Christian-Albrechts-Universität zu Kiel **SS 2022**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Henrik Valett, Fan Yu, Boy Schultz Sheet 09

Computational Finance

Exercises for all participants

C-Exercise 33 (Pricing an American option with the Longstaff-Schwartz method) (4 points)
Write a Python function

```
V0 = BS_AmPut_LSM (S0, r, sigma, T, K, m, N, l)
```

that approximates the initial price of an American put option with strike $K > 0$ and maturity $T > 0$ in the Black-Scholes model using the Longstaff-Schwartz method. Use the Laguerre polynomials (which are given in the `scipy.special` package) and test your function for

$$S0 = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 90, \quad m = 1000, \quad N = 2000.$$

Compare your result with the result from C-Exercise 09.

T-Exercise 34 (Transforming the Black-Scholes PDE to the heat equation) (4 points)
Let $v(t,x)$ denote the solution of the Black-Scholes PDE (3.26). Consider the variables $\tilde{x} = \log(x/K)$, $\tilde{t} = \sigma^2(T-t)/2$, $q = 2r/\sigma^2$ and define the function $y(\tilde{t},\tilde{x})$ via

$$y(\tilde{t},\tilde{x}) = v\Big(T - \frac{2\tilde{t}}{\sigma^2}, K\exp(\tilde{x})\Big)K^{-1}\exp\left(\frac{1}{2}(q-1)\tilde{x} + \Big(\frac{1}{4}(q-1)^2 + q\Big)\tilde{t}\right).$$

Prove that $y(\tilde{t},\tilde{x})$ solves the heat equation

$$\frac{\partial y}{\partial \tilde{t}} = \frac{\partial^2 y}{\partial \tilde{x}^2}$$

and that the initial conditions (6.2) resemble the terminal conditions for the European call and the European put, respectively.

C-Exercise 35 (Valuation of a European Call using the explicit finite difference scheme) (4 points)
Write a Python function

```
V0 = BS_EuCall_FiDi_Explicit (r, sigma, a, b, m, nu_max, T, K)
```

that approximates the option values $v(0,x_1),\dots,v(0,x_{m-1})$ of a European call option with strike $K > 0$ and maturity $T > 0$ in the Black-Scholes model using the explicit finite difference scheme. Here, $x_i = K\exp(a + i\frac{b-a}{m})$ denote the initial stock prices and a, b, m, ν_{max} are the parameters of the algorithm presented in the course. Test your function for

$$r = 0.05, \quad \sigma = 0.2, \quad a = -0.7, \quad b = 0.4, \quad m = 100, \quad \nu_{max} = 2000,$$
$$T = 1, \quad K = 100.$$

Compare your result with the exact solution using the BS-formula by plotting the difference between the finite difference approximation and the exact option price for all underlying initial stock prices.

T-Exercise 36 (for math only) (4 points)
Let $A \in \mathbb{R}^{d \times d}$ be a symmetric matrix. Show that the following properties are equivalent:

(a) $\lim_{\nu \to \infty} A^\nu z = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ for any $z \in \mathbb{R}^d$.

(b) $\lim_{\nu \to \infty} (A^\nu)_{ij} = 0$ for any $i, j \in 1, \ldots, d$.

(c) The spectral radius of A satisfies $\rho(A) < 1$.

Please include your name(s) as comment in the beginning of the file.
Do not forget to include comments in your Python-programs.
Submit until: Thu, 23.06.2022, 08:15