Assignment 1

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Chapter 1

6 There are approximately 260 million passenger vehicles registered in the United States. Assume that the battery in the average vehicle stores 540 watt-hours (W h) of energy. Estimate (in gigawatt-hours) the total energy stored in US passenger vehicles.

$$
\begin{aligned}
w_{total} &= nw_{avg} \\
&= (260,000,000)(540) \\
&= 140,400,000,000 \\
&= 140.4\,\text{GW h}
\end{aligned}
$$

11 The current at the terminals of the element is

$$
\begin{aligned}
i &= 0, && t < 0 \\
i &= 40te^{-500t}\,\text{A}, && t \geq 0
\end{aligned}
$$

a) Find the expression for the charge accumulating at the upper terminal.

$$
\begin{aligned}
q(t) &= 40 \int_0^t te^{-500t}dt \\
&= 40[(t(\tfrac{e^{-500t}}{-500}))_0^t - [\int_0^t \tfrac{e^{-500t}}{-500}dt]] \\
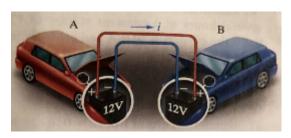&= 40[(t(\tfrac{e^{-500t}}{-500}))_0^t + \tfrac{1}{500}(\tfrac{e^{-500t}}{-500})_0^t] \\
&= \tfrac{40}{(500)^2}[(-500te^{-500t})_0^t - (e^{-500t})_0^t] \\
&= 0.16[1 - (500t + 1)e^{-500t}]\text{mC}
\end{aligned}
$$

b) Find the charge that has accumulated at $t = 1\,\text{ms}$.

$$
\begin{aligned}
q(1 \times 10^{-3}) &= 0.16[1 - (500(1 \times 10^{-3}) + 1)e^{-500(1\times 10^{-3})}] \\
&= 0.16[1 - 0.9098] \\
&= 0.014\,432\,\text{mC} \\
&= 14.432\,\mu\text{C}
\end{aligned}
$$

12 When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in the figure below. Assume the current i in the figure below is measured and found to be 40 A.

a) Which car has the dead battery?

$$
\begin{aligned}
P &= (12\,\text{V})(40\,\text{A}) \\
&= 480\,\text{W}
\end{aligned}
$$

The dead battery is in Car A

b) If this connection is maintained for $1.5\,\mathrm{min}$, how much energy is transferred to the dead battery?

$$\begin{aligned}
W(t) &= \int_0^t P\,dt \\
&= \int_0^{1.5\,\mathrm{min}} 480\,\mathrm{W}\,dt \\
&= (480\,\mathrm{W}) \times [t]\big|_0^{1.5\,\mathrm{min}} \\
&= 43.2\,\mathrm{kJ}
\end{aligned}$$

18 The voltage and current at the terminals of the circuit element are zero for $t < 0$. For $t \geq 0$ they are

$$\begin{aligned}
v &= 75 - 75e^{-1000t}\,\mathrm{V} \\
i &= 50e^{-1000t}\,\mathrm{mA}
\end{aligned}$$

a) Find the maximum value of the power delivered to the circuit.

$$\begin{aligned}
p(t) &= vi \\
&= (75 - 75e^{-1000t})(50e^{-1000t})(10^{-3}) \\
&= 50 \times \frac{1}{10^3} \times \frac{1}{e^{1000t}}(-75e^{-1000t} + 75) \\
p(625 \times 10^{-6}) &= 50 \times \frac{1}{10^3} \times \frac{1}{e^{1000(625 \times 10^{-6})}}(-75e^{-1000(625 \times 10^{-6})} + 75) \\
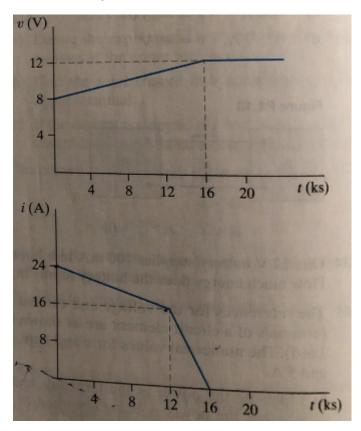&= 50 \times \frac{1}{10^3} \times \frac{1}{e^{\frac{5}{8}}}\left(-\frac{75}{e^{\frac{5}{8}}} + 75\right) \\
&= \frac{15(e^{\frac{5}{8}} - 1)}{4e^{\frac{5}{4}}} \\
&= 933\,\mathrm{mW}
\end{aligned}$$

