoregressive Moving-Average) Models*

## Description

Fits GARMA models to time series data.

## Usage

```
garma(link = "identity", p.ar.lag = 1, q.ma.lag = 0,
      coefstart = NULL, step = 1)
```

## Arguments

link	Link function applied to the mean response. The default is suitable for continuous responses. The link <a href="#">log</a> should be chosen if the data are counts. The link <a href="#">reciprocal</a> can be chosen if the data are counts and the variance assumed for this is $\mu^2$ . The links <a href="#">logit</a> , <a href="#">probit</a> , <a href="#">cloglog</a> , and <a href="#">cauchit</a> are supported and suitable for binary responses.  Note that when the log or logit link is chosen: for log and logit, zero values can be replaced by bvalue. See <a href="#">log</a> and <a href="#">logit</a> etc. for specific information about each link function.
p.ar.lag	A positive integer, the lag for the autoregressive component. Called $p$ below.
q.ma.lag	A non-negative integer, the lag for the moving-average component. Called $q$ below.
coefstart	Starting values for the coefficients. Assigning this argument is highly recommended. For technical reasons, the argument coefstart in <a href="#">vglm</a> cannot be used.
step	Numeric. Step length, e.g., 0.5 means half-stepsizing.

## Details

This function draws heavily on Benjamin *et al.* (1998). See also Benjamin *et al.* (2003). GARMA models extend the ARMA time series model to generalized responses in the exponential family, e.g., Poisson counts, binary responses. Currently, this function is rudimentary and can handle only certain continuous, count and binary responses only. The user must choose an appropriate link for the link argument.

The GARMA( $p, q$ ) model is defined by firstly having a response belonging to the exponential family

$$f(y_t|D_t) = \exp \left\{ \frac{y_t \theta_t - b(\theta_t)}{\phi/A_t} + c(y_t, \phi/A_t) \right\}$$

where  $\theta_t$  and  $\phi$  are the canonical and scale parameters respectively, and  $A_t$  are known prior weights. The mean  $\mu_t = E(Y_t|D_t) = b'(\theta_t)$  is related to the linear predictor  $\eta_t$  by the link function  $g$ . Here,  $D_t = \{x_t, \dots, x_1, y_{t-1}, \dots, y_1, \mu_{t-1}, \dots, \mu_1\}$  is the previous information set. Secondly, the GARMA( $p, q$ ) model is defined by

$$g(\mu_t) = \eta_t = x_t^T \beta + \sum_{k=1}^p \phi_k (g(y_{t-k}) - x_{t-k}^T \beta) + \sum_{k=1}^q \theta_k (g(y_{t-k}) - \eta_{t-k}).$$

Parameter vectors  $\beta$ ,  $\phi$  and  $\theta$  are estimated by maximum likelihood.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#).

### Warning

This **VGAM** family function is 'non-standard' in that the model does need some coercing to get it into the VGLM framework. Special code is required to get it running. A consequence is that some methods functions may give wrong results when applied to the fitted object.

### Note

This function is unpolished and is requires *lots* of improvements. In particular, initialization is *very poor*. Results appear *very* sensitive to quality of initial values. A limited amount of experience has shown that half-stepsizing is often needed for convergence, therefore choosing `crit = "coef"` is not recommended.

Overdispersion is not handled. For binomial responses it is currently best to input a vector of 1s and 0s rather than the `cbind(successes, failures)` because the initialize slot is rudimentary.

### Author(s)

T. W. Yee

### References

- Benjamin, M. A., Rigby, R. A. and Stasinopoulos, M. D. (1998) Fitting Non-Gaussian Time Series Models. Pages 191–196 in: *Proceedings in Computational Statistics COMPSTAT 1998* by Payne, R. and P. J. Green. Physica-Verlag.
- Benjamin, M. A., Rigby, R. A. and Stasinopoulos, M. D. (2003) Generalized Autoregressive Moving Average Models. *Journal of the American Statistical Association*, **98**: 214–223.
- Zeger, S. L. and Qaqish, B. (1988) Markov regression models for time series: a quasi-likelihood approach. *Biometrics*, **44**: 1019–1031.

### See Also

The site <http://www.stat.auckland.ac.nz/~yee> contains more documentation about this family function.

**Examples**

```

gdata <- data.frame(interspike = c(68, 41, 82, 66, 101, 66, 57, 41, 27, 78,
59, 73, 6, 44, 72, 66, 59, 60, 39, 52,
50, 29, 30, 56, 76, 55, 73, 104, 104, 52,
25, 33, 20, 60, 47, 6, 47, 22, 35, 30,
29, 58, 24, 34, 36, 34, 6, 19, 28, 16,
36, 33, 12, 26, 36, 39, 24, 14, 28, 13,
2, 30, 18, 17, 28, 9, 28, 20, 17, 12,
19, 18, 14, 23, 18, 22, 18, 19, 26, 27,
23, 24, 35, 22, 29, 28, 17, 30, 34, 17,
20, 49, 29, 35, 49, 25, 55, 42, 29, 16)) # See Zeger and Qaqish (1988)
gdata <- transform(gdata, spikenum = seq(interspike))
bvalue <- 0.1 # .Machine$double.xmin # Boundary value
fit <- vglm(interspike ~ 1, trace = TRUE, data = gdata,
           gamma(log(bvalue = bvalue),
                p = 2, coefstart = c(4, 0.3, 0.4)))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit) # A bug here
## Not run: with(gdata, plot(interspike, ylim = c(0, 120), las = 1,
xlab = "Spike Number", ylab = "Inter-Spike Time (ms)", col = "blue"))
with(gdata, lines(spikenum[-(1:fit@misc$plag)], fitted(fit), col = "orange"))
abline(h = mean(with(gdata, interspike)), lty = "dashed", col = "gray")
## End(Not run)

```

gaussianff

*Gaussian (normal) Family Function***Description**

Fits a generalized linear model to a response with Gaussian (normal) errors. The dispersion parameter may be known or unknown.

**Usage**

```
gaussianff(dispersion = 0, parallel = FALSE, zero = NULL)
```

**Arguments**

parallel	A logical or formula. If a formula, the response of the formula should be a logical and the terms of the formula indicates whether or not those terms are parallel.
dispersion	Dispersion parameter. If 0 then it is estimated and the moment estimate is put in object@misc\$dispersion; it is assigned the value

$$\sum_{i=1}^n (y_i - \eta_i)^T W_i (y_i - \eta_i) / (nM - p)$$

where  $p$  is the total number of parameters estimated (for RR-VGLMs the value used is the number of columns in the large  $X$  model matrix; this may not be correct). If the argument is assigned a positive quantity then it is assumed to be known with that value.

**zero** An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set  $\{1, 2, \dots, M\}$  where  $M$  is the number of columns of the matrix response.

## Details

This function is usually used in conjunction with [vglm](#), else `vlm` is recommended instead. The notation  $M$  is used to denote the number of linear/additive predictors. This function can handle any finite  $M$ , and the default is to use ordinary least squares. A vector linear/additive model can be fitted by minimizing

$$\sum_{i=1}^n (y_i - \eta_i)^T W_i (y_i - \eta_i)$$

where  $y_i$  is a  $M$ -vector,  $\eta_i$  is the vector of linear/additive predictors. The  $W_i$  is any positive-definite matrix, and the default is the order- $M$  identity matrix. The  $W_i$  can be inputted using the `weights` argument of `vlm`/[vglm](#)/[vgam](#) etc., and the format is the *matrix-band* format whereby it is a  $n \times A$  matrix with the diagonals are passed first, followed by next the upper band, all the way to the  $(1, M)$  element. Here,  $A$  has maximum value of  $M(M + 1)/2$  and a minimum value of  $M$ . Usually the `weights` argument of `vlm`/[vglm](#)/[vgam](#)/[rrvglm](#) is just a vector, in which case each element is multiplied by a order- $M$  identity matrix. If in doubt, type something like `weights(object, type="working")` after the model has been fitted.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Note

This **VGAM** family function is supposed to be similar to [gaussian](#) but is not compatible with [glm](#). The "ff" in the name is added to avoid any masking problems.

## Author(s)

Thomas W. Yee

## References

- McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.
- Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

## See Also

[uninormal](#), [huber2](#), [lqnrm](#), [binormal](#), [SUR](#), `vlm`, [vglm](#), [vgam](#), [rrvglm](#).

**Examples**

```

gdata <- data.frame(x2 = sort(runif(n <- 40)))
gdata <- transform(gdata, y1 = 1 + 2*x2 + rnorm(n, sd = 0.1),
                      y2 = 3 + 4*x2 + rnorm(n, sd = 0.1),
                      y3 = 7 + 4*x2 + rnorm(n, sd = 0.1))
fit <- vglm(cbind(y1,y2) ~ x2, gaussianff, data = gdata)
coef(fit, matrix = TRUE)

# For comparison:
coef( lmfit <- lm(y1 ~ x2, data = gdata))
coef(glmfit <- glm(y2 ~ x2, data = gdata, gaussian))
vcov(fit)
vcov(lmfit)

t(weights(fit, type = "prior"))      # Unweighted observations
head(weights(fit, type = "working")) # Identity matrices

# Reduced-rank VLM (rank-1)
fit2 <- rrvglm(cbind(y1, y2, y3) ~ x2, gaussianff, data = gdata)
Coef(fit2)

```

genbetaII

*Generalized Beta Distribution of the Second Kind***Description**

Maximum likelihood estimation of the 4-parameter generalized beta II distribution.

**Usage**

```

genbetaII(lshape1.a = "loge", lscale = "loge", lshape2.p = "loge", lshape3.q = "loge",
          ishape1.a = NULL, iscale = NULL, ishape2.p = 1, ishape3.q = 1,
          zero = NULL)

```

**Arguments**

lshape1.a, lscale, lshape2.p, lshape3.q	Parameter link functions applied to the shape parameter a, scale parameter scale, shape parameter p, and shape parameter q. All four parameters are positive. See <a href="#">Links</a> for more choices.
ishape1.a, iscale	Optional initial values for a and scale. A NULL means a value is computed internally.
ishape2.p, ishape3.q	Optional initial values for p and q.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2,3,4} which correspond to a, scale, p, q, respectively.

## Details

This distribution is most useful for unifying a substantial number of size distributions. For example, the Singh-Maddala, Dagum, Fisk (log-logistic), Lomax (Pareto type II), inverse Lomax, beta distribution of the second kind distributions are all special cases. Full details can be found in Kleiber and Kotz (2003), and Brazauskas (2002). The argument names given here are used by other families that are special cases of this family. Fisher scoring is used here and for the special cases too.

The 4-parameter generalized beta II distribution has density

$$f(y) = ay^{ap-1} / [b^{ap} B(p, q) \{1 + (y/b)^a\}^{p+q}]$$

for  $a > 0$ ,  $b > 0$ ,  $p > 0$ ,  $q > 0$ ,  $y \geq 0$ . Here  $B$  is the beta function, and  $b$  is the scale parameter scale, while the others are shape parameters. The mean is

$$E(Y) = b \Gamma(p + 1/a) \Gamma(q - 1/a) / (\Gamma(p) \Gamma(q))$$

provided  $-ap < 1 < aq$ ; these are returned as the fitted values.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

If the self-starting initial values fail, try experimenting with the initial value arguments, especially those whose default value is not NULL. Also, the constraint  $-ap < 1 < aq$  may be violated as the iterations progress so it pays to monitor convergence, e.g., set `trace = TRUE`. Successful convergence depends on having very good initial values. This is rather difficult for this distribution! More improvements could be made here.

## Author(s)

T. W. Yee

## References

- Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.
- Brazauskas, V. (2002) Fisher information matrix for the Feller-Pareto distribution. *Statistics & Probability Letters*, **59**, 159–167.

## See Also

[betaff](#), [betaII](#), [dagum](#), [sinmad](#), [fisk](#), [lomax](#), [invlomax](#), [paralogistic](#), [invparalogistic](#), [lino](#).

## Examples

```
## Not run:
gdata <- data.frame(y = rsinmad(3000, exp(2), exp(2), exp(1))) # A special case!
fit <- vglm(y ~ 1, genbetaII, gdata, trace = TRUE)
fit <- vglm(y ~ 1, data = gdata, trace = TRUE,
            genbetaII(ishape1.a = 4, ishape2.p = 2.2, iscale = 7, ishape3.q = 2.3))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

gengamma

*Generalized Gamma distribution family function*

## Description

Estimation of the 3-parameter generalized gamma distribution proposed by Stacy (1962).

## Usage

```
gengamma(lscale = "loge", ld = "loge", lk = "loge",
         iscale = NULL, id = NULL, ik = NULL, zero = NULL)
```

## Arguments

- `lscale`, `ld`, `lk` Parameter link function applied to each of the positive parameters  $b$ ,  $d$  and  $k$ , respectively. See [Links](#) for more choices.
- `iscale`, `id`, `ik` Initial value for  $b$ ,  $d$  and  $k$ , respectively. The defaults mean an initial value is determined internally for each.
- `zero` An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set  $\{1,2,3\}$ . The default value means none are modelled as intercept-only terms. See [CommonVGAMffArguments](#) for more information.

## Details

The probability density function can be written

$$f(y; b, d, k) = db^{-dk} y^{dk-1} \exp[-(y/b)^d] / \Gamma(k)$$

for scale parameter  $b > 0$ , and  $d > 0$ ,  $k > 0$ , and  $y > 0$ . The mean of  $Y$  is  $b \times \Gamma(k + 1/d) / \Gamma(k)$  (returned as the fitted values), which equals  $bk$  if  $d = 1$ .

There are many special cases, as given in Table 1 of Stacey and Mihram (1965). In the following, the parameters are in the order  $b, d, k$ . The special cases are: Exponential  $f(y; b, 1, 1)$ , Gamma  $f(y; b, 1, k)$ , Weibull  $f(y; b, d, 1)$ , Chi Squared  $f(y; 2, 1, a/2)$  with  $a$  degrees of freedom, Chi  $f(y; \sqrt{2}, 2, a/2)$  with  $a$  degrees of freedom, Half-normal  $f(y; \sqrt{2}, 2, 1/2)$ , Circular normal  $f(y; \sqrt{2}, 2, 1)$ , Spherical normal  $f(y; \sqrt{2}, 2, 3/2)$ , Rayleigh  $f(y; c\sqrt{2}, 2, 1)$  where  $c > 0$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

Several authors have considered maximum likelihood estimation for the generalized gamma distribution and have found that the Newton-Raphson algorithm does not work very well and that the existence of solutions to the log-likelihood equations is sometimes in doubt. Although Fisher scoring is used here, it is likely that the same problems will be encountered. It appears that large samples are required, for example, the estimator of  $k$  became asymptotically normal only with 400 or more observations. It is not uncommon for maximum likelihood estimates to fail to converge even with two or three hundred observations. With covariates, even more observations are needed to increase the chances of convergence.

**Note**

The notation used here differs from Stacy (1962) and Prentice (1974). Poor initial values may result in failure to converge so if there are covariates and there are convergence problems, try using the zero argument (e.g., `zero = 2:3`) or the `ik` argument.

**Author(s)**

T. W. Yee

**References**

- Stacy, E. W. (1962) A generalization of the gamma distribution. *Annals of Mathematical Statistics*, **33**(3), 1187–1192.
- Stacy, E. W. and Mihram, G. A. (1965) Parameter estimation for a generalized gamma distribution. *Technometrics*, **7**, 349–358.
- Prentice, R. L. (1974) A log gamma model and its maximum likelihood estimation. *Biometrika*, **61**, 539–544.

**See Also**

[rgengamma](#), [gamma1](#), [gamma2](#), [prentice74](#).

**Examples**

```
## Not run: k <- exp(-1); Scale = exp(1)
gdata <- data.frame(y = rgamma(1000, shape = k, scale = Scale))
fit <- vglm(y ~ 1, gengamma, gdata, trace = TRUE)
coef(fit, matrix = TRUE)

# Another example
gdata <- data.frame(x2 = runif(nn <- 5000))
gdata <- transform(gdata, Scale = exp(1),
                  d = exp( 0 + 1.2* x2),
```



```

      k = exp(-1 + 2 * x2))
gdata <- transform(gdata, y = rgengamma(nn, scale = Scale, d = d, k = k))
fit <- vglm(y ~ x2, gengamma(zero = 1, iscale = 6), gdata, trace = TRUE)
fit <- vglm(y ~ x2, gengamma(zero = 1), gdata, trace = TRUE, maxit = 50)
coef(fit, matrix = TRUE)

## End(Not run)

```

gengammaUC

*The Generalized Gamma Distribution***Description**

Density, distribution function, quantile function and random generation for the generalized gamma distribution with scale parameter *scale*, and parameters *d* and *k*.

**Usage**

```

dgengamma(x, scale = 1, d = 1, k = 1, log = FALSE)
pgengamma(q, scale = 1, d = 1, k = 1)
qgengamma(p, scale = 1, d = 1, k = 1)
rgengamma(n, scale = 1, d = 1, k = 1)

```

**Arguments**

<i>x</i> , <i>q</i>	vector of quantiles.
<i>p</i>	vector of probabilities.
<i>n</i>	number of observations. Positive integer of length 1.
<i>scale</i>	the (positive) scale parameter <i>b</i> .
<i>d</i> , <i>k</i>	the (positive) parameters <i>d</i> and <i>k</i> .
<i>log</i>	Logical. If <i>log</i> = TRUE then the logarithm of the density is returned.

**Details**

See [gengamma](#), the **VGAM** family function for estimating the generalized gamma distribution by maximum likelihood estimation, for formulae and other details. Apart from *n*, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

**Value**

*dgengamma* gives the density, *pgengamma* gives the distribution function, *qgengamma* gives the quantile function, and *rgengamma* generates random deviates.

**Author(s)**

T. W. Yee

References

Stacy, E. W. and Mihram, G. A. (1965) Parameter estimation for a generalized gamma distribution. *Technometrics*, **7**, 349–358.

See Also

[gengamma](#).

Examples

```
## Not run: x <- seq(0, 14, by = 0.01); d <- 1.5; Scale <- 2; k <- 6
plot(x, dgengamma(x, Scale, d, k), type = "l", col = "blue", ylim = 0:1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qgengamma(seq(0.05,0.95,by = 0.05), Scale, d, k),
      dgengamma(qgengamma(seq(0.05,0.95,by = 0.05), Scale, d, k),
                 Scale, d, k), col = "purple", lty = 3, type = "h")
lines(x, pgengamma(x, Scale, d, k), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
```

---

genpoisson	<i>Generalized Poisson distribution</i>
------------	---

---

Description

Estimation of the two parameters of a generalized Poisson distribution.

Usage

```
genpoisson(llambda = elogit(min = -1, max = 1), ltheta = "loge",
           ilambda = NULL, itheta = NULL,
           use.approx = TRUE, imethod = 1, zero = 1)
```

Arguments

llambda, ltheta	Parameter link functions for $\lambda$ and $\theta$ . See <a href="#">Links</a> for more choices. The $\lambda$ parameter lies at least within the interval $[-1, 1]$ ; see below for more details. The $\theta$ parameter is positive, therefore the default is the log link.
ilambda, itheta	Optional initial values for $\lambda$ and $\theta$ . The default is to choose values internally.
use.approx	Logical. If TRUE then an approximation to the expected information matrix is used, otherwise Newton-Raphson is used.
imethod	An integer with value 1 or 2 which specifies the initialization method for the parameters. If failure to converge occurs try another value and/or else specify a value for ilambda and/or itheta.

**zero** An integer vector, containing the value 1 or 2. If so,  $\lambda$  or  $\theta$  respectively are modelled as an intercept only. If set to NULL then both linear/additive predictors are modelled as functions of the explanatory variables.

## Details

The generalized Poisson distribution has density

$$f(y) = \theta(\theta + \lambda y)^{y-1} \exp(-\theta - \lambda y)/y!$$

for  $\theta > 0$  and  $y = 0, 1, 2, \dots$ . Now  $\max(-1, -\theta/m) \leq \lambda \leq 1$  where  $m(\geq 4)$  is the greatest positive integer satisfying  $\theta + m\lambda > 0$  when  $\lambda < 0$  [and then  $P(Y = y) = 0$  for  $y > m$ ]. Note the complicated support for this distribution means, for some data sets, the default link for `llambda` is not always appropriate.

An ordinary Poisson distribution corresponds to  $\lambda = 0$ . The mean (returned as the fitted values) is  $E(Y) = \theta/(1 - \lambda)$  and the variance is  $\theta/(1 - \lambda)^3$ .

For more information see Consul and Famoye (2006) for a summary and Consul (1989) for full details.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

This distribution is useful for dispersion modelling. Convergence problems may occur when `lambda` is very close to 0 or 1.

## Author(s)

T. W. Yee

## References

- Consul, P. C. and Famoye, F. (2006) *Lagrangian Probability Distributions*, Boston: Birkhauser.
- Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall
- Consul, P. C. (1989) *Generalized Poisson Distributions: Properties and Applications*. New York: Marcel Dekker.

## See Also

[poissonff](#).

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 200))
gdata <- transform(gdata, y = rpois(nn, exp(2 - x2))) # Ordinary Poisson data
fit <- vglm(y ~ x2, genpoisson(zero = 1), gdata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
```

---

genray

*The Generalized Rayleigh Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the generalized Rayleigh distribution.

## Usage

```
dgenray(x, shape, scale = 1, log = FALSE)
pgenray(q, shape, scale = 1)
qgenray(p, shape, scale = 1)
rgenray(n, shape, scale = 1)
```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
scale, shape	positive scale and shape parameters.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

See [genrayleigh](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

`dgenray` gives the density, `pgenray` gives the distribution function, `qgenray` gives the quantile function, and `rgenray` generates random deviates.

## Note

We define scale as the reciprocal of the scale parameter used by Kundu and Raqab (2005).

**Author(s)**

J. G. Lauder and T. W. Yee

**See Also**[genrayleigh](#), [rayleigh](#).**Examples**

```
## Not run:
shape <- 0.5; scale <- 1; nn <- 501
x <- seq(-0.10, 3.0, len = nn)
plot(x, dgenray(x, shape, scale), type = "l", las = 1, ylim = c(0, 1.2),
     ylab = paste("[dp]genray(shape = ", shape, ", scale = ", scale, ")"),
     col = "blue", cex.main = 0.8,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pgenray(x, shape, scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgenray(probs, shape, scale)
lines(Q, dgenray(Q, shape, scale), col = "purple", lty = 3, type = "h")
lines(Q, pgenray(Q, shape, scale), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pgenray(Q, shape, scale) - probs)) # Should be 0

## End(Not run)
```

genrayleigh

*Generalized Rayleigh Distribution Family Function***Description**

Estimates the two parameters of the generalized Rayleigh distribution by maximum likelihood estimation.

**Usage**

```
genrayleigh(lshape = "loge", lscale = "loge",
            ishape = NULL, iscale = NULL,
            tol12 = 1e-05, nsimEIM = 300, zero = 1)
```

**Arguments**

**lshape, lscale** Link function for the two positive parameters, shape and scale. See [Links](#) for more choices.

**ishape, iscale** Numeric. Optional initial values for the shape and scale parameters.

**nsimEIM, zero** See [CommonVGAMffArguments](#).

**tol12** Numeric and positive. Tolerance for testing whether the second shape parameter is either 1 or 2. If so then the working weights need to handle these singularities.

## Details

The generalized Rayleigh distribution has density function

$$f(y; a = \text{shape}, b = \text{scale}) = (2ay/b^2)e^{-(y/b)^2}(1 - e^{-(y/b)^2})^{a-1}$$

where  $y > 0$  and the two parameters,  $a$  and  $b$ , are positive. The mean cannot be expressed nicely so the median is returned as the fitted values. Applications of the generalized Rayleigh distribution include modeling strength data and general lifetime data. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

We define scale as the reciprocal of the scale parameter used by Kundu and Raqab (2005).

## Author(s)

J. G. Lauder and T. W. Yee

## References

Kundu, D., Raqab, M. C. (2005). Generalized Rayleigh distribution: different methods of estimations. *Computational Statistics and Data Analysis*, **49**, 187–200.

## See Also

[dgenray](#), [rayleigh](#).

## Examples

```
shape <- exp(1); scale <- exp(1)
rdata <- data.frame(y = rgenray(n = 1000, shape, scale))
fit <- vglm(y ~ 1, genrayleigh, rdata, trace = TRUE)
c(with(rdata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

geometric

*Geometric (Truncated and Untruncated) Distributions***Description**

Maximum likelihood estimation for the geometric and truncated geometric distributions.

**Usage**

```
geometric(link = "logit", expected = TRUE, imethod = 1,
          iprob = NULL, zero = NULL)
truncgeometric(upper.limit = Inf,
               link = "logit", expected = TRUE, imethod = 1,
               iprob = NULL, zero = NULL)
```

**Arguments**

link	Parameter link function applied to the probability parameter $p$ , which lies in the unit interval. See <a href="#">Links</a> for more choices.
expected	Logical. Fisher scoring is used if <code>expected = TRUE</code> , else Newton-Raphson.
iprob, imethod, zero	See <a href="#">CommonVGAMffArguments</a> for more details.
upper.limit	Numeric. Upper values. As a vector, it is recycled across responses first. The default value means both family functions should give the same result.

**Details**

A random variable  $Y$  has a 1-parameter geometric distribution if  $P(Y = y) = p(1 - p)^y$  for  $y = 0, 1, 2, \dots$ . Here,  $p$  is the probability of success, and  $Y$  is the number of (independent) trials that are fails until a success occurs. Thus the response  $Y$  should be a non-negative integer. The mean of  $Y$  is  $E(Y) = (1 - p)/p$  and its variance is  $Var(Y) = (1 - p)/p^2$ . The geometric distribution is a special case of the negative binomial distribution (see [negbinomial](#)). If  $Y$  has a geometric distribution with parameter  $p$  then  $Y + 1$  has a positive-geometric distribution with the same parameter. Multiple responses are permitted.

For `truncgeometric()`, the (upper) truncated geometric distribution can have response integer values from 0 to `upper.limit`. It has density  $\text{prob} * (1 - \text{prob})^y / [1 - (1 - \text{prob})^{(1 + \text{upper.limit})}]$ .

For a generalized truncated geometric distribution with integer values  $L$  to  $U$ , say, subtract  $L$  from the response and feed in  $U - L$  as the upper limit.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Author(s)**

T. W. Yee. Help from Viet Hoang Quoc is gratefully acknowledged.

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

[negbinomial](#), [Geometric](#), [betageometric](#), [expgeometric](#), [zageometric](#), [zigeometric](#), [rbetageom](#).

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 1000) - 0.5)
gdata <- transform(gdata, x3 = runif(nn) - 0.5,
                  x4 = runif(nn) - 0.5)
gdata <- transform(gdata, eta = -1.0 - 1.0 * x2 + 2.0 * x3)
gdata <- transform(gdata, prob = logit(eta, inverse = TRUE))
gdata <- transform(gdata, y1 = rgeom(nn, prob))
with(gdata, table(y1))
fit1 <- vglm(y1 ~ x2 + x3 + x4, geometric, gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)

# Truncated geometric (between 0 and upper.limit)
upper.limit <- 5
tdata <- subset(gdata, y1 <= upper.limit)
nrow(tdata) # Less than nn
fit2 <- vglm(y1 ~ x2 + x3 + x4, truncgeometric(upper.limit),
            data = tdata, trace = TRUE)
coef(fit2, matrix = TRUE)

# Generalized truncated geometric (between lower.limit and upper.limit)
lower.limit <- 1
upper.limit <- 8
gtdata <- subset(gdata, lower.limit <= y1 & y1 <= upper.limit)
with(gtdata, table(y1))
nrow(gtdata) # Less than nn
fit3 <- vglm(y1 - lower.limit ~ x2 + x3 + x4,
            truncgeometric(upper.limit - lower.limit),
            data = gtdata, trace = TRUE)
coef(fit3, matrix = TRUE)
```

---

get.smart

---

*Retrieve One Component of “.smart.prediction”*


---

## Description

Retrieve one component of the list `.smart.prediction` from `smartpredenv` (R) or frame 1 (S-PLUS).



**Usage**

```
get.smart()
```

**Details**

`get.smart` is used in "read" mode within a smart function: it retrieves parameters saved at the time of fitting, and is used for prediction. `get.smart` is only used in smart functions such as `poly`; `get.smart.prediction` is only used in modelling functions such as `lm` and `glm`. The function `get.smart` gets only a part of `.smart.prediction` whereas `get.smart.prediction` gets the entire `.smart.prediction`.

**Value**

Returns with one list component of `.smart.prediction` from `smartpredenv` (R) or frame 1 (S-PLUS), in fact, `.smart.prediction[[.smart.prediction.counter]]`. The whole procedure mimics a first-in first-out stack (better known as a *queue*).

**Side Effects**

The variable `.smart.prediction.counter` in `smartpredenv` (R) or frame 1 (S-PLUS) is incremented beforehand, and then written back to `smartpredenv` (R) or frame 1 (S-PLUS).

**See Also**

`get.smart.prediction`.

**Examples**

```
"my1" <- function(x, minx = min(x)) { # Here is a smart function
  x <- x # Needed for nested calls, e.g., bs(scale(x))
  if(smart.mode.is("read")) {
    smart <- get.smart()
    minx <- smart$minx          # Overwrite its value
  } else
    if(smart.mode.is("write"))
      put.smart(list(minx = minx))
  sqrt(x-minx)
}
attr(my1, "smart") <- TRUE
```

---

<code>get.smart.prediction</code>	<i>Retrieves ".smart.prediction"</i>
-----------------------------------	--------------------------------------

---

**Description**

Retrieves `.smart.prediction` from `smartpredenv` (R) or frame 1 (S-PLUS).

**Usage**

```
get.smart.prediction()
```

**Details**

A smart modelling function such as `lm` allows smart functions such as `bs` to write to a data structure called `.smart.prediction` in `smartpredenv` (R) or frame 1 (S-PLUS). At the end of fitting, `get.smart.prediction` retrieves this data structure. It is then attached to the object, and used for prediction later.

**Value**

Returns with the list `.smart.prediction` from `smartpredenv` (R) or frame 1 (S-PLUS).

**See Also**

`get.smart`, `lm`.

**Examples**

```
## Not run: # Put at the end of lm
fit$smart <- get.smart.prediction()
## End(Not run)
```

---

gev

*Generalized Extreme Value Distribution Family Function*

---

**Description**

Maximum likelihood estimation of the 3-parameter generalized extreme value (GEV) distribution.

**Usage**

```
gev(llocation = "identity", lscale = "loge", lshape = logoff(offset = 0.5),
    percentiles = c(95, 99), iscale=NULL, ishape = NULL,
    imethod = 1, gshape = c(-0.45, 0.45), tolshape0 = 0.001,
    giveWarning = TRUE, zero = 3)
egev(llocation = "identity", lscale = "loge", lshape = logoff(offset = 0.5),
    percentiles = c(95, 99), iscale=NULL, ishape = NULL,
    imethod = 1, gshape = c(-0.45, 0.45), tolshape0 = 0.001,
    giveWarning = TRUE, zero = 3)
```

## Arguments

<code>llocation, lscale, lshape</code>	Parameter link functions for $\mu$ , $\sigma$ and $\xi$ respectively. See <a href="#">Links</a> for more choices. For the shape parameter, the default <code>logoff</code> link has an offset called $A$ below; and then the linear/additive predictor is $\log(\xi + A)$ which means that $\xi > -A$ . For technical reasons (see <b>Details</b> ) it is a good idea for $A = 0.5$ .
<code>percentiles</code>	Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100. However, if <code>percentiles = NULL</code> , then the mean $\mu + \sigma(\Gamma(1 - \xi) - 1)/\xi$ is returned, and this is only defined if $\xi < 1$ .
<code>iscale, ishape</code>	Numeric. Initial value for $\sigma$ and $\xi$ . A NULL means a value is computed internally. The argument <code>ishape</code> is more important than the other two because they are initialized from the initial $\xi$ . If a failure to converge occurs, or even to obtain initial values occurs, try assigning <code>ishape</code> some value (positive or negative; the sign can be very important). Also, in general, a larger value of <code>iscale</code> is better than a smaller value.
<code>imethod</code>	Initialization method. Either the value 1 or 2. Method 1 involves choosing the best $\xi$ on a course grid with endpoints <code>gshape</code> . Method 2 is similar to the method of moments. If both methods fail try using <code>ishape</code> .
<code>gshape</code>	Numeric, of length 2. Range of $\xi$ used for a grid search for a good initial value for $\xi$ . Used only if <code>imethod</code> equals 1.
<code>tolshape0, giveWarning</code>	Passed into <a href="#">dgev</a> when computing the log-likelihood.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, 3\}$ corresponding respectively to $\mu$ , $\sigma$ , $\xi$ . If <code>zero = NULL</code> then all linear/additive predictors are modelled as a linear combination of the explanatory variables. For many data sets having <code>zero = 3</code> is a good idea.

## Details

The GEV distribution function can be written

$$G(y) = \exp(-[(y - \mu)/\sigma]_+^{-1/\xi})$$

where  $\sigma > 0$ ,  $-\infty < \mu < \infty$ , and  $1 + \xi(y - \mu)/\sigma > 0$ . Here,  $x_+ = \max(x, 0)$ . The  $\mu$ ,  $\sigma$ ,  $\xi$  are known as the *location*, *scale* and *shape* parameters respectively. The cases  $\xi > 0$ ,  $\xi < 0$ ,  $\xi = 0$  correspond to the Frechet, Weibull, and Gumbel types respectively. It can be noted that the Gumbel (or Type I) distribution accommodates many commonly-used distributions such as the normal, lognormal, logistic, gamma, exponential and Weibull.

For the GEV distribution, the  $k$ th moment about the mean exists if  $\xi < 1/k$ . Provided they exist, the mean and variance are given by  $\mu + \sigma\{\Gamma(1 - \xi) - 1\}/\xi$  and  $\sigma^2\{\Gamma(1 - 2\xi) - \Gamma^2(1 - \xi)\}/\xi^2$  respectively, where  $\Gamma$  is the gamma function.

Smith (1985) established that when  $\xi > -0.5$ , the maximum likelihood estimators are completely regular. To have some control over the estimated  $\xi$  try using `lshape = logoff(offset = 0.5)`, say, or `lshape = elogit(min = -0.5, max = 0.5)`, say.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

Currently, if an estimate of  $\xi$  is too close to zero then an error will occur for [gev\(\)](#) with multivariate responses. In general, [egev\(\)](#) is more reliable than [gev\(\)](#).

Fitting the GEV by maximum likelihood estimation can be numerically fraught. If  $1 + \xi(y - \mu)/\sigma \leq 0$  then some crude evasive action is taken but the estimation process can still fail. This is particularly the case if [vgam](#) with [s](#) is used; then smoothing is best done with [vglm](#) with regression splines ([bs](#) or [ns](#)) because [vglm](#) implements half-stepsizing whereas [vgam](#) doesn't (half-stepsizing helps handle the problem of straying outside the parameter space.)

**Note**

The **VGAM** family function [gev](#) can handle a multivariate (matrix) response. If so, each row of the matrix is sorted into descending order and NAs are put last. With a vector or one-column matrix response using [egev](#) will give the same result but be faster and it handles the  $\xi = 0$  case. The function [gev](#) implements Tawn (1988) while [egev](#) implements Prescott and Walden (1980).

The shape parameter  $\xi$  is difficult to estimate accurately unless there is a lot of data. Convergence is slow when  $\xi$  is near  $-0.5$ . Given many explanatory variables, it is often a good idea to make sure `zero = 3`. The range restrictions of the parameter  $\xi$  are not enforced; thus it is possible for a violation to occur.

Successful convergence often depends on having a reasonably good initial value for  $\xi$ . If failure occurs try various values for the argument `i shape`, and if there are covariates, having `zero = 3` is advised.

**Author(s)**

T. W. Yee

**References**

- Yee, T. W. and Stephenson, A. G. (2007) Vector generalized linear and additive extreme value models. *Extremes*, **10**, 1–19.
- Tawn, J. A. (1988) An extreme-value theory model for dependent observations. *Journal of Hydrology*, **101**, 227–250.
- Prescott, P. and Walden, A. T. (1980) Maximum likelihood estimation of the parameters of the generalized extreme-value distribution. *Biometrika*, **67**, 723–724.
- Smith, R. L. (1985) Maximum likelihood estimation in a class of nonregular cases. *Biometrika*, **72**, 67–90.

**See Also**

[rgev](#), [gumbel](#), [egumbel](#), [guplot](#), [rlplot.egev](#), [gpd](#), [weibull](#), [frechet2](#), [elogit](#), [oxtemp](#), [venice](#).

## Examples

```
## Not run:
# Multivariate example
fit1 <- vgam(cbind(r1, r2) ~ s(year, df = 3), gev(zero = 2:3),
            data = venice, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1))
par(mfrow = c(1, 2), las = 1)
plot(fit1, se = TRUE, lcol = "blue", scol = "forestgreen",
     main = "Fitted mu(year) function (centered)", cex.main = 0.8)
with(venice, matplot(year, depvar(fit1)[, 1:2], ylab = "Sea level (cm)",
                     col = 1:2, main = "Highest 2 annual sea levels", cex.main = 0.8))
with(venice, lines(year, fitted(fit1)[,1], lty = "dashed", col = "blue"))
legend("topleft", lty = "dashed", col = "blue", "Fitted 95 percentile")

# Univariate example
(fit <- vglm(maxtemp ~ 1, egev, oxtemp, trace = TRUE))
head(fitted(fit))
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
vcov(fit, untransform = TRUE)
sqrt(diag(vcov(fit))) # Approximate standard errors
rlplot(fit)

## End(Not run)
```

---

gevUC

*The Generalized Extreme Value Distribution*

---

## Description

Density, distribution function, quantile function and random generation for the generalized extreme value distribution (GEV) with location parameter location, scale parameter scale and shape parameter shape.

## Usage

```
dgev(x, location = 0, scale = 1, shape = 0, log = FALSE, tolshape0 =
     sqrt(.Machine$double.eps), oobounds.log = -Inf, giveWarning = FALSE)
pgev(q, location = 0, scale = 1, shape = 0)
qgev(p, location = 0, scale = 1, shape = 0)
rgev(n, location = 0, scale = 1, shape = 0)
```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.

n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
location	the location parameter $\mu$ .
scale	the (positive) scale parameter $\sigma$ . Must consist of positive values.
shape	the shape parameter $\xi$ .
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.
tolshape0	Positive numeric. Threshold/tolerance value for resting whether $\xi$ is zero. If the absolute value of the estimate of $\xi$ is less than this value then it will be assumed zero and a Gumbel distribution will be used.
oobounds.log, giveWarning	
	Numeric and logical. The GEV distribution has support in the region satisfying $1 + \text{shape} * (\text{x} - \text{location}) / \text{scale} > 0$ . Outside that region, the logarithm of the density is assigned <code>oobounds.log</code> , which equates to a zero density. It should not be assigned a positive number, and ideally is very negative. Since <a href="#">egev</a> uses this function it is necessary to return a finite value outside this region so as to allow for half-stepping. Both arguments are in support of this. This argument and others match those of <a href="#">egev</a> .

## Details

See [gev](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for formulae and other details. Apart from `n`, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

## Value

`dgev` gives the density, `pgev` gives the distribution function, `qgev` gives the quantile function, and `rgev` generates random deviates.

## Note

The default value of  $\xi = 0$  means the default distribution is the Gumbel.

Currently, these functions have different argument names compared with those in the **evd** package.

## Author(s)

T. W. Yee

## References

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

## See Also

[gev](#), [egev](#), [vglm.control](#).

## Examples

```
## Not run: loc <- 2; sigma <- 1; xi <- -0.4
x <- seq(loc - 3, loc + 3, by = 0.01)
plot(x, dgev(x, loc, sigma, xi), type = "l", col = "blue", ylim = c(0,1),
     main = "Blue is density, red is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", ylab = "", las = 1)
abline(h = 0, col = "blue", lty = 2)
lines(qgev(seq(0.05, 0.95, by = 0.05), loc, sigma, xi),
     dgev(qgev(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi),
     col = "purple", lty = 3, type = "h")
lines(x, pgev(x, loc, sigma, xi), type = "l", col = "red")
abline(h = 0, lty = 2)

pgev(qgev(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi)

## End(Not run)
```

---

gew

*General Electric and Westinghouse Data*


---

## Description

General Electric and Westinghouse capital data.

## Usage

```
data(gew)
```

## Format

A data frame with 20 observations on the following 7 variables. All variables are numeric vectors. Variables ending in .g correspond to General Electric and those ending in .w are Westinghouse.

**year** The observations are the years from 1934 to 1953

**invest.g, invest.w** investment figures. These are  $I$  = Gross investment = additions to plant and equipment plus maintenance and repairs in millions of dollars deflated by  $P_1$ .

**capital.g, capital.w** capital stocks. These are  $C$  = The stock of plant and equipment = accumulated sum of net additions to plant and equipment deflated by  $P_1$  minus depreciation allowance deflated by  $P_3$ .

**value.g, value.w** market values. These are  $F$  = Value of the firm = price of common and preferred shares at December 31 (or average price of December 31 and January 31 of the following year) times number of common and preferred shares outstanding plus total book value of debt at December 31 in millions of dollars deflated by  $P_2$ .

Details

These data are a subset of a table in Boot and de Wit (1960), also known as the Grunfeld data. It is used a lot in econometrics, e.g., for seemingly unrelated regressions (see SUR).  
Here,  $P_1$  = Implicit price deflator of producers durable equipment (base 1947),  $P_2$  = Implicit price deflator of G.N.P. (base 1947),  $P_3$  = Depreciation expense deflator = ten years moving average of wholesale price index of metals and metal products (base 1947).

Source

Table 10 of: Boot, J. C. G. and de Wit, G. M. (1960) Investment Demand: An Empirical Contribution to the Aggregation Problem. *International Economic Review*, **1**, 3–30.  
Grunfeld, Y. (1958) The Determinants of Corporate Investment. Unpublished PhD Thesis (Chicago).

References

Zellner, A. (1962) An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. *Journal of the American Statistical Association*, **57**, 348–368.

See Also

[SUR](http://statmath.wu.ac.at/~zeileis/grunfeld), <http://statmath.wu.ac.at/~zeileis/grunfeld>.

Examples

```
str(gew)
```

---

golf	<i>Gamma-Ordinal Link Function</i>
------	------------------------------------

---

Description

Computes the gamma-ordinal transformation, including its inverse and the first two derivatives.

Usage

```
golf(theta, lambda = 1, cutpoint = NULL,  
      inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

Arguments

theta	Numeric or character. See below for further details.
lambda, cutpoint	The former is the shape parameter in <a href="#">gamma2</a> . cutpoint is optional; if NULL then cutpoint is ignored from the GOLF definition. If given, the cutpoints should be non-negative integers. If <code>golf()</code> is used as the link function in <a href="#">cumulative</a> then, if the cutpoints are known, then one should choose <code>reverse = TRUE</code> , <code>parallel = FALSE</code> ~ -1. If the cutpoints are unknown, then choose <code>reverse = TRUE</code> , <code>parallel = TRUE</code> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .



## Details

The gamma-ordinal link function (GOLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying 2-parameter gamma distribution.

See [Links](#) for general information about **VGAM** link functions.

## Value

See Yee (2012) for details.

## Warning

Prediction may not work on [vglm](#) or [vgam](#) etc. objects if this link function is used.

## Note

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0, numerical instabilities may still arise.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the gamma distribution (see [gamma2](#)) that has been recorded as an ordinal response using known cutpoints.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2012) *Ordinal ordination with normalizing link functions for count data*, (in preparation).

## See Also

[Links](#), [gamma2](#), [polf](#), [nbolf](#), [cumulative](#).

## Examples

```
golf("p", lambda = 1, short = FALSE)
golf("p", lambda = 1, tag = TRUE)

p <- seq(0.02, 0.98, len = 201)
y <- golf(p, lambda = 1)
y. <- golf(p, lambda = 1, deriv = 1)
max(abs(golf(y, lambda = 1, inv = TRUE) - p)) # Should be 0

## Not run: par(mfrow = c(2, 1), las = 1)
plot(p, y, type = "l", col = "blue", main = "golf()")
abline(h = 0, v = 0.5, col = "orange", lty = "dashed")
plot(p, y., type = "l", col = "blue",
```

```

main = "(Reciprocal of) first GOLF derivative")

## End(Not run)

# Another example
gdata <- data.frame(x2 = sort(runif(nn <- 1000)))
gdata <- transform(gdata, x3 = runif(nn))
gdata <- transform(gdata, mymu = exp( 3 + 1 * x2 - 2 * x3))
lambda <- 4
gdata <- transform(gdata,
  y1 = rgamma(nn, shape = lambda, scale = mymu / lambda))
cutpoints <- c(-Inf, 10, 20, Inf)
gdata <- transform(gdata, cuty = Cut(y1, breaks = cutpoints))

## Not run: par(mfrow = c(1, 1), las = 1)
with(gdata, plot(x2, x3, col = cuty, pch = as.character(cuty)))
## End(Not run)
with(gdata, table(cuty) / sum(table(cuty)))
fit <- vglm(cuty ~ x2 + x3, cumulative(mv = TRUE,
  reverse = TRUE, parallel = FALSE ~ -1,
  link = golf(cutpoint = cutpoints[2:3], lambda = lambda)),
  data = gdata, trace = TRUE)
head(depvar(fit))
head(fitted(fit))
head(predict(fit))
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
fit@misc

```

---

Gompertz

---

*The Gompertz Distribution*


---

## Description

Density, cumulative distribution function, quantile function and random generation for the Gompertz distribution.

## Usage

```

dgomperz(x, shape, scale = 1, log = FALSE)
pgomperz(q, shape, scale = 1)
qgomperz(p, shape, scale = 1)
rgomperz(n, shape, scale = 1)

```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.

n                    number of observations.  
 log                  Logical. If log = TRUE then the logarithm of the density is returned.  
 shape, scale       positive shape and scale parameters.

## Details

See [gompertz](#) for details.

## Value

dgompertz gives the density, pgompertz gives the cumulative distribution function, qgompertz gives the quantile function, and rgompertz generates random deviates.

## Author(s)

T. W. Yee

## See Also

[gompertz](#), [dgumbel](#), [dmakeham](#).

## Examples

```

probs <- seq(0.01, 0.99, by = 0.01)
Shape <- exp(1); Scale <- exp(1)
max(abs(pgompertz(qgompertz(p = probs, Shape, Scale),
                  Shape, Scale) - probs)) # Should be 0

## Not run: x <- seq(-0.1, 1.0, by = 0.01)
plot(x, dgompertz(x, Shape, Scale), type = "l", col = "blue", las = 1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles",
     ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pgompertz(x, Shape, Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgompertz(probs, Shape, Scale)
lines(Q, dgompertz(Q, Shape, Scale), col = "purple", lty = 3, type = "h")
pgompertz(Q, Shape, Scale) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)

```

gompertz

*Gompertz Distribution Family Function***Description**

Maximum likelihood estimation of the 2-parameter Gompertz distribution.

**Usage**

```
gompertz(lshape = "log", lscale = "log",
         ishape = NULL, iscale = NULL,
         nsimEIM = 500, zero = NULL)
```

**Arguments**

`lshape`, `lscale` Parameter link functions applied to the shape parameter `a`, scale parameter `scale`. All parameters are positive. See [Links](#) for more choices.

`ishape`, `iscale` Optional initial values. A `NULL` means a value is computed internally.

`nsimEIM`, `zero` See [CommonVGAMffArguments](#).

**Details**

The Gompertz distribution has a cumulative distribution function

$$F(x; \alpha, \beta) = 1 - \exp[-(\alpha/\beta) \times (\exp(\beta x) - 1)]$$

which leads to a probability density function

$$f(x; \alpha, \beta) = \alpha \exp(\beta x) \exp[-(\alpha/\beta) \times (\exp(\beta x) - 1)]$$

for  $\alpha > 0$ ,  $\beta > 0$ ,  $x > 0$ . Here,  $\beta$  is called the scale parameter `scale`, and  $\alpha$  is called the shape parameter (one could refer to  $\alpha$  as a location parameter and  $\beta$  as a shape parameter—see Lenart (2012)). The mean involves an exponential integral function. Simulated Fisher scoring is used and multiple responses are handled.

The Makeham distribution has an additional parameter compared to the Gompertz distribution. If  $X$  is defined to be the result of sampling from a Gumbel distribution until a negative value  $Z$  is produced, then  $X = -Z$  has a Gompertz distribution.

**Value**

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

The same warnings in [makeham](#) apply here too.

**Author(s)**

T. W. Yee

**References**

Lenart, A. (2012) The moments of the Gompertz distribution and maximum likelihood estimation of its parameters. *Scandinavian Actuarial Journal*, in press.

**See Also**

[dgomPERTZ](#), [makeham](#).

**Examples**

```
## Not run:
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, eta1 = -1,
                  eta2 = -1 + 0.2 * x2,
                  ceta1 = 1,
                  ceta2 = -1 + 0.2 * x2)
gdata <- transform(gdata, shape1 = exp(eta1),
                  shape2 = exp(eta2),
                  scale1 = exp(ceta1),
                  scale2 = exp(ceta2))
gdata <- transform(gdata, y1 = rgompertz(nn, shape = shape1, scale = scale1),
                  y2 = rgompertz(nn, shape = shape2, scale = scale2))

fit1 <- vglm(y1 ~ 1, gompertz, data = gdata, trace = TRUE)
fit2 <- vglm(y2 ~ x2, gompertz, data = gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)
summary(fit1)
coef(fit2, matrix = TRUE)
summary(fit2)

## End(Not run)
```

gpd

*Generalized Pareto Distribution Family Function***Description**

Maximum likelihood estimation of the 2-parameter generalized Pareto distribution (GPD).

**Usage**

```
gpd(threshold = 0, lscale = "loge", lshape = logoff(offset = 0.5),
    percentiles = c(90, 95), iscale = NULL, ishape = NULL,
    tolshape0 = 0.001, giveWarning = TRUE, imethod = 1, zero = -2)
```

## Arguments

threshold	Numeric, values are recycled if necessary. The threshold value(s), called $\mu$ below.
lscale	Parameter link function for the scale parameter $\sigma$ . See <a href="#">Links</a> for more choices.
lshape	Parameter link function for the shape parameter $\xi$ . See <a href="#">Links</a> for more choices. The default constrains the parameter to be greater than $-0.5$ because if $\xi \leq -0.5$ then Fisher scoring does not work. See the Details section below for more information.  For the shape parameter, the default <a href="#">logoff</a> link has an offset called $A$ below; and then the second linear/additive predictor is $\log(\xi + A)$ which means that $\xi > -A$ . The working weight matrices are positive definite if $A = 0.5$ .
percentiles	Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100. See the example below for illustration. However, if <code>percentiles = NULL</code> then the mean $\mu + \sigma/(1 - \xi)$ is returned; this is only defined if $\xi < 1$ .
iscale, ishape	Numeric. Optional initial values for $\sigma$ and $\xi$ . The default is to use <code>imethod</code> and compute a value internally for each parameter. Values of <code>ishape</code> should be between $-0.5$ and 1. Values of <code>iscale</code> should be positive.
tolshape0, giveWarning	Passed into <a href="#">dgp</a> when computing the log-likelihood.
imethod	Method of initialization, either 1 or 2. The first is the method of moments, and the second is a variant of this. If neither work, try assigning values to arguments <code>ishape</code> and/or <code>iscale</code> .
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. For one response, the value should be from the set $\{1, 2\}$ corresponding respectively to $\sigma$ and $\xi$ . It is often a good idea for the $\sigma$ parameter only to be modelled through a linear combination of the explanatory variables because the shape parameter is probably best left as an intercept only: <code>zero = 2</code> . Setting <code>zero = NULL</code> means both parameters are modelled with explanatory variables. See <a href="#">CommonVGAMffArguments</a> for more details.

## Details

The distribution function of the GPD can be written

$$G(y) = 1 - [1 + \xi(y - \mu)/\sigma]_+^{-1/\xi}$$

where  $\mu$  is the location parameter (known, with value `threshold`),  $\sigma > 0$  is the scale parameter,  $\xi$  is the shape parameter, and  $h_+ = \max(h, 0)$ . The function  $1 - G$  is known as the *survivor function*. The limit  $\xi \rightarrow 0$  gives the *shifted exponential* as a special case:

$$G(y) = 1 - \exp[-(y - \mu)/\sigma].$$

The support is  $y > \mu$  for  $\xi > 0$ , and  $\mu < y < \mu - \sigma/\xi$  for  $\xi < 0$ .

Smith (1985) showed that if  $\xi \leq -0.5$  then this is known as the nonregular case and problems/difficulties can arise both theoretically and numerically. For the (regular) case  $\xi > -0.5$  the classical asymptotic theory of maximum likelihood estimators is applicable; this is the default.

Although for  $\xi < -0.5$  the usual asymptotic properties do not apply, the maximum likelihood estimator generally exists and is superefficient for  $-1 < \xi < -0.5$ , so it is “better” than normal. When  $\xi < -1$  the maximum likelihood estimator generally does not exist as it effectively becomes a two parameter problem.

The mean of  $Y$  does not exist unless  $\xi < 1$ , and the variance does not exist unless  $\xi < 0.5$ . So if you want to fit a model with finite variance use `lshape = "elogit"`.

### Value

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#). However, for this **VGAM** family function, [vglm](#) is probably preferred over [vgam](#) when there is smoothing.

### Warning

Fitting the GPD by maximum likelihood estimation can be numerically fraught. If  $1 + \xi(y - \mu)/\sigma \leq 0$  then some crude evasive action is taken but the estimation process can still fail. This is particularly the case if [vgam](#) with [s](#) is used. Then smoothing is best done with [vglm](#) with regression splines ([bs](#) or [ns](#)) because [vglm](#) implements half-stepsizing whereas [vgam](#) doesn’t. Half-stepsizing helps handle the problem of straying outside the parameter space.

### Note

The response in the formula of [vglm](#) and [vgam](#) is  $y$ . Internally,  $y - \mu$  is computed. This **VGAM** family function can handle a multiple responses, which is inputted as a matrix.

With functions [rgpd](#), [dgp](#), etc., the argument `location` matches with the argument `threshold` here.

### Author(s)

T. W. Yee

### References

- Yee, T. W. and Stephenson, A. G. (2007) Vector generalized linear and additive extreme value models. *Extremes*, **10**, 1–19.
- Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.
- Smith, R. L. (1985) Maximum likelihood estimation in a class of nonregular cases. *Biometrika*, **72**, 67–90.

### See Also

[rgpd](#), [meplot](#), [gev](#), [paretoff](#), [vglm](#), [vgam](#), [s](#).

## Examples

```
# Simulated data from an exponential distribution (xi = 0)
threshold <- 0.5
gdata <- data.frame(y1 = threshold + rexp(n = 3000, rate = 2))
fit <- vglm(y1 ~ 1, gpd(threshold = threshold), gdata, trace = TRUE)
head(fitted(fit))
coef(fit, matrix = TRUE) # xi should be close to 0
Coef(fit)
summary(fit)

fit@extra$threshold # Note the threshold is stored here

# Check the 90 percentile
ii <- depvar(fit) < fitted(fit)[1, "90%"]
100 * table(ii) / sum(table(ii)) # Should be 90%

# Check the 95 percentile
ii <- depvar(fit) < fitted(fit)[1, "95%"]
100 * table(ii) / sum(table(ii)) # Should be 95%

## Not run: plot(depvar(fit), col = "blue", las = 1,
                main = "Fitted 90% and 95% quantiles")
matlines(1:length(depvar(fit)), fitted(fit), lty = 2:3, lwd = 2)
## End(Not run)

# Another example
gdata <- data.frame(x2 = runif(nn <- 2000))
threshold <- 0; xi <- exp(-0.8) - 0.5
gdata <- transform(gdata, y2 = rgpd(nn, scale = exp(1 + 0.1*x2), shape = xi))
fit <- vglm(y2 ~ x2, gpd(threshold), gdata, trace = TRUE)
coef(fit, matrix = TRUE)

## Not run: # Nonparametric fits
gdata <- transform(gdata, yy = y2 + rnorm(nn, sd = 0.1))
# Not so recommended:
fit1 <- vgam(yy ~ s(x2), gpd(threshold), gdata, trace = TRUE)
par(mfrow = c(2,1))
plotvgam(fit1, se = TRUE, scol = "blue")
# More recommended:
fit2 <- vglm(yy ~ bs(x2), gpd(threshold), gdata, trace = TRUE)
plotvgam(fit2, se = TRUE, scol = "blue")
## End(Not run)
```



## Description

Density, distribution function, quantile function and random generation for the generalized Pareto distribution (GPD) with location parameter *location*, scale parameter *scale* and shape parameter *shape*.

## Usage

```
dgpd(x, location = 0, scale = 1, shape = 0, log = FALSE,
      tolshape0 = sqrt(.Machine$double.eps),
      oobounds.log = -Inf, giveWarning = FALSE)
pgpd(q, location = 0, scale = 1, shape = 0)
qgpd(p, location = 0, scale = 1, shape = 0)
rgpd(n, location = 0, scale = 1, shape = 0)
```

## Arguments

<i>x</i> , <i>q</i>	vector of quantiles.
<i>p</i>	vector of probabilities.
<i>n</i>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<i>location</i>	the location parameter $\mu$ .
<i>scale</i>	the (positive) scale parameter $\sigma$ .
<i>shape</i>	the shape parameter $\xi$ .
<i>log</i>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.
<i>tolshape0</i>	Positive numeric. Threshold/tolerance value for resting whether $\xi$ is zero. If the absolute value of the estimate of $\xi$ is less than this value then it will be assumed zero and an exponential distribution will be used.
<i>oobounds.log</i> , <i>giveWarning</i>	Numeric and logical. The GPD distribution has support in the region satisfying $(x - \text{location})/\text{scale} > 0$ and $1 + \text{shape} * (x - \text{location})/\text{scale} > 0$ . Outside that region, the logarithm of the density is assigned <code>oobounds.log</code> , which equates to a zero density. It should not be assigned a positive number, and ideally is very negative. Since <a href="#">gpd</a> uses this function it is necessary to return a finite value outside this region so as to allow for half-stepping. Both arguments are in support of this. This argument and others match those of <a href="#">gpd</a> .

## Details

See [gpd](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for formulae and other details. Apart from *n*, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

## Value

`dgpd` gives the density, `pgpd` gives the distribution function, `qgpd` gives the quantile function, and `rgpd` generates random deviates.

**Note**

The default values of all three parameters, especially  $\xi = 0$ , means the default distribution is the exponential.

Currently, these functions have different argument names compared with those in the **evd** package.

**Author(s)**

T. W. Yee

**References**

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

**See Also**

[gpd](#).

**Examples**

```
## Not run: loc <- 2; sigma <- 1; xi <- -0.4
x <- seq(loc - 0.2, loc + 3, by = 0.01)
plot(x, dgpdp(x, loc, sigma, xi), type = "l", col = "blue", ylim = c(0, 1),
     main = "Blue is density, red is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", ylab = "", las = 1)
abline(h = 0, col = "blue", lty = 2)
lines(qgpdp(seq(0.05, 0.95, by = 0.05), loc, sigma, xi),
     dgpdp(qgpdp(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi),
     col = "purple", lty = 3, type = "h")
lines(x, pgpdp(x, loc, sigma, xi), type = "l", col = "red")
abline(h = 0, lty = 2)

pgpdp(qgpdp(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi)

## End(Not run)
```

---

grain.us

---

*Grain Prices Data in USA*


---

**Description**

A 4-column matrix.

**Usage**

```
data(grain.us)
```

## Format

The columns are:

**wheat.flour** numeric

**corn** numeric

**wheat** numeric

**rye** numeric

## Details

Monthly averages of grain prices in the United States for wheat flour, corn, wheat, and rye for the period January 1961 through October 1972. The units are US dollars per 100 pound sack for wheat flour, and per bushel for corn, wheat and rye.

## Source

Ahn and Reinsel (1988).

## References

Ahn, S. K and Reinsel, G. C. (1988) Nested reduced-rank autoregressive models for multiple time series. *Journal of the American Statistical Association*, **83**, 849–856.

## Examples

```
## Not run:
cgrain <- scale(grain.us, scale = FALSE) # Center the time series only
fit <- vglm(cgrain ~ 1, rrar(Rank = c(4, 1)),
            epsilon = 1e-3, stepsize = 0.5, trace = TRUE, maxit = 50)
summary(fit)

## End(Not run)
```

---

grc

*Row-Column Interaction Models including Goodman's RC Association Model*

---

## Description

Fits a Goodman's RC association model to a matrix of counts, and more generally, a sub-class of row-column interaction models.

## Usage

```
grc(y, Rank = 1, Index.corner = 2:(1 + Rank),
    str0 = 1, summary.arg = FALSE, h.step = 1e-04, ...)
rcim(y, family = poissonff, Rank = 0, Musual = NULL,
     weights = NULL, which.linpred = 1,
     Index.corner = ifelse(is.null(str0), 0, max(str0)) + 1:Rank,
     rprefix = "Row.", cprefix = "Col.", iprefix = "X2.",
     offset = 0, str0 = if (Rank) 1 else NULL,
     summary.arg = FALSE, h.step = 0.0001,
     rbaseline = 1, cbaseline = 1,
     has.intercept = TRUE,
     M = NULL,
     rindex = 2:nrow(y),
     cindex = 2:ncol(y),
     iindex = 2:nrow(y),
     ...)
```

## Arguments

y	For grc a matrix of counts. For rcim a general matrix response depending on family. Output from table() is acceptable; it is converted into a matrix. Note that y should be at least 3 by 3 in dimension.
family	A <b>VGAM</b> family function. By default, the first linear/additive predictor is fitted using main effects plus an optional rank-Rank interaction term. Not all family functions are suitable or make sense. All other linear/additive predictors are fitted using an intercept-only, so it has a common value over all rows and columns. For example, <a href="#">zipoissonff</a> may be suitable for counts but not <a href="#">zipoisson</a> because of the ordering of the linear/additive predictors. If the <b>VGAM</b> family function does not have an infos slot then Musual needs to be inputted (the number of linear predictors for an ordinary (usually univariate) response, aka <i>M</i> ). The <b>VGAM</b> family function also needs to be able to handle multiple responses; and not all of them can do this.
Rank	An integer from the set $\{0, \dots, \min(\text{nrow}(y), \text{ncol}(y))\}$ . This is the dimension of the fit in terms of the interaction. For grc() this argument must be positive. A value of 0 means no interactions (i.e., main effects only); each row and column is represented by an indicator variable.
weights	Prior weights. Fed into <a href="#">rrvglm</a> or <a href="#">vglm</a> .
which.linpred	Single integer. Specifies which linear predictor is modelled as the sum of an intercept, row effect, column effect plus an optional interaction term. It should be one value from the set 1:Musual.
Index.corner	A vector of Rank integers. These are used to store the Rank by Rank identity matrix in the A matrix; corner constraints are used.
rprefix, cprefix, iprefix	Character, for rows and columns and interactions respectively. For labelling the indicator variables.
offset	Numeric. Either a matrix of the right dimension, else a single numeric expanded into such a matrix.

<code>str0</code>	Ignored if <code>Rank = 0</code> , else an integer from the set $\{1, \dots, \min(\text{nrow}(y), \text{ncol}(y))\}$ , specifying the row that is used as the structural zero. Passed into <code>rrvglm.control</code> if <code>Rank &gt; 0</code> . Set <code>str0 = NULL</code> for none.
<code>summary.arg</code>	Logical. If <code>TRUE</code> , a summary is returned. If <code>TRUE</code> , <code>y</code> may be the output (fitted object) of <code>grc()</code> .
<code>h.step</code>	A small positive value that is passed into <code>summary.rrvglm()</code> . Only used when <code>summary.arg = TRUE</code> .
<code>...</code>	Arguments that are passed into <code>rrvglm.control()</code> .
<code>Musual</code>	The number of linear predictors of the <b>VGAM</b> family function for an ordinary (univariate) response. Then the number of linear predictors of the <code>rcim()</code> fit is usually the number of columns of <code>y</code> multiplied by <code>Musual</code> . The default is to evaluate the <code>infos</code> slot of the <b>VGAM</b> family function to try to evaluate it; see <code>vglmff-class</code> . If this information is not yet supplied by the family function then the value needs to be inputted manually using this argument.
<code>rbaseline, cbaseline</code>	Baseline reference levels for the rows and columns. Currently stored on the object but not used.
<code>has.intercept</code>	Logical. Include an intercept?
<code>M, cindex</code>	<code>M</code> is the usual <b>VGAM</b> <code>M</code> , viz. the number of linear/additive predictors in total. Also, <code>cindex</code> means column index, and these point to the columns of <code>y</code> which are part of the vector of linear/additive predictor <i>main effects</i> . For family = multinomial it is necessary to input these arguments as <code>M = ncol(y)-1</code> and <code>cindex = 2:(ncol(y)-1)</code> .
<code>rindex, iindex</code>	<code>rindex</code> means row index, and these are similar to <code>cindex</code> . <code>iindex</code> means interaction index, and these are similar to <code>cindex</code> .

## Details

Goodman's RC association model fits a reduced-rank approximation to a table of counts. The log of each cell mean is decomposed as an intercept plus a row effect plus a column effect plus a reduced-rank component. The latter can be collectively written  $A \%*\% t(C)$ , the product of two 'thin' matrices. Indeed, `A` and `C` have `Rank` columns. By default, the first column and row of the interaction matrix  $A \%*\% t(C)$  is chosen to be structural zeros, because `str0 = 1`. This means the first row of `A` are all zeros.

This function uses `options()$contrasts` to set up the row and column indicator variables. In particular, Equation (4.5) of Yee and Hastie (2003) is used. These are called `Row.` and `Col.` (by default) followed by the row or column number.

The function `rcim()` is more general than `grc()`. Its default is a no-interaction model of `grc()`, i.e., rank-0 and a Poisson distribution. This means that each row and column has a dummy variable associated with it. The first row and column is baseline. The power of `rcim()` is that many **VGAM** family functions can be assigned to its `family` argument. For example, `uninormal` fits something in between a 2-way ANOVA with and without interactions, `alaplace2` with `Rank = 0` is something like `medpolish`. Others include `zipoissonff`, `negbinomial`. Hopefully one day *all* **VGAM** family functions will work when assigned to the `family` argument, although the result may not have meaning.

**Value**

An object of class "grc", which currently is the same as an "rrvglm" object. Currently, a rank-0 rcim() object is of class `rcim0-class`, else of class "rcim" (this may change in the future).

**Warning**

The function `rcim()` is experimental at this stage and may have bugs. Quite a lot of expertise is needed when fitting and in its interpretation thereof. For example, the constraint matrices applies the reduced-rank regression to the first (see `which.linpred`) linear predictor and the other linear predictors are intercept-only and have a common value throughout the entire data set. This means that, by default, `family = zipoissonff` is appropriate but not `family = zipoisson`. Else set `family = zipoisson` and `which.linpred = 2`. To understand what is going on, do examine the constraint matrices of the fitted object, and reconcile this with Equations (4.3) to (4.5) of Yee and Hastie (2003).

The functions temporarily create a permanent data frame called `.grc.df` or `.rcim.df`, which used to be needed by `summary.rrvglm()`. Then these data frames are deleted before exiting the function. If an error occurs, then the data frames may be present in the workspace.

**Note**

These functions set up the indicator variables etc. before calling `rrvglm` or `vglm`. The `...` is passed into `rrvglm.control` or `vglm.control`. This means, e.g., `Rank = 1` is default for `grc()`.

The data should be labelled with `rownames` and `colnames`. Setting `trace = TRUE` is recommended for monitoring convergence. Using `criterion = "coefficients"` can result in slow convergence.

If `summary = TRUE`, then `y` can be a "grc" object, in which case a summary can be returned. That is, `grc(y, summary = TRUE)` is equivalent to `summary(grc(y))`. It is not possible to plot a `grc(y, summary = TRUE)` or `rcim(y, summary = TRUE)` object.

**Author(s)**

Thomas W. Yee, with assistance from Alfian F. Hadi.

**References**

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Yee, T. W. and Hadi, A. F. (2013) Row-column interaction models *In preparation*.

Goodman, L. A. (1981) Association models and canonical correlation in the analysis of cross-classifications having ordered categories. *Journal of the American Statistical Association*, **76**, 320–334.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information about the setting up of the indicator variables.

**See Also**

`rrvglm`, `rrvglm.control`, `rrvglm-class`, `summary.grc`, `moffset`, `Rcim`, `Qvar`, `plotrcim0`, `multinomial`, `alcoff`, `crashi`, `auuc`, `olym08`, `olym12`, `poissonff`.

**Examples**

```

grc1 <- grc(auuc) # Undergraduate enrolments at Auckland University in 1990
fitted(grc1)
summary(grc1)

grc2 <- grc(auuc, Rank = 2, Index.corner = c(2, 5))
fitted(grc2)
summary(grc2)

model3 <- rcim(auuc, Rank = 1, fam = multinomial,
               M = ncol(auuc)-1, cindex = 2:(ncol(auuc)-1), trace = TRUE)
fitted(model3)
summary(model3)

# 2012 Summer Olympic Games in London
## Not run: top10 <- head(olym12, n = 10)
grc.oly1 <- with(top10, grc(cbind(gold, silver, bronze)))
round(fitted(grc.oly1))
round(resid(grc.oly1, type = "response"), digits = 1) # Response residuals
summary(grc.oly1)
Coef(grc.oly1)

## End(Not run)

# Roughly median polish
rcim0 <- rcim(auuc, fam = alaplace2(tau = 0.5, intparloc = TRUE), trace = TRUE)
round(fitted(rcim0), digits = 0)
round(100 * (fitted(rcim0) - auuc) / auuc, digits = 0) # Discrepancy
devar(rcim0)
round(coef(rcim0, matrix = TRUE), digits = 2)
Coef(rcim0, matrix = TRUE)
# constraints(rcim0)
names(constraints(rcim0))

# Compare with medpolish():
(med.a <- medpolish(auuc))
fv <- med.a$overall + outer(med.a$row, med.a$col, "+")
round(100 * (fitted(rcim0) - fv) / fv) # Hopefully should be all 0s

```

gumbel

*Gumbel Distribution Family Function***Description**

Maximum likelihood estimation of the 2-parameter Gumbel distribution.

**Usage**

```
gumbel(llocation = "identity", lscale = "loge",
       iscale = NULL, R = NA, percentiles = c(95, 99),
       mpv = FALSE, zero = NULL)
egumbel(llocation = "identity", lscale = "loge",
        iscale = NULL, R = NA, percentiles = c(95, 99),
        mpv = FALSE, zero = NULL)
```

**Arguments**

<code>llocation</code> , <code>lscale</code>	Parameter link functions for $\mu$ and $\sigma$ . See <a href="#">Links</a> for more choices.
<code>iscale</code>	Numeric and positive. Optional initial value for $\sigma$ . Recycled to the appropriate length. In general, a larger value is better than a smaller value. A NULL means an initial value is computed internally.
<code>R</code>	Numeric. Maximum number of values possible. See <b>Details</b> for more details.
<code>percentiles</code>	Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100. This argument uses the argument <code>R</code> if assigned. If <code>percentiles = NULL</code> then the mean will be returned as the fitted values.
<code>mpv</code>	Logical. If <code>mpv = TRUE</code> then the <i>median predicted value</i> (MPV) is computed and returned as the (last) column of the fitted values. This argument is ignored if <code>percentiles = NULL</code> . See <b>Details</b> for more details.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The value (possibly values) must be from the set $\{1, 2\}$ corresponding respectively to $\mu$ and $\sigma$ . By default all linear/additive predictors are modelled as a linear combination of the explanatory variables.

**Details**

The Gumbel distribution is a generalized extreme value (GEV) distribution with *shape* parameter  $\xi = 0$ . Consequently it is more easily estimated than the GEV. See [gev](#) for more details.

The quantity  $R$  is the maximum number of observations possible, for example, in the Venice data below, the top 10 daily values are recorded for each year, therefore  $R = 365$  because there are about 365 days per year. The MPV is the value of the response such that the probability of obtaining a value greater than the MPV is 0.5 out of  $R$  observations. For the Venice data, the MPV is the sea level such that there is an even chance that the highest level for a particular year exceeds the MPV. When `mpv = TRUE`, the column labelled "MPV" contains the MPVs when `fitted()` is applied to the fitted object.

The formula for the mean of a response  $Y$  is  $\mu + \sigma \times Euler$  where *Euler* is a constant that has value approximately equal to 0.5772. The formula for the percentiles are (if  $R$  is not given)  $\mu - \sigma \times \log[-\log(P/100)]$  where  $P$  is the percentile argument value(s). If  $R$  is given then the percentiles are  $\mu - \sigma \times \log[R(1 - P/100)]$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).



**Warning**

When `R` is not given (the default) the fitted percentiles are that of the data, and not of the overall population. For example, in the example below, the 50 percentile is approximately the running median through the data, however, the data are the highest sea level measurements recorded each year (it therefore equates to the median predicted value or MPV).

**Note**

`egumbel()` only handles a univariate response, and is preferred to `gumbel()` because it is faster. `gumbel()` can handle a multivariate response, i.e., a matrix with more than one column. Each row of the matrix is sorted into descending order. Missing values in the response are allowed but require `na.action = na.pass`. The response matrix needs to be padded with any missing values. With a multivariate response one has a matrix `y`, say, where `y[, 2]` contains the second order statistics etc.

**Author(s)**

T. W. Yee

**References**

- Yee, T. W. and Stephenson, A. G. (2007) Vector generalized linear and additive extreme value models. *Extremes*, **10**, 1–19.
- Smith, R. L. (1986) Extreme value theory based on the  $r$  largest annual events. *Journal of Hydrology*, **86**, 27–43.
- Rosen, O. and Cohen, A. (1996) Extreme percentile regression. In: Haerdle, W. and Schimek, M. G. (eds.), *Statistical Theory and Computational Aspects of Smoothing: Proceedings of the COMPSTAT '94 Satellite Meeting held in Semmering, Austria, 27–28 August 1994*, pp.200–214, Heidelberg: Physica-Verlag.
- Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

**See Also**

[rgumbel](#), [dgumbelII](#), [cgumbel](#), [guplot](#), [gev](#), [egev](#),  
[venice](#).

**Examples**

```
# Example 1: Simulated data
gdata <- data.frame(y = rgumbel(n = 1000, loc = 100, scale = exp(1)))
fit <- vglm(y ~ 1, egumbel(perc = NULL), gdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
with(gdata, mean(y))

# Example 2: Venice data
(fit <- vglm(cbind(r1,r2,r3,r4,r5) ~ year, data = venice,
```

```

      gumbel(R = 365, mpv = TRUE), trace = TRUE))
head(fitted(fit))
coef(fit, matrix = TRUE)
vcov(summary(fit))
sqrt(diag(vcov(summary(fit)))) # Standard errors

# Example 3: Try a nonparametric fit -----
# Use the entire data set, including missing values
y <- as.matrix(venice[, paste("r", 1:10, sep = "")])
fit1 <- vgam(y ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
            data = venice, trace = TRUE, na.action = na.pass)
depvar(fit1)[4:5, ] # NAs used to pad the matrix

## Not run:
# Plot the component functions
par(mfrow = c(2, 1), mar = c(5, 4, 0.2, 1) + 0.1, xpd = TRUE)
plot(fit1, se = TRUE, lcol = "blue", scol = "green", lty = 1,
     lwd = 2, slwd = 2, slty = "dashed")

# Quantile plot --- plots all the fitted values
par(mfrow = c(1, 1), bty = "l", mar = c(4, 4, 0.2, 3) + 0.1, xpd = TRUE, las = 1)
qtplot(fit1, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5), lwd = 2,
       pcol = "blue", tadj = 0.1, ylab = "Sea level (cm)")

# Plot the 99 percentile only
par(mfrow = c(1, 1), mar = c(3, 4, 0.2, 1) + 0.1, xpd = TRUE)
year = venice[["year"]]
matplot(year, y, ylab = "Sea level (cm)", type = "n")
matpoints(year, y, pch = "*", col = "blue")
lines(year, fitted(fit1)[,"99%"], lwd = 2, col = "red")

# Check the 99 percentiles with a smoothing spline.
# Nb. (1-0.99) * 365 = 3.65 is approx. 4, meaning the 4th order
# statistic is approximately the 99 percentile.
par(mfrow = c(1, 1), mar = c(3, 4, 2, 1) + 0.1, xpd = TRUE, lwd = 2)
plot(year, y[, 4], ylab = "Sea level (cm)", type = "n",
     main = "Red is 99 percentile, Green is a smoothing spline")
points(year, y[, 4], pch = "4", col = "blue")
lines(year, fitted(fit1)[,"99%"], lty = 1, col = "red")
lines(smooth.spline(year, y[, 4], df = 4), col = "darkgreen", lty = 2)

## End(Not run)

```

## Description

Density, cumulative distribution function, quantile function and random generation for the Gumbel-II distribution.

**Usage**

```

dgumbelII(x, shape, scale = 1, log = FALSE)
pgumbelII(q, shape, scale = 1)
qgumbelII(p, shape, scale = 1)
rgumbelII(n, shape, scale = 1)

```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations.
log	Logical. If log = TRUE then the logarithm of the density is returned.
shape, scale	positive shape and scale parameters.

**Details**

See [gumbelII](#) for details.

**Value**

dgumbelII gives the density, pgumbelII gives the cumulative distribution function, qgumbelII gives the quantile function, and rgumbelII generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[gumbelII](#), [dgumbel](#).

**Examples**

```

probs <- seq(0.01, 0.99, by = 0.01)
Shape <- exp( 0.5); Scale <- exp(1);
max(abs(pgumbelII(qgumbelII(p = probs, Shape, Scale),
                  Shape, Scale) - probs)) # Should be 0

## Not run: x <- seq(-0.1, 10, by = 0.01);
plot(x, dgumbelII(x, Shape, Scale), type = "l", col = "blue", las = 1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles",
     ylab = "", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pgumbelII(x, Shape, Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgumbelII(probs, Shape, Scale)
lines(Q, dgumbelII(Q, Shape, Scale), col = "purple", lty = 3, type = "h")
pgumbelII(Q, Shape, Scale) - probs # Should be all zero

```

```
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

gumbelII

*Gumbel-II Distribution Family Function*

## Description

Maximum likelihood estimation of the 2-parameter Gumbel-II distribution.

## Usage

```
gumbelII(lshape = "log", lscale = "log",
         ishape = NULL, iscale = NULL,
         probs.y = c(0.2, 0.5, 0.8),
         perc.out = NULL, imethod = 1, zero = -2)
```

## Arguments

`lshape`, `lscale` Parameter link functions applied to the (positive) shape parameter (called  $a$  below) and (positive) scale parameter (called  $b$  below). See [Links](#) for more choices.

`ishape`, `iscale` Optional initial values for the shape and scale parameters.

`imethod` See [weibull](#).

`zero`, `probs.y` Details at [CommonVGAMffArguments](#).

`perc.out` If the fitted values are to be quantiles then set this argument to be the percentiles of these, e.g., 50 for median.

## Details

The Gumbel-II density for a response  $Y$  is

$$f(y; a, b) = ay^{a-1} \exp[-(y/b)^a] / (b^a)$$

for  $a > 0$ ,  $b > 0$ ,  $y > 0$ . The cumulative distribution function is

$$F(y; a, b) = \exp[-(y/b)^a].$$

The mean of  $Y$  is  $b\Gamma(1 - 1/a)$  (returned as the fitted values) when  $a > 1$ , and the variance is  $b^2\Gamma(1 - 2/a)$  when  $a > 2$ . This distribution looks similar to [weibull](#), and is due to Gumbel (1954).

This **VGAM** family function currently does not handle censored data. Fisher scoring is used to estimate the two parameters. Probably similar regularity conditions hold for this distribution compared to the Weibull distribution.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

See [weibull](#). This **VGAM** family function handles multiple responses.

**Author(s)**

T. W. Yee

**References**

Gumbel, E. J. (1954). Statistical theory of extreme values and some practical applications. *Applied Mathematics Series*, volume 33, U.S. Department of Commerce, National Bureau of Standards, USA.

**See Also**

[dgumbelII](#), [gumbel](#), [gev](#).

**Examples**

```
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, eta1 = -1,
                  eta2 = -1 + 0.1 * x2,
                  ceta1 = 0,
                  ceta2 = 1)
gdata <- transform(gdata, shape1 = exp(eta1),
                  shape2 = exp(eta2),
                  scale1 = exp(ceta1),
                  scale2 = exp(ceta2))
gdata <- transform(gdata,
                  y1 = rgumbelII(nn, shape = shape1, scale = scale1),
                  y2 = rgumbelII(nn, shape = shape2, scale = scale2))

fit <- vglm(cbind(y1, y2) ~ x2,
           gumbelII(zero = c(1, 2, 4)), gdata, trace = TRUE)
coef(fit, matrix = TRUE)
vcov(fit)
summary(fit)
```

gumbelUC

*The Gumbel Distribution***Description**

Density, distribution function, quantile function and random generation for the Gumbel distribution with location parameter `location` and scale parameter `scale`.

**Usage**

```
dgumbel(x, location = 0, scale = 1, log = FALSE)
pgumbel(q, location = 0, scale = 1)
qgumbel(p, location = 0, scale = 1)
rgumbel(n, location = 0, scale = 1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>location</code>	the location parameter $\mu$ . This is not the mean of the Gumbel distribution (see <b>Details</b> below).
<code>scale</code>	the scale parameter $\sigma$ . This is not the standard deviation of the Gumbel distribution (see <b>Details</b> below).
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

The Gumbel distribution is a special case of the *generalized extreme value* (GEV) distribution where the shape parameter  $\xi = 0$ . The latter has 3 parameters, so the Gumbel distribution has two. The Gumbel distribution function is

$$G(y) = \exp \left( - \exp \left[ - \frac{y - \mu}{\sigma} \right] \right)$$

where  $-\infty < y < \infty$ ,  $-\infty < \mu < \infty$  and  $\sigma > 0$ . Its mean is

$$\mu - \sigma * \gamma$$

and its variance is

$$\sigma^2 * \pi^2 / 6$$

where  $\gamma$  is Euler's constant (which can be obtained as `-digamma(1)`).

See [gumbel](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for formulae and other details. Apart from `n`, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

**Value**

dgumbel gives the density, pgumbel gives the distribution function, qgumbel gives the quantile function, and rgumbel generates random deviates.

**Note**

The **VGAM** family function [gumbel](#) can estimate the parameters of a Gumbel distribution using maximum likelihood estimation.

**Author(s)**

T. W. Yee

**References**

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

**See Also**

[gumbel](#), [egumbel](#), [gev](#), [dgomptertz](#).

**Examples**

```
mu <- 1; sigma <- 2;
y <- rgumbel(n = 100, loc = mu, scale = sigma)
c(mean(y), mu - sigma * digamma(1)) # Sample and population means
c(var(y), sigma^2 * pi^2 / 6) # Sample and population variances

## Not run: x <- seq(-2.5, 3.5, by = 0.01)
loc <- 0; sigma <- 1
plot(x, dgumbel(x, loc, sigma), type = "l", col = "blue", ylim = c(0, 1),
     main = "Blue is density, red is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", ylab = "", las = 1)
abline(h = 0, col = "blue", lty = 2)
lines(qgumbel(seq(0.05, 0.95, by = 0.05), loc, sigma),
      dgumbel(qgumbel(seq(0.05, 0.95, by = 0.05), loc, sigma), loc, sigma),
      col = "purple", lty = 3, type = "h")
lines(x, pgumbel(x, loc, sigma), type = "l", col = "red")
abline(h = 0, lty = 2)
## End(Not run)
```

---

guplot

---

*Gumbel Plot*


---

**Description**

Produces a Gumbel plot, a diagnostic plot for checking whether the data appears to be from a Gumbel distribution.

**Usage**

```
guplot(object, ...)
guplot.default(y, main = "Gumbel Plot",
  xlab = "Reduced data", ylab = "Observed data", type = "p", ...)
guplot.vlm(object, ...)
```

**Arguments**

<code>y</code>	A numerical vector. NAs etc. are not allowed.
<code>main</code>	Character. Overall title for the plot.
<code>xlab</code>	Character. Title for the x axis.
<code>ylab</code>	Character. Title for the y axis.
<code>type</code>	Type of plot. The default means points are plotted.
<code>object</code>	An object that inherits class "vlm", usually of class <a href="#">vglm-class</a> or <a href="#">vgam-class</a> .
<code>...</code>	Graphical argument passed into <a href="#">plot</a> . See <a href="#">par</a> for an exhaustive list. The arguments <code>xlim</code> and <code>ylim</code> are particularly useful.

**Details**

If  $Y$  has a Gumbel distribution then plotting the sorted values  $y_i$  versus the *reduced values*  $r_i$  should appear linear. The reduced values are given by

$$r_i = -\log(-\log(p_i))$$

where  $p_i$  is the  $i$ th plotting position, taken here to be  $(i - 0.5)/n$ . Here,  $n$  is the number of observations. Curvature upwards/downwards may indicate a Frechet/Weibull distribution, respectively. Outliers may also be detected using this plot.

The function `guplot` is generic, and `guplot.default` and `guplot.vlm` are some methods functions for Gumbel plots.

**Value**

A list is returned invisibly with the following components.

<code>x</code>	The reduced data.
<code>y</code>	The sorted y data.

**Note**

The Gumbel distribution is a special case of the GEV distribution with shape parameter equal to zero.

**Author(s)**

T. W. Yee



## References

- Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.
- Gumbel, E. J. (1958) *Statistics of Extremes*. New York, USA: Columbia University Press.

## See Also

[gumbel](#), [egumbel](#), [gev](#), [venice](#).

## Examples

```
## Not run: guplot(rnorm(500), las = 1) -> ii
names(ii)

guplot(with(venice, r1), col = "blue") # Venice sea levels data

## End(Not run)
```

---

hatvalues

*Hat Values and Regression Deletion Diagnostics*


---

## Description

When complete, a suite of functions that can be used to compute some of the regression (leave-one-out deletion) diagnostics, for the VGLM class.

## Usage

```
hatvalues(model, ...)
hatvaluesvml(model, type = c("diagonal", "matrix", "centralBlocks"), ...)
hatplot(model, ...)
hatplot.vlm(model, multiplier = c(2, 3), lty = "dashed",
             xlab = "Observation", ylab = "Hat values", ylim = NULL, ...)
dfbetavlm(model, maxit.new = 1,
           trace.new = FALSE,
           smallno = 1.0e-8, ...)
```

## Arguments

- |            |  |
|------------|--|
| model      | an R object, typically returned by <a href="#">vglm</a> .  |
| type       | Character. The default is the first choice, which is a $nM \times nM$ matrix. If type = "matrix" then the <i>entire</i> hat matrix is returned. If type = "centralBlocks" then $n$ central $M \times M$ block matrices, in matrix-band format. |
| multiplier | Numeric, the multiplier. The usual rule-of-thumb is that values greater than two or three times the average leverage (at least for the linear model) should be checked.  |

lty, xlab, ylab, ylim  
 Graphical parameters, see [par](#) etc. The default of ylim is `c(0, max(hatvalues(model)))` which means that if the horizontal dashed lines cannot be seen then there are no particularly influential observations.

maxit.new, trace.new, smallno  
 Having `maxit.new = 1` will give a one IRLS step approximation from the ordinary solution (and no warnings!). Else having `maxit.new = 10`, say, should usually mean convergence will occur for all observations when they are removed one-at-a-time. Else having `maxit.new = 2`, say, should usually mean some lack of convergence will occur when observations are removed one-at-a-time. Setting `trace.new = TRUE` will produce some running output at each IRLS iteration and for each individual row of the model matrix. The argument `smallno` multiplies each value of the original prior weight (often unity); setting it identically to zero will result in an error, but setting a very small value effectively removes that observation.

... further arguments, for example, graphical parameters for `hatplot.vlm()`.

### Details

The invocation `hatvalues(vglmObject)` should return a  $n \times M$  matrix of the diagonal elements of the hat (projection) matrix of a [vglm](#) object. To do this, the QR decomposition of the object is retrieved or reconstructed, and then straightforward calculations are performed.

The invocation `hatplot(vglmObject)` should plot the diagonal of the hat matrix for each of the  $M$  linear/additive predictors. By default, two horizontal dashed lines are added; hat values higher than these ought to be checked.

### Note

It is hoped, soon, that the full suite of functions described at [influence.measures](#) will be written for VGLMs. This will enable general regression deletion diagnostics to be available for the entire VGLM class.

### Author(s)

T. W. Yee.

### See Also

[vglm](#), [cumulative](#), [influence.measures](#).

### Examples

```
# Proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, cumulative, data = pneumo)
hatvalues(fit) # n x M matrix, with positive values
all.equal(sum(hatvalues(fit)), fit@rank) # Should be TRUE
## Not run: par(mfrow = c(1, 2))
hatplot(fit, ylim = c(0, 1), las = 1, col = "blue")
## End(Not run)
```

---

hormone	<i>Hormone Assay Data</i>
---------	---------------------------

---

### Description

A hormone assay data set from Carroll and Ruppert (1988).

### Usage

```
data(hormone)
```

### Format

A data frame with 85 observations on the following 2 variables.

X a numeric vector, suitable as the x-axis in a scatter plot. The reference method.

Y a numeric vector, suitable as the y-axis in a scatter plot. The test method.

### Details

The data is given in Table 2.4 of Carroll and Ruppert (1988), and was downloaded from <http://www.stat.tamu.edu/~carroll>. The book describes the data as follows. The data are the results of two assay methods for hormone data; the scale of the data as presented is not particularly meaningful, and the original source of the data refused permission to divulge further information. As in a similar example of Leurgans (1980), the old or reference method is being used to predict the new or test method. The overall goal is to see whether we can reproduce the test-method measurements with the reference-method measurements. Thus calibration might be of interest for the data.

### References

Carroll, R. J. and Ruppert, D. (1988) *Transformation and Weighting in Regression*. New York, USA: Chapman & Hall.

Leurgans, S. (1980) Evaluating laboratory measurement techniques. *Biostatistics Casebook*. Eds.: Miller, R. G. Jr., and Efron, B. and Brown, B. W. Jr., and Moses, L. New York, USA: Wiley.

Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

### See Also

[uninormal](#), [rrvglm](#).

## Examples

```
## Not run:
data(hormone)
summary(hormone)

modelI <- rrvglm(Y ~ 1 + X, data = hormone, trace = TRUE,
                uninormal(zero = NULL, lsd = "identity", imethod = 2))

# Alternative way to fit modelI
modelI.other <- vglm(Y ~ 1 + X, data = hormone, trace = TRUE,
                   uninormal(zero = NULL, lsd = "identity"))

# Inferior to modelI
modelII <- vglm(Y ~ 1 + X, data = hormone, trace = TRUE,
               family = uninormal(zero = NULL))

logLik(modelI)
logLik(modelII) # Less than logLik(modelI)

# Reproduce the top 3 equations on p.65 of Carroll and Ruppert (1988).
# They are called Equations (1)--(3) here.

# Equation (1)
hormone <- transform(hormone, rX = 1 / X)
clist <- list("(Intercept)" = diag(2), X = diag(2), rX = rbind(0, 1))
fit1 <- vglm(Y ~ 1 + X + rX, family = uninormal(zero = NULL),
            constraints = clist, data = hormone, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1) # Actually, the intercepts do not seem significant
plot(Y ~ X, hormone, col = "blue")
lines(fitted(fit1) ~ X, hormone, col = "orange")

# Equation (2)
fit2 <- rrvglm(Y ~ 1 + X, uninormal(zero = NULL), hormone, trace = TRUE)
coef(fit2, matrix = TRUE)
plot(Y ~ X, hormone, col = "blue")
lines(fitted(fit2) ~ X, hormone, col = "red")
# Add +- 2 SEs
lines(fitted(fit2) + 2 * exp(predict(fit2)[, "log(sd)"]) ~ X,
      hormone, col = "orange")
lines(fitted(fit2) - 2 * exp(predict(fit2)[, "log(sd)"]) ~ X,
      hormone, col = "orange")

# Equation (3)
# Does not fit well because the loge link for the mean is not good.
fit3 <- rrvglm(Y ~ 1 + X, maxit = 300, data = hormone, trace = TRUE,
              uninormal(lmean = "loge", zero = NULL))
coef(fit3, matrix = TRUE)
plot(Y ~ X, hormone, col = "blue") # Does not look okay.
lines(exp(predict(fit3)[, 1]) ~ X, hormone, col = "red")
# Add +- 2 SEs
```

```

lines(fitted(fit3) + 2 * exp(predict(fit3)[, "log(sd)"]) ~ X,
      hormone, col = "orange")
lines(fitted(fit3) - 2 * exp(predict(fit3)[, "log(sd)"]) ~ X,
      hormone, col = "orange")

## End(Not run)

```

hspider

*Hunting Spider Data***Description**

Abundance of hunting spiders in a Dutch dune area.

**Usage**

```
data(hspider)
```

**Format**

A data frame with 28 observations (sites) on the following 18 variables.

**WaterCon** Log percentage of soil dry mass.

**BareSand** Log percentage cover of bare sand.

**FallTwig** Log percentage cover of fallen leaves and twigs.

**CoveMoss** Log percentage cover of the moss layer.

**CoveHerb** Log percentage cover of the herb layer.

**ReflLux** Reflection of the soil surface with cloudless sky.

**Alopacce** Abundance of *Alopecosa accentuata*.

**Alopcune** Abundance of *Alopecosa cuneata*.

**Alopfabr** Abundance of *Alopecosa fabrilis*.

**Arctlute** Abundance of *Arctosa lutetiana*.

**Arctperi** Abundance of *Arctosa perita*.

**Auloalbi** Abundance of *Aulonia albimana*.

**Pardlugu** Abundance of *Pardosa lugubris*.

**Pardmont** Abundance of *Pardosa monticola*.

**Pardnigr** Abundance of *Pardosa nigriceps*.

**Pardpull** Abundance of *Pardosa pullata*.

**Trocterr** Abundance of *Trochosa terricola*.

**Zoraspin** Abundance of *Zora spinimana*.

## Details

The data, which originally came from Van der Aart and Smeek-Enserink (1975) consists of abundances (numbers trapped over a 60 week period) and 6 environmental variables. There were 28 sites.

This data set has been often used to illustrate ordination, e.g., using canonical correspondence analysis (CCA). In the example below, the data is used for constrained quadratic ordination (CQO; formerly called canonical Gaussian ordination or CGO), a numerically intensive method that has many superior qualities. See [cqo](#) for details.

## References

Van der Aart, P. J. M. and Smeek-Enserink, N. (1975) Correlations between distributions of hunting spiders (Lycosidae, Ctenidae) and environmental characteristics in a dune area. *Netherlands Journal of Zoology*, **25**, 1–45.

## Examples

```
summary(hspider)

## Not run:
# Standardize the environmental variables:
hspider[, 1:6] <- scale(subset(hspider, select = WaterCon:ReflLux))

# Fit a rank-1 binomial CAO
hsbin <- hspider # Binary species data
hsbin[, -(1:6)] <- as.numeric(hsbin[, -(1:6)] > 0)
set.seed(123)
ahsb1 <- cao(cbind(Alopcune, Arctlute, Auloalbi, Zoraspin) ~
             WaterCon + ReflLux, family = binomialff(mv = TRUE),
             df1.nl = 2.2, Bestof = 3, data = hsbin)
par(mfrow = 2:1, las = 1)
lvplot(ahsb1, type = "predictors", llwd = 2, ylab = "logit p", lcol = 1:9)
persp(ahsb1, rug = TRUE, col = 1:10, lwd = 2)
coef(ahsb1)

## End(Not run)
```

---

huber2

*Huber's least favourable distribution family function*


---

## Description

M-estimation of the two parameters of Huber's least favourable distribution. The one parameter case is also implemented.

**Usage**

```
huber1(llocation = "identity", k = 0.862, imethod = 1)
huber2(llocation = "identity", lscale = "loge",
       k = 0.862, imethod = 1, zero = 2)
```

**Arguments**

<code>llocation</code> , <code>lscale</code>	Link functions applied to the location and scale parameters. See <a href="#">Links</a> for more choices.
<code>k</code>	Tuning constant. See <a href="#">rhuber</a> for more information.
<code>imethod</code> , <code>zero</code>	See <a href="#">CommonVGAMffArguments</a> for information. The default value of zero means the scale parameter is modelled as an intercept-only.

**Details**

Huber's least favourable distribution family function is popular for resistant/robust regression. The center of the distribution is normal and its tails are double exponential.

By default, the mean is the first linear/additive predictor (returned as the fitted values; this is the location parameter), and the log of the scale parameter is the second linear/additive predictor. The Fisher information matrix is diagonal; Fisher scoring is implemented.

The **VGAM** family function `huber1()` estimates only the location parameter. It assumes a scale parameter of unit value.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

Warning: actually, `huber2()` may be erroneous since the first derivative is not continuous when there are two parameters to estimate. `huber1()` is fine in this respect.

The response should be univariate.

**Author(s)**

T. W. Yee. Help was given by Arash Ardalan.

**References**

Huber, P. J. and Ronchetti, E. (2009) *Robust Statistics*, 2nd ed. New York: Wiley.

**See Also**

[rhuber](#), [uninormal](#), [gaussianff](#), [laplace](#), [CommonVGAMffArguments](#).

### Examples

```

set.seed(1231); NN <- 30; coef1 <- 1; coef2 <- 10
hdata <- data.frame(x2 = sort(runif(NN)))
hdata <- transform(hdata, y = rhuber(NN, mu = coef1 + coef2 * x2))

hdata$x2[1] <- 0.0 # Add an outlier
hdata$y[1] <- 10

fit.huber2 <- vglm(y ~ x2, huber2(imethod = 3), hdata, trace = TRUE)
fit.huber1 <- vglm(y ~ x2, huber1(imethod = 3), hdata, trace = TRUE)

coef(fit.huber2, matrix = TRUE)
summary(fit.huber2)

## Not run: # Plot the results
plot(y ~ x2, hdata, col = "blue", las = 1)
lines(fitted(fit.huber2) ~ x2, hdata, col = "darkgreen", lwd = 2)

fit.lm <- lm(y ~ x2, hdata) # Compare to a LM:
lines(fitted(fit.lm) ~ x2, hdata, col = "lavender", lwd = 3)

# Compare to truth:
lines(coef1 + coef2 * x2 ~ x2, hdata, col = "orange", lwd = 2, lty = "dashed")

legend("bottomright", legend = c("truth", "huber", "lm"),
      col = c("orange", "darkgreen", "lavender"),
      lty = c("dashed", "solid", "solid"), lwd = c(2, 2, 3))
## End(Not run)

```

---

Huggins89.t1

*Table 1 of Huggins (1989)*


---

### Description

Simulated capture data set for the linear logistic model depending on an occasion covariate and an individual covariate for 10 trapping occasions and 20 individuals.

### Usage

```

data(Huggins89table1)
data(Huggins89.t1)

```

### Format

The format is a data frame.



## Details

Table 1 of Huggins (1989) gives this toy data set. Note that variables  $t_1, \dots, t_{10}$  are occasion-specific variables. They correspond to the response variables  $y_1, \dots, y_{10}$  which have values 1 for capture and 0 for not captured.

Both Huggins89table1 and Huggins89.t1 are identical. The latter used variables beginning with  $z$ , not  $t$ , and may be withdrawn very soon.

## References

Huggins, R. M. (1989) On the statistical analysis of capture experiments. *Biometrika*, **76**, 133–140.

## Examples

```
Huggins89table1 <- transform(Huggins89table1, x3.tij = t1,
                             T2 = t2, T3 = t3, T4 = t4, T5 = t5, T6 = t6,
                             T7 = t7, T8 = t8, T9 = t9, T10 = t10)
small.table1 <- subset(Huggins89table1,
                      y1 + y2 + y3 + y4 + y5 + y6 + y7 + y8 + y9 + y10 > 0)
# fit.tbh is the bottom equation on p.133.
# It is a M_tbh model.
fit.tbh <-
  vglm(cbind(y1, y2, y3, y4, y5, y6, y7, y8, y9, y10) ~ x2 + x3.tij,
       xij = list(x3.tij ~ t1 + t2 + t3 + t4 + t5 + t6 + t7 + t8 + t9 + t10 +
                  T2 + T3 + T4 + T5 + T6 + T7 + T8 + T9 + T10 - 1),
       posbernoulli.tb(parallel.t = TRUE ~ x2 + x3.tij),
       data = small.table1, trace = TRUE,
       form2 = ~ x2 + x3.tij +
                t1 + t2 + t3 + t4 + t5 + t6 + t7 + t8 + t9 + t10 +
                T2 + T3 + T4 + T5 + T6 + T7 + T8 + T9 + T10)

# These results differ a bit from Huggins (1989), probably because
# two animals had to be removed here (they were never caught):
coef(fit.tbh) # First element is the behavioural effect
sqrt(diag(vcov(fit.tbh))) # SEs
constraints(fit.tbh, matrix = TRUE)
summary(fit.tbh, presid = FALSE)
fit.tbh@extra$N.hat # Estimate of the population size N; cf. 20.86
fit.tbh@extra$SE.N.hat # Its standard error; cf. 1.87 or 4.51

fit.th <- vglm(cbind(y1, y2, y3, y4, y5, y6, y7, y8, y9, y10) ~ x2,
              posbernoulli.t, data = small.table1, trace = TRUE)
coef(fit.th)
constraints(fit.th)
coef(fit.th, matrix = TRUE) # M_th model
summary(fit.th, presid = FALSE)
fit.th@extra$N.hat # Estimate of the population size N
fit.th@extra$SE.N.hat # Its standard error

fit.bh <- vglm(cbind(y1, y2, y3, y4, y5, y6, y7, y8, y9, y10) ~ x2,
              posbernoulli.b(I2 = FALSE), data = small.table1, trace = TRUE)
coef(fit.bh)
```

```

constraints(fit.bh)
coef(fit.bh, matrix = TRUE) # M_bh model
summary(fit.bh, presid = FALSE)
fit.bh@extra$N.hat
fit.bh@extra$SE.N.hat

fit.h <- vglm(cbind(y1, y2, y3, y4, y5, y6, y7, y8, y9, y10) ~ x2,
              posbernoulli.b, data = small.table1, trace = TRUE)
coef(fit.h, matrix = TRUE) # M_h model (version 1)
coef(fit.h)
summary(fit.h, presid = FALSE)
fit.h@extra$N.hat
fit.h@extra$SE.N.hat

Fit.h <- vglm(cbind(y1, y2, y3, y4, y5, y6, y7, y8, y9, y10) ~ x2,
              posbernoulli.t(parallel.t = TRUE ~ x2),
              data = small.table1, trace = TRUE)
coef(Fit.h)
coef(Fit.h, matrix = TRUE) # M_h model (version 2)
summary(Fit.h, presid = FALSE)
Fit.h@extra$N.hat
Fit.h@extra$SE.N.hat

```

---

hunua

*Hunua Ranges Data*


---

## Description

The hunua data frame has 392 rows and 18 columns. Altitude is explanatory, and there are binary responses (presence/absence = 1/0 respectively) for 17 plant species.

## Usage

```
data(hunua)
```

## Format

This data frame contains the following columns:

**agaaus** Agathis australis, or Kauri  
**beitaw** Beilschmiedia tawa, or Tawa  
**corlae** Corynocarpus laevigatus  
**cyadea** Cyathea dealbata  
**cyamed** Cyathea medullaris  
**daccup** Dacrydium cupressinum  
**dacdac** Dacrycarpus dacrydioides  
**eladen** Elaeocarpus dentatus

**hedarb** Hedycarya arborea  
**hohpop** Species name unknown  
**kniexc** Knightia excelsa, or Rewarewa  
**kuneri** Kunzea ericoides  
**lepsco** Leptospermum scoparium  
**metro** Metrosideros robusta  
**neslan** Nestegis lanceolata  
**rhosap** Rhopalostylis sapida  
**vitluc** Vitex lucens, or Puriri  
**altitude** meters above sea level

### Details

These were collected from the Hunua Ranges, a small forest in southern Auckland, New Zealand. At 392 sites in the forest, the presence/absence of 17 plant species was recorded, as well as the altitude. Each site was of area size  $200m^2$ .

### Source

Dr Neil Mitchell, University of Auckland.

### See Also

[waitakere](#).

### Examples

```

# Fit a GAM using vgam() and compare it with the Waitakere Ranges one
fit.h <- vgam(agaas ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run:
plot(fit.h, se = TRUE, lcol = "orange", scol = "orange",
     llwd = 2, slwd = 2, main = "Orange is Hunua, Blue is Waitakere")
## End(Not run)
head(predict(fit.h, hunua, type = "response"))

fit.w <- vgam(agaas ~ s(altitude, df = 2), binomialff, data = waitakere)
## Not run:
plot(fit.w, se = TRUE, lcol = "blue", scol = "blue", add = TRUE)
## End(Not run)
head(predict(fit.w, hunua, type = "response")) # Same as above?

```

hyperg

*Hypergeometric Family Function***Description**

Family function for a hypergeometric distribution where either the number of white balls or the total number of white and black balls are unknown.

**Usage**

```
hyperg(N = NULL, D = NULL, lprob = "logit", iprob = NULL)
```

**Arguments**

N	Total number of white and black balls in the urn. Must be a vector with positive values, and is recycled, if necessary, to the same length as the response. One of N and D must be specified.
D	Number of white balls in the urn. Must be a vector with positive values, and is recycled, if necessary, to the same length as the response. One of N and D must be specified.
lprob	Link function for the probabilities. See <a href="#">Links</a> for more choices.
iprob	Optional initial value for the probabilities. The default is to choose initial values internally.

**Details**

Consider the scenario from [dhyper](#) where there are  $N = m + n$  balls in an urn, where  $m$  are white and  $n$  are black. A simple random sample (i.e., *without* replacement) of  $k$  balls is taken. The response here is the sample *proportion* of white balls. In this document, N is  $N = m + n$ , D is  $m$  (for the number of “defectives”, in quality control terminology, or equivalently, the number of marked individuals). The parameter to be estimated is the population proportion of white balls, viz.  $prob = m/(m + n)$ .

Depending on which one of N and D is inputted, the estimate of the other parameter can be obtained from the equation  $prob = m/(m + n)$ , or equivalently,  $prob = D/N$ . However, the log-factorials are computed using [lgamma](#) and both  $m$  and  $n$  are not restricted to being integer. Thus if an integer  $N$  is to be estimated, it will be necessary to evaluate the likelihood function at integer values about the estimate, i.e., at `trunc(Nhat)` and `ceiling(Nhat)` where Nhat is the (real) estimate of  $N$ .

**Value**

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), and [cao](#).

**Warning**

No checking is done to ensure that certain values are within range, e.g.,  $k \leq N$ .

**Note**

The response can be of one of three formats: a factor (first level taken as success), a vector of proportions of success, or a 2-column matrix (first column = successes) of counts. The argument weights in the modelling function can also be specified. In particular, for a general vector of proportions, you will need to specify weights because the number of trials is needed.

**Author(s)**

Thomas W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[dhyper](#), [binomialff](#).

**Examples**

```
nn <- 100
m <- 5 # Number of white balls in the population
k <- rep(4, len = nn) # Sample sizes
n <- 4 # Number of black balls in the population
y <- rhyper(nn = nn, m = m, n = n, k = k)
yprop <- y / k # Sample proportions

# N is unknown, D is known. Both models are equivalent:
fit <- vglm(cbind(y,k-y) ~ 1, hyperg(D = m), trace = TRUE, crit = "c")
fit <- vglm(yprop ~ 1, hyperg(D = m), weight = k, trace = TRUE, crit = "c")

# N is known, D is unknown. Both models are equivalent:
fit <- vglm(cbind(y, k-y) ~ 1, hyperg(N = m+n), trace = TRUE, crit = "l")
fit <- vglm(yprop ~ 1, hyperg(N = m+n), weight = k, trace = TRUE, crit = "l")

coef(fit, matrix = TRUE)
Coef(fit) # Should be equal to the true population proportion
unique(m / (m+n)) # The true population proportion
fit@extra
head(fitted(fit))
summary(fit)
```

**Description**

Estimation of the parameter of the hyperbolic secant distribution.

**Usage**

```
hypersecant(link.theta = elogit(min = -pi/2, max = pi/2), init.theta = NULL)
hypersecant.1(link.theta = elogit(min = -pi/2, max = pi/2), init.theta = NULL)
```

**Arguments**

<code>link.theta</code>	Parameter link function applied to the parameter $\theta$ . See <a href="#">Links</a> for more choices.
<code>init.theta</code>	Optional initial value for $\theta$ . If failure to converge occurs, try some other value. The default means an initial value is determined internally.

**Details**

The probability density function of the hyperbolic secant distribution is given by

$$f(y; \theta) = \exp(\theta y + \log(\cos(\theta))) / (2 \cosh(\pi y/2)),$$

for parameter  $-\pi/2 < \theta < \pi/2$  and all real  $y$ . The mean of  $Y$  is  $\tan(\theta)$  (returned as the fitted values). Morris (1982) calls this model NEF-HS (Natural Exponential Family-Hyperbolic Secant). It is used to generate NEFs, giving rise to the class of NEF-GHS (G for Generalized).

Another parameterization is used for `hypersecant.1()`: let  $Y = (\text{logit}U)/\pi$ . Then this uses

$$f(u; \theta) = (\cos(\theta)/\pi) \times u^{-0.5+\theta/\pi} \times (1-u)^{-0.5-\theta/\pi},$$

for parameter  $-\pi/2 < \theta < \pi/2$  and  $0 < u < 1$ . Then the mean of  $U$  is  $0.5 + \theta/\pi$  (returned as the fitted values) and the variance is  $(\pi^2 - 4\theta^2)/(8\pi^2)$ .

For both parameterizations Newton-Raphson is same as Fisher scoring.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Author(s)**

T. W. Yee

**References**

- Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall.
- Morris, C. N. (1982) Natural exponential families with quadratic variance functions. *The Annals of Statistics*, **10**(1), 65–80.

**See Also**

[elogit](#).

## Examples

```
hdata <- data.frame(x2 = rnorm(nn <- 200))
hdata <- transform(hdata, y = rnorm(nn)) # Not very good data!
fit <- vglm(y ~ x2, hypersecant, hdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
fit@misc$earg

# Not recommended:
fit <- vglm(y ~ x2, hypersecant(link = "identity"), hdata, trace = TRUE)
coef(fit, matrix = TRUE)
fit@misc$earg
```

---

 Hzeta

*Haight's Zeta Distribution*


---

## Description

Density, distribution function, quantile function and random generation for Haight's zeta distribution with parameter alpha.

## Usage

```
dhzeta(x, alpha, log = FALSE)
phzeta(q, alpha)
qhzeta(p, alpha)
rhzeta(n, alpha)
```

## Arguments

x, q	Vector of quantiles. For the density, it should be a vector with positive integer values in order for the probabilities to be positive.
p	vector of probabilities.
n	number of observations. Same as <a href="#">runif</a> .
alpha	The parameter value. Must contain positive values and is recycled to the length of x or p or q if necessary.
log	Logical. If log = TRUE then the logarithm of the density is returned.

## Details

The probability function is

$$f(x) = (2x - 1)^{(-\alpha)} - (2x + 1)^{(-\alpha)},$$

where  $\alpha > 0$  and  $x = 1, 2, \dots$

**Value**

dhzeta gives the density, phzeta gives the distribution function, qhzeta gives the quantile function, and rhzeta generates random deviates.

**Note**

Given some response data, the **VGAM** family function [hzeta](#) estimates the parameter alpha.

**Author(s)**

T. W. Yee

**References**

Page 533 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey: Wiley.

**See Also**

[hzeta](#), [zeta](#), [zetaff](#).

**Examples**

```
dhzeta(1:20, 2.1)
rhzeta(20, 2.1)

round(1000 * dhzeta(1:8, 2))
table(rhzeta(1000, 2))

## Not run:  alpha <- 1.1; x <- 1:10
plot(x, dhzeta(x, alpha = alpha), type = "h", ylim = 0:1, lwd = 2,
     sub = paste("alpha =", alpha), las = 1, col = "blue", ylab = "Probability",
     main = "Haight's zeta: blue = density; orange = distribution function")
lines(x+0.1, phzeta(x, alpha = alpha), col = "orange", lty = 3, lwd = 2, type = "h")

## End(Not run)
```

---



*Haight's Zeta Family Function*


---

**Description**

Estimating the parameter of Haight's zeta distribution

**Usage**

```
hzeta(link = "loglog", ialpha = NULL, nsimEIM = 100)
```



**Arguments**

link	Parameter link function for the parameter. See <a href="#">Links</a> for more choices. Here, a log-log link keeps the parameter greater than one, meaning the mean is finite.
ialpha	Optional initial value for the (positive) parameter. The default is to obtain an initial value internally. Use this argument if the default fails.
nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information.

**Details**

The probability function is

$$f(y) = (2y - 1)^{(-\alpha)} - (2y + 1)^{(-\alpha)},$$

where the parameter  $\alpha > 0$  and  $y = 1, 2, \dots$ . The function [dhzeta](#) computes this probability function. The mean of  $Y$ , which is returned as fitted values, is  $(1 - 2^{-\alpha})\zeta(\alpha)$  provided  $\alpha > 1$ , where  $\zeta$  is Riemann's zeta function. The mean is a decreasing function of  $\alpha$ . The mean is infinite if  $\alpha \leq 1$ , and the variance is infinite if  $\alpha \leq 2$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

T. W. Yee

**References**

Page 533 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey: Wiley.

**See Also**

[Hzeta](#), [zeta](#), [zetaff](#), [loglog](#).

**Examples**

```
alpha <- exp(exp(-0.1)) # The parameter
hdata <- data.frame(y = rhzeta(n = 1000, alpha))
fit <- vglm(y ~ 1, hzeta, hdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit) # Useful for intercept-only models; should be same as alpha
c(with(hdata, mean(y)), head(fitted(fit), 1))
summary(fit)
```

iam

*Index from Array to Matrix***Description**

Maps the elements of an array containing symmetric positive-definite matrices to a matrix with sufficient columns to hold them (called matrix-band format.)

**Usage**

```
iam(j, k, M, both = FALSE, diag = TRUE)
```

**Arguments**

j	An integer from the set $\{1:M\}$ giving the row number of an element.
k	An integer from the set $\{1:M\}$ giving the column number of an element.
M	The number of linear/additive predictors. This is the dimension of each positive-definite symmetric matrix.
both	Logical. Return both the row and column indices? See below for more details.
diag	Logical. Return the indices for the diagonal elements? If FALSE then only the strictly upper triangular part of the matrix elements are used.

**Details**

Suppose we have  $n$  symmetric positive-definite square matrices, each  $M$  by  $M$ , and these are stored in an array of dimension  $c(n,M,M)$ . Then these can be more compactly represented by a matrix of dimension  $c(n,K)$  where  $K$  is an integer between  $M$  and  $M*(M+1)/2$  inclusive. The mapping between these two representations is given by this function. It firstly enumerates by the diagonal elements, followed by the band immediately above the diagonal, then the band above that one, etc. The last element is  $(1,M)$ . This function performs the mapping from elements  $(j,k)$  of symmetric positive-definite square matrices to the columns of another matrix representing such. This is called the *matrix-band* format and is used by the **VGAM** package.

**Value**

This function has a dual purpose depending on the value of both. If both=FALSE then the column number corresponding to the  $j$ - $k$  element of the matrix is returned. If both = TRUE then  $j$  and  $k$  are ignored and a list with the following components are returned.

row.index	The row indices of the upper triangular part of the matrix (This may or may not include the diagonal elements, depending on the argument diagonal).
col.index	The column indices of the upper triangular part of the matrix (This may or may not include the diagonal elements, depending on the argument diagonal).

**Note**

This function is used in the weight slot of many **VGAM** family functions (see [vglmff-class](#)), especially those whose  $M$  is determined by the data, e.g., [dirichlet](#), [multinomial](#).

**Author(s)**

T. W. Yee

**References**

The website <http://www.stat.auckland.ac.nz/~yee> contains some additional information.

**See Also**

[vglmff-class](#).

**Examples**

```
iam(1, 2, M = 3) # The 4th column represents element (1,2) of a 3x3 matrix
iam(NULL, NULL, M = 3, both = TRUE) # Return the row and column indices

dirichlet()@weight

M <- 4
temp1 <- iam(NA, NA, M = M, both = TRUE)
mat1 <- matrix(NA, M, M)
mat1[cbind(temp1$row, temp1$col)] = 1:length(temp1$row)
mat1 # More commonly used

temp2 <- iam(NA, NA, M = M, both = TRUE, diag = FALSE)
mat2 <- matrix(NA, M, M)
mat2[cbind(temp2$row, temp2$col)] = 1:length(temp2$row)
mat2 # Rarely used
```

---

identity

---

*Identity Link Function*


---

**Description**

Computes the identity transformation, including its inverse and the first two derivatives.

**Usage**

```
identity(theta, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
negidentity(theta, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

## Arguments

theta                      Numeric or character. See below for further details.  
 inverse, deriv, short, tag  
                               Details at [Links](#).

