QF 13
Rounak Bas
Adrian Bec

T - Ex. 16

$$F(x) = e^{-\left(\frac{x-m}{s}\right)^{-\alpha}}$$

$$\bar{F}(x) = 1 - e^{-\left(\frac{x-m}{s}\right)^{-\alpha}}$$

l'Hospital as both numerator and denom go to 0 for $t \to \infty$

$$\frac{\bar{F}(\lambda t)}{\bar{F}(t)} = \frac{1 - e^{-\left(\frac{\lambda t-m}{s}\right)^{-\alpha}}}{1 - e^{-\left(\frac{t-m}{s}\right)^{-\alpha}}} \xrightarrow{t}$$

$$D_t\left(1 - \exp\left(-\left(\frac{\lambda t - m}{s}\right)^{-\alpha}\right)\right) = D_t\left(-\left(\frac{\lambda t-m}{s}\right)^{-\alpha}\right) \cdot \exp\Big(\underbrace{-\left(\frac{\lambda t - m}{s}\right)^{-\alpha}}_{=:\ \theta(\lambda, t)}\Big)$$

$$= \alpha \cdot \frac{\lambda}{s}\left(\frac{\lambda t - m}{s}\right)^{-\alpha-1} \cdot \exp\left(-\left(\frac{\lambda t-m}{s}\right)^{-\alpha}\right)$$

$$\frac{\bar{F}(\lambda t)}{\bar{F}(t)} \xrightarrow{t \to \infty} \frac{\alpha \cdot \frac{\lambda}{s}\left(\frac{\lambda t-m}{s}\right)^{-\alpha-1} \cdot \exp(\theta(\lambda,t))}{\frac{\alpha}{s}\left(\frac{t-m}{s}\right)^{-\alpha-1} \cdot \exp(\theta(1,t))}$$

$$\cancel{\theta(\lambda,t)} \quad \cancel{\lambda} \cdot$$

$$= \lambda \frac{\left(\frac{\lambda t-m}{s}\right)^{-\alpha-1} \cdot \exp(\theta(\lambda,t))}{\left(\frac{t-m}{s}\right)^{-\alpha-1} \cdot \exp(\theta(1,t))} \xrightarrow{t - \infty} \lambda \frac{(\lambda t)^{-\alpha-1}}{t^{-\alpha-1}} = \lambda^{-\alpha} \quad \square$$

Index is $-\alpha$...

T - 18