Discrete Mathematics: HW8

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4.1 Divisibility and Modular Arithmetic

8 Prove or disprove that if $a|bc$, where a, b, and c are positive integers and $a \neq 0$, then $a|b$ or $a|c$.
$a|bc$ where a, b, c are positive integers
$8|40 \Rightarrow 8|4x10$ but neither $8|4$ nor $8|10$
$\therefore a|bc$ implies $a|b$ or $a|c$

14 Suppose that a and b are integers, $a \equiv 11(mod19)$, and $b \equiv 3(mod19)$. Find the integer c with $0 \leq c \leq 18$ such that
a. $c \equiv 13a(mod19)$
$c(mod19) = 13a(mod19)$
$c(mod19) = 13(11(mod19))(mod19)$
$c(mod19) = (143(mod19))(mod19)$
$c(mod19) = 10(mod19)$
$\therefore c = 10$
e. $c \equiv 2a^2 + 3b^2(mod19)$
$c(mod19) = 2a^2 + 3b^2(mod19)$
$c(mod19) = [2(11(mod19))(11(mod19)) + 3(3(mod19))(3(mod19))](mod19)$
$c(mod19) = (242(mod19) + 27(mod19))(mod19)$
$c(mod19) = (269(mod19))(mod19)$
$c(mod19) = 3(mod19)$
$\therefore c = 3$

26 List five integers that are congruent to 4 modulo 12.
$(a + b)(modm) = (a(modm) + b(modm))(modm)$
$(axb)(modm) = (a(modm)xb(modm))(modm)$
$4(mod12) = 0 + 4(mod12)$
$4(mod12) = 12(mod12) + 4(mod12)$
$4(mod12) = 16(mod12)$
$4(mod12) = 28(mod12)$
$4(mod12) = 40(mod12)$
$4(mod12) = 52(mod12)$
$4(mod12) = 64(mod12)$
$\therefore 4(mod12) = 16, 28, 40, 52, 64$

34 Show that if $a \equiv b(modm)$ and $c \equiv d(modm)$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d(modm)$.
$a = b + km$
$a - c = b - d(modm)$
$a \equiv b(modm)$ and $c \equiv d(modm)$
$\therefore a = b + mk_1$ and $c = d + mk_2$
$a - c = (b + mk_1) - (d + mk_2)$
$a - c = b + mk_1 - d - mk_2$
$a - c = b - d + m(k_1 - k_2)$

$a - c = b - d + mk$
$\therefore a - c = b - d (mod m)$

4.2 Integer Representations and Algorithms

24 Find the sum and product of each of these pairs of numbers. Express your answers as a base 3 expansion.

b. $(20CBA)_{16}$, $(A01)_{16}$

$$\begin{array}{cccccc}
 & 2 & 0 & C & B & A \\
+ & & & A & 0 & 2 \\
\hline
 & 2 & 1 & 6 & B & B
\end{array}$$

$$\begin{array}{ccccccccc}
 & & & & 2 & 0 & C & B & A \\
 & & & x & & & A & 0 & 1 \\
\hline
 & & & & 2 & 0 & C & B & A \\
 & & & 0 & 0 & 0 & 0 & 0 & \\
+ & 1 & 4 & 7 & F & 4 & 4 & & \\
\hline
 & 1 & 4 & 8 & 1 & 5 & 0 & B & A
\end{array}$$

28 Use Algorithm 5 to find $123^{1001} mod 101$

$(1001)_{10} = (1111101001)_2$
$\therefore 123^{1001} mod 101 = 22$

30 It can be shown that every integer can be uniquely represented in the form $e_k 3^k + e_{k-1} 3^{k-1} + \cdots + e_1 3 + e_0$, where $e_j = -1, 0$, or 1 for $j = 0, 1, 2, \ldots, k$. Expansions of this type are called balanced ternary expansions. Find the balanced ternary expansions of

b. 13.

$13 = 3(4) + 1$
$4 = 3(1) + 1$
$1 = 3(0) + 1$
$(13)_{10} = (111)_3$

$$\begin{array}{cccc}
 & 1 & 1 & 1 \\
+ & 1 & 1 & 1 \\
\hline
 & 2 & 2 & 2
\end{array}$$

$\therefore (1)3^2 + (1)3 + (1)$

4.3 Primes and Greatest Common Divisors

4 Find the prime factorization of each of these integers.

c. 101

$101 = 101x1$
$\therefore 101 = 101$

e. 289

$289 = 17x17$
$\therefore 289 = 17^2$

16 Determine whether the integers in each of these sets are pairwise relatively prime.

b. $14, 17, 85$

$gcd(14, 17) = 1$
$gcd(17, 85) = 17$
$gcd(14, 85) = 1$
$\therefore$ the set is not a pairwise relatively prime set.

d. $17, 18, 19, 23$

$gcd(17, 18) = 1$
$gcd(17, 19) = 1$

$gcd(17, 23) = 1$
$gcd(18, 19) = 1$
$gcd(18, 23) = 1$
$gcd(19, 23) = 1$
$\therefore$ the set is a pairwise relatively prime set.

24 What are the greatest common divisors of these pairs of integers?
c. $17, 17^{17}$
$gcd(17, 17^{17}) = 17$
d. $2^2\dot{7}, 5^3\dot{1}3$
$gcd(2^2\dot{7}, 5^3\dot{1}3) = 1$
e. $0, 5$
$gcd(0, 5) = 5$
f. $2\dot{3}\dot{5}\dot{7}, 2\dot{3}\dot{5}\dot{7}$
$gcd(2\dot{3}\dot{5}\dot{7}, 2\dot{3}\dot{5}\dot{7}) = 2\dot{3}\dot{5}\dot{7}$

26 What is the least common multiple of each pair in Exercise 24?
c. $17, 17^{17}$
$lcm(17, 17^{17}) = 17^{17}$
d. $2^2\dot{7}, 5^3\dot{1}3$
$lcm(2^2\dot{7}, 5^3\dot{1}3) = 2^2 5^3 \dot{7}\dot{1}3$
e. $0, 5$
$lcm(0, 5) = undefined$
f. $2\dot{3}\dot{5}\dot{7}, 2\dot{3}\dot{5}\dot{7}$
$lcm(2\dot{3}\dot{5}\dot{7}, 2\dot{3}\dot{5}\dot{7}) = 2\dot{3}\dot{5}\dot{7}$

30 If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?
$gcd(a, b) = 2^3 3^4 5$
$a\dot{b} = 2^7 3^8 5^2 7^{11}$
$ab = gcd(a, b)\dot{l}cm(a, b)$
$\therefore lcm(a, b) = 2^4 3^4 5^1 7^{11}$

32 Use the Euclidean algorithm to find
a. $gcd(1, 5)$
$gcd(1, 5) = 1$
b. $gcd(100, 101)$
$gcd(100, 101) = 1$
e. $gcd(1529, 14038)$
$gcd(1529, 14038) = 1$
f. $gcd(11111, 111111)$
$gcd(11111, 111111) = 1$

54 Adapt the proof in the text that there are infinitely many primes to prove that there are infinitely many primes of the form $3k + 2$, where k is a nonnegative integer. [Hint: Suppose that there are only finitely many such primes $q_1, q_2, \ldots, q_n$, and consider the number $3q_1q_2 \ldots q_n - 1$.]
$3k + 2 = (3k + 3) - 1$
$3k + 2 = 3(k + 1) - 1$
$3(k + 1) = p_1p_2p_3, \cdots, p_n$
$3k + 2 = p_1p_2p_3, \cdots, p_n - 1$
$\therefore$ there are infinitely many primes of the form $3k + 2$