## Details

The identity link function  $g(\theta) = \theta$  should be available to every parameter estimated by the **VGAM** library. However, it usually results in numerical problems because the estimates lie outside the permitted range. Consequently, the result may contain Inf, -Inf, NA or NaN.

The function `negidentity` is the negative-identity link function and corresponds to  $g(\theta) = -\theta$ . This is useful for some models, e.g., in the literature supporting the [egev](#) function it seems that half of the authors use  $\xi = -k$  for the shape parameter and the other half use  $k$  instead of  $\xi$ .

## Value

For `identity()`: for `deriv = 0`, the identity of theta, i.e., theta when `inverse = FALSE`, and if `inverse = TRUE` then theta. For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

For `negidentity()`: the results are similar to `identity()` except for a sign change in most cases.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[Links](#), [loge](#), [logit](#), [probit](#), [powerlink](#).

## Examples

```
identity((-5):5)
identity((-5):5, deriv = 1)
identity((-5):5, deriv = 2)
negidentity((-5):5)
negidentity((-5):5, deriv = 1)
negidentity((-5):5, deriv = 2)
```

Inv.gaussian

*The Inverse Gaussian Distribution***Description**

Density, distribution function and random generation for the inverse Gaussian distribution.

**Usage**

```
dinv.gaussian(x, mu, lambda, log = FALSE)
pinv.gaussian(q, mu, lambda)
rinv.gaussian(n, mu, lambda)
```

**Arguments**

x, q	vector of quantiles.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
mu	the mean parameter.
lambda	the $\lambda$ parameter.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [inv.gaussianff](#), the **VGAM** family function for estimating both parameters by maximum likelihood estimation, for the formula of the probability density function.

**Value**

`dinv.gaussian` gives the density, `pinv.gaussian` gives the distribution function, and `rinv.gaussian` generates random deviates.

**Note**

Currently `qinv.gaussian` is unavailable.

**Author(s)**

T. W. Yee

**References**

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley.

Taraldsen, G. and Lindqvist, B. H. (2005) The multiple roots simulation algorithm, the inverse Gaussian distribution, and the sufficient conditional Monte Carlo method. *Preprint Statistics No. 4/2005*, Norwegian University of Science and Technology, Trondheim, Norway.

**See Also**

[inv.gaussianff](#), [waldff](#).

**Examples**

```
## Not run: x <- seq(-0.05, 4, len = 300)
plot(x, dinv.gaussian(x, mu = 1, lambda = 1), type = "l",
     col = "blue", las = 1, main =
       "blue is density, orange is cumulative distribution function")
abline(h = 0, col = "gray", lty = 2)
lines(x, pinv.gaussian(x, mu = 1, lambda = 1), type = "l", col = "orange")
## End(Not run)
```

---

inv.gaussianff

*Inverse Gaussian Distribution Family Function*

---

**Description**

Estimates the two parameters of the inverse Gaussian distribution by maximum likelihood estimation.

**Usage**

```
inv.gaussianff(lmu = "loge", llambda = "loge",
               imethod = 1, ilambda = NULL,
               parallel = FALSE, shrinkage.init = 0.99, zero = NULL)
```

**Arguments**

`lmu`, `llambda`    Parameter link functions for the  $\mu$  and  $\lambda$  parameters. See [Links](#) for more choices.

`ilambda`, `parallel`    See [CommonVGAMffArguments](#) for more information. If `parallel = TRUE` then the constraint is not applied to the intercept.

`imethod`, `shrinkage.init`, `zero`    See [CommonVGAMffArguments](#) for more information.

**Details**

The standard (“canonical”) form of the inverse Gaussian distribution has a density that can be written as

$$f(y; \mu, \lambda) = \sqrt{\lambda / (2\pi y^3)} \exp(-\lambda(y - \mu)^2 / (2\mu^2 y))$$

where  $y > 0$ ,  $\mu > 0$ , and  $\lambda > 0$ . The mean of  $Y$  is  $\mu$  and its variance is  $\mu^3/\lambda$ . By default,  $\eta_1 = \log(\mu)$  and  $\eta_2 = \log(\lambda)$ . The mean is returned as the fitted values. This **VGAM** family function can handle multiple responses (inputted as a matrix).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Note**

The inverse Gaussian distribution can be fitted (to a certain extent) using the usual GLM framework involving a scale parameter. This family function is different from that approach in that it estimates both parameters by full maximum likelihood estimation.

**Author(s)**

T. W. Yee

**References**

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[Inv.gaussian](#), [waldff](#), [bisa](#).

The R package **SuppDists** has several functions for evaluating the density, distribution function, quantile function and generating random numbers from the inverse Gaussian distribution.

**Examples**

```
idata <- data.frame(x2 = runif(nn <- 1000))
idata <- transform(idata, mymu = exp(2 + 1 * x2),
                  Lambda = exp(2 + 1 * x2))
idata <- transform(idata, y = rinv.gaussian(nn, mu = mymu, lambda = Lambda))
fit1 <- vglm(y ~ x2, inv.gaussianff, idata, trace = TRUE)
rrig <- rrvglm(y ~ x2, inv.gaussianff, idata, trace = TRUE)
coef(fit1, matrix = TRUE)
coef(rrig, matrix = TRUE)
Coef(rrig)
summary(fit1)
```

---

invbinomial

---

Inverse Binomial Distribution Family Function

---

**Description**

Estimates the two parameters of an inverse binomial distribution by maximum likelihood estimation.

**Usage**

```
invbinomial(lrho = elogit(min = 0.5, max = 1),
            llambda = "loge", irho = NULL, ilambda = NULL, zero = NULL)
```

**Arguments**

`lrho`, `llambda` Link function for the  $\rho$  and  $\lambda$  parameters. See [Links](#) for more choices.

`irho`, `ilambda` Numeric. Optional initial values for  $\rho$  and  $\lambda$ .

`zero` See [CommonVGAMffArguments](#).

**Details**

The inverse binomial distribution of Yanagimoto (1989) has density function

$$f(y; \rho, \lambda) = \frac{\lambda \Gamma(2y + \lambda)}{\Gamma(y + 1) \Gamma(y + \lambda + 1)} \{\rho(1 - \rho)\}^y \rho^\lambda$$

where  $y = 0, 1, 2, \dots$  and  $\frac{1}{2} < \rho < 1$ , and  $\lambda > 0$ . The first two moments exist for  $\rho > \frac{1}{2}$ ; then the mean is  $\lambda(1 - \rho)/(2\rho - 1)$  (returned as the fitted values) and the variance is  $\lambda\rho(1 - \rho)/(2\rho - 1)^3$ . The inverse binomial distribution is a special case of the generalized negative binomial distribution of Jain and Consul (1971). It holds that  $Var(Y) > E(Y)$  so that the inverse binomial distribution is overdispersed compared with the Poisson distribution.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

This **VGAM** family function only works reasonably well with intercept-only models. Good initial values are needed; if convergence failure occurs use `irho` and/or `ilambda`.

Some elements of the working weight matrices use the expected information matrix while other elements use the observed information matrix. Yet to do: using the mean and the reciprocal of  $\lambda$  results in a EIM that is diagonal.

**Author(s)**

T. W. Yee

**References**

- Yanagimoto, T. (1989) The inverse binomial distribution as a statistical model. *Communications in Statistics: Theory and Methods*, **18**, 3625–3633.
- Jain, G. C. and Consul, P. C. (1971) A generalized negative binomial distribution. *SIAM Journal on Applied Mathematics*, **21**, 501–513.
- Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall



**See Also**

[negbinomial](#), [poissonff](#).

**Examples**

```
idata <- data.frame(y = rnbino(n <- 1000, mu = exp(3), size = exp(1)))
fit <- vglm(y ~ 1, invbinomial, idata, trace = TRUE)
with(idata, c(mean(y), head(fitted(fit), 1)))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
sum(weights(fit)) # Sum of the prior weights
sum(weights(fit, type = "work")) # Sum of the working weights
```

---

Invlomax

---

*The Inverse Lomax Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the inverse Lomax distribution with shape parameter  $p$  and scale parameter  $scale$ .

**Usage**

```
dinvlomax(x, scale = 1, shape2.p, log = FALSE)
pinvlomax(q, scale = 1, shape2.p)
qinvlomax(p, scale = 1, shape2.p)
rinvlomax(n, scale = 1, shape2.p)
```

**Arguments**

$x$ , $q$	vector of quantiles.
$p$	vector of probabilities.
$n$	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
$shape2.p$	shape parameter.
$scale$	scale parameter.
$log$	Logical. If $log = TRUE$ then the logarithm of the density is returned.

**Details**

See [invlomax](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

**Value**

dinvlomax gives the density, pinvlomax gives the distribution function, qinvlomax gives the quantile function, and rinvlomax generates random deviates.

**Note**

The inverse Lomax distribution is a special case of the 4-parameter generalized beta II distribution.

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[invlomax](#), [genbetaII](#).

**Examples**

```
idata <- data.frame(y = rinvlomax(n = 1000, exp(2), exp(1)))
fit <- vglm(y ~ 1, invlomax, idata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

---

invlomax	<i>Inverse Lomax Distribution Family Function</i>
----------	---

---

**Description**

Maximum likelihood estimation of the 2-parameter inverse Lomax distribution.

**Usage**

```
invlomax(lscale = "loge", lshape2.p = "loge",
         iscale = NULL, ishape2.p = 1, zero = NULL)
```

**Arguments**

- lscale, lshape2.p  
Parameter link functions applied to the (positive) scale parameter scale and (positive) shape parameter p. See [Links](#) for more choices.
- iscale, ishape2.p  
Optional initial values for scale and p.

**zero** An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2} which correspond to scale, p, respectively.

### Details

The 2-parameter inverse Lomax distribution is the 4-parameter generalized beta II distribution with shape parameters  $a = q = 1$ . It is also the 3-parameter Dagum distribution with shape parameter  $a = 1$ , as well as the beta distribution of the second kind with  $q = 1$ . More details can be found in Kleiber and Kotz (2003).

The inverse Lomax distribution has density

$$f(y) = py^{p-1} / [b^p \{1 + y/b\}^{p+1}]$$

for  $b > 0$ ,  $p > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter scale, and  $p$  is a shape parameter. The mean does not exist; NAs are returned as the fitted values.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

See the note in [genbetaII](#).

### Author(s)

T. W. Yee

### References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

### See Also

[Invlomax](#), [genbetaII](#), [betaII](#), [dagum](#), [sinmad](#), [fisk](#), [lomax](#), [paralogistic](#), [invparalogistic](#).

### Examples

```
idata <- data.frame(y = rinvlomax(n = 2000, exp(2), exp(1)))
fit <- vglm(y ~ 1, invlomax, idata, trace = TRUE)
fit <- vglm(y ~ 1, invlomax(yscale = exp(2), ishape2.p = exp(1)), idata,
            trace = TRUE, epsilon = 1e-8)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

Invparalogistic

---

*The Inverse Paralogistic Distribution*

---

**Description**

Density, distribution function, quantile function and random generation for the inverse paralogistic distribution with shape parameters  $a$  and  $p$ , and scale parameter  $scale$ .

**Usage**

```
dinvparalogistic(x, shape1.a, scale = 1, log = FALSE)
pinvparalogistic(q, shape1.a, scale = 1)
qinvparalogistic(p, shape1.a, scale = 1)
rinvparalogistic(n, shape1.a, scale = 1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>shape1.a</code>	shape parameter.
<code>scale</code>	scale parameter.
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [invparalogistic](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

**Value**

`dinvparalogistic` gives the density, `pinvparalogistic` gives the distribution function, `qinvparalogistic` gives the quantile function, and `rinvparalogistic` generates random deviates.

**Note**

The inverse paralogistic distribution is a special case of the 4-parameter generalized beta II distribution.

**Author(s)**

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

[invparalogistic](#), [genbetaII](#).

## Examples

```
idata <- data.frame(y = rinvparalogistic(n = 3000, exp(1), exp(2)))
fit <- vglm(y ~ 1, invparalogistic(ishape1.a = 2.1),
            idata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

---

invparalogistic

*Inverse Paralogistic Distribution Family Function*

---

## Description

Maximum likelihood estimation of the 2-parameter inverse paralogistic distribution.

## Usage

```
invparalogistic(lshape1.a = "loge", lscale = "loge",
                ishape1.a = 2, iscale = NULL, zero = NULL)
```

## Arguments

lshape1.a, lscale	Parameter link functions applied to the (positive) shape parameter $a$ and (positive) scale parameter scale. See <a href="#">Links</a> for more choices.
ishape1.a, iscale	Optional initial values for $a$ and scale.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set $\{1,2\}$ which correspond to $a$ , scale, respectively.

## Details

The 2-parameter inverse paralogistic distribution is the 4-parameter generalized beta II distribution with shape parameter  $q = 1$  and  $a = p$ . It is the 3-parameter Dagum distribution with  $a = p$ . More details can be found in Kleiber and Kotz (2003).

The inverse paralogistic distribution has density

$$f(y) = a^2 y^{a^2-1} / [b^{a^2} \{1 + (y/b)^a\}^{a+1}]$$

for  $a > 0$ ,  $b > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter scale, and  $a$  is the shape parameter. The mean is

$$E(Y) = b \Gamma(a + 1/a) \Gamma(1 - 1/a) / \Gamma(a)$$

provided  $a > 1$ ; these are returned as the fitted values.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

See the note in [genbetaII](#).

### Author(s)

T. W. Yee

### References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

### See Also

[Invparalogistic](#), [genbetaII](#), [betaII](#), [dagum](#), [sinmad](#), [fisk](#), [invlomax](#), [lomax](#), [paralogistic](#).

### Examples

```
idata <- data.frame(y = rinvparalogistic(n = 3000, exp(1), exp(2)))
fit <- vglm(y ~ 1, invparalogistic, idata, trace = TRUE)
fit <- vglm(y ~ 1, invparalogistic(ishape1.a = 2.7, iscale = 7.3),
            idata, trace = TRUE, epsilon = 1e-8)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

is.parallel

Parallelism Constraint Matrices

---

### Description

Returns a logical vector from a test of whether an object such as a matrix or VGLM object corresponds to a parallelism assumption.

### Usage

```
is.parallel.matrix(object, ...)
is.parallel.vglm(object, type = c("term", "lm"), ...)
```

**Arguments**

object	an object such as a constraint matrix or a <a href="#">vglm</a> object.
type	passed into <a href="#">constraints</a> .
...	additional optional arguments. Currently unused.

**Details**

These functions may be useful for categorical models such as [propodds](#), [cumulative](#), [acat](#), [cratio](#), [sratio](#), [multinomial](#).

**Value**

A vector of logicals, testing whether each constraint matrix is a one-column matrix of ones. Note that parallelism can still be thought of as holding if the constraint matrix has a non-zero but constant values, however, this is currently not implemented. No checking is done that the constraint matrices have the same number of rows.

**See Also**

[constraints](#), [vglm](#).

**Examples**

```
## Not run: require(VGAMdata)
fit <- vglm(educ ~ bs(age) * sex + ethnic,
           cumulative(parallel = TRUE), xs.nz[1:200, ])
is.parallel(fit)
is.parallel(fit, type = "lm") # For each column of the LM matrix

## End(Not run)
```

---

is.smart

*Test For a Smart Object*


---

**Description**

Tests an object to see if it is smart.

**Usage**

```
is.smart(object)
```

**Arguments**

object	a function or a fitted model.
--------	-------------------------------

### Details

If object is a function then this function looks to see whether object has the logical attribute "smart". If so then this is returned, else FALSE.

If object is a fitted model then this function looks to see whether object@smart.prediction or object\$smart.prediction exists. If it does and it is not equal to list(smart.arg=FALSE) then a TRUE is returned, else FALSE. The reason for this is because, e.g., `lm(..., smart=FALSE)` and `vglm(..., smart=FALSE)`, will return such a specific list.

Writers of smart functions manually have to assign this attribute to their smart function after it has been written.

### Value

Returns TRUE or FALSE, according to whether the object is smart or not.

### Examples

```
is.smart(my1) # TRUE
is.smart(poly) # TRUE
library(splines)
is.smart(bs) # TRUE
is.smart(ns) # TRUE
is.smart(tan) # FALSE
if(!is.R()) is.smart(lm) # TRUE
## Not run:
library(VGAM)
x <- rnorm(9)
fit1 <- vglm(rnorm(9) ~ x, normal1)
is.smart(fit1) # TRUE
fit2 <- vglm(rnorm(9) ~ x, normal1, smart = FALSE)
is.smart(fit2) # FALSE
fit2@smart.prediction

## End(Not run)
```

---

is.zero

Zero Constraint Matrices

---

### Description

Returns a logical vector from a test of whether an object such as a matrix or VGLM object corresponds to a 'zero' assumption.

### Usage

```
is.zero.matrix(object, ...)
is.zero.vglm(object, ...)
```



**Arguments**

`object` an object such as a coefficient matrix of a `vglm` object, or a `vglm` object.  
`...` additional optional arguments. Currently unused.

**Details**

These functions test the effect of the zero argument on a `vglm` object or the coefficient matrix of a `vglm` object. The latter is obtained by `coef(vglmObject, matrix = TRUE)`.

**Value**

A vector of logicals, testing whether each linear/additive predictor has the zero argument applied to it. It is TRUE if that linear/additive predictor is intercept-only, i.e., all other regression coefficients are set to zero.

No checking is done for the intercept term at all, i.e., that it was estimated in the first place.

**See Also**

`constraints`, `vglm`.

**Examples**

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age, binom2.or(zero = NULL), coalminers)
is.zero(fit)
is.zero(coef(fit, matrix = TRUE))
```

---

kendall.tau

*Kendall's Tau Statistic*

---

**Description**

Computes Kendall's Tau, which is a rank-based correlation measure, between two vectors.

**Usage**

```
kendall.tau(x, y, exact = FALSE, max.n = 3000)
```

**Arguments**

`x`, `y` Numeric vectors. Must be of equal length. Ideally their values are continuous and not too discrete. Let `length(x)` be  $N$ , say.  
`exact` Logical. If TRUE then the exact value is computed.  
`max.n` Numeric. If `exact = FALSE` and `length(x)` is more than `max.n` then a random sample of `max.n` pairs are chosen.

### Details

Kendall's tau is a measure of dependency in a bivariate distribution. Loosely, two random variables are *concordant* if large values of one random variable are associated with large values of the other random variable. Similarly, two random variables are *disconcordant* if large values of one random variable are associated with small values of the other random variable. More formally, if  $(x[i] - x[j]) * (y[i] - y[j]) > 0$  then that comparison is concordant ( $i \neq j$ ). And if  $(x[i] - x[j]) * (y[i] - y[j]) < 0$  then that comparison is disconcordant ( $i \neq j$ ). Out of  $\text{choose}(N, 2)$  comparisons, let  $c$  and  $d$  be the number of concordant and disconcordant pairs. Then Kendall's tau can be estimated by  $(c - d) / (c + d)$ . If there are ties then half the ties are deemed concordant and half disconcordant so that  $(c - d) / (c + d + t)$  is used.

### Value

Kendall's tau, which lies between  $-1$  and  $1$ .

### Warning

If  $\text{length}(x)$  is large then the cost is  $O(N^2)$ , which is expensive! Under these circumstances it is not advisable to set `exact = TRUE` or `max.n` to a very large number.

### See Also

[binormalcop, cor.](#)

### Examples

```
N <- 5000; x <- 1:N; y <- runif(N)
true.rho <- -0.8
ymat <- rbinorm(N, cov12 = true.rho) # Bivariate normal, aka N_2
x <- ymat[, 1]
y <- ymat[, 2]

## Not run: plot(x, y, col = "blue")

kendall.tau(x, y) # A random sample is taken here
kendall.tau(x, y) # A random sample is taken here

kendall.tau(x, y, exact = TRUE) # Costly if length(x) is large
kendall.tau(x, y, max.n = N)   # Same as exact = TRUE

(rhohat <- sin(kendall.tau(x, y) * pi / 2)) # This formula holds for N_2 actually
true.rho # rhohat should be near this value
```

## Description

Estimates the location and scale parameters of Koenker's distribution by maximum likelihood estimation.

## Usage

```
koenker(percentile = 50, llocation = "identity", lscale = "loge",
        ilocation = NULL, iscale = NULL, imethod = 1, zero = 2)
```

## Arguments

**percentile** A numerical vector containing values between 0 and 100, which are the quantiles and expectiles. They will be returned as 'fitted values'.

**llocation, lscale** See [Links](#) for more choices, and [CommonVGAMffArguments](#).

**ilocation, iscale, imethod, zero** See [CommonVGAMffArguments](#) for details.

## Details

Koenker (1993) solved for the distribution whose quantiles are equal to its expectiles. This is called Koenker's distribution here. Its canonical form has mean and mode at 0 and has a heavy tail (in fact, its variance is infinite).

The standard ("canonical") form of Koenker's distribution can be endowed with a location and scale parameter. The standard form has a density that can be written as

$$f(z) = 2/(4 + z^2)^{3/2}$$

for real  $y$ . Then  $z = (y - a)/b$  for location and scale parameters  $a$  and  $b > 0$ . The mean of  $Y$  is  $a$ . By default,  $\eta_1 = a$  and  $\eta_2 = \log(b)$ . The expectiles/quantiles corresponding to **percentile** are returned as the fitted values; in particular, **percentile** = 50 corresponds to the mean (0.5 expectile) and median (0.5 quantile).

Note that if  $Y$  has a standard Koenker distribution then  $Y = \sqrt{2}T_2$  where  $T_2$  has a Student-t distribution with 2 degrees of freedom. The two parameters here can also be estimated using [studentt2](#) by specifying **df** = 2 and making an adjustment for the scale parameter, however, this **VGAM** family function is more efficient since the EIM is known (Fisher scoring is implemented.)

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Author(s)

T. W. Yee

## References

Koenker, R. (1993) When are expectiles percentiles? (solution) *Econometric Theory*, **9**, 526–527.

**See Also**

[dkoenker](#), [studentt2](#).

**Examples**

```
set.seed(123); nn <- 1000
kdata <- data.frame(x2 = sort(runif(nn)))
kdata <- transform(kdata, mylocat = 1 + 3 * x2,
                   myscale = 1)
kdata <- transform(kdata, y = rkoenker(nn, loc = mylocat, scale = myscale))
fit <- vglm(y ~ x2, koenker(perc = c(1, 50, 99)), kdata, trace = TRUE)
fit2 <- vglm(y ~ x2, studentt2(df = 2), kdata, trace = TRUE) # 'same' as fit

coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))

# Nice plot of the results
## Not run: plot(y ~ x2, kdata, col = "blue", las = 1,
               sub = paste("n =", nn),
               main = "Fitted quantiles/expectiles using Koenker's distribution")
matplot(with(kdata, x2), fitted(fit), add = TRUE, type = "l", lwd = 3)
legend("bottomright", lty = 1:3, lwd = 3, legend = colnames(fitted(fit)),
       col = 1:3)
## End(Not run)

fit@extra$percentile # Sample quantiles
```

---

Kumar

---

*The Kumaraswamy Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the Kumaraswamy distribution.

**Usage**

```
dkumar(x, shape1, shape2, log = FALSE)
pkumar(q, shape1, shape2)
qkumar(p, shape1, shape2)
rkumar(n, shape1, shape2)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.

`n` number of observations. If `length(n) > 1` then the length is taken to be the number required.

`shape1, shape2` positive shape parameters.

`log` Logical. If `log = TRUE` then the logarithm of the density is returned.

### Details

See [kumar](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

### Value

`dkumar` gives the density, `pkumar` gives the distribution function, `qkumar` gives the quantile function, and `rkumar` generates random deviates.

### Author(s)

T. W. Yee

### See Also

[kumar](#).

### Examples

```
## Not run:
shape1 <- 2; shape2 <- 2; nn <- 201; # shape1 <- shape2 <- 0.5;
x <- seq(-0.05, 1.05, len = nn)
plot(x, dkumar(x, shape1, shape2), type = "l", las = 1, ylim = c(0,1.5),
     ylab = paste("fkumar(shape1 = ", shape1, ", shape2 = ", shape2, ")"),
     col = "blue", cex.main = 0.8,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pkumar(x, shape1, shape2), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qkumar(probs, shape1, shape2)
lines(Q, dkumar(Q, shape1, shape2), col = "purple", lty = 3, type = "h")
lines(Q, pkumar(Q, shape1, shape2), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pkumar(Q, shape1, shape2) - probs)) # Should be 0

## End(Not run)
```

kumar

*Kumaraswamy Distribution Family Function***Description**

Estimates the two parameters of the Kumaraswamy distribution by maximum likelihood estimation.

**Usage**

```
kumar(lshape1 = "log", lshape2 = "log",
      ishape1 = NULL, ishape2 = NULL, grid.shape1 = c(0.4, 6.0),
      tol12 = 1.0e-4, zero = NULL)
```

**Arguments**

<code>lshape1</code> , <code>lshape2</code>	Link function for the two positive shape parameters, respectively, called $a$ and $b$ below. See <a href="#">Links</a> for more choices.
<code>ishape1</code> , <code>ishape2</code>	Numeric. Optional initial values for the two positive shape parameters.
<code>tol12</code>	Numeric and positive. Tolerance for testing whether the second shape parameter is either 1 or 2. If so then the working weights need to handle these singularities.
<code>grid.shape1</code>	Lower and upper limits for a grid search for the first shape parameter.
<code>zero</code>	See <a href="#">CommonVGAMffArguments</a> .

**Details**

The Kumaraswamy distribution has density function

$$f(y; a = \text{shape1}, b = \text{shape2}) = aby^{a-1}(1 - y^a)^{b-1}$$

where  $0 < y < 1$  and the two shape parameters,  $a$  and  $b$ , are positive. The mean is  $b \times \text{Beta}(1 + 1/a, b)$  (returned as the fitted values) and the variance is  $b \times \text{Beta}(1 + 2/a, b) - (b \times \text{Beta}(1 + 1/a, b))^2$ . Applications of the Kumaraswamy distribution include the storage volume of a water reservoir. Fisher scoring is implemented. Handles multiple responses (matrix input).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

T. W. Yee

## References

- Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology*, **46**, 79–88.
- Jones, M. C. (2009). Kumaraswamy’s distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology*, **6**, 70–81.

## See Also

[dkumar](#), [betaff](#).

## Examples

```
shape1 <- exp(1); shape2 <- exp(2)
kdata <- data.frame(y = rkumar(n = 1000, shape1, shape2))
fit <- vglm(y ~ 1, kumar, kdata, trace = TRUE)
c(with(kdata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

lambertW

*The Lambert W function*


---

## Description

Computes the Lambert  $W$  function for real values.

## Usage

```
lambertW(x, tolerance = 1e-10, maxit = 50)
```

## Arguments

<code>x</code>	A vector of reals.
<code>tolerance</code>	Accuracy desired.
<code>maxit</code>	Maximum number of iterations of third-order Halley’s method.

## Details

The Lambert  $W$  function is the root of the equation  $W(z) \exp(W(z)) = z$  for complex  $z$ . It is multi-valued if  $z$  is real and  $z < -1/e$ . For real  $-1/e \leq z < 0$  it has two possible real values, and currently only the upper branch is computed.

## Value

This function returns the principal branch of the  $W$  function for *real*  $z$ . It returns  $W(z) \geq -1$ , and NA for  $z < -1/e$ .

**Note**

If convergence does not occur then increase the value of `maxit` and/or `tolerance`.

Yet to do: add an argument `lbranch = TRUE` to return the lower branch for real  $-1/e \leq z < 0$ ; this would give  $W(z) \leq -1$ .

**Author(s)**

T. W. Yee

**References**

Corless, R. M. and Gonnet, G. H. and Hare, D. E. G. and Jeffrey, D. J. and Knuth, D. E. (1996) On the Lambert  $W$  function. *Advances in Computational Mathematics*, **5**(4), 329–359.

**See Also**

[log](#), [exp](#).

**Examples**

```
## Not run:
curve(lambertW, -exp(-1), 3, xlim = c(-1, 3), ylim = c(-2, 1),
      las = 1, col = "orange")
abline(v = -exp(-1), h = -1, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")
## End(Not run)
```

---

laplace

*Laplace Distribution*


---

**Description**

Maximum likelihood estimation of the 2-parameter classical Laplace distribution.

**Usage**

```
laplace(llocation = "identity", lscale = "loge",
        ilocation = NULL, iscale = NULL, imethod = 1, zero = 2)
```

**Arguments**

`llocation`, `lscale`

Character. Parameter link functions for location parameter  $a$  and scale parameter  $b$ . See [Links](#) for more choices.

`ilocation`, `iscale`

Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.

`imethod`

Initialization method. Either the value 1 or 2.

`zero`

See [CommonVGAMffArguments](#) for more information.



**Details**

The Laplace distribution is often known as the *double-exponential* distribution and, for modelling, has heavier tail than the normal distribution. The Laplace density function is

$$f(y) = \frac{1}{2b} \exp\left(-\frac{|y-a|}{b}\right)$$

where  $-\infty < y < \infty$ ,  $-\infty < a < \infty$  and  $b > 0$ . Its mean is  $a$  and its variance is  $2b^2$ . This parameterization is called the *classical Laplace distribution* by Kotz et al. (2001), and the density is symmetric about  $a$ .

For  $y \sim 1$  (where  $y$  is the response) the maximum likelihood estimate (MLE) for the location parameter is the sample median, and the MLE for  $b$  is `mean(abs(y-location))` (replace location by its MLE if unknown).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

This family function has not been fully tested. The MLE regularity conditions do not hold for this distribution, therefore misleading inferences may result, e.g., in the summary and vcov of the object.

**Note**

This family function uses Fisher scoring. Convergence may be slow for non-intercept-only models; half-stepping is frequently required.

**Author(s)**

T. W. Yee

**References**

Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001) *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*, Boston: Birkhauser.

**See Also**

[rlaplace](#), [alaplac2](#) (which differs slightly from this parameterization), [exponential](#), [median](#).

**Examples**

```
ldata <- data.frame(y = rlaplace(nn <- 100, loc = 2, scale = exp(1)))
fit <- vglm(y ~ 1, laplace, ldata, trace = TRUE, crit = "l")
coef(fit, matrix = TRUE)
Coef(fit)
with(ldata, median(y))
```

```
ldata <- data.frame(x = runif(nn <- 1001))
ldata <- transform(ldata, y = rlaplace(nn, loc = 2, scale = exp(-1 + 1*x)))
coef(vglm(y ~ x, laplace(iloc = 0.2, imethod = 2, zero = 1), ldata,
          trace = TRUE), matrix = TRUE)
```

laplaceUC

*The Laplace Distribution*

## Description

Density, distribution function, quantile function and random generation for the Laplace distribution with location parameter location and scale parameter scale.

## Usage

```
dlaplace(x, location = 0, scale = 1, log = FALSE)
plaplace(q, location = 0, scale = 1)
qlaplace(p, location = 0, scale = 1)
rlaplace(n, location = 0, scale = 1)
```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Same as in <a href="#">runif</a> .
location	the location parameter $a$ , which is the mean.
scale	the scale parameter $b$ . Must consist of positive values.
log	Logical. If log = TRUE then the logarithm of the density is returned.

## Details

The Laplace distribution is often known as the double-exponential distribution and, for modelling, has heavier tail than the normal distribution. The Laplace density function is

$$f(y) = \frac{1}{2b} \exp\left(-\frac{|y-a|}{b}\right)$$

where  $-\infty < y < \infty$ ,  $-\infty < a < \infty$  and  $b > 0$ . The mean is  $a$  and the variance is  $2b^2$ .

See [laplace](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for formulae and details. Apart from n, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

## Value

dlaplace gives the density, plaplace gives the distribution function, qlaplace gives the quantile function, and rlaplace generates random deviates.

**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[laplace](#).

**Examples**

```
loc <- 1; b <- 2
y <- rlaplace(n = 100, loc = loc, scale = b)
mean(y) # sample mean
loc     # population mean
var(y)  # sample variance
2 * b^2 # population variance

## Not run: loc <- 0; b <- 1.5; x <- seq(-5, 5, by = 0.01)
plot(x, dlaplace(x, loc, b), type = "l", col = "blue", ylim = c(0,1),
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qlaplace(seq(0.05,0.95,by = 0.05), loc, b),
      dlaplace(qlaplace(seq(0.05, 0.95, by = 0.05), loc, b), loc, b),
      col = "purple", lty = 3, type = "h")
lines(x, plaplace(x, loc, b), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)

plaplace(qlaplace(seq(0.05, 0.95, by = 0.05), loc, b), loc, b)
```

---

latvar

*Latent Variables*


---

**Description**

Generic function for the *latent variables* of a model.

**Usage**

```
latvar(object, ...)
lv(object, ...)
```

## Arguments

object	An object for which the extraction of latent variables is meaningful.
...	Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for <a href="#">Coef</a> .

## Details

Latent variables occur in reduced-rank regression models, as well as in quadratic and additive ordination models. For the latter two, latent variable values are often called *site scores* by ecologists. Latent variables are linear combinations of the explanatory variables.

## Value

The value returned depends specifically on the methods function invoked.

## Warning

[latvar](#) and [lv](#) are identical, but the latter will be deprecated soon.

Latent variables are not really applicable to [vglm](#)/[vgam](#) models.

## Author(s)

Thomas W. Yee

## References

- Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.
- Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.
- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

## See Also

[latvar.qrrvglm](#), [latvar.rrvglm](#), [latvar.cao](#), [lvplot](#).

## Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
set.seed(123)
p1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
  WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
  family = poissonff, data = hspider, Rank = 1, df1.nl =
  c(Zoraspin = 2.5, 3), Bestof = 3, Crowlpositive = TRUE)

var(latvar(p1)) # Scaled to unit variance # Scaled to unit variance
c(latvar(p1))   # Estimated site scores

## End(Not run)
```

leipnik

*Leipnik Distribution Family Function***Description**

Estimates the two parameters of a (transformed) Leipnik distribution by maximum likelihood estimation.

**Usage**

```
leipnik(lmu = "logit", llambda = "loge", imu = NULL, ilambda = NULL)
```

**Arguments**

lmu, llambda    Link function for the  $\mu$  and  $\lambda$  parameters. See [Links](#) for more choices.  
 imu, ilambda    Numeric. Optional initial values for  $\mu$  and  $\lambda$ .

**Details**

The (transformed) Leipnik distribution has density function

$$f(y; \mu, \lambda) = \frac{\{y(1-y)\}^{-\frac{1}{2}}}{\text{Beta}(\frac{\lambda+1}{2}, \frac{1}{2})} \left[ 1 + \frac{(y-\mu)^2}{y(1-y)} \right]^{-\frac{\lambda}{2}}$$

where  $0 < y < 1$  and  $\lambda > -1$ . The mean is  $\mu$  (returned as the fitted values) and the variance is  $1/\lambda$ .

Jorgensen (1997) calls the above the **transformed** Leipnik distribution, and if  $y = (x+1)/2$  and  $\mu = (\theta+1)/2$ , then the distribution of  $X$  as a function of  $x$  and  $\theta$  is known as the (untransformed) Leipnik distribution. Here, both  $x$  and  $\theta$  are in  $(-1, 1)$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Warning**

If llambda="identity" then it is possible that the lambda estimate becomes less than  $-1$ , i.e., out of bounds. One way to stop this is to choose llambda = "loge", however, lambda is then constrained to be positive.

**Note**

Convergence may be slow or fail. Until better initial value estimates are forthcoming try assigning the argument ilambda some numerical value if it fails to converge. Currently, Newton-Raphson is implemented, not Fisher scoring. Currently, this family function probably only really works for intercept-only models, i.e.,  $y \sim 1$  in the formula.

Author(s)

T. W. Yee

References

Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) *Continuous Univariate Distributions*, 2nd edition, Volume 2, New York: Wiley. (pages 612–617).

See Also

[mccullagh89](#).

Examples

```
ldata <- data.frame(y = rnorm(n = 2000, mean = 0.5, sd = 0.1)) # Not proper data
fit <- vglm(y ~ 1, leipnik(ilambda = 1), ldata, trace = TRUE, checkwz = FALSE)
fit <- vglm(y ~ 1, leipnik(ilambda = 1, llambda = logoff(offset = 1)),
           ldata, trace = TRUE, crit = "coef")
head(fitted(fit))
with(ldata, mean(y))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)

sum(weights(fit)) # Sum of the prior weights
sum(weights(fit, type = "work")) # Sum of the working weights
```

---

lerch	<i>Lerch Phi Function</i>
-------	---------------------------

---

Description

Computes the Lerch transcendental Phi function.

Usage

```
lerch(x, s, v, tolerance = 1.0e-10, iter = 100)
```

Arguments

- |           |  |
|-----------|--|
| x, s, v   | Numeric. This function recycles values of x, s, and v if necessary.  |
| tolerance | Numeric. Accuracy required, must be positive and less than 0.01.   |
| iter      | Maximum number of iterations allowed to obtain convergence. If iter is too small then a result of NA may occur; if so, try increasing its value. |

## Details

The Lerch transcendental function is defined by

$$\Phi(x, s, v) = \sum_{n=0}^{\infty} \frac{x^n}{(n+v)^s}$$

where  $|x| < 1$  and  $v \neq 0, -1, -2, \dots$ . Actually,  $x$  may be complex but this function only works for real  $x$ . The algorithm used is based on the relation

$$\Phi(x, s, v) = x^m \Phi(x, s, v+m) + \sum_{n=0}^{m-1} \frac{x^n}{(n+v)^s}.$$

See the URL below for more information. This function is a wrapper function for the C code described below.

## Value

Returns the value of the function evaluated at the values of  $x$ ,  $s$ ,  $v$ . If the above ranges of  $x$  and  $v$  are not satisfied, or some numeric problems occur, then this function will return a NA for those values.

## Warning

This function has not been thoroughly tested and contains bugs, for example, the zeta function cannot be computed with this function even though  $\zeta(s) = \Phi(x=1, s, v=1)$ . There are many sources of problems such as lack of convergence, overflow and underflow, especially near singularities. If any problems occur then a NA will be returned.

## Note

There are a number of special cases, e.g., the Riemann zeta-function is given by  $\zeta(s) = \Phi(x=1, s, v=1)$ . The special case of  $s=1$  corresponds to the hypergeometric  $2F1$ , and this is implemented in the **gsl** package. The Lerch transcendental Phi function should not be confused with the Lerch zeta function though they are quite similar.

## Author(s)

S. V. Aksenov and U. D. Jentschura wrote the C code. The R wrapper function was written by T. W. Yee.

## References

<http://aksenov.freeshell.org/lerchphi/source/lerchphi.c>.

Bateman, H. (1953) *Higher Transcendental Functions*. Volume 1. McGraw-Hill, NY, USA.

## See Also

[zeta](#).

Examples

```
## Not run:
s <- 2; v <- 1; x <- seq(-1.1, 1.1, length = 201)
plot(x, lerch(x, s = s, v = v), type = "l", col = "blue", las = 1,
     main = paste("lerch(x, s = ", s, ", v = ", v, ")", sep = ""))
abline(v = 0, h = 1, lty = "dashed", col = "gray")

s <- rnorm(n = 100)
max(abs(zeta(s) - lerch(x = 1, s = s, v = 1))) # This fails (a bug); should be 0

## End(Not run)
```

---

leukemia	<i>Acute Myelogenous Leukemia Survival Data</i>
----------	---

---

Description

Survival in patients with Acute Myelogenous Leukemia

Usage

```
data(leukemia)
```

Format

- time: survival or censoring time
- status: censoring status
- x: maintenance chemotherapy given? (factor)

Note

This data set has been transferred from **survival** and renamed from aml to leukemia.

Source

Rupert G. Miller (1997), *Survival Analysis*. John Wiley & Sons. ISBN: 0-471-25218-2.

---

levy	<i>Levy Distribution Family Function</i>
------	--

---



**Description**

Estimates the two parameters of the Levy distribution by maximum likelihood estimation.

**Usage**

```
levy(delta = NULL, link.gamma = "loge", idelta = NULL, igamma = NULL)
```

**Arguments**

delta	Location parameter. May be assigned a known value, otherwise it is estimated (the default).
link.gamma	Parameter link function for the (positive) $\gamma$ parameter. See <a href="#">Links</a> for more choices.
idelta	Initial value for the $\delta$ parameter (if it is to be estimated). By default, an initial value is chosen internally.
igamma	Initial value for the $\gamma$ parameter. By default, an initial value is chosen internally.

**Details**

The Levy distribution is one of three stable distributions whose density function has a tractable form. The formula for the density is

$$f(y; \gamma, \delta) = \sqrt{\frac{\gamma}{2\pi}} \exp\left(\frac{-\gamma}{2(y-\delta)}\right) / (y-\delta)^{3/2}$$

where  $\delta < y < \infty$  and  $\gamma > 0$ . The mean does not exist.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

If  $\delta$  is given, then only one parameter is estimated and the default is  $\eta_1 = \log(\gamma)$ . If  $\delta$  is not given, then  $\eta_2 = \delta$ .

**Author(s)**

T. W. Yee

**References**

Nolan, J. P. (2005) *Stable Distributions: Models for Heavy Tailed Data*.

**See Also**

The Nolan article is at <http://academic2.american.edu/~jpnolan/stable/chap1.pdf>.

### Examples

```
nn <- 1000; delta <- 0
mygamma <- 1 # log link ==> 0 is the answer
ldata <- data.frame(y = delta + mygamma/rnorm(nn)^2) # Levy(mygamma, delta)

# Cf. Table 1.1 of Nolan for Levy(1,0)
with(ldata, sum(y > 1) / length(y)) # Should be 0.6827
with(ldata, sum(y > 2) / length(y)) # Should be 0.5205

fit <- vglm(y ~ 1, levy(delta = delta), ldata, trace = TRUE) # 1 parameter
fit <- vglm(y ~ 1, levy(idelta = delta, igamma = mygamma),
            ldata, trace = TRUE) # 2 parameters
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
head(weights(fit, type = "work"))
```

---

lgammaff

---

*Log-gamma Distribution Family Function*


---

### Description

Estimation of the parameter of the standard and nonstandard log-gamma distribution.

### Usage

```
lgammaff(link = "loge", init.k = NULL)
lgamma3ff(llocation = "identity", lscale = "loge", lshape = "loge",
           ilocation = NULL, iscale = NULL, ishape = 1, zero = NULL)
```

### Arguments

llocation, lscale	Parameter link function applied to the location parameter $a$ and the positive scale parameter $b$ . See <a href="#">Links</a> for more choices.
link, lshape	Parameter link function applied to the positive shape parameter $k$ . See <a href="#">Links</a> for more choices.
init.k, ishape	Initial value for $k$ . If given, it must be positive. If failure to converge occurs, try some other value. The default means an initial value is determined internally.
ilocation, iscale	Initial value for $a$ and $b$ . The defaults mean an initial value is determined internally for each.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2,3\}$ . The default value means none are modelled as intercept-only terms. See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The probability density function of the standard log-gamma distribution is given by

$$f(y; k) = \exp[ky - \exp(y)]/\Gamma(k),$$

for parameter  $k > 0$  and all real  $y$ . The mean of  $Y$  is `digamma(k)` (returned as the fitted values) and its variance is `trigamma(k)`.

For the non-standard log-gamma distribution, one replaces  $y$  by  $(y - a)/b$ , where  $a$  is the location parameter and  $b$  is the positive scale parameter. Then the density function is

$$f(y) = \exp[k(y - a)/b - \exp((y - a)/b)]/(b \Gamma(k)).$$

The mean and variance of  $Y$  are  $a + b \cdot \text{digamma}(k)$  (returned as the fitted values) and  $b^2 \cdot \text{trigamma}(k)$ , respectively.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

The standard log-gamma distribution can be viewed as a generalization of the standard type 1 extreme value density: when  $k = 1$  the distribution of  $-Y$  is the standard type 1 extreme value distribution.

The standard log-gamma distribution is fitted with `lgammaff` and the non-standard (3-parameter) log-gamma distribution is fitted with `lgamma3ff`.

## Author(s)

T. W. Yee

## References

Kotz, S. and Nadarajah, S. (2000) *Extreme Value Distributions: Theory and Applications*, pages 48–49, London: Imperial College Press.

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) *Continuous Univariate Distributions*, 2nd edition, Volume 2, p.89, New York: Wiley.

## See Also

[rlgamma](#), [gengamma](#), [prentice74](#), [gamma1](#), [lgamma](#).

## Examples

```
ldata <- data.frame(y = rlgamma(100, k = exp(1)))
fit <- vglm(y ~ 1, lgammaff, ldata, trace = TRUE, crit = "coef")
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
```

```

ldata <- data.frame(x = runif(nn <- 5000)) # Another example
ldata <- transform(ldata, loc = -1 + 2 * x, Scale = exp(1))
ldata <- transform(ldata, y = rlgamma(nn, loc, scale = Scale, k = exp(0)))
fit2 <- vglm(y ~ x, lgamma3ff(zero = 2:3), ldata, trace = TRUE, crit = "c")
coef(fit2, matrix = TRUE)

```

lgammaUC

*The Log-Gamma Distribution*

## Description

Density, distribution function, quantile function and random generation for the log-gamma distribution with location parameter location, scale parameter scale and shape parameter k.

## Usage

```

dlgamma(x, location = 0, scale = 1, k = 1, log = FALSE)
plgamma(q, location = 0, scale = 1, k = 1)
qlgamma(p, location = 0, scale = 1, k = 1)
rlgamma(n, location = 0, scale = 1, k = 1)

```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Positive integer of length 1.
location	the location parameter $a$ .
scale	the (positive) scale parameter $b$ .
k	the (positive) shape parameter $k$ .
log	Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See [lgammaff](#), the **VGAM** family function for estimating the one parameter standard log-gamma distribution by maximum likelihood estimation, for formulae and other details. Apart from n, all the above arguments may be vectors and are recycled to the appropriate length if necessary.

## Value

dlgamma gives the density, plgamma gives the distribution function, qlgamma gives the quantile function, and rlgamma generates random deviates.

## Note

The **VGAM** family function [lgamma3ff](#) is for the three parameter (nonstandard) log-gamma distribution.

**Author(s)**

T. W. Yee

**References**

Kotz, S. and Nadarajah, S. (2000) *Extreme Value Distributions: Theory and Applications*, pages 48–49, London: Imperial College Press.

**See Also**

[lgammaff](#), [prentice74](#).

**Examples**

```
## Not run: loc <- 1; Scale <- 1.5; k <- 1.4
x <- seq(-3.2, 5, by = 0.01)
plot(x, dgamma(x, loc, Scale, k), type = "l", col = "blue", ylim = 0:1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qlgamma(seq(0.05, 0.95, by = 0.05), loc, Scale, k),
      dgamma(qlgamma(seq(0.05, 0.95, by = 0.05), loc, Scale, k),
             loc, Scale, k), col = "purple", lty = 3, type = "h")
lines(x, plgamma(x, loc, Scale, k), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
```

**Description**

Density, cumulative distribution function,  
and random generation for the Lindley distribution.

**Usage**

```
dlind(x, theta, log = FALSE)
plind(q, theta)
rlind(n, theta)
```

**Arguments**

x, q	vector of quantiles.
n	number of observations.
log	Logical. If log = TRUE then the logarithm of the density is returned.
theta	positive parameter.

**Details**

See [lindley](#) for details.

**Value**

dlind gives the density, plind gives the cumulative distribution function, and  
rlind generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[lindley](#).

**Examples**

```
theta <- exp(-1); x <- seq(0.0, 17, length = 700)
dlind(0:10, theta)
## Not run:
plot(x, dlind(x, theta), type = "l", las = 1, col = "blue",
     main = "dlind(x, theta = exp(-1))")
abline(h = 1, col = "grey", lty = "dashed")
## End(Not run)
```

---

lindley

*l-parameter Gamma Distribution*


---

**Description**

Estimates the (1-parameter) Lindley distribution by maximum likelihood estimation.

**Usage**

```
lindley(link = "log", itheta = NULL, zero = NULL)
```

**Arguments**

link                      Link function applied to the (positive) parameter. See [Links](#) for more choices.  
itheta, zero              See [CommonVGAMffArguments](#) for information.

## Details

The density function is given by

$$f(y; \theta) = \theta^2(1 + y) \exp(-\theta y) / (1 + \theta)$$

for  $\theta > 0$  and  $y > 0$ . The mean of  $Y$  (returned as the fitted values) is  $\mu = (\theta + 2)/(\theta(\theta + 1))$ . The variance is  $(\theta^2 + 4\theta + 2)/(\theta(\theta + 1))^2$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

This **VGAM** family function can handle multiple responses (inputted as a matrix). Fisher scoring is implemented.

## Author(s)

T. W. Yee

## References

- Lindley, D. V. (1958) Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society, Series B, Methodological*, **20**, 102–107.
- Ghitany, M. E. and Atieh, B. and Nadarajah, S. (2008) Lindley distribution and its application. *Math. Comput. Simul.*, **78**, 493–506.

## See Also

[dlind](#), [gamma2.ab](#),

## Examples

```
ldata <- data.frame(y = rlind(n = 1000, theta = exp(3)))
fit <- vglm(y ~ 1, lindley, ldata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

**Description**

The **VGAM** package provides a number of (parameter) link functions which are described in general here. Collectively, they offer the user considerable flexibility for modelling data.

**Usage**

```
TypicalVGAMlinkFunction(theta, someParameter = 0,
                        bvalue = NULL,
                        inverse = FALSE, deriv = 0,
                        short = TRUE, tag = FALSE)
```

**Arguments**

theta	Numeric or character. Actually this can be $\theta$ (default) or $\eta$ , depending on the other arguments. If theta is character then inverse and deriv are ignored. The name theta should always be the name of the first argument.
someParameter	Some parameter, e.g., an offset.
bvalue	Boundary value, positive if given. If $0 < \text{theta}$ then values of theta which are less than or equal to 0 can be replaced by bvalue before computing the link function value. Values of theta which are greater than or equal to 1 can be replaced by 1 minus bvalue before computing the link function value. The value <code>bvalue = .Machine\$double.eps</code> is sometimes a reasonable value, or something slightly higher.
inverse	Logical. If TRUE the inverse link value $\theta$ is returned, hence the argument theta is really $\eta$ .
deriv	Integer. Either 0, 1, or 2 specifying the order of the derivative.
short, tag	Logical. These are used for labelling the blurb slot of a <code>vglmff-class</code> object. These arguments are used only if theta is character, and gives the formula for the link in character form. If tag = TRUE then the result is preceeded by a little more information.

**Details**

Almost all **VGAM** link functions have something similar to the argument list as given above. In this help file we have  $\eta = g(\theta)$  where  $g$  is the link function,  $\theta$  is the parameter and  $\eta$  is the linear/additive predictor.

The following is a brief enumeration of all **VGAM** link functions.

For parameters lying between 0 and 1 (e.g., probabilities): [logit](#), [probit](#), [cloglog](#), [cauchit](#), [fsqrt](#), [logc](#), [golf](#), [polf](#), [nbolf](#).

For positive parameters (i.e., greater than 0): [loge](#), [negloge](#), [powerlink](#).



For parameters greater than 1: [loglog](#).

For parameters between  $-1$  and  $1$ : [fisherz](#), [rhubit](#).

For parameters between  $A$  and  $B$ : [elogit](#), [logoff](#) ( $B = \infty$ ).

For unrestricted parameters (i.e., any value): [identity](#), [negidentity](#), [reciprocal](#), [negreciprocal](#).

## Value

Returns one of the link function value or its first or second derivative, the inverse link or its first or second derivative, or a character description of the link.

Here are the general details. If `inverse = FALSE` and `deriv = 0` (default) then the ordinary link function  $\eta = g(\theta)$  is returned. If `inverse = FALSE` and `deriv = 1` then it is  $d\theta/d\eta$  as a function of  $\theta$ . If `inverse = FALSE` and `deriv = 2` then it is  $d^2\theta/d\eta^2$  as a function of  $\theta$ .

If `inverse = TRUE` and `deriv = 0` then the inverse link function is returned, hence  $\theta$  is really  $\eta$ . If `inverse = TRUE` and `deriv` is positive then the *reciprocal* of the same link function with (`theta = theta`, `someParameter`, `inverse = TRUE`, `deriv = deriv`) is returned.

## Note

**VGAM** link functions are generally not compatible with other functions outside the package. In particular, they won't work with [glm](#) or any other package for fitting GAMs.

From October 2006 onwards, all **VGAM** family functions will only contain one default value for each link argument rather than giving a vector of choices. For example, rather than `binomialff(link = c("logit", "probit"))` it is now `binomialff(link = "logit", ...)`. No checking will be done to see if the user's choice is reasonable. This means that the user can write his/her own **VGAM** link function and use it within any **VGAM** family function. Altogether this provides greater flexibility. The downside is that the user must specify the *full* name of the link function, by either assigning the link argument the full name as a character string, or just the name itself. See the examples below.

From August 2012 onwards, a major change in link functions occurred. Argument `esigma` (and the like such as `earg`) used to be in **VGAM** prior to version 0.9-0 (released during the 2nd half of 2012). The major change is that arguments such as `offset` that used to be passed in via those arguments can now be done directly through the link function. For example, `gev(lshape = "logoff", eshape = list(offset = 0.5))` is replaced by `gev(lshape = logoff(offset = 0.5))`. The `@misc` slot no longer has `link` and `earg` components, but two other components replace these. Functions such as `dtheta.deta()`, `d2theta.deta2()`, `eta2theta()`, `theta2eta()` are modified.

## Author(s)

T. W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[TypicalVGAMfamilyFunction](#), [vglm](#), [vgam](#), [rrvglm](#), [cgo](#), [cao](#).

### Examples

```

logit("a")
logit("a", short = FALSE)
logit("a", short = FALSE, tag = TRUE)

logoff(1:5, offset = 1) # Same as log(1:5 + 1)
powerlink(1:5, power = 2) # Same as (1:5)^2

## Not run: # This is old and no longer works:
logoff(1:5, earg = list(offset = 1))
powerlink(1:5, earg = list(power = 2))

## End(Not run)

fit1 <- vgam(agaas ~ altitude, binomialff(link = "cloglog"), hunua) # okay
fit2 <- vgam(agaas ~ altitude, binomialff(link = "cloglog"), hunua) # okay

## Not run:
# This no longer works since "clog" is not a valid VGAM link function:
fit3 <- vgam(agaas ~ altitude, binomialff(link = "clog"), hunua) # not okay

# No matter what the link, the estimated var-cov matrix is the same
y <- rbeta(n = 1000, shape1 = exp(0), shape2 = exp(1))
fit1 <- vglm(y ~ 1, beta.ab(lshape1 = "identity", lshape2 = "identity"),
            trace = TRUE, crit = "coef")
fit2 <- vglm(y ~ 1, beta.ab(lshape1 = logoff(offset = 1.1),
                          lshape2 = logoff(offset = 1.1)),
            trace = TRUE, crit = "coef")
vcov(fit1, untransform = TRUE)
vcov(fit1, untransform = TRUE) - vcov(fit2, untransform = TRUE) # Should be all 0s
\dontrun{ # This is old:
fit1@misc$earg # Some 'special' parameters
fit2@misc$earg # Some 'special' parameters are here
}

par(mfrow = c(2, 2))
p <- seq(0.01, 0.99, len = 200)
x <- seq(-4, 4, len = 200)
plot(p, logit(p), type = "l", col = "blue")
plot(x, logit(x, inverse = TRUE), type = "l", col = "blue")
plot(p, logit(p, deriv = 1), type = "l", col = "blue") # reciprocal!
plot(p, logit(p, deriv = 2), type = "l", col = "blue") # reciprocal!

## End(Not run)

```

## Description

Density, distribution function, quantile function and random generation for the generalized beta distribution, as proposed by Libby and Novick (1982).

## Usage

```
dlino(x, shape1, shape2, lambda = 1, log = FALSE)
plino(q, shape1, shape2, lambda = 1)
qlino(p, shape1, shape2, lambda = 1)
rlino(n, shape1, shape2, lambda = 1)
```

## Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Must be a positive integer of length 1.
shape1, shape2, lambda	see <a href="#">lino</a> .
log	Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See [lino](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dlino gives the density, plino gives the distribution function, qlino gives the quantile function, and rlino generates random deviates.

## Author(s)

T. W. Yee

## See Also

[lino](#).

## Examples

```
## Not run:
lambda <- 0.4; shape1 <- exp(1.3); shape2 <- exp(1.3)
x <- seq(0.0, 1.0, len = 101)
plot(x, dlino(x, shape1 = shape1, shape2 = shape2, lambda = lambda),
     type = "l", col = "blue", las = 1, ylab = "",
     main = "Blue is density, red is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
abline(h = 0, col = "blue", lty = 2)
lines(x, plino(x, shape1 = shape1, shape2 = shape2, l = lambda), col = "red")
```

```

probs <- seq(0.1, 0.9, by = 0.1)
Q <- qlino(probs, shape1 = shape1, shape2 = shape2, lambda = lambda)
lines(Q, dlino(Q, shape1 = shape1, shape2 = shape2, lambda = lambda),
      col = "purple", lty = 3, type = "h")
plino(Q, shape1 = shape1, shape2 = shape2, l = lambda) - probs # Should be all 0

## End(Not run)

```

lino

*Generalized Beta Distribution Family Function***Description**

Maximum likelihood estimation of the 3-parameter generalized beta distribution as proposed by Libby and Novick (1982).

**Usage**

```

lino(lshape1 = "loge", lshape2 = "loge", llambda = "loge",
     ishape1 = NULL, ishape2 = NULL, ilambda = 1, zero = NULL)

```

**Arguments**

lshape1, lshape2	Parameter link functions applied to the two (positive) shape parameters $a$ and $b$ . See <a href="#">Links</a> for more choices.
llambda	Parameter link function applied to the parameter $\lambda$ . See <a href="#">Links</a> for more choices.
ishape1, ishape2, ilambda	Initial values for the parameters. A NULL value means one is computed internally. The argument <code>ilambda</code> must be numeric, and the default corresponds to a standard beta distribution.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set $\{1,2,3\}$ which correspond to $a$ , $b$ , $\lambda$ , respectively.

**Details**

Proposed by Libby and Novick (1982), this distribution has density

$$f(y; a, b, \lambda) = \frac{\lambda^a y^{a-1} (1-y)^{b-1}}{B(a, b) \{1 - (1-\lambda)y\}^{a+b}}$$

for  $a > 0$ ,  $b > 0$ ,  $\lambda > 0$ ,  $0 < y < 1$ . Here  $B$  is the beta function (see [beta](#)). The mean is a complicated function involving the Gauss hypergeometric function. If  $X$  has a lino distribution with parameters `shape1`, `shape2`, `lambda`, then  $Y = \lambda X / (1 - (1 - \lambda)X)$  has a standard beta distribution with parameters `shape1`, `shape2`.

Since  $\log(\lambda) = 0$  corresponds to the standard beta distribution, a summary of the fitted model performs a t-test for whether the data belongs to a standard beta distribution (provided the [loge](#) link for  $\lambda$  is used; this is the default).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

The fitted values, which is usually the mean, have not been implemented yet and consequently are NAs.

Although Fisher scoring is used, the working weight matrices are positive-definite only in a certain region of the parameter space. Problems with this indicate poor initial values or an ill-conditioned model or insufficient data etc.

This model is can be difficult to fit. A reasonably good value of `ilambda` seems to be needed so if the self-starting initial values fail, try experimenting with the initial value arguments. Experience suggests `ilambda` is better a little larger, rather than smaller, compared to the true value.

**Author(s)**

T. W. Yee

**References**

Libby, D. L. and Novick, M. R. (1982) Multivariate generalized beta distributions with applications to utility assessment. *Journal of Educational Statistics*, **7**, 271–294.

Gupta, A. K. and Nadarajah, S. (2004) *Handbook of Beta Distribution and Its Applications*, NY: Marcel Dekker, Inc.

**See Also**

[Lino](#), [genbetaII](#).

**Examples**

```
ldata1 <- data.frame(y = rbeta(n = 1000, exp(0.5), exp(1))) # ~ standard beta
fit <- vglm(y ~ 1, lino, ldata1, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
summary(fit)

# Nonstandard beta distribution
ldata2 <- data.frame(y = rlino(n = 1000, shape1 = 2, shape2 = 3, lambda = exp(1)))
fit <- vglm(y~1, lino(lshape1 = identity, lshape2 = identity, ilamb = 10), ldata2)
coef(fit, matrix = TRUE)
```

lirat

*Low-iron Rat Teratology Data***Description**

Low-iron rat teratology data.

**Usage**

```
data(lirat)
```

**Format**

A data frame with 58 observations on the following 4 variables.

N Litter size.

R Number of dead fetuses.

hb Hemoglobin level.

grp Group number. Group 1 is the untreated (low-iron) group, group 2 received injections on day 7 or day 10 only, group 3 received injections on days 0 and 7, and group 4 received injections weekly.

**Details**

The following description comes from Moore and Tsiatis (1991). The data comes from the experimental setup from Shepard et al. (1980), which is typical of studies of the effects of chemical agents or dietary regimens on fetal development in laboratory rats.

Female rats were put in iron-deficient diets and divided into 4 groups. One group of controls was given weekly injections of iron supplement to bring their iron intake to normal levels, while another group was given only placebo injections. Two other groups were given fewer iron-supplement injections than the controls. The rats were made pregnant, sacrificed 3 weeks later, and the total number of fetuses and the number of dead fetuses in each litter were counted.

For each litter the number of dead fetuses may be considered to be  $\text{Binomial}(N, p)$  where  $N$  is the litter size and  $p$  is the probability of a fetus dying. The parameter  $p$  is expected to vary from litter to litter, therefore the total variance of the proportions will be greater than that predicted by a binomial model, even when the covariates for hemoglobin level and experimental group are accounted for.

**Source**

Moore, D. F. and Tsiatis, A. (1991) Robust Estimation of the Variance in Moment Methods for Extra-binomial and Extra-Poisson Variation. *Biometrics*, **47**, 383–401.

**References**

Shepard, T. H., Mackler, B. and Finch, C. A. (1980) Reproductive studies in the iron-deficient rat. *Teratology*, **22**, 329–334.

## Examples

```
## Not run:
# cf. Figure 3 of Moore and Tsiatis (1991)
plot(R / N ~ hb, data = lirat, pch = as.character(grp), col = grp,
     las = 1, xlab = "Hemoglobin level", ylab = "Proportion Dead")
## End(Not run)
```

---

lms.bcg	<i>LMS Quantile Regression with a Box-Cox transformation to a Gamma Distribution</i>
---------	--

---

## Description

LMS quantile regression with the Box-Cox transformation to the gamma distribution.

## Usage

```
lms.bcg(percentiles = c(25, 50, 75), zero = c(1, 3),
        llambda = "identity", lmu = "identity", lsigma = "loge",
        dfmu.init = 4, dfsigma.init = 2, ilambda = 1, isigma = NULL)
```

## Arguments

percentiles	A numerical vector containing values between 0 and 100, which are the quantiles. They will be returned as ‘fitted values’.
zero	See <a href="#">lms.bcn</a> .
llambda, lmu, lsigma	See <a href="#">lms.bcn</a> .
dfmu.init, dfsigma.init	See <a href="#">lms.bcn</a> .
ilambda, isigma	See <a href="#">lms.bcn</a> .

## Details

Given a value of the covariate, this function applies a Box-Cox transformation to the response to best obtain a gamma distribution. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood. Similar details can be found at [lms.bcn](#).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Warning

This **VGAM** family function comes with the same warnings as [lms.bcn](#). Also, the expected value of the second derivative with respect to lambda may be incorrect (my calculations do not agree with the Lopatzidis and Green manuscript.)

**Note**

Similar notes can be found at [lms.bcn](#).

**Author(s)**

Thomas W. Yee

**References**

Lopatzidis A. and Green, P. J. (unpublished manuscript) Semiparametric quantile regression using the gamma distribution.

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[lms.bcn](#), [lms.yjn](#), [qtplot.lmscreg](#), [deplot.lmscreg](#), [cdf.lmscreg](#), [bmi.nz](#), [amlexponential](#).

**Examples**

```
# This converges, but deplot(fit) and qtplot(fit) do not work
fit0 <- vglm(BMI ~ bs(age, df = 4), lms.bcg, bmi.nz, trace = TRUE)
coef(fit0, matrix = TRUE)
## Not run:
par(mfrow = c(1, 1))
plotvgam(fit0, se = TRUE) # Plot mu function (only)

## End(Not run)

# Use a trick: fit0 is used for initial values for fit1.
fit1 <- vgam(BMI ~ s(age, df = c(4, 2)), etastart = predict(fit0),
            lms.bcg(zero = 1), bmi.nz, trace = TRUE)

# Difficult to get a model that converges.
# Here, we prematurely stop iterations because it fails near the solution.
fit2 <- vgam(BMI ~ s(age, df = c(4, 2)), maxit = 4,
            lms.bcg(zero = 1, ilam = 3), bmi.nz, trace = TRUE)
summary(fit1)
head(predict(fit1))
head(fitted(fit1))
head(bmi.nz)
# Person 1 is near the lower quartile of BMI amongst people his age
head(cdf(fit1))

## Not run:
# Quantile plot
par(bty = "l", mar=c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit1, percentiles=c(5, 50, 90, 99), main = "Quantiles",
      xlim = c(15, 90), las = 1, ylab = "BMI", lwd = 2, lcol = 4)
```



```
# Density plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit1, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
  main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit1, x0=42, y=ygrid, add=TRUE, llty=2, col="red")
aa <- deplot(fit1, x0=55, y=ygrid, add=TRUE, llty=4, col="blue", Attach=TRUE)
aa$post$deplot # Contains density function values

## End(Not run)
```

lms.bcn

*LMS Quantile Regression with a Box-Cox Transformation to Normality*

## Description

LMS quantile regression with the Box-Cox transformation to normality.

## Usage

```
lms.bcn(percentiles = c(25, 50, 75), zero = c(1, 3),
  llambda = "identity", lmu = "identity", lsigma = "loge",
  dfmu.init = 4, dfsigma.init = 2, ilambda = 1,
  isigma = NULL, tol0 = 0.001, expectiles = FALSE)
```

## Arguments

- |                      |   |
|----------------------|---|
| percentiles          | A numerical vector containing values between 0 and 100, which are the quantiles. They will be returned as ‘fitted values’.  |
| zero                 | An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set {1,2,3}. The default value usually increases the chance of successful convergence. Setting zero = NULL means they all are functions of the covariates. For more information see <a href="#">CommonVGAMffArguments</a> . |
| llambda, lmu, lsigma | Parameter link functions applied to the first, second and third linear/additive predictors. See <a href="#">Links</a> for more choices, and <a href="#">CommonVGAMffArguments</a> .   |
| dfmu.init            | Degrees of freedom for the cubic smoothing spline fit applied to get an initial estimate of mu. See <a href="#">vsmooth.spline</a> .  |
| dfsigma.init         | Degrees of freedom for the cubic smoothing spline fit applied to get an initial estimate of sigma. See <a href="#">vsmooth.spline</a> . This argument may be assigned NULL to get an initial value using some other algorithm.  |
| ilambda              | Initial value for lambda. If necessary, it is recycled to be a vector of length $n$ where $n$ is the number of (independent) observations.  |

<code>isigma</code>	Optional initial value for sigma. If necessary, it is recycled to be a vector of length $n$ . The default value, <code>NULL</code> , means an initial value is computed in the <code>@initialize</code> slot of the family function.
<code>tol0</code>	Small positive number, the tolerance for testing if lambda is equal to zero.
<code>expectiles</code>	Experimental; please do not use.

## Details

Given a value of the covariate, this function applies a Box-Cox transformation to the response to best obtain normality. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood.

In more detail, the basic idea behind this method is that, for a fixed value of  $x$ , a Box-Cox transformation of the response  $Y$  is applied to obtain standard normality. The 3 parameters ( $\lambda$ ,  $\mu$ ,  $\sigma$ , which start with the letters “L-M-S” respectively, hence its name) are chosen to maximize a penalized log-likelihood (with [vgam](#)). Then the appropriate quantiles of the standard normal distribution are back-transformed onto the original scale to get the desired quantiles. The three parameters may vary as a smooth function of  $x$ .

The Box-Cox power transformation here of the  $Y$ , given  $x$ , is

$$Z = [(Y/\mu(x))^{\lambda(x)} - 1]/(\sigma(x) \lambda(x))$$

for  $\lambda(x) \neq 0$ . (The singularity at  $\lambda(x) = 0$  is handled by a simple function involving a logarithm.) Then  $Z$  is assumed to have a standard normal distribution. The parameter  $\sigma(x)$  must be positive, therefore **VGAM** chooses  $\eta(x)^T = (\lambda(x), \mu(x), \log(\sigma(x)))$  by default. The parameter  $\mu$  is also positive, but while  $\log(\mu)$  is available, it is not the default because  $\mu$  is more directly interpretable. Given the estimated linear/additive predictors, the  $100\alpha$  percentile can be estimated by inverting the Box-Cox power transformation at the  $100\alpha$  percentile of the standard normal distribution.

Of the three functions, it is often a good idea to allow  $\mu(x)$  to be more flexible because the functions  $\lambda(x)$  and  $\sigma(x)$  usually vary more smoothly with  $x$ . This is somewhat reflected in the default value for the argument `zero`, viz. `zero = c(1, 3)`.

## Value

An object of class “`vglmff`” (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Warning

The computations are not simple, therefore convergence may fail. Set `trace = TRUE` to monitor convergence if it isn’t set already. Convergence failure will occur if, e.g., the response is bimodal at any particular value of  $x$ . In case of convergence failure, try different starting values. Also, the estimate may diverge quickly near the solution, in which case try prematurely stopping the iterations by assigning `maxits` to be the iteration number corresponding to the highest likelihood value.

One trick is to fit a simple model and use it to provide initial values for a more complex model; see in the examples below.

## Note

The response must be positive because the Box-Cox transformation cannot handle negative values. The LMS-Yeo-Johnson-normal method can handle both positive and negative values.

In general, the lambda and sigma functions should be more smoother than the mean function. Having `zero = 1`, `zero = 3` or `zero = c(1, 3)` is often a good idea. See the example below.

While it is usual to regress the response against a single covariate, it is possible to add other explanatory variables, e.g., gender. See <http://www.stat.auckland.ac.nz/~yee> for further information and examples about this feature.

## Author(s)

Thomas W. Yee

## References

Cole, T. J. and Green, P. J. (1992) Smoothing Reference Centile Curves: The LMS Method and Penalized Likelihood. *Statistics in Medicine*, **11**, 1305–1319.

Green, P. J. and Silverman, B. W. (1994) *Nonparametric Regression and Generalized Linear Models: A Roughness Penalty Approach*, London: Chapman & Hall.

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[lms.bcg](#), [lms.yjn](#), [qtplot.lmscreg](#), [deplot.lmscreg](#), [cdf.lmscreg](#),  
[alaplace1](#), [amlnormal](#), [denorm](#), [CommonVGAMffArguments](#).

## Examples

```
## Not run: require(VGAMdata)
mysubset <- subset(xs.nz, sex == "M" & ethnic == "1" & Study1)
mysubset <- transform(mysubset, BMI = weight / height^2)
BMIdata <- na.omit(mysubset)
BMIdata <- subset(BMIdata, BMI < 80 & age < 65,
                  select = c(age, BMI)) # Delete an outlier
summary(BMIdata)

fit <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1), BMIdata)

head(predict(fit))
head(fitted(fit))
head(BMIdata)
head(cdf(fit)) # Person 56 is probably overweight, given his age
100 * colMeans(c(depvar(fit)) < fitted(fit)) # Empirical proportions

# Convergence problems? Try this trick: fit0 is a simpler model used for fit1
```

```

fit0 <- vgam(BMI ~ s(age, df = 4), lms.bcn(zero = c(1, 3)), data = BMIdata)
fit1 <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1), data = BMIdata,
  etastart = predict(fit0))

## End(Not run)

## Not run:
# Quantile plot
par(bty = "l", mar = c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit, percentiles = c(5, 50, 90, 99), main = "Quantiles",
  xlim = c(15, 66), las = 1, ylab = "BMI", lwd = 2, lcol = 4)

# Density plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
  main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue",
  Attach = TRUE)
aa$post$deplot # Contains density function values

## End(Not run)

```

lms.yjn

*LMS Quantile Regression with a Yeo-Johnson Transformation to Normality*

## Description

LMS quantile regression with the Yeo-Johnson transformation to normality.

## Usage

```

lms.yjn(percentiles = c(25, 50, 75), zero = c(1,3),
  llambda = "identity", lsigma = "loge",
  dfmu.init = 4, dfsigma.init = 2,
  ilambda = 1, isigma = NULL, rule = c(10, 5),
  yoffset = NULL, diagW = FALSE, iters.diagW = 6)
lms.yjn2(percentiles=c(25,50,75), zero=c(1,3),
  llambda = "identity", lmu = "identity", lsigma = "loge",
  dfmu.init = 4, dfsigma.init = 2, ilambda = 1.0,
  isigma = NULL, yoffset = NULL, nsimEIM = 250)

```

## Arguments

percentiles	A numerical vector containing values between 0 and 100, which are the quantiles. They will be returned as ‘fitted values’.
zero	See <a href="#">lms.bcn</a> .

llambda, lmu, lsigma	See <a href="#">lms.bcn</a> .
dfmu.init, dfsigma.init	See <a href="#">lms.bcn</a> .
ilambda, isigma	See <a href="#">lms.bcn</a> .
rule	Number of abscissae used in the Gaussian integration scheme to work out elements of the weight matrices. The values given are the possible choices, with the first value being the default. The larger the value, the more accurate the approximation is likely to be but involving more computational expense.
yoffset	A value to be added to the response y, for the purpose of centering the response before fitting the model to the data. The default value, NULL, means <code>-median(y)</code> is used, so that the response actually used has median zero. The yoffset is saved on the object and used during prediction.
diagW	Logical. This argument is offered because the expected information matrix may not be positive-definite. Using the diagonal elements of this matrix results in a higher chance of it being positive-definite, however convergence will be very slow. If TRUE, then the first <code>iters.diagW</code> iterations will use the diagonal of the expected information matrix. The default is FALSE, meaning faster convergence.
iters.diagW	Integer. Number of iterations in which the diagonal elements of the expected information matrix are used. Only used if <code>diagW = TRUE</code> .
nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

Given a value of the covariate, this function applies a Yeo-Johnson transformation to the response to best obtain normality. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood. The function `lms.yjn2()` estimates the expected information matrices using simulation (and is consequently slower) while `lms.yjn()` uses numerical integration. Try the other if one function fails.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

The computations are not simple, therefore convergence may fail. In that case, try different starting values.

The generic function `predict`, when applied to a `lms.yjn` fit, does not add back the `yoffset` value.

## Note

The response may contain both positive and negative values. In contrast, the LMS-Box-Cox-normal and LMS-Box-Cox-gamma methods only handle a positive response because the Box-Cox transformation cannot handle negative values.

Some other notes can be found at [lms.bcn](#).

**Author(s)**

Thomas W. Yee

**References**

Yeo, I.-K. and Johnson, R. A. (2000) A new family of power transformations to improve normality or symmetry. *Biometrika*, **87**, 954–959.

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Yee, T. W. (2002) An Implementation for Regression Quantile Estimation. Pages 3–14. In: Haerdle, W. and Ronz, B., *Proceedings in Computational Statistics COMPSTAT 2002*. Heidelberg: Physica-Verlag.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[lms.bcn](#), [lms.bcg](#), [qtplot.lmscreg](#), [deplot.lmscreg](#), [cdf.lmscreg](#), [bmi.nz](#), [amlnormal](#).

**Examples**

```
fit <- vgam(BMI ~ s(age, df = 4), lms.yjn, bmi.nz, trace = TRUE)
head(predict(fit))
head(fitted(fit))
head(bmi.nz)
# Person 1 is near the lower quartile of BMI amongst people his age
head(cdf(fit))

## Not run:
# Quantile plot
par(bty = "l", mar = c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit, percentiles = c(5, 50, 90, 99), main = "Quantiles",
       xlim = c(15, 90), las = 1, ylab = "BMI", lwd = 2, lcol = 4)

# Density plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
             main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue", Attach = TRUE)
with(aa$post, deplot) # Contains density function values; == a$post$deplot

## End(Not run)
```

Log

*Logarithmic Distribution***Description**

Density, distribution function, and random generation for the logarithmic distribution.

**Usage**

```
dlog(x, prob, log = FALSE)
plog(q, prob, log.p = FALSE)
rlog(n, prob, Smallno = 1.0e-6)
```

**Arguments**

x, q	Vector of quantiles. For the density, it should be a vector with positive integer values in order for the probabilities to be positive.
n	number of observations. A single positive integer.
prob	The parameter value $c$ described in in <a href="#">logff</a> . Here it is called prob because $0 < c < 1$ is the range. For <code>rlog()</code> this parameter must be of length 1.
log, log.p	Logical. If <code>log.p = TRUE</code> then all probabilities $p$ are given as $\log(p)$ .
Smallno	Numeric, a small value used by the rejection method for determining the upper limit of the distribution. That is, $\text{plog}(U, \text{prob}) > 1 - \text{Smallno}$ where $U$ is the upper limit.

**Details**

The details are given in [logff](#).

**Value**

`dlog` gives the density, `plog` gives the distribution function, and `rlog` generates random deviates.

**Note**

Given some response data, the **VGAM** family function [logff](#) estimates the parameter prob. For `plog()`, if argument q contains large values and/or q is long in length then the memory requirements may be very high. Very large values in q are handled by an approximation by Owen (1965).

**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[logff](#).

**Examples**

```
dlog(1:20, 0.5)
rlog(20, 0.5)

## Not run: prob <- 0.8; x <- 1:10
plot(x, dlog(x, prob = prob), type = "h", ylim = 0:1,
      sub = "prob=0.8", las = 1, col = "blue", ylab = "Probability",
      main = "Logarithmic distribution: blue=density; orange=distribution function")
lines(x + 0.1, plog(x, prob = prob), col = "orange", lty = 3, type = "h")
## End(Not run)
```

---

log1pexp

---

*Logarithms with an Unit Offset and Exponential Term*


---

**Description**

Computes  $\log(1 + \exp(x))$  accurately.

**Usage**

```
log1pexp(x)
```

**Arguments**

**x** A vector of reals (numeric). Complex numbers not allowed since [log1p](#) does not handle these.

**Details**

Computes  $\log(1 + \exp(x))$  accurately. An adjustment is made when  $x$  is positive and large in value.

**Value**

Returns  $\log(1 + \exp(x))$ .

**See Also**

[log1p](#), [exp](#).



**Examples**

```
x <- c(10, 50, 100, 200, 400, 500, 800, 1000, 1e4, 1e5, 1e20, Inf)
log1pexp(x)
log(1 + exp(x)) # Naive; suffers from overflow
x <- -c(10, 50, 100, 200, 400, 500, 800, 1000, 1e4, 1e5, 1e20, Inf)
log1pexp(x)
log(1 + exp(x)) # Naive; suffers from inaccuracy
```

logc

*Complementary-log Link Function***Description**

Computes the complementary-log transformation, including its inverse and the first two derivatives.

**Usage**

```
logc(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
      short = TRUE, tag = FALSE)
```

**Arguments**

theta                Numeric or character. See below for further details.  
 bvalue              See [Links](#).  
 inverse, deriv, short, tag  
                      Details at [Links](#).

**Details**

The complementary-log link function is suitable for parameters that are less than unity. Numerical values of theta close to 1 or out of range result in Inf, -Inf, NA or NaN.

**Value**

For deriv = 0, the log of theta, i.e.,  $\log(1 - \text{theta})$  when inverse = FALSE, and if inverse = TRUE then  $1 - \exp(\text{theta})$ .

For deriv = 1, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if inverse = FALSE, else if inverse = TRUE then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

Numerical instability may occur when theta is close to 1. One way of overcoming this is to use bvalue.

**Author(s)**

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[Links](#), [loge](#), [cloglog](#), [loglog](#), [logoff](#).

## Examples

```
## Not run:
logc(seq(-0.2, 1.1, by = 0.1)) # Has NAs

## End(Not run)
logc(seq(-0.2, 1.1, by = 0.1), bvalue = 1 - .Machine$double.eps) # Has no NAs
```

---

loge	<i>Log link function, and variants</i>
------	--

---

## Description

Computes the log transformation, including its inverse and the first two derivatives.

## Usage

```
loge(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
      short = TRUE, tag = FALSE)
negloge(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
        short = TRUE, tag = FALSE)
logneg(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
        short = TRUE, tag = FALSE)
```

## Arguments

theta	Numeric or character. See below for further details.
bvalue	See <a href="#">Links</a> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

## Details

The log link function is very commonly used for parameters that are positive. Here, all logarithms are natural logarithms, i.e., to base  $e$ . Numerical values of theta close to 0 or out of range result in Inf, -Inf, NA or NaN.

The function loge computes  $\log(\theta)$  whereas negloge computes  $-\log(\theta) = \log(1/\theta)$ .

The function logneg computes  $\log(-\theta)$ , hence is suitable for parameters that are negative, e.g., a trap-shy effect in [posbernoulli.b](#).

**Value**

The following concerns loge. For deriv = 0, the log of theta, i.e., log(theta) when inverse = FALSE, and if inverse = TRUE then exp(theta). For deriv = 1, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if inverse = FALSE, else if inverse = TRUE then it returns the reciprocal.

**Note**

This function is called loge to avoid conflict with the [log](#) function.

Numerical instability may occur when theta is close to 0 unless bvalue is used.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [explink](#), [logit](#), [logc](#), [loglog](#), [log](#), [logoff](#), [lambertW](#), [posbernoulli.b](#).

**Examples**

```
## Not run: loge(seq(-0.2, 0.5, by = 0.1))
loge(seq(-0.2, 0.5, by = 0.1), bvalue = .Machine$double.xmin)
negloge(seq(-0.2, 0.5, by = 0.1))
negloge(seq(-0.2, 0.5, by = 0.1), bvalue = .Machine$double.xmin)
## End(Not run)
logneg(seq(-0.5, -0.2, by = 0.1))
```

---

logF

*Natural Exponential Family Generalized Hyperbolic Secant Distribution Family Function*

---

**Description**

Maximum likelihood estimation of the 1-parameter log F distribution.

**Usage**

```
logF(lshape1 = "loge", lshape2 = "loge",
     ishape1 = NULL, ishape2 = 1, imethod = 1)
```

**Arguments**

<code>lshape1</code> , <code>lshape2</code>	Parameter link functions for the shape parameters. Called $\alpha$ and $\beta$ respectively. See <a href="#">Links</a> for more choices.
<code>ishape1</code> , <code>ishape2</code>	Optional initial values for the shape parameters. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally. See <a href="#">CommonVGAMffArguments</a> for more information.
<code>imethod</code>	Initialization method. Either the value 1, 2, or ... See <a href="#">CommonVGAMffArguments</a> for more information.

**Details**

The density for this distribution is

$$f(y; \alpha, \beta) = \exp(\alpha y) / [B(\alpha, \beta)(1 + e^y)^{\alpha + \beta}]$$

where  $y$  is real,  $\alpha > 0$ ,  $\beta > 0$ ,  $B(., .)$  is the beta function [beta](#).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

Thomas W. Yee

**References**

Jones, M. C. (2008). On a class of distributions with simple exponential tails. *Statistica Sinica*, **18**(3), 1101–1110.

**See Also**

[dlogF](#), [logff](#).

**Examples**

```
nn <- 1000
ldata <- data.frame(y1 = rnorm(nn, m = +1, sd = exp(2)), # Not proper data
                    x2 = rnorm(nn, m = -1, sd = exp(2)),
                    y2 = rnorm(nn, m = -1, sd = exp(2))) # Not proper data
fit1 <- vglm(y1 ~ 1, logF, data = ldata, trace = TRUE)
fit2 <- vglm(y2 ~ x2, logF, data = ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
summary(fit2)
vcov(fit2)

head(fitted(fit1))
with(ldata, mean(y1))
max(abs(head(fitted(fit1)) - with(ldata, mean(y1))))
```

logff

*Logarithmic Distribution***Description**

Estimating the (single) parameter of the logarithmic distribution.

**Usage**

```
logff(link = "logit", init.c = NULL, zero = NULL)
```

**Arguments**

link	Parameter link function for the parameter $c$ , which lies between 0 and 1. See <a href="#">Links</a> for more choices and information.
init.c	Optional initial value for the $c$ parameter. If given, it often pays to start with a larger value, e.g., 0.95. The default is to choose an initial value internally.
zero	Details at <a href="#">CommonVGAMffArguments</a> .

**Details**

The logarithmic distribution is based on the logarithmic series, and is scaled to a probability function. Its probability function is  $f(y) = ac^y/y$ , for  $y = 1, 2, 3, \dots$ , where  $0 < c < 1$ , and  $a = -1/\log(1 - c)$ . The mean is  $ac/(1 - c)$  (returned as the fitted values) and variance is  $ac(1 - ac)/(1 - c)^2$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

The function [log](#) computes the natural logarithm. In the **VGAM** library, a link function with option [loge](#) corresponds to this.

Multiple responses are permitted.

The logarithmic distribution is sometimes confused with the log-series. The logarithmic distribution is sometimes confused with the log-series distribution. The latter was used by Fisher et al. for species abundance data, and has two parameters.

**Author(s)**

T. W. Yee

## References

Chapter 7 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey: Wiley.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

[rlog](#), [log](#), [loge](#), [logoff](#), [explogff](#).

## Examples

```
ldata <- data.frame(y = rlog(n = 1000, prob = logit(0.2, inverse = TRUE)))
fit <- vglm(y ~ 1, logff, ldata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
Coef(fit)
## Not run: with(ldata,
  hist(y, prob = TRUE, breaks = seq(0.5, max(y) + 0.5, by = 1),
    border = "blue"))
x <- seq(1, with(ldata, max(y)), by = 1)
with(ldata, lines(x, dlog(x, Coef(fit)[1]), col = "orange", type = "h", lwd = 2))
## End(Not run)

# Example: Corbet (1943) butterfly Malaya data
corbet <- data.frame(nindiv = 1:24,
  ofreq = c(118, 74, 44, 24, 29, 22, 20, 19, 20, 15, 12,
    14, 6, 12, 6, 9, 9, 6, 10, 10, 11, 5, 3, 3))
fit <- vglm(nindiv ~ 1, logff, data = corbet, weights = ofreq)
coef(fit, matrix = TRUE)
chat <- Coef(fit)["c"]
pdf2 <- dlog(x = with(corbet, nindiv), prob = chat)
print(with(corbet, cbind(nindiv, ofreq, fitted = pdf2 * sum(ofreq))), digits = 1)
```

---

logistic

---

*Logistic Distribution Family Function*


---

## Description

Estimates the location and scale parameters of the logistic distribution by maximum likelihood estimation.

## Usage

```
logistic1(llocation = "identity", scale.arg = 1, imethod = 1)
logistic2(llocation = "identity", lscale = "loge",
  ilocation = NULL, iscale = NULL, imethod = 1, zero = -2)
```

**Arguments**

- `llocation`, `lscale`      Parameter link functions applied to the location parameter  $l$  and scale parameter  $s$ . See [Links](#) for more choices, and [CommonVGAMffArguments](#) for more information.
- `scale.arg`      Known positive scale parameter (called  $s$  below).
- `ilocation`, `iscale`      See [CommonVGAMffArguments](#) for more information.
- `imethod`, `zero`      See [CommonVGAMffArguments](#) for more information.

**Details**

The two-parameter logistic distribution has a density that can be written as

$$f(y; l, s) = \frac{\exp[-(y - l)/s]}{s (1 + \exp[-(y - l)/s])^2}$$

where  $s > 0$  is the scale parameter, and  $l$  is the location parameter. The response  $-\infty < y < \infty$ . The mean of  $Y$  (which is the fitted value) is  $l$  and its variance is  $\pi^2 s^2 / 3$ .

A logistic distribution with `scale = 0.65` (see [dlogis](#)) resembles [dt](#) with `df = 7`; see [logistic1](#) and [studentt](#).

`logistic1` estimates the location parameter only while `logistic2` estimates both parameters. By default,  $\eta_1 = l$  and  $\eta_2 = \log(s)$  for `logistic2`.

`logistic2` can handle multiple responses.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Note**

Fisher scoring is used, and the Fisher information matrix is diagonal.

**Author(s)**

T. W. Yee

**References**

- Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley. Chapter 15.
- Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.
- Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience, p.130.
- deCani, J. S. and Stine, R. A. (1986) A note on Deriving the Information Matrix for a Logistic Distribution, *The American Statistician*, **40**, 220–222.

See Also

[rlogis](#), [logit](#), [cumulative](#), [bilogistic4](#).

Examples

```
# Location unknown, scale known
ldata <- data.frame(x2 = runif(nn <- 500))
ldata <- transform(ldata, y1 = rlogis(nn, loc = 1 + 5*x2, scale = exp(2)))
fit1 <- vglm(y1 ~ x2, logistic1(scale = 4), ldata, trace = TRUE, crit = "c")
coef(fit1, matrix = TRUE)

# Both location and scale unknown
ldata <- transform(ldata, y2 = rlogis(nn, loc = 1 + 5*x2, scale = exp(0 + 1*x2)))
fit2 <- vglm(cbind(y1, y2) ~ x2, logistic2, ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
vcov(fit2)
summary(fit2)
```

---

logit	<i>Logit Link Function</i>
-------	----------------------------

---

Description

Computes the logit transformation, including its inverse and the first two derivatives.

