Discrete Mathematics: HW4

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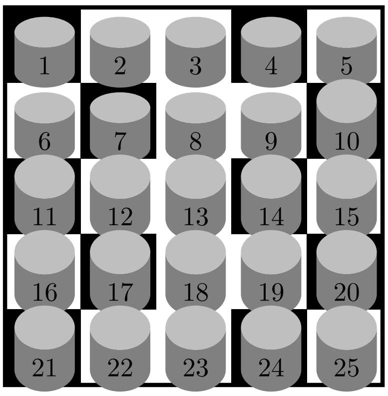
1.8 Proof Method and Strategy

16 Show that if a, b, and c are all real numbers and $a \neq 0$, then there is a unique solution of the equation $ax + b = c$.
$ax = c - b$
$x = (c - b)/a$
As long as $a \neq 0$ there always exists a unique solution

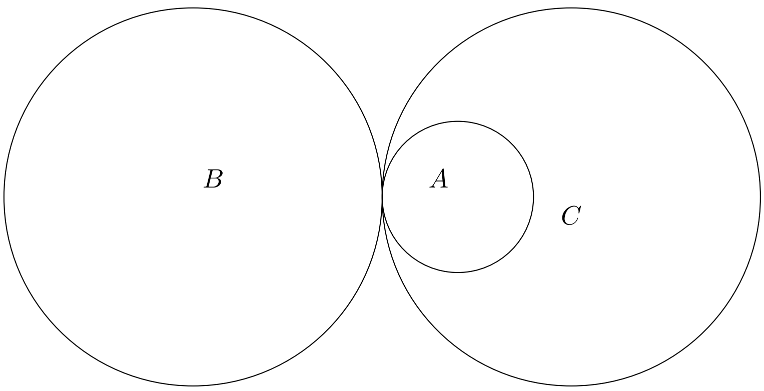
44 Prove or disprove that you can use dominoes to tile a $5x5$ checkerboard with three corners removed.



There are 13 black squares and 12 white squares, assuming we remove 3 of the corners from the grid we now have 10 black squares and 12 white squares. Since we cannot cover the whole board with checkers due to the number of black and white squares not equaling out. Our required result is found.

1 2.1 Sets

16 Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$.



20 What is the cardinality of each of these sets?

a. $\emptyset$

$|\emptyset| = 0$

c. $\{\emptyset, \{\emptyset\}\}$

$|\{\emptyset, \{\emptyset\}\}| = 2$

24 Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

b. $\{\emptyset, \{a\}\}$

For a set A if $|A| = k$ $\therefore$ the cardinality of $P(A) = 2^k$

There is only 1 element in $A = \{a\}$, so the cardinality of $P(A) = 2^1 = 2$

$\{a\}$ is a power set of $\{\emptyset, \{a\}\}$

Thus $A = \{a\}$ is a power set

c. $\{\emptyset, \{a\}, \{\emptyset, a\}\}$

There is only 1 element in $A = \{a\}$, so the cardinality of $P(A) = 2^1 = 2$

However this cannot be a power set, $\therefore$ the given set is not a power set.

26 Show that if $A \subseteq C$ and $B \subseteq D$, then $AxB \subseteq CxD$

$AxB = \{(a, b)|a \in A, b \in B\}$

$CxD = \{(c, d)|c \in C, d \in D\}$

If $a \in A \rightarrow a \in C$ as $A \subseteq C$ and $b \in B \rightarrow b \in D$ as $B \subseteq D$

Let $(a, b) \in AxB \rightarrow (a, b) \in CxD$ for all $a \in A$ and all $b \in B$

