Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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WT 2022/23

Risk Management

Exercise Sheet 6

C-Exercise 20 (4 points)

(a) Write a *Python* -function

```
e = MEF(x, u),
```

that evaluates the empirical mean excess function e_n for $n \in \mathbb{N}$ observations $x = (x_1, ..., x_n)$ and $u < \max\{x_i \colon i = 1, ..., n\}$.

(b) Write a *Python* -function

```
MEP(x),
```

that draws the mean excess plot for observations $x = (x_1, ..., x_n)$.

(c) Generate $n = 500$ simulations for

- a t-distribution with $\nu = 3$ degrees of freedom,
- a t-distribution with $\nu = 8$ degrees of freedom,
- an exponential distribution with parameter $\lambda = 1$,

and draw the corresponding mean excess plots.

(d) Write a *Python* -function

```
[beta, gamma] = PoT_estimated(x, u),
```

that estimates the parameters β and γ via maximum likelihood estimation according to section 3.3.2 in the lecture notes.

(e) Write a *Python* -function

```
[VaR, ES] = VaR_ES_PoT(x, p, u),
```

that computes the VaR and ES estimates from sections 3.3.5 and 3.3.6 for $n \in \mathbb{N}$ independent observations $x = (x_1, ..., x_n)$, $u \in (0, \infty)$ and level $p \in (0, 1)$.

(f) Take the data set from C-Exercise 19 and use a mean excess plot for a reasonable choice of u. Compute the estimates for VaR and ES at level $p = 0.98$.

Please comment your solution.

C-Exercise 21 (4 points)

(a) Write a *Python* -function

```
tau = Kendall(x)
```

which estimates and returns *Kendall's tau* $\rho_\tau(X_1, X_2)$ for i.i.d. samples of a random vector $X = (X_1, X_2)$.

(b) Write a *Python* -function

```
rho = Spearman(x),
```

which estimates and returns *Spearman's rho* $\rho_S(X_1, X_2)$ for i.i.d. samples of a random vector $X = (X_1, X_2)$.

(c) Assume that the log returns of the *Volkswagen* stock and the *Continental* stock time series on the OLAT entry of this course are i.i.d. samples from a random vector (X_1, X_2). Estimate the linear correlation coefficient $\rho_L(X_1, X_2)$, *Kendall's tau* $\rho_\tau(X_1, X_2)$ and *Spearman's rho* $\rho_S(X_1, X_2)$. Use a two-dimensional plot in order to visualize the common daily log returns.

(d) Estimate the mean μ and the covariance matrix Σ of (X_1, X_2) with appropriate estimators $\hat{\mu}$ and $\hat{\Sigma}$. Simulate $N = 5369$ i.i.d. samples of a $N(\hat{\mu}, \hat{\Sigma})$ distribution. Plot these samples and estimate *Kendall's tau* and *Spearman's rho*.

Please comment your solution.
Useful commands: `scipy.special.binom`

T-Exercise 22 (4 points)
Fix $h > 0$. Suppose that L has cdf F such that $\bar{F}_u = \bar{G}_{\gamma,\beta}$, where $G_{\gamma,\beta}$ denotes the cdf of the generalised Pareto distribution with parameters $\gamma, \beta > 0$.
Show that $\bar{F}_{u+h} = \bar{G}_{\tilde{\gamma},\tilde{\beta}}$ for some $\tilde{\gamma}, \tilde{\beta} > 0$. What are $\tilde{\gamma}, \tilde{\beta}$?

T-Exercise 23M (for mathematicians only) (4 points)
Let X_1 and X_2 be two nonnegative, independent and identically distributed random variables with cumulative distribution function F, such that $\overline{F} \in \mathrm{RV}_{-\alpha}$ for some $\alpha > 1$.

(a) Show that
$$\lim_{t\to\infty} \frac{P(X_1 + X_2 > t)}{P(2X_1 > t)} = 2^{1-\alpha}.$$
Hint: As a first step, show that for all $\epsilon \in (0, \frac{1}{2})$
$$P(X_1 + X_2 > t) \leq 2P(X_2 > (1-\epsilon)t) + P(X_1 > \epsilon t)^2.$$

(b) Conclude with (a) that for sufficiently large $p \in (0, 1)$
$$\mathrm{VaR}_p(X_1 + X_2) \leq \mathrm{VaR}_p(X_1) + \mathrm{VaR}_p(X_2).$$

Please comment your solution. Please include your name(s) as comment in the beginning of the file.

Submit until:	Wednesday, 07.12.2022, 12:00
Discussion in tutorial:	Monday, 12.12.2022 and Tuesday, 13.12.2022