Christian-Albrechts-Universität zu Kiel **WT 2022/23**
Mathematisches Seminar
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Risk Management

Exercise Sheet 1

C-Exercise 1

On the OLAT page of this course you will find a time series $s_1, ..., s_{8368}$ containing daily DAX data from 01.01.1990 to 21.10.2022.

(a) Import the time series to *Python* and plot it.
Hint: Use the data from the column Close/Schlusskurs

(b) Compute the daily log returns

$$x_n := \log\left(\frac{s_n}{s_{n-1}}\right),\ n = 2, ..., 8368,$$

and plot them.

(c) Plot a histogram of the log returns using 30 intervals.

(d) Assume that the log returns are independent and identically distributed realizations from a normal distribution with mean μ and standard deviation σ. Compute estimators for μ and σ.

(e) Plot into the histogram from (c) the density of a normal distribution using your estimates from (d) for μ and σ.

Please label the diagrams and comment your solution.
Useful *Python* commands: `numpy.genfromtxt, numpy.flip, matplotlib.pyplot.plot, numpy.log, numpy.diff, numpy.mean, numpy.var, matplotlib.pyplot.hist`

C-Exercise 2

Assume that the daily log returns of some stock are independent and normally distributed with mean $\mu = 0.0002681$ and standard deviation $\sigma = 0.0140599$.

(a) Generate a random sample $x_2, ..., x_{8368}$ of daily returns and plot it.

(b) Compute with $s_1 = 1790.37$ the stock price pertaining to this random sample and plot it.

Please label the diagrams and comment your solution.
Useful *Python* commands: `numpy.random.normal, numpy.cumsum`

T-Exercise 3

Let $S_n^{(1)}$ and $S_n^{(2)}$ denote the prices of two stocks in € at time t_n. A bank sets up a portfolio with value 1000€ at time $t_0 = 0$ that always invests 50% of the current portfolio value in each of the stocks. The bank wants to calculate the portfolio value V_n at time t_n in €. For the purpose of risk management we chose the risk factors $Z_n^{(1)} := \log(S_n^{(1)})$ and $Z_n^{(2)} := \log(S_n^{(2)})$.

(a) Derive the function that computes the portfolio value from the risk factors.

(b) Derive the risk factor changes $(X_{n+1}^{(1)}, X_{n+1}^{(2)})$ at time t_{n+1}.

(c) Derive the loss operator $l_{[n]}$, i.e. the function that computes the loss at time t_{n+1} from the risk factor changes.

(d) Derive the linearized loss operator $l_{[n]}^{\Delta}$, i.e. the function that computes the linearized loss at time t_{n+1} from the risk factor changes.

Please comment your solution. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 02.11.2022, 12:00
Discussion in tutorial: Monday, 07.11.2022 and Tuesday, 08.11.2022