$\therefore AxB \subseteq CxD$

32 Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a. $AxBxC$

$AxBxC = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)$

b. $CxBxA$

$CxBxA = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)$

38 Show that $AxB \neq BxA$, when A and B are nonempty, unless $A = B$.

$AxB = \{(x, y)|x \in A \wedge y \in B\}$

$A \neq B$

$a \in A \wedge a \notin B$ or $b \in B \wedge b \notin A$

$(a, b) \in AxB$, but $(b, a) \notin AxB$ while $a \notin B$

$b \in B \wedge a \in A$ gives you $(b, a) \in BxA$

$(b, a) \notin AxB$ and $(b, a) \in BxA$

$AxB \neq BxA$

44 Find the truth set of each of these predicates where the domain is the set of integers.

b. $Q(x) : x^2 = 2$

Truth set is $\emptyset$

c. $R(x) : x < x^2$

Truth set is $-\{0, 1\}$

2.2 Set Operations

4 Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

c. $A - B$

$A - B = \emptyset$

d. $B - A$

$B - A = \{f, g, h\}$

18 Let A, B, and C be sets. Show that

c. $(A - B) - C \subseteq A - C$

$(A - B) - C = \{x|x \in (A - B) \wedge x \notin C\}$ by definition of intersection and difference

$= \{x|(x \in A \wedge x \notin B) \wedge x \notin C\}$ by definition of difference

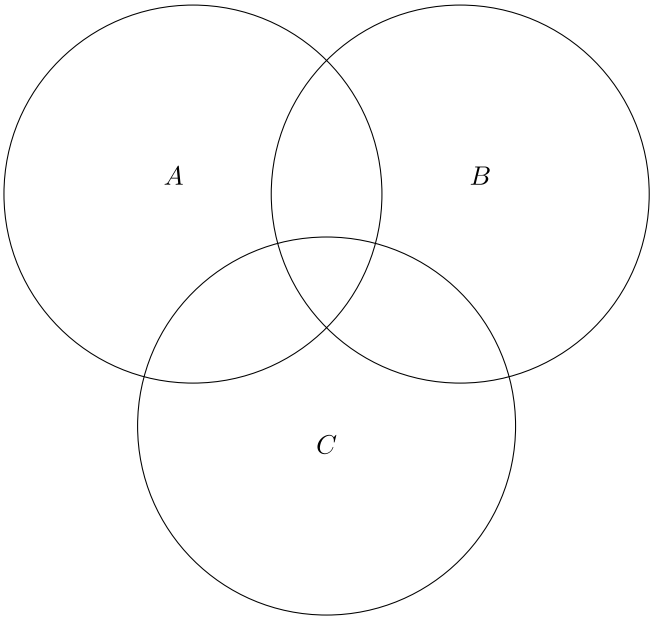
$= \{x|(x \in A \wedge (\neg x \in B)) \wedge (\neg x \in C)\}$ by definition of does not belong symbol

$= \{x | x \in A \wedge (\neg x \in B) \wedge (\neg x \in C)\}$
If $x \in A \wedge (\neg x \in B) \wedge (\neg x \in C)$ then $x \in A \wedge (\neg x \in C)$
Thus, $\{x | x \in A \wedge (\neg x \in B) \wedge (\neg x \in C)\} \subseteq \{x | x \in A \wedge (\neg x \in C)\}$
$(A - B) - C = \{x | x \in A \wedge (\neg x \in B) \wedge (\neg x \in C)\}$
$\subseteq \{x | x \in A \wedge (\neg x \in C)\}$
$\subseteq \{x | x \in A \wedge (x \notin C)\}$ by definition of does not belong symbol
$\subseteq \{x | x \in A - C\}$ by definition of difference
$= A - C$ by meaning of set builder notation
$\therefore (A - B) - C \subseteq A - C$

d. $(A - C) \cap (C - B) = \emptyset$
$(A - C) \cap (C - B) = \{x | x \in (A - C) \wedge x \in (C - B)\}$ by definition of intersection
$= \{x | (x \in A) \wedge (x \notin C) \wedge (x \in C) \wedge (x \notin B)\}$ by definition of difference
$= \{x | (x \in A) \wedge (\neg x \in B) \wedge (x \in C) \wedge (\neg x \in C)\}$ by definition of not belong symbol
$= \{x | ((x \in A) \wedge (\neg x \in B)) \wedge ((x \in C) \wedge (\neg x \in C))\}$ by definition of associative
$= \{x | ((x \in A) \wedge (\neg x \in B)) \wedge ((x \in C - C))\}$ by definition of difference
$= \{x | ((x \in A) \wedge (x \notin B)) \wedge x \in \emptyset\}$ by definition of not belong symbol
$= \{x | (x \in A - B) \wedge x \in \emptyset\}$ by definition of intersection
$= \{x \in (A - B) \cap \emptyset\}$ by definition of set builder notation
$= (A - B) \cap \emptyset$ by defintion of domination law
$= \emptyset \therefore (A - C) \cap (C - B) = \emptyset$