Usage

```
logit(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
      short = TRUE, tag = FALSE)
elogit(theta, min = 0, max = 1, bminvalue = NULL, bmaxvalue = NULL,
       inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

Arguments

- theta            Numeric or character. See below for further details.
- bvalue, bminvalue, bmaxvalue  
                 See [Links](#). These are boundary values. For elogit, values of theta less than or equal to *A* or greater than or equal to *B* can be replaced by bminvalue and bmaxvalue.
- min, max        For elogit, min gives *A*, max gives *B*, and for out of range values, bminvalue and bmaxvalue.
- inverse, deriv, short, tag  
                 Details at [Links](#).



## Details

The logit link function is very commonly used for parameters that lie in the unit interval. Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.

The *extended* logit link function `elogit` should be used more generally for parameters that lie in the interval  $(A, B)$ , say. The formula is

$$\log((\theta - A)/(B - \theta))$$

and the default values for  $A$  and  $B$  correspond to the ordinary logit function. Numerical values of theta close to  $A$  or  $B$  or out of range result in Inf, -Inf, NA or NaN. However these can be replaced by values *bminvalue* and *bmaxvalue* first before computing the link function.

## Value

For `logit` with `deriv = 0`, the logit of theta, i.e.,  $\log(\text{theta}/(1-\text{theta}))$  when `inverse = FALSE`, and if `inverse = TRUE` then  $\exp(\text{theta})/(1+\exp(\text{theta}))$ .

For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

## Note

Numerical instability may occur when theta is close to 1 or 0 (for `logit`), or close to  $A$  or  $B$  for `elogit`. One way of overcoming this is to use, e.g., `bvalue`.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the univariate logistic distribution (see [logistic](#)).

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[Links](#), [probit](#), [cloglog](#), [cauchit](#), [logistic1](#), [loge](#), [mlogit](#).

## Examples

```
p <- seq(0.01, 0.99, by = 0.01)
logit(p)
max(abs(logit(logit(p), inverse = TRUE) - p)) # Should be 0

p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
logit(p) # Has NAs
logit(p, bvalue = .Machine$double.eps) # Has no NAs
```

```

p <- seq(0.9, 2.2, by = 0.1)
elogit(p, min = 1, max = 2,
       bminvalue = 1 + .Machine$double.eps,
       bmaxvalue = 2 - .Machine$double.eps) # Has no NAs

## Not run: par(mfrow = c(2,2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
p <- seq(0.01, 0.99, by = 0.01)
for (d in 0:1) {
  matplot(p, cbind(logit(p, deriv = d), probit(p, deriv = d)),
          type = "n", col = "purple", ylab = "transformation", las = 1,
          main = if (d == 0) "Some probability link functions"
                else "First derivative")
  lines(p, logit(p, deriv = d), col = "limegreen")
  lines(p, probit(p, deriv = d), col = "purple")
  lines(p, cloglog(p, deriv = d), col = "chocolate")
  lines(p, cauchit(p, deriv = d), col = "tan")
  if (d == 0) {
    abline(v = 0.5, h = 0, lty = "dashed")
    legend(0, 4.5, c("logit", "probit", "cloglog", "cauchit"),
          col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
  } else
    abline(v = 0.5, lty = "dashed")
}

for (d in 0) {
  matplot(y, cbind(logit(y, deriv = d, inverse = TRUE),
                  probit(y, deriv = d, inverse = TRUE)), las = 1,
          type = "n", col = "purple", xlab = "transformation", ylab = "p",
          main = if (d == 0) "Some inverse probability link functions"
                else "First derivative")
  lines(y, logit(y, deriv = d, inverse = TRUE), col = "limegreen")
  lines(y, probit(y, deriv = d, inverse = TRUE), col = "purple")
  lines(y, cloglog(y, deriv = d, inverse = TRUE), col = "chocolate")
  lines(y, cauchit(y, deriv = d, inverse = TRUE), col = "tan")
  if (d == 0) {
    abline(h = 0.5, v = 0, lty = "dashed")
    legend(-4, 1, c("logit", "probit", "cloglog", "cauchit"),
          col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
  }
}

p <- seq(0.21, 0.59, by = 0.01)
plot(p, elogit(p, min = 0.2, max = 0.6),
     type = "l", col = "black", ylab = "transformation", xlim = c(0, 1),
     las = 1, main = "elogit(p, min = 0.2, max = 0.6)")
par(lwd = 1)

## End(Not run)

```

loglaplace

*Log-Laplace and Logit-Laplace Distribution Family Functions***Description**

Maximum likelihood estimation of the 1-parameter log-Laplace and the 1-parameter logit-Laplace distributions. These may be used for quantile regression for counts and proportions respectively.

**Usage**

```
loglaplace1(tau = NULL, llocation = "loge",
            ilocation = NULL, kappa = sqrt(tau/(1 - tau)), Scale.arg = 1,
            shrinkage.init = 0.95, parallelLocation = FALSE, digt = 4,
            dfmu.init = 3, rep0 = 0.5, minquantile = 0, maxquantile = Inf,
            imethod = 1, zero = NULL)
logitlaplace1(tau = NULL, llocation = "logit",
              ilocation = NULL, kappa = sqrt(tau/(1 - tau)),
              Scale.arg = 1, shrinkage.init = 0.95, parallelLocation = FALSE,
              digt = 4, dfmu.init = 3, rep01 = 0.5, imethod = 1, zero = NULL)
```

**Arguments**

tau, kappa	See <a href="#">alaplace1</a> .
llocation	Character. Parameter link functions for location parameter $\xi$ . See <a href="#">Links</a> for more choices. However, this argument should be left unchanged with count data because it restricts the quantiles to be positive. With proportions data llocation can be assigned a link such as <a href="#">logit</a> , <a href="#">probit</a> , <a href="#">cloglog</a> , etc.
ilocation	Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.
parallelLocation	Logical. Should the quantiles be parallel on the transformed scale (argument llocation)? Assigning this argument to TRUE circumvents the seriously embarrassing quantile crossing problem.
imethod	Initialization method. Either the value 1, 2, or ....
dfmu.init, shrinkage.init, Scale.arg, digt, zero	See <a href="#">alaplace1</a> .
rep0, rep01	Numeric, positive. Replacement values for 0s and 1s respectively. For count data, values of the response whose value is 0 are replaced by rep0; it avoids computing $\log(0)$ . For proportions data values of the response whose value is 0 or 1 are replaced by $\min(\text{rangey01}[1]/2, \text{rep01}/w[y <= 0])$ and $\max((1 + \text{rangey01}[2])/2, 1 - \text{rep01})$ respectively; e.g., it avoids computing $\text{logit}(0)$ or $\text{logit}(1)$ . Here, rangey01 is the 2-vector $\text{range}(y[(y > 0) \& (y < 1)])$ of the response.
minquantile, maxquantile	Numeric. The minimum and maximum values possible in the quantiles. These argument are effectively ignored by default since <a href="#">loge</a> keeps all quantiles positive. However, if llocation = logoff(offset = 1) then it is possible that the fitted quantiles have value 0 because minquantile = 0.

## Details

These **VGAM** family functions implement translations of the asymmetric Laplace distribution (ALD). The resulting variants may be suitable for quantile regression for count data or sample proportions. For example, a log link applied to count data is assumed to follow an ALD. Another example is a logit link applied to proportions data so as to follow an ALD. A positive random variable  $Y$  is said to have a log-Laplace distribution if  $Y = e^W$  where  $W$  has an ALD. There are many variants of ALDs and the one used here is described in [alaplace1](#).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

In the extra slot of the fitted object are some list components which are useful. For example, the sample proportion of values which are less than the fitted quantile curves, which is  $\text{sum}(\text{wprior}[y \leq \text{location}]) / \text{sum}(\text{wprior})$  internally. Here, `wprior` are the prior weights (called `ssize` below), `y` is the response and `location` is a fitted quantile curve. This definition comes about naturally from the transformed ALD data.

## Warning

The **VGAM** family function [logitlaplace1](#) will not handle a vector of just 0s and 1s as the response; it will only work satisfactorily if the number of trials is large.

See [alaplace1](#) for other warnings. Care is needed with tau values which are too small, e.g., for count data the sample proportion of zeros must be less than all values in tau. Similarly, this also holds with [logitlaplace1](#), which also requires all tau values to be less than the sample proportion of ones.

## Note

The form of input for [logitlaplace1](#) as response is a vector of proportions (values in  $[0, 1]$ ) and the number of trials is entered into the `weights` argument of [vglm/vgam](#). See Example 2 below. See [alaplace1](#) for other notes in general.

## Author(s)

Thomas W. Yee

## References

- Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001) *The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance*, Boston: Birkhauser.
- Kozubowski, T. J. and Podgorski, K. (2003) Log-Laplace distributions. *International Mathematical Journal*, **3**, 467–495.
- Yee, T. W. (2012) Quantile regression for counts and proportions. In preparation.

## See Also

[alaplace1](#), [dloglap](#).

## Examples

```

# Example 1: quantile regression of counts with regression splines
set.seed(123); my.k <- exp(0)
alldat <- data.frame(x2 = sort(runif(n <- 500)))
mymu <- function(x) exp( 1 + 3*sin(2*x) / (x+0.5)^2)
alldat <- transform(alldat, y = rnbino(n, mu = mymu(x2), size = my.k))
mytau <- c(0.1, 0.25, 0.5, 0.75, 0.9); mydof = 3
fitp <- vglm(y ~ bs(x2, df = mydof), data=alldat, trace = TRUE,
             loglaplace1(tau = mytau, parallelLoc = TRUE)) # halfstepping is usual

## Not run:
par(las = 1) # Plot on a log1p() scale
mylwd <- 1.5
with(alldat, plot(x2, jitter(log1p(y), factor = 1.5), col = "red", pch = "o",
                 main = "Example 1; darkgreen=truth, blue=estimated", cex = 0.75))
with(alldat, matlines(x2, log1p(fitted(fitp)), col = "blue", lty = 1, lwd = mylwd))
finexgrid <- seq(0, 1, len = 201)
for (ii in 1:length(mytau))
  lines(finexgrid, col = "darkgreen", lwd = mylwd,
        log1p(qnbinom(p = mytau[ii], mu = mymu(finexgrid), si = my.k)))

## End(Not run)
fitp@extra # Contains useful information

# Example 2: sample proportions
set.seed(123); nnn <- 1000; ssize <- 100 # ssize = 1 will not work!
alldat <- data.frame(x2 = sort(runif(nnn)))
mymu <- function(x) logit( 1.0 + 4*x, inv = TRUE)
alldat <- transform(alldat, ssize = ssize,
                   y2 = rbinom(nnn, size=ssize, prob = mymu(x2)) / ssize)

mytau <- c(0.25, 0.50, 0.75)
fit1 <- vglm(y2 ~ bs(x2, df = 3), data=alldat, weights=ssize, trace = TRUE,
            logitlaplace1(tau = mytau, lloc = "cloglog", paral = TRUE))

## Not run:
# Check the solution. Note: this may be like comparing apples with oranges.
plotvgam(fit1, se = TRUE, scol = "red", lcol = "blue", main = "Truth = 'darkgreen'")
# Centered approximately !
linkFunctionChar = as.character(fit1@misc$link)
alldat = transform(alldat, trueFunction=
                  theta2eta(theta = mymu(x2), link=linkFunctionChar))
with(alldat, lines(x2, trueFunction - mean(trueFunction), col = "darkgreen"))

# Plot the data + fitted quantiles (on the original scale)
myylim <- with(alldat, range(y2))
with(alldat, plot(x2, y2, col = "blue", ylim = myylim, las = 1, pch = ".", cex = 2.5))
with(alldat, matplot(x2, fitted(fit1), add = TRUE, lwd = 3, type = "l"))
truecol <- rep(1:3, len = fit1@misc$M) # Add the 'truth'
smallxgrid <- seq(0, 1, len = 501)

```

```

for (ii in 1:length(mytau))
  lines(smallxgrid, col = truecol[ii], lwd = 2,
        qbinom(p = mytau[ii], prob = mymu(smallxgrid), size = ssize) / ssize)

# Plot on the eta (== logit()/probit()/...) scale
with(alldat, matplot(x2, predict(fit1), add = FALSE, lwd = 3, type = "l"))
# Add the 'truth'
for (ii in 1:length(mytau)) {
  true.quant <- qbinom(p = mytau[ii], pr = mymu(smallxgrid), si = ssize) / ssize
  lines(smallxgrid, theta2eta(theta = true.quant, link = linkFunctionChar),
        col = truecol[ii], lwd = 2)
}

## End(Not run)

```

loglapUC

*The Log-Laplace Distribution***Description**

Density, distribution function, quantile function and random generation for the 3-parameter log-Laplace distribution with location parameter `location.ald`, scale parameter `scale.ald` (on the log scale), and asymmetry parameter `kappa`.

**Usage**

```

dloglap(x, location.ald = 0, scale.ald = 1,
        tau = 0.5, kappa = sqrt(tau/(1-tau)), log = FALSE)
ploglap(q, location.ald = 0, scale.ald = 1,
        tau = 0.5, kappa = sqrt(tau/(1-tau)))
qloglap(p, location.ald = 0, scale.ald = 1,
        tau = 0.5, kappa = sqrt(tau/(1-tau)))
rloglap(n, location.ald = 0, scale.ald = 1,
        tau = 0.5, kappa = sqrt(tau/(1-tau)))

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>location.ald, scale.ald</code>	the location parameter $\xi$ and the (positive) scale parameter $\sigma$ , on the log scale.
<code>tau</code>	the quantile parameter $\tau$ . Must consist of values in $(0, 1)$ . This argument is used to specify <code>kappa</code> and is ignored if <code>kappa</code> is assigned.
<code>kappa</code>	the asymmetry parameter $\kappa$ . Must consist of positive values.
<code>log</code>	if TRUE, probabilities <code>p</code> are given as <code>log(p)</code> .

## Details

A positive random variable  $Y$  is said to have a log-Laplace distribution if  $\log(Y)$  has an asymmetric Laplace distribution (ALD). There are many variants of ALDs and the one used here is described in [alaplace3](#).

## Value

dloglap gives the density, ploglap gives the distribution function, qloglap gives the quantile function, and rloglap generates random deviates.

## Author(s)

T. W. Yee

## References

Kozubowski, T. J. and Podgorski, K. (2003) Log-Laplace distributions. *International Mathematical Journal*, **3**, 467–495.

## See Also

[dalap](#), [alaplace3](#),  
[loglaplace1](#).

## Examples

```
loc <- 0; sigma <- exp(0.5); kappa <- 1
x <- seq(-0.2, 5, by = 0.01)
## Not run:
plot(x, dloglap(x, loc, sigma, kappa = kappa), type = "l", col = "blue",
     main = "Blue is density, red is cumulative distribution function",
     ylim = c(0,1), sub = "Purple are 5,10,...,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
      dloglap(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
              loc, sigma, kappa = kappa), col = "purple", lty = 3, type = "h")
lines(x, ploglap(x, loc, sigma, kappa = kappa), type = "l", col = "red")
abline(h = 0, lty = 2)

## End(Not run)
ploglap(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
        loc, sigma, kappa = kappa)
```

loglinb2

*Loglinear Model for Two Binary Responses***Description**

Fits a loglinear model to two binary responses.

**Usage**

```
loglinb2(exchangeable = FALSE, zero = 3)
```

**Arguments**

exchangeable	Logical. If TRUE, the two marginal probabilities are constrained to be equal. Should be set TRUE for ears, eyes, etc. data.
zero	Which linear/additive predictor is modelled as an intercept only? A NULL means none of them.

**Details**

The model is

$$P(Y_1 = y_1, Y_2 = y_2) = \exp(u_0 + u_1 y_1 + u_2 y_2 + u_{12} y_1 y_2)$$

where  $y_1$  and  $y_2$  are 0 or 1, and the parameters are  $u_1, u_2, u_{12}$ . The normalizing parameter  $u_0$  can be expressed as a function of the other parameters, viz.,

$$u_0 = -\log[1 + \exp(u_1) + \exp(u_2) + \exp(u_1 + u_2 + u_{12})].$$

The linear/additive predictors are  $(\eta_1, \eta_2, \eta_3)^T = (u_1, u_2, u_{12})^T$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

When fitted, the `fitted.values` slot of the object contains the four joint probabilities, labelled as  $(Y_1, Y_2) = (0,0), (0,1), (1,0), (1,1)$ , respectively.

**Note**

The response must be a two-column matrix of ones and zeros only. This is more restrictive than [binom2.or](#), which can handle more types of input formats. Note that each of the 4 combinations of the multivariate response need to appear in the data set.

**Author(s)**

Thomas W. Yee



## References

Yee, T. W. and Wild, C. J. (2001) Discussion to: “Smoothing spline ANOVA for multivariate Bernoulli observations, with application to ophthalmology data (with discussion)” by Gao, F., Wahba, G., Klein, R., Klein, B. *Journal of the American Statistical Association*, **96**, 127–160.

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[binom2.or](#), [binom2.rho](#), [loglinb3](#).

## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)

# Get the n x 4 matrix of counts
fit0 <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age, binom2.or, data = coalminers)
counts <- round(c(weights(fit0, type = "prior")) * depvar(fit0))

# Create a n x 2 matrix response for loglinb2()
# bwmat <- matrix(c(0,0, 0,1, 1,0, 1,1), 4, 2, byrow = TRUE)
bwmat <- cbind(bln = c(0,0,1,1), wheeze = c(0,1,0,1))
matof1 <- matrix(1, nrow(counts), 1)
newminers <- data.frame(bln      = kronecker(matof1, bwmat[, 1]),
                        wheeze = kronecker(matof1, bwmat[, 2]),
                        wt      = c(t(counts)),
                        Age     = with(coalminers, rep(age, rep(4, length(age)))))
newminers <- newminers[with(newminers, wt) > 0,]

fit <- vglm(cbind(bln,wheeze) ~ Age, loglinb2(zero = NULL),
            weight = wt, data = newminers)
coef(fit, matrix = TRUE) # Same! (at least for the log odds-ratio)
summary(fit)

# Try reconcile this with McCullagh and Nelder (1989), p.234
(0.166-0.131) / 0.027458 # 1.275 is approximately 1.25
```

---

loglinb3

---

*Loglinear Model for Three Binary Responses*


---

## Description

Fits a loglinear model to three binary responses.

## Usage

```
loglinb3(exchangeable = FALSE, zero = 4:6)
```

### Arguments

exchangeable	Logical. If TRUE, the three marginal probabilities are constrained to be equal.
zero	Which linear/additive predictor is modelled as an intercept only? A NULL means none.

### Details

The model is  $P(Y_1 = y_1, Y_2 = y_2, Y_3 = y_3) =$

$$\exp(u_0 + u_1 y_1 + u_2 y_2 + u_3 y_3 + u_{12} y_1 y_2 + u_{13} y_1 y_3 + u_{23} y_2 y_3)$$

where  $y_1$ ,  $y_2$  and  $y_3$  are 0 or 1, and the parameters are  $u_1$ ,  $u_2$ ,  $u_3$ ,  $u_{12}$ ,  $u_{13}$ ,  $u_{23}$ . The normalizing parameter  $u_0$  can be expressed as a function of the other parameters. Note that a third-order association parameter,  $u_{123}$  for the product  $y_1 y_2 y_3$ , is assumed to be zero for this family function.

The linear/additive predictors are  $(\eta_1, \eta_2, \dots, \eta_6)^T = (u_1, u_2, u_3, u_{12}, u_{13}, u_{23})^T$ .

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

When fitted, the `fitted.values` slot of the object contains the eight joint probabilities, labelled as  $(Y_1, Y_2, Y_3) = (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)$ , respectively.

### Note

The response must be a three-column matrix of ones and zeros only. Note that each of the 8 combinations of the multivariate response need to appear in the data set, therefore data sets will need to be large in order for this family function to work.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. and Wild, C. J. (2001) Discussion to: "Smoothing spline ANOVA for multivariate Bernoulli observations, with application to ophthalmology data (with discussion)" by Gao, F., Wahba, G., Klein, R., Klein, B. *Journal of the American Statistical Association*, **96**, 127–160.

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

### See Also

[loglinb2](#), [hunua](#).

**Examples**

```
fit <- vglm(cbind(cyadea, beita, kniexc) ~ altitude, loglinb3, data = hunua)
coef(fit, matrix = TRUE)
head(fitted(fit))
summary(fit)
```

loglog

*Log-log Link Function***Description**

Computes the log-log transformation, including its inverse and the first two derivatives.

**Usage**

```
loglog(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
       short = TRUE, tag = FALSE)
```

**Arguments**

**theta**                Numeric or character. See below for further details.

**bvalue**              Values of theta which are less than or equal to 1 can be replaced by bvalue before computing the link function value. The component name bvalue stands for “boundary value”. See [Links](#) for more information.

**inverse, deriv, short, tag**  
Details at [Links](#).

**Details**

The log-log link function is commonly used for parameters that are greater than unity. Numerical values of theta close to 1 or out of range result in Inf, -Inf, NA or NaN.

**Value**

For `deriv = 0`, the log of theta, i.e.,  $\log(\log(\theta))$  when `inverse = FALSE`, and if `inverse = TRUE` then  $\exp(\exp(\theta))$ .

For `deriv = 1`, then the function returns  $d\theta/d\eta$  as a function of theta if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

Numerical instability may occur when theta is close to 1 unless bvalue is used.

**Author(s)**

Thomas W. Yee

References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

See Also

[Links](#), [loge](#), [logoff](#).

Examples

```
x <- seq(0.8, 1.5, by = 0.1)
loglog(x) # Has NAs
loglog(x, bvalue = 1.0 + .Machine$double.eps) # Has no NAs

x <- seq(1.01, 10, len = 100)
loglog(x)
max(abs(loglog(loglog(x), inverse = TRUE) - x)) # Should be 0
```

---

lognormal	<i>Lognormal Distribution</i>
-----------	-------------------------------

---

Description

Maximum likelihood estimation of the (univariate) lognormal distribution.

Usage

```
lognormal(lmeanlog = "identity", lsdlog = "loge", zero = 2)
lognormal3(lmeanlog = "identity", lsdlog = "loge",
            powers.try = (-3):3, delta = NULL, zero = 2)
```

Arguments

lmeanlog, lsdlog	Parameter link functions applied to the mean and (positive) $\sigma$ (standard deviation) parameter. Both of these are on the log scale. See <a href="#">Links</a> for more choices.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. For <code>lognormal()</code> , the values must be from the set {1,2} which correspond to $\mu$ , $\sigma$ , respectively. For <code>lognormal3()</code> , the values must be from the set {1,2,3} where 3 is for $\lambda$ . See <a href="#">CommonVGAMffArguments</a> for more information.
powers.try	Numerical vector. The initial <i>lambda</i> is chosen as the best value from $\min(y) - 10^{\text{powers.try}}$ where $y$ is the response.
delta	Numerical vector. An alternative method for obtaining an initial <i>lambda</i> . Here, $\text{delta} = \min(y) - \text{lambda}$ . If given, this supersedes the <code>powers.try</code> argument. The value must be positive.

## Details

A random variable  $Y$  has a 2-parameter lognormal distribution if  $\log(Y)$  is distributed  $N(\mu, \sigma^2)$ . The expected value of  $Y$ , which is

$$E(Y) = \exp(\mu + 0.5\sigma^2)$$

and not  $\mu$ , make up the fitted values.

A random variable  $Y$  has a 3-parameter lognormal distribution if  $\log(Y - \lambda)$  is distributed  $N(\mu, \sigma^2)$ . Here,  $\lambda < Y$ . The expected value of  $Y$ , which is

$$E(Y) = \lambda + \exp(\mu + 0.5\sigma^2)$$

and not  $\mu$ , make up the fitted values.

`lognormal()` and `lognormal3()` fit the 2- and 3-parameter lognormal distribution respectively. Clearly, if the location parameter  $\lambda = 0$  then both distributions coincide.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

Regularity conditions are not satisfied for the 3-parameter case: results may be erroneous. May withdraw it in later versions.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

[rlnorm](#), [uninormal](#), [CommonVGAMffArguments](#).

## Examples

```
ldata <- data.frame(y1 = rlnorm(nn <- 1000, meanlog = 1.5, sdlog = exp(-0.8)))
fit1 <- vglm(y1 ~ 1, lognormal, ldata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)

ldata2 <- data.frame(x2 = runif(nn <- 1000))
ldata2 <- transform(ldata2, y2 = rlnorm(nn, mean = 0.5, sd = exp(x2)))
fit2 <- vglm(y2 ~ x2, lognormal(zero = 1), ldata2, trace = TRUE, crit = "c")
coef(fit2, matrix = TRUE)
```

```

Coef(fit2)

lambda <- 4
ldata3 <- data.frame(y3 = lambda + rlnorm(n = 1000, mean = 1.5, sd = exp(-0.8)))
fit3 <- vglm(y3 ~ 1, lognormal3, ldata3, trace = TRUE, crit = "c")
coef(fit3, matrix = TRUE)
summary(fit3)

```

logoff

*Log link function with an offset*

## Description

Computes the log transformation with an offset, including its inverse and the first two derivatives.

## Usage

```
logoff(theta, offset = 0, inverse = FALSE, deriv = 0,
       short = TRUE, tag = FALSE)
```

## Arguments

theta            Numeric or character. See below for further details.  
offset            Offset value. See [Links](#).  
inverse, deriv, short, tag  
                  Details at [Links](#).

## Details

The log-offset link function is very commonly used for parameters that are greater than a certain value. In particular, it is defined by  $\log(\text{theta} + \text{offset})$  where `offset` is the offset value. For example, if `offset = 0.5` then the value of `theta` is restricted to be greater than  $-0.5$ .

Numerical values of `theta` close to `-offset` or out of range result in `Inf`, `-Inf`, `NA` or `NaN`.

## Value

For `deriv = 0`, the log of `theta+offset`, i.e.,  $\log(\text{theta} + \text{offset})$  when `inverse = FALSE`, and if `inverse = TRUE` then  $\exp(\text{theta}) - \text{offset}$ .

For `deriv = 1`, then the function returns  $d\text{theta}/d\text{eta}$  as a function of `theta` if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

## Note

The default means this function is identical to [loge](#).

Numerical instability may occur when `theta` is close to `-offset`.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [loge](#).

**Examples**

```
## Not run:
logoff(seq(-0.2, 0.5, by = 0.1))
logoff(seq(-0.2, 0.5, by = 0.1), offset = 0.5)
  log(seq(-0.2, 0.5, by = 0.1) + 0.5)
## End(Not run)
```

---

Lomax

---

*The Lomax Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the Lomax distribution with scale parameter *scale* and shape parameter *q*.

**Usage**

```
dlomax(x, scale = 1, shape3.q, log = FALSE)
plomax(q, scale = 1, shape3.q)
qlomax(p, scale = 1, shape3.q)
rlomax(n, scale = 1, shape3.q)
```

**Arguments**

<i>x</i> , <i>q</i>	vector of quantiles.
<i>p</i>	vector of probabilities.
<i>n</i>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<i>scale</i>	scale parameter.
<i>shape3.q</i>	shape parameter.
<i>log</i>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

See [lomax](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

## Value

`dlomax` gives the density, `plomax` gives the distribution function, `qlomax` gives the quantile function, and `rlomax` generates random deviates.

## Note

The Lomax distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

[lomax](#), [genbetaII](#).

## Examples

```
probs <- seq(0.1, 0.9, by = 0.1)
max(abs(plomax(qlomax(p = probs, shape3.q = 1), shape3.q = 1) - probs)) # Should be 0

## Not run: par(mfrow = c(1, 2))
x <- seq(-0.01, 5, len = 401)
plot(x, dexp(x), type = "l", col = "black", ylab = "", las = 1, ylim = c(0, 3),
     main = "Black is standard exponential, others are dlomax(x, shape3.q)")
lines(x, dlomax(x, shape3.q = 1), col = "orange")
lines(x, dlomax(x, shape3.q = 2), col = "blue")
lines(x, dlomax(x, shape3.q = 5), col = "green")
legend("topright", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
     legend = paste("shape3.q =", c(1, 2, 5)))

plot(x, pexp(x), type = "l", col = "black", ylab = "", las = 1,
     main = "Black is standard exponential, others are plomax(x, shape3.q)")
lines(x, plomax(x, shape3.q = 1), col = "orange")
lines(x, plomax(x, shape3.q = 2), col = "blue")
lines(x, plomax(x, shape3.q = 5), col = "green")
legend("bottomright", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
     legend = paste("shape3.q =", c(1, 2, 5)))

## End(Not run)
```



---

lomax	<i>Lomax Distribution Family Function</i>
-------	---

---

## Description

Maximum likelihood estimation of the 2-parameter Lomax distribution.

## Usage

```
lomax(lscale = "loge", lshape3.q = "loge",
      iscale = NULL,  ishape3.q = NULL,
      gshape3.q = exp(-5:5), zero = NULL)
```

## Arguments

`lscale`, `lshape3.q`  
 Parameter link function applied to the (positive) parameters `scale` and `q`. See [Links](#) for more choices.

`iscale`, `ishape3.q`  
 Optional initial values for `scale` and `q`.

`gshape3.q`, `zero`  
 See [CommonVGAMffArguments](#).

## Details

The 2-parameter Lomax distribution is the 4-parameter generalized beta II distribution with shape parameters  $a = p = 1$ . It is probably more widely known as the Pareto (II) distribution. It is also the 3-parameter Singh-Maddala distribution with shape parameter  $a = 1$ , as well as the beta distribution of the second kind with  $p = 1$ . More details can be found in Kleiber and Kotz (2003).

The Lomax distribution has density

$$f(y) = q/[b\{1 + y/b\}^{1+q}]$$

for  $b > 0$ ,  $q > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter `scale`, and  $q$  is a shape parameter. The cumulative distribution function is

$$F(y) = 1 - [1 + (y/b)]^{-q}.$$

The mean is

$$E(Y) = b/(q - 1)$$

provided  $q > 1$ ; these are returned as the fitted values.

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

See the note in [genbetaII](#).

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

**See Also**

[Lomax](#), [genbetaII](#), [betaII](#), [dagum](#), [sinmad](#), [fisk](#), [invlomax](#), [paralogistic](#), [invparalogistic](#).

**Examples**

```
ldata <- data.frame(y = rlomax(n = 1000, scale = exp(1), exp(2)))
fit <- vglm(y ~ 1, lomax, ldata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

lqnorm

---

*Minimizing the L-q norm Family Function*


---

**Description**

Minimizes the L-q norm of residuals in a linear model.

**Usage**

```
lqnorm(qpower = 2, link = "identity",
       imethod = 1, imu = NULL, shrinkage.init = 0.95)
```

**Arguments**

qpower	A single numeric, must be greater than one, called $q$ below. The absolute value of residuals are raised to the power of this argument, and then summed. This quantity is minimized with respect to the regression coefficients.
link	Link function applied to the ‘mean’ $\mu$ . See <a href="#">Links</a> for more details.
imethod	Must be 1, 2 or 3. See <a href="#">CommonVGAMffArguments</a> for more information. Ignored if imu is specified.
imu	Numeric, optional initial values used for the fitted values. The default is to use imethod = 1.

`shrinkage.init` How much shrinkage is used when initializing the fitted values. The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used in conjunction with `imethod = 3`.

## Details

This function minimizes the objective function

$$\sum_{i=1}^n w_i (|y_i - \mu_i|)^q$$

where  $q$  is the argument `qpower`,  $\eta_i = g(\mu_i)$  where  $g$  is the link function, and  $\eta_i$  is the vector of linear/additive predictors. The prior weights  $w_i$  can be inputted using the `weights` argument of `vlm/vglm/vgam` etc.; it should be just a vector here since this function handles only a single vector or one-column response.

Numerical problem will occur when  $q$  is too close to one. Probably reasonable values range from 1.5 and up, say. The value  $q = 2$  corresponds to ordinary least squares while  $q = 1$  corresponds to the MLE of a double exponential (Laplace) distribution. The procedure becomes more sensitive to outliers the larger the value of  $q$ .

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as `vglm`, and `vgam`.

## Warning

Convergence failure is common, therefore the user is advised to be cautious and monitor convergence!

## Note

This **VGAM** family function is an initial attempt to provide a more robust alternative for regression and/or offer a little more flexibility than least squares. The `@misc` slot of the fitted object contains a list component called `objectiveFunction` which is the value of the objective function at the final iteration.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

## See Also

[gaussianff](#).

## Examples

```

set.seed(123)
ldata <- data.frame(x = sort(runif(nn <- 10)))
realfun <- function(x) 4 + 5*x
ldata <- transform(ldata, y = realfun(x) + rnorm(nn, sd = exp(-1)))
# Make the first observation an outlier
ldata <- transform(ldata, y = c(4*y[1], y[-1]), x = c(-1, x[-1]))
fit <- vglm(y ~ x, fam = lqnorm(qpower = 1.2), data = ldata)
coef(fit, matrix = TRUE)
head(fitted(fit))
fit@misc$qpower
fit@misc$objectiveFunction

## Not run:
# Graphical check
with(ldata, plot(x, y, main = paste("LS = red, lqnorm = blue (qpower = ",
                                   fit@misc$qpower, "), truth = black", sep = ""), col = "blue"))
lmfit <- lm(y ~ x, data = ldata)
with(ldata, lines(x, fitted(fit), col = "blue"))
with(ldata, lines(x, lmfit$fitted, col = "red"))
with(ldata, lines(x, realfun(x), col = "black"))
## End(Not run)

```

---

lrtest

*Likelihood Ratio Test of Nested Models*


---

## Description

lrtest is a generic function for carrying out likelihood ratio tests. The default method can be employed for comparing nested VGLMs (see details below).

## Usage

```
lrtest(object, ...)
```

```
lrtest_vglm(object, ..., name = NULL)
```

## Arguments

object	a <a href="#">vglm</a> object. See below for details.
...	further object specifications passed to methods. See below for details.
name	a function for extracting a suitable name/description from a fitted model object. By default the name is queried by calling <a href="#">formula</a> .

## Details

`lrtest` is intended to be a generic function for comparisons of models via asymptotic likelihood ratio tests. The default method consecutively compares the fitted model object with the models passed in `...`. Instead of passing the fitted model objects in `...`, several other specifications are possible. The updating mechanism is the same as for `waldtest`: the models in `...` can be specified as integers, characters (both for terms that should be eliminated from the previous model), update formulas or fitted model objects. Except for the last case, the existence of an `update` method is assumed. See `waldtest` for details.

Subsequently, an asymptotic likelihood ratio test for each two consecutive models is carried out: Twice the difference in log-likelihoods (as derived by the `logLik` methods) is compared with a Chi-squared distribution.

## Value

An object of class "VGAManova" which contains a slot with the log-likelihood, degrees of freedom, the difference in degrees of freedom, likelihood ratio Chi-squared statistic and corresponding p value. These are printed by `stats::print.anova()`; see `anova`.

## Warning

Several **VGAM** family functions implement distributions which do not satisfying the usual regularity conditions needed for the LRT to work. No checking or warning is given for these.

## Note

The code was adapted directly from **lmtest** (written by T. Hothorn, A. Zeileis, G. Millo, D. Mitchell) and made to work for VGLMs and S4. This help file also was adapted from **lmtest**.

*Approximate* LRTs might be applied to VGAMs, as produced by `vgam`, but it is probably better in inference to use `vglm` with regression splines (`bs` and `ns`). This methods function should not be applied to other models such as those produced by `rrvglm`, by `cqo`, by `cao`.

## See Also

`lmtest`, `vglm`.

## Examples

```
set.seed(1)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo)
fit2 <- vglm(cbind(normal, mild, severe) ~ let + x3, propodds, pneumo)
fit3 <- vglm(cbind(normal, mild, severe) ~ let, cumulative, pneumo)
# Various equivalent specifications of the LR test for testing x3
(ans1 <- lrtest(fit2, fit1))
ans2 <- lrtest(fit2, 2)
ans3 <- lrtest(fit2, "x3")
ans4 <- lrtest(fit2, . ~ . - x3)
c(all.equal(ans1, ans2), all.equal(ans1, ans3), all.equal(ans1, ans4))
```

```
# Doing it manually
(testStatistic <- 2 * (logLik(fit2) - logLik(fit1)))
(mypval <- pchisq(testStatistic, df = length(coef(fit2)) - length(coef(fit1)),
  lower.tail = FALSE))

(ans4 <- lrtest(fit3, fit1)) # Test proportional odds (parallelism) assumption
```

lvplot

*Latent Variable Plot***Description**

Generic function for a *latent variable plot* (also known as an *ordination diagram* by ecologists).

**Usage**

```
lvplot(object, ...)
```

**Arguments**

object	An object for a latent variable plot is meaningful.
...	Other arguments fed into the specific methods function of the model. They usually are graphical parameters, and sometimes they are fed into the methods function for <a href="#">Coef</a> .

**Details**

Latent variables occur in reduced-rank regression models, as well as in quadratic and additive ordination. For the latter, latent variables are often called the *site scores*. Latent variable plots were coined by Yee (2004), and have the latent variable as at least one of its axes.

**Value**

The value returned depends specifically on the methods function invoked.

**Note**

Latent variables are not really applicable to [vglm](#)/[vgam](#) models.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

**See Also**

[lvplot.qrrvglm](#), [lvplot.cao](#), [latvar](#), [trplot](#).

**Examples**

```
## Not run:
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
set.seed(123)
p1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
  WaterCon + BareSand + FallTwig +
  CoveMoss + CoveHerb + ReflLux,
  family = poissonff, data = hspider, Bestof = 3,
  df1.nl = c(Zoraspin = 2.5, 3), Crowlpositive = TRUE)
index <- 1:ncol(depvar(p1))
lvplot(p1, lcol = index, pcol = index, y = TRUE, las = 1)

## End(Not run)
```

---

lvplot.qrrvglm

*Latent Variable Plot for QO models*


---

**Description**

Produces an ordination diagram (latent variable plot) for quadratic ordination (QO) models. For rank-1 models, the x-axis is the first ordination/constrained/canonical axis. For rank-2 models, the x- and y-axis are the first and second ordination axes respectively.

**Usage**

```
lvplot.qrrvglm(object, varI.latvar = FALSE, reference = NULL,
  add = FALSE, show.plot = TRUE,
  rug = TRUE, y = FALSE, type = c("fitted.values", "predictors"),
  xlab = paste("Latent Variable", if (Rank == 1) "" else " 1", sep = ""),
  ylab = if (Rank == 1) switch(type, predictors = "Predictors",
    fitted.values = "Fitted values") else "Latent Variable 2",
  pcex = par()$cex, pcol = par()$col, pch = par()$pch,
  lty = par()$lty, lcol = par()$col, llwd = par()$lwd,
  label.arg = FALSE, adj.arg = -0.1,
  ellipse = 0.95, Absolute = FALSE,
  elty = par()$lty, ecol = par()$col, elwd = par()$lwd, egrid = 200,
  chull.arg = FALSE, clty = 2, ccol = par()$col, clwd = par()$lwd,
  cpch = " ",
  C = FALSE, OriginC = c("origin", "mean"),
  Clty = par()$lty, Ccol = par()$col, Clwd = par()$lwd,
  Ccex = par()$cex, Cadj.arg = -0.1, stretchC = 1,
  sites = FALSE, spch = NULL, scol = par()$col, scex = par()$cex,
  sfont = par()$font, check.ok = TRUE, ...)
```

**Arguments**

object	A CQO object.
varI.latvar	Logical that is fed into <a href="#">Coef.qrrvglm</a> .
reference	Integer or character that is fed into <a href="#">Coef.qrrvglm</a> .
add	Logical. Add to an existing plot? If FALSE, a new plot is made.
show.plot	Logical. Plot it?
rug	Logical. If TRUE, a rug plot is plotted at the foot of the plot (applies to rank-1 models only). These values are jittered to expose ties.
y	Logical. If TRUE, the responses will be plotted (applies only to rank-1 models and if type = "fitted.values".)
type	Either "fitted.values" or "predictors", specifies whether the y-axis is on the response or eta-scales respectively.
xlab	Caption for the x-axis. See <a href="#">par</a> .
ylab	Caption for the y-axis. See <a href="#">par</a> .
pcex	Character expansion of the points. Here, for rank-1 models, points are the response y data. For rank-2 models, points are the optima. See the cex argument in <a href="#">par</a> .
pcol	Color of the points. See the col argument in <a href="#">par</a> .
pch	Either an integer specifying a symbol or a single character to be used as the default in plotting points. See <a href="#">par</a> . The pch argument can be of length <i>M</i> , the number of species.
lty	Line type. Rank-1 models only. See the lty argument of <a href="#">par</a> .
lcol	Line color. Rank-1 models only. See the col argument of <a href="#">par</a> .
llwd	Line width. Rank-1 models only. See the lwd argument of <a href="#">par</a> .
label.arg	Logical. Label the optima and <b>C</b> ? (applies only to rank-2 models only).
adj.arg	Justification of text strings for labelling the optima (applies only to rank-2 models only). See the adj argument of <a href="#">par</a> .
ellipse	Numerical, of length 0 or 1 (applies only to rank-2 models only). If Absolute is TRUE then ellipse should be assigned a value that is used for the elliptical contouring. If Absolute is FALSE then ellipse should be assigned a value between 0 and 1, for example, setting ellipse = 0.9 means an ellipse with contour = 90% of the maximum will be plotted about each optimum. If ellipse is a negative value, then the function checks that the model is an equal-tolerances model and varI.latvar = FALSE, and if so, plots circles with radius -ellipse. For example, setting ellipse = -1 will result in circular contours that have unit radius (in latent variable units). If ellipse is NULL or FALSE then no ellipse is drawn around the optima.
Absolute	Logical. If TRUE, the contours corresponding to ellipse are on an absolute scale. If FALSE, the contours corresponding to ellipse are on a relative scale.
elty	Line type of the ellipses. See the lty argument of <a href="#">par</a> .
ecol	Line color of the ellipses. See the col argument of <a href="#">par</a> .



elwd	Line width of the ellipses. See the lwd argument of <a href="#">par</a> .
egrid	Numerical. Line resolution of the ellipses. Choosing a larger value will result in smoother ellipses. Useful when ellipses are large.
chull.arg	Logical. Add a convex hull around the site scores?
clty	Line type of the convex hull. See the lty argument of <a href="#">par</a> .
ccol	Line color of the convex hull. See the col argument of <a href="#">par</a> .
clwd	Line width of the convex hull. See the lwd argument of <a href="#">par</a> .
cpch	Character to be plotted at the intersection points of the convex hull. Having white spaces means that site labels are not obscured there. See the pch argument of <a href="#">par</a> .
C	Logical. Add C (represented by arrows emanating from OriginC) to the plot?
OriginC	Character or numeric. Where the arrows representing C emanate from. If character, it must be one of the choices given. By default the first is chosen. The value "origin" means $c(0,0)$ . The value "mean" means the sample mean of the latent variables (centroid). Alternatively, the user may specify a numerical vector of length 2.
Clty	Line type of the arrows representing C. See the lty argument of <a href="#">par</a> .
Ccol	Line color of the arrows representing C. See the col argument of <a href="#">par</a> .
Clwd	Line width of the arrows representing C. See the lwd argument of <a href="#">par</a> .
Ccex	Numeric. Character expansion of the labelling of C. See the cex argument of <a href="#">par</a> .
Cadj.arg	Justification of text strings when labelling C. See the adj argument of <a href="#">par</a> .
stretchC	Numerical. Stretching factor for C. Instead of using C, $\text{stretchC} * C$ is used.
sites	Logical. Add the site scores (aka latent variable values, nu's) to the plot? (applies only to rank-2 models only).
spch	Plotting character of the site scores. The default value of NULL means the row labels of the data frame are used. They often are the site numbers. See the pch argument of <a href="#">par</a> .
scol	Color of the site scores. See the col argument of <a href="#">par</a> .
scex	Character expansion of the site scores. See the cex argument of <a href="#">par</a> .
sfont	Font used for the site scores. See the font argument of <a href="#">par</a> .
check.ok	Logical. Whether a check is performed to see that $\text{noRRR} = \sim 1$ was used. It doesn't make sense to have a latent variable plot unless this is so.
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include xlim and ylim.

## Details

This function only works for rank-1 and rank-2 QRR-VGLMs with argument  $\text{noRRR} = \sim 1$ .

For unequal-tolerances models, the latent variable axes can be rotated so that at least one of the tolerance matrices is diagonal; see [Coef.qrrvglm](#) for details.

Arguments beginning with "p" correspond to the points e.g., pcex and pcol correspond to the size and color of the points. Such "p" arguments should be vectors of length 1, or  $n$ , the number of sites. For the rank-2 model, arguments beginning with "p" correspond to the optima.

**Value**

Returns a matrix of latent variables (site scores) regardless of whether a plot was produced or not.

**Warning**

Interpretation of a latent variable plot (CQO diagram) is potentially very misleading in terms of distances if (i) the tolerance matrices of the species are unequal and (ii) the contours of these tolerance matrices are not included in the ordination diagram.

**Note**

A species which does not have an optimum will not have an ellipse drawn even if requested, i.e., if its tolerance matrix is not positive-definite.

Plotting **C** gives a visual display of the weights (loadings) of each of the variables used in the linear combination defining each latent variable.

The arguments `elty`, `ecol` and `elwd`, may be replaced in the future by `llty`, `lcol` and `llwd`, respectively.

For rank-1 models, a similar function to this one is [perspqrvglm](#). It plots the fitted values on a more fine grid rather than at the actual site scores here. The result is a collection of smooth bell-shaped curves. However, it has the weakness that the plot is more divorced from the data; the user thinks it is the truth without an appreciation of the statistical variability in the estimates.

In the example below, the data comes from an equal-tolerances model. The species' tolerance matrices are all the identity matrix, and the optimums are at (0,0), (1,1) and (-2,0) for species 1, 2, 3 respectively.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

**See Also**

[lvplot](#), [perspqrvglm](#), [Coef.qrrvglm](#), [par](#), [cqo](#).

**Examples**

```
set.seed(123)
nn <- 200
cdata <- data.frame(x2 = rnorm(nn), # Has mean 0 (needed when ITol=TRUE)
                   x3 = rnorm(nn), # Has mean 0 (needed when ITol=TRUE)
                   x4 = rnorm(nn)) # Has mean 0 (needed when ITol=TRUE)
cdata <- transform(cdata, latvar1 = x2 + x3 - 2*x4,
                   latvar2 = -x2 + x3 + 0*x4)
# Nb. latvar2 is weakly correlated with latvar1
cdata <- transform(cdata,
```

```

        lambda1 = exp(6 - 0.5 * (latvar1-0)^2 - 0.5 * (latvar2-0)^2),
        lambda2 = exp(5 - 0.5 * (latvar1-1)^2 - 0.5 * (latvar2-1)^2),
        lambda3 = exp(5 - 0.5 * (latvar1+2)^2 - 0.5 * (latvar2-0)^2))
cdata <- transform(cdata,
  spp1 = rpois(nn, lambda1),
  spp2 = rpois(nn, lambda2),
  spp3 = rpois(nn, lambda3))
set.seed(111)
# vvv p2 <- cqc(cbind(spp1,spp2,spp3) ~ x2 + x3 + x4, poissonff,
# vvv      data = cdata,
# vvv      Rank=2, ITolerances=TRUE,
# vvv      CrowIpositive=c(TRUE,FALSE)) # deviance = 505.81
# vvv if (deviance(p2) > 506) stop("suboptimal fit obtained")
# vvv sort(p2@misc$deviance.Bestof) # A history of the fits
# vvv Coef(p2)

## Not run:
lvplot(p2, sites = TRUE, spch = "*", scol = "darkgreen", scex = 1.5,
  chull = TRUE, label = TRUE, Absolute = TRUE, ellipse = 140,
  adj = -0.5, pcol = "blue", pcex = 1.3, las = 1,
  C = TRUE, Cadj = c(-.3,-.3,1), Clwd = 2, Ccex = 1.4, Ccol = "red",
  main = paste("Contours at Abundance = 140 with",
    "convex hull of the site scores"))

## End(Not run)
# vvv var(latvar(p2)) # A diagonal matrix, i.e., uncorrelated latent variables
# vvv var(latvar(p2, varI.latvar = TRUE)) # Identity matrix
# vvv Tol(p2)[,1:2] # Identity matrix
# vvv Tol(p2, varI.latvar = TRUE)[,1:2] # A diagonal matrix

```

lvplot.rrvglm

*Latent Variable Plot for RR-VGLMs*

## Description

Produces an *ordination diagram* (also known as a *biplot* or *latent variable plot*) for *reduced-rank vector generalized linear models* (RR-VGLMs). For rank-2 models only, the x- and y-axis are the first and second canonical axes respectively.

## Usage

```

lvplot.rrvglm(object,
  A = TRUE, C = TRUE, scores = FALSE, show.plot = TRUE,
  groups = rep(1, n), gapC = sqrt(sum(par())$cxy^2)),
  scaleA = 1,
  xlab = "Latent Variable 1", ylab = "Latent Variable 2",
  Alabels = if (length(object@misc$predictors.names))
  object@misc$predictors.names else paste("LP", 1:M, sep = ""),
  Aadj = par()$adj, Acex = par()$cex, Acol = par()$col,
  Apch = NULL,

```

```

Clabels = rownames(Cmat), Cadj = par()$adj,
Ccex = par()$cex, Ccol = par()$col, Clty = par()$lty,
Clwd = par()$lwd,
chull.arg = FALSE, ccex = par()$cex, ccol = par()$col,
clty = par()$lty, clwd = par()$lwd,
spch = NULL, scex = par()$cex, scol = par()$col,
slabels = rownames(x2mat), ...)

```

## Arguments

object	Object of class "rrvglm".
A	Logical. Allow the plotting of <b>A</b> ?
C	Logical. Allow the plotting of <b>C</b> ? If TRUE then <b>C</b> is represented by arrows emanating from the origin.
scores	Logical. Allow the plotting of the $n$ scores? The scores are the values of the latent variables for each observation.
show.plot	Logical. Plot it? If FALSE, no plot is produced and the matrix of scores ( $n$ latent variable values) is returned. If TRUE, the rank of object need not be 2.
groups	A vector whose distinct values indicate which group the observation belongs to. By default, all the observations belong to a single group. Useful for the multinomial logit model (see <a href="#">multinomial</a> ).
gapC	The gap between the end of the arrow and the text labelling of <b>C</b> , in latent variable units.
scaleA	Numerical value that is multiplied by <b>A</b> , so that <b>C</b> is divided by this value.
xlab	Caption for the x-axis. See <a href="#">par</a> .
ylab	Caption for the y-axis. See <a href="#">par</a> .
Alabels	Character vector to label <b>A</b> . Must be of length $M$ .
Aadj	Justification of text strings for labelling <b>A</b> . See the adj argument of <a href="#">par</a> .
Acex	Numeric. Character expansion of the labelling of <b>A</b> . See the cex argument of <a href="#">par</a> .
Acol	Line color of the arrows representing <b>C</b> . See the col argument of <a href="#">par</a> .
Apch	Either an integer specifying a symbol or a single character to be used as the default in plotting points. See <a href="#">par</a> . The pch argument can be of length $M$ , the number of species.
Clabels	Character vector to label <b>C</b> . Must be of length $p2$ .
Cadj	Justification of text strings for labelling <b>C</b> . See the adj argument of <a href="#">par</a> .
Ccex	Numeric. Character expansion of the labelling of <b>C</b> . See the cex argument of <a href="#">par</a> .
Ccol	Line color of the arrows representing <b>C</b> . See the col argument of <a href="#">par</a> .
Clty	Line type of the arrows representing <b>C</b> . See the lty argument of <a href="#">par</a> .
Clwd	Line width of the arrows representing <b>C</b> . See the lwd argument of <a href="#">par</a> .
chull.arg	Logical. Plot the convex hull of the scores? This is done for each group (see the group argument).

ccex	Numeric. Character expansion of the labelling of the convex hull. See the cex argument of <a href="#">par</a> .
ccol	Line color of the convex hull. See the col argument of <a href="#">par</a> .
clty	Line type of the convex hull. See the lty argument of <a href="#">par</a> .
clwd	Line width of the convex hull. See the lwd argument of <a href="#">par</a> .
spch	Either an integer specifying a symbol or a single character to be used as the default in plotting points. See <a href="#">par</a> . The spch argument can be of length $M$ , the number of species.
scex	Numeric. Character expansion of the labelling of the scores. See the cex argument of <a href="#">par</a> .
scol	Line color of the arrows representing <b>C</b> . See the col argument of <a href="#">par</a> .
slabels	Character vector to label the scores. Must be of length $n$ .
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include xlim and ylim.

### Details

For RR-VGLMs, a *biplot* and a *latent variable* plot coincide. In general, many of the arguments starting with “A” refer to **A** (of length  $M$ ), “C” to **C** (of length  $p_2$ ), “c” to the convex hull (of length  $\text{length}(\text{unique}(\text{groups}))$ ), and “s” to scores (of length  $n$ ).

As the result is a biplot, its interpretation is based on the inner product.

### Value

The matrix of scores ( $n$  latent variable values) is returned regardless of whether a plot was produced or not.

### Note

The functions [lvplot.rrvglm](#) and [biplot.rrvglm](#) are equivalent.

In the example below the predictor variables are centered, which is a good idea.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

### See Also

[lvplot](#), [par](#), [rrvglm](#), [Coef.rrvglm](#), [rrvglm.control](#).

### Examples

```
nn <- nrow(pneumo) # x1, x2 and x3 are some unrelated covariates
pneumo <- transform(pneumo, slet = scale(log(exposure.time)),
                    x1 = rnorm(nn), x2 = rnorm(nn), x3 = rnorm(nn))
fit <- rrvglm(cbind(normal, mild, severe) ~ slet + x1 + x2 + x3,
             multinomial, pneumo, Rank=2, Corner=FALSE, Uncorrel=TRUE)
## Not run:
lvplot(fit, hull = TRUE, scores = TRUE, clty = 2, ccol = "blue", scol = "red",
       Ccol = "darkgreen", Clwd = 2, Ccex = 2,
       main = "Biplot of some fictional data")
## End(Not run)
```

---

`machinists`

*Machinists Accidents*

---

### Description

A small count data set involving 414 machinists from a three months study, of accidents around the end of WWI.

### Usage

```
data(machinists)
```

### Format

A data frame with the following variables.

**accidents** The number of accidents

**ofreq** Observed frequency, i.e., the number of machinists with that many accidents

### Details

The data was collected over a period of three months. There were 414 machinists in total. Also, there were data collected over six months, but it is not given here.

### Source

Incidence of Industrial Accidents. Report No. 4 (Industrial Fatigue Research Board), Stationery Office, London, 1919.

### References

Greenwood, M. and Yule, G. U. (1920). An Inquiry into the Nature of Frequency Distributions Representative of Multiple Happenings with Particular Reference to the Occurrence of Multiple Attacks of Disease or of Repeated Accidents. *Journal of the Royal Statistical Society*, **83**, 255–279.

**See Also**

[negbinomial](#), [poissonff](#).

**Examples**

```

machinists
mean(with(machinists, rep(accidents, times = ofreq)))
var(with(machinists, rep(accidents, times = ofreq)))
## Not run: barplot(with(machinists, ofreq),
                    names.arg = as.character(with(machinists, accidents)),
                    main = "Machinists accidents",
                    col = "lightblue", las = 1,
                    ylab = "Frequency", xlab = "accidents")
## End(Not run)

```

---

Makeham

*The Makeham Distribution*

---

**Description**

Density, cumulative distribution function, quantile function and random generation for the Makeham distribution.

**Usage**

```

dmakeham(x, shape, scale = 1, epsilon = 0, log = FALSE)
pmakeham(q, shape, scale = 1, epsilon = 0)
qmakeham(p, shape, scale = 1, epsilon = 0)
rmakeham(n, shape, scale = 1, epsilon = 0)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations.
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.
<code>shape, scale</code>	positive shape and scale parameters.
<code>epsilon</code>	another parameter. Must be non-negative. See below.

**Details**

See [makeham](#) for details. The default value of `epsilon = 0` corresponds to the Gompertz distribution. The function [pmakeham](#) uses [lambertW](#).

**Value**

`dmakeham` gives the density, `pmakeham` gives the cumulative distribution function, `qmakeham` gives the quantile function, and `rmakeham` generates random deviates.

**Author(s)**

T. W. Yee

**References**

Jodra, P. (2009) A closed-form expression for the quantile function of the Gompertz-Makeham distribution. *Mathematics and Computers in Simulation*, **79**, 3069–3075.

**See Also**

[makeham](#), [lambertW](#).

**Examples**

```
probs <- seq(0.01, 0.99, by = 0.01)
Shape <- exp(-1); Scale <- exp(1); eps = Epsilon <- exp(-1)
max(abs(pmakeham(qmakeham(p = probs, Shape, sca = Scale, eps = Epsilon),
                Shape, sca = Scale, eps = Epsilon) - probs)) # Should be 0

## Not run: x <- seq(-0.1, 2.0, by = 0.01);
plot(x, dmakeham(x, Shape, sca = Scale, eps = Epsilon), type = "l",
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles",
     col = "blue", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pmakeham(x, Shape, sca = Scale, eps = Epsilon), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qmakeham(probs, Shape, sca = Scale, eps = Epsilon)
lines(Q, dmakeham(Q, Shape, sca = Scale, eps = Epsilon),
     col = "purple", lty = 3, type = "h")
pmakeham(Q, Shape, sca = Scale, eps = Epsilon) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

---

makeham

---

*Makeham Distribution Family Function*


---

**Description**

Maximum likelihood estimation of the 3-parameter Makeham distribution.

**Usage**

```
makeham(lshape = "loge", lscale = "loge", lepsilon = "loge",
        ishape = NULL, iscale = NULL, iepson = NULL,
        gshape = exp(-5:5), gscale = exp(-5:5), gepson = exp(-4:1),
        nsimEIM = 500, oim.mean = TRUE, zero = NULL)
```



**Arguments**

lshape, lscale, lepsilon	Parameter link functions applied to the shape parameter shape, scale parameter scale, and other parameter epsilon. All parameters are treated as positive here (cf. <a href="#">dmakeham</a> allows epsilon = 0, etc.). See <a href="#">Links</a> for more choices.
ishape, iscale, iepson	Optional initial values. A NULL means a value is computed internally. A value must be given for iepson currently, and this is a sensitive parameter!
gshape, gscale, gepson	See <a href="#">CommonVGAMffArguments</a> .
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> . Argument probs.y is used only when imethod = 2.
oim.mean	To be currently ignored.

**Details**

The Makeham distribution, which adds another parameter to the Gompertz distribution, has cumulative distribution function

$$F(x; \alpha, \beta, \varepsilon) = 1 - \exp \left\{ -y\varepsilon + \frac{\alpha}{\beta} [1 - e^{\beta y}] \right\}$$

which leads to a probability density function

$$f(x; \alpha, \beta, \varepsilon) = [\varepsilon + \alpha e^{\beta x}] \exp \left\{ -x\varepsilon + \frac{\alpha}{\beta} [1 - e^{\beta x}] \right\},$$

for  $\alpha > 0$ ,  $\beta > 0$ ,  $\varepsilon \geq 0$ ,  $x > 0$ . Here,  $\beta$  is called the scale parameter scale, and  $\alpha$  is called a shape parameter. The moments for this distribution do not appear to be available in closed form.

Simulated Fisher scoring is used and multiple responses are handled.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

A lot of care is needed because this is a rather difficult distribution for parameter estimation, especially when the shape parameter is large relative to the scale parameter. If the self-starting initial values fail then try experimenting with the initial value arguments, especially iepson. Successful convergence depends on having very good initial values. More improvements could be made here. Also, monitor convergence by setting trace = TRUE.

A trick is to fit a [gompertz](#) distribution and use it for initial values; see below. However, this family function is currently numerically fraught.

**Author(s)**

T. W. Yee

**See Also**

[dmakeham](#), [gompertz](#).

**Examples**

```
## Not run: set.seed(123)
mdata <- data.frame(x2 = runif(nn <- 1000))
mdata <- transform(mdata, eta1 = -1,
                  ceta1 = 1,
                  eeta1 = -2)
mdata <- transform(mdata, shape1 = exp(eta1),
                  scale1 = exp(ceta1),
                  epsil1 = exp(eeta1))
mdata <- transform(mdata,
                  y1 = rmakeham(nn, shape = shape1, scale = scale1, eps = epsil1))

# A trick is to fit a Gompertz distribution first
fit0 <- vglm(y1 ~ 1, gompertz, data = mdata, trace = TRUE)
fit1 <- vglm(y1 ~ 1, makeham, data = mdata,
            etastart = cbind(predict(fit0), log(0.1)), trace = TRUE)

coef(fit1, matrix = TRUE)
summary(fit1)

## End(Not run)
```

---

margeff

---

*Marginal effects for the multinomial logit and cumulative models*


---

**Description**

Marginal effects for the multinomial logit model and cumulative logit/probit/... models: the derivative of the fitted probabilities with respect to each explanatory variable.

**Usage**

```
margeff(object, subset = NULL)
```

**Arguments**

object	A <a href="#">vglm multinomial</a> or <a href="#">cumulative</a> object.
subset	Numerical or logical vector, denoting the required observation(s). Recycling is used if possible. The default means all observations.

**Details**

Computes the derivative of the fitted probabilities of a multinomial logit model or cumulative logit/probit/... model with respect to each explanatory variable.

**Value**

A  $p$  by  $M + 1$  by  $n$  array, where  $p$  is the number of explanatory variables and the (hopefully) nominal response has  $M + 1$  levels, and there are  $n$  observations.

If `is.numeric(subset)` and `length(subset) == 1` then a  $p$  by  $M + 1$  matrix is returned.

**Warning**

Care is needed in interpretation, e.g., the change is not universally accurate for a unit change in each explanatory variable because eventually the ‘new’ probabilities may become negative or greater than unity. Also, the ‘new’ probabilities will not sum to one.

This function is not applicable for models with data-dependent terms such as `bs` and `poly`. Also the function should not be applied to models with any terms that have generated more than one column of the LM model matrix, such as `bs` and `poly`. For such try using numerical methods such as finite-differences. The formula in object should comprise of simple terms of the form  $\sim x_2 + x_3 + x_4$ , etc.

**Note**

For `multinomial` this function should handle any value of `refLevel` and also any constraint matrices. However, it does not currently handle the `xij` or `form2` arguments, nor `vgam` objects.

For `multinomial` if `subset` is numeric then the function uses a for loop over the observations (slow). The default computations use vectorization; this uses more memory than a for loop but is faster.

**Author(s)**

T. W. Yee

**See Also**

`multinomial`, `cumulative`, `vglm`.

**Examples**

```
# Not a good example for multinomial() because the response is ordinal!!
ii <- 3; hh <- 1/100
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, multinomial, pneumo)
fit <- vglm(cbind(normal, mild, severe) ~ let,
            cumulative(reverse = TRUE, parallel = TRUE),
            data = pneumo)
fitted(fit)[ii, ]

mynewdata <- with(pneumo, data.frame(let = let[ii]+hh))
(newp <- predict(fit, newdata = mynewdata, type = "response"))

# Compare the difference. Should be the same as hh --> 0.
round(digits = 3, (newp-fitted(fit)[ii, ])/hh) # Finite-difference approxn
round(digits = 3, margeff(fit, subset = ii)["let",])
```

```
# Other examples
round(digits = 3, margeff(fit))
round(digits = 3, margeff(fit, subset = 2)["let",])
round(digits = 3, margeff(fit, subset = c(FALSE, TRUE))["let",,]) # recycling
round(digits = 3, margeff(fit, subset = c(2, 4, 6, 8))["let",,])
```

---

marital.nz

*New Zealand Marital Data.*

---

## Description

Some marital data mainly from a large NZ company collected in the early 1990s.

## Usage

```
data(marital.nz)
```

## Format

A data frame with 6053 observations on the following 3 variables.

age a numeric vector, age in years

ethnicity a factor with levels European Maori Other Polynesian. Only Europeans are included in the data set.

mstatus a factor with levels Divorced/Separated, Married/Partnered, Single, Widowed.

## Details

This is a subset of a data set collected from a self-administered questionnaire administered in a large New Zealand workforce observational study conducted during 1992–3. The data were augmented by a second study consisting of retirees. The data can be considered a reasonable representation of the white male New Zealand population in the early 1990s.

## Source

Clinical Trials Research Unit, University of Auckland, New Zealand.

## References

See [bmi.nz](#) and [chest.nz](#).

## Examples

```
summary(marital.nz)
```

matched.binomial

*The Matched Binomial Distribution Family Function*

## Description

Estimation of a binomial regression in a matched case-control study.

## Usage

```
matched.binomial(mvar = NULL, link = "logit",
                 parallel = TRUE, smallno = .Machine$double.eps^(3/4))
```

## Arguments

mvar	Formula specifying the matching variable. This shows which observation belongs to which matching set. The intercept should be suppressed from the formula, and the term must be a <a href="#">factor</a> .
link	Parameter link function for the probability parameter. Information for these are at <a href="#">Links</a> and <a href="#">CommonVGAMffArguments</a> .
parallel	This should always be set TRUE otherwise there will be too many parameters to estimate. See <a href="#">CommonVGAMffArguments</a> for more information.
smallno	Numeric, a small positive value. For a specific observation, used to nullify the linear/additive predictors that are not needed.

## Details

By default, this **VGAM** family function fits a logistic regression model to a binary response from a matched case-control study. Here, each case ( $Y = 1$ ) is matched with one or more controls ( $Y = 0$ ) with respect to some matching variables (confounders). For example, the first matched set is all women aged from 20 to 25, the second matched set is women aged between 26 to 30, etc. The logistic regression has a different intercept for each matched set but the other regression coefficients are assumed to be the same across matched sets (`parallel = TRUE`).

Let  $C$  be the number of matched sets. This **VGAM** family function uses a trick by allowing  $M$ , the number of linear/additive predictors, to be equal to  $C$ , and then nullifying all but one of them for a particular observation. The term specified by the `mvar` argument must be a [factor](#). Consequently, the model matrix contains an intercept plus one column for each level of the factor (except the first (this is the default in R)). Altogether there are  $C$  columns. The algorithm here constructs a different constraint matrix for each of the  $C$  columns.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

Both the memory requirements and computational time of this **VGAM** family function grows very quickly with respect to the number of matched sets. For example, the large model matrix of a data set with 100 matched sets consisting of one case and one control per set will take up at least (about) 20Mb of memory. For a constant number of cases and controls per matched set, the memory requirements are  $O(C^3)$  and the computational time is  $O(C^4)$  flops.

The example below has been run successfully with  $n = 700$  (this corresponds to  $C = 350$ ) but only on a big machine and it took over 10 minutes. The large model matrix was 670Mb.

**Note**

The response is assumed to be in a format that can also be inputted into `binomialff`.

**Author(s)**

Thomas W. Yee

**References**

- Section 8.2 of Hastie, T. J. and Tibshirani, R. J. (1990) *Generalized Additive Models*, London: Chapman & Hall.
- Pregibon, D. (1984) Data analytic methods for matched case-control studies. *Biometrics*, **40**, 639–651.
- Chapter 7 of Breslow, N. E. and Day, N. E. (1980) *Statistical Methods in Cancer Research I: The Analysis of Case-Control Studies*. Lyon: International Agency for Research on Cancer.
- Holford, T. R. and White, C. and Kelsey, J. L. (1978) Multivariate analysis for matched case-control studies. *American Journal of Epidemiology*, **107**, 245–256.

**See Also**

`binomialff`.

**Examples**

```
## Not run:
# Cf. Hastie and Tibshirani (1990) p.209. The variable n must be even.
# Here, the intercept for each matched set accounts for x3 which is
# the confounder or matching variable.
n <- 700 # Requires a big machine with lots of memory. Expensive wrt time
n <- 100 # This requires a reasonably big machine.
mydat <- data.frame(x2 = rnorm(n), x3 = rep(rnorm(n/2), each = 2))
xmat <- with(mydat, cbind(x2, x3))
mydat <- transform(mydat, eta = -0.1 + 0.2 * x2 + 0.3 * x3)
etamat <- with(mydat, matrix(eta, n/2, 2))
condmu <- exp(etamat[, 1]) / (exp(etamat[, 1]) + exp(etamat[, 2]))
y1 <- ifelse(runif(n/2) < condmu, 1, 0)
y <- cbind(y1, 1 - y1)
mydat <- transform(mydat, y = c(y1, 1-y1),
                  ID = factor(c(row(etamat))))
```

```

fit <- vglm(y ~ 1 + ID + x2, trace = TRUE,
           matched.binomial(mvar = ~ ID - 1), data = mydat)
dimnames(coef(fit, matrix = TRUE))
coef(fit, matrix = TRUE)
summary(fit)
head(fitted(fit))
objsizeb <- function(object) round(object.size(object) / 2^20, digits = 2)
objsizeb(fit) # in Mb

VLMX <- model.matrix(fit, type = "vlm") # The big model matrix
dim(VLMX)
objsizeb(VLMX) # in Mb
rm(VLMX)
## End(Not run)

```

---

Max

---

*Maxima*


---

## Description

Generic function for the *maxima* (maximums) of a model.

## Usage

```
Max(object, ...)
```

## Arguments

object	An object for which the computation or extraction of a maximum (or maxima) is meaningful.
...	Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for <a href="#">Coef</a> .

## Details

Different models can define a maximum in different ways. Many models have no such notion or definition.

Maxima occur in quadratic and additive ordination, e.g., CQO or CAO. For these models the maximum is the fitted value at the optimum. For quadratic ordination models there is a formula for the optimum but for additive ordination models the optimum must be searched for numerically. If it occurs on the boundary, then the optimum is undefined. For a valid optimum, the fitted value at the optimum is the maximum.

## Value

The value returned depends specifically on the methods function invoked.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

**See Also**

Max.qrrvglm, [Tol](#), [Opt](#).

**Examples**

```
## Not run:
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cqp(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
               Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          quasipoissonff, Bestof = 2, data = hspider, Crow1positive = FALSE)

Max(p1)

index <- 1:ncol(depvar(p1))
persp(p1, col = index, las = 1, llwd = 2)
abline(h = Max(p1), lty = 2, col = index)

## End(Not run)
```

---

Maxwell

*The Maxwell Distribution*

---

**Description**

Density, distribution function, quantile function and random generation for the Maxwell distribution.

**Usage**

```
dmaxwell(x, a, log = FALSE)
pmaxwell(q, a)
qmaxwell(p, a)
rmaxwell(n, a)
```



**Arguments**

<code>x</code> , <code>q</code> , <code>p</code> , <code>n</code>	Same as <a href="#">Uniform</a> .
<code>a</code>	the parameter.
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [maxwell](#), the **VGAM** family function for estimating the parameter  $a$  by maximum likelihood estimation, for the formula of the probability density function.

**Value**

`dmaxwell` gives the density, `pmaxwell` gives the distribution function, `qmaxwell` gives the quantile function, and `rmaxwell` generates random deviates.

**Note**

The Maxwell distribution is related to the Rayleigh distribution.

**Author(s)**

T. W. Yee

**References**

Balakrishnan, N. and Nevzorov, V. B. (2003) *A Primer on Statistical Distributions*. Hoboken, New Jersey: Wiley.

**See Also**

[maxwell](#), [Rayleigh](#), [rayleigh](#).

**Examples**

```
## Not run: a <- 3; x <- seq(-0.5, 3, length = 100)
plot(x, dmaxwell(x, a = a), type = "l", col = "blue", las = 1, ylab = "",
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
abline(h = 0, col = "blue", lty = 2)
lines(x, pmaxwell(x, a = a), type = "l", col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qmaxwell(probs, a = a)
lines(Q, dmaxwell(Q, a), col = "purple", lty = 3, type = "h")
lines(Q, pmaxwell(Q, a), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pmaxwell(Q, a) - probs)) # Should be zero

## End(Not run)
```

maxwell

*Maxwell Distribution Family Function***Description**

Estimating the parameter of the Maxwell distribution by maximum likelihood estimation.

**Usage**

```
maxwell(link = "log", zero = NULL)
```

**Arguments**

`link`, `zero`      Parameter link function applied to  $a$ . See [Links](#) for more choices and information; a log link is the default because the parameter is positive. More information is at [CommonVGAMffArguments](#).

**Details**

The Maxwell distribution, which is used in the area of thermodynamics, has a probability density function that can be written

$$f(y; a) = \sqrt{2/\pi} a^{3/2} y^2 \exp(-0.5ay^2)$$

for  $y > 0$  and  $a > 0$ . The mean of  $Y$  is  $\sqrt{8/(a\pi)}$  (returned as the fitted values), and its variance is  $(3\pi - 8)/(\pi a)$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Note**

Fisher-scoring and Newton-Raphson are the same here. A related distribution is the Rayleigh distribution. This **VGAM** family function handles multiple responses. This **VGAM** family function can be mimicked by `poisson.points(ostatistic = 1.5, dimension = 2)`.

**Author(s)**

T. W. Yee

**References**

von Seggern, D. H. (1993) *CRC Standard Curves and Surfaces*, Boca Raton, FL.: CRC Press.

**See Also**

[Maxwell](#), [rayleigh](#), [poisson.points](#).

## Examples

```
mdata <- data.frame(y = rmaxwell(1000, a = exp(2)))
fit <- vglm(y ~ 1, maxwell, mdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

mccullagh89

*McCullagh (1989) Distribution Family Function*

## Description

Estimates the two parameters of the McCullagh (1989) distribution by maximum likelihood estimation.

## Usage

```
mccullagh89(ltheta = "rhit", lnu = logoff(offset = 0.5),
            itheta = NULL, inu = NULL, zero = NULL)
```

## Arguments

<code>ltheta</code> , <code>lnu</code>	Link functions for the $\theta$ and $\nu$ parameters. See <a href="#">Links</a> for general information.
<code>itheta</code> , <code>inu</code>	Numeric. Optional initial values for $\theta$ and $\nu$ . The default is to internally compute them.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose one value from the set $\{1,2\}$ .

## Details

The McCullagh (1989) distribution has density function

$$f(y; \theta, \nu) = \frac{\{1 - y^2\}^{\nu - \frac{1}{2}}}{(1 - 2\theta y + \theta^2)^{\nu} \text{Beta}(\nu + \frac{1}{2}, \frac{1}{2})}$$

where  $-1 < y < 1$  and  $-1 < \theta < 1$ . This distribution is equation (1) in that paper. The parameter  $\nu$  satisfies  $\nu > -1/2$ , therefore the default is to use an log-offset link with offset equal to 0.5, i.e.,  $\eta_2 = \log(\nu + 0.5)$ . The mean is of  $Y$  is  $\nu\theta/(1 + \nu)$ , and these are returned as the fitted values.

This distribution is related to the Leipnik distribution (see Johnson et al. (1995)), is related to ultraspherical functions, and under certain conditions, arises as exit distributions for Brownian motion. Fisher scoring is implemented here and it uses a diagonal matrix so the parameters are globally orthogonal in the Fisher information sense. McCullagh (1989) also states that, to some extent,  $\theta$  and  $\nu$  have the properties of a location parameter and a precision parameter, respectively.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Note**

Convergence may be slow or fail unless the initial values are reasonably close. If a failure occurs, try assigning the argument `inu` and/or `itheta`. Figure 1 of McCullagh (1989) gives a broad range of densities for different values of  $\theta$  and  $\nu$ , and this could be consulted for obtaining reasonable initial values if all else fails.

**Author(s)**

T. W. Yee

**References**

McCullagh, P. (1989) Some statistical properties of a family of continuous univariate distributions. *Journal of the American Statistical Association*, **84**, 125–129.

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995) *Continuous Univariate Distributions*, 2nd edition, Volume 2, New York: Wiley. (pages 612–617).

**See Also**

[leipnik](#), [rhobit](#), [logoff](#).

**Examples**

```
mdata <- data.frame(y = rnorm(n = 1000, sd = 0.2)) # Limit as theta = 0, nu = Inf
fit <- vglm(y ~ 1, mccullagh89, mdata, trace = TRUE)
head(fitted(fit))
with(mdata, mean(y))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
```

---

meplot

---

*Mean Excess Plot*


---

**Description**

Mean excess plot (also known as a mean residual life plot), a diagnostic plot for the generalized Pareto distribution (GPD).

**Usage**

```
meplot(object, ...)
meplot.default(y, main = "Mean Excess Plot",
  xlab = "Threshold", ylab = "Mean Excess", lty = c(2, 1:2),
  conf = 0.95, col = c("blue", "black", "blue"), type = "l", ...)
meplot.vlm(object, ...)
```

**Arguments**

<code>y</code>	A numerical vector. NAs etc. are not allowed.
<code>main, xlab, ylab</code>	Character. Overall title for the plot, and titles for the x- and y-axes.
<code>lty</code>	Line type. The second value is for the mean excess value, the first and third values are for the envelope surrounding the confidence interval.
<code>conf</code>	Confidence level. The default results in approximate 95 percent confidence intervals for each mean excess value.
<code>col</code>	Colour of the three lines.
<code>type</code>	Type of plot. The default means lines are joined between the mean excesses and also the upper and lower limits of the confidence intervals.
<code>object</code>	An object that inherits class "vlm", usually of class <code>vglm-class</code> or <code>vgam-class</code> .
<code>...</code>	Graphical argument passed into <code>plot</code> . See <code>par</code> for an exhaustive list. The arguments <code>xlim</code> and <code>ylim</code> are particularly useful.

**Details**

If  $Y$  has a GPD with scale parameter  $\sigma$  and shape parameter  $\xi < 1$ , and if  $y > 0$ , then

$$E(Y - u | Y > u) = \frac{\sigma + \xi u}{1 - \xi}.$$

It is a linear function in  $u$ , the threshold. Note that  $Y - u$  is called the *excess* and values of  $Y$  greater than  $u$  are called *exceedances*. The empirical versions used by these functions is to use sample means to estimate the left hand side of the equation. Values of  $u$  in the plot are the values of  $y$  itself. If the plot is roughly a straight line then the GPD is a good fit; this plot can be used to select an appropriate threshold value. See [gpd](#) for more details. If the plot is flat then the data may be exponential, and if it is curved then it may be Weibull or gamma. There is often a lot of variance/fluctuation at the RHS of the plot due to fewer observations.

The function `meplot` is generic, and `meplot.default` and `meplot.vlm` are some methods functions for mean excess plots.

**Value**

A list is returned invisibly with the following components.

<code>threshold</code>	The x axis values.
<code>meanExcess</code>	The y axis values. Each value is a sample mean minus a value $u$ .
<code>plusminus</code>	The amount which is added or subtracted from the mean excess to give the confidence interval. The last value is a NA because it is based on one observation.

**Note**

The function is designed for speed and not accuracy, therefore huge data sets with extremely large values may cause failure (the function `cumsum` is used.) Ties may not be well handled.

Author(s)

T. W. Yee

References

Davison, A. C. and Smith, R. L. (1990) Models for exceedances over high thresholds (with discussion). *Journal of the Royal Statistical Society, Series B, Methodological*, **52**, 393–442.

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

See Also

[gpd](#).

Examples

```
## Not run: meplot(with(venice90, sealevel), las = 1) -> ii
names(ii)
abline(h = ii$meanExcess[1], col = "orange", lty = "dashed")

par(mfrow = c(2, 2))
for (ii in 1:4)
  meplot(rgpd(1000), col = c("orange", "blue", "orange"))

## End(Not run)
```

---

micmen	<i>Michaelis-Menten Model</i>
--------	-------------------------------

---

Description

Fits a Michaelis-Menten nonlinear regression model.

Usage

```
micmen(rpar = 0.001, divisor = 10, init1 = NULL, init2 = NULL,
       imethod = 1, oim = TRUE, link1 = "identity", link2 = "identity",
       firstDeriv = c("nsimEIM", "rpar"), probs.x = c(0.15, 0.85),
       nsimEIM = 500, dispersion = 0, zero = NULL)
```

Arguments

- |         |   |
|---------|---|
| rpar    | Numeric. Initial positive ridge parameter. This is used to create positive-definite weight matrices.  |
| divisor | Numerical. The divisor used to divide the ridge parameter at each iteration until it is very small but still positive. The value of divisor should be greater than one. |

<code>init1, init2</code>	Numerical. Optional initial value for the first and second parameters, respectively. The default is to use a self-starting value.
<code>link1, link2</code>	Parameter link function applied to the first and second parameters, respectively. See <a href="#">Links</a> for more choices.
<code>dispersion</code>	Numerical. Dispersion parameter.
<code>firstDeriv</code>	Character. Algorithm for computing the first derivatives and working weights. The first is the default.
<code>imethod, probs.x</code>	See <a href="#">CommonVGAMffArguments</a> for more information.
<code>nsimEIM</code>	See <a href="#">CommonVGAMffArguments</a> for more information.
<code>oim</code>	Use the OIM? See <a href="#">CommonVGAMffArguments</a> for more information.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set {1,2}. A NULL means none. See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The Michaelis-Menten model is given by

$$E(Y_i) = (\theta_1 u_i) / (\theta_2 + u_i)$$

where  $\theta_1$  and  $\theta_2$  are the two parameters.

The relationship between iteratively reweighted least squares and the Gauss-Newton algorithm is given in Wedderburn (1974). However, the algorithm used by this family function is different. Details are given at the Author's web site.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

This function is not (nor could ever be) entirely reliable. Plotting the fitted function and monitoring convergence is recommended.

## Note

The regressor values  $u_i$  are inputted as the RHS of the `form2` argument. It should just be a simple term; no smart prediction is used. It should just a single vector, therefore omit the intercept term. The LHS of the formula `form2` is ignored.

To predict the response at new values of  $u_i$  one must assign the `@extra$Xm2` slot in the fitted object these values, e.g., see the example below.

Numerical problems may occur. If so, try setting some initial values for the parameters. In the future, several self-starting initial values will be implemented.

**Author(s)**

T. W. Yee

**References**

- Seber, G. A. F. and Wild, C. J. (1989) *Nonlinear Regression*, New York: Wiley.
- Wedderburn, R. W. M. (1974) Quasi-likelihood functions, generalized linear models, and the Gauss-Newton method. *Biometrika*, **61**, 439–447.
- Bates, D. M. and Watts, D. G. (1988) *Nonlinear Regression Analysis and Its Applications*, New York: Wiley.
- Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[enzyme](#).

**Examples**

```
fit <- vglm(velocity ~ 1, micmen, enzyme, trace = TRUE, crit = "coef",
          form2 = ~ conc - 1)
summary(fit)

## Not run: plot(velocity ~ conc, enzyme, xlab = "concentration", las = 1,
               col = "blue", main = "Michaelis-Menten equation for the enzyme data",
               ylim = c(0, max(velocity)), xlim = c(0, max(conc)))
points(fitted(fit) ~ conc, enzyme, col = "red", pch = "+", cex = 1.5)

# This predicts the response at a finer grid:
newenzyme <- data.frame(conc = seq(0, max(with(enzyme, conc)), len = 200))
fit@extra$xm2 <- newenzyme$conc # This assignment is needed for prediction
lines(predict(fit, newenzyme, "response") ~ conc, newenzyme, col = "red")
## End(Not run)
```

---

mix2exp

*Mixture of Two Exponential Distributions*

---

**Description**

Estimates the three parameters of a mixture of two exponential distributions by maximum likelihood estimation.

**Usage**

```
mix2exp(lphi = "logit", llambda = "loge", iphi = 0.5, il1 = NULL,
        il2 = NULL, qmu = c(0.8, 0.2), nsimEIM = 100, zero = 1)
```



## Arguments

- lphi, llambda    Link functions for the parameters  $\phi$  and  $\lambda$ . The latter is the rate parameter and note that the mean of an ordinary exponential distribution is  $1/\lambda$ . See [Links](#) for more choices.
- iphi, il1, il2    Initial value for  $\phi$ , and optional initial value for  $\lambda_1$  and  $\lambda_2$ . The last two have values that must be positive. The default is to compute initial values internally using the argument qmu.
- qmu                Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for  $\lambda_1$  and  $\lambda_2$ . The two values are fed in as the probs argument into [quantile](#).
- nsimEIM, zero    See [CommonVGAMffArguments](#).

## Details

The probability function can be loosely written as

$$P(Y = y) = \phi \text{Exponential}(\lambda_1) + (1 - \phi) \text{Exponential}(\lambda_2)$$

where  $\phi$  is the probability an observation belongs to the first group, and  $y > 0$ . The parameter  $\phi$  satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $\phi/\lambda_1 + (1 - \phi)/\lambda_2$  and this is returned as the fitted values. By default, the three linear/additive predictors are  $(\text{logit}(\phi), \log(\lambda_1), \log(\lambda_2))^T$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

This **VGAM** family function requires care for a successful application. In particular, good initial values are required because of the presence of local solutions. Therefore running this function with several different combinations of arguments such as iphi, il1, il2, qmu is highly recommended. Graphical methods such as [hist](#) can be used as an aid.

## Note

Fitting this model successfully to data can be difficult due to local solutions, uniqueness problems and ill-conditioned data. It pays to fit the model several times with different initial values and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as  $\lambda_1$  and  $\lambda_2$  become more different. The default control argument trace = TRUE is to encourage monitoring convergence.

## Author(s)

T. W. Yee

## See Also

[rexp](#), [exponential](#), [mix2poisson](#).

## Examples

```
## Not run: lambda1 <- exp(1); lambda2 <- exp(3)
(phi <- logit(-1, inverse = TRUE))
mdata <- data.frame(y1 = rexp(nn <- 1000, lambda1))
mdata <- transform(mdata, y2 = rexp(nn, lambda2))
mdata <- transform(mdata, Y = ifelse(runif(nn) < phi, y1, y2))
fit <- vglm(Y ~ 1, mix2exp, mdata, trace = TRUE)
coef(fit, matrix = TRUE)

# Compare the results with the truth
round(rbind('Estimated' = Coef(fit),
            'Truth' = c(phi, lambda1, lambda2)), digits = 2)

with(mdata, hist(Y, prob = TRUE, main = "Orange = estimate, blue = truth"))
abline(v = 1 / Coef(fit)[c(2, 3)], lty = 2, col = "orange", lwd = 2)
abline(v = 1 / c(lambda1, lambda2), lty = 2, col = "blue", lwd = 2)

## End(Not run)
```

---

mix2normal

*Mixture of Two Univariate Normal Distributions*


---

## Description

Estimates the five parameters of a mixture of two univariate normal distributions by maximum likelihood estimation.

## Usage

```
mix2normal(lphi = "logit", lmu = "identity", lsd = "loge",
            iphi = 0.5, imu1 = NULL, imu2 = NULL, isd1 = NULL, isd2 = NULL,
            qmu = c(0.2, 0.8), eq.sd = TRUE, nsimEIM = 100, zero = 1)
```

## Arguments

lphi, lmu, lsd	Link functions for the parameters $\phi$ , $\mu$ , and $\sigma$ . See <a href="#">Links</a> for more choices.
iphi	Initial value for $\phi$ , whose value must lie between 0 and 1.
imu1, imu2	Optional initial value for $\mu_1$ and $\mu_2$ . The default is to compute initial values internally using the argument qmu.
isd1, isd2	Optional initial value for $\sigma_1$ and $\sigma_2$ . The default is to compute initial values internally based on the argument qmu. Currently these are not great, therefore using these arguments where practical is a good idea.
qmu	Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for $\mu_1$ and $\mu_2$ . The two values are fed in as the probs argument into <a href="#">quantile</a> .
eq.sd	Logical indicating whether the two standard deviations should be constrained to be equal. If TRUE then the appropriate constraint matrices will be used.

nsimEIM	See <a href="#">CommonVGAMffArguments</a> .
zero	An integer specifying which linear/additive predictor is modelled as intercepts only. If given, the value or values must be from the set $\{1, 2, \dots, 5\}$ . The default is the first one only, meaning $\phi$ is a single parameter even when there are explanatory variables. Set zero = NULL to model all linear/additive predictors as functions of the explanatory variables. See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The probability density function can be loosely written as

$$f(y) = \phi N(\mu_1, \sigma_1) + (1 - \phi) N(\mu_2, \sigma_2)$$

where  $\phi$  is the probability an observation belongs to the first group. The parameters  $\mu_1$  and  $\mu_2$  are the means, and  $\sigma_1$  and  $\sigma_2$  are the standard deviations. The parameter  $\phi$  satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $\phi\mu_1 + (1 - \phi)\mu_2$  and this is returned as the fitted values. By default, the five linear/additive predictors are  $(\text{logit}(\phi), \mu_1, \log(\sigma_1), \mu_2, \log(\sigma_2))^T$ . If eq.sd = TRUE then  $\sigma_1 = \sigma_2$  is enforced.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

Numerical problems can occur and half-stepping is not uncommon. If failure to converge occurs, try inputting better initial values, e.g., by using iphi, qmu, imu1, imu2, isd1, isd2, etc.

This **VGAM** family function should be used with care.

## Note

Fitting this model successfully to data can be difficult due to numerical problems and ill-conditioned data. It pays to fit the model several times with different initial values and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as  $\mu_1$  and  $\mu_2$  become more different.

Convergence can be slow, especially when the two component distributions are not well separated. The default control argument trace = TRUE is to encourage monitoring convergence. Having eq.sd = TRUE often makes the overall optimization problem easier.

## Author(s)

T. W. Yee

## References

- McLachlan, G. J. and Peel, D. (2000) *Finite Mixture Models*. New York: Wiley.
- Everitt, B. S. and Hand, D. J. (1981) *Finite Mixture Distributions*. London: Chapman & Hall.

**See Also**

[uninormal](#), [Normal](#), [mix2poisson](#).

**Examples**

```
## Not run: mu1 <- 99; mu2 <- 150; nn <- 1000
sd1 <- sd2 <- exp(3)
(phi <- logit(-1, inverse = TRUE))
mdata <- data.frame(y = ifelse(runif(nn) < phi, rnorm(nn, mu1, sd1),
                                rnorm(nn, mu2, sd2)))

fit <- vglm(y ~ 1, mix2normal(eq.sd = TRUE), mdata)

# Compare the results
cfit <- coef(fit)
round(rbind('Estimated' = c(logit(cfit[1], inverse = TRUE),
                             cfit[2], exp(cfit[3]), cfit[4]),
            'Truth' = c(phi, mu1, sd1, mu2)), digits = 2)

# Plot the results
xx <- with(mdata, seq(min(y), max(y), len = 200))
plot(xx, (1-phi) * dnorm(xx, mu2, sd2), type = "l", xlab = "y",
      main = "Orange = estimate, blue = truth", col = "blue", ylab = "Density")
phi.est <- logit(coef(fit)[1], inverse = TRUE)
sd.est <- exp(coef(fit)[3])
lines(xx, phi*dnorm(xx, mu1, sd1), col = "blue")
lines(xx, phi.est * dnorm(xx, Coef(fit)[2], sd.est), col = "orange")
lines(xx, (1-phi.est) * dnorm(xx, Coef(fit)[4], sd.est), col = "orange")
abline(v = Coef(fit)[c(2,4)], lty = 2, col = "orange")
abline(v = c(mu1, mu2), lty = 2, col = "blue")

## End(Not run)
```

---

mix2poisson

*Mixture of Two Poisson Distributions*

---

**Description**

Estimates the three parameters of a mixture of two Poisson distributions by maximum likelihood estimation.

**Usage**

```
mix2poisson(lphi = "logit", llambda = "loge",
            iphi = 0.5, il1 = NULL, il2 = NULL,
            qmu = c(0.2, 0.8), nsimEIM = 100, zero = 1)
```

**Arguments**

<code>lphi, llambda</code>	Link functions for the parameter $\phi$ and $\lambda$ . See <a href="#">Links</a> for more choices.
<code>iphi</code>	Initial value for $\phi$ , whose value must lie between 0 and 1.
<code>il1, il2</code>	Optional initial value for $\lambda_1$ and $\lambda_2$ . These values must be positive. The default is to compute initial values internally using the argument <code>qmu</code> .
<code>qmu</code>	Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for $\lambda_1$ and $\lambda_2$ . The two values are fed in as the <code>probs</code> argument into <a href="#">quantile</a> .
<code>nsimEIM, zero</code>	See <a href="#">CommonVGAMffArguments</a> .

**Details**

The probability function can be loosely written as

$$P(Y = y) = \phi \text{Poisson}(\lambda_1) + (1 - \phi) \text{Poisson}(\lambda_2)$$

where  $\phi$  is the probability an observation belongs to the first group, and  $y = 0, 1, 2, \dots$ . The parameter  $\phi$  satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $\phi\lambda_1 + (1 - \phi)\lambda_2$  and this is returned as the fitted values. By default, the three linear/additive predictors are  $(\text{logit}(\phi), \log(\lambda_1), \log(\lambda_2))^T$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

This **VGAM** family function requires care for a successful application. In particular, good initial values are required because of the presence of local solutions. Therefore running this function with several different combinations of arguments such as `iphi`, `il1`, `il2`, `qmu` is highly recommended. Graphical methods such as [hist](#) can be used as an aid.

With grouped data (i.e., using the `weights` argument) one has to use a large value of `nsimEIM`; see the example below.

**Note**

The response must be integer-valued since [dpois](#) is invoked.

Fitting this model successfully to data can be difficult due to local solutions and ill-conditioned data. It pays to fit the model several times with different initial values, and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as  $\lambda_1$  and  $\lambda_2$  become more different. The default control argument `trace = TRUE` is to encourage monitoring convergence.

**Author(s)**

T. W. Yee

**See Also**

[rpois](#), [poissonff](#), [mix2normal](#).

**Examples**

```
## Not run: # Example 1: simulated data
nn <- 1000
mu1 <- exp(2.5) # also known as lambda1
mu2 <- exp(3)
(phi <- logit(-0.5, inverse = TRUE))
mdata <- data.frame(y = rpois(nn, ifelse(runif(nn) < phi, mu1, mu2)))
fit <- vglm(y ~ 1, mix2poisson, mdata)
coef(fit, matrix = TRUE)

# Compare the results with the truth
round(rbind('Estimated' = Coef(fit), 'Truth' = c(phi, mu1, mu2)), digits = 2)

ty <- with(mdata, table(y))
plot(names(ty), ty, type = "h", main = "Orange=estimate, blue=truth",
      ylab = "Frequency", xlab = "y")
abline(v = Coef(fit)[-1], lty = 2, col = "orange", lwd = 2)
abline(v = c(mu1, mu2), lty = 2, col = "blue", lwd = 2)

# Example 2: London Times data (Lange, 1997, p.31)
ltdata1 <- data.frame(deaths = 0:9,
                     freq = c(162, 267, 271, 185, 111, 61, 27, 8, 3, 1))
ltdata2 <- data.frame(y = with(ltdata1, rep(deaths, freq)))

# Usually this does not work well unless nsimEIM is large
fit <- vglm(deaths ~ 1, weight = freq, data = ltdata1,
            mix2poisson(iphi = 0.3, il1 = 1, il2 = 2.5, nsimEIM = 5000))

# This works better in general
fit <- vglm(y ~ 1, mix2poisson(iphi = 0.3, il1 = 1, il2 = 2.5), ltdata2)
coef(fit, matrix = TRUE)
Coef(fit)

## End(Not run)
```

---

mlogit

---

*Multi-logit Link Function*


---

**Description**

Computes the mlogit transformation, including its inverse and the first two derivatives.

**Usage**

```
mlogit(theta, refLevel = "last", M = NULL, whitespace = FALSE,
       bvalue = NULL, inverse = FALSE, deriv = 0,
       short = TRUE, tag = FALSE)
```

**Arguments**

theta                Numeric or character. See below for further details.  
 refLevel, M, whitespace  
                       See [multinomial](#).  
 bvalue               See [Links](#).  
 inverse, deriv, short, tag  
                       Details at [Links](#).

**Details**

The `mlogit()` link function is a generalization of the [logit](#) link to  $M$  levels/classes. It forms the basis of the [multinomial](#) logit model. It is sometimes called the *multi-logit* link or the *multinomial logit* link. When its inverse function is computed it returns values which are positive and add to unity.

**Value**

For `mlogit` with `deriv = 0`, the `mlogit` of `theta`, i.e.,  $\log(\text{theta}[, j]/\text{theta}[, M+1])$  when `inverse = FALSE`, and if `inverse = TRUE` then  $\exp(\text{theta}[, j])/(1+\text{rowSums}(\exp(\text{theta})))$ .

For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of `theta` if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base  $e$ .

**Note**

Numerical instability may occur when `theta` is close to 1 or 0 (for `mlogit`). One way of overcoming this is to use, e.g., `bvalue`. Currently `care.exp()` is used to avoid NAs being returned if the probability is too close to 1.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [multinomial](#), [logit](#), [normal.vcm](#), [CommonVGAMffArguments](#).

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let,
            multinomial, trace = TRUE, pneumo) # For illustration only!
fitted(fit)
predict(fit)
```

```

mlogit(fitted(fit))
mlogit(fitted(fit)) - predict(fit) # Should be all 0s

mlogit(predict(fit), inverse = TRUE) # rowSums() add to unity
mlogit(predict(fit), inverse = TRUE, refLevel = 1) # For illustration only
mlogit(predict(fit), inverse = TRUE) - fitted(fit) # Should be all 0s

mlogit(fitted(fit), deriv = 1)
mlogit(fitted(fit), deriv = 2)

```

---

mmt

*mmt daily maximum temperatures*


---

## Description

Melbourne daily maximum temperatures in degrees Celsius over the ten-year period 1981–1990.

## Usage

```
data(mmt)
```

## Format

A vector with 3650 observations.

## Details

This is a time series data from Melbourne, Australia. It is commonly used to give a difficult quantile regression problem since the data is bimodal. That is, a hot day is likely to be followed by either an equally hot day or one much cooler. However, an independence assumption is typically made.

## References

Hyndman, R. J. and Bashtannyk, D. M. and Grunwald, G. K. (1996). Estimating and visualizing conditional densities. *J. Comput. Graph. Statist.*, **5**(4), 315–336.

## See Also

[lms.bcn.](#)

## Examples

```

summary(mmt)
## Not run: par(mfrow = c(1, 1), mar = c(5, 4, 0.2, 0.1) + 0.1, las = 1)
melb <- data.frame(today      = mmt[-1],
                   yesterday = mmt[-length(mmt)])
plot(today ~ yesterday, data = melb,
     xlab = "Yesterday's Max Temperature",

```



```

      ylab = "Today's Max Temperature", cex = 1.4, type = "n")
points(today ~ yesterday, data = melb, pch = 0, cex = 0.50, col = "blue")
abline(a = 0, b = 1, lty = 3)

## End(Not run)

```

MNSs

*The MNSs Blood Group System***Description**

Estimates the three independent parameters of the the MNSs blood group system.

**Usage**

```
MNSs(link = "logit", imS = NULL, ims = NULL, inS = NULL)
```

**Arguments**

**link** Link function applied to the three parameters. See [Links](#) for more choices.

**imS, ims, inS** Optional initial value for mS, ms and nS respectively. A NULL means they are computed internally.

**Details**

There are three independent parameters:  $m_S$ ,  $m_s$ ,  $n_S$ , say, so that  $n_s = 1 - m_S - m_s - n_S$ . We let the eta vector (transposed) be  $(g(m_S), g(m_s), g(n_S))$  where  $g$  is the link function.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The input can be a 6-column matrix of counts, where the columns are MS, Ms, MNS, MNs, NS, Ns (in order). Alternatively, the input can be a 6-column matrix of proportions (so each row adds to 1) and the `weights` argument is used to specify the total number of counts for each row.

**Author(s)**

T. W. Yee

**References**

Elandt-Johnson, R. C. (1971) *Probability Models and Statistical Methods in Genetics*, New York: Wiley.

**See Also**

[AA.Aa.aa](#), [AB.Ab.aB.ab](#), [AB.Ab.aB.ab2](#), [ABO](#), [G1G2G3](#).

**Examples**

```
# Order matters only:
y <- cbind(MS = 295, Ms = 107, MNS = 379, MNs = 322, NS = 102, Ns = 214)
fit <- vglm(y ~ 1, MNSs("logit", .25, .28, .08), trace = TRUE)
fit <- vglm(y ~ 1, MNSs(link = logit), trace = TRUE, crit = "coef")
Coef(fit)
rbind(y, sum(y)*fitted(fit))
sqrt(diag(vcov(fit)))
```

---

model.framevlm

---

Construct the Model Frame of a VLM Object

---

**Description**

This function returns a [data.frame](#) with the variables. It is applied to an object which inherits from class "vlm" (e.g., a fitted model of class "vglm").

**Usage**

```
model.framevlm(object, setupsmart = TRUE, wrapupsmart = TRUE, ...)
```

**Arguments**

object	a model object from the <b>VGAM</b> R package that inherits from a <i>vector linear model</i> (VLM), e.g., a model of class "vglm".
...	further arguments such as data, na.action, subset. See <a href="#">model.frame</a> for more information on these.
setupsmart, wrapupsmart	Logical. Arguments to determine whether to use smart prediction.

**Details**

Since object is an object which inherits from class "vlm" (e.g., a fitted model of class "vglm"), the method will either returned the saved model frame used when fitting the model (if any, selected by argument model = TRUE) or pass the call used when fitting on to the default method.

This code implements *smart prediction* (see [smartpred](#)).

**Value**

A [data.frame](#) containing the variables used in the object plus those specified in ...

## References

Chambers, J. M. (1992) *Data for models*. Chapter 3 of *Statistical Models in S* eds J. M. Chambers and T. J. Hastie, Wadsworth & Brooks/Cole.

## See Also

[model.frame](#), [model.matrixvlm](#), [predictvglm](#), [smartpred](#).

## Examples

```
# Illustrates smart prediction
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal,mild, severe) ~ poly(c(scale(let)), 2),
           multinomial, data = pneumo, trace = TRUE, x = FALSE)
class(fit)

check1 <- head(model.frame(fit))
check1
check2 <- model.frame(fit, data = head(pneumo))
check2
all.equal(unlist(check1), unlist(check2)) # Should be TRUE

q0 <- head(predict(fit))
q1 <- head(predict(fit, newdata = pneumo))
q2 <- predict(fit, newdata = head(pneumo))
all.equal(q0, q1) # Should be TRUE
all.equal(q1, q2) # Should be TRUE
```

---

model.matrixvlm

*Construct the Design Matrix of a VLM Object*

---

## Description

Creates a design matrix. Two types can be returned: a large one (class "vlm" or one that inherits from this such as "vglm") or a small one (such as returned if it were of class "lm").

## Usage

```
model.matrixvlm(object, type = c("vlm", "lm", "lm2", "bothlm2"),
               linpred.index = NULL, ...)
```

## Arguments

object	an object of a class that inherits from the <i>vector linear model</i> (VLM).
type	Type of design matrix returned. The first is the default. The value "vlm" is the VLM model matrix corresponding to the formula argument. The value "lm" is the LM model matrix corresponding to the formula argument. The value "lm2" is the second (LM) model matrix corresponding to the form2 argument. The value "bothlm2" means both LM and VLM model matrices.

`linpred.index` Single integer. The index for a linear/additive predictor, it must have a value from the set 1:M, and `type = "lm"` must be assigned. Then it returns a subset of the VLM matrix corresponding to the `linpred.index`th linear/additive predictor; this is a LM-type matrix.

`...` further arguments passed to or from other methods. These include `data` (which is a data frame created with [model.framevlm](#)), `contrasts.arg`, and `xlev`. See [model.matrix](#) for more information.

## Details

This function creates a design matrix from object. This can be a small LM object or a big VLM object (default). The latter is constructed from the former and the constraint matrices.

This code implements *smart prediction* (see [smartpred](#)).

## Value

The design matrix for a regression model with the specified formula and data. If `type = "bothlm2"` then a list is returned with components `"X"` and `"Xm2"`.

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Chambers, J. M. (1992) *Data for models*. Chapter 3 of *Statistical Models in S* eds J. M. Chambers and T. J. Hastie, Wadsworth & Brooks/Cole.

## See Also

[model.matrix](#), [model.framevlm](#), [predictvglm](#), [smartpred](#).

## Examples

```
# Illustrates smart prediction
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ poly(c(scale(let)), 2),
           multinomial, data = pneumo, trace = TRUE, x = FALSE)
class(fit)
fit@x # Not saved on the object
model.matrix(fit)
model.matrix(fit, linpred.index = 1, type = "lm")
model.matrix(fit, linpred.index = 2, type = "lm")

(Check1 <- head(model.matrix(fit, type = "lm")))
(Check2 <- model.matrix(fit, data = head(pneumo), type = "lm"))
all.equal(c(Check1), c(Check2))

q0 <- head(predict(fit))
q1 <- head(predict(fit, newdata = pneumo))
q2 <- predict(fit, newdata = head(pneumo))
all.equal(q0, q1) # Should be TRUE
all.equal(q1, q2) # Should be TRUE
```

moffset

*Matrix Offset***Description**

Modify a matrix by shifting successive elements.

**Usage**

```
moffset(mat, roffset = 0, coffset = 0, postfix = "",
        rprefix = "Row.", cprefix = "Col.")
```

**Arguments**

mat	Data frame or matrix. This ought to have at least three rows and three columns. The elements are shifted in the order of <code>c(mat)</code> , i.e., going down successive columns, as the columns go from left to right. Wrapping of values is done.
roffset, coffset	Numeric or character. If numeric, the amount of shift (offset) for each row and column. The default is no change to <code>mat</code> . If character, the offset is computed by matching with the row or column names. For example, for the <code>alcoff</code> , put <code>roffset = "6"</code> means that we make an effective day's dataset start from 6:00 am, and this wraps around to include midnight to 05.59 am on the next day.
postfix	Character. Modified rows and columns are renamed by pasting this argument to the end of each name. The default is no change.
rprefix, cprefix	Same as <code>rcim</code> .

**Details**

This function allows a matrix to be rearranged so that element  $(\text{roffset} + 1, \text{coffset} + 1)$  becomes the  $(1, 1)$  element. The elements are assumed to be ordered in the same way as the elements of `c(mat)`,

This function is applicable to, e.g., `alcoff`, where it is useful to define the *effective day* as starting at some other hour than midnight, e.g., 6.00am. This is because partying on Friday night continues on into Saturday morning, therefore it is more interpretable to use the effective day when considering a daily effect.

This is a data preprocessing function for `rcim` and `plotrcim0`. The differences between `Rcim` and `moffset` is that `Rcim` only reorders the level of the rows and columns so that the data is shifted but not moved. That is, a value in one row stays in that row, and ditto for column. But in `moffset` values in one column can be moved to a previous column. See the examples below.

**Value**

A matrix of the same dimensional as its input.

**Note**

The input mat should have row names and column names.

**Author(s)**

T. W. Yee, Alfian F. Hadi.

**See Also**

[Rcim](#), [rcim](#), [plotrcim0](#), [alcoff](#), [crashi](#).

**Examples**

```
moffset(alcoff, 3, 2, "*") # Some day's data is moved to previous day.
Rcim(alcoff, 3 + 1, 2 + 1) # Data does not move as much.
alcoff # Original data
moffset(alcoff, 3, 2, "*") - Rcim(alcoff, 3+1, 2+1) # Note the differences

# An 'effective day' data set:
alcoff.e <- moffset(alcoff, roffset = "6", postfix = "*")
fit.o <- rcim(alcoff) # default baselines are first row and col
fit.e <- rcim(alcoff.e) # default baselines are first row and col

## Not run: par(mfrow = c(2, 2), mar = c(9, 4, 2, 1))
plot(fit.o, rsub = "Not very interpretable", csub = "Not very interpretable")
plot(fit.e, rsub = "More interpretable", csub = "More interpretable")

## End(Not run)

# Some checking
all.equal(moffset(alcoff), alcoff) # Should be no change
moffset(alcoff, 1, 1, "*")
moffset(alcoff, 2, 3, "*")
moffset(alcoff, 1, 0, "*")
moffset(alcoff, 0, 1, "*")
moffset(alcoff, "6", "Mon", "*") # This one is good

# Customise row and column baselines
fit2 <- rcim(Rcim(alcoff.e, rbaseline = "11", cbaseline = "Mon*"))
```

**Description**

Estimate the association parameter of Morgenstern's bivariate distribution by maximum likelihood estimation.

**Usage**

```
morgenstern(lapar = "rhit", iapar = NULL, tola0 = 0.01, imethod = 1)
```

**Arguments**

lapar	Link function for the association parameter $\alpha$ , which lies between $-1$ and $1$ . See <a href="#">Links</a> for more choices and other information.
iapar	Numeric. Optional initial value for $\alpha$ . By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument <code>imethod</code> .
tola0	Positive numeric. If the estimate of $\alpha$ has an absolute value less than this then it is replaced by this value. This is an attempt to fix a numerical problem when the estimate is too close to zero.
imethod	An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for <code>ia</code> .

**Details**

The cumulative distribution function is

$$P(Y_1 \leq y_1, Y_2 \leq y_2) = e^{-y_1 - y_2} (1 + \alpha[1 - e^{-y_1}][1 - e^{-y_2}]) + 1 - e^{-y_1} - e^{-y_2}$$

for  $\alpha$  between  $-1$  and  $1$ . The support of the function is for  $y_1 > 0$  and  $y_2 > 0$ . The marginal distributions are an exponential distribution with unit mean. When  $\alpha = 0$  then the random variables are independent, and this causes some problems in the estimation process since the distribution no longer depends on the parameter.

A variant of Newton-Raphson is used, which only seems to work for an intercept model. It is a very good idea to set `trace = TRUE`.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Note**

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 1. This is because each marginal distribution corresponds to a exponential distribution with unit mean.

This **VGAM** family function should be used with caution.

**Author(s)**

T. W. Yee

**References**

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005) *Extreme Value and Related Models with Applications in Engineering and Science*, Hoboken, NJ, USA: Wiley-Interscience.

See Also

[fgm](#), [bigumbell](#).

