T-Ex03

What is $r_n^{(1)}$ and $r_n^{(2)}$

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QF 13

a)

$$V_m = \frac{V_{m-1}}{2}\left(1+r_m^{(1)}\right) + \frac{V_{m-1}}{2}\left(1+r_m^{(2)}\right) = \frac{V_{m-1}}{2}\left(2+r_m^{(1)}+r_m^{(2)}\right)$$

$$= \frac{\left(\frac{V_{m-2}}{2}\left(2+r_{m-1}^{(1)}+r_{m-1}^{(2)}\right)\right)}{2}\left(2+r_m^{(1)}+r_m^{(2)}\right) = \frac{V_{m-2}}{2\cdot 2}\left(2+r_{m-1}^{(1)}+r_{m-1}^{(2)}\right)\left(2+r_m^{(1)}+r_m^{(2)}\right)$$

$$= \frac{V_{m-m}}{2^m}\prod_{i=1}^{m}\left(1+r_i^{(1)}+1+r_i^{(2)}\right) = \frac{V_0}{2^m}\prod_{i=1}^{m}\left(e^{X_m^{(1)}}+e^{X_m^{(2)}}\right) = \frac{V_0}{2^n}\prod_{i=1}^{m}\left(e^{Z_m^{(1)}-Z_{m-1}^{(1)}}+e^{Z_m^{(2)}-Z_{m-1}^{(2)}}\right)$$

$$= f(Z_m \mid \mathcal{F}_{m-1})$$

$$\boxed{1+r_m^{(1)} = \frac{S_m^{(1)}}{S_{m-1}^{(1)}} = e^{Z_m^{(1)}-Z_{m-1}^{(1)}} = e^{X_m^{(1)}}}$$

If $1+r_n^{(1)}$ denotes the change of stock 1 then you assume here that the number of shares is fixed. But we want to rebalance in every timestep.

b) $$X_{m+1}^{(1)} = Z_{m+1}^{(1)} - Z_m^{(1)} = \log\left(\frac{S_{m+1}^{(1)}}{S_m^{(1)}}\right)$$

$$\rightarrow X_{m+1}^{(2)} = \log\left(\frac{S_{m+1}^{(2)}}{S_m^{(2)}}\right)$$ ✓

c) $$L_{m+1} = -V_{m+1} + V_m = -\frac{V_m}{2}\left(e^{X_{m+1}^{(1)}}+e^{X_{m+1}^{(2)}}\right) + V_m$$ ✓

$$= V_m\left(1-\frac{e^{X_{m+1}^{(1)}}}{2}-\frac{e^{X_{m+1}^{(2)}}}{2}\right)$$ ✓

$$\Rightarrow \ell_{[m]}(x) = V_m\left(1-\frac{e^{x_1}}{2}-\frac{e^{x_2}}{2}\right)$$ ✓

d) $$L_{m+1}^{\Delta} = -\sum_{i=1}^{2} D_i f(Z_m \mid \mathcal{F}_{m-1}) X_{m+1}^{(i)} = -\sum_{i=1}^{2}\frac{V_{m-1}}{2}\left(e^{Z_m^{(i)}-Z_{m-1}^{(i)}}\right)X_{m+1}^{(i)}$$

$$\ell_{[m]}(x) = -\sum_{i=1}^{2}\frac{V_{m-1}}{2}\left(e^{Z_m^{(i)}-Z_{m-1}^{(i)}}\right)x_i$$ (✓) correct for your function f.