Christian-Albrechts-Universität zu Kiel **WT 2022/23**
Mathematisches Seminar
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Risk Management

Exercise Sheet 3

C-Exercise 8

(a) Write a *Python* function

```
[VaR, ES] = VaR_ES_var_covar (x_data, c, w, alpha),
```

that computes the estimates $\widehat{\mathrm{VaR}}_\alpha$ and $\widehat{ES}_\alpha$ of the variance covariance method for the linearized loss operator

$$l^\Delta(x) = -(c + w^T x),$$

for given historical risk factor changes $x\_data = (x_1, ..., x_n) \in \mathbb{R}^n$ and some $c \in \mathbb{R}$ and $w \in \mathbb{R}^n$.

(b) Go to http://www.ariva.de/dax-40 and download historical prices for the stocks of BMW, SAP, Volkswagen, Continental and Siemens from the german DAX from the time period from 01.01.2000 to 08.11.2022 (1.Click on "Kurse"; 2.Choose historical prices ("Historische Kurse") from Xetra; 3.Scroll down to csv-download the data on the right-hand side ("Kurse als CSV-Datei"); 4.The closing prices ("Schlusskurs") can be found in the 5th column of your csv-file). Import the time series to *Python* and compute the logarithmic returns $x_2^{(i)}, x_3^{(i)}, ...$ for $i = 1, ..., 5$ which we use as risk factor changes.

(c) Suppose you hold a portfolio of $\bar{\alpha} = (38, 31, 24, 50, 22)$ shares of the 5 stocks. Compute for each trading day $m > 254$ the estimates for *Value at Risk* and *Expected Shortfall* at level $\alpha = 0.98$ by applying the function from (a) on the last $n = 252$ risk factor changes $(x_m, x_{m-1}, ..., x_{m-n+1})$. Plot your results.

Hint: Have a look at section 1.2.3 in the lecture notes.

Please comment your solution. Do not submit the csv-files.

Useful *Python* commands: `numpy.cov`

T-Exercise 9

Let L be a random loss which follows a Gamma-distribution of the following form: L has the pdf

$$f_L(x) = \frac{\lambda^a}{\Gamma(a)} \left(x + \frac{a}{\lambda}\right)^{a-1} \exp(-\lambda x - a) \mathbb{1}_{\{x > -\frac{a}{\lambda}\}},$$

for some $a, \lambda > 0$, where $\Gamma(z) := \int_0^\infty t^{z-1} e^{-t}\, dt$ denotes the gamma function. Compute the expectation, the variance, the Value at Risk and the Expected Shortfall at level $\alpha \in (0, 1)$ of L.

C-Exercise 10

(a) Write a *Python* function

```
[VaR, ES] = VaR_ES_historic (x_data, l, alpha),
```

that computes the estimates $\widehat{\mathrm{VaR}}_\alpha(L_{n+1})$ and $\widehat{\mathrm{ES}}_\alpha(L_{n+1})$ for the one-dimensional loss operator $l\colon \mathbb{R} \to \mathbb{R}$, level $\alpha \in (0,1)$ and given historical risk factor changes $x\_data = (x_1, ..., x_n) \in \mathbb{R}^n$ using the method of historical simulation.

(b) Compute the logarithmic returns $(x_2, ..., x_{8114})$ of the DAX time series, that we use as risk factor changes. Compute for each trading day $m = 249, ..., 8368$ estimates for value at risk and expected shortfall at $\alpha = 90\%$ and $\alpha = 95\%$ by applying the function from (a) on the last $n = 248$ risk factor changes $(x_m, x_{m-1}, ..., x_{m-n+1})$. Plot your results and compare them with the results of C-Exercise 4.

Please comment your solution.
Useful *Python* commands: `numpy.sort, numpy.floor`

T-Exercise 11M (for mathematicians only)
For any real-valued random variable L and $\alpha \in (0,1)$ set

$$\mathrm{ES}_\alpha(L) := \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_p(L)\,dp.$$

(a) Show that

$$\mathrm{ES}_\alpha(L) = \frac{1}{1-\alpha}\Big(E\big(L1_{\{L>\mathrm{VaR}_\alpha(L)\}}\big) + \mathrm{VaR}_\alpha(L)\big(1-\alpha-P(L>\mathrm{VaR}_\alpha(L))\big)\Big).$$

(b) Modify the proof of Theorem 1.18 such that it covers random variables with discontinuous laws as well.

Hint: You may assume w.l.o.g. that for any measurable set A and any $\varepsilon \in [0, P(A)]$ there exists some measurable set $B \subset A$ with $P(B) = \varepsilon$.

Please comment your solution. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 16.11.2022, 12:00
Discussion in tutorial: Monday, 21.11.2022 and Tuesday, 22.11.2022