T7)

$$F_X(x) = \begin{cases} 0 & \text{if } x < -\frac{1}{3}\sqrt{3} \\ 1 - \left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-3} & \text{if } x \geq -\frac{1}{3}\sqrt{3} \end{cases}$$

$$f_X(x) = \frac{\partial F_X}{\partial x} = 3\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-4} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-4}$$

$$E[X] = \int_{-\infty}^{\infty} \frac{3\sqrt{3}x}{2}\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-4}$$

$u = x$, $du = dx$

$$dv = \frac{3\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-4}$$

$$v = -\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-3}$$

$$\Rightarrow -x\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-3}\Bigg|_{-\infty}^{\infty} + \int_{-\infty}^{\infty}\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-3} dx$$

$$\Rightarrow -x\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)\Big|_{-\infty}$$

$$\int_{-\infty}^{\infty}\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3}dx$$

$$= -x\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3}\Big|_{-\infty}^{\infty}$$

$$+ -\frac{\cancel{2}}{\sqrt{3}\cdot\cancel{2}}\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-2}\Big|_{-\infty}^{\infty}$$

$$= -x\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3} - \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-2}\Big|_{-\infty}^{\infty}$$

$$= -\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-2}\left[x\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-1}+\frac{1}{\sqrt{3}}\right)\Big|_{-\infty}^{\infty}$$

$$= -\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3}\left(x+\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)\right)\Big|_{-\infty}^{\infty}$$

$$= -\frac{x+\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{+3}}\Big|_{-\infty}^{\infty}$$

$$\infty+\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\infty+\frac{3}{2}\right)$$

$$\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)$$

$$= -\frac{\infty+\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\infty+\frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}\infty+\frac{3}{2}\right)^3} + \frac{-\infty+\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}(-\infty)+\frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}(-\infty)+\frac{3}{2}\right)^3}$$

$=0$ b/c $\frac{x}{x^3}$ for $-\infty$ and ∞ converges to 0.

$=E(X)$

$$VaR_\alpha(X) = F^{-1}(\alpha)$$

$$y = 1-\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3}$$

$$\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3} = 1-y$$

$$\frac{\sqrt{3}}{2}x+\frac{3}{2} = (1-y)^{-1/3}$$

$y = 1 - (\ldots$

$$\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-3} = 1-y$$

$$\frac{\sqrt{3}}{2}x + \frac{3}{2} = (1-y)^{-1/3}$$

$$\frac{\sqrt{3}}{2}x = (1-y)^{-1/3} - 3/2$$

$$x = \frac{2}{\sqrt{3}}(1-y)^{-1/3} - \sqrt{3}$$

$$VaR_\alpha(x) = \frac{2}{\sqrt{3}}(1-\alpha)^{-1/3} - \sqrt{3}$$

$$ES_\alpha(x) = \frac{1}{1-\alpha}\int_{VaR_\alpha(x)}^{\infty} \frac{3\sqrt{3}}{2}\, x \left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{-4} dx$$

$$= \left(\frac{1}{1-\alpha}\right) \cdot - \frac{x + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)^{3}} \Bigg|_{VaR_\alpha(x)}^{\infty}$$

$$\frac{VaR_\alpha(x) + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}VaR_\alpha(x) + \frac{3}{2}\right)}{\ldots \frac{3}{\ldots}\Big)^{3}(1-\alpha)}$$

$$-(1-\alpha) \quad \left(\frac{\sqrt{3}}{2}x + \frac{3}{2}\right)$$

$$= 0 + \frac{VaR_\alpha(x) + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}VaR_\alpha(x) + \frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}VaR_\alpha(x) + \frac{3}{2}\right)^3 \cdot (1-\alpha)}$$

$$\frac{\sqrt{3}}{2}\left[\frac{2}{\sqrt{3}}(1-\alpha)^{-1/3} - \sqrt{3}\right] + \frac{3}{2}$$

$$= (1-\alpha)^{-1/3} - \frac{3}{2} + \frac{3}{2}$$

$$= (1-\alpha)^{-1/3}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}(1-\alpha)^{-1/3} - \sqrt{3} + \frac{1}{\sqrt{3}}(1-\alpha)^{-1/3}}{\left((1-\alpha)^{-1/3}\right)^3 (1-\alpha)}$$

