Christian-Albrechts-Universität zu Kiel **SS 2022**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Henrik Valett, Fan Yu, Boy Schultz Sheet 01

Computational Finance

Exercises for participants of all programs

C-Exercise 01 (2 Bonuspoints)
Write a Python function

```
bond_value(V0, r, n, M, c)
```

that computes and returns the capital V_n if an interest of $r > 0$ has been paid on the initial endowment $V_0 > 0$ for $n \in \mathbb{N}$ years. If $c = 1$, the variable r refers to a continuous rate (i.e., $V_n = V_0 e^{rn}$), and if $c = 0$, it refers to a simple rate paid over M time periods per year (i.e., $V_n = V_0(1 + \frac{r}{M})^{(n \cdot M)}$). Test your function for

$$V_0 = 1000, \quad r = 0.05, \quad n = 10, \quad M = 4, \quad c = 0.$$

Useful Python commands: `if`, `elif`, `else`, `math.exp`

C-Exercise 02 (4 Points)

Write a Python function

```
S = CRR_stock(S_0, r, sigma, T, M)
```

that returns the stock price matrix $S \in \mathbb{R}^{(M+1)\times(M+1)}$ in the CRR-model with initial stock price $S(0) > 0$, interest rate $r > 0$ and volatility $\sigma > 0$ to a time horizon $T > 0$ with $M \in \mathbb{N}$ time steps.

Test your algorithm with

$$S(0) = 100, r = 0.05, \sigma = 0.3, T = 1, M = 500$$

Reminder: In the stock price matrix S of the CRR-model $S_{ji} = S(0)u^j d^{i-j}$ denotes the stock price at time t_i after j upward and hence $i - j$ downward movements (see Section 2.4).

T-Exercise 03 (4 Points)
In the course we fixed the relation $u = \frac{1}{d}$ in the specification of the binomial model that is used as an approximation to the Black-Scholes model. For even $M \in \mathbb{N}$, this implies $S(0)u^{\frac{M}{2}}d^{\frac{M}{2}} = S(0)$. Replace the condition (2.3), i.e. $u = \frac{1}{d}$, by

(a) $q = 0.5$,

(b) $S(0)u^{\frac{M}{2}}d^{\frac{M}{2}} = S(0)e^{rT}$,

(c) $S(0)u^{\frac{M}{2}}d^{\frac{M}{2}} = K$.

and compute the value of the parameters u, d and q for each case (*maths students*) resp. only for (a) *(QF students)*. Discuss the potential benefit of the alternative conditions and in what scenario they may be useful.

T-Exercise 04 (math only) (4 Points)
Let

$$\begin{aligned} S &= S(0)e^{\mu I + \sigma W}, \\ B &= e^{rI}, \\ D &= \delta S \bullet I \end{aligned}$$

with $\mu, \sigma, r, \delta \in \mathbb{R}$, $I(t) = t$ denoting the identity process and W a Wiener process. Show that

$$\tilde{S} := \frac{S}{B} + \frac{1}{B} \bullet D \text{ is a } Q\text{-martingale if and only if } \frac{S}{B}e^{\delta I} \text{ is a } Q\text{-martingale.}$$

Hint: Recall the following rules from Mathematical Finance:

(a) $H \bullet (K \bullet X) = (HK) \bullet X$,

(b) $XY = X(0)Y(0) + X \bullet Y + Y \bullet X + [X,Y]$ (integration by parts),

(c) $[X,Y] = 0$ if $X = H \bullet I$ or $Y = H \bullet I$ for some H,

(d) $H \bullet I(t) = \int_0^t H_s ds$,

(e) X martingale $\Rightarrow H \bullet X$ martingale,

(f) $H \bullet X = 0$ if X is constant.

Please include your name(s) as comment in the beginning of the file.
Do not forget to include comments in your Python-programs.
Submit until: Thu, 21.04.2022, 08:15