ÜF 13
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$x \geq 0$

T22

$L \sim F$

$$(*)\quad \boxed{\overline{F}_u = \overline{G}_{\gamma,\beta}}$$

$$G_{\gamma,\beta}(x) = 1 - \underbrace{\left(1+\frac{\gamma}{\beta}x\right)^{-\frac{1}{\gamma}}}_{\overline{G}_{\gamma,\beta}(x)}$$

$$\overline{F}_u(x) = 1 - F_u(x) = \frac{\overline{F}(x+u)}{\overline{F}(u)} = \overline{G}_{\gamma,\beta}$$

$$\overline{F}_{u+h}(x) = 1 - F_{u+h}(x) = \frac{\overline{F}(x+u+h)}{\overline{F}(u+h)} = \frac{1-F(x+u+h)}{1-F(u+h)} = \frac{P(X > x+u+h)}{P(X > u+h)}$$

$$= \frac{P(X-h > u+x)}{P(X-h > u)} = \cancel{\overline{F}_u(x)}$$

$$F_u(x) = P(X-u \leq x \mid X > u)$$

$$\overline{F}_u(x) = \frac{P(X > x+u)}{P(X > u)}$$

$$\overline{F}_{u+h}(x) \overset{def.}{=} \frac{\overline{F}(x+u+h)}{\overline{F}(u+h)} = \left(\frac{\overline{F}(x+u+h)}{\overline{F}(u)}\right)\cdot\left(\frac{\overline{F}(u)}{\overline{F}(u+h)}\right) = \frac{\overline{F}_u(x+h)}{\overline{F}_u(h)}$$

$$\overset{(*)}{=} \frac{\overline{G}_{\gamma,\beta}(x+h)}{\overline{G}_{\gamma,\beta}(h)} = \frac{\left(1+\frac{\gamma}{\beta}(x+h)\right)^{-\frac{1}{\gamma}}}{\left(1+\frac{\gamma}{\beta}h\right)^{-\frac{1}{\gamma}}} = \cancel{\left(\frac{1+\frac{\gamma}{\beta}x+\frac{\gamma}{\beta}h}{1+\frac{\gamma}{\beta}h}\right)}$$

$$= \left(\frac{1+\frac{\gamma}{\beta}(x+h)}{1+\frac{\gamma}{\beta}h}\right)^{-\frac{1}{\gamma}}$$

▷ Solve $\frac{1+\frac{\gamma}{\beta}(x+h)}{1+\frac{\gamma}{\beta}h} = \left(1+\frac{\gamma}{\tilde{\beta}}x\right)$ for $\tilde{\beta}$:

$\cancel{\tilde{\beta}}$ / γ $\quad \tilde{\beta} = \dfrac{\gamma x}{\left(\frac{1+\frac{\gamma}{\beta}(x+h)}{1+\frac{\gamma}{\beta}h}-1\right)}$ depends on x, not good...

$$= \frac{\gamma x}{\frac{1+\frac{\gamma}{\beta}x+\frac{\gamma}{\beta}h}{1+\frac{\gamma}{\beta}h}-1} = \frac{\gamma x}{\frac{\frac{\gamma}{\beta}x}{1+\frac{\gamma}{\beta}h}} = \frac{\gamma x}{\frac{\gamma}{\beta}x}\left(1+\frac{\gamma}{\beta}h\right) = \frac{1+\frac{\gamma}{\beta}h}{\beta}$$

Answer $\tilde{\beta} = \frac{1+\frac{\gamma}{\beta}h}{\beta}$, $\tilde{\gamma} = \gamma$

thus $\overline{F}_{u+h} = \overline{G}_{\tilde{\beta},\gamma}$