Examples

```
N <- 1000; mdata <- data.frame(y1 = rexp(N), y2 = rexp(N))
## Not run: plot(ymat)
fit <- vglm(cbind(y1, y2) ~ 1, morgenstern, mdata, trace = TRUE)
# This may fail:
fit <- vglm(cbind(y1, y2) ~ 1, morgenstern, mdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

---

multinomial	<i>Multinomial Logit Model</i>
-------------	--------------------------------

---

Description

Fits a multinomial logit model to a (preferably unordered) factor response.

Usage

```
multinomial(zero = NULL, parallel = FALSE, nointercept = NULL,
            refLevel = "last", whitespace = FALSE)
```

Arguments

- zero                   An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Any values must be from the set  $\{1, 2, \dots, M\}$ . The default value means none are modelled as intercept-only terms. See [CommonVGAMffArguments](#) for more information.
- parallel               A logical, or formula specifying which terms have equal/unequal coefficients.
- nointercept, whitespace   See [CommonVGAMffArguments](#) for more details.
- refLevel               Either a single positive integer or a value of the factor. If an integer then it specifies which column of the response matrix is the reference or baseline level. The default is the last one (the  $(M + 1)$ th one). If used, this argument will be often assigned the value 1. If inputted as a value of a factor then beware of missing values of certain levels of the factor (`drop.unused.levels = TRUE` or `drop.unused.levels = FALSE`). See the example below.



## Details

In this help file the response  $Y$  is assumed to be a factor with unordered values  $1, 2, \dots, M + 1$ , so that  $M$  is the number of linear/additive predictors  $\eta_j$ .

The default model can be written

$$\eta_j = \log(P[Y = j]/P[Y = M + 1])$$

where  $\eta_j$  is the  $j$ th linear/additive predictor. Here,  $j = 1, \dots, M$ , and  $\eta_{M+1}$  is 0 by definition. That is, the last level of the factor, or last column of the response matrix, is taken as the reference level or baseline—this is for identifiability of the parameters. The reference or baseline level can be changed with the `refLevel` argument.

In almost all the literature, the constraint matrices associated with this family of models are known. For example, setting `parallel = TRUE` will make all constraint matrices (except for the intercept) equal to a vector of  $M$  1's. If the constraint matrices are unknown and to be estimated, then this can be achieved by fitting the model as a reduced-rank vector generalized linear model (RR-VGLM; see [rrvglm](#)). In particular, a multinomial logit model with unknown constraint matrices is known as a *stereotype* model (Anderson, 1984), and can be fitted with [rrvglm](#).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Warning

No check is made to verify that the response is nominal.

See [CommonVGAMffArguments](#) for more warnings.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the `y` slot returned by [vglm/vgam/rrvglm](#) is the matrix of sample proportions.

The multinomial logit model is more appropriate for a nominal (unordered) factor response than for an ordinal (ordered) factor response. Models more suited for the latter include those based on cumulative probabilities, e.g., [cumulative](#).

`multinomial` is prone to numerical difficulties if the groups are separable and/or the fitted probabilities are close to 0 or 1. The fitted values returned are estimates of the probabilities  $P[Y = j]$  for  $j = 1, \dots, M + 1$ . See **safeBinaryRegression** for the logistic regression case.

Here is an example of the usage of the `parallel` argument. If there are covariates `x2`, `x3` and `x4`, then `parallel = TRUE ~ x2 + x3 - 1` and `parallel = FALSE ~ x4` are equivalent. This would constrain the regression coefficients for `x2` and `x3` to be equal; those of the intercepts and `x4` would be different.

In Example 4 below, a conditional logit model is fitted to an artificial data set that explores how cost and travel time affect people's decision about how to travel to work. Walking is the baseline group. The variable `Cost.car` is the difference between the cost of travel to work by car and walking, etc. The variable `Time.car` is the difference between the travel duration/time to work by car and walking, etc. For other details about the `xij` argument see [vglm.control](#) and [fill](#).

The `multinom` function in the **nnet** package uses the first level of the factor as baseline, whereas the last level of the factor is used here. Consequently the estimated regression coefficients differ.

### Author(s)

Thomas W. Yee

### References

- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.
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- Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York, USA: Wiley.
- Hastie, T. J., Tibshirani, R. J. and Friedman, J. H. (2009) *The Elements of Statistical Learning: Data Mining, Inference and Prediction*, 2nd ed. New York, USA: Springer-Verlag.
- Simonoff, J. S. (2003) *Analyzing Categorical Data*, New York, USA: Springer-Verlag.
- Anderson, J. A. (1984) Regression and ordered categorical variables. *Journal of the Royal Statistical Society, Series B, Methodological*, **46**, 1–30.
- Tutz, G. (2012) *Regression for Categorical Data*, Cambridge University Press.
- Further information and examples on categorical data analysis by the **VGAM** package can be found at <http://www.stat.auckland.ac.nz/~yee/VGAM/doc/categorical.pdf>.

### See Also

`margeff`, `cumulative`, `acat`, `cratio`, `sratio`, `dirichlet`, `dirmultinomial`, `rrvglm`, `fill1`, `Multinomial`, `mlogit`, `iris`. The author's homepage has further documentation about categorical data analysis using **VGAM**.

### Examples

```
# Example 1: fit a multinomial logit model to Edgar Anderson's iris data
data(iris)
## Not run: fit <- vglm(Species ~ ., multinomial, iris)
coef(fit, matrix = TRUE)
## End(Not run)

# Example 2a: a simple example
ycounts <- t(rmultinom(10, size = 20, prob = c(0.1, 0.2, 0.8))) # Counts
fit <- vglm(ycounts ~ 1, multinomial)
head(fitted(fit)) # Proportions
fit@prior.weights # NOT recommended for extraction of prior weights
weights(fit, type = "prior", matrix = FALSE) # The better method
depvar(fit) # Sample proportions; same as fit@
constraints(fit) # Constraint matrices
```

```

# Example 2b: Different reference level used as the baseline
fit2 <- vglm(ycounts ~ 1, multinomial(refLevel = 2))
coef(fit2, matrix = TRUE)
coef(fit , matrix = TRUE) # Easy to reconcile this output with fit2

# Example 3: The response is a factor.
nn <- 10
dframe3 <- data.frame(yfactor = gl(3, nn, labels = c("Control", "Trt1", "Trt2")),
                      x2 = runif(3 * nn))
myrefLevel <- with(dframe3, yfactor[12])
fit3a <- vglm(yfactor ~ x2, multinomial(refLevel = myrefLevel), dframe3)
fit3b <- vglm(yfactor ~ x2, multinomial(refLevel = 2), dframe3)
coef(fit3a, matrix = TRUE) # "Treatment1" is the reference level
coef(fit3b, matrix = TRUE) # "Treatment1" is the reference level
margeff(fit3b)

# Example 4: Fit a rank-1 stereotype model
data(car.all)
fit4 <- rrvglm(Country ~ Width + Height + HP, multinomial, car.all)
coef(fit4) # Contains the C matrix
constraints(fit4)$HP # The A matrix
coef(fit4, matrix = TRUE) # The B matrix
Coef(fit4)@C # The C matrix
concoef(fit4) # Better to get the C matrix this way
Coef(fit4)@A # The A matrix
svd(coef(fit4, matrix = TRUE)[-1, ])$d # This has rank 1; = C %*% t(A)
# Classification (but watch out for NAs in some of the variables):
apply(fitted(fit4), 1, which.max) # Classification
apply(predict(fit4, car.all, type = "response"), 1, which.max) # Classification
colnames(fitted(fit4))[apply(fitted(fit4), 1, which.max)] # Classification

# Example 5: The use of the xij argument (aka conditional logit model)
set.seed(111)
nn <- 100 # Number of people who travel to work
M <- 3 # There are M+1 models of transport to go to work
ycounts <- matrix(0, nn, M+1)
ycounts[cbind(1:nn, sample(x = M+1, size = nn, replace = TRUE))] = 1
dimnames(ycounts) <- list(NULL, c("bus", "train", "car", "walk"))
gotowork <- data.frame(cost.bus = runif(nn), time.bus = runif(nn),
                      cost.train= runif(nn), time.train= runif(nn),
                      cost.car = runif(nn), time.car = runif(nn),
                      cost.walk = runif(nn), time.walk = runif(nn))
gotowork <- round(gotowork, digits = 2) # For convenience
gotowork <- transform(gotowork,
                      Cost.bus = cost.bus - cost.walk,
                      Cost.car = cost.car - cost.walk,
                      Cost.train = cost.train - cost.walk,
                      Cost = cost.train - cost.walk, # for labelling
                      Time.bus = time.bus - time.walk,
                      Time.car = time.car - time.walk,
                      Time.train = time.train - time.walk,
                      Time = time.train - time.walk) # for labelling

```

```

fit <- vglm(ycounts ~ Cost + Time,
            multinomial(parall = TRUE ~ Cost + Time - 1),
            xij = list(Cost ~ Cost.bus + Cost.train + Cost.car,
                       Time ~ Time.bus + Time.train + Time.car),
            form2 = ~ Cost + Cost.bus + Cost.train + Cost.car +
                    Time + Time.bus + Time.train + Time.car,
            data = gotowork, trace = TRUE)
head(model.matrix(fit, type = "lm")) # LM model matrix
head(model.matrix(fit, type = "vlm")) # Big VLM model matrix
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
summary(fit)
max(abs(predict(fit) - predict(fit, new = gotowork))) # Should be 0

```

---

Nakagami

*Nakagami Distribution*


---

## Description

Density, cumulative distribution function, quantile function and random generation for the Nakagami distribution.

## Usage

```

dnaka(x, shape, scale = 1, log = FALSE)
pnaka(q, shape, scale = 1)
qnaka(p, shape, scale = 1, ...)
rnaka(n, shape, scale = 1, Smallno = 1.0e-6)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Must be a positive integer of length 1.
<code>shape, scale</code>	arguments for the parameters of the distribution. See <a href="#">nakagami</a> for more details. For <code>rnaka</code> , arguments <code>shape</code> and <code>scale</code> must be of length 1.
<code>Smallno</code>	Numeric, a small value used by the rejection method for determining the upper limit of the distribution. That is, $\text{pnaka}(U) > 1 - \text{Smallno}$ where $U$ is the upper limit.
<code>...</code>	Arguments that can be passed into <a href="#">uniroot</a> .
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

See [nakagami](#) for more details.

**Value**

dnaka gives the density, pnaka gives the cumulative distribution function, qnaka gives the quantile function, and rnaka generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[nakagami](#).

**Examples**

```
## Not run: x <- seq(0, 3.2, len = 200)
plot(x, dgamma(x, shape = 1), type = "n", col = "black", ylab = "",
      ylim = c(0,1.5), main = "dnaka(x, shape)")
lines(x, dnaka(x, shape = 1), col = "orange")
lines(x, dnaka(x, shape = 2), col = "blue")
lines(x, dnaka(x, shape = 3), col = "green")
legend(2, 1.0, col = c("orange","blue","green"), lty = rep(1, len = 3),
      legend = paste("shape =", c(1, 2, 3)))

plot(x, pnorm(x), type = "n", col = "black", ylab = "",
      ylim = 0:1, main = "pnaka(x, shape)")
lines(x, pnaka(x, shape = 1), col = "orange")
lines(x, pnaka(x, shape = 2), col = "blue")
lines(x, pnaka(x, shape = 3), col = "green")
legend(2, 0.6, col = c("orange","blue","green"), lty = rep(1, len = 3),
      legend = paste("shape =", c(1, 2, 3)))
## End(Not run)

probs <- seq(0.1, 0.9, by = 0.1)
pnaka(qnaka(p = probs, shape = 2), shape = 2) - probs # Should be all 0
```

---

nakagami

---

*Nakagami Distribution Family Function*


---

**Description**

Estimation of the two parameters of the Nakagami distribution by maximum likelihood estimation.

**Usage**

```
nakagami(lshape = "log", lscale = "log", ishape = NULL, iscale = 1)
```

### Arguments

- `lshape`, `lscale` Parameter link functions applied to the *shape* and *scale* parameters. Log links ensure they are positive. See [Links](#) for more choices and information.
- `ishape`, `iscale` Optional initial values for the shape and scale parameters. For `ishape`, a NULL value means it is obtained in the `initialize` slot based on the value of `iscale`. For `iscale`, assigning a NULL means a value is obtained in the `initialize` slot, however, setting another numerical value is recommended if convergence fails or is too slow.

### Details

The Nakagami distribution, which is useful for modelling wireless systems such as radio links, can be written

$$f(y) = 2(shape/scale)^{shape} y^{2 \times shape - 1} \exp(-shape \times y^2 / scale) / \Gamma(shape)$$

for  $y > 0$ ,  $shape > 0$ ,  $scale > 0$ . The mean of  $Y$  is  $\sqrt{scale/shape} \times \Gamma(shape + 0.5) / \Gamma(shape)$  and these are returned as the fitted values. By default, the linear/additive predictors are  $\eta_1 = \log(shape)$  and  $\eta_2 = \log(scale)$ . Fisher scoring is implemented.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

The Nakagami distribution is also known as the Nakagami- $m$  distribution, where  $m = shape$  here. Special cases:  $m = 0.5$  is a one-sided Gaussian distribution and  $m = 1$  is a Rayleigh distribution. The second moment is  $E(Y^2) = m$ .

If  $Y$  has a Nakagami distribution with parameters *shape* and *scale* then  $Y^2$  has a gamma distribution with shape parameter *shape* and scale parameter *scale/shape*.

### Author(s)

T. W. Yee

### References

Nakagami, M. (1960) The  $m$ -distribution: a general formula of intensity distribution of rapid fading, pp.3–36 in: *Statistical Methods in Radio Wave Propagation*. W. C. Hoffman, Ed., New York: Pergamon.

### See Also

[rnaka](#), [gamma2](#), [rayleigh](#).

## Examples

```
nn <- 1000; shape <- exp(0); Scale <- exp(1)
ndata <- data.frame(y1 = sqrt(rgamma(nn, shape = shape, scale = Scale/shape)))
fit <- vglm(y1 ~ 1, nakagami, ndata, trace = TRUE, crit = "coef")
ndata <- transform(ndata, y2 = rnaka(nn, shape = shape, scale = Scale))
fit <- vglm(y2 ~ 1, nakagami(iscale = 3), ndata, trace = TRUE)
head(fitted(fit))
with(ndata, mean(y2))
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
## Not run: with(ndata,
hist(sy <- sort(y2), prob = TRUE, main = "", xlab = "y", ylim = c(0, 0.6)))
lines(dnaka(sy, shape = Cfit[1], scale = Cfit[2]) ~ sy, ndata, col = "orange")
## End(Not run)
```

nbcancelink

*Negative binomial canonical link function*

## Description

Computes the negative binomial canonical link transformation, including its inverse and the first two derivatives.

## Usage

```
nbcancelink(theta, size = NULL, wrt.eta = NULL, bvalue = NULL,
            inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

## Arguments

theta	Numeric or character. Typically the mean of a negative binomial (NB) distribution. See below for further details.
size, wrt.eta	size contains the $k$ matrix which must be of a conformable dimension as theta. Also, if deriv > 0 then wrt.eta is either 1 or 2 (1 for with respect to the first linear predictor, and 2 for with respect to the second linear predictor (a function of $k$ )).
bvalue	Details at <a href="#">Links</a> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

## Details

The negative binomial (NB) canonical link is  $\log(\theta/(\theta + k))$  where  $\theta$  is the mean of a NB distribution. The canonical link is used for theoretically relating the NB to GLM class.

This link function was specifically written for [negbinomial](#) and [negbinomial.size](#), and should not be used elsewhere (these **VGAM** family functions have code that specifically handles `nbcancelink()`.)

**Value**

For `deriv = 0`, the above equation when `inverse = FALSE`, and if `inverse = TRUE` then `kmatrix / expm1(-theta)`. For `deriv = 1`, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of `theta` if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

**Warning**

This function currently does not work very well with [negbinomial](#)! The NB-C model is sensitive to the initial values and may converge to a local solution. Pages 210 and 309 of Hilbe (2011) notes convergence difficulties (of Newton-Raphson type algorithms), and this applies here. This function should work okay with [negbinomial.size](#). Currently trying something like `imethod = 3` or `imu`, `stepsize = 0.5`, `maxit = 100`, `zero = -2` should help; see the example below.

Standard errors may be unreliable.

**Note**

While theoretically nice, this function is not recommended in general since its value is always negative (linear predictors ought to be unbounded in general). A [loge](#) link for argument `lm` is recommended instead.

Numerical instability may occur when `theta` is close to 0 or 1. Values of `theta` which are less than or equal to 0 can be replaced by `bvalue` before computing the link function value. See [Links](#).

**Author(s)**

Thomas W. Yee

**References**

- Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.
- Hilbe, J. M. (2011) *Negative Binomial Regression*, 2nd Edition. Cambridge: Cambridge University Press.

**See Also**

[negbinomial](#), [negbinomial.size](#).

**Examples**

```
nbcancelink("mu", short = FALSE)

mymu <- 1:10 # Test some basic operations:
kmatrix <- matrix(runif(length(mymu)), length(mymu), 1)
eta1 <- nbcancelink(mymu, size = kmatrix)
ans2 <- nbcancelink(eta1, size = kmatrix, inverse = TRUE)
max(abs(ans2 - mymu)) # Should be 0

## Not run: mymu <- c(seq(0.5, 10, length = 101))
kmatrix <- matrix(10, length(mymu), 1)
```



```

plot(nbcnlink(mymu, size = kmatrix) ~ mymu, las = 1,
     type = "l", col = "blue", lwd = 1.5, xlab = expression({\mu}))

# Estimate the parameters from some simulated data (see Warning section)
set.seed(123)
ndata <- data.frame(x2 = runif(nn <- 1000 ))
size1 <- exp(1); size2 <- exp(2)
ndata <- transform(ndata, eta1 = -1 - 2 * x2, # eta1 < 0
                  size1 = size1,
                  size2 = size2)
ndata <- transform(ndata,
                  mu1 = nbcnlink(eta1, size = size1, inv = TRUE),
                  mu2 = nbcnlink(eta1, size = size2, inv = TRUE))
ndata <- transform(ndata, y1 = rnbino(n, mu = mu1, size = size1),
                  y2 = rnbino(n, mu = mu2, size = size2))

head(ndata)
summary(ndata)

fit <- vglm(cbind(y1, y2) ~ x2, negbinomial("nbcnlink", imethod = 3),
           stepsize = 0.5, ndata, # Deliberately slow the convergence rate
           maxit = 100, trace = TRUE) # Warning: may converge to a local soln
coef(fit, matrix = TRUE)
summary(fit)

## End(Not run)

```

---

nbolf

*Negative Binomial-Ordinal Link Function*


---

## Description

Computes the negative binomial-ordinal transformation, including its inverse and the first two derivatives.

## Usage

```

nbolf(theta, cutpoint = NULL, k = NULL,
      inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)

```

## Arguments

theta	Numeric or character. See below for further details.
cutpoint, k	Here, k is the $k$ parameter associated with the negative binomial distribution; see <a href="#">negbinomial</a> . The cutpoints should be non-negative integers. If <code>nbolf()</code> is used as the link function in <a href="#">cumulative</a> then one should choose <code>reverse = TRUE</code> , <code>parallel = TRUE</code> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

### Details

The negative binomial-ordinal link function (NBOLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying negative binomial distribution.

See [Links](#) for general information about **VGAM** link functions.

### Value

See Yee (2012) for details.

### Warning

Prediction may not work on [vglm](#) or [vgam](#) etc. objects if this link function is used.

### Note

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0, numerical instabilities may still arise.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the negative binomial distribution (see [negbinomial](#)) that has been recorded as an ordinal response using known cutpoints.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. (2012) *Ordinal ordination with normalizing link functions for count data*, (in preparation).

### See Also

[Links](#), [negbinomial](#), [polf](#), [golf](#), [nbolf2](#), [cumulative](#), [CommonVGAMffArguments](#).

### Examples

```
nbolf("p", cutpoint = 2, k = 1, short = FALSE)
nbolf("p", cutpoint = 2, k = 1, tag = TRUE)

p <- seq(0.02, 0.98, by = 0.01)
y <- nbolf(p, cutpoint = 2, k = 1)
y. <- nbolf(p, cutpoint = 2, k = 1, deriv = 1)
max(abs(nbolf(y, cutpoint = 2, k = 1, inv = TRUE) - p)) # Should be 0

## Not run: par(mfrow = c(2, 1), las = 1)
plot(p, y, type = "l", col = "blue", main = "nbolf()")
abline(h = 0, v = 0.5, col = "red", lty = "dashed")
```

```

plot(p, y., type = "l", col = "blue",
     main = "(Reciprocal of) first NBOLF derivative")
## End(Not run)

# Another example
nn <- 1000
x2 <- sort(runif(nn))
x3 <- runif(nn)
mymu <- exp( 3 + 1 * x2 - 2 * x3)
k <- 4
y1 <- rnbinom(nn, mu = mymu, size = k)
cutpoints <- c(-Inf, 10, 20, Inf)
cuty <- Cut(y1, breaks = cutpoints)
## Not run: plot(x2, x3, col = cuty, pch = as.character(cuty))
table(cuty) / sum(table(cuty))
fit <- vglm(cuty ~ x2 + x3, trace = TRUE,
            cumulative(reverse = TRUE, mv = TRUE,
                       parallel = TRUE,
                       link = nbolf(cutpoint = cutpoints[2:3], k = k)))

head(depvar(fit))
head(fitted(fit))
head(predict(fit))
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
fit@misc

```

negbinomial

*Negative Binomial Distribution Family Function*

## Description

Maximum likelihood estimation of the two parameters of a negative binomial distribution.

## Usage

```

negbinomial(lmu = "loge", lsize = "loge",
            imu = NULL, isize = NULL, probs.y = 0.75,
            nsimEIM = 100, cutoff = 0.995,
            Maxiter = 5000, deviance.arg = FALSE, imethod = 1,
            parallel = FALSE, shrinkage.init = 0.95, zero = -2)
polya(lprob = "logit", lsize = "loge",
      iprob = NULL, isize = NULL, probs.y = 0.75, nsimEIM = 100,
      deviance.arg = FALSE, imethod = 1, shrinkage.init = 0.95, zero = -2)

```

## Arguments

`lmu`, `lsize`, `lprob`

Link functions applied to the  $\mu$ ,  $k$  and  $p$  parameters. See [Links](#) for more choices. Note that the  $\mu$ ,  $k$  and  $p$  parameters are the `mu`, `size` and `prob` ar-

	guments of <a href="#">rnbinom</a> respectively. Common alternatives for <code>lsize</code> are <a href="#">negloge</a> and <a href="#">reciprocal</a> .
<code>imu, isize, iprob</code>	Optional initial values for the mean and $k$ and $p$ . For $k$ , if failure to converge occurs then try different values (and/or use <code>imethod</code> ). For a $S$ -column response, <code>isize</code> can be of length $S$ . A value <code>NULL</code> means an initial value for each response is computed internally using a range of values. The last argument is ignored if used within <a href="#">cqo</a> ; see the <code>iKvector</code> argument of <a href="#">qrrvglm.control</a> instead.
<code>probs.y</code>	Passed into the <code>probs</code> argument of <a href="#">quantile</a> when <code>imethod</code> = 3 to obtain an initial value for the mean.
<code>nsimEIM</code>	This argument is used for computing the diagonal element of the <i>expected information matrix</i> (EIM) corresponding to $k$ . See <a href="#">CommonVGAMffArguments</a> for more information and the note below.
<code>cutoff</code>	Used in the finite series approximation. A numeric which is close to 1 but never exactly 1. Used to specify how many terms of the infinite series for computing the second diagonal element of the EIM are actually used. The sum of the probabilities are added until they reach this value or more (but no more than <code>Maxiter</code> terms allowed). It is like specifying $p$ in an imaginary function <code>qnegbin(p)</code> .
<code>Maxiter</code>	Used in the finite series approximation. Integer. The maximum number of terms allowed when computing the second diagonal element of the EIM. In theory, the value involves an infinite series. If this argument is too small then the value may be inaccurate.
<code>deviance.arg</code>	Logical. If <code>TRUE</code> , the deviance function is attached to the object. Under ordinary circumstances, it should be left alone because it really assumes the index parameter is at the maximum likelihood estimate. Consequently, one cannot use that criterion to minimize within the IRLS algorithm. It should be set <code>TRUE</code> only when used with <a href="#">cqo</a> under the fast algorithm.
<code>imethod</code>	An integer with value 1 or 2 or 3 which specifies the initialization method for the $\mu$ parameter. If failure to converge occurs try another value and/or else specify a value for <code>shrinkage.init</code> and/or else specify a value for <code>isize</code> .
<code>parallel</code>	See <a href="#">CommonVGAMffArguments</a> for more information. Setting <code>parallel</code> = <code>TRUE</code> is useful in order to get something similar to <a href="#">quasipoissonff</a> or what is known as NB-1. The parallelism constraint does not apply to any intercept term. You should set <code>zero</code> = <code>NULL</code> too if <code>parallel</code> = <code>TRUE</code> to avoid a conflict.
<code>shrinkage.init</code>	How much shrinkage is used when initializing $\mu$ . The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used in conjunction with <code>imethod</code> . If convergence failure occurs try setting this argument to 1.
<code>zero</code>	Integer valued vector, usually assigned <code>-2</code> or <code>2</code> if used at all. Specifies which of the two linear/additive predictors are modelled as an intercept only. By default, the $k$ parameter (after <code>lsize</code> is applied) is modelled as a single unknown number that is estimated. It can be modelled as a function of the explanatory variables by setting <code>zero</code> = <code>NULL</code> ; this has been called a NB-H model by Hilbe (2011). A negative value means that the value is recycled, so setting <code>-2</code> means all $k$ are intercept-only. See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The negative binomial distribution can be motivated in several ways, e.g., as a Poisson distribution with a mean that is gamma distributed. There are several common parametrizations of the negative binomial distribution. The one used by `negbinomial()` uses the mean  $\mu$  and an *index* parameter  $k$ , both which are positive. Specifically, the density of a random variable  $Y$  is

$$f(y; \mu, k) \sim \binom{y+k-1}{y} \left( \frac{\mu}{\mu+k} \right)^y \left( \frac{k}{k+\mu} \right)^k$$

where  $y = 0, 1, 2, \dots$ , and  $\mu > 0$  and  $k > 0$ . Note that the *dispersion* parameter is  $1/k$ , so that as  $k$  approaches infinity the negative binomial distribution approaches a Poisson distribution. The response has variance  $Var(Y) = \mu + \mu^2/k$ . When fitted, the `fitted.values` slot of the object contains the estimated value of the  $\mu$  parameter, i.e., of the mean  $E(Y)$ . It is common for some to use  $\alpha = 1/k$  as the ancillary or heterogeneity parameter; so common alternatives for `lsize` are `negloge` and `reciprocal`.

For `polya` the density is

$$f(y; p, k) \sim \binom{y+k-1}{y} (1-p)^y p^k$$

where  $y = 0, 1, 2, \dots$ , and  $0 < p < 1$  and  $k > 0$ .

The negative binomial distribution can be coerced into the classical GLM framework with one of the parameters being of interest and the other treated as a nuisance/scale parameter (this is implemented in the **MASS** library). The **VGAM** family function `negbinomial` treats both parameters on the same footing, and estimates them both by full maximum likelihood estimation. Simulated Fisher scoring is employed as the default (see the `nsimEIM` argument).

The parameters  $\mu$  and  $k$  are independent (diagonal EIM), and the confidence region for  $k$  is extremely skewed so that its standard error is often of no practical use. The parameter  $1/k$  has been used as a measure of aggregation.

These **VGAM** family functions handle *multivariate* responses, so that a matrix can be used as the response. The number of columns is the number of species, say, and setting `zero = -2` means that *all* species have a  $k$  equalling a (different) intercept only.

## Value

An object of class "vglmff" (see `vglmff-class`). The object is used by modelling functions such as `vglm`, `rrvglm` and `vgam`.

## Warning

The Poisson model corresponds to  $k$  equalling infinity. If the data is Poisson or close to Poisson, numerical problems will occur. Possibly choosing a log-log link may help in such cases, otherwise use `poissonff` or `quasipoissonff`.

These functions are fragile; the maximum likelihood estimate of the index parameter is fraught (see Lawless, 1987). In general, the `quasipoissonff` is more robust. Other alternatives to `negbinomial` are to fit a NB-1 or RR-NB (aka NB-P) model; see Yee (2014). Also available are the NB-C, NB-H and NB-G. Assigning values to the `lsize` argument may lead to a local solution, and smaller values are preferred over large values when using this argument.

Yet to do: write a family function which uses the methods of moments estimator for  $k$ .

## Note

These two functions implement two common parameterizations of the negative binomial (NB). Some people called the NB with integer  $k$  the *Pascal* distribution, whereas if  $k$  is real then this is the *Polya* distribution. I don't. The one matching the details of `rnbinom` in terms of  $p$  and  $k$  is `polya()`.

For `polya()` the code may fail when  $p$  is close to 0 or 1. It is not yet compatible with `cqo` or `cao`.

Suppose the response is called `y`. For `negbinomial()` the diagonal element of the *expected information matrix* (EIM) for parameter  $k$  involves an infinite series; consequently simulated Fisher scoring (see `nsimEIM`) is the default. This algorithm should definitely be used if `max(y)` is large, e.g., `max(y) > 300` or there are any outliers in `y`. A second algorithm involving a finite series approximation can be invoked by setting `nsimEIM = NULL`. Then the arguments `Maxiter` and `cutoff` are pertinent.

Regardless of the algorithm used, convergence problems may occur, especially when the response has large outliers or is large in magnitude. If convergence failure occurs, try using arguments (in recommended decreasing order) `nsimEIM`, `shrinkage.init`, `imethod`, `Maxiter`, `cutoff`, `isize`, `zero`.

The function `negbinomial` can be used by the fast algorithm in `cqo`, however, setting `EqualTolerances = TRUE` and `ITolerances = FALSE` is recommended.

In the first example below (Bliss and Fisher, 1953), from each of 6 McIntosh apple trees in an orchard that had been sprayed, 25 leaves were randomly selected. On each of the leaves, the number of adult female European red mites were counted.

There are two special uses of `negbinomial` for handling count data. Firstly, when used by `rrvglm` this results in a continuum of models in between and inclusive of quasi-Poisson and negative binomial regression. This is known as a reduced-rank negative binomial model (*RR-NB*). It fits a negative binomial log-linear regression with variance function  $Var(Y) = \mu + \delta_1 \mu^{\delta_2}$  where  $\delta_1$  and  $\delta_2$  are parameters to be estimated by MLE. Confidence intervals are available for  $\delta_2$ , therefore it can be decided upon whether the data are quasi-Poisson or negative binomial, if any.

Secondly, the use of `negbinomial` with `parallel = TRUE` inside `vglm` can result in a model similar to `quasipoissonff`. This is named the *NB-I* model. The dispersion parameter is estimated by MLE whereas `glm` uses the method of moments. In particular, it fits a negative binomial log-linear regression with variance function  $Var(Y) = \phi_0 \mu$  where  $\phi_0$  is a parameter to be estimated by MLE. Confidence intervals are available for  $\phi_0$ .

## Author(s)

Thomas W. Yee

## References

- Lawless, J. F. (1987) Negative binomial and mixed Poisson regression. *The Canadian Journal of Statistics* **15**, 209–225.
- Hilbe, J. M. (2011) *Negative Binomial Regression*, 2nd Edition. Cambridge: Cambridge University Press.
- Bliss, C. and Fisher, R. A. (1953) Fitting the negative binomial distribution to biological data. *Biometrics* **9**, 174–200.

Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

### See Also

[quasipoissonff](#), [poissonff](#), [zinegbinomial](#), [negbinomial.size](#) (e.g., NB-G), [nbcanlink](#) (NB-C), [posnegbinomial](#), [invbinomial](#),  
[rnbinom](#), [nboldf](#), [rrvglm](#), [cao](#), [cqo](#), [CommonVGAMffArguments](#).

### Examples

```
# Example 1: apple tree data
appletree <- data.frame(y = 0:7, w = c(70, 38, 17, 10, 9, 3, 2, 1))
fit <- vglm(y ~ 1, negbinomial, appletree, weights = w)
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)

# Example 2: simulated data with multivariate response
ndata <- data.frame(x2 = runif(nn <- 500))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = exp(3+x2), size = exp(1)),
                    y2 = rnbinom(nn, mu = exp(2-x2), size = exp(0)))
fit1 <- vglm(cbind(y1, y2) ~ x2, negbinomial, ndata, trace = TRUE)
coef(fit1, matrix = TRUE)

# Example 3: large counts so definitely use the nsimEIM argument
ndata <- transform(ndata, y3 = rnbinom(nn, mu = exp(12+x2), size = exp(1)))
with(ndata, range(y3)) # Large counts
fit2 <- vglm(y3 ~ x2, negbinomial(nsimEIM = 100), ndata, trace = TRUE)
coef(fit2, matrix = TRUE)

# Example 4: a NB-1 to estimate a negative binomial with  $\text{Var}(Y) = \phi_0 * \mu$ 
nn <- 1000 # Number of observations
phi0 <- 10 # Specify this; should be greater than unity
delta0 <- 1 / (phi0 - 1)
mydata <- data.frame(x2 = runif(nn), x3 = runif(nn))
mydata <- transform(mydata, mu = exp(2 + 3 * x2 + 0 * x3))
mydata <- transform(mydata, y3 = rnbinom(nn, mu = mu, size = delta0 * mu))
## Not run:
plot(y3 ~ x2, data = mydata, pch = "+", col = 'blue',
     main = paste("Var(Y) = ", phi0, " * mu", sep = ""), las = 1)
## End(Not run)
nb1 <- vglm(y3 ~ x2 + x3, negbinomial(parallel = TRUE, zero = NULL),
            mydata, trace = TRUE)
# Extracting out some quantities:
cnb1 <- coef(nb1, matrix = TRUE)
mydiff <- (cnb1["(Intercept)", "log(size)"] -
           cnb1["(Intercept)", "log(mu)"])
delta0.hat <- exp(mydiff)
(phi.hat <- 1 + 1 / delta0.hat) # MLE of phi
summary(nb1)
# Obtain a 95 percent confidence interval for phi0:
```

```

myvec <- rbind(-1, 1, 0, 0)
(se.mydiff <- sqrt(t(myvec) %*% vcov(nb1) %*% myvec))
ci.mydiff <- mydiff + c(-1.96, 1.96) * se.mydiff
ci.delta0 <- ci.exp.mydiff <- exp(ci.mydiff)
(ci.phi0 <- 1 + 1 / rev(ci.delta0)) # The 95 percent conf. interval for phi0

Confint.nb1(nb1) # Quick way to get it

summary(glm(y3 ~ x2 + x3, quasipoisson, mydata))$disper # cf. moment estimator

```

---

negbinomial.size	<i>Negative Binomial Distribution Family Function With Known Size</i>
------------------	---

---

## Description

Maximum likelihood estimation of the mean parameter of a negative binomial distribution with known size parameter.

## Usage

```

negbinomial.size(size = Inf, lmu = "loge", imu = NULL,
                 probs.y = 0.75, imethod = 1,
                 shrinkage.init = 0.95, zero = NULL)

```

## Arguments

size	Numeric, positive. Same as argument size of <a href="#">rnbinom</a> . If the response is a matrix then this is recycled to a matrix of the same dimension, by row ( <a href="#">matrix</a> with <code>byrow = TRUE</code> ).
lmu, imu	Same as <a href="#">negbinomial</a> .
probs.y	Same as <a href="#">negbinomial</a> .
imethod, zero	Same as <a href="#">negbinomial</a> .
shrinkage.init	Same as <a href="#">negbinomial</a> .

## Details

This **VGAM** family function estimates only the mean parameter of the negative binomial distribution. See [negbinomial](#) for general information. Setting `size = 1` gives what I call the NB-G (geometric model; see Hilbe (2011)). The default, `size = Inf`, corresponds to the Poisson distribution.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).



**Note**

If `lmu = "nbcnlink"` in `negbinomial.size()` then the `size` argument here should be assigned.

**Author(s)**

Thomas W. Yee

**References**

- Hilbe, J. M. (2011) *Negative Binomial Regression*, 2nd Edition. Cambridge: Cambridge University Press.
- Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

**See Also**

[negbinomial](#), [nbcnlink](#) (NB-C model), [quasipoissonff](#), [poissonff](#),  
[rnbinom](#).

**Examples**

```
# Simulated data with various multiple responses
size1 <- exp(1); size2 <- exp(2); size3 <- exp(0); size4 <- Inf
ndata <- data.frame(x2 = runif(nn <- 1000))
ndata <- transform(ndata, eta1 = -1 - 2 * x2, # eta1 must be negative
                  size1 = size1)
ndata <- transform(ndata,
                  mu1 = nbcnlink(eta1, size = size1, inv = TRUE))
ndata <- transform(ndata,
                  y1 = rnbinom(nn, mu = mu1, size = size1), # NB-C
                  y2 = rnbinom(nn, mu = exp(2 - x2), size = size2),
                  y3 = rnbinom(nn, mu = exp(3 + x2), size = size3), # NB-G
                  y4 = rpois (nn, la = exp(1 + x2)))

# Also known as NB-C with size known (Hilbe, 2011)
fit1 <- vglm(y1 ~ x2, negbinomial.size(size = size1, lmu = "nbcnlink"),
            ndata, trace = TRUE, crit = "coef")
coef(fit1, matrix = TRUE)
head(fit1@misc$size) # size saved here

fit2 <- vglm(cbind(y2, y3, y4) ~ x2,
            negbinomial.size(size = c(size2, size3, size4)),
            ndata, trace = TRUE)
coef(fit2, matrix = TRUE)
head(fit2@misc$size) # size saved here
```

normal.vcm

*Univariate Normal Distribution as a Varying-Coefficient Model*

## Description

Maximum likelihood estimation of all the coefficients of a LM where each of the usual regression coefficients is modelled with other explanatory variables via parameter link functions. Thus this is a basic varying-coefficient model.

## Usage

```
normal.vcm(link.list = list("(Default)" = "identity"),
           earg.list = list("(Default)" = list()),
           lsd = "loge", lvar = "loge",
           esd = list(), evar = list(),
           var.arg = FALSE, imethod = 1,
           icoefficients = NULL, isd = NULL, zero = "M")
```

## Arguments

- link.list, earg.list  
Link functions and extra arguments applied to the coefficients of the LM, excluding the standard deviation/variance. See [CommonVGAMffArguments](#) for more information. The default is for an identity link to be applied to each of the regression coefficients.
- lsd, esd, lvar, evar  
Link function and extra argument applied to the standard deviation/variance. See [CommonVGAMffArguments](#) for more information. Same as [uninormal](#).
- icoefficients  
Optional initial values for the coefficients. Recycled to length  $M - 1$  (does not include the standard deviation/variance). Try using this argument if there is a link function that is not programmed explicitly to handle range restrictions in the initialize slot.
- var.arg, imethod, isd  
Same as [uninormal](#).
- zero  
See [CommonVGAMffArguments](#) for more information. The default applies to the last one, viz. the standard deviation/variance.

## Details

This function allows all the usual LM regression coefficients to be modelled as functions of other explanatory variables via parameter link functions. For example, we may want some of them to be positive. Or we may want a subset of them to be positive and add to unity. So a class of such models have been named *varying-coefficient models* (VCMs).

The usual linear model is specified through argument form2. As with all other **VGAM** family functions, the linear/additive predictors are specified through argument formula.

The `mlogit` link allows a subset of the coefficients to be positive and add to unity. Either none or more than one call to `mlogit` is allowed. The last variable will be used as the baseline/reference group, and therefore excluded from the estimation.

By default, the log of the standard deviation is the last linear/additive predictor. It is recommended that this parameter be estimated as intercept-only, for numerical stability.

Technically, the Fisher information matrix is of unit-rank for all but the last parameter (the standard deviation/variance). Hence an approximation is used that pools over all the observations.

This **VGAM** family function cannot handle multiple responses. Also, this function will probably not have the full capabilities of the class of varying-coefficient models as described by Hastie and Tibshirani (1993). However, it should be able to manage some simple models, especially involving the following links: `identity`, `loge`, `logoff`, `loglog`, `logit`, `probit`, `cauchit`, `cloglog`, `rhobit`, `fisherz`.

### Value

An object of class "vglmff" (see `vglmff-class`). The object is used by modelling functions such as `vglm`, and `vgam`.

### Warning

This **VGAM** family function is fragile. One should monitor convergence, and possibly enter initial values especially when there are non-`identity`-link functions. If the initial value of the standard deviation/variance is too small then numerical problems may occur. One trick is to fit an intercept-only model and feed its `predict()` output into argument `etastart` of a more complicated model. The use of the zero argument is recommended in order to keep models as simple as possible.

### Note

The standard deviation/variance parameter is best modelled as intercept-only.

Yet to do: allow an argument such as `parallel` that enables many of the coefficients to be equal.  
Fix a bug: `Coef()` does not work for intercept-only models.

### Author(s)

T. W. Yee

### References

Hastie, T. and Tibshirani, R. (1993) Varying-coefficient models. *J. Roy. Statist. Soc. Ser. B*, **55**, 757–796.

### See Also

`uninormal`, `lm`.

## Examples

```

ndata <- data.frame(x2 = runif(nn <- 2000))
# Note that coeff1 + coeff2 + coeff5 == 1. So try a "mlogit" link.
myoffset <- 10
ndata <- transform(ndata,
  coeff1 = 0.25, # "mlogit" link
  coeff2 = 0.25, # "mlogit" link
  coeff3 = exp(-0.5), # "loge" link
  coeff4 = logoff(+0.5, offset = myoffset, inverse = TRUE), # "logoff" link
  coeff5 = 0.50, # "mlogit" link
  coeff6 = 1.00, # "identity" link
  v2 = runif(nn),
  v3 = runif(nn),
  v4 = runif(nn),
  v5 = rnorm(nn),
  v6 = rnorm(nn))
ndata <- transform(ndata,
  Coeff1 = 0.25 - 0 * x2,
  Coeff2 = 0.25 - 0 * x2,
  Coeff3 = logit(-0.5 - 1 * x2, inverse = TRUE),
  Coeff4 = loglog( 0.5 - 1 * x2, inverse = TRUE),
  Coeff5 = 0.50 - 0 * x2,
  Coeff6 = 1.00 + 1 * x2)
ndata <- transform(ndata,
  y1 = coeff1 * 1 +
    coeff2 * v2 +
    coeff3 * v3 +
    coeff4 * v4 +
    coeff5 * v5 +
    coeff6 * v6 + rnorm(nn, sd = exp(0)),
  y2 = Coeff1 * 1 +
    Coeff2 * v2 +
    Coeff3 * v3 +
    Coeff4 * v4 +
    Coeff5 * v5 +
    Coeff6 * v6 + rnorm(nn, sd = exp(0)))

# An intercept-only model
fit1 <- vglm(y1 ~ 1,
  form2 = ~ 1 + v2 + v3 + v4 + v5 + v6,
  normal.vcm(link.list = list("(Intercept)" = "mlogit",
    "v2" = "mlogit",
    "v3" = "loge",
    "v4" = "logoff",
    "(Default)" = "identity",
    "v5" = "mlogit"),
  earg.list = list("(Intercept)" = list(),
    "v2" = list(),
    "v4" = list(offset = myoffset),
    "v3" = list(),
    "(Default)" = list(),
    "v5" = list()),

```

```

                                zero = c(1:2, 6)),
                                data = ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
# This works only for intercept-only models:
mlogit(rbind(coef(fit1, matrix = TRUE)[1, c(1, 2)]), inverse = TRUE)

# A model with covariate x2 for the regression coefficients
fit2 <- vglm(y2 ~ 1 + x2,
             form2 = ~ 1 + v2 + v3 + v4 + v5 + v6,
             normal.vcm(link.list = list("(Intercept)" = "mlogit",
                                           "v2"           = "mlogit",
                                           "v3"           = "logit",
                                           "v4"           = "loglog",
                                           "(Default)"     = "identity",
                                           "v5"           = "mlogit"),
             earg.list = list("(Intercept)" = list(),
                               "v2"         = list(),
                               "v3"         = list(),
                               "v4"         = list(),
                               "(Default)"  = list(),
                               "v5"         = list()),
                                zero = c(1:2, 6)),
                                data = ndata, trace = TRUE)

coef(fit2, matrix = TRUE)
summary(fit2)

```

olympics

2008 and 2012 Summer Olympic Final Medal Count Data

## Description

Final medal count, by country, for the Summer 2008 and 2012 Olympic Games.

## Usage

```
data(olym08)
data(olym12)
```

## Format

A data frame with 87 or 85 observations on the following 6 variables.

rank a numeric vector, overall ranking of the countries.

country a factor.

gold a numeric vector, number of gold medals.

silver a numeric vector, number of silver medals.

bronze a numeric vector, number of bronze medals.

totalmedal a numeric vector, total number of medals.

### Details

The events were held during (i) August 8–24, 2008, in Beijing; and (ii) 27 July–12 August, 2012, in London.

### Source

[http://www.associatedcontent.com/article/979484/2008\\_summer\\_olympic\\_medal\\_count\\_total.html](http://www.associatedcontent.com/article/979484/2008_summer_olympic_medal_count_total.html), <http://www.london2012.com/medals/medal-count/>.

### References

The official English website was/is <http://en.beijing2008.cn> and <http://www.london2012.com>. Help from Viet Hoang Quoc is gratefully acknowledged.

### See Also

[grc](#).

### Examples

```
summary(olym08)
summary(olym12)
## maybe str(olym08) ; plot(olym08) ...
## Not run: par(mfrow = c(1, 2))
myylim <- c(0, 55)
with(head(olym08, n = 8),
barplot(rbind(gold, silver, bronze),
        col = c("gold", "grey", "brown"), # No "silver" or "bronze"!
        # "gold", "grey71", "chocolate4",
        names.arg = country, cex.names = 0.5, ylim = myylim,
        beside = TRUE, main = "2008 Summer Olympic Final Medal Count",
        ylab = "Medal count", las = 1,
        sub = "Top 8 countries; 'gold'=gold, 'grey'=silver, 'brown'=bronze"))
with(head(olym12, n = 8),
barplot(rbind(gold, silver, bronze),
        col = c("gold", "grey", "brown"), # No "silver" or "bronze"!
        names.arg = country, cex.names = 0.5, ylim = myylim,
        beside = TRUE, main = "2012 Summer Olympic Final Medal Count",
        ylab = "Medal count", las = 1,
        sub = "Top 8 countries; 'gold'=gold, 'grey'=silver, 'brown'=bronze"))
## End(Not run)
```

---

Opt

*Maxima*

---

### Description

Generic function for the *optima* (or optimums) of a model.

**Usage**

Opt(object, ...)

**Arguments**

object	An object for which the computation or extraction of an optimum (or optima) is meaningful.
...	Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for <a href="#">Coef</a> .

**Details**

Different models can define an optimum in different ways. Many models have no such notion or definition.

Optima occur in quadratic and additive ordination, e.g., CQO or CAO. For these models the optimum is the value of the latent variable where the maximum occurs, i.e., where the fitted value achieves its highest value. For quadratic ordination models there is a formula for the optimum but for additive ordination models the optimum must be searched for numerically. If it occurs on the boundary, then the optimum is undefined. At an optimum, the fitted value of the response is called the *maximum*.

**Value**

The value returned depends specifically on the methods function invoked.

**Note**

In ordination, the optimum of a species is sometimes called the *species score*.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

**See Also**

Opt.qrrvglm, [Max](#), [Tol](#).

Examples

```
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
# vvv p1 = cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
# vvv          Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
# vvv          Trocterr, Zoraspin) ~
# vvv          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
# vvv          Bestof = 2,
# vvv          fam = quasipoissonff, data = hspider, Crowlpositive=FALSE)
# vvv Opt(p1)

## Not run:
index <- 1:ncol(depvar(p1))
persp(p1, col = index, las = 1, lwd = 2, main = "Vertical lines at the optima")
abline(v = Opt(p1), lty = 2, col = index)

## End(Not run)
```

---

ordpoisson	<i>Ordinal Poisson Family Function</i>
------------	--

---

Description

Fits a Poisson regression where the response is ordinal (the Poisson counts are grouped between known cutpoints).

Usage

```
ordpoisson(cutpoints, countdata = FALSE, NOS = NULL,
           Levels = NULL, init.mu = NULL, parallel = FALSE,
           zero = NULL, link = "loge")
```

Arguments

cutpoints	Numeric. The cutpoints, $K_l$ . These must be non-negative integers. Inf values may be included. See below for further details.
countdata	Logical. Is the response (LHS of formula) in count-data format? If not then the response is a matrix or vector with values 1, 2, ..., L, say, where L is the number of levels. Such input can be generated with <a href="#">cut</a> with argument labels = FALSE. If countdata = TRUE then the response is expected to be in the same format as depvar(fit) where fit is a fitted model with ordpoisson as the <b>VGAM</b> family function. That is, the response is matrix of counts with L columns (if NOS = 1).
NOS	Integer. The number of species, or more generally, the number of response random variates. This argument must be specified when countdata = TRUE. Usually NOS = 1.



Levels	Integer vector, recycled to length NOS if necessary. The number of levels for each response random variate. This argument should agree with cutpoints. This argument must be specified when <code>countdata = TRUE</code> .
init.mu	Numeric. Initial values for the means of the Poisson regressions. Recycled to length NOS if necessary. Use this argument if the default initial values fail (the default is to compute an initial value internally).
parallel, zero, link	See <a href="#">poissonff</a> .

## Details

This **VGAM** family function uses maximum likelihood estimation (Fisher scoring) to fit a Poisson regression to each column of a matrix response. The data, however, is ordinal, and is obtained from known integer cutpoints. Here,  $l = 1, \dots, L$  where  $L$  ( $L \geq 2$ ) is the number of levels. In more detail, let  $Y^* = l$  if  $K_{l-1} < Y \leq K_l$  where the  $K_l$  are the cutpoints. We have  $K_0 = -\infty$  and  $K_L = \infty$ . The response for this family function corresponds to  $Y^*$  but we are really interested in the Poisson regression of  $Y$ .

If NOS=1 then the argument cutpoints is a vector  $(K_1, K_2, \dots, K_L)$  where the last value (Inf) is optional. If NOS>1 then the vector should have NOS-1 Inf values separating the cutpoints. For example, if there are NOS=3 responses, then something like `ordpoisson(cut = c(0, 5, 10, Inf, 20, 30, Inf, 0, 10, 40, Inf))` is valid.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

The input requires care as little to no checking is done. If `fit` is the fitted object, have a look at `fit@extra` and `devar(fit)` to check.

## Note

Sometimes there are no observations between two cutpoints. If so, the arguments `Levels` and `NOS` need to be specified too. See below for an example.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2012) *Ordinal ordination with normalizing link functions for count data*, (in preparation).

## See Also

[poissonff](#), [polf](#), [ordered](#).

## Examples

```

set.seed(123) # Example 1
x2 <- runif(n <- 1000); x3 <- runif(n)
mymu <- exp(3 - 1 * x2 + 2 * x3)
y1 <- rpois(n, lambda = mymu)
cutpts <- c(-Inf, 20, 30, Inf)
fcutpts <- cutpts[is.finite(cutpts)] # finite cutpoints
ystar <- cut(y1, breaks = cutpts, labels = FALSE)
## Not run:
plot(x2, x3, col = ystar, pch = as.character(ystar))

## End(Not run)
table(ystar) / sum(table(ystar))
fit <- vglm(ystar ~ x2 + x3, fam = ordpoisson(cutpoi = fcutpts))
head(depvar(fit)) # This can be input if countdata = TRUE
head(fitted(fit))
head(predict(fit))
coef(fit, matrix = TRUE)
fit@extra

# Example 2: multivariate and there are no obsns between some cutpoints
cutpts2 <- c(-Inf, 0, 9, 10, 20, 70, 200, 201, Inf)
fcutpts2 <- cutpts2[is.finite(cutpts2)] # finite cutpoints
y2 <- rpois(n, lambda = mymu) # Same model as y1
ystar2 <- cut(y2, breaks = cutpts2, labels = FALSE)
table(ystar2) / sum(table(ystar2))
fit <- vglm(cbind(ystar, ystar2) ~ x2 + x3, fam =
  ordpoisson(cutpoi = c(fcutpts, Inf, fcutpts2, Inf),
    Levels = c(length(fcutpts)+1, length(fcutpts2)+1),
    parallel = TRUE), trace = TRUE)
coef(fit, matrix = TRUE)
fit@extra
constraints(fit)
summary(depvar(fit)) # Some columns have all zeros

```

---

 oxtemp

*Oxford Temperature Data*


---

## Description

Annual maximum temperatures collected at Oxford, UK.

## Usage

```
data(oxtemp)
```

**Format**

A data frame with 80 observations on the following 2 variables.

**maxtemp** Annual maximum temperatures (in degrees Fahrenheit).

**year** The values 1901 to 1980.

**Details**

The data were collected from 1901 to 1980.

**Source**

Unknown.

**Examples**

```
## Not run: fit <- vglm(maxtemp ~ 1, egev, data = otemp, trace = TRUE)
```

---

Paralogistic

*The Paralogistic Distribution*

---

**Description**

Density, distribution function, quantile function and random generation for the paralogistic distribution with shape parameter  $a$  and scale parameter  $scale$ .

**Usage**

```
dparalogistic(x, shape1.a, scale = 1, log = FALSE)
pparalogistic(q, shape1.a, scale = 1)
qparalogistic(p, shape1.a, scale = 1)
rparalogistic(n, shape1.a, scale = 1)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> , the length is taken to be the number required.
<code>shape1.a</code>	shape parameter.
<code>scale</code>	scale parameter.
<code>log</code>	Logical. If <code>log=TRUE</code> then the logarithm of the density is returned.

**Details**

See [paralogistic](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

Value

dparalogistic gives the density, pparalogistic gives the distribution function, qparalogistic gives the quantile function, and rparalogistic generates random deviates.

Note

The paralogistic distribution is a special case of the 4-parameter generalized beta II distribution.

Author(s)

T. W. Yee

References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

See Also

[paralogistic](#), [genbetaII](#).

Examples

```
pdata <- data.frame(y = rparalogistic(n = 3000, exp(1), exp(2)))
fit <- vglm(y ~ 1, paralogistic(ishape1.a = 2.1), pdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

---

paralogistic	<i>Paralogistic Distribution Family Function</i>
--------------	--

---

Description

Maximum likelihood estimation of the 2-parameter paralogistic distribution.

Usage

```
paralogistic(lshape1.a = "loge", lscale = "loge",
             ishape1.a = 2, iscale = NULL, zero = NULL)
```

Arguments

- lshape1.a, lscale  
Parameter link functions applied to the (positive) shape parameter a and (positive) scale parameter scale. See [Links](#) for more choices.
- ishape1.a, iscale  
Optional initial values for a and scale.

**zero** An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2} which correspond to a, scale, respectively.

### Details

The 2-parameter paralogistic distribution is the 4-parameter generalized beta II distribution with shape parameter  $p = 1$  and  $a = q$ . It is the 3-parameter Singh-Maddala distribution with  $a = q$ . More details can be found in Kleiber and Kotz (2003).

The 2-parameter paralogistic has density

$$f(y) = a^2 y^{a-1} / [b^a \{1 + (y/b)^a\}^{1+a}]$$

for  $a > 0$ ,  $b > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter scale, and  $a$  is the shape parameter. The mean is

$$E(Y) = b \Gamma(1 + 1/a) \Gamma(a - 1/a) / \Gamma(a)$$

provided  $a > 1$ ; these are returned as the fitted values.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

See the note in [genbetaII](#).

### Author(s)

T. W. Yee

### References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

### See Also

[Paralogistic](#), [genbetaII](#), [betaII](#), [dagum](#), [fisk](#), [invlomax](#), [lomax](#), [invparalogistic](#).

### Examples

```
pdata <- data.frame(y = rparalogistic(n = 3000, exp(1), exp(2)))
fit <- vglm(y ~ 1, paralogistic, pdata, trace = TRUE)
fit <- vglm(y ~ 1, paralogistic(ishape1.a = 2.3, iscale = 7),
           pdata, trace = TRUE, epsilon = 1e-8)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

**Description**

Density, distribution function, quantile function and random generation for the Pareto(I) distribution with parameters location and shape.

**Usage**

```
dpareto(x, location, shape, log = FALSE)
ppareto(q, location, shape)
qpareto(p, location, shape)
rpareto(n, location, shape)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Must be a single positive integer.
location, shape	the $\alpha$ and $k$ parameters.
log	Logical. If log = TRUE then the logarithm of the density is returned.

**Details**

See [paretoff](#), the **VGAM** family function for estimating the parameter  $k$  by maximum likelihood estimation, for the formula of the probability density function and the range restrictions imposed on the parameters.

**Value**

dpareto gives the density, ppareto gives the distribution function, qpareto gives the quantile function, and rpareto generates random deviates.

**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[paretoff](#), [ParetoIV](#).

### Examples

```
alpha <- 3; k <- exp(1); x <- seq(2.8, 8, len = 300)
## Not run:
plot(x, dpareto(x, location = alpha, shape = k), type = "l",
     main = "Pareto density split into 10 equal areas")
abline(h = 0, col = "blue", lty = 2)
qvec <- qpareto(seq(0.1,0.9,by = 0.1),location = alpha,shape = k)
lines(qvec, dpareto(qvec, loc = alpha, shape = k),
      col = "purple", lty = 3, type = "h")

## End(Not run)
pvec <- seq(0.1, 0.9, by = 0.1)
qvec <- qpareto(pvec, location = alpha, shape = k)
ppareto(qvec, location = alpha, shape = k)
qpareto(ppareto(qvec, loc = alpha, shape = k),
      loc = alpha, shape = k) - qvec # Should be 0
```

---

paretoff

*Pareto and Truncated Pareto Distribution Family Functions*

---

### Description

Estimates one of the parameters of the Pareto(I) distribution by maximum likelihood estimation. Also includes the upper truncated Pareto(I) distribution.

### Usage

```
paretoff(lshape = "log", location = NULL)
truncpareto(lower, upper, lshape = "log", ishape = NULL, imethod = 1)
```

### Arguments

lshape	Parameter link function applied to the parameter $k$ . See <a href="#">Links</a> for more choices. A log link is the default because $k$ is positive.
location	Numeric. The parameter $\alpha$ below. If the user inputs a number then it is assumed known with this value. The default means it is estimated by maximum likelihood estimation, which means $\min(y)$ is used, where $y$ is the response vector.
lower, upper	Numeric. Lower and upper limits for the truncated Pareto distribution. Each must be positive and of length 1. They are called $\alpha$ and $U$ below.
ishape	Numeric. Optional initial value for the shape parameter. A NULL means a value is obtained internally. If failure to converge occurs try specifying a value, e.g., 1 or 2.
imethod	See <a href="#">CommonVGAMffArguments</a> for information. If failure to converge occurs then try specifying a value for ishape.

## Details

A random variable  $Y$  has a Pareto distribution if

$$P[Y > y] = C/y^k$$

for some positive  $k$  and  $C$ . This model is important in many applications due to the power law probability tail, especially for large values of  $y$ .

The Pareto distribution, which is used a lot in economics, has a probability density function that can be written

$$f(y; k, \alpha) = k\alpha^k / y^{k+1}$$

for  $0 < k$  and  $0 < \alpha < y$ . The  $\alpha$  is called the location parameter, and it is either assumed *known* or else  $\min(y)$  is used. The parameter  $k$  is called the shape parameter. The mean of  $Y$  is  $\alpha k / (k - 1)$  provided  $k > 1$ . Its variance is  $\alpha^2 k / ((k - 1)^2 (k - 2))$  provided  $k > 2$ .

The upper truncated Pareto distribution has a probability density function that can be written

$$f(y) = k\alpha^k / [y^{k+1} (1 - (\alpha/U)^k)]$$

for  $0 < \alpha < y < U < \infty$  and  $k > 0$ . Possibly, better names for  $k$  are the *index* and *tail* parameters. Here,  $\alpha$  and  $U$  are known. The mean of  $Y$  is  $k\alpha^k (U^{1-k} - \alpha^{1-k}) / [(1 - k)(1 - (\alpha/U)^k)]$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

The usual or unbounded Pareto distribution has two parameters (called  $\alpha$  and  $k$  here) but the family function `paretoff` estimates only  $k$  using iteratively reweighted least squares. The MLE of the  $\alpha$  parameter lies on the boundary and is  $\min(y)$  where  $y$  is the response. Consequently, using the default argument values, the standard errors are incorrect when one does a summary on the fitted object. If the user inputs a value for `alpha` then it is assumed known with this value and then summary on the fitted object should be correct. Numerical problems may occur for small  $k$ , e.g.,  $k < 1$ .

## Note

Outside of economics, the Pareto distribution is known as the Bradford distribution.

For `paretoff`, if the estimate of  $k$  is less than or equal to unity then the fitted values will be NAs. Also, `paretoff` fits the Pareto(I) distribution. See [paretoIV](#) for the more general Pareto(IV/III/II) distributions, but there is a slight change in notation:  $s = k$  and  $b = \alpha$ .

In some applications the Pareto law is truncated by a natural upper bound on the probability tail. The upper truncated Pareto distribution has three parameters (called  $\alpha$ ,  $U$  and  $k$  here) but the family function `truncpareto()` estimates only  $k$ . With known lower and upper limits, the ML estimator of  $k$  has the usual properties of MLEs. Aban (2006) discusses other inferential details.

## Author(s)

T. W. Yee



## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

Aban, I. B., Meerschaert, M. M. and Panorska, A. K. (2006) Parameter estimation for the truncated Pareto distribution, *Journal of the American Statistical Association*, **101**(473), 270–277.

## See Also

[Pareto](#), [Truncpareto](#), [paretoIV](#), [gpd](#).

## Examples

```
alpha <- 2; kay <- exp(3)
pdata <- data.frame(y = rpareto(n = 1000, location = alpha, shape = kay))
fit <- vglm(y ~ 1, paretoff, pdata, trace = TRUE)
fit@extra # The estimate of alpha is here
head(fitted(fit))
with(pdata, mean(y))
coef(fit, matrix = TRUE)
summary(fit) # Standard errors are incorrect!!

# Here, alpha is assumed known
fit2 <- vglm(y ~ 1, paretoff(location = alpha), pdata, trace = TRUE)
fit2@extra # alpha stored here
head(fitted(fit2))
coef(fit2, matrix = TRUE)
summary(fit2) # Standard errors are okay

# Upper truncated Pareto distribution
lower <- 2; upper <- 8; kay <- exp(2)
pdata3 <- data.frame(y = rtruncpareto(n = 100, lower = lower,
                                     upper = upper, shape = kay))
fit3 <- vglm(y ~ 1, truncpareto(lower, upper), pdata3, trace = TRUE)
coef(fit3, matrix = TRUE)
c(fit3@misc$lower, fit3@misc$upper)
```

## Description

Density, distribution function, quantile function and random generation for the Pareto(IV/III/II) distributions.

**Usage**

```

dparetoIV(x, location = 0, scale = 1, inequality = 1, shape = 1, log = FALSE)
pparetoIV(q, location = 0, scale = 1, inequality = 1, shape = 1)
qparetoIV(p, location = 0, scale = 1, inequality = 1, shape = 1)
rparetoIV(n, location = 0, scale = 1, inequality = 1, shape = 1)
dparetoIII(x, location = 0, scale = 1, inequality = 1, log = FALSE)
pparetoIII(q, location = 0, scale = 1, inequality = 1)
qparetoIII(p, location = 0, scale = 1, inequality = 1)
rparetoIII(n, location = 0, scale = 1, inequality = 1)
dparetoII(x, location = 0, scale = 1, shape = 1, log = FALSE)
pparetoII(q, location = 0, scale = 1, shape = 1)
qparetoII(p, location = 0, scale = 1, shape = 1)
rparetoII(n, location = 0, scale = 1, shape = 1)
dparetoI(x, scale = 1, shape = 1, log = FALSE)
pparetoI(q, scale = 1, shape = 1)
qparetoI(p, scale = 1, shape = 1)
rparetoI(n, scale = 1, shape = 1)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Must be a single positive integer.
<code>location</code>	the location parameter.
<code>scale, shape, inequality</code>	the (positive) scale, inequality and shape parameters.
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

For the formulas and other details see [paretoIV](#).

**Value**

Functions beginning with the letter `d` give the density, functions beginning with the letter `p` give the distribution function, functions beginning with the letter `q` give the quantile function, and functions beginning with the letter `r` generates random deviates.

**Note**

The functions `[dpqr]paretoI` are the same as `[dpqr]pareto` except for a slight change in notation:  $s = k$  and  $b = \alpha$ ; see [Pareto](#).

**Author(s)**

T. W. Yee

## References

Brazauskas, V. (2003) Information matrix for Pareto(IV), Burr, and related distributions. *Comm. Statist. Theory and Methods* **32**, 315–325.

Arnold, B. C. (1983) *Pareto Distributions*. Fairland, Maryland: International Cooperative Publishing House.

## See Also

[paretoIV](#), [Pareto](#).

## Examples

```
## Not run:
x <- seq(-0.2, 4, by = 0.01)
loc <- 0; Scale <- 1; ineq <- 1; shape <- 1.0
plot(x, dparetoIV(x, loc, Scale, ineq, shape), type = "l", col = "blue",
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple are 5,10,...,95 percentiles", ylim = 0:1, las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
Q <- qparetoIV(seq(0.05, 0.95, by = 0.05), loc, Scale, ineq, shape)
lines(Q, dparetoIV(Q, loc, Scale, ineq, shape), col = "purple", lty = 3, type = "h")
lines(x, pparetoIV(x, loc, Scale, ineq, shape), col = "orange")
abline(h = 0, lty = 2)

## End(Not run)
```

---

paretoIV

*Pareto(IV/III/II) Distribution Family Functions*

---

## Description

Estimates three of the parameters of the Pareto(IV) distribution by maximum likelihood estimation. Some special cases of this distribution are also handled.

## Usage

```
paretoIV(location = 0, lscale = "loge", linequality = "loge",
         lshape = "loge",
         iscale = 1, iinequality = 1, ishape = NULL, imethod = 1)
paretoIII(location = 0, lscale = "loge", linequality = "loge",
          iscale = NULL, iinequality = NULL)
paretoII(location = 0, lscale = "loge", lshape = "loge",
          iscale = NULL, ishape = NULL)
```

## Arguments

location	Location parameter, called $a$ below. It is assumed known.
lscale, linequality, lshape	Parameter link functions for the scale parameter (called $b$ below), inequality parameter (called $g$ below), and shape parameter (called $s$ below). See <a href="#">Links</a> for more choices. A log link is the default for all because all these parameters are positive.
iscale, iinequality, ishape	Initial values for the parameters. A NULL value means that it is obtained internally. If convergence failure occurs, use these arguments to input some alternative initial values.
imethod	Method of initialization for the shape parameter. Currently only values 1 and 2 are available. Try the other value if convergence failure occurs.

## Details

The Pareto(IV) distribution, which is used in actuarial science, economics, finance and telecommunications, has a cumulative distribution function that can be written

$$F(y) = 1 - [1 + ((y - a)/b)^{1/g}]^{-s}$$

for  $y > a$ ,  $b > 0$ ,  $g > 0$  and  $s > 0$ . The  $a$  is called the *location* parameter,  $b$  the *scale* parameter,  $g$  the *inequality* parameter, and  $s$  the *shape* parameter.

The location parameter is assumed known otherwise the Pareto(IV) distribution will not be a regular family. This assumption is not too restrictive in modelling because in typical applications this parameter is known, e.g., in insurance and reinsurance it is pre-defined by a contract and can be represented as a deductible or a retention level.

The inequality parameter is so-called because of its interpretation in the economics context. If we choose a unit shape parameter value and a zero location parameter value then the inequality parameter is the Gini index of inequality, provided  $g \leq 1$ .

The fitted values are currently NA because I haven't worked out what the mean of  $Y$  is yet.

There are a number of special cases of the Pareto(IV) distribution. These include the Pareto(I), Pareto(II), Pareto(III), and Burr family of distributions. Denoting  $PIV(a, b, g, s)$  as the Pareto(IV) distribution, the Burr distribution  $Burr(b, g, s)$  is  $PIV(a = 0, b, 1/g, s)$ , the Pareto(III) distribution  $PIII(a, b, g)$  is  $PIV(a, b, g, s = 1)$ , the Pareto(II) distribution  $PII(a, b, s)$  is  $PIV(a, b, g = 1, s)$ , and the Pareto(I) distribution  $PI(b, s)$  is  $PIV(b, b, g = 1, s)$ . Thus the Burr distribution can be fitted using the [negloge](#) link function and using the default `location=0` argument. The Pareto(I) distribution can be fitted using [paretoff](#) but there is a slight change in notation:  $s = k$  and  $b = \alpha$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

The extra slot of the fitted object has a component called "location" which stores the location parameter value(s).

**Author(s)**

T. W. Yee

**References**

Brazauskas, V. (2003) Information matrix for Pareto(IV), Burr, and related distributions. *Comm. Statist. Theory and Methods* **32**, 315–325.

Arnold, B. C. (1983) *Pareto Distributions*. Fairland, Maryland: International Cooperative Publishing House.

**See Also**

[ParetoIV](#), [paretoff](#), [gpd](#).

**Examples**

```
pdata <- data.frame(y = rparetoIV(2000, scale = exp(1),
                                ineq = exp(-0.3), shape = exp(1)))
## Not run: par(mfrow = c(2, 1))
with(pdata, hist(y)); with(pdata, hist(log(y)))
## End(Not run)
fit <- vglm(y ~ 1, paretoIV, data = pdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

Perks

*The Perks Distribution*

---

**Description**

Density, cumulative distribution function, quantile function and random generation for the Perks distribution.

**Usage**

```
dperks(x, shape, scale = 1, log = FALSE)
pperks(q, shape, scale = 1)
qperks(p, shape, scale = 1)
rperks(n, shape, scale = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations.
log	Logical. If log = TRUE then the logarithm of the density is returned.
shape, scale	positive shape and scale parameters.

**Details**

See [perks](#) for details.

**Value**

dperks gives the density, pperks gives the cumulative distribution function, qperks gives the quantile function, and rperks generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[perks](#).

**Examples**

```
probs <- seq(0.01, 0.99, by = 0.01)
Shape <- exp(-1.0); Scale <- exp(1);
max(abs(pperks(qperks(p = probs, Shape, Scale),
                Shape, Scale) - probs)) # Should be 0

## Not run: x <- seq(-0.1, 0.7, by = 0.01);
plot(x, dperks(x, Shape, Scale), type = "l", col = "blue", las = 1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles",
     ylab = "", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pperks(x, Shape, Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qperks(probs, Shape, Scale)
lines(Q, dperks(Q, Shape, Scale), col = "purple", lty = 3, type = "h")
pperks(Q, Shape, Scale) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

---

perks

*Perks Distribution Family Function*


---

**Description**

Maximum likelihood estimation of the 2-parameter Perks distribution.

**Usage**

```
perks(lshape = "loge", lscale = "loge",
     ishape = NULL, iscale = NULL,
     gshape = exp(-5:5), gscale = exp(-5:5),
     nsimEIM = 500, oim.mean = FALSE, zero = NULL)
```

**Arguments**

lshape, lscale	Parameter link functions applied to the shape parameter shape, scale parameter scale. All parameters are treated as positive here See <a href="#">Links</a> for more choices.
ishape, iscale	Optional initial values. A NULL means a value is computed internally.
gshape, gscale	See <a href="#">CommonVGAMffArguments</a> .
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> .
oim.mean	To be currently ignored.

**Details**

The Perks distribution has cumulative distribution function

$$F(x; \alpha, \beta) = 1 - \left\{ \frac{1 + \alpha}{1 + \alpha e^{\beta y}} \right\}^{1/\beta}$$

which leads to a probability density function

$$f(x; \alpha, \beta) = [1 + \alpha]^{1/\beta} \alpha e^{\beta y} / (1 + \alpha e^{\beta y})^{1+1/\beta}$$

for  $\alpha > 0$ ,  $\beta > 0$ ,  $x > 0$ . Here,  $\beta$  is called the scale parameter scale, and  $\alpha$  is called a shape parameter. The moments for this distribution do not appear to be available in closed form.

Simulated Fisher scoring is used and multiple responses are handled.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

A lot of care is needed because this is a rather difficult distribution for parameter estimation. If the self-starting initial values fail then try experimenting with the initial value arguments, especially iscale. Successful convergence depends on having very good initial values. Also, monitor convergence by setting trace = TRUE.

**Author(s)**

T. W. Yee

**References**

- Perks, W. (1932) On some experiments in the graduation of mortality statistics. *Journal of the Institute of Actuaries*, **63**, 12–40.
- Richards, S. J. (2012) A handbook of parametric survival models for actuarial use. *Scandinavian Actuarial Journal*. 1–25.

**See Also**

[dperks](#).

## Examples

```
## Not run: set.seed(123)
pdata <- data.frame(x2 = runif(nn <- 1000)) # x2 unused
pdata <- transform(pdata, eta1 = -1,
                  ceta1 = 1)
pdata <- transform(pdata, shape1 = exp(eta1),
                  scale1 = exp(ceta1))
pdata <- transform(pdata,
                  y1 = rperks(nn, shape = shape1, scale = scale1))

fit1 <- vglm(y1 ~ 1, perks, data = pdata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)

## End(Not run)
```

---

perspqrvglm

*Perspective plot for QRR-VGLMs*

---

## Description

Produces a perspective plot for a CQO model (QRR-VGLM). It is only applicable for rank-1 or rank-2 models with argument `noRRR = ~ 1`.

## Usage

```
perspqrvglm(x, varI.latvar = FALSE, reference = NULL, show.plot = TRUE,
            xlim = NULL, ylim = NULL, zlim = NULL,
            gridlength = if (Rank == 1) 301 else c(51,51),
            which.species = NULL,
            xlab = if (Rank == 1) "Latent Variable" else "Latent Variable 1",
            ylab = if (Rank == 1) "Expected Value" else "Latent Variable 2",
            zlab = "Expected value", labelSpecies = FALSE,
            stretch = 1.05, main = "", ticktype = "detailed",
            col = if (Rank == 1) par()$col else "white",
            llty = par()$lty, llwd = par()$lwd,
            add1 = FALSE, ...)
```

## Arguments

<code>x</code>	Object of class "qrrvglm", i.e., a constrained quadratic ordination (CQO) object.
<code>varI.latvar</code>	Logical that is fed into <code>Coef.qrrvglm</code> .
<code>reference</code>	Integer or character that is fed into <code>Coef.qrrvglm</code> .
<code>show.plot</code>	Logical. Plot it?
<code>xlim, ylim</code>	Limits of the x- and y-axis. Both are numeric of length 2. See <code>par</code> .



zlim	Limits of the z-axis. Numeric of length 2. Ignored if rank is 1. See <a href="#">par</a> .
gridlength	Numeric. The fitted values are evaluated on a grid, and this argument regulates the fineness of the grid. If Rank = 2 then the argument is recycled to length 2, and the two numbers are the number of grid points on the x- and y-axes respectively.
which.species	Numeric or character vector. Indicates which species are to be plotted. The default is to plot all of them. If numeric, it should contain values in the set $\{1, 2, \dots, S\}$ where $S$ is the number of species.
xlab, ylab	Character caption for the x-axis and y-axis. By default, a suitable caption is found. See the xlab argument in <a href="#">plot</a> or <a href="#">title</a> .
zlab	Character caption for the z-axis. Used only if Rank = 2. By default, a suitable caption is found. See the xlab argument in <a href="#">plot</a> or <a href="#">title</a> .
labelSpecies	Logical. Whether the species should be labelled with their names. Used for Rank = 1 only. The position of the label is just above the species' maximum.
stretch	Numeric. A value slightly more than 1, this argument adjusts the height of the y-axis. Used for Rank = 1 only.
main	Character, giving the title of the plot. See the main argument in <a href="#">plot</a> or <a href="#">title</a> .
ticktype	Tick type. Used only if Rank = 2. See <a href="#">persp</a> for more information.
col	Color. See <a href="#">persp</a> for more information.
lty	Line type. Rank-1 models only. See the lty argument of <a href="#">par</a> .
lwd	Line width. Rank-1 models only. See the lwd argument of <a href="#">par</a> .
add1	Logical. Add to an existing plot? Used only for rank-1 models.
...	Arguments passed into <a href="#">persp</a> . Useful arguments here include theta and phi, which control the position of the eye.

## Details

For a rank-1 model, a perspective plot is similar to [lvplot.qrrvglm](#) but plots the curves along a fine grid and there is no rugplot to show the site scores.

For a rank-2 model, a perspective plot has the first latent variable as the x-axis, the second latent variable as the y-axis, and the expected value (fitted value) as the z-axis. The result of a CQO is that each species has a response surface with elliptical contours. This function will, at each grid point, work out the maximum fitted value over all the species. The resulting response surface is plotted. Thus rare species will be obscured and abundant species will dominate the plot. To view rare species, use the `which.species` argument to select a subset of the species.

A perspective plot will be performed if `noRRR` = ~ 1, and Rank = 1 or 2. Also, all the tolerance matrices of those species to be plotted must be positive-definite.

## Value

For a rank-2 model, a list with the following components.

fitted	A $(G_1 \times G_2)$ by $M$ matrix of fitted values on the grid. Here, $G_1$ and $G_2$ are the two values of <code>gridlength</code> .
--------	--

latvar1grid, latvar2grid  
 The grid points for the x-axis and y-axis.

max.fitted  
 A  $G_1$  by  $G_2$  matrix of maximum of the fitted values over all species. These are the values that are plotted on the z-axis.

For a rank-1 model, the components latvar2grid and max.fitted are NULL.

### Note

Yee (2004) does not refer to perspective plots. Instead, contour plots via [lvplot.qrrvglm](#) are used. For rank-1 models, a similar function to this one is [lvplot.qrrvglm](#). It plots the fitted values at the actual site score values rather than on a fine grid here. The result has the advantage that the user sees the curves as a direct result from a model fitted to data whereas here, it is easy to think that the smooth bell-shaped curves are the truth because the data is more of a distance away.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

### See Also

[persp](#), [cqo](#), [Coef.qrrvglm](#), [lvplot.qrrvglm](#), [par](#), [title](#).

### Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Good idea when ITolerances = TRUE
set.seed(111)
r1 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardmont, Pardnigr, Pardpull, Trocterr) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          poissonff, data = hspider, trace = FALSE, ITolerances = TRUE)
set.seed(111) # r2 below is an ill-conditioned model
r2 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardmont, Pardnigr, Pardpull, Trocterr) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          isd.lv = c(2.4, 1.0), Muxfactor = 3.0, trace = FALSE,
          poissonff, data = hspider, Rank = 2, EqualTolerances = TRUE)

sort(r1@misc$deviance.Bestof) # A history of the fits
sort(r2@misc$deviance.Bestof) # A history of the fits
if (deviance(r2) > 857) stop("suboptimal fit obtained")

persp(r1, xlim = c(-6,5), col = 1:4, label = TRUE)

# Involves all species
```

```

persp(r2, xlim = c(-6,5), ylim = c(-4, 5), theta = 10, phi = 20, zlim = c(0, 220))
# Omit the two dominant species to see what is behind them
persp(r2, xlim = c(-6,5), ylim = c(-4, 5), theta = 10, phi = 20, zlim = c(0, 220),
      which = (1:10)[-c(8, 10)]) # Use zlim to retain the original z-scale

## End(Not run)

```

pgamma.deriv

*Derivatives of the Incomplete Gamma Integral***Description**

The first two derivatives of the incomplete gamma integral.

**Usage**

```
pgamma.deriv(q, shape, tmax = 100)
```

**Arguments**

**q, shape** As in [pgamma](#) but these must be vectors of positive values only and finite.  
**tmax** Maximum number of iterations allowed in the computation (per q value).

**Details**

Write  $x = q$  and  $\text{shape} = a$ . The first and second derivatives with respect to  $q$  and  $a$  are returned. This function is similar in spirit to [pgamma](#); define

$$P(a, x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt$$

so that  $P(a, x)$  is `pgamma(x, a)`. Currently a 6-column matrix is returned (in the future this may change and an argument may be supplied so that only what is required by the user is computed.)

The computations use a series expansion for  $a \leq x \leq 1$  or  $x < a$ , else otherwise a continued fraction expansion. Machine overflow can occur for large values of  $x$  when  $x$  is much greater than  $a$ .

**Value**

The first 5 columns, running from left to right, are the derivatives with respect to:  $x$ ,  $x^2$ ,  $a$ ,  $a^2$ ,  $xa$ . The 6th column is  $P(a, x)$  (but it is not as accurate as calling [pgamma](#) directly).

**Note**

If convergence does not occur then try increasing the value of `tmax`.

Yet to do: add more arguments to give greater flexibility in the accuracy desired and to compute only quantities that are required by the user.

**Author(s)**

T. W. Yee wrote the wrapper function to the Fortran subroutine written by R. J. Moore. The subroutine was modified to run using double precision. The original code came from <http://lib.stat.cmu.edu/apstat/187>.

**References**

Moore, R. J. (1982) Algorithm AS 187: Derivatives of the Incomplete Gamma Integral. *Journal of the Royal Statistical Society, Series C (Applied Statistics)*, **31**(3), 330–335.

**See Also**

[pgamma.deriv.unscaled](#), [pgamma](#).

**Examples**

```
x <- seq(2, 10, length = 501)
head(ans <- pgamma.deriv(x, 2))
## Not run: par(mfrow = c(2, 3))
for (jay in 1:6)
  plot(x, ans[, jay], type = "l", col = "blue", cex.lab = 1.5,
       cex.axis = 1.5, las = 1, log = "x",
       main = colnames(ans)[jay], xlab = "q", ylab = "")
## End(Not run)
```

---

`pgamma.deriv.unscaled` *Derivatives of the Incomplete Gamma Integral (Unscaled Version)*

---

**Description**

The first two derivatives of the incomplete gamma integral with scaling.

**Usage**

```
pgamma.deriv.unscaled(q, shape)
```

**Arguments**

`q`, `shape` As in [pgamma](#) and [pgamma.deriv](#) but these must be vectors of positive values only and finite.

**Details**

Define

$$G(x, a) = \int_0^x t^{a-1} e^{-t} dt$$

so that  $G(x, a)$  is `pgamma(x, a) * gamma(a)`. Write  $x = q$  and  $\text{shape} = a$ . The 0th and first and second derivatives with respect to  $a$  of  $G$  are returned. This function is similar in spirit to [pgamma.deriv](#) but here there is no gamma function to scale things. Currently a 3-column matrix is returned (in the future this may change and an argument may be supplied so that only what is required by the user is computed.) This function is based on Wingo (1989).

**Value**

The 3 columns, running from left to right, are the 0:2th derivatives with respect to  $a$ .

**Warning**

These function seems inaccurate for  $q = 1$  and  $q = 2$ ; see the plot below.

**Author(s)**

T. W. Yee.

**References**

See [truncweibull](#).

**See Also**

[pgamma.deriv](#), [pgamma](#).

**Examples**

```
x <- 3; aa <- seq(0.3, 04, by = 0.01)
ans.u <- pgamma.deriv.unscaled(x, aa)
head(ans.u)

## Not run: par(mfrow = c(1, 3))
for (jay in 1:3) {
  plot(aa, ans.u[, jay], type = "l", col = "blue", cex.lab = 1.5,
        cex.axis = 1.5, las = 1, main = colnames(ans.u)[jay],
        log = "", xlab = "shape", ylab = "")
  abline(h = 0, v = 1:2, lty = "dashed", col = "gray") # Inaccurate at 1 and 2
}

## End(Not run)
```

---

Plackett*Plackett's Bivariate Distribution*

---

**Description**

Density, distribution function, and random generation for the (one parameter) bivariate Plackett distribution.

**Usage**

```
dplack(x1, x2, oratio, log = FALSE)
pplack(q1, q2, oratio)
rplack(n, oratio)
```

**Arguments**

`x1`, `x2`, `q1`, `q2` vector of quantiles.  
`n` number of observations. Must be a positive integer of length 1.  
`oratio` the positive odds ratio  $\psi$ .  
`log` Logical. If TRUE then the logarithm is returned.

**Details**

See [plackett](#), the **VGAM** family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

**Value**

`dplack` gives the density, `pplack` gives the distribution function, and `rplack` generates random deviates (a two-column matrix).

**Author(s)**

T. W. Yee

**References**

Mardia, K. V. (1967) Some contributions to contingency-type distributions. *Biometrika*, **54**, 235–249.

**See Also**

[plackett](#), [bifrankcop](#).

**Examples**

```
## Not run: N <- 101; oratio <- exp(1)
x <- seq(0.0, 1.0, len = N)
ox <- expand.grid(x, x)
zedd <- dplack(ox[, 1], ox[, 2], oratio = oratio)
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pplack(ox[, 1], ox[, 2], oratio = oratio)
contour(x, x, matrix(zedd, N, N), col = "blue")

plot(rr <- rplack(n = 3000, oratio = oratio))
par(mfrow = c(1, 2))
hist(rr[, 1]) # Should be uniform
hist(rr[, 2]) # Should be uniform

## End(Not run)
```

plackett

*Plackett's Bivariate Distribution Family Function***Description**

Estimate the association parameter of Plackett's bivariate distribution by maximum likelihood estimation.

**Usage**

```
plackett(link = "loge", ioratio = NULL, imethod = 1, nsimEIM = 200)
```

**Arguments**

**link** Link function applied to the (positive) odds ratio  $\psi$ . See [Links](#) for more choices and information.

**ioratio** Numeric. Optional initial value for  $\psi$ . If a convergence failure occurs try assigning a value or a different value.

**imethod, nsimEIM** See [CommonVGAMffArguments](#).

**Details**

The defining equation is

$$\psi = H \times (1 - y_1 - y_2 + H) / ((y_1 - H) \times (y_2 - H))$$

where  $P(Y_1 \leq y_1, Y_2 \leq y_2) = H_\psi(y_1, y_2)$  is the cumulative distribution function. The density function is  $h_\psi(y_1, y_2) =$

$$\psi[1 + (\psi - 1)(y_1 + y_2 - 2y_1y_2)] / ([1 + (\psi - 1)(y_1 + y_2)]^2 - 4\psi(\psi - 1)y_1y_2)^{3/2}$$

for  $\psi > 0$ . Some writers call  $\psi$  the *cross product ratio* but it is called the *odds ratio* here. The support of the function is the unit square. The marginal distributions here are the standard uniform although it is commonly generalized to other distributions.

If  $\psi = 1$  then  $h_\psi(y_1, y_2) = y_1 y_2$ , i.e., independence. As the odds ratio tends to infinity one has  $y_1 = y_2$ . As the odds ratio tends to 0 one has  $y_2 = 1 - y_1$ .

Fisher scoring is implemented using [rplack](#). Convergence is often quite slow.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

The response must be a two-column matrix. Currently, the fitted value is a 2-column matrix with 0.5 values because the marginal distributions correspond to a standard uniform distribution.

## Author(s)

T. W. Yee

## References

Plackett, R. L. (1965) A class of bivariate distributions. *Journal of the American Statistical Association*, **60**, 516–522.

## See Also

[rplack](#), [bifrankcop](#).

## Examples

```
## Not run:
ymat <- rplack(n = 2000, oratio = exp(2))
plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = plackett, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
head(fitted(fit))
summary(fit)

## End(Not run)
```



---

plotdeplot.lmscreg      *Density Plot for LMS Quantile Regression*


---

**Description**

Plots a probability density function associated with a LMS quantile regression.

**Usage**

```
plotdeplot.lmscreg(answer, y.arg, add.arg = FALSE,
  xlab = "", ylab = "density", xlim = NULL, ylim = NULL,
  lty.arg = par()$lty, col.arg = par()$col,
  llwd.arg = par()$lwd, ...)
```

**Arguments**

answer	Output from functions of the form deplot.??? where ??? is the name of the <b>VGAM</b> LMS family function, e.g., lms.yjn. See below for details.
y.arg	Numerical vector. The values of the response variable at which to evaluate the density. This should be a grid that is fine enough to ensure the plotted curves are smooth.
add.arg	Logical. Add the density to an existing plot?
xlab, ylab	Caption for the x- and y-axes. See <a href="#">par</a> .
xlim, ylim	Limits of the x- and y-axes. See <a href="#">par</a> .
lty.arg	Line type. See the lty argument of <a href="#">par</a> .
col.arg	Line color. See the col argument of <a href="#">par</a> .
llwd.arg	Line width. See the lwd argument of <a href="#">par</a> .
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include main and las.

**Details**

The above graphical parameters offer some flexibility when plotting the quantiles.

**Value**

The list answer, which has components

newdata	The argument newdata above from the argument list of <a href="#">deplot.lmscreg</a> , or a one-row data frame constructed out of the x0 argument.
y	The argument y.arg above.
density	Vector of the density function values evaluated at y.arg.

**Note**

While the graphical arguments of this function are useful to the user, this function should not be called directly.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[deplot.lmscreg](#).

**Examples**

```
fit <- vgam(BMI ~ s(age, df = c(4,2)), lms.bcn(zero = 1), bmi.nz)
## Not run: y = seq(15, 43, by = 0.25)
deplot(fit, x0 = 20, y = y, xlab = "BMI", col = "green", llwd = 2,
       main = "BMI distribution at ages 20 (green), 40 (blue), 60 (orange)")
deplot(fit, x0 = 40, y = y, add = TRUE, col = "blue", llwd = 2)
deplot(fit, x0 = 60, y = y, add = TRUE, col = "orange", llwd = 2) -> aa

names(aa$post$deplot)
aa$post$deplot$newdata
head(aa$post$deplot$y)
head(aa$post$deplot$density)
## End(Not run)
```

---

plotqrrvglm

---

*Model Diagnostic Plots for QRR-VGLMs*


---

**Description**

The residuals of a QRR-VGLM are plotted for model diagnostic purposes.

**Usage**

```
plotqrrvglm(object, rtype = c("response", "pearson", "deviance", "working"),
            ask = FALSE,
            main = paste(Rtype, "residuals vs latent variable(s)"),
            xlab = "Latent Variable",
            ITolerances = object@control$EqualTolerances, ...)
```

**Arguments**

object	An object of class "qrrvglm".
rtype	Character string giving residual type. By default, the first one is chosen.
ask	Logical. If TRUE, the user is asked to hit the return key for the next plot.
main	Character string giving the title of the plot.
xlab	Character string giving the x-axis caption.
ITolerances	Logical. This argument is fed into <code>Coef(object, ITolerances = ITolerances)</code> .
...	Other plotting arguments (see <a href="#">par</a> ).

**Details**

Plotting the residuals can be potentially very useful for checking that the model fit is adequate.

**Value**

The original object.

**Note**

An ordination plot of a QRR-VGLM can be obtained by [lvplot.qrrvglm](#).

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

**See Also**

[lvplot.qrrvglm](#), [cqo](#).

**Examples**

```
## Not run:
# QRR-VGLM on the hunting spiders data
# This is computationally expensive
set.seed(111) # This leads to the global solution
# hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardize the environmental variables
p1 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
               Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          quasipoissonff, data = hspider, Crowlpositive = FALSE)
par(mfrow = c(3, 4))
plot(p1, rtype = "response", col = "blue", pch = 4, las = 1, main = "")

## End(Not run)
```

---

plotqtplot.lmscreg	<i>Quantile Plot for LMS Quantile Regression</i>
--------------------	--

---

## Description

Plots the quantiles associated with a LMS quantile regression.

## Usage

```
plotqtplot.lmscreg(fitted.values, object, newdata = NULL,
  percentiles = object@misc$percentiles, lp = NULL,
  add.arg = FALSE, y = if (length(newdata)) FALSE else TRUE,
  spline.fit = FALSE, label = TRUE, size.label = 0.06,
  xlab = NULL, ylab = "",
  pch = par()$pch, pcex = par()$cex, pcol.arg = par()$col,
  xlim = NULL, ylim = NULL,
  llty.arg = par()$lty, lcol.arg = par()$col, llwd.arg = par()$lwd,
  tcol.arg = par()$col, tadj = 1, ...)
```

## Arguments

fitted.values	Matrix of fitted values.
object	A <b>VGAM</b> quantile regression model, i.e., an object produced by modelling functions such as <a href="#">vglm</a> and <a href="#">vgam</a> with a family function beginning with "lms.", e.g., <a href="#">lms.yjn</a> .
newdata	Data frame at which predictions are made. By default, the original data are used.
percentiles	Numerical vector with values between 0 and 100 that specify the percentiles (quantiles). The default is to use the percentiles when fitting the model. For example, the value 50 corresponds to the median.
lp	Length of percentiles.
add.arg	Logical. Add the quantiles to an existing plot?
y	Logical. Add the response as points to the plot?
spline.fit	Logical. Add a spline curve to the plot?
label	Logical. Add the percentiles (as text) to the plot?
size.label	Numeric. How much room to leave at the RHS for the label. It is in percent (of the range of the primary variable).
xlab	Caption for the x-axis. See <a href="#">par</a> .
ylab	Caption for the y-axis. See <a href="#">par</a> .
pch	Plotting character. See <a href="#">par</a> .
pcex	Character expansion of the points. See <a href="#">par</a> .
pcol.arg	Color of the points. See the col argument of <a href="#">par</a> .
xlim	Limits of the x-axis. See <a href="#">par</a> .

ylim	Limits of the y-axis. See <a href="#">par</a> .
lty.arg	Line type. Line type. See the lty argument of <a href="#">par</a> .
lcol.arg	Color of the lines. See the col argument of <a href="#">par</a> .
llwd.arg	Line width. See the lwd argument of <a href="#">par</a> .
tcol.arg	Color of the text (if label is TRUE). See the col argument of <a href="#">par</a> .
tadj	Text justification. See the adj argument of <a href="#">par</a> .
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include main and las.

### Details

The above graphical parameters offer some flexibility when plotting the quantiles.

### Value

The matrix of fitted values.

### Note

While the graphical arguments of this function are useful to the user, this function should not be called directly.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

### See Also

[qtplot.lmscreg](#).

### Examples

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4,2)), lms.bcn(zero = 1), data = bmi.nz)
qtplot(fit)
qtplot(fit, perc = c(25,50,75,95), lcol = "blue", tcol = "blue", llwd = 2)

## End(Not run)
```

plotrcim0

*Main effects plot for a Row-Column Interaction Model (RCIM)***Description**

Produces a main effects plot for Row-Column Interaction Models (RCIMs).

**Usage**

```
plotrcim0(object, centered = TRUE, which.plots = c(1, 2),
          hline0 = TRUE, hlty = "dashed", hcol = par()$col, hlwd = par()$lwd,
          rfirst = 1, cfirst = 1,
          rtype = "h", ctype = "h",
          rcex.lab = 1, rcex.axis = 1, rtick = FALSE,
          ccex.lab = 1, ccex.axis = 1, ctick = FALSE,
          rmain = "Row effects", rsub = "",
          rxlab = "", rylab = "Row effects",
          cmain = "Column effects", csub = "",
          cxlab = "", cylab = "Column effects",
          rcol = par()$col, ccol = par()$col,
          no.warning = FALSE, ...)
```

**Arguments**

object	An <a href="#">rcim</a> object. This should be of rank-0, i.e., main effects only and no interactions.
which.plots	Numeric, describing which plots are to be plotted. The row effects plot is 1 and the column effects plot is 2. Set the value 0, say, for no plots at all.
centered	Logical. If TRUE then the row and column effects are centered (but not scaled) by <a href="#">scale</a> . If FALSE then the raw effects are used (of which the first are zero by definition).
hline0, hlty, hcol, hlwd	hline0 is logical. If TRUE then a horizontal line is plotted at 0 and the other arguments describe this line. Probably having hline0 = TRUE only makes sense when centered = TRUE.
rfirst, cfirst	rfirst is the level of row that is placed first in the row effects plot, etc.
rmain, cmain	Character. rmain is the main label in the row effects plot, etc.
rtype, ctype, rsub, csub	See the type and sub arguments of <a href="#">plot</a> .
rxlab, rylab, cxlab, cylab	Character. For the row effects plot, rxlab is xlab and rylab is ylab; see <a href="#">par</a> . Ditto for cxlab and cylab for the column effects plot.
rcex.lab, ccex.lab	Numeric. rcex.lab is cex for the row effects plot label, etc.

<code>rcex.axis, ccex.axis</code>	Numeric. <code>rcex.axis</code> is the <code>cex</code> argument for the row effects axis label, etc.
<code>rtick, ctick</code>	Logical. If <code>rtick = TRUE</code> then add ticks to the row effects plot, etc.
<code>rcol, ccol</code>	<code>rcol</code> give a colour for the row effects plot, etc.
<code>no.warning</code>	Logical. If <code>TRUE</code> then no warning is issued if the model is not rank-0.
<code>...</code>	Arguments fed into both <a href="#">plot</a> calls.

## Details

This function plots the row and column effects of a rank-0 RCIM. As the result is a main effects plot of a regression analysis, its interpretation when `centered = FALSE` is relative to the baseline (reference level) of a row and column, and should also be considered in light of the link function used. Many arguments that start with "r" refer to the row effects plot, and "c" for the column effects plot.

## Value

The original object with the `post` slot assigned additional information from the plot.

## Note

This function should be only used to plot the object of rank-0 RCIM. If the rank is positive then it will issue a warning.

Using an argument `ylim` will mean the row and column effects are plotted on a common scale; see [plot.window](#).

## Author(s)

T. W. Yee, A. F. Hadi.

## See Also

[moffset Rcim](#), [rcim](#).

## Examples

```
alcoff.e <- moffset(alcoff, "6", "Mon", postfix = "*") # Effective day
fit0 <- rcim(alcoff.e, family = poissonff)
## Not run: par(oma = c(0, 0, 4, 0), mfrow = 1:2) # For all plots below too
ii = plot(fit0, rcol = "blue", ccol = "orange",
          lwd = 4, ylim = c(-2, 2), # A common ylim
          cylab = "Effective daily effects", rylab = "Hourly effects",
          rxlab = "Hour", cxlab = "Effective day")
ii@post # Endowed with additional information

## End(Not run)

# Negative binomial example
fit1 <- rcim(alcoff.e, negbinomial, trace = TRUE)
## Not run: plot(fit1, ylim = c(-2, 2))
```

```
# Univariate normal example
fit2 <- rcim(alcoff.e, uninormal, trace = TRUE)
## Not run: plot(fit2, ylim = c(-200, 400))

# Median-polish example
fit3 <- rcim(alcoff.e, alaplace2(tau = 0.5, intparloc = TRUE),
            trace = TRUE)
## Not run: plot(fit3, ylim = c(-200, 250))

# Zero-inflated Poisson example on "crashp" (no 0s in alcoff)
cbind(rowSums(crashp)) # Easy to see the data
cbind(colSums(crashp)) # Easy to see the data
fit4 <- rcim(Rcim(crashp, rbaseline = "5", cbaseline = "Sun"),
            zipoissonff, trace = TRUE)
## Not run: plot(fit4, ylim = c(-3, 3))
```

plotvgam

*Default VGAM Plotting*

## Description

Component functions of a **vgam-class** object can be plotted with `plotvgam()`. These are on the scale of the linear/additive predictor.

## Usage

```
plotvgam(x, newdata = NULL, y = NULL, residuals = NULL,
         rugplot = TRUE, se = FALSE, scale = 0, raw = TRUE,
         offset.arg = 0, deriv.arg = 0, overlay = FALSE,
         type.residuals = c("deviance", "working", "pearson", "response"),
         plot.arg = TRUE, which.term = NULL, which.cf = NULL,
         control = plotvgam.control(...), varxij = 1, ...)
```

## Arguments

<code>x</code>	A fitted <b>VGAM</b> object, e.g., produced by <code>vgam</code> , <code>vglm</code> , or <code>rrvglm</code> .
<code>newdata</code>	Data frame. May be used to reconstruct the original data set.
<code>y</code>	Unused.
<code>residuals</code>	Logical. If TRUE then residuals are plotted. See <code>type.residuals</code>
<code>rugplot</code>	Logical. If TRUE then a rug plot is plotted at the foot of each plot. These values are jittered to expose ties.
<code>se</code>	Logical. If TRUE then approximate $\pm 2$ pointwise standard error bands are included in the plot.
<code>scale</code>	Numerical. By default, each plot will have its own y-axis scale. However, by specifying a value, each plot's y-axis scale will be at least scale wide.



<code>raw</code>	Logical. If TRUE then the smooth functions are those obtained directly by the algorithm, and are plotted without having to premultiply with the constraint matrices. If FALSE then the smooth functions have been premultiply by the constraint matrices. The <code>raw</code> argument is directly fed into <code>predict.vgam()</code> .
<code>offset.arg</code>	Numerical vector of length $r$ . These are added to the component functions. Useful for separating out the functions when <code>overlay</code> is TRUE. If <code>overlay</code> is TRUE and there is one covariate then using the intercept values as the offsets can be a good idea.
<code>deriv.arg</code>	Numerical. The order of the derivative. Should be assigned an small integer such as 0, 1, 2. Only applying to <code>s()</code> terms, it plots the derivative.
<code>overlay</code>	Logical. If TRUE then component functions of the same covariate are overlaid on each other. The functions are centered, so <code>offset.arg</code> can be useful when <code>overlay</code> is TRUE.
<code>type.residuals</code>	if <code>residuals</code> is TRUE then the first possible value of this vector, is used to specify the type of residual.
<code>plot.arg</code>	Logical. If FALSE then no plot is produced.
<code>which.term</code>	Character or integer vector containing all terms to be plotted, e.g., <code>which.term = c("s(age)", "s(height)")</code> or <code>which.term = c(2, 5, 9)</code> . By default, all are plotted.
<code>which.cf</code>	An integer-valued vector specifying which linear/additive predictors are to be plotted. The values must be from the set $\{1, 2, \dots, r\}$ . By default, all are plotted.
<code>control</code>	Other control parameters. See <a href="#">plotvgam.control</a> .
<code>...</code>	Other arguments that can be fed into <a href="#">plotvgam.control</a> . This includes line colors, line widths, line types, etc.
<code>varxij</code>	Positive integer. Used if <code>xij</code> of <a href="#">vglm.control</a> was used, this chooses which inner argument the component is plotted against. This argument is related to <code>raw = TRUE</code> and terms such as <code>NS(dum1, dum2)</code> and constraint matrices that have more than one column. The default would plot the smooth against <code>dum1</code> but setting <code>varxij = 2</code> could mean plotting the smooth against <code>dum2</code> . See the <b>VGAM</b> website for further information.

## Details

In this help file  $M$  is the number of linear/additive predictors, and  $r$  is the number of columns of the constraint matrix of interest.

Many of `plotvgam()`'s options can be found in [plotvgam.control](#), e.g., line types, line widths, colors.

## Value

The original object, but with the `preplot` slot of the object assigned information regarding the plot.

## Note

While `plot(fit)` will work if `class(fit)` is `"vgam"`, it is necessary to use `plotvgam(fit)` explicitly otherwise.

`plotvgam()` is quite buggy at the moment.

**Author(s)**

Thomas W. Yee

**See Also**[vgam](#), [plotvgam.control](#), [predict.vgam](#), [plotvglm](#), [vglm](#).**Examples**

```

coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vgam(cbind(nBnW, nBW, BnW, BW) ~ s(Age),
            binom2.or(zero = NULL), coalminers)
## Not run: par(mfrow = c(1,3))
plot(fit, se = TRUE, ylim = c(-3, 2), las = 1)
plot(fit, se = TRUE, which.cf = 1:2, lcol = "blue", scol = "orange",
     ylim = c(-3, 2))
plot(fit, se = TRUE, which.cf = 1:2, lcol = "blue", scol = "orange",
     overlay = TRUE)
## End(Not run)

```

plotvgam.control

*Control Function for plotvgam()***Description**

Provides default values for many arguments available for plotvgam().

**Usage**

```

plotvgam.control(which.cf = NULL,
                 xlim = NULL, ylim = NULL, llty = par()$lty,
                 slty = "dashed", pcex = par()$cex,
                 pch = par()$pch, pcol = par()$col,
                 lcol = par()$col, rcol = par()$col,
                 scol = par()$col, llwd = par()$lwd, slwd = par()$lwd,
                 add.arg = FALSE, one.at.a.time = FALSE,
                 .include.dots = TRUE, noxmean = FALSE, ...)

```

**Arguments**

which.cf	Integer vector specifying which component functions are to be plotted (for each covariate). Must have values from the set $\{1, 2, \dots, M\}$ .
xlim	Range for the x-axis.
ylim	Range for the y-axis.
llty	Line type for the fitted functions (lines). Fed into par(lty).
slty	Line type for the standard error bands. Fed into par(lty).

pcex	Character expansion for the points (residuals). Fed into <code>par(cex)</code> .
pch	Character used for the points (residuals). Same as <code>par(pch)</code> .
pcol	Color of the points. Fed into <code>par(col)</code> .
lcol	Color of the fitted functions (lines). Fed into <code>par(col)</code> .
rcol	Color of the rug plot. Fed into <code>par(col)</code> .
scol	Color of the standard error bands. Fed into <code>par(col)</code> .
llwd	Line width of the fitted functions (lines). Fed into <code>par(lwd)</code> .
slwd	Line width of the standard error bands. Fed into <code>par(lwd)</code> .
add.arg	Logical. If TRUE then the plot will be added to an existing plot, otherwise a new plot will be made.
one.at.a.time	Logical. If TRUE then the plots are done one at a time, with the user having to hit the return key between the plots.
.include.dots	Not to be used by the user.
noxmean	Logical. If TRUE then the point at the mean of $x$ , which is added when standard errors are specified and it thinks the function is linear, is not added. One might use this argument if <code>ylab</code> is specified.
...	Other arguments that may be fed into <code>par()</code> . In the above, $M$ is the number of linear/additive predictors.

## Details

The most obvious features of `plotvgam` can be controlled by the above arguments.

## Value

A list with values matching the arguments.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

## See Also

`plotvgam`.

## Examples

```
plotvgam.control(lcol = c("red", "blue"), scol = "darkgreen", se = TRUE)
```

plotvglm

*Plots for VGLMs***Description**

Currently not working, this function can be used to feed the object to the VGAM plotting function. In the future some diagnostic plots will be plotted.

**Usage**

```
plotvglm(x, type = c("vglm", "vgam"),
         newdata = NULL, y = NULL, residuals = NULL,
         rugplot = TRUE, se = FALSE, scale = 0, raw = TRUE,
         offset.arg = 0, deriv.arg = 0, overlay = FALSE,
         type.residuals = c("deviance", "working", "pearson", "response"),
         plot.arg = TRUE, which.term = NULL, which.cf = NULL,
         control = plotvgam.control(...), varxij = 1, ...)
```

**Arguments**

x	Same as <a href="#">plotvgam</a> .
type	Default is the first choice. Currently the first choice gives an error (not written yet). If "vgam" then all the arguments are fed into <a href="#">plotvgam</a> .
newdata, y, residuals, rugplot	Same as <a href="#">plotvgam</a> .
se, scale, raw, offset.arg	Same as <a href="#">plotvgam</a> .
deriv.arg, overlay, type.residuals	Same as <a href="#">plotvgam</a> .
plot.arg, which.term, which.cf, control	Same as <a href="#">plotvgam</a> .
..., varxij	Same as <a href="#">plotvgam</a> .

**Details**

Currently this function has not been written. When this is done some diagnostic plots based on residuals and hatvalues will be done. In the meanwhile, this function can be used to call the plotting function for [vgam](#) objects.

**Value**

Same as [plotvgam](#).

**See Also**

[plotvgam](#), [plotvgam.control](#), [vglm](#).

## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW, nBW, BnW, BW) ~ bs(Age),
            binom2.or(zero = NULL), coalminers)
## Not run: par(mfrow = c(1, 3))
plot(fit, type = "vgam", se = TRUE, ylim = c(-3, 2), las = 1)
plot(fit, type = "vgam", se = TRUE, which.cf = 1:2,
     lcol = "blue", scol = "orange", ylim = c(-3, 2))
plot(fit, type = "vgam", se = TRUE, which.cf = 1:2,
     lcol = "blue", scol = "orange", overlay = TRUE)
## End(Not run)
```

---

pneumo

*Pneumoconiosis in Coalminers Data*


---

## Description

The pneumo data frame has 8 rows and 4 columns. Exposure time is explanatory, and there are 3 ordinal response variables.

