T22 Hedging in the Heston model

QF26
Adrian Bauer
Dieu Hue Vu

ⓐ $D_x k(t,x,v,u) = \frac{iu}{x} k(t,x,v,u)$

$D_v k(t,x,v,u) = \varphi_2(t)\, k(t,x,v,u)$ with $\varphi_2(t)$ ~~formula~~ from p. 41.

ⓑ $$\varphi_2(t) = \frac{\partial_3 v(t,S(t),\gamma(t))}{\partial_3 c(t,S(t),\gamma(t))} = \frac{\int_0^\infty Re\left(\tilde f_K(R+iu)\, D_v k(t,S(t),\gamma(t),u-iR)\right)du}{\int_0^\infty Re\left(f_K(R+iu)\, D_v k(t,S(t),\gamma(t),u-iR)\right)du}$$

$$\cancel{\int_0^\infty Re\left(\frac{K^{1-R-iu}}{(R+iu)}\right)}$$

$$= \frac{\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z}}{z(z-1)} k(t,S(t),\gamma(t),-iz)\,\varphi_2(t)\right)dz}{\int_{R+i0}^{R+i\infty} Re\left(\frac{K^{1-z}}{z(z-1)} k(t,S(t),\gamma(t),-iz)\,\varphi_2(t)\right)dz}$$

$$\varphi_1(t) = \partial_x v(t,x,\gamma) - \varphi_2(t)\cdot \partial_x c(t,x,\gamma) \qquad \frac{i(-iz)}{x} = \frac{z}{x}$$

$$= \frac{e^{-r(T-t)}}{\pi i}\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z}}{z(z-1)} k(t,x,\gamma,-iz)\frac{z}{x}\right)dz - \varphi_2(t)\frac{e^{-r(T-t)}}{\pi i}\int_{R+i0}^{R+i\infty} Re\left(\frac{K^{1-z}}{z(z-1)} k(\dots)\frac{z}{x}\right)dz$$

$$= \frac{e^{-r(T-t)}}{\pi i}\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z} - \varphi_2(t)K^{1-z}}{z(z-1)} k(\dots)\frac{z}{x}\right)dz$$

$$\varphi_0(t) = \frac{v(t) - \varphi_1(t)S(t) - \varphi_2(t)c(t)}{B(t)} = e^{-rt}\left[\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z} - \varphi_2(t)K^{1-z}}{z(z-1)} k(\dots)\right)\frac{e^{-r(T-t)}}{i\pi}dz - \frac{S(t)}{i\pi e^{r(T-t)}}\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z} - \varphi_2(t)K^{1-z}}{z(z-1)} k(\dots)\frac{z}{x}\right)dz\right]$$

$$e^{-r(T-t)-rt} = e^{-rT}$$

$$= e^{-r\cdot T}\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z} - \varphi_2(t)K^{1-z}}{z(z-1)} k(\cdot)\left(\frac{1}{i\pi} - \frac{z\,S(t)}{i\pi\, S(t)}\right)\right)dz$$

$$= \frac{e^{-r\cdot T}}{i\pi}\int_{R+i0}^{R+i\infty} Re\left(\frac{\tilde K^{1-z} - \varphi_2(t)K^{1-z}}{z(z-1)}\, k(\,)\,(1-z)\right)dz = \frac{e^{-2r(T-t)}}{i\pi}\int_{R+i0}^{R+i\infty} Re\left(\left(\tilde f_K(z) - \varphi_2(t)\hat f_K(z)\right)\cdot k(\dots)(1-z)\right)dz$$