Christian-Albrechts-Universität zu Kiel **SS 2022**
Mathematisches Seminar
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Henrik Valett, Fan Yu, Boy Schultz Sheet 06

Computational Finance

Exercises for all participants

C-Exercise 21 (Valuation of European options in the Black-Scholes model using Monte-Carlo) (4 points)
Write a Python function

```
Eu_Option_BS_MC (S0, r, sigma, T, K, M, f)
```

that computes the initial price $V(0) = e^{-rT}\mathbb{E}_Q[f(S(T))]$ of a European option with payoff $f(S(T))$ at maturity T for some strike price in the Black-Scholes model and the asymptotic 95%-confidence interval $[c_1, c_2]$ via the Monte-Carlo approach using $M \in \mathbb{N}$ simulations. In the B-S model we have the formula

$$S(T) = S(0)\exp\left((r-\sigma^2/2)T + \sigma\sqrt{T}X\right),$$

where X has law $N(0,1)$ under Q.
Test your function for a call option $f(x) = (x-100)^+$, $S_0 = 110$, $r = 0.03$, $\sigma = 0.2$, $T = 1$ and $M = 10000$ and compare the price with the BS-Formula.

Useful Python command: `numpy.random.normal`
Hint: The Black-Scholes formula for the European Call is given on exercise sheet 02.

T-Exercise 22 (Hedging in the Heston model) (4 points)
The characteristic function of $X(T)$ given $\mathscr{F}_t$ in the Heston model has the representation $\chi_t(u) = k(t, S(t), \nu(t), u)$ with

$$k(t,x,\nu,u) = e^{iu(\log(x)+r(T-t))}\left(\frac{e^{\lambda(T-t)/2}}{\cosh(d(u)(T-t)/2) + \lambda\sinh(d(u)(T-t)/2)/d(u)}\right)^{2\kappa/\tilde{\sigma}^2} \times \exp\left(-\nu\frac{(iu+u^2)\sinh(d(u)(T-t)/2)/d(u)}{\cosh(d(u)(T-t)/2) + \lambda\sinh(d(u)(T-t)/2)/d(u)}\right)$$

and

$$d(u) := \sqrt{\lambda^2 + \tilde{\sigma}^2(iu+u^2)},$$

see formula 4.8 in the lecture notes.

(a) Compute the partial derivatives of $k(t,x,\nu,u)$ with respect to x and ν.

(b) Consider a market where in addition to bond and stock we have a liquidly traded European call option with strike price K and maturity T. We want to find the perfect hedge for a second European call with strike price $\tilde{K}$ and same maturity T. To this end calculate the perfection hedge via formulas (3.47) and (3.48) using the approach presented in Section 4.3.

C-Exercise 23 (Implied volatility in the Heston-model) (4 points)

(a) On Exercise sheet 02 you implemented the Black-Scholes formula, which computes the fair price of an option under some parameters. Instead of computing the option price $V(0)$ for a given volatility σ, we now look for an algorithm computing the volatility σ that yields a given option price $V(0)$ using the Black-Scholes formula. To this end, write a Python function

```
ImplVol(S0, r, T, K, V)
```

that computes and returns this so-called *implied volatility*. Test your function with

$$S(0) = 100, r = 0.05, T = 1, K = 100, V(0) = 6.09$$

Useful Python commands: `scipy.optimize.minimize`

(b) Write a Python function

```
Heston_EuPut_Laplace(S0, r, nu0, kappa, lambda, sigma_tilde,
                     T, K, R)
```

that computes the fair price of the European put in the Heston model using the Laplace transform approach. Choose an appropriate R and test your function for

$$S(0) = 100, r = 0.05, \nu(0) = 0.3^2, \kappa = 0.3^2, \lambda = 2.5, \tilde{\sigma} = 0.2, T = 1, K = [50 : 150]$$

Useful Python commands: `cmath.exp`, `complex`, `scipy.integrate.quad`, `real`

(c) We now want to combine our functions from part a) and b) to calculate the *implied volatility* in the Heston model. To this end, write a Python function

```
ImplVol_Heston(S0, r, nu0, kappa, lambda, sigma_tilde, T, K,
               R)
```

that computes put option prices in the Heston model (using the function from part b) and then calculates the associated *implied volatility* (using the function from part a). Test your function with the same parameters as in part b) and plot the results for σ against the corresponding K.

T-Exercise 24 (for math only) (4 points)
Use Slutsky's theorem to show the statement concerning (5.6) in the lecture notes, namely that

$$\frac{\sqrt{N}(\hat{V}_N - V)}{\sqrt{\hat{\sigma}_N^2(f(X))}}$$

converges in law to a standard Gaussian random variable as $N \to \infty$.

Please include your name(s) as comment in the beginning of the file.
Do not forget to include comments in your Python-programs.
Submit until: Thu, 26.05.2022, 08:15