24 Let $A, B,$ and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.
$(A - C) - (B - C) = \{x | x \in (A - C) \wedge x \notin \underline{(B - C)}\}$ by the definition of difference of two sets
$(A - C) - (B - C) = \{x | x \in (A - C) \wedge x \in \overline{(B - C)}\}$ by definition of complement of sets
$(A - C) - (B - C) = \{x | x \in (A \cap \overline{C}) \wedge x \in \overline{(B \cap \overline{C})}\}$ since $A - C = A \cap \overline{C}$ and $B - C = B \cap \overline{C}$
$(A - C) - (B - C) = \{x | x \in (A \cap \overline{C}) \wedge x \in (\overline{B} \cup C)\}$ by DeMorgan's law and complementation law
$(A - C) - (B - C) = \{x | x \in (A \cap \overline{C}) \cap (\overline{B} \cup C)\}$ by definition of intersection of two sets
$(A \cap \overline{C}) \cap (\overline{B} \cup C) = D \cap (\overline{B} \cup C)$ by taking $A \cap \overline{C} = D$
$(A \cap \overline{C}) \cap (\overline{B} \cup C) = (D \cap \overline{B}) \cup (D \cap C)$ by distributive laws
$(A \cap \overline{C}) \cap (\overline{B} \cup C) = ((A \cap \overline{C}) \cap \overline{B}) \cup ((A \cap \overline{C}) \cap C)$ since $A \cap \overline{C} = D$
$(A - C) - (B - C) = \{x | x \in (A \cap \overline{C}) \cap (\overline{B} \cup C)\}$
$(A - C) - (B - C) = \{x | x \in ((A \cap \overline{C}) \cap \overline{B}) \vee x \in ((A \cap \overline{C}) \cap C)\}$ by defintion of union of two sets
$(A - C) - (B - C) = \{x | x \in ((A \cap (\overline{C} \cap \overline{B})) \vee x \in (A \cap (\overline{C} \cap C))\}$ by associative law
$(A - C) - (B - C) = \{x | x \in (A \cap (\overline{B} \cap \overline{C})) \vee x \in (A \cap \phi)\}$ by commutative law and complement law
$(A - C) - (B - C) = \{x | x \in (A \cap \overline{B}) \cap \overline{C} \vee x \in \phi\}$ by associative law
$= \{x | x \in (A - B) \cap \overline{C}\}$ since $A - B = A \cap \overline{B}$
$= \{x | x \in (A - B) - C\}$ since $(A - B) - C = (A - B) \cap \overline{C}$
$= (A - B) - C$ by the meaning of set builder notation
$\therefore (A - C) - (B - C) = (A - B) - C$

26 Draw the Venn diagrams for each of these combinations of the sets $A, B,$ and C.
b. $\overline{A} \cap \overline{B} \cap \overline{C}$
by DeMorgan's Law $\overline{A} \cap \overline{B} = \overline{A \cup B}$
$\overline{A \cup B \cup C}$



c. $(A - B) \cup (A - C) \cup (B - C)$
$(A \cap \overline{B}) \cup (A \cap \overline{C}) \cup (B \cap \overline{C})$
$\{(A \cap \overline{B}) \cup (A \cap \overline{C})\} \cup \{(A \cap \overline{C}) \cup (B \cap \overline{C})\} (\because X \cup X = X)$
by Distributive property $\{A \cap (\overline{B} \cup \overline{C})\} \cup \{\{A \cup B\} \cap \overline{C}\}$