b) Find the total energy delivered to the element.

$$\begin{aligned}
w_{total}(t) &= \int_0^\infty p(t)\,dt \\
&= \int_0^\infty \left(50 \times \frac{1}{10^3} \times \frac{1}{e^{1000t}}(-75e^{-1000t} + 75)\right)dt \\
&= -\frac{3}{1600}(-e^{-2000t} + 2e^{-1000t})\big|_0^\infty \\
&= \frac{3}{1600} \\
&= 1875\,\mathrm{\mu J}
\end{aligned}$$

27 The voltage and the current at the terminals of an automobile battery during a charge cycle are shown in the figure below.

a) Calculate the total charge transferred to the battery.

$$\begin{aligned} q &= \text{area under } i \text{ vs } t \text{ plot} \\ &= \tfrac{1}{2}(8)(12000) + (16)(12000) + \tfrac{1}{2}(16)(4000) \\ &= 272.000\,\text{C} \end{aligned}$$

b) Calculate the total energy transferred to the battery.

$$W = \int p\,dt = \int vi\,dt$$

For $0 \le t \le 12.000\,\text{sec}$:

$$\begin{aligned} v &= 250 \times 10^{-6}t + 8 \\ i &= 24 - 666.67 \times 10^{-6}t \\ p &= 192 + 666.67 \times 10^{-6}t - 166.67 \times 10^{-9}t^2 \\ W_1 &= \int_0^{12,000}(192 + 666.67 \times 10^{-6}t - 166.67 \times 10^{-9}t^2)dt \\ W_1 &= 2256\,\text{kJ} \end{aligned}$$

For $12,000 \le t \le 16.000\,\text{sec}$:

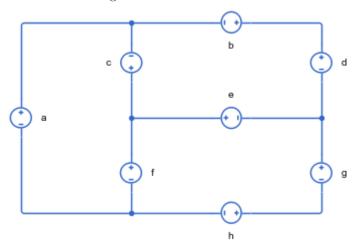$$\begin{aligned} v &= 250 \times 10^{-6}t + 8 \\ i &= 64 - 4 \times 10^{-3}t \\ p &= 512 - 16 \times 10^{-3}t - \times 10^{-6}t^2 \\ W_2 &= \int_{12,000}^{16,000}(512 - 16 \times 10^{-3}t - \times 10^{-6}t^2)dt \\ W_2 &= 2256\,\text{kJ} \\ W_T &= 2256 + 362.667 = 2618.667\,\text{kJ} \end{aligned}$$

33 Assume you are an engineer in charge of a project and one of your subordinate engineers reports that the interconnection in the figure below does not pass the power check. The data for the interconnection are given in the table below.

Element	Voltage (V)	Current (A)	Power (V × A = W)
a	900	-22.5	-20250
b	105	-52.5	5512.5
c	-600	-30.0	-18000
d	585	-52.5	-30712.5
e	-120	30.0	3600
f	300	60.0	18000
g	585	82.5	-48262.5
h	-165	82.5	13612.5
Total	1590	97.5	-36000

a) Is the subordinate correct? Explain your answer.
The subordinate is correct, due the net power being $-36\,000\,\text{W}s$ and interconnection must consume power for each atom.

b) If the subordinate is correct, can you find the error in the data?

Elements c, d, and g supply extra power. To fix we just reduce the power to match the required load of the system.