$$= \frac{\sqrt{3}(1-\alpha)^{-1/3} - \sqrt{3}}{(1-\alpha)^{-1}(1-\alpha)}$$

$$)^{-1/3} - \sqrt{3}$$

$$= \frac{\sqrt{3}\,(1-\alpha)^{-1/3} - \sqrt{3}\,(1-\alpha)}{(1-\alpha)^{-1}\,(1-\alpha)}$$

$$= \sqrt{3}\,(1-\alpha)^{-1/3} - \sqrt{3}$$

$$= \sqrt{3}\left((1-\alpha)^{-1/3} - 1\right)$$

$$= ES_\alpha(X)$$

b) $\tilde{X} \sim N(0,1)$

i) $VaR_\alpha(X) = \frac{2}{\sqrt{3}}(1-\alpha)^{-1/3} - \sqrt{3}$

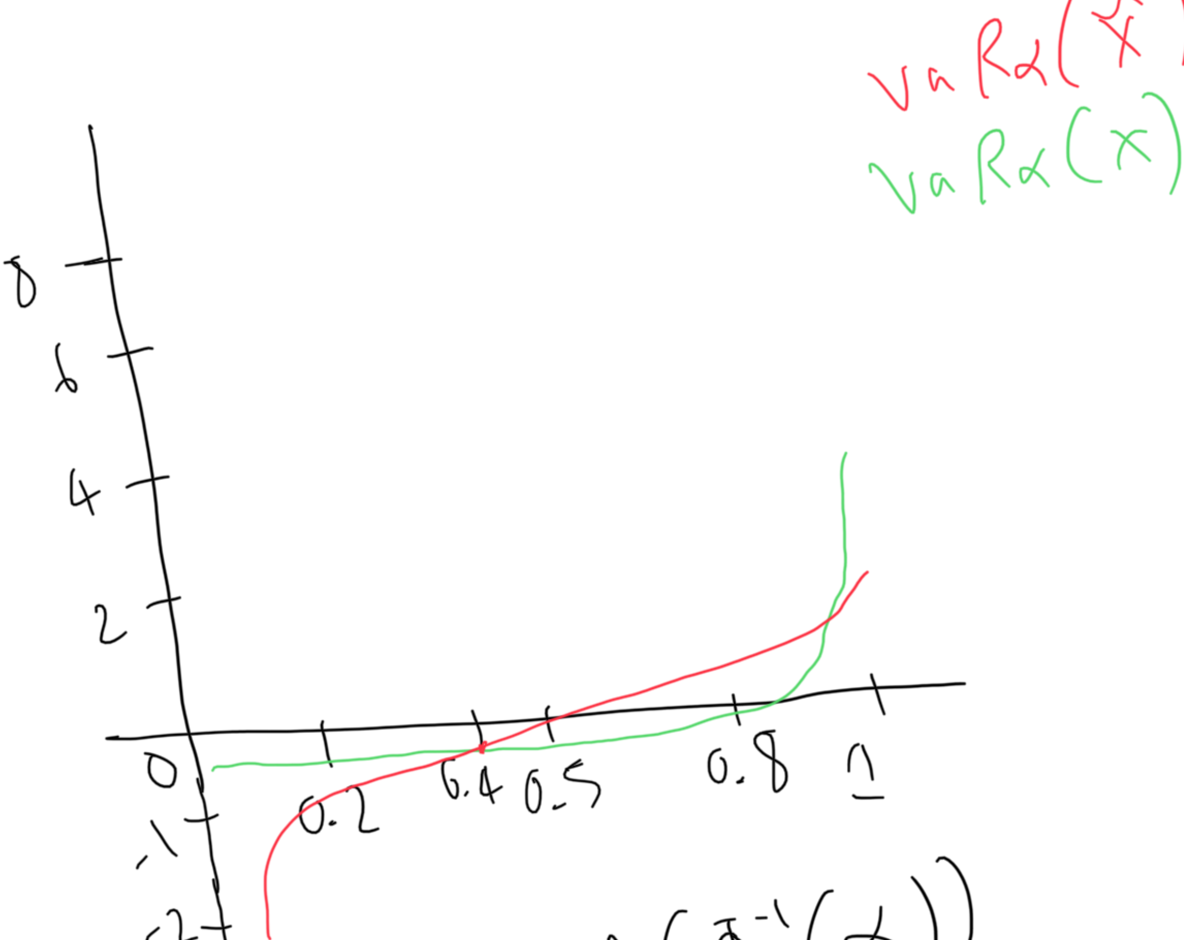
$VaR_\alpha(\tilde{X}) = \Phi^{-1}(\alpha)$