## Usage

```
data(pneumo)
```

## Format

This data frame contains the following columns:

**exposure.time** a numeric vector, in years

**normal** a numeric vector, counts

**mild** a numeric vector, counts

**severe** a numeric vector, counts

## Details

These were collected from coalface workers. In the original data set, the two most severe categories were combined.

## Source

Ashford, J.R., 1959. An approach to the analysis of data for semi-quantal responses in biological assay. *Biometrics*, **15**, 573–581.

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

See Also

[cumulative](#).

Examples

```
# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo)
```

---

poisson.points	<i>Poisson-points-on-a-plane/volume Distances Distribution</i>
----------------	--

---

Description

Estimating the density parameter of the distances from a fixed point to the  $u$ -th nearest point, in a plane or volume.

Usage

```
poisson.points(ostatistic, dimension = 2, link = "loge",
               idensity = NULL, imethod = 1)
```

Arguments

ostatistic	Order statistic. A single positive value, usually an integer. For example, the value 5 means the response are the distances of the fifth nearest value to that point (usually over many planes or volumes). Non-integers are allowed because the value 1.5 coincides with <a href="#">maxwell</a> when <code>dimension = 2</code> . Note: if <code>ostatistic = 1</code> and <code>dimension = 2</code> then this <b>VGAM</b> family function coincides with <a href="#">rayleigh</a> .
dimension	The value 2 or 3; 2 meaning a plane and 3 meaning a volume.
link	Parameter link function applied to the (positive) density parameter, called $\lambda$ below. See <a href="#">Links</a> for more choices.
idensity	Optional initial value for the parameter. A NULL value means a value is obtained internally. Use this argument if convergence failure occurs.
imethod	An integer with value 1 or 2 which specifies the initialization method for $\lambda$ . If failure to converge occurs try another value and/or else specify a value for <code>idensity</code> .

Details

Suppose the number of points in any region of area  $A$  of the plane is a Poisson random variable with mean  $\lambda A$  (i.e.,  $\lambda$  is the *density* of the points). Given a fixed point  $P$ , define  $D_1, D_2, \dots$  to be the distance to the nearest point to  $P$ , second nearest to  $P$ , etc. This **VGAM** family function estimates  $\lambda$  since the probability density function for  $D_u$  is easily derived,  $u = 1, 2, \dots$ . Here,  $u$  corresponds to the argument `ostatistic`.

Similarly, suppose the number of points in any volume  $V$  is a Poisson random variable with mean  $\lambda V$  where, once again,  $\lambda$  is the *density* of the points. This **VGAM** family function estimates  $\lambda$  by specifying the argument `ostatistic` and using `dimension = 3`.

The mean of  $D_u$  is returned as the fitted values. Newton-Raphson is the same as Fisher-scoring.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

### Warning

Convergence may be slow if the initial values are far from the solution. This often corresponds to the situation when the response values are all close to zero, i.e., there is a high density of points.

Formulae such as the means have not been fully checked.

### Author(s)

T. W. Yee

### See Also

[poissonff](#), [maxwell](#), [rayleigh](#).

### Examples

```
pdata <- data.frame(y = rgamma(10, shape = exp(-1))) # Not proper data!
ostat <- 2
fit <- vglm(y ~ 1, poisson.points(ostat, 2), pdata,
            trace = TRUE, crit = "coef")
fit <- vglm(y ~ 1, poisson.points(ostat, 3), pdata,
            trace = TRUE, crit = "coef") # Slow convergence?
fit <- vglm(y ~ 1, poisson.points(ostat, 3, idensi = 1), pdata,
            trace = TRUE, crit = "coef")
head(fitted(fit))
with(pdata, mean(y))
coef(fit, matrix = TRUE)
Coef(fit)
```

---

poissonff

*Poisson Family Function*

---

### Description

Family function for a generalized linear model fitted to Poisson responses. The dispersion parameters may be known or unknown.

**Usage**

```
poissonff(link = "log", dispersion = 1, onedpar = FALSE, imu = NULL,
          imethod = 1, parallel = FALSE, zero = NULL, bred = FALSE,
          earg.link = FALSE)
```

**Arguments**

link	Link function applied to the mean or means. See <a href="#">Links</a> for more choices and information.
dispersion	Dispersion parameter. By default, maximum likelihood is used to estimate the model because it is known. However, the user can specify <code>dispersion = 0</code> to have it estimated, or else specify a known positive value (or values if the response is a matrix—one value per column).
onedpar	One dispersion parameter? If the response is a matrix, then a separate dispersion parameter will be computed for each response (column), by default. Setting <code>onedpar=TRUE</code> will pool them so that there is only one dispersion parameter to be estimated.
parallel	A logical or formula. Used only if the response is a matrix.
imu, imethod	See <a href="#">CommonVGAMffArguments</a> for more information.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ , where $M$ is the number of columns of the matrix response. See <a href="#">CommonVGAMffArguments</a> for more information.
bred, earg.link	Details at <a href="#">CommonVGAMffArguments</a> . Setting <code>bred = TRUE</code> should work for multiple responses and all <b>VGAM</b> link functions; it has been tested for <a href="#">log</a> , <a href="#">identity</a> but further testing is required.

**Details**

$M$  defined above is the number of linear/additive predictors.

If the dispersion parameter is unknown, then the resulting estimate is not fully a maximum likelihood estimate.

A dispersion parameter that is less/greater than unity corresponds to under-/over-dispersion relative to the Poisson model. Over-dispersion is more common in practice.

When fitting a Quadratic RR-VGLM (see [cqo](#)), the response is a matrix of  $M$ , say, columns (e.g., one column per species). Then there will be  $M$  dispersion parameters (one per column of the response matrix) if `dispersion = 0` and `onedpar = FALSE`.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), and [cao](#).



**Warning**

With a multivariate response, assigning a known dispersion parameter for *each* response is not handled well yet. Currently, only a single known dispersion parameter is handled well.

**Note**

This function will handle a matrix response automatically.

The call `poissonff(dispersion=0, ...)` is equivalent to `quasipoissonff(...)`. The latter was written so that R users of `quasipoisson()` would only need to add a “ff” to the end of the family function name.

Regardless of whether the dispersion parameter is to be estimated or not, its value can be seen from the output from the `summary()` of the object.

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [quasipoissonff](#), [genpoisson](#), [zipoisson](#), [skellam](#), [mix2poisson](#), [cenpoisson](#), [ordpoisson](#), [amlpoisson](#), [invbinomial](#), [loge](#), [polg](#), [rrvglm](#), [cqo](#), [cao](#), [binomialff](#), [quasibinomialff](#), [poisson](#), [poisson.points](#), [ruge](#), [V1](#).

**Examples**

```
poissonff()

set.seed(123)
pdata <- data.frame(x2 = rnorm(nn <- 100))
pdata <- transform(pdata, y1 = rpois(nn, exp(1 + x2)),
                  y2 = rpois(nn, exp(1 + x2)))
(fit1 <- vglm(cbind(y1, y2) ~ x2, family = poissonff, pdata))
(fit2 <- vglm(y1 ~ x2, family = poissonff(bred = TRUE), pdata))
coef(fit1, matrix = TRUE)
coef(fit2, matrix = TRUE)

nn <- 200
cdata <- data.frame(x2 = rnorm(nn), x3 = rnorm(nn), x4 = rnorm(nn))
cdata <- transform(cdata, lv1 = 0 + x3 - 2*x4)
cdata <- transform(cdata, lambda1 = exp(3 - 0.5 * (lv1-0)^2),
                  lambda2 = exp(2 - 0.5 * (lv1-1)^2),
                  lambda3 = exp(2 - 0.5 * ((lv1+4)/2)^2))
cdata <- transform(cdata, y1 = rpois(nn, lambda1),
                  y2 = rpois(nn, lambda2),
                  y3 = rpois(nn, lambda3))
```

```
## Not run: lvplot(p1, y = TRUE, lcol = 2:4, pch = 2:4, pcol = 2:4, rug = FALSE)
```

---

PoissonPoints

*Poisson Points Distribution*


---

## Description

Density  
for the PoissonPoints distribution.

## Usage

```
dpois.points(x, lambda, ostatic, dimension = 2, log = FALSE)
```

## Arguments

x	vector of quantiles.
lambda	the mean density of points.
ostatic	positive values, usually integers.
dimension	Either 2 and/or 3.
log	Logical; if TRUE, the logarithm is returned.

## Details

See [poisson.points](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dpois.points gives the density.

## See Also

[poisson.points](#), [dpois](#), [Maxwell](#).

## Examples

```
## Not run: lambda <- 1; xvec <- seq(0, 2, length = 400)
plot(xvec, dpois.points(xvec, lambda, ostatic = 1, dimension = 2),
     type = "l", las = 1, col = "blue",
     sub = "First order statistic",
     main = paste("PDF of PoissonPoints distribution with lambda = ",
                  lambda, " and on the plane", sep = ""))
## End(Not run)
```

polf

*Poisson-Ordinal Link Function***Description**

Computes the Poisson-ordinal transformation, including its inverse and the first two derivatives.

**Usage**

```
polf(theta, cutpoint = NULL,
      inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

**Arguments**

theta	Numeric or character. See below for further details.
cutpoint	The cutpoints should be non-negative integers. If polf() is used as the link function in <a href="#">cumulative</a> then one should choose reverse = TRUE, parallel = TRUE.
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

**Details**

The Poisson-ordinal link function (POLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying Poisson distribution. If the cutpoint is zero then a complementary log-log link is used.

See [Links](#) for general information about **VGAM** link functions.

**Value**

See Yee (2012) for details.

**Warning**

Prediction may not work on [vglm](#) or [vgam](#) etc. objects if this link function is used.

**Note**

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0, numerical instabilities may still arise.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the Poisson distribution (see [poissonff](#)) that has been recorded as an ordinal response using known cutpoints.

**Author(s)**

Thomas W. Yee

## References

Yee, T. W. (2012) *Ordinal ordination with normalizing link functions for count data*, (in preparation).

## See Also

[Links](#), [ordpoisson](#), [poissonff](#), [nbolf](#), [golf](#), [cumulative](#).

## Examples

```
polf("p", cutpoint = 2, short = FALSE)
polf("p", cutpoint = 2, tag = TRUE)

p <- seq(0.01, 0.99, by = 0.01)
y <- polf(p, cutpoint = 2)
y. <- polf(p, cutpoint = 2, deriv = 1)
max(abs(polf(y, cutpoint = 2, inv = TRUE) - p)) # Should be 0

## Not run: par(mfrow = c(2, 1), las = 1)
plot(p, y, type = "l", col = "blue", main = "polf()")
abline(h = 0, v = 0.5, col = "orange", lty = "dashed")

plot(p, y., type = "l", col = "blue",
      main = "(Reciprocal of) first POLF derivative")
## End(Not run)

# Rutherford and Geiger data
ruge <- data.frame(yy = rep(0:14,
  times = c(57,203,383,525,532,408,273,139,45,27,10,4,0,1,1)))
with(ruge, length(yy)) # 2608 1/8-minute intervals
cutpoint <- 5
ruge <- transform(ruge, yy01 = ifelse(yy <= cutpoint, 0, 1))
fit <- vglm(yy01 ~ 1, binomialff(link = polf(cutpoint = cutpoint)), ruge)
coef(fit, matrix = TRUE)
exp(coef(fit))

# Another example
pdata <- data.frame(x2 = sort(runif(nn <- 1000)))
pdata <- transform(pdata, x3 = runif(nn))
pdata <- transform(pdata, mymu = exp( 3 + 1 * x2 - 2 * x3))
pdata <- transform(pdata, y1 = rpois(nn, lambda = mymu))
cutpoints <- c(-Inf, 10, 20, Inf)
pdata <- transform(pdata, cuty = Cut(y1, breaks = cutpoints))
## Not run: with(pdata, plot(x2, x3, col = cuty, pch = as.character(cuty)))
with(pdata, table(cuty) / sum(table(cuty)))
fit <- vglm(cuty ~ x2 + x3, data = pdata, trace = TRUE,
  cumulative(reverse = TRUE,
    parallel = TRUE,
    link = polf(cutpoint = cutpoints[2:3]),
    mv = TRUE))
```

```

head(depvar(fit))
head(fitted(fit))
head(predict(fit))
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
fit@misc$earg

```

---

Polono

---

*The Poisson Lognormal Distribution*


---

## Description

Density, distribution function and random generation for the Poisson lognormal distribution.

## Usage

```

dpolono(x, meanlog = 0, sdlog = 1, bigx = 170, ...)
ppolono(q, meanlog = 0, sdlog = 1,
        isOne = 1 - sqrt( .Machine$double.eps ), ...)
rpolono(n, meanlog = 0, sdlog = 1)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>meanlog, sdlog</code>	the mean and standard deviation of the normal distribution (on the log scale). They match the arguments in <a href="#">Lognormal</a> .
<code>bigx</code>	Numeric. This argument is for handling large values of <code>x</code> and/or when <a href="#">integrate</a> fails. A first order Taylor series approximation [Equation (7) of Bulmer (1974)] is used at values of <code>x</code> that are greater or equal to this argument. For <code>bigx = 10</code> , he showed that the approximation has a relative error less than 0.001 for values of <code>meanlog</code> and <code>sdlog</code> “likely to be encountered in practice”. The argument can be assigned <code>Inf</code> in which case the approximation is not used.
<code>isOne</code>	Used to test whether the cumulative probabilities have effectively reached unity.
<code>...</code>	Arguments passed into <a href="#">integrate</a> .

## Details

The Poisson lognormal distribution is similar to the negative binomial in that it can be motivated by a Poisson distribution whose mean parameter comes from a right skewed distribution (gamma for the negative binomial and lognormal for the Poisson lognormal distribution).

**Value**

`dpolono` gives the density, `ppolono` gives the distribution function, and `rpolono` generates random deviates.

**Note**

By default, `dpolono` involves numerical integration that is performed using [integrate](#). Consequently, computations are very slow and numerical problems may occur (if so then the use of `...` may be needed). Alternatively, for extreme values of `x`, `meanlog`, `sdlog`, etc., the use of `bigx = Inf` avoids the call to [integrate](#), however the answer may be a little inaccurate.

For the maximum likelihood estimation of the 2 parameters a **VGAM** family function called `polono()`, say, has not been written yet.

**Author(s)**

T. W. Yee. Some anonymous soul kindly wrote `ppolono()` and improved the original `dpolono()`.

**References**

Bulmer, M. G. (1974) On fitting the Poisson lognormal distribution to species-abundance data. *Biometrics*, **30**, 101–110.

**See Also**

[lognormal](#), [poissonff](#), [negbinomial](#).

**Examples**

```
meanlog <- 0.5; sdlog <- 0.5; yy <- 0:19
sum(proby <- dpolono(yy, m = meanlog, sd = sdlog)) # Should be 1
max(abs(cumsum(proby) - ppolono(yy, m = meanlog, sd = sdlog))) # Should be 0

## Not run: opar = par(no.readonly = TRUE)
par(mfrow = c(2, 2))
plot(yy, proby, type = "h", col = "blue", ylab = "P[Y=y]", log = "",
      main = paste("Poisson lognormal(m = ", meanlog,
                  ", sdl = ", sdlog, ")", sep = ""))

y <- 0:190 # More extreme values; use the approximation and plot on a log scale
(sum(proby <- dpolono(y, m = meanlog, sd = sdlog, bigx = 100))) # Should be 1
plot(y, proby, type = "h", col = "blue", ylab = "P[Y=y] (log)", log = "y",
      main = paste("Poisson lognormal(m = ", meanlog,
                  ", sdl = ", sdlog, ")", sep = "")) # Note the kink at bigx

# Random number generation
table(y <- rpolono(n = 1000, m = meanlog, sd = sdlog))
hist(y, breaks = ((-1):max(y))+0.5, prob = TRUE, border = "blue")
par(opar)
## End(Not run)
```

posbernoulli.b

*Positive Bernoulli Family Function with Behavioural Effects***Description**

Fits a GLM-/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Capture history behavioural effects are accommodated.

**Usage**

```
posbernoulli.b(link = "logit", drop.b = FALSE ~ 1,
               type.fitted = c("likelihood.cond", "mean.uncond"), I2 = FALSE,
               ipcapture = NULL, iprecapture = NULL,
               p.small = 1e-4, no.warning = FALSE)
```

**Arguments**

link, drop.b, ipcapture, iprecapture

See [CommonVGAMffArguments](#) for information about these arguments. By default the parallelism assumption does not apply to the intercept. With an intercept-only model setting drop.b = TRUE ~ 1 results in the  $M_0/M_h$  model.

I2 Logical. This argument is used for terms that are not parallel. If TRUE then the constraint matrix `diag(2)` (the general default constraint matrix in **VGAM**) is used, else `cbind(0:1, 1)`. The latter means the first element/column corresponds to the behavioural effect. Consequently it and its standard error etc. can be accessed directly without subtracting two quantities.

type.fitted Details at [posbernoulli.tb](#).

p.small, no.warning

See [posbernoulli.t](#).

**Details**

This model (commonly known as  $M_b/M_{bh}$  in the capture–recapture literature) operates on a capture history matrix response of 0s and 1s ( $n \times \tau$ ). See [posbernoulli.t](#) for details, e.g., common assumptions with other models. Once an animal is captured for the first time, it is marked/tagged so that its future capture history can be recorded. The effect of the recapture probability is modelled through a second linear/additive predictor. It is well-known that some species of animals are affected by capture, e.g., trap-shy or trap-happy. This **VGAM** family function *does* allow the capture history to be modelled via such behavioural effects. So does [posbernoulli.tb](#) but [posbernoulli.t](#) cannot.

The number of linear/additive predictors is  $M = 2$ , and the default links are  $(\text{logit } p_c, \text{logit } p_r)^T$  where  $p_c$  is the probability of capture and  $p_r$  is the probability of recapture. The fitted value returned is of the same dimension as the response matrix, and depends on the capture history: prior to being first captured, it is pcapture. Afterwards, it is precapture.

By default, the constraint matrices for the intercept term and the other covariates are set up so that  $p_r$  differs from  $p_c$  by a simple binary effect, on a logit scale. However, this difference (the behavioural effect) is more directly estimated by having `I2 = FALSE`. Then it allows an estimate of the trap-happy/trap-shy effect; these are positive/negative values respectively. If `I2 = FALSE` then the (nonstandard) constraint matrix used is `cbind(0:1, 1)`, meaning the first element can be interpreted as the behavioural effect.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

The dependent variable is *not* scaled to row proportions. This is the same as [posbernoulli.t](#) and [posbernoulli.tb](#) but different from [posbinomial](#) and [binomialff](#).

## Author(s)

Thomas W. Yee.

## References

See [posbernoulli.t](#).

## See Also

[posbernoulli.t](#) and [posbernoulli.tb](#) (including estimating  $N$ ), [deermice](#), [dposbern](#), [rposbern](#), [posbinomial](#), [aux.posbernoulli.t](#), [prinia](#).

## Examples

```
# deermice data ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

# Fit a M_b model
M.b <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1,
            posbernoulli.b, data = deermice, trace = TRUE)
coef(M.b)["(Intercept):1"] # Behavioural effect on the logit scale
coef(M.b, matrix = TRUE)
constraints(M.b, matrix = TRUE)
summary(M.b, presid = FALSE)

# Fit a M_bh model
M.bh <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
             posbernoulli.b, data = deermice, trace = TRUE)
coef(M.bh, matrix = TRUE)
coef(M.bh)["(Intercept):1"] # Behavioural effect on the logit scale
constraints(M.bh) # (2,1) element of "(Intercept)" is for the behavioural effect
summary(M.bh, presid = FALSE) # Significant positive (trap-happy) behavioural effect
# Approx. 95 percent confidence for the behavioural effect:
SE.M.bh <- coef(summary(M.bh))["(Intercept):1", "Std. Error"]
```



```

coef(M.bh)[ "(Intercept):1" ] + c(-1, 1) * 1.96 * SE.M.bh

# Fit a M_h model
M.h <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
            posbernoulli.b(drop.b = TRUE ~ sex + weight),
            data = deermice, trace = TRUE)
coef(M.h, matrix = TRUE)
constraints(M.h, matrix = TRUE)
summary(M.h, presid = FALSE)

# Fit a M_0 model
M.0 <- vglm(cbind(      y1 + y2 + y3 + y4 + y5 + y6,
                    6 - y1 - y2 - y3 - y4 - y5 - y6) ~ 1,
            posbinomial, data = deermice, trace = TRUE)
coef(M.0, matrix = TRUE)
summary(M.0, presid = FALSE)

# Simulated data set ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
set.seed(123); nTimePts <- 5; N <- 1000 # N is the popn size
pdata <- rposbern(n = N, nTimePts = nTimePts, pvars = 2, is.popn = TRUE)
nrow(pdata) # Less than N (because some animals were never captured)
# The truth: xcoeffs are c(-2, 1, 2) and cap.effect = +1

M.bh.2 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
               posbernoulli.b, data = pdata, trace = TRUE)
coef(M.bh.2)
coef(M.bh.2, matrix = TRUE)
constraints(M.bh.2, matrix = TRUE)
summary(M.bh.2, presid = FALSE)
head(depvar(M.bh.2)) # Capture history response matrix
head(M.bh.2@extra$cap.hist1) # Info on its capture history
head(M.bh.2@extra$cap1) # When it was first captured
head(fitted(M.bh.2)) # Depends on capture history
(trap.effect <- coef(M.bh.2)[ "(Intercept):1" ]) # Should be +1
head(model.matrix(M.bh.2, type = "vlm"), 21)
head(pdata)
summary(pdata)
dim(depvar(M.bh.2))
vcov(M.bh.2)

M.bh.2@extra$N.hat # Estimate of the population size; should be about N
M.bh.2@extra$SE.N.hat # SE of the estimate of the population size
# An approximate 95 percent confidence interval:
round(M.bh.2@extra$N.hat + c(-1, 1) * 1.96 * M.bh.2@extra$SE.N.hat, 1)

```

## Description

Fits a GLM/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Sampling occasion effects are accommodated.

## Usage

```
posbernoulli.t(link = "logit", parallel.t = FALSE ~ 1, iprob = NULL,
               p.small = 1e-4, no.warning = FALSE)
```

## Arguments

link, iprob, parallel.t

See [CommonVGAMffArguments](#) for information. By default, the parallelism assumption does not apply to the intercept. Setting `parallel.t = FALSE ~ -1`, or equivalently `parallel.t = FALSE ~ 0`, results in the  $M_0/M_h$  model.

p.small, no.warning

A small probability value used to give a warning for the Horvitz–Thompson estimator. Any estimated probability value less than `p.small` will result in a warning, however, setting `no.warning = TRUE` will suppress this warning if it occurs. This is because the Horvitz–Thompson estimator is the sum of the reciprocal of such probabilities, therefore any probability that is too close to 0 will result in an unstable estimate.

## Details

These models (commonly known as  $M_t$  or  $M_{th}$  (no prefix  $h$  means it is an intercept-only model) in the capture–recapture literature) operate on a capture history matrix response of 0s and 1s ( $n \times \tau$ ). Each column is a sampling occasion where animals are potentially captured (e.g., a field trip), and each row is an individual animal. Capture is a 1, else a 0. No removal of animals from the population is made (closed population), e.g., no immigration or emigration. Each row of the response matrix has at least one capture. Once an animal is captured for the first time, it is marked/tagged so that its future capture history can be recorded. Then it is released immediately back into the population to remix. It is released immediately after each recapture too. It is assumed that the animals are independent and that, for a given animal, each sampling occasion is independent. And animals do not lose their marks/tags, and all marks/tags are correctly recorded.

The number of linear/additive predictors is equal to the number of sampling occasions, i.e.,  $M = \tau$ , say. The default link functions are  $(\text{logit } p_1, \dots, \text{logit } p_\tau)^T$  where each  $p_j$  denotes the probability of capture at time point  $j$ . The fitted value returned is a matrix of probabilities of the same dimension as the response matrix.

A conditional likelihood is maximized here using Fisher scoring. Each sampling occasion has a separate probability that is modelled here. The probabilities can be constrained to be equal by setting `parallel.t = FALSE ~ 0`; then the results are effectively the same as [posbinomial](#) except the binomial constants are not included in the log-likelihood. If `parallel.t = TRUE ~ 0` then each column should have at least one 1 and at least one 0.

It is well-known that some species of animals are affected by capture, e.g., trap-shy or trap-happy. This **VGAM** family function does *not* allow any behavioral effect to be modelled ([posbernoulli.b](#) and [posbernoulli.tb](#) do) because the denominator of the likelihood function must be free of behavioral effects.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

Upon fitting the extra slot has a (list) component called `N.hat` which is a point estimate of the population size  $N$  (it is the Horvitz-Thompson (1952) estimator). And there is a component called `SE.N.hat` containing its standard error.

**Note**

The `weights` argument of [vglm](#) need not be assigned, and the default is just a matrix of ones.

Fewer numerical problems are likely to occur for `parallel.t = TRUE`. Data-wise, each sampling occasion may need at least one success (capture) and one failure. Less stringent conditions in the data are needed when `parallel.t = TRUE`. Ditto when parallelism is applied to the intercept too.

The response matrix is returned unchanged; i.e., not converted into proportions like [posbinomial](#). If the response matrix has column names then these are used in the labelling, else `prob1`, `prob2`, etc. are used.

Using `AIC()` or `BIC()` to compare [posbernoulli.t](#), [posbernoulli.b](#), [posbernoulli.tb](#) models with a [posbinomial](#) model requires `posbinomial(omit.constant = TRUE)` because one needs to remove the normalizing constant from the log-likelihood function. See [posbinomial](#) for an example.

**Author(s)**

Thomas W. Yee.

**References**

- Huggins, R. M. (1991) Some practical aspects of a conditional likelihood approach to capture experiments. *Biometrics*, **47**, 725–732.
- Huggins, R. M. and Hwang, W.-H. (2011) A review of the use of conditional likelihood in capture–recapture experiments. *International Statistical Review*, **79**, 385–400.
- Otis, D. L. and Burnham, K. P. and White, G. C. and Anderson, D. R. (1978) Statistical inference from capture data on closed animal populations, *Wildlife Monographs*, **62**, 3–135.

**See Also**

[posbernoulli.b](#), [posbernoulli.tb](#), [deermice](#), [Huggins89table1](#), [Huggins89.t1](#), [dposbern](#), [rposbern](#), [posbinomial](#), [AICvglm](#), [BICvglm](#), [prinia](#).

**Examples**

```
M.t <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1, posbernoulli.t,
            data = deermice, trace = TRUE)
coef(M.t, matrix = TRUE)
constraints(M.t, matrix = TRUE)
summary(M.t, presid = FALSE)

M.h.1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight, trace = TRUE,
```

```

      posbernoulli.t(parallel.t = FALSE ~ -1), data = deermice)
coef(M.h.1, matrix = TRUE)
constraints(M.h.1)
summary(M.h.1, presid = FALSE)
head(depvar(M.h.1)) # Response capture history matrix
dim(depvar(M.h.1))

M.th.2 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight, trace = TRUE,
              posbernoulli.t(parallel.t = FALSE), data = deermice)
lrtest(M.h.1, M.th.2) # Test the parallelism assumption wrt sex and weight
coef(M.th.2)
coef(M.th.2, matrix = TRUE)
constraints(M.th.2)
summary(M.th.2, presid = FALSE)
head(model.matrix(M.th.2, type = "vlm"), 21)

M.th.2@extra$N.hat      # Estimate of the population size; should be about N
M.th.2@extra$SE.N.hat   # SE of the estimate of the population size
# An approximate 95 percent confidence interval:
round(M.th.2@extra$N.hat + c(-1, 1) * 1.96 * M.th.2@extra$SE.N.hat, 1)

# Fit a M_h model, effectively the parallel M_t model, using posbinomial()
deermice <- transform(deermice, ysum = y1 + y2 + y3 + y4 + y5 + y6,
                      tau = 6)
M.h.3 <- vglm(cbind(ysum, tau - ysum) ~ sex + weight,
              posbinomial(omit.constant = TRUE), data = deermice, trace = TRUE)
max(abs(coef(M.h.1) - coef(M.h.3))) # Should be zero
logLik(M.h.3) - logLik(M.h.1) # Difference is due to the binomial constants

```

---

posbernoulli.tb

---

*Positive Bernoulli Family Function with Time and Behavioural Effects*


---

## Description

Fits a GLM/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Sampling occasion effects and behavioural effects are accommodated.

## Usage

```

posbernoulli.tb(link = "logit",
                parallel.t = FALSE ~ 1,
                parallel.b = FALSE ~ 0,
                drop.b      = FALSE ~ 1,
                type.fitted = c("likelihood.cond", "mean.uncond"),
                imethod = 1, iprob = NULL,
                p.small = 1e-4, no.warning = FALSE,
                ridge.constant = 0.01, ridge.power = -4)

```

## Arguments

link, imethod, iprob

See [CommonVGAMffArguments](#) for information.

parallel.t, parallel.b, drop.b

A logical, or formula with a logical as the response. See [CommonVGAMffArguments](#) for information. The parallel.-type arguments specify whether the constraint matrices have a parallelism assumption for the temporal and behavioural effects. Argument parallel.t means parallel with respect to time, and matches the same argument name in [posbernoulli.t](#).

Suppose the model is intercept-only. Setting parallel.t = FALSE ~ 0 results in the  $M_b$  model. Setting drop.b = FALSE ~ 0 results in the  $M_t$  model because it drops columns off the constraint matrices corresponding to any behavioural effect. Setting parallel.t = FALSE ~ 0 and setting parallel.b = FALSE ~ 0 results in the  $M_b$  model. Setting parallel.t = FALSE ~ 0, parallel.b = FALSE ~ 0 and drop.b = FALSE ~ 0 results in the  $M_0$  model. Note the default for parallel.t and parallel.b may be unsuitable for data sets which have a large  $\tau$  because of the large number of parameters; it might be too flexible. If it is desired to have the behaviour affect some of the other covariates then set drop.b = TRUE ~ 0.

The default model has a different intercept for each sampling occasion, a time-parallelism assumption for all other covariates, and a dummy variable representing a single behavioural effect (also in the intercept).

The most flexible model is to set parallel.b = TRUE ~ 0, parallel.t = TRUE ~ 0 and drop.b = TRUE ~ 0. This means that all possible temporal and behavioural effects are estimated, for the intercepts and other covariates. Such a model is *not* recommended; it will contain a lot of parameters.

type.fitted

Character, one of the choices for the type of fitted value returned. The default is the first one. Partial matching is okay. For "likelihood.cond": the probability defined by the conditional likelihood. For "mean.uncond": the unconditional mean, which should agree with [colMeans](#) applied to the response matrix for intercept-only models.

ridge.constant, ridge.power

Determines the ridge parameters at each IRLS iteration. They are the constant and power (exponent) for the ridge adjustment for the working weight matrices (the capture probability block matrix, hence the first  $\tau$  diagonal values). At iteration  $a$  of the IRLS algorithm a positive value is added to the first  $\tau$  diagonal elements of the working weight matrices to make them positive-definite. This adjustment is the mean of the diagonal elements of  $wz$  multiplied by  $K \times a^p$  where  $K$  is ridge.constant and  $p$  is ridge.power. This is always positive but decays to zero as iterations proceed (provided  $p$  is negative etc.).

p.small, no.warning

See [posbernoulli.t](#).

## Details

This model (commonly known as  $M_{tb}/M_{tbb}$  in the capture–recapture literature) operates on a response matrix of 0s and 1s ( $n \times \tau$ ). See [posbernoulli.t](#) for information that is in common. It allows time and behavioural effects to be modelled.

Evidently, the expected information matrix (EIM) seems *not* of full rank (especially in early iterations), so `ridge.constant` and `ridge.power` are used to *try* fix up the problem. The default link functions are  $(\text{logit } p_{c1}, \dots, \text{logit } p_{c\tau}, \text{logit } p_{r2}, \dots, \text{logit } p_{r\tau})^T$  where the subscript  $c$  denotes capture, the subscript  $r$  denotes recapture, and it is not possible to recapture the animal at sampling occasion 1. Thus  $M = 2\tau - 1$ . The parameters are currently prefixed by `pcapture` and `precapture` for the capture and recapture probabilities. This **VGAM** family function may be further modified in the future.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

It is a good idea to apply the parallelism assumption to each sampling occasion except possibly with respect to the intercepts. Also, a simple behavioural effect such as being modelled using the intercept is recommended; if the behavioural effect is not parallel and/or allowed to apply to other covariates then there will probably be too many parameters, and hence, numerical problems. See `M_tbh.1` below.

It is a good idea to monitor convergence. Simpler models such as the  $M_0/M_h$  models are best fitted with [posbernoulli.t](#) or [posbernoulli.b](#) or [posbinomial](#).

### Author(s)

Thomas W. Yee.

### References

See [posbernoulli.t](#).

### See Also

[posbernoulli.b](#) (including `N.hat`), [posbernoulli.t](#), [posbinomial](#), [Huggins89table1](#), [Huggins89.t1](#), [deermice.prinia](#).

### Examples

```
## Not run:
# Example 1: simulated data
nTimePts <- 5 # (aka tau == # of sampling occasions)
nnn <- 1000 # Number of animals
pdata <- rposbern(n = nnn, nTimePts = nTimePts, pvars = 2)
dim(pdata); head(pdata)

M_tbh.1 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
               posbernoulli.tb, data = pdata, trace = TRUE)
coef(M_tbh.1) # First element is the behavioural effect
coef(M_tbh.1, matrix = TRUE)
constraints(M_tbh.1, matrix = TRUE)
summary(M_tbh.1, presid = FALSE) # Standard errors are approximate
```

```

head(fitted(M_tbh.1))
head(model.matrix(M_tbh.1, type = "vlm"), 21)
dim(depvar(M_tbh.1))

M_tbh.2 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
               posbernoulli.tb(parallel.t = FALSE ~ 0),
               data = pdata, trace = TRUE)
coef(M_tbh.2) # First element is the behavioural effect
coef(M_tbh.2, matrix = TRUE)
constraints(M_tbh.2, matrix = TRUE)
summary(M_tbh.2, presid = FALSE) # Standard errors are approximate
head(fitted(M_tbh.2))
head(model.matrix(M_tbh.2, type = "vlm"), 21)
dim(depvar(M_tbh.2))

# Example 2: deermice subset data
fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
             posbernoulli.t, data = deermice, trace = TRUE)
coef(fit1)
coef(fit1, matrix = TRUE)
constraints(fit1, matrix = TRUE)
summary(fit1, presid = FALSE) # Standard errors are approximate

# fit1 is the same as Fit1 (a M_{th} model):
Fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
             posbernoulli.tb(drop.b = TRUE ~ sex + weight,
                             parallel.t = TRUE), # No parallelism for the intercept
             data = deermice, trace = TRUE)
constraints(Fit1)

## End(Not run)

```

posbernUC

*Positive Bernoulli Sequence Model***Description**

Density, and random generation for multiple Bernoulli responses where each row in the response matrix has at least one success.

**Usage**

```

rposbern(n, nTimePts = 5, pvars = length(xcoeff), xcoeff = c(-2, 1, 2),
         cap.effect = 1, is.popn = FALSE, link = "logit", earg.link = FALSE)
dposbern(x, prob, prob0 = prob, log = FALSE)

```

**Arguments**

**x** response vector or matrix. Should only have 0 and 1 values, at least two columns, and each row should have at least one 1.

nTimePts	Number of sampling occasions. Called $\tau$ in <a href="#">posbernoulli.b</a> and <a href="#">posbernoulli.t</a> .
n	number of observations. Usually a single positive integer, else the length of the vector is used. See argument <code>is.popn</code> .
is.popn	Logical. If TRUE then argument <code>n</code> is the population size and what is returned may have substantially less rows than <code>n</code> . That is, if an animal has at least one one in its sequence then it is returned, else that animal is not returned because it never was captured.
cap.effect	Numeric, the capture effect. Added to the linear predictor if captured previously. A positive or negative value corresponds to a trap-happy and trap-shy effect respectively.
pvars	Number of other numeric covariates that make up the linear predictor. Labelled <code>x1</code> , <code>x2</code> , ..., where the first is an intercept, and the others are independent standard <a href="#">runif</a> random variates. The first <code>pvars</code> elements of <code>xcoeff</code> are used.
xcoeff	The regression coefficients of the linear predictor. These correspond to <code>x1</code> , <code>x2</code> , ..., and the first is for the intercept. The length of <code>xcoeff</code> must be at least <code>pvars</code> .
link, earg.link	The former is used to generate the probabilities for capture at each occasion. Other details at <a href="#">CommonVGAMffArguments</a> .
prob, prob0	Matrix of probabilities for the numerator and denominators respectively. The default does <i>not</i> correspond to the $M_b$ model since the $M_b$ model has a denominator which involves the capture history.
log	Logical. Return the logarithm of the answer?

### Details

The form of the conditional likelihood is described in [posbernoulli.b](#) and/or [posbernoulli.t](#) and/or [posbernoulli.tb](#). The denominator is equally shared among the elements of the matrix `x`.

### Value

`rposbern` returns a data frame with some attributes. The function generates random deviates ( $\tau$  columns labelled `y1`, `y2`, ...) for the response. Some indicator columns are also included (those starting with `ch` are for previous capture history). The default setting corresponds to a  $M_{bh}$  model that has a single trap-happy effect. Covariates `x1`, `x2`, ... have the same affect on capture/recapture at every sampling occasion (see the argument `parallel.t` in, e.g., [posbernoulli.tb](#)).

The function `dposbern` gives the density,

### Note

The `r`-type function is experimental only and does not follow the usual conventions of `r`-type R functions. It may change a lot in the future. The `d`-type function is more conventional and is less likely to change.

### Author(s)

Thomas W. Yee.



**See Also**

[posbernoulli.tb](#), [posbernoulli.b](#), [posbernoulli.t](#).

**Examples**

```
rposbern(n = 10)
attributes(pdata <- rposbern(n = 100))
M.bh <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2 + x3, posbernoulli.b(I2 = FALSE),
            data = pdata, trace = TRUE)
constraints(M.bh)
summary(M.bh)
```

---

Posbinom

---

*Positive-Binomial Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the positive-binomial distribution.

**Usage**

```
dposbinom(x, size, prob, log = FALSE)
pposbinom(q, size, prob)
qposbinom(p, size, prob)
rposbinom(n, size, prob)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Fed into <a href="#">runif</a> .
size	number of trials. It is the $N$ symbol in the formula given in <a href="#">posbinomial</a> .
prob	probability of success on each trial. Should be in $(0, 1)$ .
log	See <a href="#">dbinom</a> .

**Details**

The positive-binomial distribution is a binomial distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$\mu / (1 - (1 - \mu)^N)$$

where  $\mu$  is the argument prob above. As  $\mu$  increases, the positive-binomial and binomial distributions become more similar. Unlike similar functions for the binomial distribution, a zero value of prob is not permitted here.

**Value**

dposbinom gives the density, pposbinom gives the distribution function, qposbinom gives the quantile function, and rposbinom generates random deviates.

**Note**

For dposbinom(), if arguments size or prob equal 0 then a NaN is returned.

The family function [posbinomial](#) estimates the parameters by maximum likelihood estimation.

**Author(s)**

T. W. Yee.

**See Also**

[posbinomial](#), [dposbern](#), [zabinomial](#), [zibinomial](#), [rbinom](#).

**Examples**

```
prob <- 0.2; size <- 10
table(y <- rposbinom(n = 1000, size, prob))
mean(y) # Sample mean
size * prob / (1 - (1 - prob)^size) # Population mean

(ii <- dposbinom(0:size, size, prob))
cumsum(ii) - pposbinom(0:size, size, prob) # Should be 0s
table(rposbinom(100, size, prob))

table(qposbinom(runif(1000), size, prob))
round(dposbinom(1:10, size, prob) * 1000) # Should be similar

## Not run: barplot(rbind(dposbinom(x = 0:size, size, prob),
                        dbinom(x = 0:size, size, prob)),
                  beside = TRUE, col = c("blue", "green"),
                  main = paste("Positive-binomial(", size, ",",
                              prob, ") (blue) vs",
                              " Binomial(", size, ",",
                              prob, ") (green)", sep = ""),
                  names.arg = as.character(0:size), las = 1)
## End(Not run)

# Simulated data example
nn <- 1000; sizeval1 <- 10; sizeval2 <- 20
pdata <- data.frame(x2 = seq(0, 1, length = nn))
pdata <- transform(pdata, prob1 = logit(-2 + 2 * x2, inverse = TRUE),
                  prob2 = logit(-1 + 1 * x2, inverse = TRUE),
                  sizev1 = rep(sizeval1, len = nn),
                  sizev2 = rep(sizeval2, len = nn))
pdata <- transform(pdata, y1 = rposbinom(nn, size = sizev1, prob = prob1),
                  y2 = rposbinom(nn, size = sizev2, prob = prob2))
with(pdata, table(y1))
with(pdata, table(y2))
```

```
# Multivariate response
fit2 <- vglm(cbind(y1, y2) ~ x2, posbinomial(mv = TRUE),
            trace = TRUE, pdata, weight = cbind(sizev1, sizev2))
coef(fit2, matrix = TRUE)
```

posbinomial

*Positive Binomial Distribution Family Function***Description**

Fits a positive binomial distribution.

**Usage**

```
posbinomial(link = "logit", mv = FALSE, parallel = FALSE,
            omit.constant = FALSE, p.small = 1e-4, no.warning = FALSE,
            zero = NULL)
```

**Arguments**

link, mv, parallel, zero

Details at [CommonVGAMffArguments](#).

omit.constant Logical. If TRUE then the constant (`lchoose(size, size * yprop)`) is omitted from the loglikelihood calculation. If the model is to be compared using `AIC()` or `BIC()` (see [AICv1m](#) or [BICv1m](#)) to the likes of [posbernoulli.tb](#) etc. then it is important to set `omit.constant = TRUE` because all models then will not have any normalizing constants in the likelihood function. Hence they become comparable. This is because the  $M_0$  Otis et al. (1978) model coincides with `posbinomial()`. See below for an example. Also see [posbernoulli.t](#) regarding estimating the population size (`N.hat` and `SE.N.hat`) if the number of trials is the same for all observations.

p.small, no.warning

See [posbernoulli.t](#).

**Details**

The positive binomial distribution is the ordinary binomial distribution but with the probability of zero being zero. Thus the other probabilities are scaled up (i.e., divided by  $1 - P(Y = 0)$ ). The fitted values are the ordinary binomial distribution fitted values, i.e., the usual mean.

In the capture–recapture literature this model is called the  $M_0$  if it is an intercept-only model. Otherwise it is called the  $M_h$  when there are covariates. It arises from a sum of a sequence of  $\tau$ -Bernoulli random variates subject to at least one success (capture). Here, each animal has the same probability of capture or recapture, regardless of the  $\tau$  sampling occasions. Independence between animals and between sampling occasions etc. is assumed.



```

constraints(M.0, matrix = TRUE)
summary(M.0)
c( N.hat = M.0@extra$N.hat,      # Since tau = 6, i.e., 6 Bernoulli trials per
  SE.N.hat = M.0@extra$SE.N.hat) # observation is the same for each observation

# Compare it to the M_b using AIC and BIC
M.b <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1, trace = TRUE,
            posbernoulli.b, data = deermice)
sort(c(M.0 = AIC(M.0), M.b = AIC(M.b))) # Okay since omit.constant = TRUE
sort(c(M.0 = BIC(M.0), M.b = BIC(M.b))) # Okay since omit.constant = TRUE

```

---

Posgeom

---

*Positive-geometric Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the positive-geometric distribution.

## Usage

```

dposgeom(x, prob, log = FALSE)
pposgeom(q, prob)
qposgeom(p, prob)
rposgeom(n, prob)

```

## Arguments

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Fed into <a href="#">runif</a> .
<code>prob</code>	vector of probabilities of success (of an ordinary geometric distribution). Short vectors are recycled.
<code>log</code>	logical.

## Details

The positive-geometric distribution is a geometric distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is  $1/prob$ .

As *prob* decreases, the positive-geometric and geometric distributions become more similar. Like similar functions for the geometric distribution, a zero value of *prob* is not permitted here.

## Value

`dposgeom` gives the density, `pposgeom` gives the distribution function, `qposgeom` gives the quantile function, and `rposgeom` generates random deviates.

**Author(s)**

T. W. Yee

**See Also**[zageometric](#), [zigeometric](#), [rgeom](#).**Examples**

```

prob <- 0.75; y = rposgeom(n = 1000, prob)
table(y)
mean(y) # Sample mean
1 / prob # Population mean

(ii <- dposgeom(0:7, prob))
cumsum(ii) - pposgeom(0:7, prob) # Should be 0s
table(rposgeom(100, prob))

table(qposgeom(runif(1000), prob))
round(dposgeom(1:10, prob) * 1000) # Should be similar

## Not run:
x <- 0:5
barplot(rbind(dposgeom(x, prob), dgeom(x, prob)),
        beside = TRUE, col = c("blue", "orange"),
        main = paste("Positive geometric(", prob, ") (blue) vs",
                      "geometric(", prob, ") (orange)", sep = ""),
        names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)

```

---

Posnegbin

---

*Positive-Negative Binomial Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the positive-negative binomial distribution.

**Usage**

```

dposnegbin(x, size, prob = NULL, munb = NULL, log = FALSE)
pposnegbin(q, size, prob = NULL, munb = NULL)
qposnegbin(p, size, prob = NULL, munb = NULL)
rposnegbin(n, size, prob = NULL, munb = NULL)

```

**Arguments**

`x, q`                vector of quantiles.  
`p`                    vector of probabilities.  
`n`                    number of observations. Fed into [runif](#).  
`size, prob, munb, log`  
                       Same arguments as that of an ordinary negative binomial distribution (see [dnbinom](#)).  
                       Some arguments have been renamed slightly.  
                       Short vectors are recycled. The parameter `1/size` is known as a dispersion  
                       parameter; as `size` approaches infinity, the negative binomial distribution ap-  
                       proaches a Poisson distribution.

**Details**

The positive-negative binomial distribution is a negative binomial distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$\mu/(1 - p(0))$$

where  $\mu$  the mean of an ordinary negative binomial distribution.

**Value**

`dposnegbin` gives the density, `pposnegbin` gives the distribution function, `qposnegbin` gives the quantile function, and `rposnegbin` generates  $n$  random deviates.

**Author(s)**

T. W. Yee

**References**

Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996) Modelling the abundances of rare species: statistical models for counts with extra zeros. *Ecological Modelling*, **88**, 297–308.

**See Also**

[posnegbinomial](#), [zanegbinomial](#), [zinegbinomial](#), [rnbinom](#).

**Examples**

```
munb <- 5; size <- 4; n <- 1000
table(y <- rposnegbin(n, munb = munb, size = size))
mean(y) # sample mean
munb / (1 - (size / (size + munb))^size) # population mean
munb / pnbinom(0, mu = munb, size = size, lower.tail = FALSE) # same as before

x <- (-1):17
(ii <- dposnegbin(x, munb = munb, size = size))
max(abs(cumsum(ii) - pposnegbin(x, munb = munb, size = size))) # Should be 0
```

```
## Not run:
x <- 0:10
barplot(rbind(dposnegbin(x, munb = munb, size = size),
              dnbinom(x, mu = munb, size = size)),
        beside = TRUE, col = c("blue", "green"),
        main = paste("dposnegbin(munb = ", munb, ", size = ", size, ") (blue) vs",
                     "dnbinom(mu = ", munb, ", size = ", size, ") (green)", sep = ""),
        names.arg = as.character(x))
## End(Not run)

# Another test for pposnegbin()
nn <- 5000
mytab <- cumsum(table(rposnegbin(nn, munb = munb, size = size))) / nn
myans <- pposnegbin(sort(as.numeric(names(mytab))), munb = munb, size = size)
max(abs(mytab - myans)) # Should be 0
```

---

posnegbinomial

---

*Positive Negative Binomial Distribution Family Function*


---

## Description

Maximum likelihood estimation of the two parameters of a positive negative binomial distribution.

## Usage

```
posnegbinomial(lmunb = "loge", lsize = "loge",
              isize = NULL, zero = -2, nsimEIM = 250,
              shrinkage.init = 0.95, imethod = 1)
```

## Arguments

lmunb	Link function applied to the munb parameter, which is the mean $\mu_{nb}$ of an ordinary negative binomial distribution. See <a href="#">Links</a> for more choices.
lsize	Parameter link function applied to the dispersion parameter, called k. See <a href="#">Links</a> for more choices.
isize	Optional initial value for k, an index parameter. The value $1/k$ is known as a dispersion parameter. If failure to converge occurs try different values (and/or use imethod). If necessary this vector is recycled to length equal to the number of responses. A value NULL means an initial value for each response is computed internally using a range of values.
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> .
shrinkage.init, imethod	See <a href="#">negbinomial</a> .



## Details

The positive negative binomial distribution is an ordinary negative binomial distribution but with the probability of a zero response being zero. The other probabilities are scaled to sum to unity.

This family function is based on [negbinomial](#) and most details can be found there. To avoid confusion, the parameter `munb` here corresponds to the mean of an ordinary negative binomial distribution [negbinomial](#). The mean of `posnegbinomial` is

$$\mu_{nb}/(1 - p(0))$$

where  $p(0) = (k/(k + \mu_{nb}))^k$  is the probability an ordinary negative binomial distribution has a zero value.

The parameters `munb` and `k` are not independent in the positive negative binomial distribution, whereas they are in the ordinary negative binomial distribution.

This function handles *multivariate* responses, so that a matrix can be used as the response. The number of columns is the number of species, say, and setting `zero = -2` means that *all* species have a `k` equalling a (different) intercept only.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

## Warning

The Poisson model corresponds to `k` equalling infinity. If the data is Poisson or close to Poisson, numerical problems may occur. Possibly a loglog link could be added in the future to try help handle this problem.

This **VGAM** family function is computationally expensive and usually runs slowly; setting `trace = TRUE` is useful for monitoring convergence.

## Note

This family function handles multiple responses.

## Author(s)

Thomas W. Yee

## References

- Barry, S. C. and Welsh, A. H. (2002) Generalized additive modelling and zero inflated count data. *Ecological Modelling*, **157**, 179–188.
- Williamson, E. and Bretherton, M. H. (1964) Tables of the logarithmic series distribution. *Annals of Mathematical Statistics*, **35**, 284–297.

## See Also

[rposnegbin](#), [pospoisson](#), [negbinomial](#), [zanegbinomial](#),  
[rnbinom](#), [CommonVGAMffArguments](#), [corbet](#), [logff](#).

## Examples

```
## Not run:
pdata <- data.frame(x2 = runif(nn <- 1000))
pdata <- transform(pdata, y1 = rposnegbin(nn, munb = exp(0+2*x2), size = exp(1)),
                  y2 = rposnegbin(nn, munb = exp(1+2*x2), size = exp(3)))
fit <- vglm(cbind(y1, y2) ~ x2, posnegbinomial, pdata, trace = TRUE)
coef(fit, matrix = TRUE)
dim(depvar(fit)) # dim(fit@y) is not as good

# Another artificial data example
pdata2 <- data.frame(munb = exp(2), size = exp(3)); nn <- 1000
pdata2 <- transform(pdata2, y3 = rposnegbin(nn, munb = munb, size = size))
with(pdata2, table(y3))
fit <- vglm(y3 ~ 1, posnegbinomial, pdata2, trace = TRUE)
coef(fit, matrix = TRUE)
with(pdata2, mean(y3)) # Sample mean
head(with(pdata2, munb/(1-(size/(size+munb))^size)), 1) # Population mean
head(fitted(fit), 3)
head(predict(fit), 3)

# Example: Corbet (1943) butterfly Malaya data
fit <- vglm(ofreq ~ 1, posnegbinomial, weights = species, data = corbet)
coef(fit, matrix = TRUE)
Coef(fit)
(khat <- Coef(fit)["size"])
pdf2 <- dposnegbin(x = with(corbet, ofreq), mu = fitted(fit), size = khat)
print( with(corbet, cbind(ofreq, species, fitted = pdf2*sum(species))), digits = 1)
with(corbet,
matplot(ofreq, cbind(species, fitted = pdf2*sum(species)), las = 1,
        xlab = "Observed frequency (of individual butterflies)",
        type = "b", ylab = "Number of species", col = c("blue", "orange"),
        main = "blue 1s = observe; orange 2s = fitted"))

## End(Not run)
```

---

Posnorm

---

*The Positive-Normal Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the univariate positive-normal distribution.

## Usage

```
dposnorm(x, mean = 0, sd = 1, log = FALSE)
pposnorm(q, mean = 0, sd = 1)
qposnorm(p, mean = 0, sd = 1)
rposnorm(n, mean = 0, sd = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
mean, sd, log	see <a href="#">rnorm</a> .

**Details**

See [posnormal](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

**Value**

`dposnorm` gives the density, `pposnorm` gives the distribution function, `qposnorm` gives the quantile function, and `rposnorm` generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[posnormal](#).

**Examples**

```
## Not run: m <- 0.8; x <- seq(-1, 4, len = 501)
plot(x, dposnorm(x, m = m), type = "l", ylim = 0:1, las = 1,
     ylab = paste("posnorm(m = ", m, ", sd = 1)"), col = "blue",
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pposnorm(x, m = m), col = "orange")
abline(h = 0, col = "grey")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qposnorm(probs, m = m)
lines(Q, dposnorm(Q, m = m), col = "purple", lty = 3, type = "h")
lines(Q, pposnorm(Q, m = m), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pposnorm(Q, m = m) - probs)) # Should be 0

## End(Not run)
```

posnormal

*Positive Normal Distribution Family Function***Description**

Fits a positive (univariate) normal distribution.

**Usage**

```
posnormal(lmean = "identity", lsd = "loge",
          imean = NULL, isd = NULL, nsimEIM = 100, zero = NULL)
```

**Arguments**

lmean, lsd	Link functions for the mean and standard deviation parameters of the usual univariate normal distribution. They are $\mu$ and $\sigma$ respectively. See <a href="#">Links</a> for more choices.
imean, isd	Optional initial values for $\mu$ and $\sigma$ . A NULL means a value is computed internally.
nsimEIM	See <a href="#">CommonVGAMffArguments</a> for more information.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set {1,2} corresponding respectively to $\mu$ , $\sigma$ . If zero = NULL then all linear/additive predictors are modelled as a linear combination of the explanatory variables. For many data sets having zero = 2 is a good idea.

**Details**

The positive normal distribution is the ordinary normal distribution but with the probability of zero or less being zero. The rest of the probability density function is scaled up. Hence the probability density function can be written

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}(y - \mu)^2/\sigma^2\right) / [1 - \Phi(-\mu/\sigma)]$$

where  $\Phi()$  is the cumulative distribution function of a standard normal ([pnorm](#)). Equivalently, this is

$$f(y) = \frac{1}{\sigma} \frac{\phi((y - \mu)/\sigma)}{1 - \Phi(-\mu/\sigma)}.$$

where  $\phi()$  is the probability density function of a standard normal distribution ([dnorm](#)).

The mean of  $Y$  is

$$E(Y) = \mu + \sigma \frac{\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)}.$$

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

Under- or over-flow may occur if the data is ill-conditioned.

**Note**

The response variable for this family function is the same as `uninormal` except positive values are required. Reasonably good initial values are needed. Fisher scoring is implemented.

The distribution of the reciprocal of a positive normal random variable is known as an alpha distribution.

**Author(s)**

Thomas W. Yee

**References**

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

`uninormal`, `tobit`.

**Examples**

```
pdata <- data.frame(m = 1.0, SD = exp(1.0))
pdata <- transform(pdata, y = rposnorm(n <- 1000, m = m, sd = SD))

## Not run: with(pdata, hist(y, prob = TRUE, border = "blue",
  main = paste("posnorm(m =", m[1], ", sd =", round(SD[1], 2), ")"))
## End(Not run)
fit <- vglm(y ~ 1, fam = posnormal, pdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
mygrid <- with(pdata, seq(min(y), max(y), len = 200)) # Add the fit to the histogram
## Not run: lines(mygrid, dposnorm(mygrid, Cfit[1], Cfit[2]), col = "red")
```

---

Pospois

---

*Positive-Poisson Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the positive-Poisson distribution.

**Usage**

```
dpospois(x, lambda, log = FALSE)
ppospois(q, lambda)
qpospois(p, lambda)
rpospois(n, lambda)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Fed into <a href="#">runif</a> .
lambda	vector of positive means (of an ordinary Poisson distribution). Short vectors are recycled.
log	logical.

**Details**

The positive-Poisson distribution is a Poisson distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$\lambda / (1 - \exp(-\lambda)).$$

As  $\lambda$  increases, the positive-Poisson and Poisson distributions become more similar. Unlike similar functions for the Poisson distribution, a zero value of lambda is not permitted here.

**Value**

dpospois gives the density, ppospois gives the distribution function, qpospois gives the quantile function, and rpospois generates random deviates.

**Note**

The family function [pospoisson](#) estimates  $\lambda$  by maximum likelihood estimation.

**Author(s)**

T. W. Yee

**See Also**

[pospoisson](#), [zapoisson](#), [zipoisson](#), [rpois](#).

**Examples**

```
lambda <- 2; y = rpospois(n = 1000, lambda)
table(y)
mean(y) # Sample mean
lambda / (1 - exp(-lambda)) # Population mean
```

```
(ii <- dpospois(0:7, lambda))
cumsum(ii) - ppospois(0:7, lambda) # Should be 0s
table(rpospois(100, lambda))

table(qpospois(runif(1000), lambda))
round(dpospois(1:10, lambda) * 1000) # Should be similar

## Not run: x <- 0:7
barplot(rbind(dpospois(x, lambda), dpois(x, lambda)),
        beside = TRUE, col = c("blue", "orange"),
        main = paste("Positive Poisson(", lambda, ") (blue) vs",
        " Poisson(", lambda, ") (orange)", sep = ""),
        names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)
```

pospoisson

*Positive Poisson Distribution Family Function***Description**

Fits a positive Poisson distribution.

**Usage**

```
pospoisson(link = "log", expected = TRUE,
           ilambda = NULL, imethod = 1, zero = NULL)
```

**Arguments**

**link** Link function for the usual mean ( $\lambda$ ) parameter of an ordinary Poisson distribution. See [Links](#) for more choices.

**expected** Logical. Fisher scoring is used if `expected = TRUE`, else Newton-Raphson.

**ilambda, imethod, zero** See [CommonVGAMffArguments](#) for more information.

**Details**

The positive Poisson distribution is the ordinary Poisson distribution but with the probability of zero being zero. Thus the other probabilities are scaled up (i.e., divided by  $1 - P[Y = 0]$ ). The mean,  $\lambda / (1 - \exp(-\lambda))$ , can be obtained by the extractor function `fitted` applied to the object.

A related distribution is the zero-inflated Poisson, in which the probability  $P[Y = 0]$  involves another parameter  $\phi$ . See [zipoisson](#).

**Value**

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Warning**

Under- or over-flow may occur if the data is ill-conditioned.

**Note**

This family function can handle a multivariate response.

Yet to be done: a quasi .pospoisson which estimates a dispersion parameter.

**Author(s)**

Thomas W. Yee

**References**

Coleman, J. S. and James, J. (1961) The equilibrium size distribution of freely-forming groups. *Sociometry*, **24**, 36–45.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[Pospois](#), [posnegbinomial](#), [poissonff](#), [zipoisson](#).

**Examples**

```
# Data from Coleman and James (1961)
cjdata <- data.frame(y = 1:6, freq = c(1486, 694, 195, 37, 10, 1))
fit <- vglm(y ~ 1, pospoisson, cjdata, weights = freq)
Coef(fit)
summary(fit)
fitted(fit)

pdata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
pdata <- transform(pdata, lambda = exp(1 - 2 * x2))
pdata <- transform(pdata, y1 = rpospois(nn, lambda))
with(pdata, table(y1))
fit <- vglm(y1 ~ x2, pospoisson, pdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
```

---

powerlink

*Power Link Function*

---

**Description**

Computes the power transformation, including its inverse and the first two derivatives.



**Usage**

```
powerlink(theta, power = 1, inverse = FALSE, deriv = 0,
          short = TRUE, tag = FALSE)
```

**Arguments**

theta                Numeric or character. See below for further details.  
 power               This denotes the power or exponent.  
 inverse, deriv, short, tag  
                      Details at [Links](#).

**Details**

The power link function raises a parameter by a certain value of power. Care is needed because it is very easy to get numerical problems, e.g., if power=0.5 and theta is negative.

**Value**

For powerlink with deriv = 0, then theta raised to the power of power. And if inverse = TRUE then theta raised to the power of 1/power.

For deriv = 1, then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if inverse = FALSE, else if inverse = TRUE then it returns the reciprocal.

**Note**

Numerical problems may occur for certain combinations of theta and power. Consequently this link function should be used with caution.

**Author(s)**

Thomas W. Yee

**See Also**

[Links](#), [loge](#).

**Examples**

```
powerlink("a", power = 2, short = FALSE, tag = TRUE)
powerlink(x <- 1:5)
powerlink(x, power = 2)
max(abs(powerlink(powerlink(x, power = 2),
                  power = 2, inverse = TRUE) - x)) # Should be 0
powerlink(x <- (-5):5, power = 0.5) # Has NAs

# 1/2 = 0.5
pdata <- data.frame(y = rbeta(n = 1000, shape1 = 2^2, shape2 = 3^2))
fit <- vglm(y ~ 1, beta.ab(lshape1 = powerlink(power = 0.5), i1 = 3,
                    lshape2 = powerlink(power = 0.5), i2 = 7), pdata)
t(coef(fit, matrix = TRUE))
```

```
Coef(fit) # Useful for intercept-only models
vcov(fit, untransform = TRUE)
```

prats

*Pregnant Rats Toxological Experiment Data*

## Description

A small toxological experiment data. The subjects are fetuses from two randomized groups of pregnant rats, and they were given a placebo or chemical treatment. The number with birth defects were recorded, as well as each litter size.

## Usage

```
data(prats)
```

## Format

A data frame with the following variables.

**treatment** A 0 means control; a 1 means the chemical treatment.

**alive, litter.size** The number of fetuses alive at 21 days, out of the number of fetuses alive at 4 days (the litter size).

## Details

The data concerns a toxological experiment where the subjects are fetuses from two randomized groups of 16 pregnant rats each, and they were given a placebo or chemical treatment. The number with birth defects and the litter size were recorded. Half the rats were fed a control diet during pregnancy and lactation, and the diet of the other half was treated with a chemical. For each litter the number of pups alive at 4 days and the number of pups that survived the 21 day lactation period, were recorded.

## Source

Weil, C. S. (1970) Selection of the valid number of sampling units and a consideration of their combination in toxicological studies involving reproduction, teratogenesis or carcinogenesis. *Food and Cosmetics Toxicology*, **8**(2), 177–182.

## References

Williams, D. A. (1975) The Analysis of Binary Responses From Toxicological Experiments Involving Reproduction and Teratogenicity. *Biometrics*, **31**(4), 949–952.

## See Also

[betabinomial](#), [betabinomial.ab](#).

**Examples**

```
prats
colSums(subset(prats, treatment == 0))
colSums(subset(prats, treatment == 1))
summary(prats)
```

predictqrrvglm

*Predict Method for a CQO fit***Description**

Predicted values based on a constrained quadratic ordination (CQO) object.

**Usage**

```
predictqrrvglm(object, newdata=NULL,
               type = c("link", "response", "latvar", "terms"),
               se.fit = FALSE, deriv = 0, dispersion = NULL,
               extra = object@extra, varI.latvar = FALSE, reference = NULL, ...)
```

**Arguments**

object	Object of class inheriting from "qrrvglm".
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type, se.fit, dispersion, extra	See <a href="#">predictvglm</a> .
deriv	Derivative. Currently only 0 is handled.
varI.latvar, reference	Arguments passed into <a href="#">Coef.qrrvglm</a> .
...	Currently undocumented.

**Details**

Obtains predictions from a fitted CQO object. Currently there are lots of limitations of this function; it is unfinished.

**Value**

See [predictvglm](#).

**Note**

This function is not robust and has not been checked fully.

**Author(s)**

T. W. Yee

**References**

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

**See Also**

[cqo](#).

**Examples**

```
hspider[,1:6]=scale(hspider[,1:6]) # Standardize the environmental variables
set.seed(1234)
# vvv p1 = cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi, Auloalbi,
# vvv          Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
# vvv          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
# vvv          fam=poissonff, data=hspider, Crowlpositive=FALSE, ITol=TRUE)
# vvv sort(p1@misc$deviance.Bestof) # A history of all the iterations

# vvv head(predict(p1))

# The following should be all zeros
# vvv max(abs(predict(p1, new=head(hspider)) - head(predict(p1))))
# vvv max(abs(predict(p1, new=head(hspider), type="res") - head(fitted(p1))))
```

---

predictvglm

---

*Predict Method for a VGLM fit*


---

**Description**

Predicted values based on a vector generalized linear model (VGLM) object.

**Usage**

```
predictvglm(object, newdata = NULL,
            type = c("link", "response", "terms"),
            se.fit = FALSE, deriv = 0, dispersion = NULL,
            untransform = FALSE, extra = object@extra, ...)
```

**Arguments**

object	Object of class inheriting from "vlm", e.g., <a href="#">vglm</a> .
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.

type	<p>The value of this argument can be abbreviated. The type of prediction required. The default is the first one, meaning on the scale of the linear predictors. This should be a <math>n \times M</math> matrix.</p> <p>The alternative "response" is on the scale of the response variable, and depending on the family function, this may or may not be the mean. Often this is the fitted value, e.g., <code>fitted(vglmObject)</code> (see <a href="#">fittedvglm</a>). Note that the response is output from the <code>@linkinv</code> slot, where the <code>eta</code> argument is the <math>n \times M</math> matrix of linear predictors.</p> <p>The "terms" option returns a matrix giving the fitted values of each term in the model formula on the linear predictor scale. The terms have been centered.</p>
se.fit	logical: return standard errors?
deriv	Non-negative integer. Currently this must be zero. Later, this may be implemented for general values.
dispersion	Dispersion parameter. This may be inputted at this stage, but the default is to use the dispersion parameter of the fitted model.
extra	A list containing extra information. This argument should be ignored.
untransform	Logical. Reverses any parameter link function. This argument only works if <code>type = "link"</code> , <code>se.fit = FALSE</code> , <code>deriv = 0</code> .
...	Arguments passed into <code>predictvglm</code> .

### Details

Obtains predictions and optionally estimates standard errors of those predictions from a fitted [vglm](#) object.

This code implements *smart prediction* (see [smartpred](#)).

### Value

If `se.fit = FALSE`, a vector or matrix of predictions. If `se.fit = TRUE`, a list with components

<code>fitted.values</code>	Predictions
<code>se.fit</code>	Estimated standard errors
<code>df</code>	Degrees of freedom
<code>sigma</code>	The square root of the dispersion parameter

### Warning

This function may change in the future.

### Note

Setting `se.fit = TRUE` and `type = "response"` will generate an error.

### Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

## See Also

[predict](#), [vglm](#), [predictvglm](#), [smartpred](#).

## Examples

```
# Illustrates smart prediction
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ poly(c(scale(let)), 2),
            propodds, data = pneumo, trace = TRUE, x.arg = FALSE)
class(fit)

(q0 <- head(predict(fit)))
(q1 <- predict(fit, newdata = head(pneumo)))
(q2 <- predict(fit, newdata = head(pneumo)))
all.equal(q0, q1) # Should be TRUE
all.equal(q1, q2) # Should be TRUE

head(predict(fit))
head(predict(fit, untransform = TRUE))

p0 <- head(predict(fit, type = "response"))
p1 <- head(predict(fit, type = "response", newdata = pneumo))
p2 <- head(predict(fit, type = "response", newdata = pneumo))
p3 <- head(fitted(fit))
all.equal(p0, p1) # Should be TRUE
all.equal(p1, p2) # Should be TRUE
all.equal(p2, p3) # Should be TRUE

predict(fit, type = "terms", se = TRUE)
```

---

```
prentice74
```

---

```
Prentice (1974) Log-gamma Distribution
```

---

## Description

Estimation of a 3-parameter log-gamma distribution described by Prentice (1974).

## Usage

```
prentice74(llocation = "identity", lscale = "loge", lshape = "identity",
           ilocation = NULL, iscale = NULL, ishape = NULL, zero = 2:3)
```

## Arguments

<code>llocation</code> , <code>lscale</code> , <code>lshape</code>	Parameter link function applied to the location parameter $a$ , positive scale parameter $b$ and the shape parameter $q$ , respectively. See <a href="#">Links</a> for more choices.
<code>ilocation</code> , <code>iscale</code>	Initial value for $a$ and $b$ , respectively. The defaults mean an initial value is determined internally for each.
<code>ishape</code>	Initial value for $q$ . If failure to converge occurs, try some other value. The default means an initial value is determined internally.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts-only. The values must be from the set $\{1,2,3\}$ . See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

The probability density function is given by

$$f(y; a, b, q) = |q| \exp(w/q^2 - e^w) / (b \Gamma(1/q^2)),$$

for shape parameter  $q \neq 0$ , positive scale parameter  $b > 0$ , location parameter  $a$ , and all real  $y$ . Here,  $w = (y - a)q/b + \psi(1/q^2)$  where  $\psi$  is the digamma function, [digamma](#). The mean of  $Y$  is  $a$  (returned as the fitted values). This is a different parameterization compared to [lgamma3ff](#).

Special cases:  $q = 0$  is the normal distribution with standard deviation  $b$ ,  $q = -1$  is the extreme value distribution for maxima,  $q = 1$  is the extreme value distribution for minima (Weibull). If  $q > 0$  then the distribution is left skew, else  $q < 0$  is right skew.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

The special case  $q = 0$  is not handled, therefore estimates of  $q$  too close to zero may cause numerical problems.

## Note

The notation used here differs from Prentice (1974):  $\alpha = a$ ,  $\sigma = b$ . Fisher scoring is used.

## Author(s)

T. W. Yee

## References

Prentice, R. L. (1974) A log gamma model and its maximum likelihood estimation. *Biometrika*, **61**, 539–544.

See Also

[lgamma3ff](#), [lgamma](#), [gengamma](#).

Examples

```
pdata <- data.frame(x2 = runif(nn <- 1000))
pdata <- transform(pdata, loc = -1 + 2*x2, Scale = exp(1))
pdata <- transform(pdata, y = rlgamma(nn, loc = loc, scale = Scale, k = 1))
fit <- vglm(y ~ x2, prentice74(zero = 2:3), pdata, trace = TRUE)
coef(fit, matrix = TRUE) # Note the coefficients for location
```

---

prinia	<i>Yellow-bellied Prinia</i>
--------	------------------------------

---

Description

A data frame with yellow-bellied Prinia.

Usage

```
data(prinia)
```

Format

A data frame with 151 observations on the following 23 variables.

- length** a numeric vector, the scaled wing length (zero mean and unit variance).
- fat** a numeric vector, fat index; originally 1 (no fat) to 4 (very fat) but converted to 0 (no fat) versus 1 otherwise.
- cap** a numeric vector, number of times the bird was captured or recaptured.
- noncap** a numeric vector, number of times the bird was not captured.
- y1, y2, y3, y4, y5, y6** a numeric vector of 0s and 1s; for noncapture and capture resp.
- y7, y8, y9, y10, y11, y12** same as above.
- y13, y14, y15, y16, y17, y18, y19** same as above.

Details

The yellow-bellied Prinia *Prinia flaviventris* is a common bird species located in Southeast Asia. A capture-recapture experiment was conducted at the Mai Po Nature Reserve in Hong Kong during 1991, where captured individuals had their wing lengths measured and fat index recorded. A total of 19 weekly capture occasions were considered, where 151 distinct birds were captured.

More generally, the prinias are a genus of small insectivorous birds, and are sometimes referred to as *wren-warblers*. They are a little-known group of the tropical and subtropical Old World, the roughly 30 species being divided fairly equally between Africa and Asia.



## Source

Thanks to Paul Yip for permission to make this data available.

Hwang, W.-H. and Huggins, R. M. (2007) Application of semiparametric regression models in the analysis of capture–recapture experiments. *Australian and New Zealand Journal of Statistics* **49**, 191–202.

## Examples

```
head(prinia)
summary(prinia)
rowSums(prinia[, c("cap", "noncap")]) # 19s

# Fit a positive-binomial distribution (M.h) to the data:
fit1 <- vglm(cbind(cap, noncap) ~ length + fat, posbinomial, data = prinia)

# Fit another positive-binomial distribution (M.h) to the data:
# The response input is suitable for posbernoulli.*-type functions.
fit2 <- vglm(cbind( y1, y2, y3, y4, y5, y6, y7, y8, y9,
                   y10, y11, y12, y13, y14, y15, y16, y17, y18, y19) ~
             length + fat, posbernoulli.b(drop.b = FALSE ~ 0), data = prinia)
```

---

probit

---

*Probit Link Function*


---

## Description

Computes the probit transformation, including its inverse and the first two derivatives.

## Usage

```
probit(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
       short = TRUE, tag = FALSE)
```

## Arguments

theta	Numeric or character. See below for further details.
bvalue	See <a href="#">Links</a> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

## Details

The probit link function is commonly used for parameters that lie in the unit interval. Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.

**Value**

For `deriv = 0`, the probit of `theta`, i.e., `qnorm(theta)` when `inverse = FALSE`, and if `inverse = TRUE` then `pnorm(theta)`.

For `deriv = 1`, then the function returns  $d\theta/d\eta$  as a function of `theta` if `inverse = FALSE`, else if `inverse = TRUE` then it returns the reciprocal.

**Note**

Numerical instability may occur when `theta` is close to 1 or 0. One way of overcoming this is to use `bvalue`.

In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the univariate normal distribution (see [uninormal](#)).

**Author(s)**

Thomas W. Yee

**References**

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

**See Also**

[Links](#), [logit](#), [cloglog](#), [cauchit](#).

**Examples**

```
p <- seq(0.01, 0.99, by = 0.01)
probit(p)
max(abs(probit(probit(p), inverse = TRUE) - p)) # Should be 0

p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
probit(p) # Has NAs
probit(p, bvalue = .Machine$double.eps) # Has no NAs

## Not run: p <- seq(0.01, 0.99, by = 0.01); par(lwd = (mylwd <- 2))
plot(p, logit(p), type = "l", col = "limegreen", ylab = "transformation",
     las = 1, main = "Some probability link functions")
lines(p, probit(p), col = "purple")
lines(p, cloglog(p), col = "chocolate")
lines(p, cauchit(p), col = "tan")
abline(v = 0.5, h = 0, lty = "dashed")
legend(0.1, 4.0, c("logit", "probit", "cloglog", "cauchit"),
     col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
par(lwd = 1)
## End(Not run)
```

---

propodds*Proportional Odds Model for Ordinal Regression*

---

### Description

Fits the proportional odds model to a (preferably ordered) factor response.

### Usage

```
propodds(reverse = TRUE, whitespace = FALSE)
```

### Arguments

reverse, whitespace

Logical. Fed into arguments of the same name in [cumulative](#).

### Details

The *proportional odds model* is a special case from the class of *cumulative link models*. It involves a logit link applied to cumulative probabilities and a strong *parallelism* assumption. A parallelism assumption means there is less chance of numerical problems because the fitted probabilities will remain between 0 and 1; however the *parallelism* assumption ought to be checked, e.g., via a likelihood ratio test. This **VGAM** family function is merely a shortcut for `cumulative(reverse = reverse, link = "logit",` Please see [cumulative](#) for more details on this model.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Warning

No check is made to verify that the response is ordinal if the response is a matrix; see [ordered](#).

### Author(s)

Thomas W. Yee

### References

- Agresti, A. (2010) *Analysis of Ordinal Categorical Data*, 2nd ed. New York: Wiley.
- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.
- Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[cumulative](#).

**Examples**

```
# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo))
depvar(fit) # Sample proportions
weights(fit, type = "prior") # Number of observations
coef(fit, matrix = TRUE)
constraints(fit) # Constraint matrices
summary(fit)

# Check that the model is linear in let -----
fit2 <- vgam(cbind(normal, mild, severe) ~ s(let, df = 2), propodds, pneumo)
## Not run: plot(fit2, se = TRUE, lcol = 2, scol = 2)

# Check the proportional odds assumption with a LRT -----
(fit3 <- vglm(cbind(normal, mild, severe) ~ let,
              cumulative(parallel = FALSE, reverse = TRUE), pneumo))
pchisq(deviance(fit) - deviance(fit3),
        df = df.residual(fit) - df.residual(fit3), lower.tail = FALSE)
lrtest(fit3, fit) # Easier
```

---

prplot

---

*Probability Plots for Categorical Data Analysis*


---

**Description**

Plots the fitted probabilities for some very simplified special cases of categorical data analysis models.

**Usage**

```
prplot(object, control = prplot.control(...), ...)

prplot.control(xlab = NULL, ylab = "Probability", main = NULL, xlim = NULL,
               ylim = NULL, lty = par()$lty, col = par()$col, rcol = par()$col,
               lwd = par()$lwd, rlwd = par()$lwd, las = par()$las, rug.arg = FALSE, ...)
```

**Arguments**

object	Currently only an <a href="#">cumulative</a> object. This includes a <a href="#">propodds</a> object since that <b>VGAM</b> family function is a special case of <a href="#">cumulative</a> .
control	List containing some basic graphical parameters.

xlab, ylab, main, xlim, ylim, lty  
     See [par](#) and ... below.

col, rcol, lwd, rlwd, las, rug.arg  
     See [par](#) and ... below. Arguments starting with r refer to the rug. Argument rug.arg is logical: add a rug for the distinct values of the explanatory variable?

...      Arguments such as xlab which are fed into `prplot.control()`. Only a small selection of graphical arguments from [par](#) are offered.

### Details

For models involving one term in the RHS of the formula this function plots the fitted probabilities against the single explanatory variable.

### Value

The object is returned invisibly with the `preplot` slot assigned. This is obtained by a call to `plotvgam()`.

### Note

This function is rudimentary.

### See Also

[cumulative](#).

### Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo)
M <- npred(fit) # Or fit@misc$M
## Not run: prplot(fit)
prplot(fit, lty = 1:M, col = (1:M)+2, rug = TRUE, las = 1,
       ylim = c(0, 1), rlwd = 2)
## End(Not run)
```

---

put.smart

*Adds a List to the End of the List ".smart.prediction"*

---

### Description

Adds a list to the end of the list `.smart.prediction` in `smartpredenv` (R) or frame 1 (S-PLUS).

### Usage

```
put.smart(smart)
```

## Arguments

`smart` a list containing parameters needed later for smart prediction.

## Details

`put.smart` is used in "write" mode within a smart function. It saves parameters at the time of model fitting, which are later used for prediction. The function `put.smart` is the opposite of `get.smart`, and both deal with the same contents.

## Value

Nothing is returned.

## Side Effects

The variable `.smart.prediction.counter` in `smartpredenv` (R) or frame 1 (S-PLUS) is incremented beforehand, and `.smart.prediction[[.smart.prediction.counter]]` is assigned the list `smart`. If the list `.smart.prediction` in `smartpredenv` (R) or frame 1 (S-PLUS) is not long enough to hold `smart`, then it is made larger, and the variable `.max.smart` in `smartpredenv` (R) or frame 1 (S-PLUS) is adjusted accordingly.

## See Also

`get.smart`.

## Examples

```
"my1" <- function(x, minx=min(x)) { # Here is a smart function
  x <- x # Needed for nested calls, e.g., bs(scale(x))
  if(smart.mode.is("read")) {
    smart <- get.smart()
    minx <- smart$minx # Overwrite its value
  } else
  if(smart.mode.is("write"))
    put.smart(list(minx=minx))
  sqrt(x-minx)
}
attr(my1, "smart") <- TRUE
```

---

qrrvglm.control

*Control function for QRR-VGLMs (CQO)*

---

## Description

Algorithmic constants and parameters for a constrained quadratic ordination (CQO), by fitting a *quadratic reduced-rank vector generalized linear model* (QRR-VGLM), are set using this function. It is the control function for `cqo`.

**Usage**

```
qrrvglm.control(Rank = 1,
  Bestof = if(length(Cinit)) 1 else 10,
  checkwz = TRUE,
  Cinit = NULL,
  Crowlpositive = TRUE,
  epsilon = 1.0e-06,
  EqualTolerances = TRUE,
  Etamat.colmax = 10,
  FastAlgorithm = TRUE,
  GradientFunction = TRUE,
  Hstep = 0.001,
  isd.latvar = rep(c(2, 1, rep(0.5, length = Rank)), length = Rank),
  iKvector = 0.1,
  iShape = 0.1,
  ITolerances = FALSE,
  maxitl = 40,
  imethod = 1,
  Maxit.optim = 250,
  MUXfactor = rep(7, length = Rank),
  noRRR = ~ 1, Norrr = NA,
  optim.maxit = 20,
  Parscale = if(ITolerances) 0.001 else 1.0,
  sd.Cinit = 0.02,
  SmallNo = 5.0e-13,
  trace = TRUE,
  Use.Init.Poisson.Q0 = TRUE,
  wzepsilon = .Machine$double.eps^0.75, ...)
```

**Arguments**

In the following,  $R$  is the Rank,  $M$  is the number of linear predictors, and  $S$  is the number of responses (species). Thus  $M = S$  for binomial and Poisson responses, and  $M = 2S$  for the negative binomial and 2-parameter gamma distributions.

The numerical rank  $R$  of the model, i.e., the number of ordination axes. Must be an element from the set  $\{1, 2, \dots, \min(M, p_2)\}$  where the vector of explanatory variables  $x$  is partitioned into  $(x_1, x_2)$ , which is of dimension  $p_1 + p_2$ . The variables making up  $x_1$  are given by the terms in the `noRRR` argument, and the rest of the terms comprise  $x_2$ .

<b>Bestof</b>	Integer. The best of <code>Bestof</code> models fitted is returned. This argument helps guard against local solutions by (hopefully) finding the global solution from many fits. The argument has value 1 if an initial value for $C$ is inputted using <code>Cinit</code> .
<code>checkwz</code>	logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than <code>wzepsilon</code> . If not, any values less than <code>wzepsilon</code> are replaced with this value.

Cinit	Optional initial $C$ matrix, which must be a $p_2$ by $R$ matrix. The default is to apply <code>.Init.Poisson.Q0()</code> to obtain initial values.
Crow1positive	Logical vector of length Rank (recycled if necessary): are the elements of the first row of $C$ positive? For example, if Rank is 4, then specifying <code>Crow1positive = c(FALSE, TRUE, TRUE, TRUE)</code> will force $C[1, 1]$ and $C[1, 3]$ to be negative, and $C[1, 2]$ and $C[1, 4]$ to be positive. This argument allows for a reflection in the ordination axes because the coefficients of the latent variables are unique up to a sign.
epsilon	Positive numeric. Used to test for convergence for GLMs fitted in C. Larger values mean a loosening of the convergence criterion. If an error code of 3 is reported, try increasing this value.
EqualTolerances	Logical indicating whether each (quadratic) predictor will have equal tolerances. Having <code>EqualTolerances = TRUE</code> can help avoid numerical problems, especially with binary data. Note that the estimated (common) tolerance matrix may or may not be positive-definite. If it is then it can be scaled to the $R$ by $R$ identity matrix, i.e., made equivalent to <code>ITolerances = TRUE</code> . Setting <code>ITolerances = TRUE</code> will <i>force</i> a common $R$ by $R$ identity matrix as the tolerance matrix to the data even if it is not appropriate. In general, setting <code>ITolerances = TRUE</code> is preferred over <code>EqualTolerances = TRUE</code> because, if it works, it is much faster and uses less memory. However, <code>ITolerances = TRUE</code> requires the environmental variables to be scaled appropriately. See <b>Details</b> for more details.
Etamat.colmax	Positive integer, no smaller than Rank. Controls the amount of memory used by <code>.Init.Poisson.Q0()</code> . It is the maximum number of columns allowed for the pseudo-response and its weights. In general, the larger the value, the better the initial value. Used only if <code>Use.Init.Poisson.Q0 = TRUE</code> .
FastAlgorithm	Logical. Whether a new fast algorithm is to be used. The fast algorithm results in a large speed increases compared to Yee (2004). Some details of the fast algorithm are found in Appendix A of Yee (2006). Setting <code>FastAlgorithm = FALSE</code> will give an error.
GradientFunction	Logical. Whether <code>optim</code> 's argument <code>gr</code> is used or not, i.e., to compute gradient values. Used only if <code>FastAlgorithm</code> is TRUE. The default value is usually faster on most problems.
Hstep	Positive value. Used as the step size in the finite difference approximation to the derivatives by <code>optim</code> .
isd.latvar	Initial standard deviations for the latent variables (site scores). Numeric, positive and of length $R$ (recycled if necessary). This argument is used only if <code>ITolerances = TRUE</code> . Used by <code>.Init.Poisson.Q0()</code> to obtain initial values for the constrained coefficients $C$ adjusted to a reasonable value. It adjusts the spread of the site scores relative to a common species tolerance of 1 for each ordination axis. A value between 0.5 and 10 is recommended; a value such as 10 means that the range of the environmental space is very large relative to the niche width of the species. The successive values should decrease because the first ordination axis should have the most spread of site scores, followed by the second ordination axis, etc.



ikvector, iShape	Numeric, recycled to length $S$ if necessary. Initial values used for estimating the positive $k$ and $\lambda$ parameters of the negative binomial and 2-parameter gamma distributions respectively. For further information see <a href="#">negbinomial</a> and <a href="#">gamma2</a> . These arguments override the ik and iShape arguments in <a href="#">negbinomial</a> and <a href="#">gamma2</a> .
ITolerances	Logical. If TRUE then the (common) tolerance matrix is the $R$ by $R$ identity matrix by definition. Note that having ITolerances = TRUE implies EqualTolerances = TRUE, but not vice versa. Internally, the quadratic terms will be treated as offsets (in GLM jargon) and so the models can potentially be fitted very efficiently. <i>However, it is a very good idea to center and scale all numerical variables in the <math>x_2</math> vector.</i> See <b>Details</b> for more details. The success of ITolerances = TRUE often depends on suitable values for <code>isd.latvar</code> and/or <code>MUXfactor</code> .
maxitl	Maximum number of times the optimizer is called or restarted. Most users should ignore this argument.
imethod	Method of initialization. A positive integer 1 or 2 or 3 etc. depending on the <b>VGAM</b> family function. Currently it is used for <a href="#">negbinomial</a> and <a href="#">gamma2</a> only, and used within the C.
Maxit.optim	Positive integer. Number of iterations given to the function <a href="#">optim</a> at each of the <code>optim.maxit</code> iterations.
MUXfactor	Multiplication factor for detecting large offset values. Numeric, positive and of length $R$ (recycled if necessary). This argument is used only if ITolerances = TRUE. Offsets are $-0.5$ multiplied by the sum of the squares of all $R$ latent variable values. If the latent variable values are too large then this will result in numerical problems. By too large, it is meant that the standard deviation of the latent variable values are greater than <code>MUXfactor[r] * isd.latvar[r]</code> for $r=1:\text{Rank}$ (this is why centering and scaling all the numerical predictor variables in $x_2$ is recommended). A value about 3 or 4 is recommended. If failure to converge occurs, try a slightly lower value.
optim.maxit	Positive integer. Number of times <a href="#">optim</a> is invoked. At iteration $i$ , the $i$ th value of <code>Maxit.optim</code> is fed into <a href="#">optim</a> .
noRRR	Formula giving terms that are <i>not</i> to be included in the reduced-rank regression (or formation of the latent variables), i.e., those belong to $x_1$ . Those variables which do not make up the latent variable (reduced-rank regression) correspond to the $B_1$ matrix. The default is to omit the intercept term from the latent variables.
Norrr	Defunct. Please use noRRR. Use of Norrr will become an error soon.
Parscale	Numerical and positive-valued vector of length $C$ (recycled if necessary). Passed into <code>optim(..., control = list(parscale = Parscale))</code> ; the elements of $C$ become $C / \text{Parscale}$ . Setting ITolerances = TRUE results in line searches that are very large, therefore $C$ has to be scaled accordingly to avoid large step sizes. See <b>Details</b> for more information. It's probably best to leave this argument alone.
sd.Cinit	Standard deviation of the initial values for the elements of $C$ . These are normally distributed with mean zero. This argument is used only if <code>Use.Init.Poisson.Q0 = FALSE</code> and $C$ is not inputted using <code>Cinit</code> .

trace	Logical indicating if output should be produced for each iteration. The default is TRUE because the calculations are numerically intensive, meaning it may take a long time, so that the user might think the computer has locked up if trace = FALSE.
SmallNo	Positive numeric between .Machine\$double.eps and 0.0001. Used to avoid under- or over-flow in the IRLS algorithm. Used only if FastAlgorithm is TRUE.
Use.Init.Poisson.QQ	Logical. If TRUE then the function .Init.Poisson.QQ() is used to obtain initial values for the canonical coefficients $C$ . If FALSE then random numbers are used instead.
wzepsilon	Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive.
...	Ignored at present.

## Details

Recall that the central formula for CQO is

$$\eta = B_1^T x_1 + A\nu + \sum_{m=1}^M (\nu^T D_m \nu) e_m$$

where  $x_1$  is a vector (usually just a 1 for an intercept),  $x_2$  is a vector of environmental variables,  $\nu = C^T x_2$  is a  $R$ -vector of latent variables,  $e_m$  is a vector of 0s but with a 1 in the  $m$ th position. QRR-VGLMs are an extension of RR-VGLMs and allow for maximum likelihood solutions to constrained quadratic ordination (CQO) models.

Having ITolerances = TRUE means all the tolerance matrices are the order- $R$  identity matrix, i.e., it *forces* bell-shaped curves/surfaces on all species. This results in a more difficult optimization problem (especially for 2-parameter models such as the negative binomial and gamma) because of overflow errors and it appears there are more local solutions. To help avoid the overflow errors, scaling  $C$  by the factor Parscale can help enormously. Even better, scaling  $C$  by specifying `isd.latvar` is more understandable to humans. If failure to converge occurs, try adjusting Parscale, or better, setting EqualTolerances = TRUE (and hope that the estimated tolerance matrix is positive-definite). To fit an equal-tolerances model, it is firstly best to try setting ITolerances = TRUE and varying `isd.latvar` and/or `MUXfactor` if it fails to converge. If it still fails to converge after many attempts, try setting EqualTolerances = TRUE, however this will usually be a lot slower because it requires a lot more memory.

With a  $R > 1$  model, the latent variables are always uncorrelated, i.e., the variance-covariance matrix of the site scores is a diagonal matrix.

If setting EqualTolerances = TRUE is used and the common estimated tolerance matrix is positive-definite then that model is effectively the same as the ITolerances = TRUE model (the two are transformations of each other). In general, ITolerances = TRUE is numerically more unstable and presents a more difficult problem to optimize; the arguments `isd.latvar` and/or `MUXfactor` often must be assigned some good value(s) (possibly found by trial and error) in order for convergence to occur. Setting ITolerances = TRUE *forces* a bell-shaped curve or surface onto all the species data, therefore this option should be used with deliberation. If unsuitable, the resulting fit may be very misleading. Usually it is a good idea for the user to set EqualTolerances = FALSE to see

which species appear to have a bell-shaped curve or surface. Improvements to the fit can often be achieved using transformations, e.g., nitrogen concentration to log nitrogen concentration.

Fitting a CAO model (see [cao](#)) first is a good idea for pre-examining the data and checking whether it is appropriate to fit a CQO model.

### Value

A list with components matching the input names.

### Warning

The default value of `Bestof` is a bare minimum for many datasets, therefore it will be necessary to increase its value to increase the chances of obtaining the global solution.

### Note

When `ITolerances = TRUE` it is a good idea to apply [scale](#) to all the numerical variables that make up the latent variable, i.e., those of  $x_2$ . This is to make them have mean 0, and hence avoid large offset values which cause numerical problems.

This function has many arguments that are common with [rrvglm.control](#) and [vglm.control](#).

It is usually a good idea to try fitting a model with `ITolerances = TRUE` first, and if convergence is unsuccessful, then try `EqualTolerances = TRUE` and `ITolerances = FALSE`. Ordination diagrams with `EqualTolerances = TRUE` have a natural interpretation, but with `EqualTolerances = FALSE` they are more complicated and requires, e.g., contours to be overlaid on the ordination diagram (see [lvplot.qrrvglm](#)).

In the example below, an equal-tolerances CQO model is fitted to the hunting spiders data. Because `ITolerances = TRUE`, it is a good idea to center all the  $x_2$  variables first. Upon fitting the model, the actual standard deviation of the site scores are computed. Ideally, the `isd.latvar` argument should have had this value for the best chances of getting good initial values. For comparison, the model is refitted with that value and it should run more faster and reliably.

### Author(s)

Thomas W. Yee

### References

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

### See Also

[cqo](#), [rcqo](#), [Coef.qrrvglm](#), [Coef.qrrvglm-class](#),  
[optim](#), [binomialff](#), [poissonff](#), [negbinomial](#), [gamma2](#), [gaussianff](#).

## Examples

```
## Not run: # Poisson CQO with equal tolerances
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Good idea when ITolerances = TRUE
p1 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi, Auloalbi,
               Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          quasipoissonff, data = hspider, EqualTolerances = TRUE)
sort(p1@misc$deviance.Bestof) # A history of all the iterations

(isd.latvar <- apply(latvar(p1), 2, sd)) # Should be approx isd.latvar

# Refit the model with better initial values
set.seed(111) # This leads to the global solution
p1 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi, Auloalbi,
               Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          ITolerances = TRUE, quasipoissonff, data = hspider,
          isd.latvar = isd.latvar) # Note the use of isd.latvar here
sort(p1@misc$deviance.Bestof) # A history of all the iterations

## End(Not run)
```

---

qtplot.gumbel

*Quantile Plot for Gumbel Regression*


---

## Description

Plots quantiles associated with a Gumbel model.

## Usage

```
qtplot.gumbel(object, show.plot = TRUE,
              y.arg = TRUE, spline.fit = FALSE, label = TRUE,
              R = object@misc$R, percentiles = object@misc$percentiles,
              add.arg = FALSE, mpv = object@misc$mpv,
              xlab = NULL, ylab = "", main = "",
              pch = par()$pch, pcol.arg = par()$col,
              lty.arg = par()$lty, lcol.arg = par()$col, llwd.arg = par()$lwd,
              tcol.arg = par()$col, tadj = 1, ...)
```

## Arguments

object	A <b>VGAM</b> extremes model of the Gumbel type, produced by modelling functions such as <b>vglm</b> and <b>vgam</b> with a family function either "gumbel" or "egumbel".
show.plot	Logical. Plot it? If FALSE no plot will be done.
y.arg	Logical. Add the raw data on to the plot?

spline.fit	Logical. Use a spline fit through the fitted percentiles? This can be useful if there are large gaps between some values along the covariate.
label	Logical. Label the percentiles?
R	See <a href="#">gumbel</a> .
percentiles	See <a href="#">gumbel</a> .
add.arg	Logical. Add the plot to an existing plot?
mpv	See <a href="#">gumbel</a> .
xlab	Caption for the x-axis. See <a href="#">par</a> .
ylab	Caption for the y-axis. See <a href="#">par</a> .
main	Title of the plot. See <a href="#">title</a> .
pch	Plotting character. See <a href="#">par</a> .
pcol.arg	Color of the points. See the col argument of <a href="#">par</a> .
lty.arg	Line type. Line type. See the lty argument of <a href="#">par</a> .
lcol.arg	Color of the lines. See the col argument of <a href="#">par</a> .
lwd.arg	Line width. See the lwd argument of <a href="#">par</a> .
tcol.arg	Color of the text (if label is TRUE). See the col argument of <a href="#">par</a> .
tadj	Text justification. See the adj argument of <a href="#">par</a> .
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include sub and las.

## Details

There should be a single covariate such as time. The quantiles specified by percentiles are plotted.

## Value

The object with a list called qtplot in the post slot of object. (If show.plot = FALSE then just the list is returned.) The list contains components

fitted.values	The percentiles of the response, possibly including the MPV.
percentiles	The percentiles (small vector of values between 0 and 100.

## Note

Unlike [gumbel](#), one cannot have percentiles = NULL.

## Author(s)

Thomas W. Yee

## See Also

[gumbel](#).

### Examples

```
ymat <- as.matrix(venice[, paste("r", 1:10, sep = "")])
fit1 <- vgam(ymat ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
            data = venice, trace = TRUE, na.action = na.pass)
head(fitted(fit1))

## Not run: par(mfrow = c(1, 1), bty = "l", xpd = TRUE, las = 1)
qtplot(fit1, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5),
       lwd = 2, pcol = "blue", tadj = 0.4, ylab = "Sea level (cm)")

qtplot(fit1, perc = 97, mpv = FALSE, lcol = 3, tcol = 3,
       lwd = 2, tadj = 0.4, add = TRUE) -> saved
head(saved$post$qtplot$fitted)

## End(Not run)
```

---

qtplot.lmscreg

*Quantile Plot for LMS Quantile Regression*


---

### Description

Plots quantiles associated with a LMS quantile regression.

### Usage

```
qtplot.lmscreg(object, newdata = NULL,
               percentiles = object@misc$percentiles,
               show.plot = TRUE, ...)
```

### Arguments

object	A <b>VGAM</b> quantile regression model, i.e., an object produced by modelling functions such as <a href="#">vglm</a> and <a href="#">vgam</a> with a family function beginning with "lms.", e.g., <a href="#">lms.yjn</a> .
newdata	Optional data frame for computing the quantiles. If missing, the original data is used.
percentiles	Numerical vector with values between 0 and 100 that specify the percentiles (quantiles). The default are the percentiles used when the model was fitted.
show.plot	Logical. Plot it? If FALSE no plot will be done.
...	Graphical parameter that are passed into <a href="#">plotqtplot.lmscreg</a> .

### Details

The 'primary' variable is defined as the main covariate upon which the regression or smoothing is performed. For example, in medical studies, it is often the age. In **VGAM**, it is possible to handle more than one covariate, however, the primary variable must be the first term after the intercept.

**Value**

A list with the following components.

`fitted.values`    A vector of fitted percentile values.  
`percentiles`      The percentiles used.

**Note**

`plotqtplot.lmscreg` does the actual plotting.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

`plotqtplot.lmscreg`, `deplot.lmscreg`, `lms.bcn`, `lms.bcg`, `lms.yjn`.

