Christian-Albrechts-Universität zu Kiel **SS 2022**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Henrik Valett, Fan Yu, Boy Schultz Sheet 03

Computational Finance

Exercises for all participants

C-Exercise 09 (Options in the CRR model) (4 points)

(a) Write a Python function

```
V_0 = CRR_AmEuPut (S_0, r, sigma, T, M, K, EU)
```

that computes and returns an approximation to the price of a European or an American put option with strike $K > 0$ and maturity $T > 0$ in the CRR model with initial stock price $S(0) > 0$, interest rate $r > 0$ and volatility $\sigma > 0$. The parameter "EU" is 1 if the price of an European put shall be computed or is 0 in the American case. Use the binomial method as presented in the course with $M \in \mathbb{N}$ time steps.

(b) As $M \to \infty$ we would expect convergence of the price in the binomial model towards the price in the Black-Scholes model. To show this implement the BS-Formula for European put options as a Python function:

```
V_0 = BlackScholes_EuPut (t, S_t, r, sigma, T, K)
```

(c) Test your algorithm with

$$S(0) = 100,\ r = 0.05,\ \sigma = 0.3,\ T = 1,\ M = 10, \dots, 500,\ K = 120.$$

- Plot the price of a European put option (EU = 1) in the binomial model in dependence on the number of steps M.
- Plot the fair price in the BS-model into the same plot using the same parameters.
- Print the price of an American put option (EU = 0) with the same $S(0), r, \sigma, T, K$ as above and $M = 500$ steps in the console.

Useful Python commands: `numpy.maximum`

T-Exercise 10 (4 points)
Let W_1, W_2 be independent standard Brownian motions. Consider a market with three assets S_0, S_1, S_2, which follow the equations

$$\begin{aligned} S_0(t) &= 1, \\ \mathrm{d}S_1(t) &= S_1(t)\left(3\mathrm{d}t + \mathrm{d}W_1(t) - \mathrm{d}W_2(t)\right), \\ \mathrm{d}S_2(t) &= S_2(t)\left(1\mathrm{d}t - \mathrm{d}W_1(t) + \mathrm{d}W_2(t)\right). \end{aligned}$$

Construct an arbitrage in this market.

Hint: Find φ with $d\hat{V}_\varphi(t) = \mu(t)dt$ for some $\mu > 0$.

T-Exercise 11 (Vasiček model for interest rates) (4 points)
Let W be a standard Brownian motion and let x, κ, λ and σ real numbers. Show as in the lecture that the process X with

$$dX(t) := (\kappa - \lambda X(t))dt + \sigma dW(t)$$

and $X(0) = x$ solves the equation

$$X(t) = xe^{-\lambda t} + \frac{\kappa}{\lambda}(1 - e^{-\lambda t}) + \int_0^t e^{-\lambda(t-s)}\sigma dW(s).$$

T-Exercise 12 (for math only) (4 points)
Let W be a standard Brownian motion and $T > 0$. Assume that the underlying filtration $(\mathscr{F}_t)_{t\geq 0}$ is generated by W. Let Y be an integrable $\mathscr{F}_T$-measurable random variable. Show that there exist $x \in \mathbb{R}$ and a process H such that the process

$$X = x + \int_0^\cdot H(s)dW(s)$$

fulfills

$$X(T) = Y.$$

Determine x and H explicitly for

(a) $Y = (W(T))^2$,

(b) $Y = \int_0^T W(s)\mathrm{d}s$ and

(c) $Y = (W(T))^3$,

respectively.

Hint: martingale representation theorem

Please include your name(s) as comment in the beginning of the file.
Do not forget to include comments in your Python-programs.
Submit until: Thu, 05.05.2022, 08:15