$VaR_\alpha(\tilde{X})$

b) $\tilde{X} \sim N(0,1)$

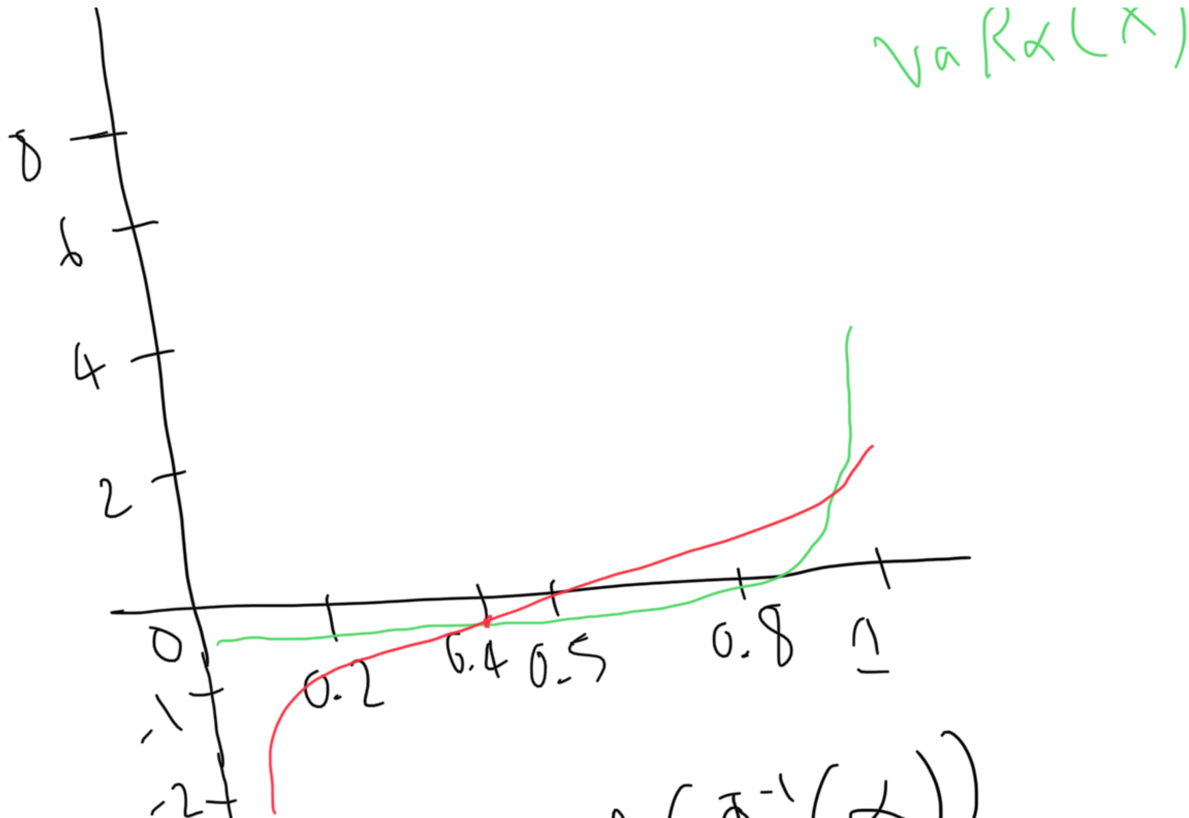
i) $VaR_\alpha(X) = \frac{2}{\sqrt{3}}(1-\alpha)^{-1/3} - \sqrt{3}$

$VaR_\alpha(\tilde{X}) = \Phi^{-1}(\alpha)$



ii) $ES_\alpha(\tilde{X}) = \frac{\varphi(\Phi^{-1}(\alpha))}{1-\alpha}$

$ES_\alpha(X) = \sqrt{3}\left((1-\alpha)^{-1/3} - 1\right)$



ii) $ES_\alpha(\tilde{X}) = \dfrac{\varphi(\Phi^{-1}(\alpha))}{1-\alpha}$

$$ES_\alpha(X) = \sqrt{3}\left((1-\alpha)^{-1/3}-1\right)$$