**Examples**

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero=1), data = bmi.nz)
qtplot(fit)
qtplot(fit, perc = c(25, 50, 75, 95), lcol = "blue", tcol = "blue", llwd = 2)

## End(Not run)
```

---

quasibinomialff

*Quasi-Binomial Family Function*


---

**Description**

Family function for fitting generalized linear models to binomial responses, where the dispersion parameters are unknown.

**Usage**

```
quasibinomialff(link = "logit", mv = FALSE, onedpar = !mv,
                 parallel = FALSE, zero = NULL)
```

## Arguments

link	Link function. See <a href="#">Links</a> for more choices.
mv	Multivariate response? If TRUE, then the response is interpreted as $M$ binary responses, where $M$ is the number of columns of the response matrix. In this case, the response matrix should have zero/one values only. If FALSE and the response is a (2-column) matrix, then the number of successes is given in the first column and the second column is the number of failures.
onedpar	One dispersion parameter? If mv, then a separate dispersion parameter will be computed for each response (column), by default. Setting onedpar=TRUE will pool them so that there is only one dispersion parameter to be estimated.
parallel	A logical or formula. Used only if mv is TRUE. This argument allows for the parallelism assumption whereby the regression coefficients for a variable is constrained to be equal over the $M$ linear/additive predictors.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ , where $M$ is the number of columns of the matrix response.

## Details

The final model is not fully estimated by maximum likelihood since the dispersion parameter is unknown (see pp.124–8 of McCullagh and Nelder (1989) for more details).

A dispersion parameter that is less/greater than unity corresponds to under-/over-dispersion relative to the binomial model. Over-dispersion is more common in practice.

Setting mv=TRUE is necessary when fitting a Quadratic RR-VGLM (see [cqo](#)) because the response will be a matrix of  $M$  columns (e.g., one column per species). Then there will be  $M$  dispersion parameters (one per column of the response).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), and [cao](#).

## Warning

The log-likelihood pertaining to the ordinary family is used to test for convergence during estimation, and is printed out in the summary.

## Note

If mv is FALSE (the default), then the response can be of one of three formats: a factor (first level taken as success), a vector of proportions of success, or a 2-column matrix (first column = successes) of counts. The argument weights in the modelling function can also be specified. In particular, for a general vector of proportions, you will need to specify weights because the number of trials is needed.

If mv is TRUE, then the matrix response can only be of one format: a matrix of 1's and 0's (1=success).



This function is only a front-end to the **VGAM** family function `binomialff()`; indeed, `quasibinomialff(...)` is equivalent to `binomialff(..., dispersion=0)`. Here, the argument `dispersion=0` signifies that the dispersion parameter is to be estimated.

Regardless of whether the dispersion parameter is to be estimated or not, its value can be seen from the output from the `summary()` of the object.

### Author(s)

Thomas W. Yee

### References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

### See Also

[binomialff](#), [rrvglm](#), [cqo](#), [cao](#), [logit](#), [probit](#), [cloglog](#), [cauchit](#), [poissonff](#), [quasipoissonff](#), [quasibinomial](#).

### Examples

```
quasibinomialff()
quasibinomialff(link = "probit")

# Nonparametric logistic regression
hunua <- transform(hunua, a.5 = sqrt(altitude)) # Transformation of altitude
fit1 <- vglm(agaau ~ poly(a.5, 2), quasibinomialff, hunua)
fit2 <- vgam(agaau ~ s(a.5, df = 2), quasibinomialff, hunua)
## Not run:
plot(fit2, se = TRUE, llwd = 2, lcol = "orange", scol = "orange",
     xlab = "sqrt(altitude)", ylim = c(-3, 1),
     main = "GAM and quadratic GLM fitted to species data")
plotvgam(fit1, se = TRUE, lcol = "blue", scol = "blue", add = TRUE, llwd = 2)

## End(Not run)
fit1@misc$dispersion # dispersion parameter
logLik(fit1)

# Here, the dispersion parameter defaults to 1
fit0 <- vglm(agaau ~ poly(a.5, 2), binomialff, hunua)
fit0@misc$dispersion # dispersion parameter
```

---

quasipoissonff

*Quasi-Poisson Family Function*

---

### Description

Fits a generalized linear model to a Poisson response, where the dispersion parameter is unknown.

**Usage**

```
quasipoissonff(link = "log", onedpar = FALSE,
               parallel = FALSE, zero = NULL)
```

**Arguments**

<code>link</code>	Link function. See <a href="#">Links</a> for more choices.
<code>onedpar</code>	One dispersion parameter? If the response is a matrix, then a separate dispersion parameter will be computed for each response (column), by default. Setting <code>onedpar=TRUE</code> will pool them so that there is only one dispersion parameter to be estimated.
<code>parallel</code>	A logical or formula. Used only if the response is a matrix.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ , where $M$ is the number of columns of the matrix response.

**Details**

$M$  defined above is the number of linear/additive predictors.

If the dispersion parameter is unknown, then the resulting estimate is not fully a maximum likelihood estimate.

A dispersion parameter that is less/greater than unity corresponds to under-/over-dispersion relative to the Poisson model. Over-dispersion is more common in practice.

When fitting a Quadratic RR-VGLM, the response is a matrix of  $M$ , say, columns (e.g., one column per species). Then there will be  $M$  dispersion parameters (one per column of the response matrix).

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [vgam](#), [rrvglm](#), [cqo](#), and [cao](#).

**Warning**

See the warning in [quasibinomialff](#).

**Note**

This function will handle a matrix response automatically.

The call `poissonff(dispersion = 0, ...)` is equivalent to `quasipoissonff(...)`. The latter was written so that R users of `quasipoisson()` would only need to add a "ff" to the end of the family function name.

Regardless of whether the dispersion parameter is to be estimated or not, its value can be seen from the output from the `summary()` of the object.

**Author(s)**

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

## See Also

[poissonff](#), [negbinomial](#), [loge](#), [rrvglm](#), [cqo](#), [cao](#), [binomialff](#), [quasibinomialff](#), [quasipoisson](#).

## Examples

```
quasipoissonff()

## Not run: n <- 200; p <- 5; S <- 5
mydata <- rcqo(n, p, S, fam = "poisson", eq.tol = FALSE)
myform <- attr(mydata, "formula")
p1 <- cqo(myform, fam = quasipoissonff, EqualTol = FALSE, data = mydata)
sort(p1@misc$deviance.Bestof) # A history of all the iterations
lvplot(p1, y = TRUE, lcol = 1:S, pch = 1:S, pcol = 1:S)
summary(p1) # The dispersion parameters are estimated

## End(Not run)
```

---

Qvar

---

*Quasi-variances Preprocessing Function*


---

## Description

Takes a [vglm](#) fit or a variance-covariance matrix, and preprocesses it for [rcim](#) and [uinormal](#) so that quasi-variances can be computed.

## Usage

```
Qvar(object, factorname = NULL, which.linpred = 1,
      coef.indices = NULL, labels = NULL,
      dispersion = NULL, reference.name = "(reference)", estimates = NULL)
```

## Arguments

- |               |  |
|---------------|--|
| object        | A " <a href="#">vglm</a> " object or a variance-covariance matrix, e.g., <code>vcov(vglm.object)</code> . The former is preferred since it contains all the information needed. If a matrix then <code>factorname</code> and/or <code>coef.indices</code> should be specified to identify the factor.  |
| which.linpred | A single integer from the set 1:M. Specifies which linear predictor to use. Let the value of <code>which.linpred</code> be called <i>j</i> . Then the factor should appear in that linear predictor, hence the <i>j</i> th row of the constraint matrix corresponding to the factor should have at least one nonzero value. Currently the <i>j</i> th row must have exactly one nonzero value because programming it for more than one nonzero value is difficult. |

<code>factorname</code>	Character. If the <code>vglm</code> object contains more than one factor as explanatory variable then this argument should be the name of the factor of interest. If object is a variance-covariance matrix then this argument should also be specified.
<code>labels</code>	Character. Optional, for labelling the variance-covariance matrix.
<code>dispersion</code>	Numeric. Optional, passed into <code>vcov()</code> with the same argument name.
<code>reference.name</code>	Character. Label for for the reference level.
<code>coef.indices</code>	Optional numeric vector of length at least 3 specifying the indices of the factor from the variance-covariance matrix.
<code>estimates</code>	an optional vector of estimated coefficients (redundant if object is a model).

### Details

Suppose a factor with  $L$  levels is an explanatory variable in a regression model. By default, R treats the first level as baseline so that its coefficient is set to zero. It estimates the other  $L - 1$  coefficients, and with its associated standard errors, this is the conventional output. From the complete variance-covariance matrix one can compute  $L$  quasi-variances based on all pairwise difference of the coefficients. They are based on an approximation, and can be treated as uncorrelated. In minimizing the relative (not absolute) errors it is not hard to see that the estimation involves a RCIM (`rcim`) with an exponential link function (`explink`).

If object is a model, then at least one of `factorname` or `coef.indices` must be non-NULL. The value of `coef.indices`, if non-NULL, determines which rows and columns of the model's variance-covariance matrix to use. If `coef.indices` contains a zero, an extra row and column are included at the indicated position, to represent the zero variances and covariances associated with a reference level. If `coef.indices` is NULL, then `factorname` should be the name of a factor effect in the model, and is used in order to extract the necessary variance-covariance estimates.

Quasi-variances were first implemented in R with **qvcalc**. This implementation draws heavily from that.

### Value

A  $L$  by  $L$  matrix whose  $i$ - $j$  element is the logarithm of the variance of the  $i$ th coefficient minus the  $j$ th coefficient, for all values of  $i$  and  $j$ . The diagonal elements are arbitrary and are set to zero.

The matrix has an attribute that corresponds to the prior weight matrix; it is accessed by `uninormal` and replaces the usual `weights` argument. of `vglm`. This weight matrix has ones on the off-diagonals and some small positive number on the diagonals.

### Warning

Negative quasi-variances may occur (one of them and only one), though they are rare in practice. If so then numerical problems may occur. See `qvcalc()` for more information.

### Note

This is an adaptation of `qvcalc()` in **qvcalc**. It should work for all `vglm` models with one linear predictor, i.e.,  $M = 1$ . For  $M > 1$  the factor should appear only in one of the linear predictors.

It is important to set `maxit` to be larger than usual for `rcim` since convergence is slow. Upon successful convergence the  $i$ th row effect and the  $i$ th column effect should be equal. A simple

computation involving the fitted and predicted values allows the quasi-variances to be extracted (see example below).

A function to plot *comparison intervals* has not been written here.

### Author(s)

T. W. Yee, based heavily on `qvcalc()` in **qvcalc** written by David Firth.

### References

Firth, D. (2003) Overcoming the reference category problem in the presentation of statistical models. *Sociological Methodology* **33**, 1–18.

Firth, D. and de Menezes, R. X. (2004) Quasi-variances. *Biometrika* **91**, 65–80.

### See Also

[rcim](#), [vglm](#), [qvar](#), [uninormal](#), [explink](#), `qvcalc()` in **qvcalc**, [ships](#).

### Examples

```
# Example 1
data("ships", package = "MASS")

Shipmodel <- vglm(incidents ~ type + year + period,
                 quasipoissonff, offset = log(service),
#                 trace = TRUE, model = TRUE,
                 data = ships, subset = (service > 0))

# Easiest form of input
fit1 <- rcim(Qvar(Shipmodel, "type"), uninormal("explink"), maxit = 99)
qvar(fit1)           # Easy method to get the quasi-variances
qvar(fit1, se = TRUE) # Easy method to get the quasi-standard errors

(quasiVar <- exp(diag(fitted(fit1))) / 2)           # Version 1
(quasiVar <- diag(predict(fit1)[, c(TRUE, FALSE)]) / 2) # Version 2
(quasiSE <- sqrt(quasiVar))

# Another form of input
fit2 <- rcim(Qvar(Shipmodel, coef.ind = c(0,2:5), reference.name = "typeA"),
            uninormal("explink"), maxit = 99)
## Not run: plotqvar(fit2, col = "green", lwd = 3, scol = "blue", slwd = 2, las = 1)

# The variance-covariance matrix is another form of input (not recommended)
fit3 <- rcim(Qvar(cbind(0, rbind(0, vcov(Shipmodel)[2:5, 2:5])),
                labels = c("typeA", "typeB", "typeC", "typeD", "typeE"),
                estimates = c(typeA = 0, coef(Shipmodel)[2:5])),
            uninormal("explink"), maxit = 99)
(QuasiVar <- exp(diag(fitted(fit3))) / 2)           # Version 1
(QuasiVar <- diag(predict(fit3)[, c(TRUE, FALSE)]) / 2) # Version 2
(QuasiSE <- sqrt(QuasiVar))
## Not run: plotqvar(fit3)
```

```

# Example 2: a model with M > 1 linear predictors
## Not run: require(VGAMdata)
xs.nz.f <- subset(xs.nz, sex == "F")
xs.nz.f <- subset(xs.nz.f, !is.na(babies) & !is.na(age) & !is.na(ethnic))
xs.nz.f$babies <- as.numeric(as.character(xs.nz.f$babies))
xs.nz.f <- subset(xs.nz.f, babies >= 0)
xs.nz.f <- subset(xs.nz.f, as.numeric(as.character(ethnic)) <= 2)

clist <- list("bs(age, df = 4)" = rbind(1, 0),
             "bs(age, df = 3)" = rbind(0, 1),
             "ethnic" = diag(2),
             "(Intercept)" = diag(2))
fit1 <- vglm(babies ~ bs(age, df = 4) + bs(age, df = 3) + ethnic,
            zipoissonff(zero = NULL), xs.nz.f,
            constraints = clist, trace = TRUE)
Fit1 <- rcim(Qvar(fit1, "ethnic", which.linpred = 1),
            uninormal("explink", imethod = 1), maxit = 99, trace = TRUE)
Fit2 <- rcim(Qvar(fit1, "ethnic", which.linpred = 2),
            uninormal("explink", imethod = 1), maxit = 99, trace = TRUE)

## End(Not run)
## Not run: par(mfrow = c(1, 2))
plotqvar(Fit1, scol = "blue", pch = 16,
         main = expression(eta[1]),
         slwd = 1.5, las = 1, length.arrows = 0.07)
plotqvar(Fit2, scol = "blue", pch = 16,
         main = expression(eta[2]),
         slwd = 1.5, las = 1, length.arrows = 0.07)

## End(Not run)

```

---

qvar

---

*Quasi-variances Extraction Function*


---

## Description

Takes a [rcim](#) fit of the appropriate format and returns either the quasi-variances or quasi-standard errors.

## Usage

```
qvar(object, se = FALSE, ...)
```

## Arguments

object	A <a href="#">rcim</a> object that has family function <a href="#">uninormal</a> with the <a href="#">explink</a> link. See below for an example.
--------	---

<code>se</code>	Logical. If TRUE then the quasi-variances are returned, else the square root of them, called quasi-standard errors.
<code>...</code>	Currently unused.

### Details

This simple function is ad hoc and simply is equivalent to computing the quasi-variances by `diag(predict(fit1)[, c(TRUE`  
This function is for convenience only. Serious users of quasi-variances ought to understand why  
and how this function works.

### Value

A vector of quasi-variances or quasi-standard errors.

### Author(s)

T. W. Yee.

### See Also

[rcim](#), [uninormal](#), [explink](#), [Qvar](#), [ships](#).

### Examples

```
data("ships", package = "MASS")
Shipmodel <- vglm(incidents ~ type + year + period,
                 quasipoissonff, offset = log(service),
                 data = ships, subset = (service > 0))

# Easiest form of input
fit1 <- rcim(Qvar(Shipmodel, "type"), uninormal("explink"), maxit = 99)
qvar(fit1)           # Quasi-variances
qvar(fit1, se = FALSE) # Quasi-standard errors

# Manually compute them:
(quasiVar <- exp(diag(fitted(fit1))) / 2)           # Version 1
(quasiVar <- diag(predict(fit1)[, c(TRUE, FALSE)]) / 2) # Version 2
(quasiSE <- sqrt(quasiVar))

## Not run: plotqvar(fit1, col = "green", lwd = 3, scol = "blue", slwd = 2, las = 1)
```

### Description

Density, distribution function, quantile function and random generation for the Rayleigh distribution with parameter  $a$ .

**Usage**

```
drayleigh(x, scale = 1, log = FALSE)
prayleigh(q, scale = 1)
qrayleigh(p, scale = 1)
rrayleigh(n, scale = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Fed into <a href="#">runif</a> .
scale	the scale parameter $b$ .
log	Logical. If log = TRUE then the logarithm of the density is returned.

**Details**

See [rayleigh](#), the **VGAM** family function for estimating the scale parameter  $b$  by maximum likelihood estimation, for the formula of the probability density function and range restrictions on the parameter  $b$ .

**Value**

`drayleigh` gives the density, `prayleigh` gives the distribution function, `qrayleigh` gives the quantile function, and `rrayleigh` generates random deviates.

**Note**

The Rayleigh distribution is related to the Maxwell distribution.

**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[rayleigh](#), [maxwell](#).



## Examples

```
## Not run:  Scale <- 2; x <- seq(-1, 8, by = 0.1)
plot(x, drayleigh(x, scale = Scale), type = "l", ylim = c(0,1),
     las = 1, ylab = "",
     main = "Rayleigh density divided into 10 equal areas; orange = cdf")
abline(h = 0, col = "blue", lty = 2)
qq <- grayleigh(seq(0.1, 0.9, by = 0.1), scale = Scale)
lines(qq, drayleigh(qq, scale = Scale), col = "purple", lty = 3, type = "h")
lines(x, prayleigh(x, scale = Scale), col = "orange")
## End(Not run)
```

rayleigh

*Rayleigh Distribution Family Function*

## Description

Estimating the parameter of the Rayleigh distribution by maximum likelihood estimation. Right-censoring is allowed.

## Usage

```
rayleigh(lscale = "log", nrfs = 1/3 + 0.01,
         oim.mean = TRUE, zero = NULL)
cenrayleigh(lscale = "log", oim = TRUE)
```

## Arguments

lscale	Parameter link function applied to the scale parameter $b$ . See <a href="#">Links</a> for more choices. A log link is the default because $b$ is positive.
nrfs	Numeric, of length one, with value in $[0, 1]$ . Weighting factor between Newton-Raphson and Fisher scoring. The value 0 means pure Newton-Raphson, while 1 means pure Fisher scoring. The default value uses a mixture of the two algorithms, and retaining positive-definite working weights.
oim.mean	Logical, used only for intercept-only models. TRUE means the mean of the OIM elements are used as working weights. If TRUE then this argument has top priority for working out the working weights. FALSE means use another algorithm.
oim	Logical. For censored data only, TRUE means the Newton-Raphson algorithm, and FALSE means Fisher scoring.
zero	Details at <a href="#">CommonVGAMffArguments</a> .

## Details

The Rayleigh distribution, which is used in physics, has a probability density function that can be written

$$f(y) = y \exp(-0.5(y/b)^2)/b^2$$

for  $y > 0$  and  $b > 0$ . The mean of  $Y$  is  $b\sqrt{\pi/2}$  and its variance is  $b^2(4 - \pi)/2$ .

The **VGAM** family function `cenrayleigh` handles right-censored data (the true value is greater than the observed value). To indicate which type of censoring, input `extra = list(rightcensored = vec2)` where `vec2` is a logical vector the same length as the response. If the component of this list is missing then the logical values are taken to be `FALSE`. The fitted object has this component stored in the `extra` slot.

The **VGAM** family function `rayleigh` handles multiple responses.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

### Warning

The theory behind the argument `oim` is not fully complete.

### Note

The [poisson.points](#) family function is more general so that if `ostatistic = 1` and `dimension = 2` then it coincides with [rayleigh](#). Another related distribution is the Maxwell distribution.

### Author(s)

T. W. Yee

### References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

### See Also

[Rayleigh](#), [genrayleigh](#), [riceff](#), [maxwell](#), [poisson.points](#).

### Examples

```
nn <- 1000; Scale <- exp(2)
rdata <- data.frame(ystar = rrayleigh(nn, scale = Scale))
fit <- vglm(ystar ~ 1, rayleigh, rdata, trace = TRUE, crit = "c")
head(fitted(fit))
with(rdata, mean(ystar))
coef(fit, matrix = TRUE)
Coef(fit)

# Censored data
rdata <- transform(rdata, U = runif(nn, 5, 15))
rdata <- transform(rdata, y = pmin(U, ystar))
## Not run: par(mfrow = c(1,2)); hist(with(rdata, ystar)); hist(with(rdata, y))
extra <- with(rdata, list(rightcensored = ystar > U))
fit <- vglm(y ~ 1, cenrayleigh, rdata, trace = TRUE, extra = extra)
table(fit@extra$rightcen)
```

```
coef(fit, matrix = TRUE)
head(fitted(fit))
```

---

Rcim

---

*Mark the Baseline of Row and Column on a Matrix data*


---

## Description

Rearrange the rows and columns of the input so that the first row and first column are baseline. This function is for rank-zero row-column interaction models (RCIMs; i.e., general main effects models).

## Usage

```
Rcim(mat, rbaseline = 1, cbaseline = 1)
```

## Arguments

mat	Matrix, of dimension $r$ by $c$ . It is best that it is labelled with row and column names.
rbaseline, cbaseline	Numeric (row number of the matrix <code>mat</code> ) or character (matching a row name of <code>mat</code> ) that the user wants as the row baseline or reference level. Similarly cbaseline for the column.

## Details

This is a data preprocessing function for [rcim](#). For rank-zero row-column interaction models this function establishes the baseline (or reference) levels of the matrix response with respect to the row and columns—these become the new first row and column.

## Value

Matrix of the same dimension as the input, with `rbaseline` and `cbaseline` specifying the first rows and columns. The default is no change in `mat`.

## Note

This function is similar to [moffset](#); see [moffset](#) for information about the differences. If numeric, the arguments `rbaseline` and `cbaseline` differ from arguments `roffset` and `coffset` in [moffset](#) by 1 (when elements of the matrix agree).

## Author(s)

Alfian F. Hadi and T. W. Yee.

See Also

[moffset](#), [rcim](#), [plotrcim0](#).

Examples

```
(alcoff.e <- moffset(alcoff, roffset = "6", postfix = "*"))
(aa <- Rcim(alcoff, rbaseline = "11", cbaseline = "Sun"))
(bb <- moffset(alcoff, "11", "Sun", postfix = "*"))
aa - bb # Note the difference!
```

---

rcqo	<i>Constrained Quadratic Ordination</i>
------	---

---

Description

Random generation for constrained quadratic ordination (CQO).

Usage

```
rcqo(n, p, S, Rank = 1,
     family = c("poisson", "negbinomial", "binomial-poisson",
                 "Binomial-negbinomial", "ordinal-poisson",
                 "Ordinal-negbinomial", "gamma2"),
     eq.maxima = FALSE, eq.tolerances = TRUE, es.optima = FALSE,
     lo.abundance = if (eq.maxima) hi.abundance else 10,
     hi.abundance = 100, sd.latvar = head(1.5/2^(0:3), Rank),
     sd.optima = ifelse(es.optima, 1.5/Rank, 1) *
                 ifelse(scale.latvar, sd.latvar, 1),
     sd.tolerances = 0.25, Kvector = 1, Shape = 1,
     sqrt.arg = FALSE, Log = FALSE, rhox = 0.5, breaks = 4,
     seed = NULL, optima1.arg = NULL, Crowlpositive = TRUE,
     xmat = NULL, scale.latvar = TRUE)
```

Arguments

n	Number of sites. It is denoted by $n$ below.
p	Number of environmental variables, including an intercept term. It is denoted by $p$ below. Must be no less than $1 + R$ in value.
S	Number of species. It is denoted by $S$ below.
Rank	The rank or the number of latent variables or true dimension of the data on the reduced space. This must be either 1, 2, 3 or 4. It is denoted by $R$ .
family	What type of species data is to be returned. The first choice is the default. If binomial then a 0 means absence and 1 means presence. If ordinal then the breaks argument is passed into the breaks argument of <a href="#">cut</a> . Note that either the Poisson or negative binomial distributions are used to generate binomial and ordinal data, and that an upper-case choice is used for the negative binomial

	distribution (this makes it easier for the user). If "gamma2" then this is the 2-parameter gamma distribution.
eq.maxima	Logical. Does each species have the same maxima? See arguments lo.abundance and hi.abundance.
eq.tolerances	Logical. Does each species have the same tolerance? If TRUE then the common value is 1 along every latent variable, i.e., all species' tolerance matrices are the order- $R$ identity matrix.
es.optima	Logical. Do the species have equally spaced optima? If TRUE then the quantity $S^{1/R}$ must be an integer with value 2 or more. That is, there has to be an appropriate number of species in total. This is so that a grid of optimum values is possible in $R$ -dimensional latent variable space in order to place the species' optima. Also see the argument sd.tolerances.
lo.abundance, hi.abundance	Numeric. These are recycled to a vector of length $S$ . The species have a maximum between lo.abundance and hi.abundance. That is, at their optimal environment, the mean abundance of each species is between the two component-wise values. If eq.maxima is TRUE then lo.abundance and hi.abundance must have the same values. If eq.maxima is FALSE then the logarithm of the maxima are uniformly distributed between $\log(\text{lo.abundance})$ and $\log(\text{hi.abundance})$ .
sd.latvar	Numeric, of length $R$ (recycled if necessary). Site scores along each latent variable have these standard deviation values. This must be a decreasing sequence of values because the first ordination axis contains the greatest spread of the species' site scores, followed by the second axis, followed by the third axis, etc.
sd.optima	Numeric, of length $R$ (recycled if necessary). If es.optima = FALSE then, for the $r$ th latent variable axis, the optima of the species are generated from a normal distribution centered about 0. If es.optima = TRUE then the $S$ optima are equally spaced about 0 along every latent variable axis. Regardless of the value of es.optima, the optima are then scaled to give standard deviation sd.optima[r].
sd.tolerances	Logical. If eq.tolerances = FALSE then, for the $r$ th latent variable, the species' tolerances are chosen from a normal distribution with mean 1 and standard deviation sd.tolerances[r]. However, the first species y1 has its tolerance matrix set equal to the order- $R$ identity matrix. All tolerance matrices for all species are diagonal in this function. This argument is ignored if eq.tolerances is TRUE, otherwise it is recycled to length $R$ if necessary.
Kvector	A vector of positive $k$ values (recycled to length $S$ if necessary) for the negative binomial distribution (see <a href="#">negbinomial</a> for details). Note that a natural default value does not exist, however the default value here is probably a realistic one, and that for large values of $\mu$ one has $\text{Var}(Y) = \mu^2/k$ approximately.
Shape	A vector of positive $\lambda$ values (recycled to length $S$ if necessary) for the 2-parameter gamma distribution (see <a href="#">gamma2</a> for details). Note that a natural default value does not exist, however the default value here is probably a realistic one, and that $\text{Var}(Y) = \mu^2/\lambda$ .
sqrt.arg	Logical. Take the square-root of the negative binomial counts? Assigning sqrt.arg = TRUE when family="negbinomial" means that the resulting

	species data can be considered very crudely to be approximately Poisson distributed. They will not integers in general but much easier (less numerical problems) to estimate using something like <code>cqo(..., family="poissonff")</code> .
Log	Logical. Take the logarithm of the gamma random variates? Assigning <code>Log = TRUE</code> when <code>family="gamma2"</code> means that the resulting species data can be considered very crudely to be approximately Gaussian distributed about its (quadratic) mean. The result is that it is much easier (less numerical problems) to estimate using something like <code>cqo(..., family="gaussianff")</code> .
rhox	Numeric, less than 1 in absolute value. The correlation between the environmental variables. The correlation matrix is a matrix of 1's along the diagonal and <code>rhox</code> in the off-diagonals. Note that each environmental variable is normally distributed with mean 0. The standard deviation of each environmental variable is chosen so that the site scores have the determined standard deviation, as given by argument <code>sd.latvar</code> .
breaks	If <code>family</code> is assigned an ordinal value then this argument is used to define the cutpoints. It is fed into the <code>breaks</code> argument of <code>cut</code> .
seed	If given, it is passed into <code>set.seed</code> . This argument can be used to obtain reproducible results. If set, the value is saved as the "seed" attribute of the returned value. The default will not change the random generator state, and return <code>.Random.seed</code> as "seed" attribute.
optimal.arg	If assigned and <code>Rank = 1</code> then these are the explicit optima. Recycled to length <code>S</code> .
Crow1positive	See <code>qrrvglm.control</code> for details.
xmat	The $n$ by $p - 1$ environmental matrix can be inputted.
scale.latvar	Logical. If <code>FALSE</code> the argument <code>sd.latvar</code> is ignored and no scaling of the latent variable values is performed.

## Details

This function generates data coming from a constrained quadratic ordination (CQO) model. In particular, data coming from a *species packing model* can be generated with this function. The species packing model states that species have equal tolerances, equal maxima, and optima which are uniformly distributed over the latent variable space. This can be achieved by assigning the arguments `es.optima = TRUE`, `eq.maxima = TRUE`, `eq.tolerances = TRUE`.

At present, the Poisson and negative binomial abundances are generated first using `lo.abundance` and `hi.abundance`, and if `family` is binomial or ordinal then it is converted into these forms.

In CQO theory the  $n$  by  $p$  matrix  $X$  is partitioned into two parts  $X_1$  and  $X_2$ . The matrix  $X_2$  contains the 'real' environmental variables whereas the variables in  $X_1$  are just for adjustment purposes; they contain the intercept terms and other variables that one wants to adjust for when (primarily) looking at the variables in  $X_2$ . This function has  $X_1$  only being a matrix of ones, i.e., containing an intercept only.

## Value

A  $n$  by  $p - 1 + S$  data frame with components and attributes. In the following the attributes are labelled with double quotes.

- $x_2, x_3, x_4, \dots, x_p$   
The environmental variables. This makes up the  $n$  by  $p - 1$   $X_2$  matrix. Note that  $x_1$  is not present; it is effectively a vector of ones since it corresponds to an intercept term when `cqo` is applied to the data.
- $y_1, y_2, x_3, \dots, y_S$   
The species data. This makes up the  $n$  by  $S$  matrix  $Y$ . This will be of the form described by the argument family.
- "concoefficients"  
The  $p - 1$  by  $R$  matrix of constrained coefficients (or canonical coefficients). These are also known as weights or loadings.
- "formula"  
The formula involving the species and environmental variable names. This can be used directly in the formula argument of `cqo`.
- "logmaxima"  
The  $S$ -vector of species' maxima, on a log scale. These are uniformly distributed between  $\log(\text{lo.abundance})$  and  $\log(\text{hi.abundance})$ .
- "latvar"  
The  $n$  by  $R$  matrix of site scores. Each successive column (latent variable) has sample standard deviation equal to successive values of `sd.latvar`.
- "eta"  
The linear/additive predictor value.
- "optima"  
The  $S$  by  $R$  matrix of species' optima.
- "tolerances"  
The  $S$  by  $R$  matrix of species' tolerances. These are the square root of the diagonal elements of the tolerance matrices (recall that all tolerance matrices are restricted to being diagonal in this function).
- Other attributes are "break", "family", "Rank", "lo.abundance", "hi.abundance", "eq.tolerances", "eq.maxima", "seed" as used.

### Note

This function is under development and is not finished yet. There may be a few bugs.

Yet to do: add an argument that allows absences to be equal to the first level if ordinal data is requested.

### Author(s)

T. W. Yee

### References

- Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.
- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.
- ter Braak, C. J. F. and Prentice, I. C. (1988) A theory of gradient analysis. *Advances in Ecological Research*, **18**, 271–317.

### See Also

`cqo`, `qrrvglm.control`, `cut`, `binomialff`, `poissonff`, `negbinomial`, `gamma2`, `gaussianff`.

## Examples

```
## Not run:
# Example 1: Species packing model:
n <- 100; p <- 5; S <- 5
mydata <- rcqo(n, p, S, es.opt = TRUE, eq.max = TRUE)
names(mydata)
(myform <- attr(mydata, "formula"))
fit <- cqo(myform, poissonff, mydata, Bestof = 3) # eq.tol = TRUE
matplot(attr(mydata, "latvar"), mydata[,-(1:(p-1))], col = 1:S)
persp(fit, col = 1:S, add = TRUE)
lvplot(fit, lcol = 1:S, y = TRUE, pcol = 1:S) # The same plot as above

# Compare the fitted model with the 'truth'
concoef(fit) # The fitted model
attr(mydata, "concoefficients") # The 'truth'

c(apply(attr(mydata, "latvar"), 2, sd),
  apply(latvar(fit), 2, sd)) # Both values should be approx equal

# Example 2: negative binomial data fitted using a Poisson model:
n <- 200; p <- 5; S <- 5
mydata <- rcqo(n, p, S, fam = "negbin", sqrt = TRUE)
myform <- attr(mydata, "formula")
fit <- cqo(myform, fam = poissonff, dat = mydata) # ITol = TRUE,
lvplot(fit, lcol = 1:S, y = TRUE, pcol = 1:S)
# Compare the fitted model with the 'truth'
concoef(fit) # The fitted model
attr(mydata, "concoefficients") # The 'truth'

# Example 3: gamma2 data fitted using a Gaussian model:
n <- 200; p <- 5; S <- 3
mydata <- rcqo(n, p, S, fam = "gamma2", Log = TRUE)
fit <- cqo(attr(mydata, "formula"),
  fam = gaussianff, data = mydata) # ITol = TRUE,
matplot(attr(mydata, "latvar"),
  exp(mydata[, -(1:(p-1))]), col = 1:S) # 'raw' data
# Fitted model to transformed data:
lvplot(fit, lcol = 1:S, y = TRUE, pcol = 1:S)
# Compare the fitted model with the 'truth'
concoef(fit) # The fitted model
attr(mydata, "concoefficients") # The 'truth'

## End(Not run)
```



**Description**

Generates Dirichlet random variates.

**Usage**

```
rdiric(n, shape, dimension = NULL)
```

**Arguments**

n	number of observations.
shape	the shape parameters. These must be positive. If dimension is specified, values are recycled if necessary to length dimension.
dimension	the dimension of the distribution. If dimension is not numeric then it is taken to be length(shape).

**Details**

This function is based on a relationship between the gamma and Dirichlet distribution. Random gamma variates are generated, and then Dirichlet random variates are formed from these.

**Value**

A n by dimension matrix of Dirichlet random variates. Each element is positive, and each row will sum to unity.

**Author(s)**

Thomas W. Yee

**References**

Lange, K. (2002) *Mathematical and Statistical Methods for Genetic Analysis*, 2nd ed. New York: Springer-Verlag.

**See Also**

[dirichlet](#) is a **VGAM** family function for fitting a Dirichlet distribution to data.

**Examples**

```
y <- rdiric(n = 1000, shape = c(3, 1, 4))
fit <- vglm(y ~ 1, dirichlet, trace = TRUE, crit = "c")
Coef(fit)
coef(fit, matrix = TRUE)
```

---

recexp1*Upper Record Values from a 1-parameter Exponential Distribution*

---

**Description**

Maximum likelihood estimation of the rate parameter of a 1-parameter exponential distribution when the observations are upper record values.

**Usage**

```
recexp1(lrate = "loge", irate = NULL, imethod = 1)
```

**Arguments**

lrate	Link function applied to the rate parameter. See <a href="#">Links</a> for more choices.
irate	Numeric. Optional initial values for the rate. The default value NULL means they are computed internally, with the help of imethod.
imethod	Integer, either 1 or 2 or 3. Initial method, three algorithms are implemented. Choose the another value if convergence fails, or use irate.

**Details**

The response must be a vector or one-column matrix with strictly increasing values.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

By default, this family function has the intercept-only MLE as the initial value, therefore convergence may only take one iteration. Fisher scoring is used.

**Author(s)**

T. W. Yee

**References**

Arnold, B. C. and Balakrishnan, N. and Nagaraja, H. N. (1998) *Records*, New York: John Wiley & Sons.

**See Also**

[exponential](#).

**Examples**

```

rawy <- rexp(n <- 10000, rate = exp(1))
y <- unique(cummax(rawy)) # Keep only the records

length(y) / y[length(y)] # MLE of rate

fit <- vglm(y ~ 1, recexp1, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)

```

---

reciprocal	<i>Reciprocal link function</i>
------------	---------------------------------

---

**Description**

Computes the reciprocal transformation, including its inverse and the first two derivatives.

**Usage**

```

reciprocal(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
           short = TRUE, tag = FALSE)
negreciprocal(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
              short = TRUE, tag = FALSE)

```

**Arguments**

theta	Numeric or character. See below for further details.
bvalue	See <a href="#">Links</a> .
inverse, deriv, short, tag	Details at <a href="#">Links</a> .

**Details**

The reciprocal link function is a special case of the power link function. Numerical values of theta close to 0 result in Inf, -Inf, NA or NaN.

The negreciprocal link function computes the negative reciprocal, i.e.,  $-1/\theta$ .

**Value**

For reciprocal: for deriv = 0, the reciprocal of theta, i.e.,  $1/\theta$  when inverse = FALSE, and if inverse = TRUE then  $1/\theta$ . For deriv = 1, then the function returns  $d\theta/d\eta$  as a function of theta if inverse = FALSE, else if inverse = TRUE then it returns the reciprocal.

**Note**

Numerical instability may occur when theta is close to 0.

Author(s)

Thomas W. Yee

References

McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.

See Also

[identity](#), [powerlink](#).

Examples

```
reciprocal(1:5)
reciprocal(1:5, inverse = TRUE, deriv = 2)
negreciprocal(1:5)
negreciprocal(1:5, inverse = TRUE, deriv = 2)

x <- (-3):3
reciprocal(x) # Has Inf
reciprocal(x, bvalue = .Machine$double.eps) # Has no Inf
```

---

recnormal	<i>Upper Record Values from a Univariate Normal Distribution</i>
-----------	--

---

Description

Maximum likelihood estimation of the two parameters of a univariate normal distribution when the observations are upper record values.

Usage

```
recnormal(lmean = "identity", lsd = "loge",
          imean = NULL, isd = NULL, imethod = 1, zero = NULL)
```

Arguments

lmean, lsd	Link functions applied to the mean and sd parameters. See <a href="#">Links</a> for more choices.
imean, isd	Numeric. Optional initial values for the mean and sd. The default value NULL means they are computed internally, with the help of imethod.
imethod	Integer, either 1 or 2 or 3. Initial method, three algorithms are implemented. Choose the another value if convergence fails, or use imean and/or isd.
zero	An integer vector, containing the value 1 or 2. If so, the mean or standard deviation respectively are modelled as an intercept only. Usually, setting zero = 2 will be used, if used at all. The default value NULL means both linear/additive predictors are modelled as functions of the explanatory variables.

## Details

The response must be a vector or one-column matrix with strictly increasing values.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

This family function tries to solve a difficult problem, and the larger the data set the better. Convergence failure can commonly occur, and convergence may be very slow, so set `maxit = 200`, `trace = TRUE`, say. Inputting good initial values are advised.

This family function uses the BFGS quasi-Newton update formula for the working weight matrices. Consequently the estimated variance-covariance matrix may be inaccurate or simply wrong! The standard errors must be therefore treated with caution; these are computed in functions such as `vcov()` and `summary()`.

## Author(s)

T. W. Yee

## References

Arnold, B. C. and Balakrishnan, N. and Nagaraja, H. N. (1998) *Records*, New York: John Wiley & Sons.

## See Also

[uninormal](#), [double.cennormal](#).

## Examples

```
nn <- 10000; mymean <- 100
# First value is reference value or trivial record
Rdata <- data.frame(rawy = c(mymean, rnorm(nn, me = mymean, sd = exp(3))))
# Keep only observations that are records:
rdata <- data.frame(y = unique(cummax(with(Rdata, rawy))))

fit <- vglm(y ~ 1, recnormal, rdata, trace = TRUE, maxit = 200)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

rhobit

*Rhobit Link Function***Description**

Computes the rhobit link transformation, including its inverse and the first two derivatives.

**Usage**

```
rhobit(theta, bminvalue = NULL, bmaxvalue = NULL,
        inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

**Arguments**

**theta**                      Numeric or character. See below for further details.

**bminvalue, bmaxvalue**                      Optional boundary values, e.g., values of theta which are less than or equal to -1 can be replaced by bminvalue before computing the link function value. And values of theta which are greater than or equal to 1 can be replaced by bmaxvalue before computing the link function value. See [Links](#).

**inverse, deriv, short, tag**                      Details at [Links](#).

**Details**

The rhobit link function is commonly used for parameters that lie between  $-1$  and  $1$ . Numerical values of theta close to  $-1$  or  $1$  or out of range result in Inf, -Inf, NA or NaN.

**Value**

For  $\text{deriv} = 0$ , the rhobit of theta, i.e.,  $\log((1 + \text{theta})/(1 - \text{theta}))$  when  $\text{inverse} = \text{FALSE}$ , and if  $\text{inverse} = \text{TRUE}$  then  $(\exp(\text{theta}) - 1)/(\exp(\text{theta}) + 1)$ .

For  $\text{deriv} = 1$ , then the function returns  $d \text{ theta} / d \text{ eta}$  as a function of theta if  $\text{inverse} = \text{FALSE}$ , else if  $\text{inverse} = \text{TRUE}$  then it returns the reciprocal.

**Note**

Numerical instability may occur when theta is close to  $-1$  or  $1$ . One way of overcoming this is to use bminvalue, etc.

The correlation parameter of a standard bivariate normal distribution lies between  $-1$  and  $1$ , therefore this function can be used for modelling this parameter as a function of explanatory variables.

The link function rhobit is very similar to [fisherz](#), e.g., just twice the value of [fisherz](#).

**Author(s)**

Thomas W. Yee

## References

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[Links](#), [binom2.rho](#), [fisherz](#).

## Examples

```
theta <- seq(-0.99, 0.99, by = 0.01)
y <- rhobit(theta)
## Not run:
plot(theta, y, type = "l", las = 1, ylab = "", main = "rhobit(theta)")
abline(v = 0, h = 0, lty = 2)

## End(Not run)

x <- c(seq(-1.02, -0.98, by = 0.01), seq(0.97, 1.02, by = 0.01))
rhobit(x) # Has NAs
rhobit(x, bminvalue = -1 + .Machine$double.eps,
        bmaxvalue = 1 - .Machine$double.eps) # Has no NAs
```

---

Rice

*The Rice Distribution*

---

## Description

Density  
and random generation for the Rician distribution.

## Usage

```
drice(x, vee, sigma, log = FALSE)
rrice(n, vee, sigma)
```

## Arguments

<code>x</code>	vector of quantiles.
<code>n</code>	number of observations. Must be a positive integer of length 1.
<code>vee, sigma</code>	See <a href="#">riceff</a> .
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

See [riceff](#), the **VGAM** family function for estimating the two parameters, for the formula of the probability density function and other details.

Value

drice gives the density,  
rrice generates random deviates.

Author(s)

T. W. Yee

See Also

[riceff](#).

Examples

```
## Not run: x <- seq(0.01, 7, len = 201)
plot(x, drice(x, vee = 0, sigma = 1), type = "n", las = 1,, ylab = "",
      main = "Density of Rice distribution for various values of v")
sigma <- 1; vee <- c(0, 0.5, 1, 2, 4)
for (ii in 1:length(vee))
  lines(x, drice(x, vee[ii], sigma), col = ii)
legend(x = 5, y = 0.6, legend = as.character(vee),
       col = 1:length(vee), lty = 1)

## End(Not run)
```

---

riceff	<i>Rice Distribution Family Function</i>
--------	--

---

Description

Estimates the two parameters of a Rice distribution by maximum likelihood estimation.

Usage

```
riceff(lvee = "loge", lsigma = "loge",
       ivee = NULL, isigma = NULL, nsimEIM = 100, zero = NULL)
```

Arguments

- |               |  |
|---------------|--|
| lvee, lsigma  | Link functions for the $v$ and $\sigma$ parameters. See <a href="#">Links</a> for more choices and for general information.  |
| ivee, isigma  | Optional initial values for the parameters. See <a href="#">CommonVGAMffArguments</a> for more information. If convergence failure occurs (this <b>VGAM</b> family function seems to require good initial values) try using these arguments. |
| nsimEIM, zero | See <a href="#">CommonVGAMffArguments</a> for more information.  |



## Details

The Rician distribution has density function

$$f(y; v, \sigma) = \frac{y}{\sigma^2} \exp(-(y^2 + v^2)/(2\sigma^2)) I_0(yv/\sigma^2)$$

where  $y > 0$ ,  $v > 0$ ,  $\sigma > 0$  and  $I_0$  is the modified Bessel function of the first kind with order zero. When  $v = 0$  the Rice distribution reduces to a Rayleigh distribution. The mean is  $\sigma\sqrt{\pi/2} \exp(z/2)((1 - z)I_0(-z/2) - zI_1(-z/2))$  (returned as the fitted values) where  $z = -v^2/(2\sigma^2)$ . Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

Convergence problems may occur for data where  $v = 0$ ; if so, use [rayleigh](#) or possibly use an [identity](#) link.

When  $v$  is large (greater than 3, say) then the mean is approximately  $v$  and the standard deviation is approximately  $\sigma$ .

## Author(s)

T. W. Yee

## References

Rice, S. O. (1945) Mathematical Analysis of Random Noise. *Bell System Technical Journal*, **24**, 46–156.

## See Also

[drice](#), [rayleigh](#), [bessell](#).

## Examples

```
## Not run: vee <- exp(2); sigma <- exp(1)
rdata <- data.frame(y = rrice(n <- 1000, vee, sigma))
fit <- vglm(y ~ 1, riceff, data = rdata, trace = TRUE, crit = "coef")
c(with(rdata, mean(y)), fitted(fit)[1])
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

---

`rigff`*Reciprocal Inverse Gaussian distribution*

---

**Description**

Estimation of the parameters of a reciprocal inverse Gaussian distribution.

**Usage**

```
rigff(lmu = "identity", llambda = "loge", imu = NULL, ilambda = 1)
```

**Arguments**

`lmu`, `llambda`    Link functions for `mu` and `lambda`. See [Links](#) for more choices.

`imu`, `ilambda`    Initial values for `mu` and `lambda`. A NULL means a value is computed internally.

**Details**

See Jorgensen (1997) for details.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

This distribution is potentially useful for dispersion modelling.

**Author(s)**

T. W. Yee

**References**

Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall

**See Also**

[simplex](#).

**Examples**

```
rdata <- data.frame(y = rchisq(n = 100, df = 14)) # Not 'proper' data!!
fit <- vglm(y ~ 1, rigff, rdata, trace = TRUE)
fit <- vglm(y ~ 1, rigff, rdata, trace = TRUE, eps = 1e-9, crit = "coef")
summary(fit)
```

rlplot.egev

*Return Level Plot for GEV Fits***Description**

A return level plot is constructed for a GEV-type model.

**Usage**

```
rlplot.egev(object, show.plot = TRUE,
  probability = c((1:9)/100, (1:9)/10, 0.95, 0.99, 0.995, 0.999),
  add.arg = FALSE, xlab = "Return Period", ylab = "Return Level",
  main = "Return Level Plot",
  pch = par()$pch, pcol.arg = par()$col, pcex = par()$cex,
  llty.arg = par()$lty, lcol.arg = par()$col, llwd.arg = par()$lwd,
  slty.arg = par()$lty, scol.arg = par()$col, slwd.arg = par()$lwd,
  ylim = NULL, log = TRUE, CI = TRUE, epsilon = 1e-05, ...)
```

**Arguments**

object	A <b>VGAM</b> extremes model of the GEV-type, produced by <a href="#">vglm</a> with a family function either "gev" or "egev".
show.plot	Logical. Plot it? If FALSE no plot will be done.
probability	Numeric vector of probabilities used.
add.arg	Logical. Add the plot to an existing plot?
xlab	Caption for the x-axis. See <a href="#">par</a> .
ylab	Caption for the y-axis. See <a href="#">par</a> .
main	Title of the plot. See <a href="#">title</a> .
pch	Plotting character. See <a href="#">par</a> .
pcol.arg	Color of the points. See the col argument of <a href="#">par</a> .
pcex	Character expansion of the points. See the cex argument of <a href="#">par</a> .
llty.arg	Line type. Line type. See the lty argument of <a href="#">par</a> .
lcol.arg	Color of the lines. See the col argument of <a href="#">par</a> .
llwd.arg	Line width. See the lwd argument of <a href="#">par</a> .
slty.arg, scol.arg, slwd.arg	Corresponding arguments for the lines used for the confidence intervals. Used only if CI=TRUE.
ylim	Limits for the y-axis. Numeric of length 2.
log	Logical. If TRUE then log="x" otherwise log="y". This changes the labelling of the x-axis only.
CI	Logical. Add in a 95 percent confidence interval?

epsilon	Numeric, close to zero. Used for the finite-difference approximation to the first derivatives with respect to each parameter. If too small, numerical problems will occur.
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include <code>sub</code> and <code>las</code> .

### Details

A return level plot plots  $z_p$  versus  $\log(y_p)$ . It is linear if the shape parameter  $\xi = 0$ . If  $\xi < 0$  then the plot is convex with asymptotic limit as  $p$  approaches zero at  $\mu - \sigma/\xi$ . And if  $\xi > 0$  then the plot is concave and has no finite bound. Here,  $G(z_p) = 1 - p$  where  $0 < p < 1$  ( $p$  corresponds to the argument probability) and  $G$  is the cumulative distribution function of the GEV distribution. The quantity  $z_p$  is known as the *return level* associated with the *return period*  $1/p$ . For many applications, this means  $z_p$  is exceeded by the annual maximum in any particular year with probability  $p$ .

The points in the plot are the actual data.

### Value

In the `post` slot of the object is a list called `rlplot` with list components

<code>yp</code>	<code>-log(probability)</code> , which is used on the x-axis.
<code>zp</code>	values which are used for the y-axis
<code>lower, upper</code>	lower and upper confidence limits for the 95 percent confidence intervals evaluated at the values of probability (if <code>CI=TRUE</code> ).

### Note

The confidence intervals are approximate, being based on finite-difference approximations to derivatives.

### Author(s)

T. W. Yee

### References

Coles, S. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

### See Also

[egev](#).

## Examples

```
gdata <- data.frame(y = rgev(n <- 100, scale = 2, shape = -0.1))
fit <- vglm(y ~ 1, egev, gdata, trace = TRUE)

# Identity link for all parameters:
fit2 <- vglm(y ~ 1, egev(lshape = identity, lscale = identity,
                        iscale = 10), gdata, trace = TRUE)
coef(fit2, matrix = TRUE)
## Not run:
par(mfrow = c(1, 2))
rlplot(fit) -> i1
rlplot(fit2, pcol = "darkorange", lcol = "blue", log = FALSE,
       scol = "darkgreen", slty = "dashed", las = 1) -> i2
range(i2$post$rlplot$upper - i1$post$rlplot$upper) # Should be near 0
range(i2$post$rlplot$lower - i1$post$rlplot$lower) # Should be near 0

## End(Not run)
```

---

rrar

---

*Nested reduced-rank autoregressive models for multiple time series*


---

## Description

Estimates the parameters of a nested reduced-rank autoregressive model for multiple time series.

## Usage

```
rrar(Ranks = 1, coefstart = NULL)
```

## Arguments

Ranks	Vector of integers: the ranks of the model. Each value must be at least one and no more than M, where M is the number of response variables in the time series. The length of Ranks is the <i>lag</i> , which is often denoted by the symbol <i>L</i> in the literature.
coefstart	Optional numerical vector of initial values for the coefficients. By default, the family function chooses these automatically.

## Details

Full details are given in Ahn and Reinsel (1988). Convergence may be very slow, so setting `maxits = 50`, say, may help. If convergence is not obtained, you might like to try inputting different initial values.

Setting `trace = TRUE` in `vglm` is useful for monitoring the progress at each iteration.

## Value

An object of class `"vglmff"` (see `vglmff-class`). The object is used by modelling functions such as `vglm` and `vgam`.

**Note**

This family function should be used within `vglm` and not with `rrvglm` because it does not fit into the RR-VGLM framework exactly. Instead, the reduced-rank model is formulated as a VGLM!

A methods function `Coef.rrar`, say, has yet to be written. It would return the quantities  $Ak_1$ ,  $C$ ,  $D$ ,  $\hat{\omega}$ ,  $\Phi$ , etc. as slots, and then `show.Coef.rrar` would also need to be written.

**Author(s)**

T. W. Yee

**References**

Ahn, S. and Reinsel, G. C. (1988) Nested reduced-rank autoregressive models for multiple time series. *Journal of the American Statistical Association*, **83**, 849–856.

**See Also**

`vglm`, `grain.us`.

**Examples**

```
## Not run:
year <- seq(1961 + 1/12, 1972 + 10/12, by = 1/12)
par(mar = c(4, 4, 2, 2) + 0.1, mfrow = c(2, 2))
for (ii in 1:4) {
  plot(year, grain.us[, ii], main = names(grain.us)[ii], las = 1,
       type = "l", xlab = "", ylab = "", col = "blue")
  points(year, grain.us[, ii], pch = "*", col = "blue")
}
apply(grain.us, 2, mean) # mu vector
cgrain <- scale(grain.us, scale = FALSE) # Center the time series only
fit <- vglm(cgrain ~ 1, rrar(Ranks = c(4, 1)), trace = TRUE)
summary(fit)

print(fit@misc$Ak1, digits = 2)
print(fit@misc$Cmatrices, digits = 3)
print(fit@misc$Dmatrices, digits = 3)
print(fit@misc$omegahat, digits = 3)
print(fit@misc$Phimatrices, digits = 2)

par(mar = c(4, 4, 2, 2) + 0.1, mfrow = c(4, 1))
for (ii in 1:4) {
  plot(year, fit@misc$Z[, ii], main = paste("Z", ii, sep = ""),
       type = "l", xlab = "", ylab = "", las = 1, col = "blue")
  points(year, fit@misc$Z[, ii], pch = "*", col = "blue")
}

## End(Not run)
```

---

rrvglm	<i>Fitting Reduced-Rank Vector Generalized Linear Models (RR-VGLMs)</i>
--------	---

---

## Description

A *reduced-rank vector generalized linear model* (RR-VGLM) is fitted. RR-VGLMs are VGLMs but some of the constraint matrices are estimated. In this documentation,  $M$  is the number of linear predictors.

## Usage

```
rrvglm(formula, family, data = list(), weights = NULL, subset = NULL,
       na.action = na.fail, etastart = NULL, mustart = NULL,
       coefstart = NULL, control = rrvglm.control(...), offset = NULL,
       method = "rrvglm.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
       contrasts = NULL, constraints = NULL, extra = NULL,
       qr.arg = FALSE, smart = TRUE, ...)
```

## Arguments

formula, family, weights	
	See <a href="#">vglm</a> .
data	an optional data frame containing the variables in the model. By default the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>rrvglm</code> is called.
subset, na.action	See <a href="#">vglm</a> .
etastart, mustart, coefstart	See <a href="#">vglm</a> .
control	a list of parameters for controlling the fitting process. See <a href="#">rrvglm.control</a> for details.
offset, model, contrasts	See <a href="#">vglm</a> .
method	the method to be used in fitting the model. The default (and presently only) method <code>rrvglm.fit</code> uses iteratively reweighted least squares (IRLS).
x.arg, y.arg	logical values indicating whether the model matrix and response vector/matrix used in the fitting process should be assigned in the x and y slots. Note the model matrix is the LM model matrix; to get the VGLM model matrix type <code>model.matrix(vglmfit)</code> where <code>vglmfit</code> is a <code>vglm</code> object.
constraints	See <a href="#">vglm</a> .
extra, smart, qr.arg	See <a href="#">vglm</a> .
...	further arguments passed into <a href="#">rrvglm.control</a> .

## Details

The central formula is given by

$$\eta = B_1^T x_1 + A\nu$$

where  $x_1$  is a vector (usually just a 1 for an intercept),  $x_2$  is another vector of explanatory variables, and  $\nu = C^T x_2$  is an  $R$ -vector of latent variables. Here,  $\eta$  is a vector of linear predictors, e.g., the  $m$ th element is  $\eta_m = \log(E[Y_m])$  for the  $m$ th Poisson response. The matrices  $B_1$ ,  $A$  and  $C$  are estimated from the data, i.e., contain the regression coefficients. For ecologists, the central formula represents a *constrained linear ordination* (CLO) since it is linear in the latent variables. It means that the response is a monotonically increasing or decreasing function of the latent variables.

For identifiability it is common to enforce *corner constraints* on  $A$ : by default, the top  $R$  by  $R$  submatrix is fixed to be the order- $R$  identity matrix and the remainder of  $A$  is estimated.

The underlying algorithm of RR-VGLMs is iteratively reweighted least squares (IRLS) with an optimizing algorithm applied within each IRLS iteration (e.g., alternating algorithm).

In theory, any **VGAM** family function that works for `vglm` and `vgam` should work for `rrvglm` too. The function that actually does the work is `rrvglm.fit`; it is `vglm.fit` with some extra code.

## Value

An object of class "rrvglm", which has the the same slots as a "vglm" object. The only difference is that the some of the constraint matrices are estimates rather than known. But **VGAM** stores the models the same internally. The slots of "vglm" objects are described in [vglm-class](#).

## Note

The arguments of `rrvglm` are in general the same as those of `vglm` but with some extras in `rrvglm.control`.

The smart prediction ([smartpred](#)) library is packed with the **VGAM** library.

In an example below, a rank-1 *stereotype* model of Anderson (1984) is fitted to some car data. The reduced-rank regression is performed, adjusting for two covariates. Setting a trivial constraint matrix for the latent variable variables in  $x_2$  avoids a warning message when it is overwritten by a (common) estimated constraint matrix. It shows that German cars tend to be more expensive than American cars, given a car of fixed weight and width.

If `fit <- rrvglm(..., data = mydata)` then `summary(fit)` requires corner constraints and no missing values in `mydata`. Often the estimated variance-covariance matrix of the parameters is not positive-definite; if this occurs, try refitting the model with a different value for `Index.corner`.

For *constrained quadratic ordination* (CQO) see [cqo](#) for more details about QRR-VGLMs.

With multivariate binary responses, one must use `binomialff(mv = TRUE)` to indicate that the response (matrix) is multivariate. Otherwise, it is interpreted as a single binary response variable.

## Author(s)

Thomas W. Yee



## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.

Anderson, J. A. (1984) Regression and ordered categorical variables. *Journal of the Royal Statistical Society, Series B, Methodological*, **46**, 1–30.

Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

`rrvglm.control`, `lvplot.rrvglm` (same as `biplot.rrvglm`), `rrvglm-class`, `grc`, `cqo`, `vglmff-class`, `vglm`, `vglm-class`, `smartpred`, `rrvglm.fit`. Special family functions include `negbinomial`, `zipoisson` and `zinegbinomial`. (see Yee (2014) and **COZIGAM**). Methods functions include `Coef.rrvglm`, `summary.rrvglm`, etc. Data include `crashi`.

## Examples

```
## Not run:
# Example 1: RR negative binomial (RR-NB) with Var(Y) = mu + delta1 * mu^delta2
nn <- 1000      # Number of observations
delta1 <- 3.0   # Specify this
delta2 <- 1.5   # Specify this; should be greater than unity
a21 <- 2 - delta2
mydata <- data.frame(x2 = runif(nn), x3 = runif(nn))
mydata <- transform(mydata, mu = exp(2 + 3 * x2 + 0 * x3))
mydata <- transform(mydata, y2 = rnbinom(nn, mu=mu, size=(1/delta1)*mu^a21))
plot(y2 ~ x2, data = mydata, pch = "+", col = 'blue', las = 1,
     main = paste("Var(Y) = mu + ", delta1, " * mu^", delta2, sep = ""))
rrnb2 <- rrvglm(y2 ~ x2 + x3, negbinomial(zero = NULL), mydata, trace = TRUE)

a21.hat <- (Coef(rrnb2)@A)["log(size)", 1]
beta11.hat <- Coef(rrnb2)@B1["(Intercept)", "log(mu)"]
beta21.hat <- Coef(rrnb2)@B1["(Intercept)", "log(size)"]
(delta1.hat <- exp(a21.hat * beta11.hat - beta21.hat))
(delta2.hat <- 2 - a21.hat)
# exp(a21.hat * predict(rrnb2)[1,1] - predict(rrnb2)[1,2]) # delta1.hat
summary(rrnb2)

# Obtain a 95 percent confidence interval for delta2:
se.a21.hat <- sqrt(vcov(rrnb2)["I(latvar.mat)", "I(latvar.mat)"])
ci.a21 <- a21.hat + c(-1, 1) * 1.96 * se.a21.hat
(ci.delta2 <- 2 - rev(ci.a21)) # The 95 percent confidence interval

Confint(rrnb(rrnb2)) # Quick way to get it
```

```

# Plot the abundances and fitted values against the latent variable
plot(y2 ~ latvar(rrnb2), data = mydata, col = "blue",
     xlab = "Latent variable", las = 1)
ooo <- order(latvar(rrnb2))
lines(fitted(rrnb2)[ooo] ~ latvar(rrnb2)[ooo], col = "red")

# Example 2: stereotype model (reduced-rank multinomial logit model)
data(car.all)
index <- with(car.all, Country == "Germany" | Country == "USA" |
              Country == "Japan" | Country == "Korea")
scar <- car.all[index, ] # standardized car data
fcols <- c(13,14,18:20,22:26,29:31,33,34,36) # These are factors
scar[, -fcols] = scale(scar[, -fcols]) # Standardize all numerical vars
ones <- matrix(1, 3, 1)
clist <- list("(Intercept)" = diag(3), Width = ones, Weight = ones,
              Disp. = diag(3), Tank = diag(3), Price = diag(3),
              Frt.Leg.Room = diag(3))

set.seed(111)
fit <- rrvglm(Country ~ Width + Weight + Disp. + Tank + Price + Frt.Leg.Room,
              multinomial, data = scar, Rank = 2, trace = TRUE,
              constraints = clist, noRRR = ~ 1 + Width + Weight,
              Uncor = TRUE, Corner = FALSE, Bestof = 2)
fit@misc$deviance # A history of the fits
Coef(fit)
biplot(fit, chull = TRUE, scores = TRUE, clty = 2, Ccex = 2,
       ccol = "blue", scol = "orange", Ccol = "darkgreen", Clwd = 2,
       main = "1=Germany, 2=Japan, 3=Korea, 4=USA")

## End(Not run)

```

rrvglm-class

Class “rrvglm”

## Description

Reduced-rank vector generalized linear models.

## Objects from the Class

Objects can be created by calls to [rrvglm](#).

## Slots

**extra:** Object of class “list”; the extra argument on entry to `vglm`. This contains any extra information that might be needed by the family function.

**family:** Object of class “vglmff”. The family function.

**iter:** Object of class “numeric”. The number of IRLS iterations used.

**predictors:** Object of class “matrix” with  $M$  columns which holds the  $M$  linear predictors.

**assign:** Object of class "list", from class "vlm". This named list gives information matching the columns and the (LM) model matrix terms.

**call:** Object of class "call", from class "vlm". The matched call.

**coefficients:** Object of class "numeric", from class "vlm". A named vector of coefficients.

**constraints:** Object of class "list", from class "vlm". A named list of constraint matrices used in the fitting.

**contrasts:** Object of class "list", from class "vlm". The contrasts used (if any).

**control:** Object of class "list", from class "vlm". A list of parameters for controlling the fitting process. See [vglm.control](#) for details.

**criterion:** Object of class "list", from class "vlm". List of convergence criterion evaluated at the final IRLS iteration.

**df.residual:** Object of class "numeric", from class "vlm". The residual degrees of freedom.

**df.total:** Object of class "numeric", from class "vlm". The total degrees of freedom.

**dispersion:** Object of class "numeric", from class "vlm". The scaling parameter.

**effects:** Object of class "numeric", from class "vlm". The effects.

**fitted.values:** Object of class "matrix", from class "vlm". The fitted values. This is usually the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.

**misc:** Object of class "list", from class "vlm". A named list to hold miscellaneous parameters.

**model:** Object of class "data.frame", from class "vlm". The model frame.

**na.action:** Object of class "list", from class "vlm". A list holding information about missing values.

**offset:** Object of class "matrix", from class "vlm". If non-zero, a  $M$ -column matrix of offsets.

**post:** Object of class "list", from class "vlm" where post-analysis results may be put.

**preplot:** Object of class "list", from class "vlm" used by [plotvgam](#); the plotting parameters may be put here.

**prior.weights:** Object of class "matrix", from class "vlm" holding the initially supplied weights.

**qr:** Object of class "list", from class "vlm". QR decomposition at the final iteration.

**R:** Object of class "matrix", from class "vlm". The  $\mathbf{R}$  matrix in the QR decomposition used in the fitting.

**rank:** Object of class "integer", from class "vlm". Numerical rank of the fitted model.

**residuals:** Object of class "matrix", from class "vlm". The *working* residuals at the final IRLS iteration.

**res.ss:** Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.

**smart.prediction:** Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.

**terms:** Object of class "list", from class "vlm". The [terms](#) object used.

**weights:** Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.

**x:** Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).

**xlevels:** Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.

**y:** Object of class "matrix", from class "vlm". The response, in matrix form.

**Xm2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**Ym2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**callXm2:** Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vglm", directly. Class "vlm", by class "vglm".

## Methods

**biplot** signature(x = "rrvglm"): biplot.

**Coef** signature(object = "rrvglm"): more detailed coefficients giving **A**, **B**<sub>1</sub>, **C**, etc.

**biplot** signature(object = "rrvglm"): biplot.

**print** signature(x = "rrvglm"): short summary of the object.

**summary** signature(object = "rrvglm"): a more detailed summary of the object.

## Note

The slots of "rrvglm" objects are currently identical to "vglm" objects.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

<http://www.stat.auckland.ac.nz/~yee>

## See Also

[rrvglm](#), [lvplot.rrvglm](#), [vglmff-class](#).

## Examples

```
## Not run: # Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time),
                    x3 = runif(nrow(pneumo))) # x3 is unrelated
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3,
             multinomial, pneumo, Rank = 1)
Coef(fit)

## End(Not run)
```

---

rrvglm.control                      *Control function for rrvglm*


---

## Description

Algorithmic constants and parameters for running rrvglm are set using this function.

## Usage

```
rrvglm.control(Rank = 1, Algorithm = c("alternating", "derivative"),
  Corner = TRUE, Uncorrelated.latvar = FALSE,
  Wmat = NULL, Svd.arg = FALSE,
  Index.corner = if (length(str0))
    head((1:1000)[-str0], Rank) else 1:Rank,
  Ainit = NULL, Alpha = 0.5, Bestof = 1, Cinit = NULL,
  Etamat.colmax = 10,
  sd.Ainit = 0.02, sd.Cinit = 0.02, str0 = NULL,
  noRRR = ~1, Norrr = NA,
  noWarning = FALSE,
  trace = FALSE, Use.Init.Poisson.Q0 = FALSE,
  checkwz = TRUE, Check.rank = TRUE,
  wzepsilon = .Machine$double.eps^0.75, ...)
```

## Arguments

Rank	The numerical rank $R$ of the model. Must be an element from the set $\{1, 2, \dots, \min(M, p2)\}$ . Here, the vector of explanatory variables $\mathbf{x}$ is partitioned into $(\mathbf{x1}, \mathbf{x2})$ , which is of dimension $p1 + p2$ . The variables making up $\mathbf{x1}$ are given by the terms in noRRR argument, and the rest of the terms comprise $\mathbf{x2}$ .
Algorithm	Character string indicating what algorithm is to be used. The default is the first one.
Corner	Logical indicating whether corner constraints are to be used. This is one method for ensuring a unique solution. If TRUE, Index.corner specifies the $R$ rows of the constraint matrices that are use as the corner constraints, i.e., they hold an order- $R$ identity matrix.
Uncorrelated.latvar	Logical indicating whether uncorrelated latent variables are to be used. This is normalization forces the variance-covariance matrix of the latent variables to be $\text{diag}(\text{Rank})$ , i.e., unit variance and uncorrelated. This constraint does not lead to a unique solution because it can be rotated.
Wmat	Yet to be done.
Svd.arg	Logical indicating whether a singular value decomposition of the outer product is to computed. This is another normalization which ensures uniqueness. See the argument Alpha below.
Index.corner	Specifies the $R$ rows of the constraint matrices that are used for the corner constraints, i.e., they hold an order- $R$ identity matrix.

Alpha	The exponent in the singular value decomposition that is used in the first part: if the SVD is $UDV^T$ then the first and second parts are $UD^\alpha$ and $D^{1-\alpha}V^T$ respectively. A value of 0.5 is 'symmetrical'. This argument is used only when <code>Svd.arg=TRUE</code> .
Bestof	Integer. The best of Bestof models fitted is returned. This argument helps guard against local solutions by (hopefully) finding the global solution from many fits. The argument works only when the function generates its own initial value for <b>C</b> , i.e., when <b>C</b> is <i>not</i> passed in as initial values.
Ainit, Cinit	Initial <b>A</b> and <b>C</b> matrices which may speed up convergence. They must be of the correct dimension.
Etamat.colmax	Positive integer, no smaller than Rank. Controls the amount of memory used by <code>.Init.Poisson.Q0()</code> . It is the maximum number of columns allowed for the pseudo-response and its weights. In general, the larger the value, the better the initial value. Used only if <code>Use.Init.Poisson.Q0=TRUE</code> .
str0	Integer vector specifying which rows of the estimated constraint matrices ( <b>A</b> ) are to be all zeros. These are called <i>structural zeros</i> . Must not have any common value with <code>Index.corner</code> , and be a subset of the vector 1:M. The default, <code>str0 = NULL</code> , means no structural zero rows at all.
sd.Ainit, sd.Cinit	Standard deviation of the initial values for the elements of <b>A</b> and <b>C</b> . These are normally distributed with mean zero. This argument is used only if <code>Use.Init.Poisson.Q0 = FALSE</code> .
noRRR	Formula giving terms that are <i>not</i> to be included in the reduced-rank regression. That is, noRRR specifies which explanatory variables are in the $x_1$ vector of <code>rrvglm</code> , and the rest go into $x_2$ . The $x_1$ variables constitute the $B_1$ matrix in Yee and Hastie (2003). Those $x_2$ variables which are subject to the reduced-rank regression correspond to the $B_2$ matrix. Set <code>noRRR = NULL</code> for the reduced-rank regression to be applied to every explanatory variable including the intercept.
Norrr	Defunct. Please use noRRR. Use of Norrr will become an error soon.
trace	Logical indicating if output should be produced for each iteration.
Use.Init.Poisson.Q0	Logical indicating whether the <code>.Init.Poisson.Q0()</code> should be used to obtain initial values for the <b>C</b> . The function uses a new method that can work well if the data are Poisson counts coming from an equal-tolerances QRR-VGLM (CQO). This option is less realistic for RR-VGLMs compared to QRR-VGLMs.
checkwz	logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than <code>wzepsilon</code> . If not, any values less than <code>wzepsilon</code> are replaced with this value.
noWarning, Check.rank	Same as <code>vglm.control</code> .
wzepsilon	Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive.
...	Variables in ... are passed into <code>vglm.control</code> . If the derivative algorithm is used, then ... are also passed into <code>rrvglm.optim.control</code> . In the above, $R$ is the Rank and $M$ is the number of linear predictors.

## Details

**VGAM** supports three normalizations to ensure a unique solution. Of these, only corner constraints will work with summary of RR-VGLM objects.

## Value

A list with components matching the input names. Some error checking is done, but not much.

## Note

The arguments in this function begin with an upper case letter to help avoid interference with those of [vglm.control](#).

In the example below a rank-1 *stereotype* model (Anderson, 1984) is fitted.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

## See Also

[rrvglm](#), [rrvglm.optim.control](#), [rrvglm-class](#), [vglm](#), [vglm.control](#), [cqo](#).

## Examples

```
## Not run:
set.seed(111)
pneumo <- transform(pneumo, let = log(exposure.time),
                    x3 = runif(nrow(pneumo))) # x3 is random noise
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3,
             multinomial, pneumo, Rank = 1, Index.corner = 2)
constraints(fit)
vcov(fit)
summary(fit)

## End(Not run)
```

---

rrvglm.optim.control     *Control function for rrvglm() calling optim()*


---

## Description

Algorithmic constants and parameters for running `optim` within `rrvglm` are set using this function.

## Usage

```
rrvglm.optim.control(Fnscale = 1, Maxit = 100,
                     Switch.optimizer = 3, Abstol = -Inf,
                     Reltol = sqrt(.Machine$double.eps), ...)
```

## Arguments

<code>Fnscale</code>	Passed into <code>optim</code> as <code>fnscale</code> .
<code>Maxit</code>	Passed into <code>optim</code> as <code>maxit</code> .
<code>Switch.optimizer</code>	Iteration number when the "Nelder-Mead" method of <code>optim</code> is switched to the quasi-Newton "BFGS" method. Assigning <code>Switch.optimizer</code> a negative number means always BFGS, while assigning <code>Switch.optimizer</code> a value greater than <code>maxits</code> means always use Nelder-Mead.
<code>Abstol</code>	Passed into <code>optim</code> as <code>abstol</code> .
<code>Reltol</code>	Passed into <code>optim</code> as <code>reltol</code> .
<code>...</code>	Ignored.

## Details

See [optim](#) for more details.

## Value

A list with components equal to the arguments.

## Note

The transition between optimization methods may be unstable, so users may have to vary the value of `Switch.optimizer`.

Practical experience with `Switch.optimizer` shows that setting it to too large a value may lead to a local solution, whereas setting it to a low value will obtain the global solution. It appears that, if BFGS kicks in too late when the Nelder-Mead algorithm is starting to converge to a local solution, then switching to BFGS will not be sufficient to bypass convergence to that local solution.

## Author(s)

Thomas W. Yee



**See Also**

[rrvglm.control](#), [optim](#).

---

ruge

*Rutherford-Geiger Polonium Data*


---

**Description**

Decay counts of polonium recorded by Rutherford and Geiger (1910).

**Usage**

```
data(ruge)
```

**Format**

This data frame contains the following columns:

**counts** a numeric vector, counts or frequencies

**number** a numeric vector, the number of decays

**Details**

These are the radioactive decay counts of polonium recorded by Rutherford and Geiger (1910) representing the number of scintillations in 2608 1/8 minute intervals. For example, there were 57 frequencies of zero counts. The counts can be thought of as being approximately Poisson distributed.

**Source**

Rutherford, E. and Geiger, H. (1910) The Probability Variations in the Distribution of alpha Particles, *Philosophical Magazine*, **20**, 698–704.

**Examples**

```
lambdahat <- with(ruge, weighted.mean(number, w = counts))
(N <- with(ruge, sum(counts)))
with(ruge, cbind(number, counts,
                  fitted = round(N * dpois(number, lam = lambdahat))))
```

s

*Defining smooths in VGAM formulae***Description**

s is used in the definition of (vector) smooth terms within vgam formulae.

**Usage**

```
s(x, df = 4, spar = 0, ...)
```

**Arguments**

x	covariate (abscissae) to be smoothed. Note that x must be a <i>single</i> variable and not a function of a variable. For example, s(x) is fine but s(log(x)) will fail. In this case, let logx <- log(x) (in the data frame), say, and then use s(logx). At this stage bivariate smoothers (x would be a two-column matrix) are not implemented.
df	numerical vector of length <i>r</i> . Effective degrees of freedom: must lie between 1 (linear fit) and <i>n</i> (interpolation). Thus one could say that df-1 is the <i>nonlinear degrees of freedom</i> of the smooth. Recycling of values will be used if df is not of length <i>r</i> . If spar is positive then this argument is ignored.
spar	numerical vector of length <i>r</i> . Positive smoothing parameters (after scaling) . Larger values mean more smoothing so that the solution approaches a linear fit for that component function. A zero value means that df is used. Recycling of values will be used if spar is not of length <i>r</i> .
...	Ignored for now.

**Details**

In this help file *M* is the number of additive predictors and *r* is the number of component functions to be estimated (so that *r* is an element from the set {1,2,...,*M*}). Also, if *n* is the number of *distinct* abscissae, then s will fail if *n* < 7.

s, which is symbolic and does not perform any smoothing itself, only handles a single covariate. Note that s works in **vgam** only. It has no effect in **vglm** (actually, it is similar to the identity function **I** so that s(x2) is the same as x2 in the LM model matrix). It differs from the s of the **gam** and **mgcv** packages; they should not be mixed together. Also, terms involving s should be simple additive terms, and not involving interactions and nesting etc. For example, myfactor : s(x2) is not a good idea.

**Value**

A vector with attributes that are (only) used by vgam.

**Note**

The vector cubic smoothing spline which `s()` represents is computationally demanding for large  $M$ . The cost is approximately  $O(nM^3)$  where  $n$  is the number of unique abscissae.

An alternative to using `s` with `vgam` is `bs` and/or `ns` with `vglm`. The latter implements half-stepping, which is helpful if convergence is difficult.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

**See Also**

[vgam](#), [vsmooth.spline](#).

**Examples**

```
# Nonparametric logistic regression
fit <- vgam(agaas ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run: plot(fit, se = TRUE)

# Bivariate logistic model with artificial data
nn <- 300
bdata <- data.frame(x1 = runif(nn), x2 = runif(nn))
bdata <- transform(bdata,
  y1 = rbinom(nn, size = 1, prob = logit(sin(2 * x2), inverse = TRUE)),
  y2 = rbinom(nn, size = 1, prob = logit(sin(2 * x2), inverse = TRUE)))
fit <- vgam(cbind(y1, y2) ~ x1 + s(x2, 3), trace = TRUE,
  binom2.or(exchangeable = TRUE ~ s(x2, 3)), data = bdata)
coef(fit, matrix = TRUE) # Hard to interpret
## Not run: plot(fit, se = TRUE, which.term = 2, scol = "blue")
```

---

seq2binomial

---

*The Two-stage Sequential Binomial Distribution Family Function*


---

**Description**

Estimation of the probabilities of a two-stage binomial distribution.

**Usage**

```
seq2binomial(lprob1 = "logit", lprob2 = "logit",
  iprob1 = NULL, iprob2 = NULL,
  parallel = FALSE, zero = NULL)
```

## Arguments

- lprob1, lprob2 Parameter link functions applied to the two probabilities, called  $p$  and  $q$  below. See [Links](#) for more choices.
- iprob1, iprob2 Optional initial value for the first and second probabilities respectively. A NULL means a value is obtained in the initialize slot.
- parallel, zero Details at [Links](#). If parallel = TRUE then the constraint also applies to the intercept.

## Details

This **VGAM** family function fits the model described by Crowder and Sweeting (1989) which is described as follows. Each of  $m$  spores has a probability  $p$  of germinating. Of the  $y_1$  spores that germinate, each has a probability  $q$  of bending in a particular direction. Let  $y_2$  be the number that bend in the specified direction. The probability model for this data is  $P(y_1, y_2) =$

$$\binom{m}{y_1} p^{y_1} (1-p)^{m-y_1} \binom{y_1}{y_2} q^{y_2} (1-q)^{y_1-y_2}$$

for  $0 < p < 1$ ,  $0 < q < 1$ ,  $y_1 = 1, \dots, m$  and  $y_2 = 1, \dots, y_1$ . Here,  $p$  is prob1,  $q$  is prob2.

Although the Authors refer to this as the *bivariate binomial* model, I have named it the *(two-stage) sequential binomial* model. Fisher scoring is used.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

The response must be a two-column matrix of sample proportions corresponding to  $y_1$  and  $y_2$ . The  $m$  values should be inputted with the weights argument of [vglm](#) and [vgam](#). The fitted value is a two-column matrix of estimated probabilities  $p$  and  $q$ . A common form of error is when there are no trials for  $y_1$ , e.g., if mvector below has some values which are zero.

## Author(s)

Thomas W. Yee

## References

Crowder, M. and Sweeting, T. (1989). Bayesian inference for a bivariate binomial distribution. *Biometrika*, **76**, 599–603.

## See Also

[binomialff](#).

## Examples

```
sdata <- data.frame(mvector = round(rnorm(nn <- 100, m = 10, sd = 2)),
                    x2 = runif(nn))
sdata <- transform(sdata, prob1 = logit(+2 - x2, inverse = TRUE),
                  prob2 = logit(-2 + x2, inverse = TRUE))
sdata <- transform(sdata, successes1 = rbinom(nn, size = mvector, prob = prob1))
sdata <- transform(sdata, successes2 = rbinom(nn, size = successes1, prob = prob2))
sdata <- transform(sdata, y1 = successes1 / mvector)
sdata <- transform(sdata, y2 = successes2 / successes1)
fit <- vglm(cbind(y1, y2) ~ x2, seq2binomial, weight = mvector,
            data = sdata, trace = TRUE)

coef(fit)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(depvar(fit))
head(weights(fit, type = "prior")) # Same as with(sdata, mvector)
# Number of first successes:
head(depvar(fit)[, 1] * c(weights(fit, type = "prior"))
# Number of second successes:
head(depvar(fit)[, 2] * c(weights(fit, type = "prior")) *
      depvar(fit)[, 1])
```

---

 setup.smart

*Smart Prediction Setup*


---

## Description

Sets up smart prediction in one of two modes: "write" and "read".

## Usage

```
setup.smart(mode.arg, smart.prediction = NULL, max.smart = 30)
```

## Arguments

mode.arg	mode.arg must be "write" or "read". If in "read" mode then smart.prediction must be assigned the data structure .smart.prediction that was created while fitting. This is stored in object@smart.prediction or object\$smart.prediction where object is the name of the fitted object.
smart.prediction	If in "read" mode then smart.prediction must be assigned the list of data dependent parameters, which is stored on the fitted object. Otherwise, smart.prediction is ignored.
max.smart	max.smart is the initial length of the list .smart.prediction. It is not important because .smart.prediction is made larger if needed.

## Details

This function is only required by programmers writing a modelling function such as `lm` and `glm`, or a prediction functions of such, e.g., `predict.lm`. The function `setup.smart` operates by mimicking the operations of a first-in first-out stack (better known as a *queue*).

## Value

Nothing is returned.

## Side Effects

In "write" mode `.smart.prediction` in `smartpredenv` (R) or frame 1 (S-PLUS) is assigned an empty list with `max.smart` components. In "read" mode `.smart.prediction` in `smartpredenv` (R) or frame 1 (S-PLUS) is assigned `smart.prediction`. In both cases, `.smart.prediction.counter` in `smartpredenv` (R) or frame 1 (S-PLUS) is assigned the value 0, and `.smart.prediction.mode` and `.max.smart` are written to `smartpredenv` (R) or frame 1 (S-PLUS) too.

## See Also

`lm`, `predict.lm`.

## Examples

```
## Not run: # Put at the beginning of lm
setup.smart("write")

## End(Not run)

## Not run: # Put at the beginning of predict.lm
setup.smart("read", smart.prediction=object$smart.prediction)

## End(Not run)
```

---

Simplex

*Simplex Distribution*

---

## Description

Density function, and random generation for the simplex distribution.

## Usage

```
dsimplex(x, mu = 0.5, dispersion = 1, log = FALSE)
rsimplex(n, mu = 0.5, dispersion = 1)
```

**Arguments**

<code>x</code>	Vector of quantiles. The support of the distribution is the interval $(0, 1)$ .
<code>mu</code> , <code>dispersion</code>	Mean and dispersion parameters. The former lies in the interval $(0, 1)$ and the latter is positive.
<code>n</code> , <code>log</code>	Same usage as <code>runif</code> .

**Details**

The **VGAM** family function `simplex` fits this model; see that online help for more information. For `rsimplex()` the rejection method is used; it may be very slow if the density is highly peaked, and will fail if the density asymptotes at the boundary.

**Value**

`dsimplex(x)` gives the density function, `rsimplex(n)` gives  $n$  random variates.

**Author(s)**

T. W. Yee

**See Also**

`simplex`.

**Examples**

```
sigma <- c(4, 2, 1) # Dispersion parameter
mymu <- c(0.1, 0.5, 0.7); xxx <- seq(0, 1, len = 501)
## Not run: par(mfrow = c(3, 3)) # Figure 2.1 of Song (2007)
for (iii in 1:3)
  for (jjj in 1:3) {
    plot(xxx, dsimplex(xxx, mymu[jjj], sigma[iii]),
         type = "l", col = "blue", xlab = "", ylab = "", main =
           paste("mu = ", mymu[jjj], ", sigma = ", sigma[iii], sep = "")) }
## End(Not run)
```

---

simplex

---

*Simplex Distribution Family Function*


---

**Description**

The two parameters of the univariate standard simplex distribution are estimated by full maximum likelihood estimation.

**Usage**

```
simplex(lmu = "logit", lsigma = "loge",
       imu = NULL, isigma = NULL,
       imethod = 1, shrinkage.init = 0.95, zero = 2)
```

**Arguments**

`lmu`, `lsigma`      Link function for  $\mu$  and  $\sigma$ . See [Links](#) for more choices.

`imu`, `isigma`      Optional initial values for  $\mu$  and  $\sigma$ . A NULL means a value is obtained internally.

`imethod`, `shrinkage.init`, `zero`  
See [CommonVGAMffArguments](#) for more information.

**Details**

The probability density function can be written

$$f(y; \mu, \sigma) = [2\pi\sigma^2(y(1-y))^3]^{-0.5} \exp[-0.5(y - \mu)^2 / (\sigma^2 y(1-y)\mu^2(1-\mu)^2)]$$

for  $0 < y < 1$ ,  $0 < \mu < 1$ , and  $\sigma > 0$ . The mean of  $Y$  is  $\mu$  (called `mu`, and returned as the fitted values).

The second parameter, `sigma`, of this standard simplex distribution is known as the dispersion parameter. The unit variance function is  $V(\mu) = \mu^3(1 - \mu)^3$ . Fisher scoring is applied to both parameters.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

This distribution is potentially useful for dispersion modelling. Numerical problems may occur when  $\mu$  is very close to 0 or 1.

**Author(s)**

T. W. Yee

**References**

Jorgensen, B. (1997) *The Theory of Dispersion Models*. London: Chapman & Hall

Song, P. X.-K. (2007) *Correlated Data Analysis: Modeling, Analytics, and Applications*. Springer.

**See Also**

[dsimplex](#), [dirichlet](#), [rig](#), [binomialff](#).



Examples

```
sdata <- data.frame(x2 = runif(nn <- 1000))
sdata <- transform(sdata, eta1 = 1 + 2 * x2,
                   eta2 = 1 - 2 * x2)
sdata <- transform(sdata, y = rsimplex(nn, mu = logit(eta1, inverse = TRUE),
                                     dispersion = exp(eta2)))
(fit <- vglm(y ~ x2, simplex(zero = NULL), sdata, trace = TRUE))
coef(fit, matrix = TRUE)
summary(fit)
```

---

Sinmad	<i>The Singh-Maddala Distribution</i>
--------	---------------------------------------

---

Description

Density, distribution function, quantile function and random generation for the Singh-Maddala distribution with shape parameters  $a$  and  $q$ , and scale parameter  $scale$ .

Usage

```
dsinmad(x, shape1.a, scale = 1, shape3.q, log = FALSE)
psinmad(q, shape1.a, scale = 1, shape3.q)
qsinmad(p, shape1.a, scale = 1, shape3.q)
rsinmad(n, shape1.a, scale = 1, shape3.q)
```

Arguments

$x$ , $q$	vector of quantiles.
$p$	vector of probabilities.
$n$	number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
$shape1.a$ , $shape3.q$	shape parameters.
$scale$	scale parameter.
$log$	Logical. If $log = TRUE$ then the logarithm of the density is returned.

Details

See [sinmad](#), which is the **VGAM** family function for estimating the parameters by maximum likelihood estimation.

Value

`dsinmad` gives the density, `psinmad` gives the distribution function, `qsinmad` gives the quantile function, and `rsinmad` generates random deviates.

**Note**

The Singh-Maddala distribution is a special case of the 4-parameter generalized beta II distribution.

**Author(s)**

T. W. Yee

**References**

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ: Wiley-Interscience.

**See Also**

[sinmad](#), [genbetaII](#).

**Examples**

```
sdata <- data.frame(y = rsinmad(n = 3000, exp(1), exp(2), exp(1)))
fit <- vglm(y ~ 1, sinmad(ishape1.a = 2.1), sdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

---

sinmad	<i>Singh-Maddala Distribution Family Function</i>
--------	---

---

**Description**

Maximum likelihood estimation of the 3-parameter Singh-Maddala distribution.

**Usage**

```
sinmad(lshape1.a = "loge", lscale = "loge", lshape3.q = "loge",
       ishape1.a = NULL, iscale = NULL, ishape3.q = 1, zero = NULL)
```

**Arguments**

- |                              |  |
|------------------------------|--|
| lshape1.a, lscale, lshape3.q | Parameter link functions applied to the (positive) parameters a, scale, and q. See <a href="#">Links</a> for more choices.   |
| ishape1.a, iscale, ishape3.q | Optional initial values for a, scale, and q.   |
| zero                         | An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set {1,2,3} which correspond to a, scale, q, respectively. |

## Details

The 3-parameter Singh-Maddala distribution is the 4-parameter generalized beta II distribution with shape parameter  $p = 1$ . It is known under various other names, such as the Burr XII (or just the Burr distribution), Pareto IV, beta-P, and generalized log-logistic distribution. More details can be found in Kleiber and Kotz (2003).

Some distributions which are special cases of the 3-parameter Singh-Maddala are the Lomax ( $a = 1$ ), Fisk ( $q = 1$ ), and paralogistic ( $a = q$ ).

The Singh-Maddala distribution has density

$$f(y) = aqy^{a-1}/[b^a\{1 + (y/b)^a\}^{1+q}]$$

for  $a > 0$ ,  $b > 0$ ,  $q > 0$ ,  $y \geq 0$ . Here,  $b$  is the scale parameter scale, and the others are shape parameters. The cumulative distribution function is

$$F(y) = 1 - [1 + (y/b)^a]^{-q}.$$

The mean is

$$E(Y) = b\Gamma(1 + 1/a)\Gamma(q - 1/a)/\Gamma(q)$$

provided  $-a < 1 < aq$ ; these are returned as the fitted values.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

See the note in [genbetaII](#).

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

[Sinmad](#), [genbetaII](#), [betaII](#), [dagum](#), [fisk](#), [invlomax](#), [lomax](#), [paralogistic](#), [invparalogistic](#).

## Examples

```
sdata <- data.frame(y = rsinmad(n = 1000, exp(1), exp(2), exp(0)))
fit <- vglm(y ~ 1, sinmad, sdata, trace = TRUE)
fit <- vglm(y ~ 1, sinmad(ishapel.a = exp(1)), sdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

```
# Harder problem (has the shape3.q parameter going to infinity)

set.seed(3)
sdata <- data.frame(y1 = rbeta(1000, 6, 6))
# hist(with(sdata, y1))
if (FALSE) {
  # This fails
  fit1 <- vglm(y1 ~ 1, sinmad, data = sdata, trace = TRUE)
  fit1 <- vglm(y1 ~ 1, sinmad, data = sdata, trace = TRUE, maxit = 6,
               crit = "coef")
  Coef(fit1)
}
# Try this remedy:
fit2 <- vglm(y1 ~ 1, sinmad(ishape3.q = 3, lshape3.q = "loglog"),
             data = sdata, trace = TRUE, stepsize = 0.05, maxit = 99)
coef(fit2, matrix = TRUE)
Coef(fit2)
```

---

Skellam

*The Skellam Distribution*


---

## Description

Density  
and random generation for the Skellam distribution.

## Usage

```
dskellam(x, mu1, mu2, log = FALSE)
rskellam(n, mu1, mu2)
```

## Arguments

x	vector of quantiles.
n	number of observations. Same as <a href="#">runif</a> .
mu1, mu2	See <a href="#">skellam</a>
	.
log	Logical; if TRUE, the logarithm is returned.

## Details

See [skellam](#), the **VGAM** family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dskellam gives the density, and  
rskellam generates random deviates.

**Warning**

Numerical problems may occur for data if  $\mu_1$  and/or  $\mu_2$  are large. The normal approximation for this case has not been implemented yet.

**See Also**

[skellam](#), [dpois](#).

**Examples**

```
## Not run: mu1 <- 1; mu2 <- 2; x <- (-7):7
plot(x, dskellam(x, mu1, mu2), type = "h", las = 1, col = "blue",
     main = paste("Density of Skellam distribution with mu1 = ", mu1,
                  " and mu2 = ", mu2, sep = ""))
## End(Not run)
```

skellam

*Skellam Distribution Family Function***Description**

Estimates the two parameters of a Skellam distribution by maximum likelihood estimation.

**Usage**

```
skellam(lmu1 = "loge", lmu2 = "loge", imu1 = NULL, imu2 = NULL,
       nsimEIM = 100, parallel = FALSE, zero = NULL)
```

**Arguments**

`lmu1`, `lmu2` Link functions for the  $\mu_1$  and  $\mu_2$  parameters. See [Links](#) for more choices and for general information.

`imu1`, `imu2` Optional initial values for the parameters. See [CommonVGAMffArguments](#) for more information. If convergence failure occurs (this **VGAM** family function seems to require good initial values) try using these arguments.

`nsimEIM`, `parallel`, `zero` See [CommonVGAMffArguments](#) for more information. In particular, setting `parallel=TRUE` will constrain the two means to be equal.

**Details**

The Skellam distribution models the difference between two independent Poisson distributions (with means  $\mu_j$ , say). It has density function

$$f(y; \mu_1, \mu_2) = \left(\frac{\mu_1}{\mu_2}\right)^{y/2} \exp(-\mu_1 - \mu_2) I_{|y|}(2\sqrt{\mu_1\mu_2})$$

where  $y$  is an integer,  $\mu_1 > 0$ ,  $\mu_2 > 0$ . Here,  $I_v$  is the modified Bessel function of the first kind with order  $v$ .

The mean is  $\mu_1 - \mu_2$  (returned as the fitted values), and the variance is  $\mu_1 + \mu_2$ . Simulated Fisher scoring is implemented.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

### Warning

This **VGAM** family function seems fragile and very sensitive to the initial values. Use very cautiously!!

### Note

Numerical problems may occur for data if  $\mu_1$  and/or  $\mu_2$  are large.

### References

Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates belonging to different populations. *Journal of the Royal Statistical Society, Series A*, **109**, 296.

### See Also

[dskellam](#), [dpois](#), [poissonff](#).

### Examples

```
## Not run:
sdata <- data.frame(x2 = runif(nn <- 1000))
sdata <- transform(sdata, mu1 = exp(1 + x2), mu2 = exp(1 + x2))
sdata <- transform(sdata, y = rskellam(nn, mu1, mu2))
fit1 <- vglm(y ~ x2, skellam, sdata, trace = TRUE, crit = "coef")
fit2 <- vglm(y ~ x2, skellam(parallel = TRUE), sdata, trace = TRUE)
coef(fit1, matrix = TRUE)
coef(fit2, matrix = TRUE)
summary(fit1)
# Likelihood ratio test for equal means:
pchisq(2 * (logLik(fit1) - logLik(fit2)),
      df = fit2@df.residual - fit1@df.residual, lower.tail = FALSE)
lrtest(fit1, fit2) # Alternative

## End(Not run)
```

---

skewnorm*Skew-Normal Distribution*

---

**Description**

Density and  
random generation for the univariate skew-normal distribution.

**Usage**

```
dskewnorm(x, location = 0, scale = 1, shape = 0, log = FALSE)
rskewnorm(n, location = 0, scale = 1, shape = 0)
```

**Arguments**

x	vector of quantiles.
n	number of observations. Same as <a href="#">runif</a> .
location	The location parameter $\xi$ . A vector.
scale	The scale parameter $\omega$ . A positive vector.
shape	The shape parameter. It is called $\alpha$ in <a href="#">skewnormal</a> .
log	Logical. If log=TRUE then the logarithm of the density is returned.

**Details**

See [skewnormal](#), which currently only estimates the shape parameter. More generally here,  $Z = \xi + \omega Y$  where  $Y$  has a standard skew-normal distribution (see [skewnormal](#)),  $\xi$  is the location parameter and  $\omega$  is the scale parameter.

**Value**

dskewnorm gives the density,  
rskewnorm generates random deviates.

**Note**

The default values of all three parameters corresponds to the skew-normal being the standard normal distribution.

**Author(s)**

T. W. Yee

**References**

<http://tango.stat.unipd.it/SN>.

**See Also**

[skewnormal1](#).

**Examples**

```
## Not run: N <- 200 # Grid resolution
shape <- 7; x <- seq(-4, 4, len = N)
plot(x, dskewnorm(x, shape = shape), type = "l", col = "blue", las = 1,
      ylab = "", lty = 1, lwd = 2)
abline(v = 0, h = 0, col = "grey")
lines(x, dnorm(x), col = "orange", lty = 2, lwd = 2)
legend("topleft", leg = c(paste("Blue = dskewnorm(x, ", shape, ")", sep = ""),
                           "Orange = standard normal density"), lty = 1:2, lwd = 2,
      col = c("blue", "orange"))
## End(Not run)
```

---

skewnormal

*Univariate Skew-Normal Distribution Family Function*


---

**Description**

Maximum likelihood estimation of the shape parameter of a univariate skew-normal distribution.

**Usage**

```
skewnormal(lshape = "identity", ishape = NULL, nsimEIM = NULL)
```

**Arguments**

lshape, ishape, nsimEIM

See [Links](#) and [CommonVGAMffArguments](#).

**Details**

The univariate skew-normal distribution has a density function that can be written

$$f(y) = 2\phi(y)\Phi(\alpha y)$$

where  $\alpha$  is the shape parameter. Here,  $\phi$  is the standard normal density and  $\Phi$  its cumulative distribution function. When  $\alpha = 0$  the result is a standard normal distribution. When  $\alpha = 1$  it models the distribution of the maximum of two independent standard normal variates. When the absolute value of the shape parameter increases the skewness of the distribution increases. The limit as the shape parameter tends to positive infinity results in the folded normal distribution or half-normal distribution. When the shape parameter changes its sign, the density is reflected about  $y = 0$ .

The mean of the distribution is  $\mu = \alpha\sqrt{2/(\pi(1 + \alpha^2))}$  and these are returned as the fitted values. The variance of the distribution is  $1 - \mu^2$ . The Newton-Raphson algorithm is used unless the nsimEIM argument is used.



**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

It is well known that the EIM of Azzalini's skew-normal distribution is singular for skewness parameter tending to zero, and thus produces influential problems.

**Note**

It is a good idea to use several different initial values to ensure that the global solution is obtained.  
This family function will be modified (hopefully soon) to handle a location and scale parameter too.

**Author(s)**

Thomas W. Yee

**References**

- Azzalini, A. A. (1985) A class of distributions which include the normal. *Scandinavian Journal of Statistics*, **12**, 171–178.
- Azzalini, A. and Capitanio, A. (1999) Statistical applications of the multivariate skew-normal distribution. *Journal of the Royal Statistical Society, Series B, Methodological*, **61**, 579–602.

**See Also**

[skewnorm](#), [uninormal](#), [foldnormal](#).

**Examples**

```
sdata <- data.frame(y1 = rskewnorm(nn <- 1000, shape = 5))
fit1 <- vglm(y1 ~ 1, skewnormal, sdata, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1), 1)
with(sdata, mean(y1))
## Not run: with(sdata, hist(y1, prob = TRUE))
x <- with(sdata, seq(min(y1), max(y1), len = 200))
with(sdata, lines(x, dskewnorm(x, shape = Coef(fit1)), col = "blue"))
## End(Not run)

sdata <- data.frame(x2 = runif(nn))
sdata <- transform(sdata, y2 = rskewnorm(nn, shape = 1 + 2*x2))
fit2 <- vglm(y2 ~ x2, skewnormal, sdata, trace = TRUE, crit = "coef")
summary(fit2)
```

Slash

*Slash Distribution*

---

**Description**

Density function, distribution function, and random generation for the slash distribution.

**Usage**

```
dslash(x, mu = 0, sigma = 1, log = FALSE, smallno = .Machine$double.eps*1000)
pslash(q, mu = 0, sigma = 1)
rslash(n, mu = 0, sigma = 1)
```

**Arguments**

x, q	vector of quantiles.
n	number of observations. Must be a single positive integer.
mu, sigma	the mean and standard deviation of the univariate normal distribution.
log	Logical. If TRUE then the logarithm of the density is returned.
smallno	See <a href="#">slash</a> .

**Details**

See [slash](#), the **VGAM** family function for estimating the two parameters by maximum likelihood estimation, for the formula of the probability density function and other details.

**Value**

dslash gives the density, and pslash gives the distribution function, rslash generates random deviates.

**Note**

pslash is very slow.

**Author(s)**

Thomas W. Yee and C. S. Chee

**See Also**

[slash](#).

Examples

```
## Not run:
curve(dslash, col = "blue", ylab = "f(x)", -5, 5, ylim = c(0, 0.4),
      main = "Standard slash, normal and Cauchy densities", lwd = 2)
curve(dnorm, col = "black", lty = 2, lwd = 2, add = TRUE)
curve(dcauchy, col = "orange", lty = 3, lwd = 2, add = TRUE)
legend(x = 2, y = 0.3, c("slash", "normal", "Cauchy"), lty = 1:3,
      col = c("blue", "black", "orange"), lwd = 2)

curve(pslash, col = "blue", -5, 5, ylim = 0:1)

## End(Not run)
```

---

slash	<i>Slash Distribution Family Function</i>
-------	---

---

Description

Estimates the two parameters of the slash distribution by maximum likelihood estimation.

Usage

```
slash(lmu = "identity", lsigma = "log",
      imu = NULL, isigma = NULL, iprobs = c(0.1, 0.9), nsimEIM = 250,
      zero = NULL, smallno = .Machine$double.eps*1000)
```

Arguments

lmu, lsigma	Parameter link functions applied to the $\mu$ and $\sigma$ parameters, respectively. See <a href="#">Links</a> for more choices.
imu, isigma	Initial values. A NULL means an initial value is chosen internally. See <a href="#">CommonVGAMffArguments</a> for more information.
iprobs	Used to compute the initial values for mu. This argument is fed into the probs argument of <a href="#">quantile</a> , and then a grid between these two points is used to evaluate the log-likelihood. This argument must be of length two and have values between 0 and 1.
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> for more information.
smallno	Small positive number, used to test for the singularity.

Details

The standard slash distribution is the distribution of the ratio of a standard normal variable to an independent standard uniform(0,1) variable. It is mainly of use in simulation studies. One of its properties is that it has heavy tails, similar to those of the Cauchy.

The general slash distribution can be obtained by replacing the univariate normal variable by a general normal  $N(\mu, \sigma)$  random variable. It has a density that can be written as

$$f(y) = \begin{cases} 1/(2\sigma\sqrt{(2\pi)}) & \text{if } y = \mu, \\ 1 - \exp(-(((y - \mu)/\sigma)^2)/2))/(\sqrt{(2\pi)}\sigma((y - \mu)/\sigma)^2) & \text{if } y \neq \mu. \end{cases}$$

where  $\mu$  and  $\sigma$  are the mean and standard deviation of the univariate normal distribution respectively.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

Fisher scoring using simulation is used. Convergence is often quite slow. Numerical problems may occur.

### Author(s)

T. W. Yee and C. S. Chee

### References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley.

Kafadar, K. (1982) A Biweight Approach to the One-Sample Problem *Journal of the American Statistical Association*, **77**, 416–424.

### See Also

[rslash](#).

### Examples

```
## Not run:
sdata <- data.frame(y = rslash(n = 1000, mu = 4, sigma = exp(2)))
fit <- vglm(y ~ 1, slash, sdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)

## End(Not run)
```

---

smart.expression	<i>S Expression for Smart Functions</i>
------------------	---

---

## Description

smart.expression is an S expression for a smart function to call itself. It is best if you go through it line by line, but most users will not need to know anything about it. It requires the primary argument of the smart function to be called "x".

The list component match.call must be assigned the value of match.call() in the smart function; this is so that the smart function can call itself later.

## See Also

[match.call.](#)

## Examples

```
"my2" <- function(x, minx=min(x)) { # Here is a smart function
  x <- x # Needed for nested calls, e.g., bs(scale(x))
  if(smart.mode.is("read")) {
    return(eval(smart.expression))
  } else
  if(smart.mode.is("write"))
    put.smart(list(minx=minx, match.call=match.call()))
  (x-minx)^2
}
attr(my2, "smart") <- TRUE
```

---

smart.mode.is	<i>Determine What Mode the Smart Prediction is In</i>
---------------	---

---

## Description

Determine which of three modes the smart prediction is currently in.

## Usage

```
smart.mode.is(mode.arg=NULL)
```

## Arguments

mode.arg            a character string, either "read", "write" or "neutral".

## Details

Smart functions such as `bs` and `poly` need to know what mode smart prediction is in. If it is in "write" mode then the parameters are saved to `.smart.prediction` using `put.smart`. If in "read" mode then the parameters are read in using `get.smart`. If in "neutral" mode then the smart function behaves like an ordinary function.

## Value

If `mode.arg` is given, then either TRUE or FALSE is returned. If `mode.arg` is not given, then the mode ("neutral", "read" or "write") is returned. Usually, the mode is "neutral".

## See Also

`put.smart`, `bs`, `poly`.

## Examples

```
my1 <- function(x, minx = min(x)) { # Here is a smart function
  x <- x # Needed for nested calls, e.g., bs(scale(x))
  if(smart.mode.is("read")) {
    smart <- get.smart()
    minx <- smart$minx # Overwrite its value
  } else if(smart.mode.is("write"))
    put.smart(list(minx = minx))
  sqrt(x - minx)
}
attr(my1, "smart") <- TRUE

smart.mode.is() # Returns "neutral"
smart.mode.is(smart.mode.is()) # Returns TRUE
```

---

smartpred

*Smart Prediction*

---

## Description

Data-dependent parameters in formula terms can cause problems in when predicting. The **smart-pred** package for R and S-PLUS saves data-dependent parameters on the object so that the bug is fixed. The `lm` and `glm` functions have been fixed properly. Note that the **VGAM** package by T. W. Yee automatically comes with smart prediction.

## Details

R version 1.6.0 introduced a partial fix for the prediction problem because it does not work all the time, e.g., for terms such as `I(poly(x, 3))`, `poly(c(scale(x)), 3)`, `bs(scale(x), 3)`, `scale(scale(x))`. See the examples below. Smart prediction, however, will always work.

The basic idea is that the functions in the formula are now smart, and the modelling functions make use of these smart functions. Smart prediction works in two ways: using `smart.expression`, or using a combination of `put.smart` and `get.smart`.

**Value**

Returns the usual object, but with one list/slot component called `smart.prediction` containing any data-dependent parameters.

**Side Effects**

The variables `.max.smart`, `.smart.prediction` and `.smart.prediction.counter` are created while the model is being fitted. In R they are created in a new environment called `smartpredenv`. In S-PLUS they are created in frame 1. These variables are deleted after the model has been fitted. However, in R, if there is an error in the model fitting function or the fitting model is killed (e.g., by typing control-C) then these variables will be left in `smartpredenv`. At the beginning of model fitting, these variables are deleted if present in `smartpredenv`.

During prediction, the variables `.smart.prediction` and `.smart.prediction.counter` are reconstructed and read by the smart functions when the model frame is re-evaluated. After prediction, these variables are deleted.

If the modelling function is used with argument `smart = FALSE` (e.g., `vglm(..., smart = FALSE)`) then smart prediction will not be used, and the results should match with the original R or S-PLUS functions.

**WARNING**

In S-PLUS, if the "bigdata" library is loaded then it is `detach()`'ed. This is done because scale cannot be made smart if "bigdata" is loaded (it is loaded by default in the Windows version of Splus 8.0, but not in Linux/Unix). The function [search](#) tells what is currently attached.

In R and S-PLUS the functions [predict.bs](#) and `predict.ns` are not smart. That is because they operate on objects that contain attributes only and do not have list components or slots. In R the function [predict.poly](#) is not smart.

**Note**

In S-PLUS you will need to load in the **smartpred** library with the argument `first = T`, e.g., `library(smartpred, lib = "./mys8libs", first = T)`. Here, `mys8libs` is the name of a directory of installed packages. To install the smartpred package in Linux/Unix, type something like `Splus8 INSTALL -l ./mys8libs ./smartpred_0.8-2.tar.gz`.

**Author(s)**

T. W. Yee and T. J. Hastie

**See Also**

[get.smart.prediction](#), [get.smart](#), [put.smart](#), [smart.expression](#), [smart.mode.is](#), [setup.smart](#), [wrapup.smart](#). Commonly used data-dependent functions include [scale](#), [poly](#), [bs](#), [ns](#). In R, the functions [bs](#) and [ns](#) are in the **splines** package, and this library is automatically loaded in because it contains compiled code that [bs](#) and [ns](#) call.

The website <http://www.stat.auckland.ac.nz/~yee> contains more information such as how to write a smart function, and other technical details.

The functions `vglm`, `vgam`, `rrvglm` and `cqo` in T. W. Yee's **VGAM** package are examples of modelling functions that employ smart prediction.

### Examples

```
# Create some data first
n <- 20
set.seed(86) # For reproducibility of the random numbers
x <- sort(runif(n))
y <- sort(runif(n))
## Not run: if(is.R()) library(splines) # To get ns() in R

# This will work for R 1.6.0 and later, but fail for S-PLUS
fit <- lm(y ~ ns(x, df = 5))
## Not run: plot(x, y)
lines(x, fitted(fit))
newx <- seq(0, 1, len = n)
points(newx, predict(fit, data.frame(x = newx)), type = "b",
       col = 2, err = -1)
## End(Not run)

# The following fails for R 1.6.x and later but works with smart prediction
fit <- lm(y ~ ns(scale(x), df = 5))
## Not run: fit$smart.prediction
plot(x, y)
lines(x, fitted(fit))
newx <- seq(0, 1, len = n)
points(newx, predict(fit, data.frame(x = newx)), type = "b",
       col = 2, err = -1)
## End(Not run)

# The following requires the VGAM package to be loaded
## Not run: library(VGAM)
fit <- vlm(y ~ ns(scale(x), df = 5))
fit@smart.prediction
plot(x, y)
lines(x, fitted(fit))
newx <- seq(0, 1, len = n)
points(newx, predict(fit, data.frame(x = newx)), type = "b",
       col = 2, err = -1)
## End(Not run)
```

---

sratio

---

*Ordinal Regression with Stopping Ratios*


---

### Description

Fits a stopping ratio logit/probit/cloglog/cauchit/... regression model to an ordered (preferably) factor response.



**Usage**

```
sratio(link = "logit", parallel = FALSE, reverse = FALSE,
       zero = NULL, whitespace = FALSE)
```

**Arguments**

link	Link function applied to the $M$ stopping ratio probabilities. See <a href="#">Links</a> for more choices.
parallel	A logical, or formula specifying which terms have equal/unequal coefficients.
reverse	Logical. By default, the stopping ratios used are $\eta_j = \text{logit}(P[Y = j   Y \geq j])$ for $j = 1, \dots, M$ . If reverse is TRUE, then $\eta_j = \text{logit}(P[Y = j+1   Y \leq j+1])$ will be used.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2, \dots, M\}$ . The default value means none are modelled as intercept-only terms.
whitespace	See <a href="#">CommonVGAMffArguments</a> for information.

**Details**

In this help file the response  $Y$  is assumed to be a factor with ordered values  $1, 2, \dots, M + 1$ , so that  $M$  is the number of linear/additive predictors  $\eta_j$ .

There are a number of definitions for the *continuation ratio* in the literature. To make life easier, in the **VGAM** package, we use *continuation* ratios (see [cratio](#)) and *stopping* ratios. Continuation ratios deal with quantities such as  $\text{logit}(P[Y > j | Y \geq j])$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

**Warning**

No check is made to verify that the response is ordinal if the response is a matrix; see [ordered](#).

**Note**

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the y slot returned by [vglm](#)/[vgam](#)/[rrvglm](#) is the matrix of counts.

For a nominal (unordered) factor response, the multinomial logit model ([multinomial](#)) is more appropriate.

Here is an example of the usage of the parallel argument. If there are covariates  $x_1$ ,  $x_2$  and  $x_3$ , then `parallel = TRUE ~ x1 + x2 -1` and `parallel = FALSE ~ x3` are equivalent. This would constrain the regression coefficients for  $x_1$  and  $x_2$  to be equal; those of the intercepts and  $x_3$  would be different.

**Author(s)**

Thomas W. Yee

## References

- Agresti, A. (2002) *Categorical Data Analysis*, 2nd ed. New York: Wiley.
- Simonoff, J. S. (2003) *Analyzing Categorical Data*, New York: Springer-Verlag.
- McCullagh, P. and Nelder, J. A. (1989) *Generalized Linear Models*, 2nd ed. London: Chapman & Hall.
- Yee, T. W. (2010) The **VGAM** package for categorical data analysis. *Journal of Statistical Software*, **32**, 1–34. <http://www.jstatsoft.org/v32/i10/>.
- Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[cratio](#), [acat](#), [cumulative](#), [multinomial](#), [pneumo](#), [logit](#), [probit](#), [cloglog](#), [cauchit](#).

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
             sratio(parallel = TRUE), data = pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
predict(fit)
predict(fit, untransform = TRUE)
```

---

studentt

*Student t Distribution*

---

## Description

Estimating the parameters of a Student t distribution.

