Christian-Albrechts-Universität zu Kiel **WT 2022/23**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Henrik Valett

Risk Management

Exercise Sheet 5

T-Exercise 16 (4 points)
Let X be a Fréchet-distributed random variable. Show that X is regularly varying and compute the corresponding index.

Hint: L'Hospital's Rule

C-Exercise 17 (2 points)

(a) Write a *Python* -function

```
qqplot(x, F_inv),
```

which draws a quantile-quantile plot for $n \in \mathbb{N}$ given observations $x = (x_1, \dots, x_n)$ and the quantile function $F^{\leftarrow}$ of a reference cumulative distribution function F.

(b) Consider the historical prices of one of the stocks from C-Exercise 08. Use a quantile-quantile plot in order to examine the tail behaviour of the log returns. As reference distributions choose the standard normal distribution and t- distributions with at least two different degrees of freedom. Compare the results.

T-Exercise 18M (for mathematicians only) (4 points)

(a) Find a cdf F on $\mathbb{R}_+$ such that $\bar{F}$ is slowly varying. Show that $E[X^\beta] = \infty$ for all $\beta > 0$ if X has cdf F.

(b) Suppose that F is differentiable and $\bar{F} \in RV_{-\alpha}$ for some $\alpha > 0$. Show that $E[X^\beta] < \infty$ for $\beta \in [0, \alpha)$ and $E[X^\beta] = \infty$ for $\beta \in (\alpha, \infty)$.

C-Exercise 19 (6 points)

(a) Write a *Python* -function

```
alpha = Hill_Estimator(x, k),
```

which computes the Hill estimator $\hat{\alpha}_{k,n}$ for $n \in \mathbb{N}$ independent observations $x = (x_1, ..., x_n)$ and $k \in \{1, ..., n-1\}$.

(b) Write a *Python* -function

```
Hill_Plot(x),
```

which draws the corresponding Hill plot for $n \in \mathbb{N}$ independent observations $x = (x_1, ..., x_n)$.

(c) Generate $n = 500$ simulations for

- a t-distribution with $\nu = 3$ degrees of freedom,
- a t-distribution with $\nu = 8$ degrees of freedom,
- an exponential distribution with parameter $\lambda = 1$,

and draw the corresponding Hill plots.

(d) Write a *Python* -function

```
[VaR, ES] = VaR_ES_Hill(x, p, k)
```

that computes the VaR and ES estimates from sections 3.2.3 and 3.2.4 for $n \in \mathbb{N}$ independent observations $x = (x_1, ..., x_n)$, $k \in \{1, ..., n-1\}$ and level $p \in (0, 1)$.

(e) On OLAT you will find a data set with $n = 500$ i.i.d. simulations of a regularly varying random variable. Use a Hill plot for a reasonable choice of k. Compute the estimates for VaR and ES at level $p = 0.98$.

Please comment your solution.
Useful *Python* commands: `numpy.random.standard_t`

Please comment your solution. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 30.11.2022, 12:00
Discussion in tutorial: Monday, 05.12.2022 and Tuesday, 06.12.2022