OF13
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T13

$L_n \;,\; n = 1, \dots, 250$

ⓐ $\mathbb{P}(\max\{L_1, \dots, L_{250}\} > t) = 1 - \mathbb{P}(\max\{L_1, \dots, L_{250}\} \leq t)$

$\overset{\text{iid.}}{=} 1 - \mathbb{P}(L_1 \leq t)^{250} = 1 - F(t)^{250}$

$1 - F(t)^{250} < 0.05 \iff \underline{\sqrt[250]{0.95} < F(t)} \iff t > F^{-1}\left(\sqrt[250]{0.95}\right)$

$a := \sqrt[250]{0.95}$. $\quad a < \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{t} e^{-\frac{x^2}{2\sigma^2}} dx$

$u = \frac{x}{\sigma} = \varphi(x)$

$a < \frac{\sigma}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{t} e^{-\frac{x^2}{2\sigma^2}} \cdot \frac{1}{\sigma}\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t/\sigma} e^{-\frac{u^2}{2}} du$

$\frac{du}{dx} = \varphi'(x) = \frac{1}{\sigma}$

$dx = du \cdot \sigma$

$= \phi\left(\frac{t}{\sigma}\right)$.

$du = dx \cdot \frac{1}{\sigma}$

$\boxed{\Rightarrow t > \phi^{-1}(a) \cdot \sigma}$

ⓑ $a < F_\lambda(t) = 1 - e^{-\lambda t} \iff e^{-\lambda t} < 1 - a$

$\iff \boxed{t > -\frac{1}{\lambda} \ln(1 - a)}$