## Usage

```
studentt(ldf = "loglog", idf = NULL, tol1 = 0.1, imethod = 1)
studentt2(df = Inf, llocation = "identity", lscale = "loge",
          ilocation = NULL, iscale = NULL, imethod = 1, zero = -2)
studentt3(llocation = "identity", lscale = "loge", ldf = "loglog",
          ilocation = NULL, iscale = NULL, idf = NULL,
          imethod = 1, zero = -(2:3))
```

## Arguments

`llocation`, `lscale`, `ldf`

Parameter link functions for each parameter, e.g., for degrees of freedom  $\nu$ . See [Links](#) for more choices. The defaults ensures the parameters are in range. A [loglog](#) link keeps the degrees of freedom greater than unity; see below.

ilocation, iscale, idf	Optional initial values. If given, the values must be in range. The default is to compute an initial value internally.
tol1	A positive value, the tolerance for testing whether an initial value is 1. Best to leave this argument alone.
df	Numeric, user-specified degrees of freedom. It may be of length equal to the number of columns of a response matrix.
imethod, zero	See <a href="#">CommonVGAMffArguments</a> .

## Details

The Student  $t$  density function is

$$f(y; \nu) = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \left(1 + \frac{y^2}{\nu}\right)^{-(\nu+1)/2}$$

for all real  $y$ . Then  $E(Y) = 0$  if  $\nu > 1$  (returned as the fitted values), and  $Var(Y) = \nu/(\nu - 2)$  for  $\nu > 2$ . When  $\nu = 1$  then the Student  $t$ -distribution corresponds to the standard Cauchy distribution, [cauchy1](#). When  $\nu = 2$  with a scale parameter of  $\sqrt{2}$  then the Student  $t$ -distribution corresponds to the standard Koenker distribution, [koenker](#). The degrees of freedom can be treated as a parameter to be estimated, and as a real and not an integer. The Student  $t$  distribution is used for a variety of reasons in statistics, including robust regression.

Let  $Y = (T - \mu)/\sigma$  where  $\mu$  and  $\sigma$  are the location and scale parameters respectively. Then `studentt3` estimates the location, scale and degrees of freedom parameters. And `studentt2` estimates the location, scale parameters for a user-specified degrees of freedom, `df`. And `studentt` estimates the degrees of freedom parameter only. The fitted values are the location parameters. By default the linear/additive predictors are  $(\mu, \log(\sigma), \log \log(\nu))^T$  or subsets thereof.

In general convergence can be slow, especially when there are covariates.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Note

`studentt3()` and `studentt2()` can handle multiple responses.

Practical experience has shown reasonably good initial values are required. If convergence failure occurs try using arguments such as `idf`. Local solutions are also possible, especially when the degrees of freedom is close to unity or the scale parameter is close to zero.

A standard normal distribution corresponds to a  $t$  distribution with infinite degrees of freedom. Consequently, if the data is close to normal, there may be convergence problems; best to use [uninormal](#) instead.

## Author(s)

T. W. Yee

References

Student (1908) The probable error of a mean. *Biometrika*, **6**, 1–25.

Zhu, D. and Galbraith, J. W. (2010) A generalized asymmetric Student-*t* distribution with application to financial econometrics. *Journal of Econometrics*, **157**, 297–305.

See Also

[uninormal](#), [cauchy1](#), [logistic](#), [huber2](#), [koenker](#), [TDist](#).

Examples

```
tdata <- data.frame(x2 = runif(nn <- 1000))
tdata <- transform(tdata, y1 = rt(nn, df = exp(exp(0.5 - x2))),
                  y2 = rt(nn, df = exp(exp(0.5 - x2))))
fit1 <- vglm(y1 ~ x2, studentt, tdata, trace = TRUE)
coef(fit1, matrix = TRUE)

fit2 <- vglm(cbind(y1, y2) ~ x2, studentt3, tdata, trace = TRUE)
coef(fit2, matrix = TRUE)
```

---

SUR	<i>Seemingly Unrelated Regressions</i>
-----	--

---

Description

Fits a system of seemingly unrelated regressions.

Usage

```
SUR(mle.normal = FALSE,
    divisor = c("n", "n-max(pj,pk)", "sqrt((n-pj)*(n-pk))"),
    parallel = FALSE, Varcov = NULL, matrix.arg = FALSE)
```

Arguments

<code>mle.normal</code>	Logical. If TRUE then the MLE, assuming multivariate normal errors, is computed; the effect is just to add a loglikelihood slot to the returned object. Then it results in the <i>maximum likelihood estimator</i> .
<code>divisor</code>	Character, partial matching allowed and the first choice is the default. The divisor for the estimate of the covariances. If "n" then the estimate will be biased. If the others then the estimate will be unbiased for some elements. If <code>mle.normal = TRUE</code> and this argument is not "n" then a warning or an error will result.
<code>parallel</code>	See <a href="#">CommonVGAMffArguments</a> . If <code>parallel = TRUE</code> then the constraint applies to the intercept too.

Varcov	Numeric. This may be assigned a variance-covariance of the errors. If <code>matrix.arg</code> then this is a $M \times M$ matrix. If <code>!matrix.arg</code> then this is a $M \times M$ matrix in matrix-band format (a vector with at least $M$ and at most $M*(M+1)/2$ elements).
<code>matrix.arg</code>	Logical. Of single length.

### Details

Proposed by Zellner (1962), the basic seemingly unrelated regressions (SUR) model is a set of LMs ( $M > 1$  of them) tied together at the error term level. Each LM's model matrix may potentially have its own set of predictor variables.

Zellner's efficient (ZEF) estimator (also known as *Zellner's two-stage Aitken estimator*) can be obtained by setting `maxit = 1` (and possibly `divisor = "sqrt"` or `divisor = "n-max"`).

The default value of `maxit` (in `vglm.control`) probably means *iterative GLS* (IGLS) estimator is computed because IRLS will probably iterate to convergence. IGLS means, at each iteration, the residuals are used to estimate the error variance-covariance matrix, and then the matrix is used in the GLS. The IGLS estimator is also known as *Zellner's iterative Aitken estimator*, or IZEF.

### Value

An object of class `"vglmff"` (see `vglmff-class`). The object is used by modelling functions such as `vglm` and `vgam`.

### Warning

The default convergence criterion may be a little loose. Try setting `epsilon = 1e-11`, especially with `mle.normal = TRUE`.

### Note

The fitted object has slot `@extra$ncols.X.lm` which is a  $M$  vector with the number of parameters for each LM. Also, `@misc$values.divisor` is the  $M$ -vector of divisor values.

Constraint matrices are needed in order to specify which response variables that each term on the RHS of the formula is a regressor for. See the `constraints` argument of `vglm` for more information.

### Author(s)

T. W. Yee.

### References

Zellner, A. (1962) An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias. *J. Amer. Statist. Assoc.*, **57**(298), 348–368.

Kmenta, J. and Gilbert, R. F. (1968) Small Sample Properties of Alternative Estimators of Seemingly Unrelated Regressions. *J. Amer. Statist. Assoc.*, **63**(324), 1180–1200.

### See Also

`uninormal`, `gew`.

## Examples

```
# Obtain some of the results of p.1199 of Kmenta and Gilbert (1968)
clist <- list("(Intercept)" = diag(2),
             "capital.g"   = rbind(1, 0),
             "value.g"     = rbind(1, 0),
             "capital.w"   = rbind(0, 1),
             "value.w"     = rbind(0, 1))
zef1 <- vglm(cbind(invest.g, invest.w) ~
            capital.g + value.g + capital.w + value.w,
            SUR(divisor = "sqrt"), maxit = 1,
            data = gew, trace = TRUE, constraints = clist)

round(coef(zef1, matrix = TRUE), digits = 4) # ZEF
zef1@extra$ncols.X.lm
zef1@misc$divisor
zef1@misc$values.divisor
round(sqrt(diag(vcov(zef1))), digits = 4) # SEs

mle1 <- vglm(cbind(invest.g, invest.w) ~
            capital.g + value.g + capital.w + value.w,
            SUR(mle.normal = TRUE, divisor = "n-max"),
            epsilon = 1e-11,
            data = gew, trace = TRUE, constraints = clist)
round(coef(mle1, matrix = TRUE), digits = 4) # MLE
round(sqrt(diag(vcov(mle1))), digits = 4) # SEs
```

---

SurvS4

---

*Create a Survival Object*


---

## Description

Create a survival object, usually used as a response variable in a model formula.

## Usage

```
SurvS4(time, time2, event, type =, origin = 0)
is.SurvS4(x)
```

## Arguments

time	for right censored data, this is the follow up time. For interval data, the first argument is the starting time for the interval.
x	any R object.
event	The status indicator, normally 0=alive, 1=dead. Other choices are TRUE/FALSE (TRUE = death) or 1/2 (2=death). For interval censored data, the status indicator is 0=right censored, 1=event at time, 2=left censored, 3=interval censored. Although unusual, the event indicator can be omitted, in which case all subjects are assumed to have an event.

time2	ending time of the interval for interval censored or counting process data only. Intervals are assumed to be open on the left and closed on the right, (start, end]. For counting process data, event indicates whether an event occurred at the end of the interval.
type	character string specifying the type of censoring. Possible values are "right", "left", "counting", "interval", or "interval2". The default is "right" or "counting" depending on whether the time2 argument is absent or present, respectively.
origin	for counting process data, the hazard function origin. This is most often used in conjunction with a model containing time dependent strata in order to align the subjects properly when they cross over from one strata to another.

## Details

Typical usages are

```
SurvS4(time, event)
SurvS4(time, time2, event, type=, origin=0)
```

In theory it is possible to represent interval censored data without a third column containing the explicit status. Exact, right censored, left censored and interval censored observation would be represented as intervals of (a,a), (a, infinity), (-infinity,b), and (a,b) respectively; each specifying the interval within which the event is known to have occurred.

If type = "interval2" then the representation given above is assumed, with NA taking the place of infinity. If 'type="interval" event must be given. If event is 0, 1, or 2, the relevant information is assumed to be contained in time, the value in time2 is ignored, and the second column of the result will contain a placeholder.

Presently, the only methods allowing interval censored data are the parametric models computed by [survreg](#), so the distinction between open and closed intervals is unimportant. The distinction is important for counting process data and the Cox model.

The function tries to distinguish between the use of 0/1 and 1/2 coding for left and right censored data using `if (max(status)==2)`. If 1/2 coding is used and all the subjects are censored, it will guess wrong. Use 0/1 coding in this case.

## Value

An object of class SurvS4 (formerly Surv). There are methods for print, is.na, and subscripting survival objects. SurvS4 objects are implemented as a matrix of 2 or 3 columns.

In the case of `is.SurvS4`, a logical value TRUE if x inherits from class "SurvS4", otherwise a FALSE.

## Note

The purpose of having SurvS4 in **VGAM** is so that the same input can be fed into [vglm](#) as functions in **survival** such as [survreg](#). The class name has been changed from "Surv" to "SurvS4"; see [SurvS4-class](#).

The format J+ is interpreted in **VGAM** as  $\geq J$ . If type="interval" then these should not be used in **VGAM**: (L, U-] or (L, U+].

**Author(s)**

The code and documentation comes from **survival**. Slight modifications have been made for conversion to S4 by T. W. Yee. Also, for "interval" data, `as.character.SurvS4()` has been modified to print intervals of the form (start, end] and not [start, end] as previously. (This makes a difference for discrete data, such as for [cenpoisson](#)). All **VGAM** family functions beginning with "cen" require the packaging function `Surv` to format the input.

**See Also**

[SurvS4-class](#), [cenpoisson](#),  
[survreg](#), [leukemia](#).

**Examples**

```
with(leukemia, SurvS4(time, status))
class(with(leukemia, SurvS4(time, status)))
```

---

SurvS4-class	<i>Class "SurvS4"</i>
--------------	-----------------------

---

**Description**

S4 version of the `Surv` class.

**Objects from the Class**

A virtual Class: No objects may be created from it.

**Extends**

Class "Surv", directly. Class "[matrix](#)", directly. Class "[oldClass](#)", by class "Surv", distance 2. Class "[structure](#)", by class "matrix", distance 2. Class "[array](#)", by class "matrix", distance 2. Class "[vector](#)", by class "matrix", distance 3, with explicit coerce. Class "[vector](#)", by class "matrix", distance 4, with explicit coerce.

**Methods**

```
show signature(object = "SurvS4"): ...
```

**Warning**

This code has not been thoroughly tested.

**Note**

The purpose of having [SurvS4](#) in **VGAM** is so that the same input can be fed into [vglm](#) as functions in **survival** such as [survreg](#).



**Author(s)**

T. W. Yee.

**References**

See **survival**.

**See Also**

[SurvS4](#).

**Examples**

```
showClass("SurvS4")
```

---

Tikuv	<i>A Short-tailed Symmetric Distribution</i>
-------	--

---

**Description**

Density, cumulative distribution function, quantile function and random generation for the short-tailed symmetric distribution of Tiku and Vaughan (1999).

**Usage**

```
dtikuv(x, d, mean = 0, sigma = 1, log = FALSE)
ptikuv(q, d, mean = 0, sigma = 1)
qtikuv(p, d, mean = 0, sigma = 1, ...)
rtikuv(n, d, mean = 0, sigma = 1, Smallno = 1.0e-6)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Must be a positive integer of length 1.
d, mean, sigma	arguments for the parameters of the distribution. See <a href="#">tikuv</a> for more details. For <code>rtikuv</code> , arguments <code>mean</code> and <code>sigma</code> must be of length 1.
Smallno	Numeric, a small value used by the rejection method for determining the lower and upper limits of the distribution. That is, <code>ptikuv(L) &lt; Smallno</code> and <code>ptikuv(U) &gt; 1-Smallno</code> where <code>L</code> and <code>U</code> are the lower and upper limits respectively.
...	Arguments that can be passed into <a href="#">uniroot</a> .
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

**Details**

See [tikuv](#) for more details.

**Value**

dtikuv gives the density, ptikuv gives the cumulative distribution function, qtikuv gives the quantile function, and rtikuv generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[tikuv](#).

**Examples**

```
## Not run: par(mfrow = c(2, 1))
x <- seq(-5, 5, len = 401)
plot(x, dnorm(x), type = "l", col = "black", ylab = "", las = 1,
      main = "Black is standard normal, others are dtikuv(x, d)")
lines(x, dtikuv(x, d = -10), col = "orange")
lines(x, dtikuv(x, d = -1 ), col = "blue")
lines(x, dtikuv(x, d = 1 ), col = "green")
legend("topleft", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
      legend = paste("d =", c(-10, -1, 1)))

plot(x, pnorm(x), type = "l", col = "black", ylab = "", las = 1,
      main = "Black is standard normal, others are ptikuv(x, d)")
lines(x, ptikuv(x, d = -10), col = "orange")
lines(x, ptikuv(x, d = -1 ), col = "blue")
lines(x, ptikuv(x, d = 1 ), col = "green")
legend("topleft", col = c("orange", "blue", "green"), lty = rep(1, len = 3),
      legend = paste("d =", c(-10, -1, 1)))
## End(Not run)

probs <- seq(0.1, 0.9, by = 0.1)
ptikuv(qtikuv(p = probs, d = 1), d = 1) - probs # Should be all 0
```

---

tikuv

---

*Short-tailed Symmetric Distribution Family Function*


---

**Description**

Fits the short-tailed symmetric distribution of Tiku and Vaughan (1999).

**Usage**

```
tikuv(d, lmean = "identity", lsigma = "loge",
      isigma = NULL, zero = 2)
```

**Arguments**

<code>d</code>	The $d$ parameter. It must be a single numeric value less than 2. Then $h = 2 - d > 0$ is another parameter.
<code>lmean, lsigma</code>	Link functions for the mean and standard deviation parameters of the usual univariate normal distribution (see <b>Details</b> below). They are $\mu$ and $\sigma$ respectively. See <a href="#">Links</a> for more choices.
<code>isigma</code>	Optional initial value for $\sigma$ . A NULL means a value is computed internally.
<code>zero</code>	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1, 2\}$ corresponding respectively to $\mu, \sigma$ . If <code>zero = NULL</code> then all linear/additive predictors are modelled as a linear combination of the explanatory variables. For many data sets having <code>zero = 2</code> is a good idea.

**Details**

The short-tailed symmetric distribution of Tiku and Vaughan (1999) has a probability density function that can be written

$$f(y) = \frac{K}{\sqrt{2\pi}\sigma} \left[ 1 + \frac{1}{2h} \left( \frac{y - \mu}{\sigma} \right)^2 \right]^2 \exp \left( -\frac{1}{2} (y - \mu)^2 / \sigma^2 \right)$$

where  $h = 2 - d > 0$ ,  $K$  is a function of  $h$ ,  $-\infty < y < \infty$ ,  $\sigma > 0$ . The mean of  $Y$  is  $E(Y) = \mu$  and this is returned as the fitted values.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

Under- or over-flow may occur if the data is ill-conditioned, e.g., when  $d$  is very close to 2 or approaches  $-\text{Inf}$ .

**Note**

The density function is the product of a univariate normal density and a polynomial in the response  $y$ . The distribution is bimodal if  $d > 0$ , else is unimodal. A normal distribution arises as the limit as  $d$  approaches  $-\infty$ , i.e., as  $h$  approaches  $\infty$ . Fisher scoring is implemented. After fitting the value of  $d$  is stored in `@misc` with component name `d`.

**Author(s)**

Thomas W. Yee

## References

- Akkaya, A. D. and Tiku, M. L. (2008) Short-tailed distributions and inliers. *Test*, **17**, 282–296.
- Tiku, M. L. and Vaughan, D. C. (1999) A family of short-tailed symmetric distributions. *Technical report, McMaster University, Canada*.

## See Also

[dtikuv](#), [uninormal](#).

## Examples

```
m <- 1.0; sigma <- exp(0.5)
tdata <- data.frame(y = rtikuv(n = 1000, d = 1, m = m, s = sigma))
tdata <- transform(tdata, sy = sort(y))
fit <- vglm(y ~ 1, tikuv(d = 1), data = tdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
with(tdata, mean(y))
## Not run: with(tdata, hist(y, prob = TRUE))
lines(dtikuv(sy, d = 1, m = Cfit[1], s = Cfit[2]) ~ sy, tdata, col = "orange")
## End(Not run)
```

---

Tobit

---

*The Tobit Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the Tobit model.

## Usage

```
dtobit(x, mean = 0, sd = 1, Lower = 0, Upper = Inf, log = FALSE)
ptobit(q, mean = 0, sd = 1, Lower = 0, Upper = Inf,
       lower.tail = TRUE, log.p = FALSE)
qtobit(p, mean = 0, sd = 1, Lower = 0, Upper = Inf)
rtobit(n, mean = 0, sd = 1, Lower = 0, Upper = Inf)
```

## Arguments

**x, q**                vector of quantiles.

**p**                    vector of probabilities.

**n**                    number of observations. If `length(n) > 1` then the length is taken to be the number required.

**Lower, Upper**        vector of lower and upper thresholds.

**mean, sd, lower.tail, log, log.p**  
                       see [rnorm](#).

## Details

See [tobit](#), the **VGAM** family function for estimating the parameters, for details. Note that the density at Lower and Upper is the value of [dnorm](#) evaluated there plus the area to the left/right of that point too. Thus there are two spikes; see the example below.

## Value

`dtobit` gives the density, `ptobit` gives the distribution function, `qtobit` gives the quantile function, and `rtobit` generates random deviates.

## Author(s)

T. W. Yee

## See Also

[tobit](#).

## Examples

```
## Not run: m <- 0.5; x <- seq(-2, 4, len = 501)
Lower <- -1; Upper <- 2.5
plot(x, ptobit(x, m = m, Lower = Lower, Upper = Upper),
     type = "l", ylim = 0:1, las = 1, col = "orange",
     ylab = paste("ptobit(m = ", m, ", sd = 1, Lower =", Lower,
                  ", Upper =", Upper, ")"),
     main = "Orange is cumulative distribution function; blue is density",
     sub = "Purple lines are the 10,20,...,90 percentiles")
abline(h = 0)
lines(x, dtobit(x, m = m, Lower = Lower, Upper = Upper), col = "blue")

probs <- seq(0.1, 0.9, by = 0.1)
Q <- qtobit(probs, m = m, Lower = Lower, Upper = Upper)
lines(Q, ptobit(Q, m = m, Lower = Lower, Upper = Upper),
     col = "purple", lty = "dashed", type = "h")
lines(Q, dtobit(Q, m = m, Lower = Lower, Upper = Upper),
     col = "darkgreen", lty = "dashed", type = "h")
abline(h = probs, col = "purple", lty = "dashed")
max(abs(ptobit(Q, m = m, Lower = Lower, Upper = Upper) - probs)) # Should be 0

endpts <- c(Lower, Upper) # Endpoints have a spike
lines(endpts, dtobit(endpts, m = m, Lower = Lower, Upper = Upper),
     col = "blue", lwd = 2, type = "h")

## End(Not run)
```

---

tobit	<i>Tobit Model</i>
-------	--------------------

---

**Description**

Fits a Tobit model.

**Usage**

```
tobit(Lower = 0, Upper = Inf, lmu = "identity", lsd = "loge",
      nsimEIM = 250, imu = NULL, isd = NULL,
      type.fitted = c("uncensored", "censored", "mean.obs"),
      imethod = 1, zero = -2)
```

**Arguments**

Lower	Numeric. It is the value $L$ described below. Any value of the linear model $x_i^T \beta$ that is less than this lowerbound is assigned this value. Hence this should be the smallest possible value in the response variable. May be a vector (see below for more information).
Upper	Numeric. It is the value $U$ described below. Any value of the linear model $x_i^T \beta$ that is greater than this upperbound is assigned this value. Hence this should be the largest possible value in the response variable. May be a vector (see below for more information).
lmu, lsd	Parameter link functions for the mean and standard deviation parameters. See <a href="#">Links</a> for more choices. The standard deviation is a positive quantity, therefore a log link is its default.
imu, isd	See <a href="#">CommonVGAMffArguments</a> for information.
type.fitted	Type of fitted value returned. The first choice is default and is the ordinary uncensored or unbounded linear model. If "censored" then the fitted values in the interval $[L, U]$ . If "mean.obs" then the mean of the observations is returned; this is a doubly truncated normal distribution augmented by point masses at the truncation points (see <a href="#">dtobit</a> ).
imethod	Initialization method. Either 1 or 2, this specifies two methods for obtaining initial values for the parameters.
nsimEIM	Used if nonstandard Tobit model. See <a href="#">CommonVGAMffArguments</a> for information.
zero	An integer vector, containing the value 1 or 2. If so, the mean or standard deviation respectively are modelled as an intercept-only. Setting zero = NULL means both linear/additive predictors are modelled as functions of the explanatory variables.

## Details

The Tobit model can be written

$$y_i^* = x_i^T \beta + \varepsilon_i$$

where the  $\varepsilon_i \sim N(0, \sigma^2)$  independently and  $i = 1, \dots, n$ . However, we measure  $y_i = y_i^*$  only if  $y_i^* > L$  and  $y_i^* < U$  for some cutpoints  $L$  and  $U$ . Otherwise we let  $y_i = L$  or  $y_i = U$ , whatever is closer. The Tobit model is thus a multiple linear regression but with censored responses if it is below or above certain cutpoints.

The defaults for Lower and Upper and lmu correspond to the *standard* Tobit model. Then Fisher scoring is used, else simulated Fisher scoring. By default, the mean  $x_i^T \beta$  is the first linear/additive predictor, and the log of the standard deviation is the second linear/additive predictor. The Fisher information matrix for uncensored data is diagonal. The fitted values are the estimates of  $x_i^T \beta$ .

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

## Warning

Convergence is often slow. Setting `crit = "coeff"` is recommended since premature convergence of the log-likelihood is common. Simulated Fisher scoring is implemented for the nonstandard Tobit model. For this, the working weight matrices for some observations are prone to not being positive-definite; if so then some checking of the final model is recommended and/or try inputting some initial values.

## Note

The response can be a matrix. If so, then Lower and Upper are recycled into a matrix with the number of columns equal to the number of responses, and the recycling is done row-wise (`byrow = TRUE`). For example, these are returned in `fit4@misc$Lower` and `fit4@misc$Upper` below.

If there is no censoring then [uninormal](#) is recommended instead. Any value of the response less than Lower or greater than Upper will be assigned the value Lower and Upper respectively, and a warning will be issued. The fitted object has components `censoredL` and `censoredU` in the extra slot which specifies whether observations are censored in that direction. The function [cennormal](#) is an alternative to `tobit()`.

## Author(s)

Thomas W. Yee

## References

Tobin, J. (1958) Estimation of relationships for limited dependent variables. *Econometrica* **26**, 24–36.

**See Also**

[rtobit](#), [cennormal](#), [uninormal](#), [double.cennormal](#), [posnormal](#), [rnorm](#).

**Examples**

```
## Not run:
# Here, fit1 is a standard Tobit model and fit2 is a nonstandard Tobit model
tdata <- data.frame(x2 = seq(-1, 1, length = (nn <- 100)))
set.seed(1)
Lower <- 1; Upper <- 4 # For the nonstandard Tobit model
tdata <- transform(tdata,
  Lower.vec = rnorm(nn, Lower, 0.5),
  Upper.vec = rnorm(nn, Upper, 0.5))

meanfun1 <- function(x) 0 + 2*x
meanfun2 <- function(x) 2 + 2*x
meanfun3 <- function(x) 2 + 2*x
meanfun4 <- function(x) 3 + 2*x
tdata <- transform(tdata,
  y1 = rtobit(nn, mean = meanfun1(x2)), # Standard Tobit model
  y2 = rtobit(nn, mean = meanfun2(x2), Lower = Lower, Upper = Upper),
  y3 = rtobit(nn, mean = meanfun3(x2), Lower = Lower.vec, Upper = Upper.vec),
  y4 = rtobit(nn, mean = meanfun4(x2), Lower = Lower.vec, Upper = Upper.vec))
with(tdata, table(y1 == 0)) # How many censored values?
with(tdata, table(y2 == Lower | y2 == Upper)) # How many censored values?
with(tdata, table(attr(y2, "cenL")))
with(tdata, table(attr(y2, "cenU")))

fit1 <- vglm(y1 ~ x2, tobit, tdata, trace = TRUE,
  crit = "coeff") # crit = "coeff" is recommended
coef(fit1, matrix = TRUE)
summary(fit1)

fit2 <- vglm(y2 ~ x2, tobit(Lower = Lower, Upper = Upper, type.f = "cens"),
  tdata, crit = "coeff", trace = TRUE) # ditto
table(fit2@extra$censoredL)
table(fit2@extra$censoredU)
coef(fit2, matrix = TRUE)

fit3 <- vglm(y3 ~ x2,
  tobit(Lower = with(tdata, Lower.vec),
    Upper = with(tdata, Upper.vec), type.f = "cens"),
  tdata, crit = "coeff", trace = TRUE) # ditto
table(fit3@extra$censoredL)
table(fit3@extra$censoredU)
coef(fit3, matrix = TRUE)

# fit4 is fit3 but with type.fitted = "uncen".
fit4 <- vglm(cbind(y3, y4) ~ x2,
  tobit(Lower = rep(with(tdata, Lower.vec), each = 2),
    Upper = rep(with(tdata, Upper.vec), each = 2)),
  tdata, crit = "coeff", trace = TRUE) # ditto
head(fit4@extra$censoredL) # A matrix
```



```

head(fit4@extra$censoredU) # A matrix
head(fit4@misc$Lower)      # A matrix
head(fit4@misc$Upper)      # A matrix
coef(fit4, matrix = TRUE)

## End(Not run)

## Not run: # Plot the results
par(mfrow = c(2, 2))
# Plot fit1
plot(y1 ~ x2, tdata, las = 1, main = "Standard Tobit model",
     col = as.numeric(attr(y1, "cenL")) + 3,
     pch = as.numeric(attr(y1, "cenL")) + 1)
legend(x = "topleft", leg = c("censored", "uncensored"),
      pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 2.5, c("Truth", "Estimate", "Naive"),
      col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun1(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit1) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y1 ~ x2, tdata)) ~ x2, tdata, col = "black",
      lty = 2, lwd = 2) # This is simplest but wrong!

# Plot fit2
plot(y2 ~ x2, tdata, las = 1, main = "Tobit model",
     col = as.numeric(attr(y2, "cenL")) + 3 +
           as.numeric(attr(y2, "cenU")),
     pch = as.numeric(attr(y2, "cenL")) + 1 +
           as.numeric(attr(y2, "cenU")))
legend(x = "topleft", leg = c("censored", "uncensored"),
      pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
      col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun2(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit2) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y2 ~ x2, tdata)) ~ x2, tdata, col = "black",
      lty = 2, lwd = 2) # This is simplest but wrong!

# Plot fit3
plot(y3 ~ x2, tdata, las = 1,
     main = "Tobit model with nonconstant censor levels",
     col = as.numeric(attr(y3, "cenL")) + 3 +
           as.numeric(attr(y3, "cenU")),
     pch = as.numeric(attr(y3, "cenL")) + 1 +
           as.numeric(attr(y3, "cenU")))
legend(x = "topleft", leg = c("censored", "uncensored"),
      pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
      col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun3(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit3) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y3 ~ x2, tdata)) ~ x2, tdata, col = "black",
      lty = 2, lwd = 2) # This is simplest but wrong!

```

```
# Plot fit4
plot(y3 ~ x2, tdata, las = 1,
      main = "Tobit model with nonconstant censor levels",
      col = as.numeric(attr(y3, "cenL")) + 3 +
            as.numeric(attr(y3, "cenU")),
      pch = as.numeric(attr(y3, "cenL")) + 1 +
            as.numeric(attr(y3, "cenU")))
legend(x = "topleft", leg = c("censored", "uncensored"),
       pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
       col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun3(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit4)[, 1] ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y3 ~ x2, tdata)) ~ x2, tdata, col = "black",
      lty = 2, lwd = 2) # This is simplest but wrong!

## End(Not run)
```

---

Tol	<i>Tolerances</i>
-----	-------------------

---

**Description**

Generic function for the *tolerances* of a model.

**Usage**

```
Tol(object, ...)
```

**Arguments**

- object            An object for which the computation or extraction of a tolerance or tolerances is meaningful.
- ...              Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for [Coef](#).

**Details**

Different models can define an optimum in different ways. Many models have no such notion or definition.

Tolerances occur in quadratic ordination, i.e., CQO. They have ecological meaning because a high tolerance for a species means the species can survive over a large environmental range (stenoecous species), whereas a small tolerance means the species' niche is small (eurycous species). Mathematically, the tolerance is like the variance of a normal distribution.

**Value**

The value returned depends specifically on the methods function invoked.

**Warning**

There is a direct inverse relationship between the scaling of the latent variables (site scores) and the tolerances. One normalization is for the latent variables to have unit variance. Another normalization is for all the tolerances to be unit. These two normalization cannot simultaneously hold in general. For rank- $R > I$  models it becomes more complicated because the latent variables are also uncorrelated. An important argument when fitting quadratic ordination models is whether `EqualTolerances` is `TRUE` or `FALSE`. See Yee (2004) for details.

**Note**

Tolerances are undefined for ‘linear’ and additive ordination models. They are well-defined for quadratic ordination models.

**Author(s)**

Thomas W. Yee

**References**

- Yee, T. W. (2004) A new technique for maximum-likelihood canonical Gaussian ordination. *Ecological Monographs*, **74**, 685–701.
- Yee, T. W. (2006) Constrained additive ordination. *Ecology*, **87**, 203–213.

**See Also**

`Tol.qrrvglm`. [Max](#), [Opt](#).

**Examples**

```
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cgo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
               Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
               Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          Bestof = 2, quasipoissonff, data = hspider, Crowlpositive = FALSE)

Tol(p1)
```

---

toxop

*Toxoplasmosis Data*

---

**Description**

Toxoplasmosis data in 34 cities in El Salvador.

**Usage**

```
data(toxop)
```

**Format**

A data frame with 34 observations on the following 4 variables.

`rainfall` a numeric vector; the amount of rainfall in each city.

`ssize` a numeric vector; sample size.

`cityNo` a numeric vector; the city number.

`positive` a numeric vector; the number of subjects testing positive for the disease.

**Details**

See the references for details.

**Source**

See the references for details.

**References**

Efron, B. (1978) Regression and ANOVA With zero-one data: measures of residual variation. *Journal of the American Statistical Association*, **73**, 113–121.

Efron, B. (1986) Double exponential families and their use in generalized linear regression. *Journal of the American Statistical Association*, **81**, 709–721.

**See Also**

[double.expbinoial](#).

**Examples**

```
## Not run: with(toxop, plot(rainfall, positive / ssize, col = "blue"))
plot(toxop, col = "blue")
## End(Not run)
```

---

Triangle

---

*The Triangle Distribution*


---

**Description**

Density, distribution function, quantile function and random generation for the Triangle distribution with parameter `theta`.

**Usage**

```
dtriangle(x, theta, lower = 0, upper = 1, log = FALSE)
ptriangle(q, theta, lower = 0, upper = 1)
qtriangle(p, theta, lower = 0, upper = 1)
rttriangle(n, theta, lower = 0, upper = 1)
```

**Arguments**

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. Same as <a href="#">runif</a> .
theta	the theta parameter which lies between lower and upper.
lower, upper	lower and upper limits of the distribution. Must be finite.
log	Logical. If log = TRUE then the logarithm of the density is returned.

**Details**

See [triangle](#), the **VGAM** family function for estimating the parameter  $\theta$  by maximum likelihood estimation.

**Value**

dtriangle gives the density, pttriangle gives the distribution function, qtriangle gives the quantile function, and rttriangle generates random deviates.

**Author(s)**

T. W. Yee

**See Also**

[triangle](#).

**Examples**

```
## Not run: x <- seq(-0.1, 1.1, by = 0.01); theta <- 0.75
plot(x, dtriangle(x, theta = theta), type = "l", col = "blue", las = 1,
     main = "Blue is density, orange is cumulative distribution function",
     sub = "Purple lines are the 10,20,...,90 percentiles",
     ylim = c(0,2), ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pttriangle(x, theta = theta), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qtriangle(probs, theta = theta)
lines(Q, dtriangle(Q, theta = theta), col = "purple", lty = 3, type = "h")
pttriangle(Q, theta = theta) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

triangle

*Triangle Distribution Family Function***Description**

Estimating the parameter of the triangle distribution by maximum likelihood estimation.

**Usage**

```
triangle(lower = 0, upper = 1,
         link = elogit(min = 0, max = 1), itheta = NULL)
```

**Arguments**

lower, upper	lower and upper limits of the distribution. Must be finite. Called $A$ and $B$ respectively below.
link	Parameter link function applied to the parameter $\theta$ , which lies in $(A, B)$ . See <a href="#">Links</a> for more choices. The default constrains the estimate to lie in the interval.
itheta	Optional initial value for the parameter. The default is to compute the value internally.

**Details**

The triangle distribution has a probability density function that consists of two lines joined at  $\theta$ , which is the location of the mode. The lines intersect the  $y = 0$  axis at  $A$  and  $B$ . Here, Fisher scoring is used.

On fitting, the extra slot has components called lower and upper which contains the values of the above arguments (recycled to the right length). The fitted values are the mean of the distribution, which is  $(A + B + \theta)/3$ .

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Warning**

The MLE regularity conditions do not seem to hold for this distribution so that misleading inferences may result, e.g., in the summary and vcov of the object. Additionally, convergence to the MLE often appears to fail.

**Note**

The response must contain values in  $(A, B)$ . For most data sets (especially small ones) it is very common for half-stepping to occur.

Arguments lower and upper and link must match. For example, setting lower = 0.2 and upper = 4 and link = elogit(min = 0.2, max = 4.1) will result in an error. Ideally link = elogit(min = lower, max = upper) ought to work but it does not (yet)! Minimal error checking is done for this deficiency.

**Author(s)**

T. W. Yee

**References**

Kotz, S. and van Dorp, J. R. (2004) Beyond Beta: Other Continuous Families of Distributions with Bounded Support and Applications. Chapter 1. World Scientific: Singapore.

**See Also**

[Triangle](#).

**Examples**

```
# Example 1
tdata <- data.frame(y = rtriangle(n <- 3000, theta = 3/4))
fit <- vglm(y ~ 1, triangle(link = "identity"), tdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fit@extra$lower)
head(fitted(fit))
with(tdata, mean(y))

# Example 2; Kotz and van Dorp (2004), p.14
rdata <- data.frame(y = c(0.1, 0.25, 0.3, 0.4, 0.45, 0.6, 0.75, 0.8))
fit <- vglm(y ~ 1, triangle(link = "identity"), rdata, trace = TRUE,
            crit = "coef", maxit = 1000)
Coef(fit) # The MLE is the 3rd order statistic, which is 0.3.
fit <- vglm(y ~ 1, triangle(link = "identity"), rdata, trace = TRUE,
            crit = "coef", maxit = 1001)
Coef(fit) # The MLE is the 3rd order statistic, which is 0.3.
```

---

trplot

*Trajectory Plot*


---

**Description**

Generic function for a trajectory plot.

**Usage**

```
trplot(object, ...)
```

**Arguments**

object	An object for which a trajectory plot is meaningful.
...	Other arguments fed into the specific methods function of the model. They usually are graphical parameters, and sometimes they are fed into the methods function for <a href="#">Coef</a> .

**Details**

Trajectory plots can be defined in different ways for different models. Many models have no such notion or definition.

For quadratic and additive ordination models they plot the fitted values of two species against each other (more than two is theoretically possible, but not implemented in this software yet).

**Value**

The value returned depends specifically on the methods function invoked.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2012) On constrained and unconstrained quadratic ordination. *Manuscript in preparation*.

**See Also**

[trplot.qrrvglm](#), [perspqrrvglm](#), [lvplot](#).

**Examples**

```
## Not run: set.seed(123)
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
p1cqo <- cqq(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi,
                  Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
                  Trocterr, Zoraspin) ~
            WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
            quasipoissonff, data = hspider, Crowlpositive = FALSE)

nos <- ncol(depvar(p1cqo))
clr <- 1:nos # OR (1:(nos+1))[-7] to omit yellow

trplot(p1cqo, which.species = 1:3, log = "xy",
       col = c("blue", "orange", "green"), lwd = 2, label = TRUE) -> ii
legend(0.00005, 0.3, paste(ii$species[, 1], ii$species[, 2], sep = " and " ),
```



```

      lwd = 2, lty = 1, col = c("blue", "orange", "green"))
abline(a = 0, b = 1, lty = "dashed", col = "grey")
## End(Not run)

```

trplot.qrrvglm

Trajectory plot for QRR-VGLMs

## Description

Produces a trajectory plot for *quadratic reduced-rank vector generalized linear models* (QRR-VGLMs). It is only applicable for rank-1 models with argument `noRRR = ~ 1`.

## Usage

```

trplot.qrrvglm(object, which.species = NULL, add = FALSE, show.plot = TRUE,
  label.sites = FALSE, sitenames = rownames(object@y),
  axes.equal = TRUE, cex = par()$cex,
  col = 1:(nos * (nos - 1)/2), log = "",
  lty = rep(par()$lty, length.out = nos * (nos - 1)/2),
  lwd = rep(par()$lwd, length.out = nos * (nos - 1)/2),
  tcol = rep(par()$col, length.out = nos * (nos - 1)/2),
  xlab = NULL, ylab = NULL,
  main = "", type = "b", check.ok = TRUE, ...)

```

## Arguments

<code>object</code>	Object of class "qrrvglm", i.e., a CQO object.
<code>which.species</code>	Integer or character vector specifying the species to be plotted. If integer, these are the columns of the response matrix. If character, these must match exactly with the species' names. The default is to use all species.
<code>add</code>	Logical. Add to an existing plot? If FALSE (default), a new plot is made.
<code>show.plot</code>	Logical. Plot it?
<code>label.sites</code>	Logical. If TRUE, the points on the curves/trajectories are labelled with the <code>sitenames</code> .
<code>sitenames</code>	Character vector. The names of the sites.
<code>axes.equal</code>	Logical. If TRUE, the x- and y-axes will be on the same scale.
<code>cex</code>	Character expansion of the labelling of the site names. Used only if <code>label.sites</code> is TRUE. See the <code>cex</code> argument in <a href="#">par</a> .
<code>col</code>	Color of the lines. See the <code>col</code> argument in <a href="#">par</a> . Here, <code>nos</code> is the number of species.
<code>log</code>	Character, specifying which (if any) of the x- and y-axes are to be on a logarithmic scale. See the <code>log</code> argument in <a href="#">par</a> .
<code>lty</code>	Line type. See the <code>lty</code> argument of <a href="#">par</a> .
<code>lwd</code>	Line width. See the <code>lwd</code> argument of <a href="#">par</a> .

tcol	Color of the text for the site names. See the col argument in <a href="#">par</a> . Used only if label.sites is TRUE.
xlab	Character caption for the x-axis. By default, a suitable caption is found. See the xlab argument in <a href="#">plot</a> or <a href="#">title</a> .
ylab	Character caption for the y-axis. By default, a suitable caption is found. See the xlab argument in <a href="#">plot</a> or <a href="#">title</a> .
main	Character, giving the title of the plot. See the main argument in <a href="#">plot</a> or <a href="#">title</a> .
type	Character, giving the type of plot. A common option is to use type="l" for lines only. See the type argument of <a href="#">plot</a> .
check.ok	Logical. Whether a check is performed to see that noRRR = ~ 1 was used. It doesn't make sense to have a trace plot unless this is so.
...	Arguments passed into the plot function when setting up the entire plot. Useful arguments here include xlim and ylim.

### Details

A trajectory plot plots the fitted values of a 'second' species against a 'first' species. The argument which.species must therefore contain at least two species. By default, all of the species that were fitted in object are plotted. With more than a few species the resulting plot will be very congested, and so it is recommended that only a few species be selected for plotting.

In the above,  $M$  is the number of species selected for plotting, so there will be  $M(M - 1)/2$  curves/trajectories in total.

A trajectory plot will be fitted only if noRRR = ~ 1 because otherwise the trajectory will not be a smooth function of the latent variables.

### Value

A list with the following components.

species.names	A matrix of characters giving the 'first' and 'second' species. The number of different combinations of species is given by the number of rows. This is useful for creating a legend.
sitenames	A character vector of site names, sorted by the latent variable (from low to high).

### Note

Plotting the axes on a log scale is often a good idea. The use of xlim and ylim to control the axis limits is also a good idea, so as to limit the extent of the curves at low abundances or probabilities. Setting label.sites = TRUE is a good idea only if the number of sites is small, otherwise there is too much clutter.

### Author(s)

Thomas W. Yee

## References

Yee, T. W. (2012) On constrained and unconstrained quadratic ordination. *Manuscript in preparation*.

## See Also

`cqo`, `par`, `title`.

## Examples

```
## Not run: set.seed(111) # This leads to the global solution
# hspider[,1:6] <- scale(hspider[,1:6]) # Standardize the environmental variables
p1 <- cqo(cbind(Alopacce, Alopcone, Alopfabr, Arctlute, Arctperi, Auloalbi,
               Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
          WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
          poissonff, data = hspider, trace = FALSE)

trplot(p1, which.species = 1:3, log = "xy", type = "b", lty = 1,
       main = "Trajectory plot of three hunting spiders species",
       col = c("blue", "red", "green"), lwd = 2, label = TRUE) -> ii
legend(0.00005, 0.3, lwd = 2, lty = 1, col = c("blue", "red", "green"),
      with(ii, paste(species.names[1], species.names[2], sep = " and ")))
abline(a = 0, b = 1, lty = "dashed", col = "grey") # Useful reference line

## End(Not run)
```

---

Truncpareto

*The Truncated Pareto Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the upper truncated Pareto(I) distribution with parameters lower, upper and shape.

## Usage

```
dtruncpareto(x, lower, upper, shape, log = FALSE)
ptruncpareto(q, lower, upper, shape)
qtruncpareto(p, lower, upper, shape)
rtruncpareto(n, lower, upper, shape)
```

## Arguments

<code>x</code> , <code>q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code> , <code>log</code>	Same meaning as <code>runif</code> .
<code>lower</code> , <code>upper</code> , <code>shape</code>	the lower, upper and shape ( $k$ ) parameters. If necessary, values are recycled.

## Details

See [truncpareto](#), the **VGAM** family function for estimating the parameter  $k$  by maximum likelihood estimation, for the formula of the probability density function and the range restrictions imposed on the parameters.

## Value

`dtruncpareto` gives the density, `ptruncpareto` gives the distribution function, `qtruncpareto` gives the quantile function, and `rtruncpareto` generates random deviates.

## Author(s)

T. W. Yee

## References

Aban, I. B., Meerschaert, M. M. and Panorska, A. K. (2006) Parameter estimation for the truncated Pareto distribution, *Journal of the American Statistical Association*, **101**(473), 270–277.

## See Also

[truncpareto](#).

## Examples

```
lower <- 3; upper <- 8; kay <- exp(0.5)
## Not run: xx <- seq(lower - 0.5, upper + 0.5, len = 401)
plot(xx, dtruncpareto(xx, low = lower, upp = upper, shape = kay),
     main = "Truncated Pareto density split into 10 equal areas",
     type = "l", ylim = 0:1, xlab = "x")
abline(h = 0, col = "blue", lty = 2)
qq <- qtruncpareto(seq(0.1, 0.9, by = 0.1), low = lower, upp = upper,
                  shape = kay)
lines(qq, dtruncpareto(qq, low = lower, upp = upper, shape = kay),
      col = "purple", lty = 3, type = "h")
lines(xx, ptruncpareto(xx, low = lower, upp = upper, shape = kay),
      col = "orange")
## End(Not run)
pp <- seq(0.1, 0.9, by = 0.1)
qq <- qtruncpareto(pp, low = lower, upp = upper, shape = kay)

ptruncpareto(qq, low = lower, upp = upper, shape = kay)
qtruncpareto(ptruncpareto(qq, low = lower, upp = upper, shape = kay),
             low = lower, upp = upper, shape = kay) - qq # Should be all 0
```

truncweibull

*Truncated Weibull Distribution Family Function*

## Description

Maximum likelihood estimation of the 2-parameter Weibull distribution with lower truncation. No observations should be censored.

## Usage

```
truncweibull(lower.limit = 1e-5,
             lAlpha = "log", lBetaa = "log",
             iAlpha = NULL, iBetaa = NULL,
             nrfs = 1, probs.y = c(0.2, 0.5, 0.8),
             imethod = 1, zero = -2)
```

## Arguments

`lower.limit` Positive lower truncation limits. Recycled to the same dimension as the response, going across rows first. The default, being close to 0, should mean effectively the same results as [weibull](#) if there are no response values that are smaller.

`lAlpha`, `lBetaa` Parameter link functions applied to the (positive) parameters Alpha (called  $\alpha$  below) and (positive) Betaa (called  $\beta$  below). See [Links](#) for more choices.

`iAlpha`, `iBetaa` See [CommonVGAMffArguments](#).

`imethod`, `nrfs`, `zero`, `probs.y` Details at [weibull](#).

## Details

MLE of the two parameters of the Weibull distribution are computed, subject to lower truncation. That is, all response values are greater than `lower.limit`, element-wise. For a particular observation this is any known positive value. This function is currently based directly on Wingo (1989) and his parameterization is used (it differs from [weibull](#).) In particular,  $\beta = a$  and  $\alpha = (1/b)^a$  where  $a$  and  $b$  are as in [weibull](#) and [dweibull](#).

Upon fitting the extra slot has a component called `lower.limit` which is of the same dimension as the response. The fitted values are the mean, which are computed using [pgamma.deriv](#) and [pgamma.deriv.unscaled](#).

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

This function may be converted to the same parameterization as [weibull](#) at any time. Yet to do: one element of the EIM may be wrong (due to two interpretations of a formula; but it seems to work). Convergence is slower than usual and this may imply something is wrong; use argument `maxit`. In fact, it's probably because [pgamma.deriv.unscaled](#) is inaccurate at  $q = 1$  and  $q = 2$ . Also, convergence should be monitored, especially if the truncation means that a large proportion of the data is lost compared to an ordinary Weibull distribution.

**Note**

More improvements need to be made, e.g., initial values are currently based on no truncation. This **VGAM** family function handles multiple responses.

**Author(s)**

T. W. Yee

**References**

Wingo, D. R. (1989) The left-truncated Weibull distribution: theory and computation. *Statistical Papers*, **30**(1), 39–48.

**See Also**

[weibull](#), [dweibull](#), [pgamma.deriv](#), [pgamma.deriv.unscaled](#).

**Examples**

```
nn <- 5000; prop.lost <- 0.40 # Proportion lost to truncation
wdata <- data.frame(x2 = runif(nn)) # Complete Weibull data
wdata <- transform(wdata,
                   Betaa = exp(1)) # > 2 is okay (satisfies regularity conds)
wdata <- transform(wdata, Alpha = exp(0.5 - 1 * x2))
wdata <- transform(wdata, Shape = Betaa,
#                   aaa = Betaa,
#                   bbb = 1 / Alpha^(1 / Betaa),
#                   Scale = 1 / Alpha^(1 / Betaa))
wdata <- transform(wdata, y2 = rweibull(nn, shape = Shape, scale = Scale))
summary(wdata)

lower.limit2 <- with(wdata, quantile(y2, prob = prop.lost)) # Proportion lost
wdata <- subset(wdata, y2 > lower.limit2) # Smaller due to truncation

fit1 <- vglm(y2 ~ x2, maxit = 100, trace = TRUE,
             truncweibull(lower.limit = lower.limit2), data = wdata)
coef(fit1, matrix = TRUE)
summary(fit1)
vcov(fit1)
head(fit1@extra$lower.limit)
```

---

ucberk*University California Berkeley Graduate Admissions*

---

## Description

University California Berkeley Graduate Admissions: counts cross-classified by acceptance/rejection and gender, for the six largest departments.

## Usage

```
data(ucberk)
```

## Format

A data frame with 6 departmental groups with the following 5 columns.

**m.deny** Counts of men denied admission.

**m.admit** Counts of men admitted.

**w.deny** Counts of women denied admission.

**w.admit** Counts of women admitted.

**dept** Department (the six largest), called A, codeB, . . . , codeF.

## Details

From Bickel et al. (1975), the data consists of applications for admission to graduate study at the University of California, Berkeley, for the fall 1973 quarter. In the admissions cycle for that quarter, the Graduate Division at Berkeley received approximately 15,000 applications, some of which were later withdrawn or transferred to a different proposed entry quarter by the applicants. Of the applications finally remaining for the fall 1973 cycle 12,763 were sufficiently complete to permit a decision. There were about 101 graduate department and interdepartmental graduate majors. There were 8442 male applicants and 4321 female applicants. About 44 percent of the males and about 35 percent of the females were admitted. The data are well-known for illustrating Simpson's paradox.

## References

Bickel, P. J., Hammel, E. A. and O'Connell, J. W. (1975) Sex bias in graduate admissions: data from Berkeley. *Science*, **187**(4175): 398–404.

Freedman, D., Pisani, R. and Purves, R. (1998) Chapter 2 of *Statistics*, 3rd. ed., W. W. Norton & Company.

## Examples

```
summary(ucberk)
```

uninormal

*Univariate Normal Distribution***Description**

Maximum likelihood estimation of the two parameters of a univariate normal distribution.

**Usage**

```
uninormal(lmean = "identity", lsd = "loge", lvar = "loge",
          var.arg = FALSE, imethod = 1, isd = NULL, parallel = FALSE,
          smallno = 1e-05, zero = -2)
```

**Arguments**

<code>lmean, lsd, lvar</code>	Link functions applied to the mean and standard deviation/variance. See <a href="#">Links</a> for more choices. Being positive quantities, a log link is the default for the standard deviation and variance (see <code>var.arg</code> ).
<code>var.arg</code>	Logical. If TRUE then the second parameter is the variance and <code>lsd</code> and <code>esd</code> are ignored, else the standard deviation is used and <code>lvar</code> and <code>evvar</code> are ignored.
<code>smallno</code>	Numeric, positive but close to 0. Used specifically for quasi-variances; if the link for the mean is <a href="#">explink</a> then any non-positive value of $\eta$ is replaced by this quantity (hopefully, temporarily and only during early iterations).
<code>imethod, parallel, isd, zero</code>	See <a href="#">CommonVGAMffArguments</a> for more information. If <code>lmean = loge</code> then try <code>imethod = 2</code> . If <code>parallel = TRUE</code> then the parallelism constraint is not applied to the intercept.

**Details**

This fits a linear model (LM) as the first linear/additive predictor. So, by default, this is just the mean. By default, the log of the standard deviation is the second linear/additive predictor. The Fisher information matrix is diagonal. This **VGAM** family function can handle multiple responses.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

`uninormal()` is the new name; `normal1()` is old and will be decommissioned soon.

**Note**

Yet to do: allow an argument such as `eq.sd` that enables the standard deviations to be the same.



**Author(s)**

T. W. Yee

**References**

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[gaussianff](#), [posnormal](#), [mix2normal](#), [normal.vcm](#), [Qvar](#), [tobit](#), [cennormal](#), [foldnormal](#), [skewnormal](#), [double.cennormal](#), [SUR](#), [huber2](#), [studentt](#), [binormal](#), [dnorm](#).

**Examples**

```
ndata <- data.frame(x2 = rnorm(nn <- 200))
ndata <- transform(ndata,
  y1 = rnorm(nn, m = 1 - 3*x2, sd = exp(1 + 0.2*x2)),
  y2a = rnorm(nn, m = 1 + 2*x2, sd = exp(1 + 2.0*x2)^0.5),
  y2b = rnorm(nn, m = 1 + 2*x2, sd = exp(1 + 2.0*x2)^0.5))
fit1 <- vglm(y1 ~ x2, uninormal(zero = NULL), ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
fit2 <- vglm(cbind(y2a, y2b) ~ x2, data = ndata, trace = TRUE,
  uninormal(var = TRUE, parallel = TRUE ~ x2,
    zero = NULL))
coef(fit2, matrix = TRUE)

# Generate data from N(mu = theta = 10, sigma = theta) and estimate theta.
theta <- 10
ndata <- data.frame(y3 = rnorm(100, m = theta, sd = theta))
fit3a <- vglm(y3 ~ 1, uninormal(lsd = "identity"), ndata,
  constraints = list("(Intercept)" = rbind(1, 1)))
fit3b <- vglm(y3 ~ 1, uninormal(lsd = "identity", parallel = TRUE ~ 1,
  zero = NULL), ndata)
coef(fit3a, matrix = TRUE)
coef(fit3b, matrix = TRUE) # Same as fit3a
```

**Description**

A small count data set. During WWII V1 flying-bombs were fired from sites in France (Pas-de-Calais) and Dutch coasts towards London. The number of hits per square grid around London were recorded.

**Usage**

```
data(V1)
```

## Format

A data frame with the following variables.

**hits** Values between 0 and 4, and 7. Actually, the 7 is really imputed from the paper (it was recorded as "5 and over").

**ofreq** Observed frequency, i.e., the number of grids with that many hits.

## Details

The data concerns 576 square grids each of 0.25 square kms about south London. The area was selected comprising 144 square kms over which the basic probability function of the distribution was very nearly constant. V1s, which were one type of flying-bomb, were a "Vergeltungswaffen" or vengeance weapon fired during the summer of 1944 at London. The V1s were informally called Buzz Bombs or Doodlebugs, and they were pulse-jet-powered with a warhead of 850 kg of explosives. Over 9500 were launched at London, and many were shot down by artillery and the RAF. Over the period considered the total number of bombs within the area was 537.

It was asserted that the bombs tended to be grouped in clusters. However, a basic Poisson analysis shows this is not the case. Their guidance system being rather primitive, the data is consistent with a Poisson distribution (random).

## Source

Clarke, R. D. (1946). An application of the Poisson distribution. *Journal of the Institute of Actuaries*, **72**(3), 481.

## References

Feller, W. (1970). *An Introduction to Probability Theory and Its Applications*, Vol. 1, Third Edition. John Wiley and Sons: New York, USA.

## See Also

[poissonff](#).

## Examples

```
V1
mean(with(V1, rep(hits, times = ofreq)))
var(with(V1, rep(hits, times = ofreq)))
sum(with(V1, rep(hits, times = ofreq)))
## Not run: barplot(with(V1, ofreq),
  names.arg = as.character(with(V1, hits)),
  main = "London V1 buzz bomb hits",
  col = "lightblue", las = 1,
  ylab = "Frequency", xlab = "Hits")
## End(Not run)
```

venice

*Venice Maximum Sea Levels Data***Description**

Some sea levels data sets recorded at Venice, Italy.

**Usage**

```
data(venice)
data(venice90)
```

**Format**

**venice** is a data frame with 51 observations on the following 11 variables. It concerns the maximum heights of sea levels between 1931 and 1981.

**year** a numeric vector.

**r1,r2,r3,r4,r5,r6,r7,r8,r9,r10** numeric vectors; r1 is the highest recorded value, r2 is the second highest recorded value, etc.

**venice90** is a data frame with 455 observations on the following 7 variables.

**year, month, day, hour** numeric vectors; actual time of the recording.

**sealevel** numeric; sea level.

**ohour** numeric; number of hours since the midnight of 31 Dec 1939 and 1 Jan 1940.

**Year** numeric vector; approximate year as a real number. The formula is `start.year + ohour / (365.26 * 24)` where `start.year` is 1940. One can treat **Year** as continuous whereas **year** can be treated as both continuous and discrete.

**Details**

Sea levels are in cm. For **venice90**, the value 0 corresponds to a fixed reference point (e.g., the mean sea level in 1897 at an old palace of Venice). Clearly since the relative (perceived) mean sea level has been increasing in trend over time (more than an overall 0.4 m increase by 2010), therefore the value 0 is (now) a very low and unusual measurement.

For **venice**, in 1935 only the top six values were recorded.

For **venice90**, this is a subset of a data set provided by Paolo Pirazzoli consisting of hourly sea levels from 1940 to 2009. Values greater than 90 cm were extracted, and then declustered (each cluster provides no more than one value, and each value is at least 24 hours apart). Thus the values are more likely to be independent. Of the original  $(2009-1940+1)*365.26*24$  values about 7 percent of these comprise **venice90**.

Yet to do: check for consistency between the data sets. Some external data sets elsewhere have some extremes recorded at times not exactly on the hour.

## Source

Pirazzoli, P. (1982) Maree estreme a Venezia (periodo 1872–1981). *Acqua Aria*, **10**, 1023–1039.

Thanks to Paolo Pirazzoli and Alberto Tomasin for the `venice90` data.

## References

Smith, R. L. (1986) Extreme value theory based on the  $r$  largest annual events. *Journal of Hydrology*, **86**, 27–43.

Battistin, D. and Canestrelli, P. (2006). *La serie storica delle maree a Venezia, 1872–2004* (in Italian), Comune di Venezia. Istituzione Centro Previsione e Segnalazioni Maree.

## See Also

[guplot](#), [gev](#), [gpd](#).

## Examples

```
## Not run:
matplot(venice[["year"]], venice[, -1], xlab = "Year",
        ylab = "Sea level (cm)", type = "l")

ymat <- as.matrix(venice[, paste("r", 1:10, sep = "")])
fit1 <- vgam(ymat ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
           data = venice, trace = TRUE, na.action = na.pass)
head(fitted(fit1))

par(mfrow = c(2, 1), xpd = TRUE)
plot(fit1, se = TRUE, lcol = "blue", llwd = 2, slty = "dashed")

par(mfrow = c(1,1), bty = "l", xpd = TRUE, las = 1)
qtplot(fit1, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5),
       llwd = 2, pcol = "blue", tadj = 0.1)

plot(sealevel ~ Year, data = venice90, type = "h", col = "blue")
summary(venice90)
dim(venice90)
round(100 * nrow(venice90) / ((2009 - 1940 + 1) * 365.26 * 24), digits = 3)

## End(Not run)
```

## Description

Fit a vector generalized additive model (VGAM). This is a large class of models that includes generalized additive models (GAMs) and vector generalized linear models (VGLMs) as special cases.

**Usage**

```
vgam(formula, family, data = list(), weights = NULL, subset = NULL,
      na.action = na.fail, etastart = NULL, mustart = NULL,
      coefstart = NULL, control = vgam.control(...), offset = NULL,
      method = "vgam.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
      contrasts = NULL, constraints = NULL,
      extra = list(), form2 = NULL, qr.arg = FALSE, smart = TRUE, ...)
```

**Arguments**

formula	a symbolic description of the model to be fit. The RHS of the formula is applied to each linear/additive predictor, and usually includes at least one <a href="#">s</a> term. Different variables in each linear/additive predictor can be chosen by specifying constraint matrices.
family	Same as for <a href="#">vglm</a> .
data	an optional data frame containing the variables in the model. By default the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>vgam</code> is called.
weights, subset, na.action	Same as for <a href="#">vglm</a> .
etastart, mustart, coefstart	Same as for <a href="#">vglm</a> .
control	a list of parameters for controlling the fitting process. See <a href="#">vgam.control</a> for details.
method	the method to be used in fitting the model. The default (and presently only) method <code>vgam.fit</code> uses iteratively reweighted least squares (IRLS).
constraints, model, offset	Same as for <a href="#">vglm</a> .
x.arg, y.arg	logical values indicating whether the model matrix and response vector/matrix used in the fitting process should be assigned in the <code>x</code> and <code>y</code> slots. Note the model matrix is the LM model matrix; to get the VGAM model matrix type <code>model.matrix(vgamfit)</code> where <code>vgamfit</code> is a <code>vgam</code> object.
contrasts, extra, form2, qr.arg, smart	Same as for <a href="#">vglm</a> .
...	further arguments passed into <a href="#">vgam.control</a> .

**Details**

A vector generalized additive model (VGAM) is loosely defined as a statistical model that is a function of  $M$  additive predictors. The central formula is given by

$$\eta_j = \sum_{k=1}^p f_{(j)k}(x_k)$$

where  $x_k$  is the  $k$ th explanatory variable (almost always  $x_1 = 1$  for the intercept term), and  $f_{(j)k}$  are smooth functions of  $x_k$  that are estimated by smoothers. The first term in the summation is

just the intercept. Currently only one type of smoother is implemented and this is called a *vector (cubic smoothing spline) smoother*. Here,  $j = 1, \dots, M$  where  $M$  is finite. If all the functions are constrained to be linear then the resulting model is a vector generalized linear model (VGLM). VGLMs are best fitted with [vglm](#).

Vector (cubic smoothing spline) smoothers are represented by `s()` (see [s](#)). Local regression via `lo()` is *not* supported. The results of `vgam` will differ from the `gam()` (in the **gam**) because `vgam()` uses a different knot selection algorithm. In general, fewer knots are chosen because the computation becomes expensive when the number of additive predictors  $M$  is large.

The underlying algorithm of VGAMs is iteratively reweighted least squares (IRLS) and modified vector backfitting using vector splines. B-splines are used as the basis functions for the vector (smoothing) splines. `vgam.fit()` is the function that actually does the work. The smoothing code is based on F. O'Sullivan's BART code.

A closely related methodology based on VGAMs called *constrained additive ordination* (CAO) first forms a linear combination of the explanatory variables (called *latent variables*) and then fits a GAM to these. This is implemented in the function [cao](#) for a very limited choice of family functions.

## Value

An object of class "vgam" (see [vgam-class](#) for further information).

## WARNING

Currently `vgam` can only handle constraint matrices `cmat`, say, such that `crossprod(cmat)` is diagonal. This is a bug that I will try to fix up soon.

See warnings in [vglm.control](#).

## Note

This function can fit a wide variety of statistical models. Some of these are harder to fit than others because of inherent numerical difficulties associated with some of them. Successful model fitting benefits from cumulative experience. Varying the values of arguments in the **VGAM** family function itself is a good first step if difficulties arise, especially if initial values can be inputted. A second, more general step, is to vary the values of arguments in [vgam.control](#). A third step is to make use of arguments such as `etastart`, `coefstart` and `mustart`.

Some **VGAM** family functions end in "ff" to avoid interference with other functions, e.g., [binomialff](#), [poissonff](#), [gaussianff](#), [gammaff](#). This is because **VGAM** family functions are incompatible with [glm](#) (and also [gam](#) in the **gam** library and [gam](#) in the **mgeV** library).

The smart prediction ([smartpred](#)) library is packed with the **VGAM** library.

The theory behind the scaling parameter is currently being made more rigorous, but it should give the same value as the scale parameter for GLMs.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

Yee, T. W. (2008) The VGAM Package. *R News*, **8**, 28–39.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

[vgam.control](#), [vgam-class](#), [vglmff-class](#), [plotvgam](#), [vglm](#), [s](#), [vsmooth.spline](#), [cao](#).

## Examples

```
# Nonparametric proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let),
      cumulative(parallel = TRUE), data = pneumo)

# Nonparametric logistic regression
fit <- vgam(agaas ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run: plot(fit, se = TRUE)
pfit <- predict(fit, type = "terms", raw = TRUE, se = TRUE)
names(pfit)
head(pfit$fitted)
head(pfit$se.fit)
pfit$df
pfit$sigma

# Fit two species simultaneously
fit2 <- vgam(cbind(agaas, kniexc) ~ s(altitude, df = c(2, 3)),
             binomialff(mv = TRUE), data = hunua)
coef(fit2, matrix = TRUE) # Not really interpretable
## Not run: plot(fit2, se = TRUE, overlay = TRUE, lcol = 1:2, scol = 1:2)

ooo <- with(hunua, order(altitude))
with(hunua, matplot(altitude[ooo], fitted(fit2)[ooo,], ylim = c(0, 0.8),
                    xlab = "Altitude (m)", ylab = "Probability of presence", las = 1,
                    main = "Two plant species' response curves", type = "l", lwd = 2))
with(hunua, rug(altitude))
## End(Not run)
```

---

vgam-class

Class “vgam”

---

## Description

Vector generalized additive models.

## Objects from the Class

Objects can be created by calls of the form `vgam(...)`.

## Slots

**nl.chisq:** Object of class "numeric". Nonlinear chi-squared values.

**nl.df:** Object of class "numeric". Nonlinear chi-squared degrees of freedom values.

**spar:** Object of class "numeric" containing the (scaled) smoothing parameters.

**s.xargument:** Object of class "character" holding the variable name of any `s()` terms.

**var:** Object of class "matrix" holding approximate pointwise standard error information.

**Bspline:** Object of class "list" holding the scaled (internal and boundary) knots, and the fitted B-spline coefficients. These are used for prediction.

**extra:** Object of class "list"; the extra argument on entry to `vglm`. This contains any extra information that might be needed by the family function.

**family:** Object of class "vglmff". The family function.

**iter:** Object of class "numeric". The number of IRLS iterations used.

**predictors:** Object of class "matrix" with  $M$  columns which holds the  $M$  linear predictors.

**assign:** Object of class "list", from class "vlm". This named list gives information matching the columns and the (LM) model matrix terms.

**call:** Object of class "call", from class "vlm". The matched call.

**coefficients:** Object of class "numeric", from class "vlm". A named vector of coefficients.

**constraints:** Object of class "list", from class "vlm". A named list of constraint matrices used in the fitting.

**contrasts:** Object of class "list", from class "vlm". The contrasts used (if any).

**control:** Object of class "list", from class "vlm". A list of parameters for controlling the fitting process. See [vglm.control](#) for details.

**criterion:** Object of class "list", from class "vlm". List of convergence criterion evaluated at the final IRLS iteration.

**df.residual:** Object of class "numeric", from class "vlm". The residual degrees of freedom.

**df.total:** Object of class "numeric", from class "vlm". The total degrees of freedom.

**dispersion:** Object of class "numeric", from class "vlm". The scaling parameter.

**effects:** Object of class "numeric", from class "vlm". The effects.

**fitted.values:** Object of class "matrix", from class "vlm". The fitted values. This is usually the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.

**misc:** Object of class "list", from class "vlm". A named list to hold miscellaneous parameters.

**model:** Object of class "data.frame", from class "vlm". The model frame.

**na.action:** Object of class "list", from class "vlm". A list holding information about missing values.

**offset:** Object of class "matrix", from class "vlm". If non-zero, a  $M$ -column matrix of offsets.

**post:** Object of class "list", from class "vlm" where post-analysis results may be put.



**preplot:** Object of class "list", from class "vlm" used by [plotvgam](#); the plotting parameters may be put here.

**prior.weights:** Object of class "matrix", from class "vlm" holding the initially supplied weights.

**qr:** Object of class "list", from class "vlm". QR decomposition at the final iteration.

**R:** Object of class "matrix", from class "vlm". The **R** matrix in the QR decomposition used in the fitting.

**rank:** Object of class "integer", from class "vlm". Numerical rank of the fitted model.

**residuals:** Object of class "matrix", from class "vlm". The *working* residuals at the final IRLS iteration.

**res.ss:** Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.

**smart.prediction:** Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.

**terms:** Object of class "list", from class "vlm". The [terms](#) object used.

**weights:** Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.

**x:** Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).

**xlevels:** Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.

**y:** Object of class "matrix", from class "vlm". The response, in matrix form.

**Xm2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**Ym2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**callXm2:** Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vglm", directly. Class "vlm", by class "vglm".

## Methods

**cdf** signature(object = "vglm"): cumulative distribution function. Useful for quantile regression and extreme value data models.

**deplot** signature(object = "vglm"): density plot. Useful for quantile regression models.

**deviance** signature(object = "vglm"): deviance of the model (where applicable).

**plot** signature(x = "vglm"): diagnostic plots.

**predict** signature(object = "vglm"): extract the additive predictors or predict the additive predictors at a new data frame.

**print** signature(x = "vglm"): short summary of the object.

**qtplot** signature(object = "vglm"): quantile plot (only applicable to some models).

**resid** signature(object = "vglm"): residuals. There are various types of these.

**residuals** signature(object = "vglm"): residuals. Shorthand for resid.

**rlplot** signature(object = "vglm"): return level plot. Useful for extreme value data models.

**summary** signature(object = "vglm"): a more detailed summary of the object.

**Note**

VGAMs have all the slots that `vglm` objects have (`vglm-class`), plus the first few slots described in the section above.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

<http://www.stat.auckland.ac.nz/~yee>

**See Also**

`vgam.control`, `vglm`, `s`, `vglm-class`, `vglmff-class`.

**Examples**

```
# Fit a nonparametric proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let),
      cumulative(parallel = TRUE), data = pneumo)
```

---

`vgam.control`

*Control function for vgam*

---

**Description**

Algorithmic constants and parameters for running `vgam` are set using this function.

**Usage**

```
vgam.control(all.knots = FALSE, bf.epsilon = 1e-07, bf.maxit = 30,
             checkwz=TRUE, criterion = names(.min.criterion.VGAM),
             epsilon = 1e-07, maxit = 30, na.action = na.fail,
             nk = NULL, save.weight = FALSE, se.fit = TRUE,
             trace = FALSE, wzepsilon = .Machine$double.eps^0.75,
             ...)
```

**Arguments**

<code>all.knots</code>	logical indicating if all distinct points of the smoothing variables are to be used as knots. By default, <code>all.knots=TRUE</code> for $n \leq 40$ , and for $n > 40$ , the number of knots is approximately $40 + (n - 40)^{0.25}$ . This increases very slowly with $n$ so that the number of knots is approximately between 50 and 60 for large $n$ .
<code>bf.epsilon</code>	tolerance used by the modified vector backfitting algorithm for testing convergence. Must be a positive number.
<code>bf.maxit</code>	maximum number of iterations allowed in the modified vector backfitting algorithm. Must be a positive integer.
<code>checkwz</code>	logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than <code>wzepsilon</code> . If not, any values less than <code>wzepsilon</code> are replaced with this value.
<code>criterion</code>	character variable describing what criterion is to be used to test for convergence. The possibilities are listed in <code>.min.criterion.VGAM</code> , but most family functions only implement a few of these.
<code>epsilon</code>	positive convergence tolerance <code>epsilon</code> . Roughly speaking, the Newton-Raphson/Fisher-scoring/local-scoring iterations are assumed to have converged when two successive criterion values are within <code>epsilon</code> of each other.
<code>maxit</code>	maximum number of Newton-Raphson/Fisher-scoring/local-scoring iterations allowed.
<code>na.action</code>	how to handle missing values. Unlike the <code>SPLUS gam</code> function, <code>vgam</code> cannot handle NAs when smoothing.
<code>nk</code>	vector of length $d$ containing positive integers, where $d$ be the number of <code>s</code> terms in the formula. Recycling is used if necessary. The $i$ th value is the number of B-spline coefficients to be estimated for each component function of the $i$ th <code>s()</code> term. <code>nk</code> differs from the number of knots by some constant. If specified, <code>nk</code> overrides the automatic knot selection procedure.
<code>save.weight</code>	logical indicating whether the weights slot of a "vglm" object will be saved on the object. If not, it will be reconstructed when needed, e.g., <code>summary</code> .
<code>se.fit</code>	logical indicating whether approximate pointwise standard errors are to be saved on the object. If <code>TRUE</code> , then these can be plotted with <code>plot(..., se = TRUE)</code> .
<code>trace</code>	logical indicating if output should be produced for each iteration.
<code>wzepsilon</code>	Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive.
<code>...</code>	other parameters that may be picked up from control functions that are specific to the <b>VGAM</b> family function.

**Details**

Most of the control parameters are used within `vgam.fit` and you will have to look at that to understand the full details. Many of the control parameters are used in a similar manner by `vglm.fit` (`vglm`) because the algorithm (IRLS) is very similar.

Setting `save.weight=FALSE` is useful for some models because the `weights` slot of the object is often the largest and so less memory is used to store the object. However, for some **VGAM** family function, it is necessary to set `save.weight=TRUE` because the `weights` slot cannot be reconstructed later.

**Value**

A list with components matching the input names. A little error checking is done, but not much. The list is assigned to the control slot of `vgam` objects.

**Warning**

See `vglm.control`.

**Note**

`vgam` does not implement half-stepsizing, therefore parametric models should be fitted with `vglm`. Also, `vgam` is slower than `vglm` too.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

**See Also**

`vgam`, `vglm.control`, `vsmooth.spline`, `vglm`.

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let, df = 2), multinomial,
     data = pneumo, trace = TRUE, eps = 1e-4, maxit = 10)
```

---

vglm

---

*Fitting Vector Generalized Linear Models*


---

**Description**

`vglm` is used to fit vector generalized linear models (VGLMs). This is a very large class of models that includes generalized linear models (GLMs) as a special case.

**Usage**

```
vglm(formula, family, data = list(), weights = NULL, subset = NULL,
     na.action = na.fail, etastart = NULL, mustart = NULL,
     coefstart = NULL, control = vglm.control(...), offset = NULL,
     method = "vglm.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
     contrasts = NULL, constraints = NULL, extra = list(),
     form2 = NULL, qr.arg = TRUE, smart = TRUE, ...)
```

**Arguments**

formula	a symbolic description of the model to be fit. The RHS of the formula is applied to each linear predictor. Different variables in each linear predictor can be chosen by specifying constraint matrices.
family	a function of class "vglmff" (see <a href="#">vglmff-class</a> ) describing what statistical model is to be fitted. This is called a “ <b>VGAM</b> family function”. See <a href="#">CommonVGAMffArguments</a> for general information about many types of arguments found in this type of function.
data	an optional data frame containing the variables in the model. By default the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>vglm</code> is called.
weights	an optional vector or matrix of (prior) weights to be used in the fitting process. If the <b>VGAM</b> family function handles multiple responses ( $q > 1$ of them, say) then weights can be a matrix with $q$ columns. Each column matches the respective response. If it is a vector (the usually case) then it is recycled into a matrix with $q$ columns. The values of weights must be positive; try setting a very small value such as $1.0\text{e-}8$ to effectively delete an observation.
subset	an optional logical vector specifying a subset of observations to be used in the fitting process.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the <code>na.action</code> setting of <a href="#">options</a> , and is <code>na.fail</code> if that is unset. The “factory-fresh” default is <code>na.omit</code> .
etastart	starting values for the linear predictors. It is a $M$ -column matrix with the same number of rows as the response. If $M = 1$ then it may be a vector. Note that <code>etastart</code> and the output of <code>predict(fit)</code> should be comparable. Here, <code>fit</code> is the fitted object.
mustart	starting values for the fitted values. It can be a vector or a matrix; if a matrix, then it has the same number of rows as the response. Usually <code>mustart</code> and the output of <code>fitted(fit)</code> should be comparable. Some family functions do not make use of this argument.
coefstart	starting values for the coefficient vector. The length and order must match that of <code>coef(fit)</code> .
control	a list of parameters for controlling the fitting process. See <a href="#">vglm.control</a> for details.
offset	a vector or $M$ -column matrix of offset values. These are <i>a priori</i> known and are added to the linear/additive predictors during fitting.
method	the method to be used in fitting the model. The default (and presently only) method <code>vglm.fit()</code> uses iteratively reweighted least squares (IRLS).
model	a logical value indicating whether the <i>model frame</i> should be assigned in the <code>model</code> slot.
x.arg, y.arg	logical values indicating whether the model matrix and response vector/matrix used in the fitting process should be assigned in the <code>x</code> and <code>y</code> slots. Note the model matrix is the LM model matrix; to get the VGLM model matrix type <code>model.matrix(vglmfit)</code> where <code>vglmfit</code> is a <code>vglm</code> object.

contrasts	an optional list. See the <code>contrasts.arg</code> of <code>model.matrix.default</code> .
constraints	<p>an optional list of constraint matrices. The components of the list must be named with the term it corresponds to (and it must match in character format exactly). There are two types of input: "lm"-type and "vlm"-type. The former is a subset of the latter. The former has a matrix for each term of the LM matrix. The latter has a matrix for each column of the VLM matrix. After fitting, the <code>constraints</code> extractor function may be applied; it returns the "vlm"-type list of constraint matrices by default. If "lm"-type are returned by <code>constraints</code> then these can be fed into this argument and it should give the same model as before.</p> <p>Each constraint matrix must have <math>M</math> rows, and be of full-column rank. By default, constraint matrices are the <math>M</math> by <math>M</math> identity matrix unless arguments in the family function itself override these values, e.g., <code>parallel</code> (see <code>CommonVGAMffArguments</code>). If <code>constraints</code> is used it must contain <i>all</i> the terms; an incomplete list is not accepted.</p>
extra	an optional list with any extra information that might be needed by the <b>VGAM</b> family function.
form2	The second (optional) formula. If argument <code>xij</code> is used (see <code>vglm.control</code> ) then <code>form2</code> needs to have <i>all</i> terms in the model. Also, some <b>VGAM</b> family functions such as <code>micmen</code> use this argument to input the regressor variable. If given, the slots <code>@Xm2</code> and <code>@Ym2</code> may be assigned. Note that smart prediction applies to terms in <code>form2</code> too.
qr.arg	logical value indicating whether the slot <code>qr</code> , which returns the QR decomposition of the VLM model matrix, is returned on the object.
smart	logical value indicating whether smart prediction ( <code>smartpred</code> ) will be used.
...	further arguments passed into <code>vglm.control</code> .

## Details

A vector generalized linear model (VGLM) is loosely defined as a statistical model that is a function of  $M$  linear predictors. The central formula is given by

$$\eta_j = \beta_j^T x$$

where  $x$  is a vector of explanatory variables (sometimes just a 1 for an intercept), and  $\beta_j$  is a vector of regression coefficients to be estimated. Here,  $j = 1, \dots, M$ , where  $M$  is finite. Then one can write  $\eta = (\eta_1, \dots, \eta_M)^T$  as a vector of linear predictors.

Most users will find `vglm` similar in flavour to `glm`. The function `vglm.fit` actually does the work.

## Value

An object of class "vglm", which has the following slots. Some of these may not be assigned to save space, and will be recreated if necessary later.

extra	the list <code>extra</code> at the end of fitting.
family	the family function (of class "vglmff").
iter	the number of IRLS iterations used.
predictors	a $M$ -column matrix of linear predictors.

assign	a named list which matches the columns and the (LM) model matrix terms.
call	the matched call.
coefficients	a named vector of coefficients.
constraints	a named list of constraint matrices used in the fitting.
contrasts	the contrasts used (if any).
control	list of control parameter used in the fitting.
criterion	list of convergence criterion evaluated at the final IRLS iteration.
df.residual	the residual degrees of freedom.
df.total	the total degrees of freedom.
dispersion	the scaling parameter.
effects	the effects.
fitted.values	the fitted values, as a matrix. This is often the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.
misc	a list to hold miscellaneous parameters.
model	the model frame.
na.action	a list holding information about missing values.
offset	if non-zero, a $M$ -column matrix of offsets.
post	a list where post-analysis results may be put.
preplot	used by <a href="#">plotvgam</a> , the plotting parameters may be put here.
prior.weights	initially supplied weights (the <code>weights</code> argument). Also see <a href="#">weightsvglm</a> .
qr	the QR decomposition used in the fitting.
R	the <b>R</b> matrix in the QR decomposition used in the fitting.
rank	numerical rank of the fitted model.
residuals	the <i>working</i> residuals at the final IRLS iteration.
res.ss	residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.
smart.prediction	a list of data-dependent parameters (if any) that are used by smart prediction.
terms	the <a href="#">terms</a> object used.
weights	the working weight matrices at the final IRLS iteration. This is in matrix-band form.
x	the model matrix (linear model LM, not VGLM).
xlevels	the levels of the factors, if any, used in fitting.
y	the response, in matrix form.

This slot information is repeated at [vglm-class](#).

## WARNING

See warnings in [vglm.control](#).

## Note

This function can fit a wide variety of statistical models. Some of these are harder to fit than others because of inherent numerical difficulties associated with some of them. Successful model fitting benefits from cumulative experience. Varying the values of arguments in the **VGAM** family function itself is a good first step if difficulties arise, especially if initial values can be inputted. A second, more general step, is to vary the values of arguments in `vglm.control`. A third step is to make use of arguments such as `etastart`, `coefstart` and `mustart`.

Some **VGAM** family functions end in "ff" to avoid interference with other functions, e.g., `binomialff`, `poissonff`, `gaussianff`, `gammaff`. This is because **VGAM** family functions are incompatible with `glm` (and also `gam` in the **gam** library and `gam` in the **mgcv** library).

The smart prediction (`smartpred`) library is incorporated within the **VGAM** library.

The theory behind the scaling parameter is currently being made more rigorous, but it should give the same value as the scale parameter for GLMs.

In Example 5 below, the `xij` argument is used to illustrate covariates that are specific to a linear predictor. Here, `lop/rop` are the ocular pressures of the left/right eye (artificial data). Variables `leye` and `reye` might be the presence/absence of a particular disease on the LHS/RHS eye respectively. See `vglm.control` and `fill` for more details and examples.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

Yee, T. W. (2008) The VGAM Package. *R News*, **8**, 28–39.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

## See Also

`vglm.control`, `vglm-class`, `vglmff-class`, `smartpred`, `vglm.fit`, `fill`, `rrvglm`, `vgam`. Methods functions include `coef.vlm`, `constraints.vlm`, `hatvaluesvlm`, `predictvglm`, `summary.vglm`, `AIC.vglm`, `lrtest.vglm`, etc.

## Examples

```
# Example 1. See help(glm)
print(d.AD <- data.frame(treatment = gl(3, 3),
                        outcome = gl(3, 1, 9),
                        counts = c(18,17,15,20,10,20,25,13,12)))
vglm.D93 <- vglm(counts ~ outcome + treatment, family = poissonff,
                data = d.AD, trace = TRUE)
summary(vglm.D93)
```



```

# Example 2. Multinomial logit model
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, pneumo)

# Example 3. Proportional odds model
fit3 <- vglm(cbind(normal, mild, severe) ~ let, propodds, pneumo)
coef(fit3, matrix = TRUE)
constraints(fit3)
model.matrix(fit3, type = "lm") # LM model matrix
model.matrix(fit3)             # Larger VGLM (or VLM) model matrix

# Example 4. Bivariate logistic model
fit4 <- vglm(cbind(nBnW, nBW, BnW, BW) ~ age, binom2.or, coalminers)
coef(fit4, matrix = TRUE)
depvar(fit4) # Response are proportions
weights(fit4, type = "prior")

# Example 5. The use of the xij argument (simple case).
# The constraint matrix for 'op' has one column.
nn <- 1000
eyesdat <- round(data.frame(lop = runif(nn),
                           rop = runif(nn),
                           op = runif(nn)), digits = 2)
eyesdat <- transform(eyesdat, eta1 = -1 + 2 * lop,
                    eta2 = -1 + 2 * lop)
eyesdat <- transform(eyesdat,
                    leye = rbinom(nn, size = 1, prob = logit(eta1, inverse = TRUE)),
                    reye = rbinom(nn, size = 1, prob = logit(eta2, inverse = TRUE)))
head(eyesdat)
fit5 <- vglm(cbind(leye, reye) ~ op,
            binom2.or(exchangeable = TRUE, zero = 3),
            data = eyesdat, trace = TRUE,
            xij = list(op ~ lop + rop + fill(lop)),
            form2 = ~ op + lop + rop + fill(lop))
coef(fit5)
coef(fit5, matrix = TRUE)
constraints(fit5)

```

vglm-class

Class "vglm"

## Description

Vector generalized linear models.

## Objects from the Class

Objects can be created by calls of the form `vglm(...)`.

## Slots

In the following,  $M$  is the number of linear predictors.

**extra:** Object of class "list"; the extra argument on entry to `vglm`. This contains any extra information that might be needed by the family function.

**family:** Object of class "vglmff". The family function.

**iter:** Object of class "numeric". The number of IRLS iterations used.

**predictors:** Object of class "matrix" with  $M$  columns which holds the  $M$  linear predictors.

**assign:** Object of class "list", from class "vglm". This named list gives information matching the columns and the (LM) model matrix terms.

**call:** Object of class "call", from class "vglm". The matched call.

**coefficients:** Object of class "numeric", from class "vglm". A named vector of coefficients.

**constraints:** Object of class "list", from class "vglm". A named list of constraint matrices used in the fitting.

**contrasts:** Object of class "list", from class "vglm". The contrasts used (if any).

**control:** Object of class "list", from class "vglm". A list of parameters for controlling the fitting process. See [vglm.control](#) for details.

**criterion:** Object of class "list", from class "vglm". List of convergence criterion evaluated at the final IRLS iteration.

**df.residual:** Object of class "numeric", from class "vglm". The residual degrees of freedom.

**df.total:** Object of class "numeric", from class "vglm". The total degrees of freedom.

**dispersion:** Object of class "numeric", from class "vglm". The scaling parameter.

**effects:** Object of class "numeric", from class "vglm". The effects.

**fitted.values:** Object of class "matrix", from class "vglm". The fitted values.

**misc:** Object of class "list", from class "vglm". A named list to hold miscellaneous parameters.

**model:** Object of class "data.frame", from class "vglm". The model frame.

**na.action:** Object of class "list", from class "vglm". A list holding information about missing values.

**offset:** Object of class "matrix", from class "vglm". If non-zero, a  $M$ -column matrix of offsets.

**post:** Object of class "list", from class "vglm" where post-analysis results may be put.

**preplot:** Object of class "list", from class "vglm" used by [plotvgam](#); the plotting parameters may be put here.

**prior.weights:** Object of class "matrix", from class "vglm" holding the initially supplied weights.

**qr:** Object of class "list", from class "vglm". QR decomposition at the final iteration.

**R:** Object of class "matrix", from class "vglm". The  $\mathbf{R}$  matrix in the QR decomposition used in the fitting.

**rank:** Object of class "integer", from class "vglm". Numerical rank of the fitted model.

**residuals:** Object of class "matrix", from class "vlm". The *working* residuals at the final IRLS iteration.

**res.ss:** Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.

**smart.prediction:** Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.

**terms:** Object of class "list", from class "vlm". The [terms](#) object used.

**weights:** Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.

**x:** Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).

**xlevels:** Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.

**y:** Object of class "matrix", from class "vlm". The response, in matrix form.

**Xm2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**Ym2:** Object of class "matrix", from class "vlm". See [vglm-class](#)).

**callXm2:** Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vlm", directly.

## Methods

**cdf** signature(object = "vglm"): cumulative distribution function. Applicable to, e.g., quantile regression and extreme value data models.

**deplot** signature(object = "vglm"): Applicable to, e.g., quantile regression.

**deviance** signature(object = "vglm"): deviance of the model (where applicable).

**plot** signature(x = "vglm"): diagnostic plots.

**predict** signature(object = "vglm"): extract the linear predictors or predict the linear predictors at a new data frame.

**print** signature(x = "vglm"): short summary of the object.

**qtplot** signature(object = "vglm"): quantile plot (only applicable to some models).

**resid** signature(object = "vglm"): residuals. There are various types of these.

**residuals** signature(object = "vglm"): residuals. Shorthand for resid.

**rlplot** signature(object = "vglm"): return level plot. Useful for extreme value data models.

**summary** signature(object = "vglm"): a more detailed summary of the object.

## Author(s)

Thomas W. Yee

References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

<http://www.stat.auckland.ac.nz/~yee>

See Also

[vglm](#), [vglmff-class](#), [vgam-class](#).

Examples

```
# Multinomial logit model
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, pneumo)
```

---

vglm.control	<i>Control function for vglm</i>
--------------	----------------------------------

---

Description

Algorithmic constants and parameters for running vglm are set using this function.

Usage

```
vglm.control(checkwz = TRUE, Check.rank = TRUE,
             criterion = names(.min.criterion.VGAM),
             epsilon = 1e-07, half.stepsizing = TRUE,
             maxit = 30, noWarning = FALSE,
             stepsize = 1, save.weight = FALSE,
             trace = FALSE, wzepsilon = .Machine$double.eps^0.75,
             xij = NULL, ...)
```

Arguments

checkwz	logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than wzepsilon. If not, any values less than wzepsilon are replaced with this value.
Check.rank	logical indicating whether the rank of the VLM matrix should be checked. If this is not of full column rank then the results are not to be trusted. The default is to give an error message if the VLM matrix is not of full column rank.
criterion	character variable describing what criterion is to be used to test for convergence. The possibilities are listed in .min.criterion.VGAM, but most family functions only implement a few of these.

epsilon	positive convergence tolerance epsilon. Roughly speaking, the Newton-Raphson/Fisher-scoring iterations are assumed to have converged when two successive criterion values are within epsilon of each other.
half.stepsizing	logical indicating if half-stepsizing is allowed. For example, in maximizing a log-likelihood, if the next iteration has a log-likelihood that is less than the current value of the log-likelihood, then a half step will be taken. If the log-likelihood is still less than at the current position, a quarter-step will be taken etc. Eventually a step will be taken so that an improvement is made to the convergence criterion. half.stepsizing is ignored if criterion == "coefficients".
maxit	maximum number of (usually Fisher-scoring) iterations allowed. Sometimes Newton-Raphson is used.
noWarning	logical indicating whether to suppress a warning if convergence is not obtained within maxit iterations. This is ignored if maxit = 1 is set.
stepsize	usual step size to be taken between each Newton-Raphson/Fisher-scoring iteration. It should be a value between 0 and 1, where a value of unity corresponds to an ordinary step. A value of 0.5 means half-steps are taken. Setting a value near zero will cause convergence to be generally slow but may help increase the chances of successful convergence for some family functions.
save.weight	logical indicating whether the weights slot of a "vglm" object will be saved on the object. If not, it will be reconstructed when needed, e.g., summary. Some family functions have save.weight = TRUE and others have save.weight = FALSE in their control functions.
trace	logical indicating if output should be produced for each iteration. Setting trace = TRUE is recommended in general because <b>VGAM</b> fits a very broad variety of models and distributions, and for some of them, convergence is intrinsically more difficult. Monitoring convergence can help check that the solution is reasonable or that a problem has occurred. It may suggest better initial values are needed, the making of invalid assumptions, or that the model is inappropriate for the data, etc.
wzepsilon	small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive.
xij	A formula or a list of formulas. Each formula has a RHS giving $M$ terms making up a covariate-dependent term (whose name is the response). That is, it creates a variable that takes on different values for each linear/additive predictor, e.g., the ocular pressure of each eye. The $M$ terms must be unique; use fill1, fill2, fill3, etc. if necessary. Each formula should have a response which is taken as the name of that variable, and the $M$ terms are enumerated in sequential order. Each of the $M$ terms multiply each successive row of the constraint matrix. When xij is used, the use of form2 is also required to give <i>every</i> term used by the model.
...	other parameters that may be picked up from control functions that are specific to the <b>VGAM</b> family function.

## Details

Most of the control parameters are used within `vglm.fit` and you will have to look at that to understand the full details.

Setting `save.weight = FALSE` is useful for some models because the `weights` slot of the object is the largest and so less memory is used to store the object. However, for some **VGAM** family function, it is necessary to set `save.weight = TRUE` because the `weights` slot cannot be reconstructed later.

## Value

A list with components matching the input names. A little error checking is done, but not much. The list is assigned to the `control` slot of `vglm` objects.

## Warning

For some applications the default convergence criterion should be tightened. Setting something like `criterion = "coef"`, `epsilon = 1e-09` is one way to achieve this, and also add `trace = TRUE` to monitor the convergence. Setting `maxit` to some higher number is usually not needed, and needing to do so suggests something is wrong, e.g., an ill-conditioned model, over-fitting or under-fitting.

## Note

Reiterating from above, setting `trace = TRUE` is recommended in general.

In Example 2 below there are two covariates that have linear/additive predictor specific values. These are handled using the `xij` argument.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

## See Also

[vglm, fill](#). The author's homepage has further documentation about the `xij` argument.

## Examples

```
# Example 1.
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo,
     crit = "coef", step = 0.5, trace = TRUE, epsil = 1e-8, maxit = 40)

# Example 2. The use of the xij argument (simple case).
ymat <- rdiric(n <- 1000, shape = rep(exp(2), len = 4))
mydat <- data.frame(x1 = runif(n), x2 = runif(n), x3 = runif(n), x4 = runif(n),
```

```

      z1 = runif(n), z2 = runif(n), z3 = runif(n), z4 = runif(n))
mydat <- transform(mydat, X = x1, Z = z1)
mydat <- round(mydat, digits = 2)
fit2 <- vglm(ymat ~ X + Z,
  dirichlet(parallel = TRUE), data = mydat, trace = TRUE,
  xij = list(Z ~ z1 + z2 + z3 + z4,
    X ~ x1 + x2 + x3 + x4),
  form2 = ~ Z + z1 + z2 + z3 + z4 +
    X + x1 + x2 + x3 + x4)
head(model.matrix(fit2, type = "lm")) # LM model matrix
head(model.matrix(fit2, type = "vlm")) # Big VLM model matrix
coef(fit2)
coef(fit2, matrix = TRUE)
max(abs(predict(fit2)-predict(fit2, new = mydat))) # Predicts correctly
summary(fit2)
## Not run:
# plotvgam(fit2, se = TRUE, xlab = "x1", which.term = 1) # Bug!
# plotvgam(fit2, se = TRUE, xlab = "z1", which.term = 2) # Bug!
plotvgam(fit2, xlab = "x1") # Correct
plotvgam(fit2, xlab = "z1") # Correct

## End(Not run)

# Example 3. The use of the xij argument (complex case).
set.seed(123)
coalminers <- transform(coalminers,
  Age = (age - 42) / 5,
  dum1 = round(runif(nrow(coalminers)), digits = 2),
  dum2 = round(runif(nrow(coalminers)), digits = 2),
  dum3 = round(runif(nrow(coalminers)), digits = 2),
  dumm = round(runif(nrow(coalminers)), digits = 2))
BS <- function(x, ..., df = 3) bs(c(x,...), df = df)[1:length(x),drop = FALSE]
NS <- function(x, ..., df = 3) ns(c(x,...), df = df)[1:length(x),drop = FALSE]

# Equivalently...
BS <- function(x, ..., df = 3) head(bs(c(x,...), df = df), length(x), drop = FALSE)
NS <- function(x, ..., df = 3) head(ns(c(x,...), df = df), length(x), drop = FALSE)

fit3 <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age + NS(dum1, dum2),
  fam = binom2.or(exchangeable = TRUE, zero = 3),
  xij = list(NS(dum1, dum2) ~ NS(dum1, dum2) +
    NS(dum2, dum1) +
    fill(NS( dum1))),
  form2 = ~ NS(dum1, dum2) + NS(dum2, dum1) + fill(NS(dum1)) +
    dum1 + dum2 + dum3 + Age + age + dumm,
  data = coalminers, trace = TRUE)
head(model.matrix(fit3, type = "lm")) # LM model matrix
head(model.matrix(fit3, type = "vlm")) # Big VLM model matrix
coef(fit3)
coef(fit3, matrix = TRUE)
## Not run: plotvgam(fit3, se = TRUE, lcol = "red", scol = "blue", xlab = "dum1")

```

---

 vglmff-class

 Class "vglmff"
 

---

## Description

Family functions for the **VGAM** package

## Objects from the Class

Objects can be created by calls of the form `new("vglmff", ...)`.

## Slots

In the following,  $M$  is the number of linear/additive predictors.

**blurb:** Object of class "character" giving a small description of the model. Important arguments such as parameter link functions can be expressed here.

**constraints:** Object of class "expression" which sets up any constraint matrices defined by arguments in the family function. A zero argument is always fed into `cm.zero.vgam`, whereas other constraints are fed into `cm.vgam`.

**deviance:** Object of class "function" returning the deviance of the model. This slot is optional. If present, the function must have arguments `function(mu, y, w, residuals = FALSE, eta, extra = NULL)`. Deviance residuals are returned if `residuals = TRUE`.

**fini:** Object of class "expression" to insert code at a special position in `vglm.fit` or `vgam.fit`. This code is evaluated immediately after the fitting.

**first:** Object of class "expression" to insert code at a special position in `vglm` or `vgam`.

**infos:** Object of class "function" which returns a list with components such as `Musual`. At present only a very few **VGAM** family functions have this feature implemented. Those that do do not require specifying the `Musual` argument when used with `rcim`.

**initialize:** Object of class "expression" used to perform error checking (especially for the variable `y`) and obtain starting values for the model. In general, `etastart` or `mustart` are assigned values based on the variables `y`, `x` and `w`.

**linkinv:** Object of class "function" which returns the fitted values, given the linear/additive predictors. The function must have arguments `function(eta, extra = NULL)`.

**last:** Object of class "expression" to insert code at a special position (at the very end) of `vglm.fit()` or `vgam.fit()`. This code is evaluated after the fitting. The list `misc` is often assigned components in this slot, which becomes the `misc` slot on the fitted object.

**linkfun:** Object of class "function" which, given the fitted values, returns the linear/additive predictors. If present, the function must have arguments `function(mu, extra = NULL)`. Most **VGAM** family functions do not have a `linkfun` function. They largely are for classical exponential families, i.e., GLMs.

**loglikelihood:** Object of class "function" returning the log-likelihood of the model. This slot is optional. If present, the function must have arguments `function(mu, y, w, residuals = FALSE, eta, extra = NULL)`. The argument `residuals` can be ignored because log-likelihood residuals aren't defined.



- middle:** Object of class "expression" to insert code at a special position in `vglm.fit` or `vgam.fit`.
- middle2:** Object of class "expression" to insert code at a special position in `vglm.fit` or `vgam.fit`.
- summary.dispersion:** Object of class "logical" indicating whether the general VGLM formula (based on a residual sum of squares) can be used for computing the scaling/dispersion parameter. It is TRUE for most models except for nonlinear regression models.
- vfamly:** Object of class "character" giving class information about the family function. Although not developed at this stage, more flexible classes are planned in the future. For example, family functions `sratio`, `cratio`, `cumulative`, and `acat` all operate on categorical data, therefore will have a special class called "VGAMcat", say. Then if `fit` was a `vglm` object, then `coef(fit)` would print out the `vglm` coefficients plus "VGAMcat" information as well.
- deriv:** Object of class "expression" which returns a  $M$ -column matrix of first derivatives of the log-likelihood function with respect to the linear/additive predictors, i.e., the score vector. In Yee and Wild (1996) this is the  $d_i$  vector. Thus each row of the matrix returned by this slot is such a vector.
- weight:** Object of class "expression" which returns the second derivatives of the log-likelihood function with respect to the linear/additive predictors. This can be either the observed or expected information matrix, i.e., Newton-Raphson or Fisher-scoring respectively. In Yee and Wild (1996) this is the  $W_i$  matrix. Thus each row of the matrix returned by this slot is such a matrix. Like the weights slot of `vglm/vgam`, it is stored in *matrix-band* form, whereby the first  $M$  columns of the matrix are the diagonals, followed by the upper-diagonal band, followed by the band above that, etc. In this case, there can be up to  $M(M + 1)$  columns, with the last column corresponding to the  $(1, M)$  elements of the weight matrices.

## Methods

**print** signature(`x = "vglmff"`): short summary of the family function.

## Warning

**VGAM** family functions are not compatible with `glm`, nor `gam` (from either **gam** or **mgcv** packages).

## Note

With link functions etc., one must use `substitute` to embed the options into the code. There are two different forms: `eval(substitute(expression({...}), list(...)))` for expressions, and `eval(substitute(function(...) { ... }, list(...)))` for functions.

The extra argument in `linkinv`, `linkfun`, `deviance`, `loglikelihood`, etc. matches with the argument `extra` in `vglm`, `vgam` and `rrvglm`. This allows input to be fed into all slots of a **VGAM** family function.

The expression derivative is evaluated immediately prior to `weight`, so there is provision for re-use of variables etc. Programmers must be careful to choose variable names that do not interfere with `vglm.fit`, `vgam.fit()` etc.

Programmers of **VGAM** family functions are encouraged to keep to previous conventions regarding the naming of arguments, e.g., `link` is the argument for parameter link functions, `zero` for allowing some of the linear/additive predictors to be an intercept term only, etc.

In general, Fisher-scoring is recommended over Newton-Raphson where tractable. Although usually slightly slower in convergence, the weight matrices from using the expected information are positive-definite over a larger parameter space.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.  
<http://www.stat.auckland.ac.nz/~yee> contains further information on how to write **VGAM** family functions. The file is amongst other **VGAM** PDF documentation.

**See Also**

[vglm](#), [vgam](#), [rrvglm](#), [rcim](#).

**Examples**

```
cratio()  
cratio(link = "cloglog")  
cratio(link = "cloglog", reverse = TRUE)
```

---

vonmises	<i>von Mises Distribution Family Function</i>
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**Description**

Estimates the location and scale parameters of the von Mises distribution by maximum likelihood estimation.

**Usage**

```
vonmises(llocation = elogit(min = 0, max = 2 * pi), lscale = "loge",  
         ilocation = NULL, iscale = NULL, imethod = 1, zero = NULL)
```

**Arguments**

- |                   |   |
|-------------------|---|
| llocation, lscale | Parameter link functions applied to the location $a$ parameter and scale parameter $k$ , respectively. See <a href="#">Links</a> for more choices. For $k$ , a log link is the default because the parameter is positive. |
| ilocation         | Initial value for the location $a$ parameter. By default, an initial value is chosen internally using imethod. Assigning a value will override the argument imethod.  |

iscale	Initial value for the scale $k$ parameter. By default, an initial value is chosen internally using imethod. Assigning a value will override the argument imethod.
imethod	An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ilocation and iscale.
zero	An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose one value from the set $\{1,2\}$ .

### Details

The (two-parameter) von Mises is the most commonly used distribution in practice for circular data. It has a density that can be written as

$$f(y; a, k) = \frac{\exp[k \cos(y - a)]}{2\pi I_0(k)}$$

where  $0 \leq y < 2\pi$ ,  $k > 0$  is the scale parameter,  $a$  is the location parameter, and  $I_0(k)$  is the modified Bessel function of order 0 evaluated at  $k$ . The mean of  $Y$  (which is the fitted value) is  $a$  and the circular variance is  $1 - I_1(k)/I_0(k)$  where  $I_1(k)$  is the modified Bessel function of order 1. By default,  $\eta_1 = \log(a/(2\pi - a))$  and  $\eta_2 = \log(k)$  for this family function.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

### Warning

Numerically, the von Mises can be difficult to fit because of a log-likelihood having multiple maxima. The user is therefore encouraged to try different starting values, i.e., make use of ilocation and iscale.

### Note

The response and the fitted values are scaled so that  $0 \leq y < 2\pi$ . The linear/additive predictors are left alone. Fisher scoring is used.

### Author(s)

T. W. Yee

### References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) *Statistical Distributions*, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

**See Also**

[Bessel](#), [cardioid](#).

**CircStats** and **circular** currently have a lot more R functions for circular data than the **VGAM** package.

**Examples**

```
vdata <- data.frame(x2 = runif(nn <- 1000))
vdata <- transform(vdata, y = rnorm(nn, m = 2+x2, sd = exp(0.2))) # Bad data!!
fit <- vglm(y ~ x2, vonmises(zero = 2), vdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
with(vdata, range(y)) # Original data
range(depvar(fit))    # Processed data is in [0,2*pi)
```

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vsmooth.spline

---

*Vector cubic smoothing spline*


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**Description**

Fits a vector cubic smoothing spline.

**Usage**

```
vsmooth.spline(x, y, w = NULL, df = rep(5, M), spar = NULL,
               all.knots = FALSE, iconstraint = diag(M),
               xconstraint = diag(M),
               constraints = list("(Intercepts)" = diag(M), x = diag(M)),
               var.arg = FALSE, scale.w = TRUE, nk = NULL,
               control.spar = list())
```

**Arguments**

- |   |   |
|---|---|
| x | A vector, matrix or a list. If a list, the x component is used. If a matrix, the first column is used. x may also be a complex vector, in which case the real part is used, and the imaginary part is used for the response. In this help file, n is the number of unique values of x.                                    |
| y | A vector, matrix or a list. If a list, the y component is used. If a matrix, all but the first column is used. In this help file, M is the number of columns of y if there are no constraints on the functions.   |
| w | The weight matrices or the number of observations. If the weight matrices, then this must be a n-row matrix with the elements in matrix-band form (see <code>iam</code> ). If a vector, then these are the number of observations. By default, w is the M by M identity matrix, denoted by <code>matrix(1, n, M)</code> . |

df	Numerical vector containing the degrees of freedom for each component function (smooth). If necessary, the vector is recycled to have length equal to the number of component functions to be estimated (M if there are no constraints), which equals the number of columns of the x-constraint matrix. A value of 2 means a linear fit, and each element of df should lie between 2 and n. The larger the values of df the more wiggly the smooths.
spar	Numerical vector containing the non-negative smoothing parameters for each component function (smooth). If necessary, the vector is recycled to have length equal to the number of component functions to be estimated (M if there are no constraints), which equals the number of columns of the x-constraint matrix. A value of zero means the smooth goes through the data and hence is wiggly. A value of Inf may be assigned, meaning the smooth will be linear. By default, the NULL value of spar means df is used to determine the smoothing parameters.
all.knots	Logical. If TRUE then each distinct value of x will be a knot. By default, only a subset of the unique values of x are used; typically, the number of knots is $O(n^{0.25})$ for n large, but if $n \leq 40$ then all the unique values of x are used.
iconstraint	A M-row constraint matrix for the intercepts. It must be of full column rank. By default, the constraint matrix for the intercepts is the M by M identity matrix, meaning no constraints.
xconstraint	A M-row constraint matrix for x. It must be of full column rank. By default, the constraint matrix for the intercepts is the M by M identity matrix, meaning no constraints.
constraints	An alternative to specifying iconstraint and xconstraint, this is a list with two components corresponding to the intercept and x respectively. They must both be a M-row constraint matrix with full column rank.
var.arg	Logical: return the pointwise variances of the fit? Currently, this corresponds only to the nonlinear part of the fit, and may be wrong.
scale.w	Logical. By default, the weights w are scaled so that the diagonal elements have mean 1.
nk	Number of knots. If used, this argument overrides all.knots, and must lie between 6 and n+2 inclusive.
control.spar	See <a href="#">smooth.spline</a> .

## Details

The algorithm implemented is detailed in Yee (2000). It involves decomposing the component functions into a linear and nonlinear part, and using B-splines. The cost of the computation is  $O(n M^3)$ .

The argument spar contains *scaled* smoothing parameters.

## Value

An object of class "vsmooth.spline" (see vsmooth.spline-class).

## WARNING

See [vgam](#) for information about an important bug.

**Note**

This function is quite similar to [smooth.spline](#) but offers less functionality. For example, cross validation is not implemented here. For  $M = 1$ , the results will be generally different, mainly due to the different way the knots are selected.

The vector cubic smoothing spline which `s()` represents is computationally demanding for large  $M$ . The cost is approximately  $O(nM^3)$  where  $n$  is the number of unique abscissae.

Yet to be done: return the *unscaled* smoothing parameters.

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. (2000) Vector Splines and Other Vector Smoothers. Pages 529–534. In: Bethlehem, J. G. and van der Heijde, P. G. M. *Proceedings in Computational Statistics COMPSTAT 2000*. Heidelberg: Physica-Verlag.

**See Also**

`vsmooth.spline-class`, `plot.vsmooth.spline`, `predict.vsmooth.spline`, `iam`, [s](#), [smooth.spline](#).

**Examples**

```
nn <- 20; x <- 2 + 5*(nn:1)/nn
x[2:4] <- x[5:7] # Allow duplication
y1 <- sin(x) + rnorm(nn, sd = 0.13)
y2 <- cos(x) + rnorm(nn, sd = 0.13)
y3 <- 1 + sin(x) + rnorm(nn, sd = 0.13) # Run this for constraints
y <- cbind(y1, y2, y3)
ww <- cbind(rep(3, nn), 4, (1:nn)/nn)

(fit <- vsmooth.spline(x, y, w = ww, df = 5))
## Not run:
plot(fit) # The 1st and 3rd functions do not differ by a constant

## End(Not run)

mat <- matrix(c(1,0,1, 0,1,0), 3, 2)
(fit2 <- vsmooth.spline(x, y, w = ww, df = 5, iconstr = mat, xconstr = mat))
# The 1st and 3rd functions do differ by a constant:
mycols <- c("orange", "blue", "orange")
## Not run: plot(fit2, lcol = mycols, pcol = mycols, las = 1)

p <- predict(fit, x = model.matrix(fit, type = "lm"), deriv = 0)
max(abs(depvar(fit) - with(p, y))) # Should be 0; and fit@y is not good

par(mfrow = c(3, 1))
ux <- seq(1, 8, len = 100)
for (dd in 1:3) {
```

```

    pp <- predict(fit, x = ux, deriv = dd)
## Not run: with(pp, matplot(x, y, type = "l", main = paste("deriv =", dd),
                        lwd = 2, ylab = "", cex.axis = 1.5,
                        cex.lab = 1.5, cex.main = 1.5))

## End(Not run)
}

```

waitakere

*Waitakere Ranges Data*

### Description

The waitakere data frame has 579 rows and 18 columns. Altitude is explanatory, and there are binary responses (presence/absence = 1/0 respectively) for 17 plant species.

### Usage

```
data(waitakere)
```

### Format

This data frame contains the following columns:

**agaaus** Agathis australis, or Kauri  
**beitaw** Beilschmiedia tawa, or Tawa  
**corlae** Corynocarpus laevigatus  
**cyadea** Cyathea dealbata  
**cyamed** Cyathea medullaris  
**daccup** Dacrydium cupressinum  
**dacdac** Dacrycarpus dacrydioides  
**eladen** Elaeocarpus dentatus  
**hedarb** Hedycarya arborea  
**hohpop** Species name unknown  
**kniexc** Knightia excelsa, or Rewarewa  
**kuneri** Kunzea ericoides  
**lepsco** Leptospermum scoparium  
**metro** Metrosideros robusta  
**neslan** Nestegis lanceolata  
**rhosap** Rhopalostylis sapida  
**vitluc** Vitex lucens, or Puriri  
**altitude** meters above sea level

### Details

These were collected from the Waitakere Ranges, a small forest in northern Auckland, New Zealand. At 579 sites in the forest, the presence/absence of 17 plant species was recorded, as well as the altitude. Each site was of area size  $200m^2$ .

### Source

Dr Neil Mitchell, University of Auckland.

### See Also

[hunua](#).

### Examples

```
fit <- vgam(agaas ~ s(altitude, df = 2), binomialff, waitakere)
head(predict(fit, waitakere, type = "response"))
## Not run: plot(fit, se = TRUE, lcol = "orange", scol = "blue")
```

---

waldff

*Wald Distribution Family Function*

---

### Description

Estimates the parameter of the standard Wald distribution by maximum likelihood estimation.

### Usage

```
waldff(link.lambda = "log", init.lambda = NULL)
```

### Arguments

link.lambda	Parameter link function for the $\lambda$ parameter. See <a href="#">Links</a> for more choices and general information.
init.lambda	Initial value for the $\lambda$ parameter. The default means an initial value is chosen internally.

### Details

The standard Wald distribution is a special case of the inverse Gaussian distribution with  $\mu = 1$ . It has a density that can be written as

$$f(y; \lambda) = \sqrt{\lambda/(2\pi y^3)} \exp(-\lambda(y-1)^2/(2y))$$

where  $y > 0$  and  $\lambda > 0$ . The mean of  $Y$  is 1 (returned as the fitted values) and its variance is  $1/\lambda$ . By default,  $\eta = \log(\lambda)$ .



**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Note**

The **VGAM** family function [inv.gaussianff](#) estimates the location parameter  $\mu$  too.

**Author(s)**

T. W. Yee

**References**

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley.

**See Also**

[inv.gaussianff](#), [rinv.gaussian](#).

**Examples**

```
wdata <- data.frame(y = rinv.gaussian(n = 1000, mu = 1, lambda = exp(1)))
fit <- vglm(y ~ 1, waldff(init = 0.2), wdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

---

weibull

*Weibull Distribution Family Function*

---

**Description**

Maximum likelihood estimation of the 2-parameter Weibull distribution. No observations should be censored.

**Usage**

```
weibull(lshape = "loge", lscale = "loge",
        ishape = NULL, iscale = NULL, nrfs = 1,
        probs.y = c(0.2, 0.5, 0.8), imethod = 1, zero = -2)
```

## Arguments

<code>lshape</code> , <code>lscale</code>	Parameter link functions applied to the (positive) shape parameter (called $a$ below) and (positive) scale parameter (called $b$ below). See <a href="#">Links</a> for more choices.
<code>ishape</code> , <code>iscale</code>	Optional initial values for the shape and scale parameters.
<code>nrfs</code>	Currently this argument is ignored. Numeric, of length one, with value in $[0, 1]$ . Weighting factor between Newton-Raphson and Fisher scoring. The value 0 means pure Newton-Raphson, while 1 means pure Fisher scoring. The default value uses a mixture of the two algorithms, and retaining positive-definite working weights.
<code>imethod</code>	Initialization method used if there are censored observations. Currently only the values 1 and 2 are allowed.
<code>zero</code> , <code>probs.y</code>	Details at <a href="#">CommonVGAMffArguments</a> .

## Details

The Weibull density for a response  $Y$  is

$$f(y; a, b) = ay^{a-1} \exp[-(y/b)^a]/(b^a)$$

for  $a > 0$ ,  $b > 0$ ,  $y > 0$ . The cumulative distribution function is

$$F(y; a, b) = 1 - \exp[-(y/b)^a].$$

The mean of  $Y$  is  $b\Gamma(1 + 1/a)$  (returned as the fitted values), and the mode is at  $b(1 - 1/a)^{1/a}$  when  $a > 1$ . The density is unbounded for  $a < 1$ . The  $k$ th moment about the origin is  $E(Y^k) = b^k \Gamma(1 + k/a)$ . The hazard function is  $at^{a-1}/b^a$ .

This **VGAM** family function currently does not handle censored data. Fisher scoring is used to estimate the two parameters. Although the expected information matrices used here are valid in all regions of the parameter space, the regularity conditions for maximum likelihood estimation are satisfied only if  $a > 2$  (according to Kleiber and Kotz (2003)). If this is violated then a warning message is issued. One can enforce  $a > 2$  by choosing `lshape = logoff(offset = -2)`. Common values of the shape parameter lie between 0.5 and 3.5.

Summarized in Harper et al. (2011), for inference, there are 4 cases to consider. If  $a \leq 1$  then the MLEs are not consistent (and the smallest observation becomes a hyperefficient solution for the location parameter in the 3-parameter case). If  $1 < a < 2$  then MLEs exist but are not asymptotically normal. If  $a = 2$  then the MLEs exist and are normal and asymptotically efficient but with a slower convergence rate than when  $a > 2$ . If  $a > 2$  then MLEs have classical asymptotic properties.

The 3-parameter (location is the third parameter) Weibull can be estimated by maximizing a profile log-likelihood (see, e.g., Harper et al. (2011) and Lawless (2003)), else try [gev](#) which is a better parameterization.

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

**Warning**

This function is under development to handle other censoring situations. The version of this function which will handle censored data will be called `cenweibull()`. It is currently being written and will use [SurvS4](#) as input. It should be released in later versions of **VGAM**.

If the shape parameter is less than two then misleading inference may result, e.g., in the summary and `vcov` of the object.

**Note**

Successful convergence depends on having reasonably good initial values. If the initial values chosen by this function are not good, make use the two initial value arguments.

This **VGAM** family function handles multiple responses.

The Weibull distribution is often an alternative to the lognormal distribution. The inverse Weibull distribution, which is that of  $1/Y$  where  $Y$  has a  $\text{Weibull}(a, b)$  distribution, is known as the log-Gompertz distribution.

There are problems implementing the three-parameter Weibull distribution. These are because the classical regularity conditions for the asymptotic properties of the MLEs are not satisfied because the support of the distribution depends on one of the parameters.

**Author(s)**

T. W. Yee

**References**

- Kleiber, C. and Kotz, S. (2003) *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, NJ, USA: Wiley-Interscience.
- Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994) *Continuous Univariate Distributions*, 2nd edition, Volume 1, New York: Wiley.
- Lawless, J. F. (2003) *Statistical Models and Methods for Lifetime Data*, 2nd ed. Hoboken, NJ, USA: John Wiley & Sons.
- Rinne, Horst. (2009) *The Weibull Distribution: A Handbook*. Boca Raton, FL, USA: CRC Press.
- Gupta, R. D. and Kundu, D. (2006) On the comparison of Fisher information of the Weibull and GE distributions, *Journal of Statistical Planning and Inference*, **136**, 3130–3144.
- Harper, W. V. and Eschenbach, T. G. and James, T. R. (2011) Concerns about Maximum Likelihood Estimation for the Three-Parameter Weibull Distribution: Case Study of Statistical Software, *The American Statistician*, **65**(1), 44–54.
- Smith, R. L. (1985) Maximum likelihood estimation in a class of nonregular cases. *Biometrika*, **72**, 67–90.
- Smith, R. L. and Naylor, J. C. (1987) A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distribution. *Applied Statistics*, **36**, 358–369.

**See Also**

[dweibull](#), [truncweibull](#), [gev](#), [lognormal](#), [expexp.gumbelII](#).

## Examples

```
wdata <- data.frame(x2 = runif(nn <- 1000)) # Complete data
wdata <- transform(wdata,
  y1 = rweibull(nn, shape = exp(1 + x2), scale = exp(-2)),
  y2 = rweibull(nn, shape = exp(2 - x2), scale = exp( 1)))
fit <- vglm(cbind(y1, y2) ~ x2, weibull, wdata, trace = TRUE)
coef(fit, matrix = TRUE)
vcov(fit)
summary(fit)
```

---

weightsvglm

---

*Prior and Working Weights of a VGLM fit*


---

## Description

Returns either the prior weights or working weights of a VGLM object.

## Usage

```
weightsvglm(object, type = c("prior", "working"),
  matrix.arg = TRUE, ignore.slot = FALSE,
  deriv.arg = FALSE, ...)
```

## Arguments

object	a model object from the <b>VGAM</b> R package that inherits from a <i>vector generalized linear model</i> (VGLM), e.g., a model of class "vglm".
type	Character, which type of weight is to be returned? The default is the first one.
matrix.arg	Logical, whether the answer is returned as a matrix. If not, it will be a vector.
ignore.slot	Logical. If TRUE then object@weights is ignored even if it has been assigned, and the long calculation for object@weights is repeated. This may give a slightly different answer because of the final IRLS step at convergence may or may not assign the latest value of quantities such as the mean and weights.
deriv.arg	Logical. If TRUE then a list with components deriv and weights is returned. See below for more details.
...	Currently ignored.

## Details

Prior weights are usually inputted with the weights argument in functions such as [vglm](#) and [vgam](#). It may refer to frequencies of the individual data or be weight matrices specified beforehand.

Working weights are used by the IRLS algorithm. They correspond to the second derivatives of the log-likelihood function with respect to the linear predictors. The working weights correspond to positive-definite weight matrices and are returned in *matrix-band* form, e.g., the first  $M$  columns correspond to the diagonals, etc.

**Value**

If `type = "working"` and `deriv = TRUE` then a list is returned with the two components described below. Otherwise the prior or working weights are returned depending on the value of `type`.

<code>deriv</code>	Typically the first derivative of the log-likelihood with respect to the linear predictors. For example, this is the variable <code>deriv.mu</code> in <code>vglm.fit()</code> , or equivalently, the matrix returned in the <code>"deriv"</code> slot of a <b>VGAM</b> family function.
<code>weights</code>	The working weights.

**Note**

This function is intended to be similar to `weights.glm` (see [glm](#)).

**Author(s)**

Thomas W. Yee

**References**

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

Chambers, J. M. and T. J. Hastie (eds) (1992) *Statistical Models in S*. Wadsworth & Brooks/Cole.

**See Also**

[glm](#), [vglmff-class](#), [vglm](#).

**Examples**

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
             cumulative(parallel = TRUE, reverse = TRUE), pneumo))
depvar(fit) # These are sample proportions
weights(fit, type = "prior", matrix = FALSE) # Number of observations

# Look at the working residuals
nn <- nrow(model.matrix(fit, type = "lm"))
M <- ncol(predict(fit))

temp <- weights(fit, type = "working", deriv = TRUE)
wz <- m2adefault(temp$weights, M = M) # In array format
wzinv <- array(apply(wz, 3, solve), c(M, M, nn))
wresid <- matrix(NA, nn, M) # Working residuals
for (ii in 1:nn)
  wresid[ii,] <- wzinv[, , ii, drop = TRUE] %*% temp$deriv[ii, ]
max(abs(c(resid(fit, type = "work")) - c(wresid))) # Should be 0

(zedd <- predict(fit) + wresid) # Adjusted dependent vector
```

---

wrapup.smart

*Cleans Up After Smart Prediction*


---

### Description

wrapup.smart deletes any variables used by smart prediction. Needed by both the modelling function and the prediction function.

### Usage

```
wrapup.smart()
```

### Details

The variables to be deleted are .smart.prediction, .smart.prediction.counter, and .smart.prediction.mode. The function wrapup.smart is useful in R because these variables are held in smartpredenv. In S-PLUS, wrapup.smart is not really necessary because the variables are placed in frame 1, which disappears when finished anyway.

### References

See the technical help file at <http://www.stat.auckland.ac.nz/~yee> for details.

### See Also

[setup.smart.](#)

### Examples

```
## Not run: # Place this inside modelling functions such as lm, glm, vglm.
wrapup.smart() # Put at the end of lm

## End(Not run)
```

---

yeo.johnson

*Yeo-Johnson Transformation*


---

### Description

Computes the Yeo-Johnson transformation, which is a normalizing transformation.

### Usage

```
yeo.johnson(y, lambda, derivative = 0,
            epsilon = sqrt(.Machine$double.eps), inverse = FALSE)
```

**Arguments**

<code>y</code>	Numeric, a vector or matrix.
<code>lambda</code>	Numeric. It is recycled to the same length as <code>y</code> if necessary.
<code>derivative</code>	Non-negative integer. The default is the ordinary function evaluation, otherwise the derivative with respect to <code>lambda</code> .
<code>epsilon</code>	Numeric and positive value. The tolerance given to values of <code>lambda</code> when comparing it to 0 or 2.
<code>inverse</code>	Logical. Return the inverse transformation?

**Details**

The Yeo-Johnson transformation can be thought of as an extension of the Box-Cox transformation. It handles both positive and negative values, whereas the Box-Cox transformation only handles positive values. Both can be used to transform the data so as to improve normality. They can be used to perform LMS quantile regression.

**Value**

The Yeo-Johnson transformation or its inverse, or its derivatives with respect to `lambda`, of `y`.

**Note**

If `inverse = TRUE` then the argument `derivative = 0` is required.

**Author(s)**

Thomas W. Yee

**References**

Yeo, I.-K. and Johnson, R. A. (2000) A new family of power transformations to improve normality or symmetry. *Biometrika*, **87**, 954–959.

Yee, T. W. (2004) Quantile regression via vector generalized additive models. *Statistics in Medicine*, **23**, 2295–2315.

**See Also**

[lms.yjn](#), [boxcox](#).

**Examples**

```
y <- seq(-4, 4, len = (nn <- 200))
ltry <- c(0, 0.5, 1, 1.5, 2) # Try these values of lambda
lltry <- length(ltry)
psi <- matrix(as.numeric(NA), nn, lltry)
for (ii in 1:lltry)
  psi[, ii] <- yeo.johnson(y, lambda = ltry[ii])

## Not run:
```

```

matplot(y, psi, type = "l", ylim = c(-4, 4), lwd = 2, lty = 1:11try,
        ylab = "Yeo-Johnson transformation", col = 1:11try, las = 1,
        main = "Yeo-Johnson transformation with some values of lambda")
abline(v = 0, h = 0)
legend(x = 1, y = -0.5, lty = 1:11try, legend = as.character(1try),
       lwd = 2, col = 1:11try)
## End(Not run)

```

yip88

*Zero-Inflated Poisson Distribution (Yip (1988) algorithm)*

## Description

Fits a zero-inflated Poisson distribution based on Yip (1988).

## Usage

```
yip88(link = "loge", n.arg = NULL)
```

## Arguments

link	Link function for the usual $\lambda$ parameter. See <a href="#">Links</a> for more choices.
n.arg	The total number of observations in the data set. Needed when the response variable has all the zeros deleted from it, so that the number of zeros can be determined.

## Details

The method implemented here, Yip (1988), maximizes a *conditional* likelihood. Consequently, the methodology used here deletes the zeros from the data set, and is thus related to the positive Poisson distribution (where  $P(Y = 0) = 0$ ).

The probability function of  $Y$  is 0 with probability  $\phi$ , and  $\text{Poisson}(\lambda)$  with probability  $1 - \phi$ . Thus

$$P(Y = 0) = \phi + (1 - \phi)P(W = 0)$$

where  $W$  is  $\text{Poisson}(\lambda)$ . The mean,  $(1 - \phi)\lambda$ , can be obtained by the extractor function `fitted` applied to the object.

This family function treats  $\phi$  as a scalar. If you want to model both  $\phi$  and  $\lambda$  as a function of covariates, try [zipoisson](#).

## Value

An object of class `"vglmff"` (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).



**Warning**

Under- or over-flow may occur if the data is ill-conditioned. Yip (1988) only considered  $\phi$  being a scalar and not modelled as a function of covariates. To get around this limitation, try [zipoisson](#).

Inference obtained from `summary.vglm` and `summary.vgam` may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

**Note**

The data may be inputted in two ways. The first is when the response is a vector of positive values, with the argument `n` in `yip88` specifying the total number of observations. The second is simply include all the data in the response. In this case, the zeros are trimmed off during the computation, and the `x` and `y` slots of the object, if assigned, will reflect this.

The estimate of  $\phi$  is placed in the `misc` slot as `@misc$pstr0`. However, this estimate is computed only for intercept models, i.e., the formula is of the form  $y \sim 1$ .

**Author(s)**

Thomas W. Yee

**References**

Yip, P. (1988) Inference about the mean of a Poisson distribution in the presence of a nuisance parameter. *The Australian Journal of Statistics*, **30**, 299–306.

Angers, J-F. and Biswas, A. (2003) A Bayesian analysis of zero-inflated generalized Poisson model. *Computational Statistics & Data Analysis*, **42**, 37–46.

**See Also**

[zipoisson](#), [Zipois](#), [zipoisson](#), [pospoisson](#), [poissonff](#), [dzipois](#).

**Examples**

```
phi <- 0.35; lambda <- 2 # Generate some artificial data
y <- rzipois(n <- 1000, lambda, phi)
table(y)

# Two equivalent ways of fitting the same model
fit1 <- vglm(y ~ 1, yip88(n = length(y)), subset = y > 0)
fit2 <- vglm(y ~ 1, yip88, trace = TRUE, crit = "coef")
(true.mean <- (1-phi) * lambda)
mean(y)
head(fitted(fit1))
fit1@misc$pstr0 # The estimate of phi

# Compare the ZIP with the positive Poisson distribution
pp <- vglm(y ~ 1, pospoisson, subset = y > 0, crit = "c")
coef(pp)
Coef(pp)
```

```

coef(fit1) - coef(pp)          # Same
head(fitted(fit1) - fitted(pp)) # Different

# Another example (Angers and Biswas, 2003) -----
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)

yy <- with(abdata, rep(y, w))
fit3 <- vglm(yy ~ 1, yip88(n = length(yy)), subset = yy > 0)
fit3@misc$pstr0 # Estimate of phi (they get 0.5154 with SE 0.0707)
coef(fit3, matrix = TRUE)
Coef(fit3) # Estimate of lambda (they get 0.6997 with SE 0.1520)
head(fitted(fit3))
mean(yy) # Compare this with fitted(fit3)

```

---

Yules

---

*Yule-Simon Distribution*


---

## Description

Density, distribution function, quantile function and random generation for the Yule-Simon distribution.

## Usage

```

dyules(x, rho, log = FALSE)
pyules(q, rho)

ryules(n, rho)

```

## Arguments

<code>x, q</code>	Vector of quantiles. For the density, it should be a vector with positive integer values in order for the probabilities to be positive.
<code>n</code>	number of observations. A single positive integer.
<code>rho</code>	See <a href="#">yulesimon</a> .
<code>log</code>	logical; if TRUE, the logarithm is returned.

## Details

See [yulesimon](#), the **VGAM** family function for estimating the parameter, for the formula of the probability density function and other details.

## Value

`dyules` gives the density, `pyules` gives the distribution function, and `ryules` generates random deviates.

**Author(s)**

T. W. Yee

**See Also**[yulesimon.](#)**Examples**

```

dyules(1:20, 2.1)
ryules(20, 2.1)

round(1000 * dyules(1:8, 2))
table(ryules(1000, 2))

## Not run: x <- 0:6
plot(x, dyules(x, rho = 2.2), type = "h", las = 1, col = "blue")
## End(Not run)

```

yulesimon

*Yule-Simon Family Function***Description**

Estimating the parameter of the Yule-Simon distribution.

**Usage**

```
yulesimon(link = "log", irho = NULL, nsimEIM = 200, zero = NULL)
```

**Arguments**

link	Link function for the $\rho$ parameter. See <a href="#">Links</a> for more choices and for general information.
irho	Optional initial value for the (positive) parameter. See <a href="#">CommonVGAMffArguments</a> for more information. The default is to obtain an initial value internally. Use this argument if the default fails.
nsimEIM, zero	See <a href="#">CommonVGAMffArguments</a> for more information.

**Details**

The probability function is

$$f(y; \rho) = \rho * \text{beta}(y, \rho + 1),$$

where the parameter  $\rho > 0$ , *beta* is the [beta](#) function, and  $y = 1, 2, \dots$ . The function [dyules](#) computes this probability function. The mean of  $Y$ , which is returned as fitted values, is  $\rho/(\rho - 1)$  provided  $\rho > 1$ . The variance of  $Y$  is  $\rho^2/((\rho - 1)^2(\rho - 2))$  provided  $\rho > 2$ .

The distribution was named after Udny Yule and Herbert A. Simon. Simon originally called it the Yule distribution. This family function can handle multiple responses.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

**Author(s)**

T. W. Yee

**References**

Simon, H. A. (1955) On a class of skew distribution functions. *Biometrika*, **42**, 425–440.

**See Also**

[ryules](#).

**Examples**

```
ydata <- data.frame(x2 = runif(nn <- 1000))
ydata <- transform(ydata, y = ryules(nn, rho = exp(1.5 - x2)))
with(ydata, table(y))
fit <- vglm(y ~ x2, yulesimon, ydata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
```

---

Zabinom	<i>Zero-Altered Binomial Distribution</i>
---------	---

---

**Description**

Density, distribution function, quantile function and random generation for the zero-altered binomial distribution with parameter `pobs0`.

**Usage**

```
dzabinom(x, size, prob, pobs0 = 0, log = FALSE)
pzabinom(q, size, prob, pobs0 = 0)
qzabinom(p, size, prob, pobs0 = 0)
rzabinom(n, size, prob, pobs0 = 0)
```

**Arguments**

- `x, q` vector of quantiles.
- `p` vector of probabilities.
- `n` number of observations. If `length(n) > 1` then the length is taken to be the number required.

size, prob, log	Parameters from the ordinary binomial distribution (see <a href="#">dbinom</a> ).
pobs0	Probability of (an observed) zero, called <i>pobs0</i> . The default value of pobs0 = 0 corresponds to the response having a positive binomial distribution.

## Details

The probability function of  $Y$  is 0 with probability pobs0, else a positive binomial(size, prob) distribution.

## Value

dzabinom gives the density and pzabinom gives the distribution function, qzabinom gives the quantile function, and rzabinom generates random deviates.

## Note

The argument pobs0 is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

## Author(s)

T. W. Yee

## See Also

[zibinomial](#), [rposbinom](#).

## Examples

```
size <- 10; prob <- 0.15; pobs0 <- 0.05; x <- (-1):7
dzabinom(x, size = size, prob = prob, pobs0 = pobs0)
table(rzabinom(100, size = size, prob = prob, pobs0 = pobs0))

## Not run: x <- 0:10
barplot(rbind(dzabinom(x, size = size, prob = prob, pobs0 = pobs0),
              dbinom(x, size = size, prob = prob)),
        beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
        ylab = "Probability", names.arg = as.character(x),
        main = paste("ZAB(size = ", size, ", prob = ", prob, ", pobs0 = ", pobs0,
                    ") [blue] vs", " Binom(size = ", size, ", prob = ", prob,
                    ") [orange] densities", sep = ""))

## End(Not run)
```

zabinomial

*Zero-Altered Binomial Distribution***Description**

Fits a zero-altered binomial distribution based on a conditional model involving a Bernoulli distribution and a positive-binomial distribution.

**Usage**

```
zabinomial(lpobs0 = "logit", lprob = "logit",
            type.fitted = c("mean", "pobs0"),
            ipobs0 = NULL, iprob = NULL, imethod = 1, zero = NULL)
zabinomialff(lprob = "logit", lonempobs0 = "logit",
             type.fitted = c("mean", "pobs0", "onempobs0"),
             iprob = NULL, ionempobs0 = NULL, imethod = 1, zero = 2)
```

**Arguments**

lprob	Parameter link function applied to the probability parameter of the binomial distribution. See <a href="#">Links</a> for more choices.
lpobs0	Link function for the parameter $p_0$ , called pobs0 here. See <a href="#">Links</a> for more choices.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvlm</a> for information.
iprob, ipobs0	See <a href="#">CommonVGAMffArguments</a> .
lonempobs0, ionempobs0	Corresponding argument for the other parameterization. See details below.
imethod, zero	See <a href="#">CommonVGAMffArguments</a> .

**Details**

The response  $Y$  is zero with probability  $p_0$ , else  $Y$  has a positive-binomial distribution with probability  $1 - p_0$ . Thus  $0 < p_0 < 1$ , which may be modelled as a function of the covariates. The zero-altered binomial distribution differs from the zero-inflated binomial distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the binomial distribution too. The zero-inflated binomial distribution is implemented in [zibinomial](#). Some people call the zero-altered binomial a *hurdle* model.

The input is currently a vector or one-column matrix. By default, the two linear/additive predictors for `zabinomial()` are  $(\text{logit}(p_0), \log(p))^T$ .

The **VGAM** family function `zabinomialff()` has a few changes compared to `zabinomial()`. These are: (i) the order of the linear/additive predictors is switched so the binomial probability comes first; (ii) argument `onempobs0` is now 1 minus the probability of an observed 0, i.e., the probability of the positive binomial distribution, i.e., `onempobs0` is  $1 - p_{\text{obs0}}$ ; (iii) argument `zero` has a new default so that the `onempobs0` is intercept-only by default. Now `zabinomialff()` is generally recommended over `zabinomial()`. Both functions implement Fisher scoring and neither can handle multiple responses.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean  $\mu$  (default) which is given by

$$\mu = (1 - p_0)\mu_b / [1 - (1 - \mu_b)^N]$$

where  $\mu_b$  is the usual binomial mean. If type.fitted = "pobs0" then  $p_0$  is returned.

**Note**

The response should be a two-column matrix of counts, with first column giving the number of successes.

Note this family function allows  $p_0$  to be modelled as functions of the covariates by having zero = NULL. It is a conditional model, not a mixture model.

These family functions effectively combine [posbinomial](#) and [binomialff](#) into one family function.

**Author(s)**

T. W. Yee

**See Also**

[dzabinom](#), [zibinomial](#), [posbinomial](#), [binomialff](#), [dbinom](#), [CommonVGAMffArguments](#).

**Examples**

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, size = 10,
                  prob = logit(-2 + 3*x2, inverse = TRUE),
                  pobs0 = logit(-1 + 2*x2, inverse = TRUE))
zdata <- transform(zdata,
                  y1 = rzabinom(nn, size = size, prob = prob, pobs0 = pobs0))
with(zdata, table(y1))

fit <- vglm(cbind(y1, size - y1) ~ x2, zabinomial(zero = NULL), zdata, trace = TRUE)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))
summary(fit)
```

Zageom

*Zero-Altered Geometric Distribution***Description**

Density, distribution function, quantile function and random generation for the zero-altered geometric distribution with parameter `pobs0`.

**Usage**

```
dzageom(x, prob, pobs0 = 0, log = FALSE)
pzageom(q, prob, pobs0 = 0)
qzageom(p, prob, pobs0 = 0)
rzageom(n, prob, pobs0 = 0)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>prob, log</code>	Parameters from the ordinary geometric distribution (see <a href="#">dgeom</a> ).
<code>pobs0</code>	Probability of (an observed) zero, called <i>pobs0</i> . The default value of <code>pobs0 = 0</code> corresponds to the response having a positive geometric distribution.

**Details**

The probability function of  $Y$  is 0 with probability `pobs0`, else a positive `geometric(prob)` distribution.

**Value**

`dzageom` gives the density and `pzageom` gives the distribution function, `qzageom` gives the quantile function, and `rzageom` generates random deviates.

**Note**

The argument `pobs0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

**Author(s)**

T. W. Yee

**See Also**

[zageometric](#), [zigeometric](#), [rposgeom](#).



## Examples

```

prob <- 0.35; pobs0 <- 0.05; x <- (-1):7
dzageom(x, prob = prob, pobs0 = pobs0)
table(rzageom(100, prob = prob, pobs0 = pobs0))

## Not run: x <- 0:10
barplot(rbind(dzageom(x, prob = prob, pobs0 = pobs0),
              dgeom(x, prob = prob)),
        beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
        ylab = "Probability", names.arg = as.character(x),
        main = paste("ZAG(prob = ", prob, ", pobs0 = ", pobs0,
                     ") [blue] vs", " Geometric(prob = ", prob,
                     ") [orange] densities", sep = ""))
## End(Not run)

```

---

zageometric

Zero-Altered Geometric Distribution

---

## Description

Fits a zero-altered geometric distribution based on a conditional model involving a Bernoulli distribution and a positive-geometric distribution.

## Usage

```

zageometric(lpobs0 = "logit", lprob = "logit",
            type.fitted = c("mean", "pobs0", "onempobs0"),
            imethod = 1, ipobs0 = NULL, iprob = NULL, zero = NULL)
zageometricff(lprob = "logit", lonempobs0 = "logit",
              type.fitted = c("mean", "pobs0", "onempobs0"),
              imethod = 1, iprob = NULL, ionempobs0 = NULL, zero = -2)

```

## Arguments

lpobs0	Link function for the parameter $p_0$ or $\phi$ , called pobs0 or phi here. See <a href="#">Links</a> for more choices.
lprob	Parameter link function applied to the probability of success, called prob or $p$ . See <a href="#">Links</a> for more choices.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvglm</a> for information.
ipobs0, iprob	Optional initial values for the parameters. If given, they must be in range. For multi-column responses, these are recycled sideways.
lonempobs0, ionempobs0	Corresponding argument for the other parameterization. See details below.
zero, imethod	See <a href="#">CommonVGAMffArguments</a> .

## Details

The response  $Y$  is zero with probability  $p_0$ , or  $Y$  has a positive-geometric distribution with probability  $1 - p_0$ . Thus  $0 < p_0 < 1$ , which is modelled as a function of the covariates. The zero-altered geometric distribution differs from the zero-inflated geometric distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the geometric distribution too. The zero-inflated geometric distribution is implemented in the **VGAM** package. Some people call the zero-altered geometric a *hurdle* model.

The input can be a matrix (multiple responses). By default, the two linear/additive predictors of `zageometric` are  $(\text{logit}(\phi), \text{logit}(p))^T$ .

The **VGAM** family function `zageometricff()` has a few changes compared to `zageometric()`. These are: (i) the order of the linear/additive predictors is switched so the geometric probability comes first; (ii) argument `onempobs0` is now 1 minus the probability of an observed 0, i.e., the probability of the positive geometric distribution, i.e., `onempobs0` is  $1 - \text{pobs0}$ ; (iii) argument `zero` has a new default so that the `pobs0` is intercept-only by default. Now `zageometricff()` is generally recommended over `zageometric()`. Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

The `fitted.values` slot of the fitted object, which should be extracted by the generic function `fitted`, returns the mean  $\mu$  (default) which is given by

$$\mu = (1 - \phi)/p.$$

If `type.fitted = "pobs0"` then  $p_0$  is returned.

## Warning

Convergence for this **VGAM** family function seems to depend quite strongly on providing good initial values.

Inference obtained from `summary.vglm` and `summary.vgam` may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

## Note

Note this family function allows  $p_0$  to be modelled as functions of the covariates. It is a conditional model, not a mixture model.

This family function effectively combines [binomialff](#) and `posgeometric()` and [geometric](#) into one family function. However, `posgeometric()` is not written because it is trivially related to [geometric](#).

## Author(s)

T. W. Yee

**See Also**

[dzageom](#), [geometric](#), [zigeometric](#), [dgeom](#), [CommonVGAMffArguments](#).

**Examples**

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pobs0 = logit(-1 + 2*x2, inverse = TRUE),
                  prob = logit(-2 + 3*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzageom(nn, prob = prob, pobs0 = pobs0),
                  y2 = rzageom(nn, prob = prob, pobs0 = pobs0))
with(zdata, table(y1))

fit <- vglm(cbind(y1, y2) ~ x2, zageometric, data = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))
summary(fit)
```

---

Zanegbin

---

Zero-Altered Negative Binomial Distribution

---

**Description**

Density, distribution function, quantile function and random generation for the zero-altered negative binomial distribution with parameter `pobs0`.

**Usage**

```
dzanegbin(x, size, prob = NULL, munb = NULL, pobs0 = 0, log = FALSE)
pzanegbin(q, size, prob = NULL, munb = NULL, pobs0 = 0)
qzanegbin(p, size, prob = NULL, munb = NULL, pobs0 = 0)
rzanegbin(n, size, prob = NULL, munb = NULL, pobs0 = 0)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<code>size, prob, munb, log</code>	Parameters from the ordinary negative binomial distribution (see <a href="#">dnbinom</a> ). Some arguments have been renamed slightly.
<code>pobs0</code>	Probability of zero, called <i>pobs0</i> . The default value of <code>pobs0 = 0</code> corresponds to the response having a positive negative binomial distribution.

**Details**

The probability function of  $Y$  is 0 with probability `pobs0`, else a positive negative binomial( $\mu_{nb}$ , `size`) distribution.

**Value**

`dzanegbin` gives the density and `pzanegbin` gives the distribution function, `qzanegbin` gives the quantile function, and `rzanegbin` generates random deviates.

**Note**

The argument `pobs0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

**Author(s)**

T. W. Yee

**See Also**

[zanegbinomial](#), [rposnegbin](#).

**Examples**

```
munb <- 3; size <- 4; pobs0 <- 0.3; x <- (-1):7
dzanegbin(x, munb = munb, size = size, pobs0 = pobs0)
table(rzanegbin(100, munb = munb, size = size, pobs0 = pobs0))

## Not run: x <- 0:10
barplot(rbind(dzanegbin(x, munb = munb, size = size, pobs0 = pobs0),
              dnbinom(x, mu = munb, size = size)),
        beside = TRUE, col = c("blue", "green"), cex.main = 0.7, las = 1,
        ylab = "Probability", names.arg = as.character(x),
        main = paste("ZANB(munb = ", munb, ", size = ", size, ",
                     pobs0 = ", pobs0,
                     ") [blue] vs", " NB(mu = ", munb, ", size = ", size,
                     ") [green] densities", sep = ""))

## End(Not run)
```

---

zanegbinomial

*Zero-Altered Negative Binomial Distribution*

---

**Description**

Fits a zero-altered negative binomial distribution based on a conditional model involving a binomial distribution and a positive-negative binomial distribution.

**Usage**

```
zanegbinomial(lpobs0 = "logit", lmunb = "loge", lsize = "loge",
              type.fitted = c("mean", "pobs0"),
              ipobs0 = NULL, isize = NULL, zero = -3, imethod = 1,
              nsimEIM = 250, shrinkage.init = 0.95)
zanegbinomialff(lmunb = "loge", lsize = "loge", lonempobs0 = "logit",
                type.fitted = c("mean", "pobs0", "onempobs0"),
                isize = NULL, ionempobs0 = NULL, zero = c(-2, -3),
                imethod = 1, nsimEIM = 250, shrinkage.init = 0.95)
```

**Arguments**

lpobs0	Link function for the parameter $p_0$ , called pobs0 here. See <a href="#">Links</a> for more choices.
lmunb	Link function applied to the munb parameter, which is the mean $\mu_{nb}$ of an ordinary negative binomial distribution. See <a href="#">Links</a> for more choices.
lsize	Parameter link function applied to the reciprocal of the dispersion parameter, called k. That is, as k increases, the variance of the response decreases. See <a href="#">Links</a> for more choices.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvlm</a> for information.
lonempobs0, ionempobs0	Corresponding argument for the other parameterization. See details below.
ipobs0, isize	Optional initial values for $p_0$ and k. If given then it is okay to give one value for each response/species by inputting a vector whose length is the number of columns of the response matrix.
zero	Specifies which of the three linear predictors are modelled as an intercept only. All parameters can be modelled as a function of the explanatory variables by setting zero = NULL (not recommended). A negative value means that the value is recycled, e.g., setting -3 means all k are intercept-only for zanebinomial. See <a href="#">CommonVGAMffArguments</a> for more information.
nsimEIM, imethod	See <a href="#">CommonVGAMffArguments</a> .
shrinkage.init	See <a href="#">negbinomial</a> and <a href="#">CommonVGAMffArguments</a> .

**Details**

The response  $Y$  is zero with probability  $p_0$ , or  $Y$  has a positive-negative binomial distribution with probability  $1 - p_0$ . Thus  $0 < p_0 < 1$ , which is modelled as a function of the covariates. The zero-altered negative binomial distribution differs from the zero-inflated negative binomial distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the negative binomial distribution too. The zero-inflated negative binomial distribution is implemented in the **VGAM** package. Some people call the zero-altered negative binomial a *hurdle* model.

For one response/species, by default, the three linear/additive predictors for `zanegbinomial()` are  $(\text{logit}(p_0), \log(\mu_{nb}), \log(k))^T$ . This vector is recycled for multiple species.

The **VGAM** family function `zanegbinomialff()` has a few changes compared to `zanegbinomial()`. These are: (i) the order of the linear/additive predictors is switched so the negative binomial mean

comes first; (ii) argument `onempobs0` is now 1 minus the probability of an observed 0, i.e., the probability of the positive negative binomial distribution, i.e., `onempobs0` is `1-pobs0`; (iii) argument zero has a new default so that the `pobs0` is intercept-only by default. Now `zanegbinomialff()` is generally recommended over `zanegbinomial()`. Both functions implement Fisher scoring and can handle multiple responses.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

The `fitted.values` slot of the fitted object, which should be extracted by the generic function `fitted`, returns the mean  $\mu$  (default) which is given by

$$\mu = (1 - p_0)\mu_{nb}/[1 - (k/(k + \mu_{nb}))^k].$$

If `type.fitted = "pobs0"` then  $p_0$  is returned.

### Warning

Convergence for this **VGAM** family function seems to depend quite strongly on providing good initial values.

This **VGAM** family function is computationally expensive and usually runs slowly; setting `trace = TRUE` is useful for monitoring convergence.

Inference obtained from `summary.vglm` and `summary.vgam` may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

### Note

Note this family function allows  $p_0$  to be modelled as functions of the covariates provided zero is set correctly. It is a conditional model, not a mixture model. Simulated Fisher scoring is the algorithm.

This family function effectively combines [posnegbinomial](#) and [binomialff](#) into one family function.

This family function can handle a multivariate response, e.g., more than one species.

### Author(s)

T. W. Yee

### References

- Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996) Modelling the abundances of rare species: statistical models for counts with extra zeros. *Ecological Modelling*, **88**, 297–308.
- Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

See Also

[dzanegbin](#), [posnegbinomial](#), [negbinomial](#), [binomialff](#), [rposnegbin](#), [zinegbinomial](#), [zipoisson](#), [dnbinom](#), [CommonVGAMffArguments](#).

Examples

```
## Not run:
zdata <- data.frame(x2 = runif(nn <- 2000))
zdata <- transform(zdata, pobs0 = logit(-1 + 2*x2, inverse = TRUE))
zdata <- transform(zdata,
  y1 = rzanegbin(nn, munb = exp(0+2*x2), size = exp(1), pobs0 = pobs0),
  y2 = rzanegbin(nn, munb = exp(1+2*x2), size = exp(1), pobs0 = pobs0))
with(zdata, table(y1))
with(zdata, table(y2))

fit <- vglm(cbind(y1, y2) ~ x2, zanegbinomial, data = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))

## End(Not run)
```

---

Zapois	<i>Zero-Altered Poisson Distribution</i>
--------	--

---

Description

Density, distribution function, quantile function and random generation for the zero-altered Poisson distribution with parameter *pobs0*.

Usage

```
dzapois(x, lambda, pobs0 = 0, log = FALSE)
pzapois(q, lambda, pobs0 = 0)
qzapois(p, lambda, pobs0 = 0)
rzapois(n, lambda, pobs0 = 0)
```

Arguments

<i>x</i> , <i>q</i>	vector of quantiles.
<i>p</i>	vector of probabilities.
<i>n</i>	number of observations. If <code>length(n) &gt; 1</code> then the length is taken to be the number required.
<i>lambda</i>	Vector of positive means.
<i>pobs0</i>	Probability of zero, called <i>pobs0</i> . The default value of <i>pobs0</i> = 0 corresponds to the response having a positive Poisson distribution.
<i>log</i>	Logical. Return the logarithm of the answer?

**Details**

The probability function of  $Y$  is 0 with probability `pobs0`, else a positive *Poisson*( $\lambda$ ).

**Value**

`dzapois` gives the density, `pzapois` gives the distribution function, `qzapois` gives the quantile function, and `rzapois` generates random deviates.

**Note**

The argument `pobs0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

**Author(s)**

T. W. Yee

**See Also**

[zapoisson](#), [dzipois](#).

**Examples**

```
lambda <- 3; pobs0 <- 0.2; x <- (-1):7
(ii <- dzapois(x, lambda, pobs0))
max(abs(cumsum(ii) - pzapois(x, lambda, pobs0))) # Should be 0
table(rzapois(100, lambda, pobs0))
table(qzapois(runif(100), lambda, pobs0))
round(dzapois(0:10, lambda, pobs0) * 100) # Should be similar

## Not run: x <- 0:10
barplot(rbind(dzapois(x, lambda, pobs0), dpois(x, lambda)),
        beside = TRUE, col = c("blue", "green"), las = 1,
        main = paste("ZAP(", lambda, ", ", pobs0 = ", pobs0, ") [blue] vs",
                      " Poisson(", lambda, ") [green] densities", sep = ""),
        names.arg = as.character(x), ylab = "Probability")
## End(Not run)
```

---

zapoisson

---

*Zero-Altered Poisson Distribution*


---

**Description**

Fits a zero-altered Poisson distribution based on a conditional model involving a Bernoulli distribution and a positive-Poisson distribution.



**Usage**

```
zapoisson(lpobs0 = "logit", llambda = "loge",
           type.fitted = c("mean", "pobs0", "onempobs0"), zero = NULL)
zapoissonff(llambda = "loge", lonempobs0 = "logit",
             type.fitted = c("mean", "pobs0", "onempobs0"), zero = -2)
```

**Arguments**

lpobs0	Link function for the parameter $p_0$ , called pobs0 here. See <a href="#">Links</a> for more choices.
llambda	Link function for the usual $\lambda$ parameter. See <a href="#">Links</a> for more choices.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvglm</a> for information.
lonempobs0	Corresponding argument for the other parameterization. See details below.
zero	See <a href="#">CommonVGAMffArguments</a> for more information.

**Details**

The response  $Y$  is zero with probability  $p_0$ , else  $Y$  has a positive-Poisson( $\lambda$ ) distribution with probability  $1 - p_0$ . Thus  $0 < p_0 < 1$ , which is modelled as a function of the covariates. The zero-altered Poisson distribution differs from the zero-inflated Poisson distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the Poisson distribution too. Some people call the zero-altered Poisson a *hurdle* model.

For one response/species, by default, the two linear/additive predictors for `zapoisson()` are  $(\text{logit}(p_0), \log(\lambda))^T$ .

The **VGAM** family function `zapoissonff()` has a few changes compared to `zapoisson()`. These are: (i) the order of the linear/additive predictors is switched so the Poisson mean comes first; (ii) argument `onempobs0` is now 1 minus the probability of an observed 0, i.e., the probability of the positive Poisson distribution, i.e., `onempobs0` is  $1 - p_0$ ; (iii) argument `zero` has a new default so that the `onempobs0` is intercept-only by default. Now `zapoissonff()` is generally recommended over `zapoisson()`. Both functions implement Fisher scoring and can handle multiple responses.

**Value**

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as `vglm`, and `vgam`.

The `fitted.values` slot of the fitted object, which should be extracted by the generic function `fitted`, returns the mean  $\mu$  (default) which is given by

$$\mu = (1 - p_0)\lambda / [1 - \exp(-\lambda)].$$

If `type.fitted = "pobs0"` then  $p_0$  is returned.

**Note**

There are subtle differences between this family function and [zipoisson](#) and [yip88](#). In particular, [zipoisson](#) is a *mixture* model whereas `zapoisson()` and [yip88](#) are *conditional* models.

Note this family function allows  $p_0$  to be modelled as functions of the covariates.

This family function effectively combines [pospoisson](#) and [binomialff](#) into one family function. This family function can handle a multivariate response, e.g., more than one species.

**Author(s)**

T. W. Yee

**References**

Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996) Modelling the abundances of rare species: statistical models for counts with extra zeros. *Ecological Modelling*, **88**, 297–308.

Angers, J-F. and Biswas, A. (2003) A Bayesian analysis of zero-inflated generalized Poisson model. *Computational Statistics & Data Analysis*, **42**, 37–46.

Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

Documentation accompanying the **VGAM** package at <http://www.stat.auckland.ac.nz/~yee> contains further information and examples.

**See Also**

[rzapois](#), [zipoisson](#), [pospoisson](#), [posnegbinomial](#), [binomialff](#), [rpospois](#), [CommonVGAMffArguments](#).

**Examples**

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pobs0 = logit(-1 + 1*x2, inverse = TRUE),
                  lambda = loge(-0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y = rzapois(nn, lambda, pobs0 = pobs0))

with(zdata, table(y))
fit <- vglm(y ~ x2, zapoisson, data = zdata, trace = TRUE)
fit <- vglm(y ~ x2, zapoisson, data = zdata, trace = TRUE, crit = "coef")
head(fitted(fit))
head(predict(fit))
head(predict(fit, untransform = TRUE))
coef(fit, matrix = TRUE)
summary(fit)

# Another example -----
# Data from Angers and Biswas (2003)
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)
yy <- with(abdata, rep(y, w))
fit3 <- vglm(yy ~ 1, zapoisson, trace = TRUE, crit = "coef")
coef(fit3, matrix = TRUE)
Coef(fit3) # Estimate lambda (they get 0.6997 with SE 0.1520)
head(fitted(fit3), 1)
mean(yy) # Compare this with fitted(fit3)
```

---

zero

*The zero Argument in VGAM Family Functions*

---

### Description

The zero argument allows users to conveniently model certain linear/additive predictors as intercepts only.

### Details

Often a certain parameter needs to be modelled simply while other parameters in the model may be more complex, for example, the  $\lambda$  parameter in LMS-Box-Cox quantile regression should be modelled more simply compared to its  $\mu$  parameter. Another example is the  $\xi$  parameter in a GEV distribution which is should be modelled simpler than its  $\mu$  parameter. Using the zero argument allows this to be fitted conveniently without having to input all the constraint matrices explicitly.

The zero argument should be assigned an integer vector from the set  $\{1:M\}$  where  $M$  is the number of linear/additive predictors. Full details about constraint matrices can be found in the references.

### Value

Nothing is returned. It is simply a convenient argument for constraining certain linear/additive predictors to be an intercept only.

### Warning

The use of other arguments may conflict with the zero argument. For example, using constraints to input constraint matrices may conflict with the zero argument. Another example is the argument `parallel`. In general users should not assume any particular order of precedence when there is potential conflict of definition. Currently no checking for consistency is made.

The argument `zero` may be renamed in the future to something better.

### Side Effects

The argument creates the appropriate constraint matrices internally.

### Note

In all **VGAM** family functions `zero = NULL` means none of the linear/additive predictors are modelled as intercepts-only. Almost all **VGAM** family function have `zero = NULL` as the default, but there are some exceptions, e.g., [binom2.or](#).

Typing something like `coef(fit, matrix = TRUE)` is a useful way to ensure that the zero argument has worked as expected.

### Author(s)

T. W. Yee

## References

Yee, T. W. and Wild, C. J. (1996) Vector generalized additive models. *Journal of the Royal Statistical Society, Series B, Methodological*, **58**, 481–493.

Yee, T. W. and Hastie, T. J. (2003) Reduced-rank vector generalized linear models. *Statistical Modelling*, **3**, 15–41.

<http://www.stat.auckland.ac.nz/~yee>

## See Also

[constraints](#).

## Examples

```
args(multinomial)
args(binom2.or)
args(gpd)

#LMS quantile regression example
fit <- vglm(BMI ~ bs(age, df = 4), lms.bcg(zero = c(1,3)),
            bmi.nz, trace = TRUE)
coef(fit, matrix = TRUE)
```

---

Zeta

*The Zeta Distribution*

---

## Description

Density for the zeta distribution.

## Usage

```
dzeta(x, p, log = FALSE)
```

## Arguments

x	Numerical vector/matrix to evaluate the density.
p	The parameter $p$ . This must be positive.
log	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

## Details

The density function of the zeta distribution is given by

$$y^{-p-1}/\zeta(p+1)$$

where  $p > 0$ ,  $y = 1, 2, \dots$ , and  $\zeta$  is Riemann's zeta function.

**Value**

Returns the density evaluated at  $x$ .

**Warning**

This function has not been fully tested.

**Note**

The **VGAM** family function `zetaaff` estimates the parameter  $p$ .

**Author(s)**

T. W. Yee

**References**

Johnson N. L., Kotz S., and Balakrishnan N. (1993) *Univariate Discrete Distributions*, 2nd ed. New York: Wiley.

**See Also**

`zeta`, `zetaaff`.

**Examples**

```
dzeta(1:20, p = 2)
## Not run: plot(1:6, dzeta(1:6, p = 4), type = "h", las = 1, ylab = "Probability",
  main = "zeta probability function; black: p = 4; blue: p = 2")
points(0.10 + 1:6, dzeta(1:6, p = 2), type = "h", col = "blue")
## End(Not run)
```

---

zeta

*Riemann's Zeta Function*

---

**Description**

Computes Riemann's zeta function and its first two derivatives.

**Usage**

```
zeta(x, deriv = 0)
```

**Arguments**

x	A complex-valued vector/matrix whose real values must be $\geq 1$ . Otherwise, if x may be real. If deriv is 1 or 2 then x must be real and positive.
deriv	An integer equalling 0 or 1 or 2, which is the order of the derivative. The default means it is computed ordinarily.

**Details**

While the usual definition involves an infinite series, more efficient methods have been devised to compute the value. In particular, this function uses Euler-Maclaurin summation. Theoretically, the zeta function can be computed over the whole complex plane because of analytic continuation.

The formula used here for analytic continuation is

$$\zeta(s) = 2^s \pi^{s-1} \sin(\pi s/2) \Gamma(1-s) \zeta(1-s).$$

This is actually one of several formulas, but this one was discovered by Riemann himself and is called the *functional equation*.

**Value**

A vector/matrix of computed values.

**Warning**

This function has not been fully tested, especially the derivatives. In particular, analytic continuation does not work here for complex x with  $\text{Re}(x) < 1$  because currently the [gamma](#) function does not handle complex arguments.

**Note**

Estimation of the parameter of the zeta distribution can be achieved with [zetaaff](#).

**Author(s)**

T. W. Yee, with the help of Garry J. Tee.

**References**

- Riemann, B. (1859) Ueber die Anzahl der Primzahlen unter einer gegebenen Grosse. *Monatsberichte der Berliner Akademie*, November 1859.
- Edwards, H. M. (1974) *Riemann's Zeta Function*. Academic Press: New York.
- Markman, B. (1965) The Riemann zeta function. *BIT*, **5**, 138–141.
- Abramowitz, M. and Stegun, I. A. (1972) *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover Publications Inc.

**See Also**

[zetaaff](#), [lerch](#), [gamma](#).

## Examples

```

zeta(2:10)

## Not run:
curve(zeta, -13, 0.8, xlim = c(-12, 10), ylim = c(-1, 4), col = "orange",
      las = 1, main = expression({zeta}(x)))
curve(zeta, 1.2, 12, add = TRUE, col = "orange")
abline(v = 0, h = c(0, 1), lty = "dashed", col = "gray")

# Close up plot:
curve(zeta, -14, -0.4, col = "orange", main = expression({zeta}(x)))
abline(v = 0, h = 0, lty = "dashed", col = "gray")

x <- seq(0.04, 0.8, len = 100) # Plot of the first derivative
plot(x, zeta(x, deriv = 1), type = "l", las = 1, col = "blue",
      xlim = c(0.04, 3), ylim = c(-6, 0), main = "zeta'(x)")
x <- seq(1.2, 3, len = 100)
lines(x, zeta(x, deriv = 1), col = "blue")
abline(v = 0, h = 0, lty = "dashed", col = "gray")
## End(Not run)

zeta(2) - pi^2 / 6      # Should be zero
zeta(4) - pi^4 / 90     # Should be zero
zeta(6) - pi^6 / 945    # Should be 0
zeta(8) - pi^8 / 9450   # Should be 0
# zeta(0, deriv = 1) + 0.5 * log(2*pi) # Should be 0

```

---

zetaff

*Zeta Distribution Family Function*


---

## Description

Estimates the parameter of the zeta distribution.

## Usage

```
zetaff(link = "log", init.p = NULL, zero = NULL)
```

## Arguments

link, init.p, zero

See [CommonVGAMffArguments](#) for more information. These arguments apply to the (positive) parameter  $p$ . See [Links](#) for more choices. Choosing **loglog** constrains  $p > 1$ , but may fail if the maximum likelihood estimate is less than one.

### Details

In this long tailed distribution the response must be a positive integer. The probability function for a response  $Y$  is

$$P(Y = y) = 1/[y^{p+1}\zeta(p+1)], \quad p > 0, \quad y = 1, 2, \dots$$

where  $\zeta$  is Riemann's zeta function. The parameter  $p$  is positive, therefore a log link is the default. The mean of  $Y$  is  $\mu = \zeta(p)/\zeta(p+1)$  (provided  $p > 1$ ) and these are the fitted values. The variance of  $Y$  is  $\zeta(p-1)/\zeta(p+1) - \mu^2$  provided  $p > 2$ .

It appears that good initial values are needed for successful convergence. If convergence is not obtained, try several values ranging from values near 0 to values about 10 or more.

Multiple responses are handled.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Note

The [zeta](#) function may be used to compute values of the zeta function.

### Author(s)

T. W. Yee

### References

pp.527– of Chapter 11 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey: Wiley.

Knight, K. (2000) *Mathematical Statistics*. Boca Raton: Chapman & Hall/CRC Press.

### See Also

[zeta](#), [dzeta](#), [hzeta](#), [zipf](#).

### Examples

```
zdata <- data.frame(y = 1:5, w = c(63, 14, 5, 1, 2)) # Knight, p.304
fit <- vglm(y ~ 1, zetaff, zdata, trace = TRUE, weight = w, crit = "coef")
(phat <- Coef(fit)) # 1.682557
with(zdata, cbind(round(dzeta(y, phat) * sum(w), 1), w))

with(zdata, weighted.mean(y, w))
fitted(fit, matrix = FALSE)
predict(fit)

# The following should be zero at the MLE:
with(zdata, mean(log(rep(y, w))) + zeta(1+phat, deriv = 1) / zeta(1+phat))
```



**Description**

Density, distribution function, quantile function and random generation for the zero-inflated binomial distribution with parameter `pstr0`.

**Usage**

```
dzibinom(x, size, prob, pstr0 = 0, log = FALSE)
pzibinom(q, size, prob, pstr0 = 0, lower.tail = TRUE, log.p = FALSE)
qzibinom(p, size, prob, pstr0 = 0, lower.tail = TRUE, log.p = FALSE)
rzibinom(n, size, prob, pstr0 = 0)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>size</code>	number of trials. It is the $N$ symbol in the formula given in <a href="#">zibinomial</a> .
<code>prob</code>	probability of success on each trial.
<code>n</code>	number of observations. Must be a single positive integer.
<code>log, log.p, lower.tail</code>	Arguments that are passed on to <a href="#">pbinom</a> .
<code>pstr0</code>	Probability of a structural zero (i.e., ignoring the binomial distribution), called $\phi$ . The default value of $\phi = 0$ corresponds to the response having an ordinary binomial distribution.

**Details**

The probability function of  $Y$  is 0 with probability  $\phi$ , and  $\text{Binomial}(\text{size}, \text{prob})$  with probability  $1 - \phi$ . Thus

$$P(Y = 0) = \phi + (1 - \phi)P(W = 0)$$

where  $W$  is distributed  $\text{Binomial}(\text{size}, \text{prob})$ .

**Value**

`dzibinom` gives the density, `pzibinom` gives the distribution function, `qzibinom` gives the quantile function, and `rzibinom` generates random deviates.

**Note**

The argument `pstr0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

These functions actually allow for *zero-deflation*. That is, the resulting probability of a zero count is *less than* the nominal value of the parent distribution. See [Zipois](#) for more information.

**Author(s)**

T. W. Yee

**See Also**[zibinomial](#), [dbinom](#).**Examples**

```

prob <- 0.2; size <- 10; pstr0 <- 0.5
(ii <- dzibinom(0:size, size, prob, pstr0 = pstr0))
max(abs(cumsum(ii) - pzinom(0:size, size, prob, pstr0 = pstr0))) # Should be 0
table(rzibinom(100, size, prob, pstr0 = pstr0))

table(qzibinom(runif(100), size, prob, pstr0 = pstr0))
round(dzibinom(0:10, size, prob, pstr0 = pstr0) * 100) # Should be similar

## Not run: x <- 0:size
barplot(rbind(dzibinom(x, size, prob, pstr0 = pstr0),
             dbinom(x, size, prob)),
        beside = TRUE, col = c("blue", "green"), ylab = "Probability",
        main = paste("ZIB(", size, ", ", prob, ", pstr0 = ", pstr0, ") (blue) vs",
                     " Binomial(", size, ", ", prob, ") (green)", sep=""),
        names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)

```

zibinomial

*Zero-Inflated Binomial Distribution Family Function***Description**

Fits a zero-inflated binomial distribution by maximum likelihood estimation.

**Usage**

```

zibinomial(lpstr0 = "logit", lprob = "logit",
           type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
           ipstr0 = NULL, zero = NULL, mv = FALSE, imethod = 1)
zibinomialff(lprob = "logit", lonempstr0 = "logit",
            type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
            ionempstr0 = NULL, zero = 2, mv = FALSE, imethod = 1)

```

**Arguments**

**lpstr0, lprob** Link functions for the parameter  $\phi$  and the usual binomial probability  $\mu$  parameter. See [Links](#) for more choices. For the zero-deflated model see below.

**type.fitted** See [CommonVGAMffArguments](#) and [fittedv1m](#).

<code>ipstr0</code>	Optional initial values for $\phi$ , whose values must lie between 0 and 1. The default is to compute an initial value internally. If a vector then recycling is used.
<code>lonempstr0</code> , <code>ionempstr0</code>	Corresponding arguments for the other parameterization. See details below.
<code>mv</code>	Logical. Currently it must be FALSE to mean the function does not handle multivariate responses. This is to remain compatible with the same argument in <a href="#">binomialff</a> .
<code>zero</code> , <code>imethod</code>	See <a href="#">CommonVGAMffArguments</a> for information. Argument <code>zero</code> has changed its default value for version 0.9-2.

## Details

These functions are based on

$$P(Y = 0) = \phi + (1 - \phi)(1 - \mu)^N,$$

for  $y = 0$ , and

$$P(Y = y) = (1 - \phi) \binom{N}{Ny} \mu^{Ny} (1 - \mu)^{N(1-y)}.$$

for  $y = 1/N, 2/N, \dots, 1$ . That is, the response is a sample proportion out of  $N$  trials, and the argument size in [rzibinom](#) is  $N$  here. The parameter  $\phi$  is the probability of a structural zero, and it satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $E(Y) = (1 - \phi)\mu$  and these are returned as the fitted values by default. By default, the two linear/additive predictors for `zibinomial()` are  $(\text{logit}(\phi), \text{logit}(\mu))^T$ .

The **VGAM** family function `zibinomialff()` has a few changes compared to `zibinomial()`. These are: (i) the order of the linear/additive predictors is switched so the binomial probability comes first; (ii) argument `onempstr0` is now 1 minus the probability of a structural zero, i.e., the probability of the parent (binomial) component, i.e., `onempstr0` is `1-pstr0`; (iii) argument `zero` has a new default so that the `onempstr0` is intercept-only by default. Now `zibinomialff()` is generally recommended over `zibinomial()`. Both functions implement Fisher scoring.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Warning

Numerical problems can occur. Half-stepping is not uncommon. If failure to converge occurs, make use of the argument `ipstr0` or `ionempstr0`, or `imethod`.

## Note

The response variable must have one of the formats described by [binomialff](#), e.g., a factor or two column matrix or a vector of sample proportions with the `weights` argument specifying the values of  $N$ .

To work well, one needs large values of  $N$  and  $\mu > 0$ , i.e., the larger  $N$  and  $\mu$  are, the better. If  $N = 1$  then the model is unidentifiable since the number of parameters is excessive.

Setting `stepsize = 0.5`, say, may aid convergence.

Estimated probabilities of a structural zero and an observed zero are returned, as in [zipoisson](#).

The *zero-deflated* binomial distribution might be fitted by setting `lpstr0 = identity`, albeit, not entirely reliably. See [zipoisson](#) for information that can be applied here. Else try the zero-altered binomial distribution (see [zabinomial](#)).

### Author(s)

T. W. Yee

### References

Welsh, A. H., Lindenmayer, D. B. and Donnelly, C. F. (2013) Fitting and interpreting occupancy models. *PLOS One*, **8**, 1–21.

### See Also

[rzipinom](#), [binomialff](#), [posbinomial](#), [rbinom](#).

### Examples

```
size <- 10 # Number of trials; N in the notation above
nn <- 200
zdata <- data.frame(pstr0 = logit( 0, inverse = TRUE), # 0.50
                  mubin = logit(-1, inverse = TRUE), # Mean of usual binomial
                  sv     = rep(size, length = nn))
zdata <- transform(zdata,
                  y = rzipinom(nn, size = sv, prob = mubin, pstr0 = pstr0))
with(zdata, table(y))
fit <- vglm(cbind(y, sv - y) ~ 1, zibinomialff, zdata, trace = TRUE)
fit <- vglm(cbind(y, sv - y) ~ 1, zibinomialff, zdata, trace = TRUE, stepsize = 0.5)

coef(fit, matrix = TRUE)
Coef(fit) # Useful for intercept-only models
fitted(fit, type = "pobs0") # Estimate of P(Y = 0)
head(fitted(fit))
with(zdata, mean(y)) # Compare this with fitted(fit)
summary(fit)
```

### Description

Density, and random generation for the zero-inflated geometric distribution with parameter `pstr0`.

**Usage**

```
dzigeom(x, prob, pstr0 = 0, log = FALSE)
pzigeom(q, prob, pstr0 = 0)
qzigeom(p, prob, pstr0 = 0)
rzigeom(n, prob, pstr0 = 0)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>prob</code>	see <a href="#">dgeom</a> .
<code>n</code>	number of observations.
<code>pstr0</code>	Probability of structural zero (ignoring the geometric distribution), called $\phi$ . The default value corresponds to the response having an ordinary geometric distribution.
<code>log</code>	Logical. Return the logarithm of the answer?

**Details**

The probability function of  $Y$  is 0 with probability  $\phi$ , and *geometric*(*prob*) with probability  $1 - \phi$ . Thus

$$P(Y = 0) = \phi + (1 - \phi)P(W = 0)$$

where  $W$  is distributed *geometric*(*prob*).

**Value**

`dzigeom` gives the density, `pzigeom` gives the distribution function, `qzigeom` gives the quantile function, and `rzigeom` generates random deviates.

**Note**

The argument `pstr0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

These functions actually allow for *zero-deflation*. That is, the resulting probability of a zero count is *less than* the nominal value of the parent distribution. See [Zipois](#) for more information.

**Author(s)**

T. W. Yee

**See Also**

[zigeometric](#), [dgeom](#).

## Examples

```

prob <- 0.5; pstr0 <- 0.2; x <- (-1):20
(ii <- dzigeom(x, prob, pstr0))
max(abs(cumsum(ii) - pzigeom(x, prob, pstr0))) # Should be 0
table(rzigeom(1000, prob, pstr0))

## Not run: x <- 0:10
barplot(rbind(dzigeom(x, prob, pstr0), dgeom(x, prob)),
        beside = TRUE, col = c("blue", "orange"),
        ylab = "P[Y = y]", xlab = "y", las = 1,
        main = paste("zigeometric(", prob, ", pstr0 = ", pstr0,
                      ") (blue) vs",
                      "geometric(", prob, ") (orange)", sep = ""),
        names.arg = as.character(x))
## End(Not run)

```

---

zigeometric

Zero-Inflated Geometric Distribution Family Function

---

## Description

Fits a zero-inflated geometric distribution by maximum likelihood estimation.

## Usage

```

zigeometric(lpstr0 = "logit", lprob = "logit",
            type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
            ipstr0 = NULL, iprob = NULL,
            imethod = 1, bias.red = 0.5, zero = NULL)
zigeometricff(lprob = "logit", lonempstr0 = "logit",
             type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
             iprob = NULL, ionempstr0 = NULL,
             imethod = 1, bias.red = 0.5, zero = -2)

```

## Arguments

lpstr0, lprob	Link functions for the parameters $\phi$ and $p$ (prob). The usual geometric probability parameter is the latter. The probability of a structural zero is the former. See <a href="#">Links</a> for more choices. For the zero-deflated model see below.
lonempstr0, ionempstr0	Corresponding arguments for the other parameterization. See details below.
bias.red	A constant used in the initialization process of pstr0. It should lie between 0 and 1, with 1 having no effect.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedv1m</a> for information.
ipstr0, iprob	See <a href="#">CommonVGAMffArguments</a> for information.
zero, imethod	See <a href="#">CommonVGAMffArguments</a> for information.



```

                                prob3 = logit(eta3, inverse = TRUE))
gdata <- transform(gdata, y1 = rzigeom(nn, prob1, pstr0 = prob3),
                                y2 = rzigeom(nn, prob2, pstr0 = prob3),
                                y3 = rzigeom(nn, prob2, pstr0 = prob3))

with(gdata, table(y1))
with(gdata, table(y2))
with(gdata, table(y3))
head(gdata)

fit1 <- vglm(y1 ~ x2 + x3 + x4, zigeometric(zero = 1), data = gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1, type = "pstr0"))

fit2 <- vglm(cbind(y2, y3) ~ 1, zigeometric(zero = 1), data = gdata, trace = TRUE)
coef(fit2, matrix = TRUE)
summary(fit2)

```

Zinegbin

*Zero-Inflated Negative Binomial Distribution***Description**

Density, distribution function, quantile function and random generation for the zero-inflated negative binomial distribution with parameter  $pstr0$ .

**Usage**

```

dzinegbin(x, size, prob = NULL, munb = NULL, pstr0 = 0, log = FALSE)
pzinegbin(q, size, prob = NULL, munb = NULL, pstr0 = 0)
qzinegbin(p, size, prob = NULL, munb = NULL, pstr0 = 0)
rzinegbin(n, size, prob = NULL, munb = NULL, pstr0 = 0)

```

**Arguments**

$x$ , $q$	vector of quantiles.
$p$	vector of probabilities.
$n$	number of observations. Must be a single positive integer.
$size$ , $prob$ , $munb$ , $log$	Arguments matching <a href="#">dnbinom</a> . The argument $munb$ corresponds to $\mu$ in <a href="#">dnbinom</a> and has been renamed to emphasize the fact that it is the mean of the negative binomial <i>component</i> .
$pstr0$	Probability of structural zero (i.e., ignoring the negative binomial distribution), called $\phi$ .



**Details**

The probability function of  $Y$  is 0 with probability  $\phi$ , and a negative binomial distribution with probability  $1 - \phi$ . Thus

$$P(Y = 0) = \phi + (1 - \phi)P(W = 0)$$

where  $W$  is distributed as a negative binomial distribution (see [rnbinom](#).) See [negbinomial](#), a **VGAM** family function, for the formula of the probability density function and other details of the negative binomial distribution.

**Value**

`dzinegbin` gives the density, `pzinegbin` gives the distribution function, `qzinegbin` gives the quantile function, and `rzinegbin` generates random deviates.

**Note**

The argument `pstr0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

These functions actually allow for *zero-deflation*. That is, the resulting probability of a zero count is *less than* the nominal value of the parent distribution. See [Zipois](#) for more information.

**Author(s)**

T. W. Yee

**See Also**

[zinegbinomial](#), [rnbinom](#), [rzipois](#).

**Examples**

```
munb <- 3; pstr0 <- 0.2; size <- k <- 10; x <- 0:10
(ii <- dzinegbin(x, pstr0 = pstr0, mu = munb, size = k))
max(abs(cumsum(ii) - pzinegbin(x, pstr0 = pstr0, mu = munb, size = k))) # 0
table(rzinegbin(100, pstr0 = pstr0, mu = munb, size = k))

table(qzinegbin(runif(1000), pstr0 = pstr0, mu = munb, size = k))
round(dzinegbin(x, pstr0 = pstr0, mu = munb, size = k) * 1000) # Should be similar

## Not run: barplot(rbind(dzinegbin(x, pstr0 = pstr0, mu = munb, size = k),
  dnbinom(x, mu = munb, size = k)), las = 1,
  beside = TRUE, col = c("blue", "green"), ylab = "Probability",
  main = paste("ZINB(mu = ", munb, ", k = ", k, ", pstr0 = ", pstr0,
    ") (blue) vs NB(mu = ", munb,
    ", size = ", k, ") (green)", sep = ""),
  names.arg = as.character(x))
## End(Not run)
```

zinegbinomial

*Zero-Inflated Negative Binomial Distribution Family Function***Description**

Fits a zero-inflated negative binomial distribution by full maximum likelihood estimation.

**Usage**

```
zinegbinomial(lpstr0 = "logit", lmunb = "loge", lsize = "loge",
              type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
              ipstr0 = NULL, isize = NULL, zero = -3,
              imethod = 1, shrinkage.init = 0.95, nsimEIM = 250)
zinegbinomialff(lmunb = "loge", lsize = "loge", lonempstr0 = "logit",
                type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
                isize = NULL, ionempstr0 = NULL, zero = c(-2, -3),
                imethod = 1, shrinkage.init = 0.95, nsimEIM = 250)
```

**Arguments**

lpstr0, lmunb, lsize	Link functions for the parameters $\phi$ , the mean and $k$ ; see <a href="#">negbinomial</a> for details, and <a href="#">Links</a> for more choices. For the <i>zero-deflated</i> model see below.
type.fitted	See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvlm</a> for more information.
ipstr0, isize	Optional initial values for $\phi$ and $k$ . The default is to compute an initial value internally for both. If a vector then recycling is used.
lonempstr0, ionempstr0	Corresponding arguments for the other parameterization. See details below.
imethod	An integer with value 1 or 2 or 3 which specifies the initialization method for the mean parameter. If failure to converge occurs try another value and/or else specify a value for shrinkage.init.
zero	Integers specifying which linear/additive predictor is modelled as intercepts only. If given, their absolute values must be either 1 or 2 or 3. The default is the $\phi$ and $k$ parameters (both for each response). See <a href="#">CommonVGAMffArguments</a> for more information.
shrinkage.init, nsimEIM	See <a href="#">CommonVGAMffArguments</a> for information.

**Details**

These functions are based on

$$P(Y = 0) = \phi + (1 - \phi)(k/(k + \mu))^k,$$

and for  $y = 1, 2, \dots$ ,

$$P(Y = y) = (1 - \phi) \text{dnbinom}(y, \mu, k).$$

The parameter  $\phi$  satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $(1 - \phi)\mu$  (returned as the fitted values). By default, the three linear/additive predictors for `zinegbinomial()` are  $(\text{logit}(\phi), \log(\mu), \log(k))^T$ . See [negbinomial](#), another **VGAM** family function, for the formula of the probability density function and other details of the negative binomial distribution.

Independent multivariate responses are handled. If so then arguments `ipstr0` and `isize` may be vectors with length equal to the number of responses.

The **VGAM** family function `zinegbinomialff()` has a few changes compared to `zinegbinomial()`. These are: (i) the order of the linear/additive predictors is switched so the NB mean comes first; (ii) `onempstr0` is now 1 minus the probability of a structural 0, i.e., the probability of the parent (NB) component, i.e., `onempstr0` is `1-pstr0`; (iii) argument `zero` has a new default so that the `onempstr0` is intercept-only by default. Now `zinegbinomialff()` is generally recommended over `zinegbinomial()`. Both functions implement Fisher scoring and can handle multiple responses.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), and [vgam](#).

### Warning

Numerical problems can occur, e.g., when the probability of zero is actually less than, not more than, the nominal probability of zero. Half-stepping is not uncommon. If failure to converge occurs, try using combinations of arguments `stepsize` (in [vglm.control](#)), `imethod`, `shrinkage.init`, `ipstr0`, `isize`, and/or `zero` if there are explanatory variables.

An infinite loop might occur if some of the fitted values (the means) are too close to 0.

This **VGAM** family function is computationally expensive and usually runs slowly; setting `trace = TRUE` is useful for monitoring convergence.

### Note

Estimated probabilities of a structural zero and an observed zero can be returned, as in [zipoisson](#); see [fittedvglm](#) for more information.

If  $k$  is large then the use of **VGAM** family function [zipoisson](#) is probably preferable. This follows because the Poisson is the limiting distribution of a negative binomial as  $k$  tends to infinity.

The *zero-deflated* negative binomial distribution might be fitted by setting `lpstr0 = identity`, albeit, not entirely reliably. See [zipoisson](#) for information that can be applied here. Else try the zero-altered negative binomial distribution (see [zanegbinomial](#)).

### Author(s)

T. W. Yee

### See Also

[Zinegbin](#), [negbinomial](#), [rpois](#), [CommonVGAMffArguments](#).

## Examples

```
## Not run: # Example 1
ndata <- data.frame(x2 = runif(nn <- 1000))
ndata <- transform(ndata, pstr0 = logit(-0.5 + 1 * x2, inverse = TRUE),
                  munb = exp( 3 + 1 * x2),
                  size = exp( 0 + 2 * x2))
ndata <- transform(ndata,
                  y1 = rzinegbin(nn, mu = munb, size = size, pstr0 = pstr0),
                  y2 = rzinegbin(nn, mu = munb, size = size, pstr0 = pstr0))
with(ndata, table(y1)["0"] / sum(table(y1)))
fit <- vglm(cbind(y1, y2) ~ x2, zinegbinomial(zero = NULL), data = ndata)
coef(fit, matrix = TRUE)
summary(fit)
head(cbind(fitted(fit), with(ndata, (1 - pstr0) * munb)))
round(vcov(fit), 3)

# Example 2: RR-ZINB could also be called a COZIVGLM-ZINB-2
ndata <- data.frame(x2 = runif(nn <- 2000))
ndata <- transform(ndata, x3 = runif(nn))
ndata <- transform(ndata, eta1 = 3 + 1 * x2 + 2 * x3)
ndata <- transform(ndata, pstr0 = logit(-1.5 + 0.5 * eta1, inverse = TRUE),
                  munb = exp(eta1),
                  size = exp(4))
ndata <- transform(ndata,
                  y1 = rzinegbin(nn, pstr0 = pstr0, mu = munb, size = size))
with(ndata, table(y1)["0"] / sum(table(y1)))
rrzinb <- rrvglm(y1 ~ x2 + x3, zinegbinomial(zero = NULL), data = ndata,
                Index.corner = 2, str0 = 3, trace = TRUE)
coef(rrzinb, matrix = TRUE)
Coef(rrzinb)

## End(Not run)
```

---

zipebcom

---

*Exchangeable bivariate cloglog odds-ratio model from a zero-inflated Poisson distribution*


---

## Description

Fits an exchangeable bivariate odds-ratio model to two binary responses with a complementary log-log link. The data are assumed to come from a zero-inflated Poisson distribution that has been converted to presence/absence.

## Usage

```
zipebcom(lmu12 = "cloglog", lphi12 = "logit", loratio = "loge",
        imu12 = NULL, iphi12 = NULL, ioratio = NULL,
        zero = 2:3, tol = 0.001, addRidge = 0.001)
```

## Arguments

lmu12, imu12	Link function, extra argument and optional initial values for the first (and second) marginal probabilities. Argument lmu12 should be left alone. Argument imu12 may be of length 2 (one element for each response).
lphi12	Link function applied to the $\phi$ parameter of the zero-inflated Poisson distribution (see <a href="#">zipoisson</a> ). See <a href="#">Links</a> for more choices.
loratio	Link function applied to the odds ratio. See <a href="#">Links</a> for more choices.
iphi12, ioratio	Optional initial values for $\phi$ and the odds ratio. See <a href="#">CommonVGAMffArguments</a> for more details. In general, good initial values (especially for iphi12) are often required, therefore use these arguments if convergence failure occurs. If inputted, the value of iphi12 cannot be more than the sample proportions of zeros in either response.
zero	Which linear/additive predictor is modelled as an intercept only? A NULL means none. The default has both $\phi$ and the odds ratio as not being modelled as a function of the explanatory variables (apart from an intercept).
tol	Tolerance for testing independence. Should be some small positive numerical value.
addRidge	Some small positive numerical value. The first two diagonal elements of the working weight matrices are multiplied by 1+addRidge to make it diagonally dominant, therefore positive-definite.

## Details

This **VGAM** family function fits an exchangeable bivariate odds ratio model ([binom2.or](#)) with a [cloglog](#) link. The data are assumed to come from a zero-inflated Poisson (ZIP) distribution that has been converted to presence/absence. Explicitly, the default model is

$$\text{cloglog}[P(Y_j = 1)/(1 - \phi)] = \eta_1, \quad j = 1, 2$$

for the (exchangeable) marginals, and

$$\text{logit}[\phi] = \eta_2,$$

for the mixing parameter, and

$$\log[P(Y_{00} = 1)P(Y_{11} = 1)/(P(Y_{01} = 1)P(Y_{10} = 1))] = \eta_3,$$

specifies the dependency between the two responses. Here, the responses equal 1 for a success and a 0 for a failure, and the odds ratio is often written  $\psi = p_{00}p_{11}/(p_{10}p_{01})$ . We have  $p_{10} = p_{01}$  because of the exchangeability.

The second linear/additive predictor models the  $\phi$  parameter (see [zipoisson](#)). The third linear/additive predictor is the same as [binom2.or](#), viz., the log odds ratio.

Suppose a dataset1 comes from a Poisson distribution that has been converted to presence/absence, and that both marginal probabilities are the same (exchangeable). Then `binom2.or("cloglog", exch=TRUE)` is appropriate. Now suppose a dataset2 comes from a *zero-inflated* Poisson distribution. The first linear/additive predictor of `zipebcom()` applied to dataset2 is the same as that of `binom2.or("cloglog", exch=TRUE)`

applied to dataset1. That is, the  $\phi$  has been taken care of by `zipebcom()` so that it is just like the simpler `binom2.or`.

Note that, for  $\eta_1, \mu_{12} = \text{prob}_{12} / (1 - \phi_{12})$  where  $\text{prob}_{12}$  is the probability of a 1 under the ZIP model. Here,  $\mu_{12}$  correspond to  $\mu_1$  and  $\mu_2$  in the `binom2.or`-Poisson model.

If  $\phi = 0$  then `zipebcom()` should be equivalent to `binom2.or("cloglog", exch=TRUE)`. Full details are given in Yee and Dirnbock (2009).

The leading  $2 \times 2$  submatrix of the expected information matrix (EIM) is of rank-1, not 2! This is due to the fact that the parameters corresponding to the first two linear/additive predictors are unidentifiable. The quick fix around this problem is to use the `addRidge` adjustment. The model is fitted by maximum likelihood estimation since the full likelihood is specified. Fisher scoring is implemented.

The default models  $\eta_2$  and  $\eta_3$  as single parameters only, but this can be circumvented by setting `zero=NULL` in order to model the  $\phi$  and odds ratio as a function of all the explanatory variables.

### Value

An object of class "vglmff" (see `vglmff-class`). The object is used by modelling functions such as `vglm` and `vgam`.

When fitted, the `fitted.values` slot of the object contains the four joint probabilities, labelled as  $(Y_1, Y_2) = (0,0), (0,1), (1,0), (1,1)$ , respectively. These estimated probabilities should be extracted with the fitted generic function.

### Warning

The fact that the EIM is not of full rank may mean the model is naturally ill-conditioned. Not sure whether there are any negative consequences wrt theory. For now it is certainly safer to fit `binom2.or` to bivariate binary responses.

### Note

The "12" in the argument names reinforce the user about the exchangeability assumption. The name of this **VGAM** family function stands for *zero-inflated Poisson exchangeable bivariate complementary log-log odds-ratio model* or ZIP-EBCOM.

See `binom2.or` for details that are pertinent to this **VGAM** family function too. Even better initial values are usually needed here.

The `xij` (see `vglm.control`) argument enables environmental variables with different values at the two time points to be entered into an exchangeable `binom2.or` model. See the author's webpage for sample code.

### References

Yee, T. W. and Dirnbock, T. (2009) Models for analysing species' presence/absence data at two time points. *Journal of Theoretical Biology*, **259**(4), 684–694.

### See Also

`binom2.or`, `zipoisson`, `cloglog`, `CommonVGAMffArguments`.

**Examples**

```

zdata <- data.frame(x2 = seq(0, 1, len = (nsites <- 2000)))
zdata <- transform(zdata, eta1 = -3 + 5 * x2,
                  phi1 = logit(-1, inverse = TRUE),
                  oratio = exp(2))
zdata <- transform(zdata, mu12 = cloglog(eta1, inverse = TRUE) * (1-phi1))
tmat <- with(zdata, rbinom2.or(nsites, mu1 = mu12, oratio = oratio, exch = TRUE))
zdata <- transform(zdata, ybin1 = tmat[, 1], ybin2 = tmat[, 2])

with(zdata, table(ybin1, ybin2)) / nsites # For interest only
## Not run:
# Various plots of the data, for interest only
par(mfrow = c(2, 2))
plot(jitter(ybin1) ~ x2, data = zdata, col = "blue")

plot(jitter(ybin2) ~ jitter(ybin1), data = zdata, col = "blue")

plot(mu12 ~ x2, data = zdata, col = "blue", type = "l", ylim = 0:1,
     ylab = "Probability", main = "Marginal probability and phi")
with(zdata, abline(h = phi1[1], col = "red", lty = "dashed"))

tmat2 <- with(zdata, dbinom2.or(mu1 = mu12, oratio = oratio, exch = TRUE))
with(zdata, matplot(x2, tmat2, col = 1:4, type = "l", ylim = 0:1,
                  ylab = "Probability", main = "Joint probabilities"))
## End(Not run)

# Now fit the model to the data.
fit <- vglm(cbind(ybin1, ybin2) ~ x2, zipfcom, dat = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
vcov(fit)

```

---

Zipf

---

*The Zipf Distribution*


---

**Description**

Density, and cumulative distribution function for the Zipf distribution.

**Usage**

```

dzipf(x, N, s, log = FALSE)
pzipf(q, N, s)

```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>N, s</code>	the number of elements, and the exponent characterizing the distribution. See <a href="#">zipf</a> for more details.
<code>log</code>	Logical. If <code>log = TRUE</code> then the logarithm of the density is returned.

Details

This is a finite version of the zeta distribution. See [zipf](#) for more details.

Value

dzipf gives the density, and pzipf gives the cumulative distribution function.

Author(s)

T. W. Yee

See Also

[zipf](#).

Examples

```
N <- 10; s <- 0.5; y <- 1:N
proby <- dzipf(y, N = N, s = s)
## Not run: plot(proby ~ y, type = "h", col = "blue", ylab = "Probability",
  ylim = c(0, 0.2), main = paste("Zipf(N = ",N," s = ",s,")", sep = "" ),
  lwd = 2, las = 1)
## End(Not run)
sum(proby) # Should be 1
max(abs(cumsum(proby) - pzipf(y, N = N, s = s))) # Should be 0
```

---

zipf	<i>Zipf Distribution Family Function</i>
------	--

---

Description

Estimates the parameter of the Zipf distribution.

Usage

```
zipf(N = NULL, link = "loge", init.s = NULL)
```

Arguments

N	Number of elements, an integer satisfying $1 < N < \text{Inf}$ . The default is to use the maximum value of the response. If given, N must be no less than the largest response value. If $N = \text{Inf}$ and $s > 1$ then this is the zeta distribution (use <a href="#">zetaaff</a> instead).
link	Parameter link function applied to the (positive) parameter $s$ . See <a href="#">Links</a> for more choices.
init.s	Optional initial value for the parameter $s$ . The default is to choose an initial value internally. If converge failure occurs use this argument to input a value.



## Details

The probability function for a response  $Y$  is

$$P(Y = y) = y^{-s} / \sum_{i=1}^N i^{-s}, \quad s > 0, \quad y = 1, 2, \dots, N,$$

where  $s$  is the exponent characterizing the distribution. The mean of  $Y$ , which are returned as the fitted values, is  $\mu = H_{N,s-1}/H_{N,s}$  where  $H_{n,m} = \sum_{i=1}^n i^{-m}$  is the  $n$ th generalized harmonic number.

Zipf's law is an experimental law which is often applied to the study of the frequency of words in a corpus of natural language utterances. It states that the frequency of any word is inversely proportional to its rank in the frequency table. For example, "the" and "of" are first two most common words, and Zipf's law states that "the" is twice as common as "of". Many other natural phenomena conform to Zipf's law.

## Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#) and [vgam](#).

## Note

Upon convergence, the  $N$  is stored as `@misc$N`.

## Author(s)

T. W. Yee

## References

pp.526– of Chapter 11 of Johnson N. L., Kemp, A. W. and Kotz S. (2005) *Univariate Discrete Distributions*, 3rd edition, Hoboken, New Jersey, USA: Wiley.

## See Also

[dzipf](#), [zetaff](#).

## Examples

```
zdata <- data.frame(y = 1:5, ofreq = c(63, 14, 5, 1, 2))
fit <- vglm(y ~ 1, zipf, zdata, trace = TRUE, weight = ofreq, crit = "coef")
fit <- vglm(y ~ 1, zipf(link = identity, init = 3.4), zdata,
            trace = TRUE, weight = ofreq)
fit@misc$N
(shat <- Coef(fit))
with(zdata, weighted.mean(y, ofreq))
fitted(fit, matrix = FALSE)
```

## Zipois

*Zero-Inflated Poisson Distribution***Description**

Density, distribution function, quantile function and random generation for the zero-inflated Poisson distribution with parameter `pstr0`.

**Usage**

```
dzipois(x, lambda, pstr0 = 0, log = FALSE)
pzipois(q, lambda, pstr0 = 0)
qzipois(p, lambda, pstr0 = 0)
rzipois(n, lambda, pstr0 = 0)
```

**Arguments**

<code>x, q</code>	vector of quantiles.
<code>p</code>	vector of probabilities.
<code>n</code>	number of observations. Must be a single positive integer.
<code>lambda</code>	Vector of positive means.
<code>pstr0</code>	Probability of a structural zero (i.e., ignoring the Poisson distribution), called $\phi$ . The default value of $\phi = 0$ corresponds to the response having an ordinary Poisson distribution.
<code>log</code>	Logical. Return the logarithm of the answer?

**Details**

The probability function of  $Y$  is 0 with probability  $\phi$ , and  $Poisson(\lambda)$  with probability  $1 - \phi$ . Thus

$$P(Y = 0) = \phi + (1 - \phi)P(W = 0)$$

where  $W$  is distributed  $Poisson(\lambda)$ .

**Value**

`dzipois` gives the density, `pzipois` gives the distribution function, `qzipois` gives the quantile function, and `rzipois` generates random deviates.

**Note**

The argument `pstr0` is recycled to the required length, and must have values which lie in the interval  $[0, 1]$ .

These functions actually allow for the *zero-deflated Poisson* distribution. Here, `pstr0` is also permitted to lie in the interval  $[-1/\expm1(\lambda), 0]$ . The resulting probability of a zero count is *less than* the nominal Poisson value, and the use of `pstr0` to stand for the probability of a structural zero loses its meaning. When `pstr0` equals  $-1/\expm1(\lambda)$  this corresponds to the positive-Poisson distribution (e.g., see [dpospois](#)).

**Author(s)**

T. W. Yee

**See Also**[zipoisson](#), [dpois](#), [rzinegbin](#).**Examples**

```

lambda <- 3; pstr0 <- 0.2; x <- (-1):7
(ii <- dzipois(x, lambda, pstr0 = pstr0))
max(abs(cumsum(ii) - pzipois(x, lambda, pstr0 = pstr0))) # Should be 0
table(rzipois(100, lambda, pstr0 = pstr0))

table(qzipois(runif(100), lambda, pstr0))
round(dzipois(0:10, lambda, pstr0 = pstr0) * 100) # Should be similar

## Not run: x <- 0:10
par(mfrow = c(2, 1)) # Zero-inflated Poisson
barplot(rbind(dzipois(x, lambda, pstr0 = pstr0), dpois(x, lambda)),
  beside = TRUE, col = c("blue", "orange"),
  main = paste("ZIP(", lambda, ", ", pstr0 = ", pstr0, ") (blue) vs",
    " Poisson(", lambda, ") (orange)", sep = ""),
  names.arg = as.character(x))

deflat.limit <- -1 / expm1(lambda) # Zero-deflated Poisson
newpstr0 <- round(deflat.limit / 1.5, 3)
barplot(rbind(dzipois(x, lambda, pstr0 = newpstr0),
  dpois(x, lambda)),
  beside = TRUE, col = c("blue", "orange"),
  main = paste("ZDP(", lambda, ", ", pstr0 = ", newpstr0, ") (blue) vs",
    " Poisson(", lambda, ") (orange)", sep = ""),
  names.arg = as.character(x))
## End(Not run)

```

zipoisson

*Zero-Inflated Poisson Distribution Family Function***Description**

Fits a zero-inflated Poisson distribution by full maximum likelihood estimation.

**Usage**

```

zipoisson(lpstr0 = "logit", llambda = "loge",
  type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
  ipstr0 = NULL, ilambda = NULL,
  imethod = 1, shrinkage.init = 0.8, zero = NULL)
zipoissonff(llambda = "loge", lonempstr0 = "logit",

```

```

type.fitted = c("mean", "pobs0", "pstr0", "onempstr0"),
ilambda = NULL, ionempstr0 = NULL,
imethod = 1, shrinkage.init = 0.8, zero = -2)

```

## Arguments

lpstr0, llambda	Link function for the parameter $\phi$ and the usual $\lambda$ parameter. See <a href="#">Links</a> for more choices; see <a href="#">CommonVGAMffArguments</a> for more information. For the zero-deflated model see below.
ipstr0, ilambda	Optional initial values for $\phi$ , whose values must lie between 0 and 1. Optional initial values for $\lambda$ , whose values must be positive. The defaults are to compute an initial value internally for each. If a vector then recycling is used.
lonempstr0, ionempstr0	Corresponding arguments for the other parameterization. See details below.
type.fitted	Character. The type of fitted value to be returned. The first choice (the expected value) is the default. The estimated probability of an observed 0 is an alternative, else the estimated probability of a structural 0, or one minus the estimated probability of a structural 0. See <a href="#">CommonVGAMffArguments</a> and <a href="#">fittedvglm</a> for more information.
imethod	An integer with value 1 or 2 which specifies the initialization method for $\lambda$ . If failure to converge occurs try another value and/or else specify a value for shrinkage.init and/or else specify a value for ipstr0. See <a href="#">CommonVGAMffArguments</a> for more information.
shrinkage.init	How much shrinkage is used when initializing $\lambda$ . The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used in conjunction with imethod. See <a href="#">CommonVGAMffArguments</a> for more information.
zero	An integer specifying which linear/additive predictor is modelled as intercepts only. If given, the value must be either 1 or 2, and the default is none of them. Setting zero = 1 makes $\phi$ a single parameter. See <a href="#">CommonVGAMffArguments</a> for more information.

## Details

These models are a mixture of a Poisson distribution and the value 0; it has value 0 with probability  $\phi$  else is  $\text{Poisson}(\lambda)$  distributed. Thus there are two sources for zero values, and  $\phi$  is the probability of a *structural zero*. The model for zipoisson() can be written

$$P(Y = 0) = \phi + (1 - \phi) \exp(-\lambda),$$

and for  $y = 1, 2, \dots$ ,

$$P(Y = y) = (1 - \phi) \exp(-\lambda) \lambda^y / y!.$$

Here, the parameter  $\phi$  satisfies  $0 < \phi < 1$ . The mean of  $Y$  is  $(1 - \phi)\lambda$  and these are returned as the fitted values, by default. The variance of  $Y$  is  $(1 - \phi)\lambda(1 + \phi\lambda)$ . By default, the two linear/additive predictors of zipoisson() are  $(\text{logit}(\phi), \log(\lambda))^T$ .

The **VGAM** family function `zipoissonff()` has a few changes compared to `zipoisson()`. These are: (i) the order of the linear/additive predictors is switched so the Poisson mean comes first; (ii) `onempstr0` is now 1 minus the probability of a structural 0, i.e., the probability of the parent (Poisson) component, i.e., `onempstr0` is  $1 - \text{pstr0}$ ; (iii) argument `zero` has a new default so that the `onempstr0` is intercept-only by default. Now `zipoissonff()` is generally recommended over `zipoisson()` (and definitely recommended over [yip88](#)). Both functions implement Fisher scoring and can handle multiple responses.

### Value

An object of class "vglmff" (see [vglmff-class](#)). The object is used by modelling functions such as [vglm](#), [rrvglm](#) and [vgam](#).

### Warning

Numerical problems can occur, e.g., when the probability of zero is actually less than, not more than, the nominal probability of zero. For example, in the Angers and Biswas (2003) data below, replacing 182 by 1 results in nonconvergence. Half-stepping is not uncommon. If failure to converge occurs, try using combinations of `imethod`, `shrinkage.init`, `ipstr0`, and/or `zipoisson(zero = 1)` if there are explanatory variables. The default for `zipoissonff()` is to model the structural zero probability as an intercept-only.

### Note

Although the functions in [Zipois](#) can handle the zero-deflated Poisson distribution, this family function cannot estimate this very well in general. One sets `lpstr0 = identity`, however, the iterations might fall outside the parameter space. Practically, it is restricted to intercept-models only (see example below). Also, one might need inputting good initial values or using a simpler model to obtain initial values.

A (somewhat) similar and more reliable method for zero-deflation is to try the zero-altered Poisson model (see [zapoisson](#)).

The use of this **VGAM** family function with [rrvglm](#) can result in a so-called COZIGAM or COZIGLM. That is, a reduced-rank zero-inflated Poisson model (RR-ZIP) is a constrained zero-inflated generalized linear model. See **COZIGAM**. A RR-ZINB model can also be fitted easily; see [zinegbinomial](#). Jargon-wise, a COZIGLM might be better described as a COZIVGLM-ZIP.

### Author(s)

T. W. Yee

### References

- Thas, O. and Rayner, J. C. W. (2005) Smooth tests for the zero-inflated Poisson distribution. *Biometrics*, **61**, 808–815.
- Data: Angers, J-F. and Biswas, A. (2003) A Bayesian analysis of zero-inflated generalized Poisson model. *Computational Statistics & Data Analysis*, **42**, 37–46.
- Cameron, A. C. and Trivedi, P. K. (1998) *Regression Analysis of Count Data*. Cambridge University Press: Cambridge.

Yee, T. W. (2014) Reduced-rank vector generalized linear models with two linear predictors. *Computational Statistics and Data Analysis*.

### See Also

[zipoisson](#), [Zipois](#), [yip88](#), [rrvglm](#), [zipebcom](#), [rpois](#).

### Examples

```
# Example 1: simulated ZIP data
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pstr01 = logit(-0.5 + 1*x2, inverse = TRUE),
                    pstr02 = logit( 0.5 - 1*x2, inverse = TRUE),
                    Ps01 = logit(-0.5          , inverse = TRUE),
                    Ps02 = logit( 0.5          , inverse = TRUE),
                    lambda1 = loge(-0.5 + 2*x2, inverse = TRUE),
                    lambda2 = loge( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(nn, lambda = lambda1, pstr0 = Ps01),
                    y2 = rzipois(nn, lambda = lambda2, pstr0 = Ps02))

with(zdata, table(y1)) # Eyeball the data
with(zdata, table(y2))
fit1 <- vglm(y1 ~ x2, zipoisson(zero = 1), data = zdata, crit = "coef")
fit2 <- vglm(y2 ~ x2, zipoisson(zero = 1), data = zdata, crit = "coef")
coef(fit1, matrix = TRUE) # These should agree with the above values
coef(fit2, matrix = TRUE) # These should agree with the above values

# Fit all two simultaneously, using a different parameterization:
fit12 <- vglm(cbind(y1, y2) ~ x2, zipoissonff, data = zdata, crit = "coef")
coef(fit12, matrix = TRUE) # These should agree with the above values

# For the first observation compute the probability that y1 is
# due to a structural zero.
(fitted(fit1, type = "pstr0") / fitted(fit1, type = "pobs0"))[1]

# Example 2: McKendrick (1926). Data from 223 Indian village households
cholera <- data.frame(ncases = 0:4, # Number of cholera cases,
                      wfreq = c(168, 32, 16, 6, 1)) # Frequencies
fit <- vglm(ncases ~ 1, zipoisson, wei = wfreq, cholera, trace = TRUE)
coef(fit, matrix = TRUE)
with(cholera, cbind(actual = wfreq,
                    fitted = round(dzipois(ncases, lambda = Coef(fit)[2],
                                           pstr0 = Coef(fit)[1]) *
                                   sum(wfreq), digits = 2)))

# Example 3: data from Angers and Biswas (2003)
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)
fit <- vglm(y ~ 1, zipoisson(lpstr0 = probit, ipstr0 = 0.8),
            abdata, weight = w, trace = TRUE)
fitted(fit, type = "pobs0") # Estimate of P(Y = 0)
coef(fit, matrix = TRUE)
```

```

Coef(fit) # Estimate of pstr0 and lambda
fitted(fit)
with(abdata, weighted.mean(y, w)) # Compare this with fitted(fit)
summary(fit)

# Example 4: zero-deflated model for an intercept-only data
zdata <- transform(zdata, lambda3 = loge(0.0, inverse = TRUE))
zdata <- transform(zdata, deflat.limit = -1 / expm1(lambda3)) # Boundary
# The 'pstr0' parameter is negative and in parameter space:
zdata <- transform(zdata, usepstr0 = deflat.limit / 1.5)
zdata <- transform(zdata, y3 = rzipois(nn, lambda3, pstr0 = usepstr0))
head(zdata)
with(zdata, table(y3)) # A lot of deflation
fit3 <- vglm(y3 ~ 1, zipoisson(zero = -1, lpstr0 = identity),
             data = zdata, trace = TRUE, crit = "coef")
coef(fit3, matrix = TRUE)
# Check how accurate it was:
zdata[1, "usepstr0"] # Answer
coef(fit3)[1]       # Estimate
Coef(fit3)

# Example 5: This RR-ZIP is known as a COZIGAM or COZIVGLM-ZIP
set.seed(123)
rrzip <- rrvglm(Alopacce ~ bs(WaterCon, df = 3), zipoisson(zero = NULL),
               hspider, trace = TRUE, Index.corner = 2)
coef(rrzip, matrix = TRUE)
Coef(rrzip)
summary(rrzip)
## Not run: plotvgam(rrzip, lcol = "blue")

```

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