Homework Assignment #2: Chapters 1 & 2 and (A Little Bit of) Chapter 3

This Homework Assignment is based on Chapters 1 & 2 and (A Little Bit of) Chapter 3 from your course textbook. It will be an opportunity for you to review problems from calculus. Remember, this Homework Assignment is **not collected or graded**! But you are advised to do it anyway because the problems for Homework Quiz #2 will be chosen from these problems![1]

1. Let A and B be events associated with the same sample space with probability P. Decide if the following statement is: always true, sometimes true, or never true.

   If $P(A) = 1/3$ and $P(B) = 1/3$ then $P(A \cap B) = 1/2$.

   Remember, $A \cap B$ is the set intersection operation:

   $$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

2. Let A and B be events associated with the same sample space with probability P. Decide if the following statement is: always true, sometimes true, or never true.

   If $P(A) = 1/3$ and $P(B) = 1/3$ then $P(A \cup B) = 1/2$.

   Remember, $A \cup B$ is the set union operation:

   $$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

3. Let A and B be events associated with the same sample space with probability P. Decide if the following statement is: always true, sometimes true, or never true.

   If $P(A) = 1/2$, $P(B) = 1/2$ and $P(A \cap B) = 1/4$ then $P(A \cup B) = 3/4$.

4. We flip a fair coin two times. The flips are independent. Remember, a fair coin lands on heads H with probability 1/2 and tails (T) with probability 1/2. If the outcomes are the same (HH or TT) we win; otherwise we lose.

   Define the following events:

   - A: The first coin comes up heads
   - B: The second coin comes up heads
   - C: The event we win.

   What is $P(A \cap B \cap C)$?

5. You receive 10 letters, 5 of them come in red envelopes and 5 of them in blue envelopes. You randomly shuffle them and place them in a stack.

   Let A be the event that there is at least 1 blue envelope in the top three letters of the stack.

   What is $P(A)$? (Your answer should be accurate to 3 decimal places.)

6. Consider the following experiment involving a fair coin and a pair of 6 sided dice. You flip a coin:

   - If you get heads, roll one die and record the number.
   - If you get tails, roll a pair of dice and record the sum.

   Define A to be the event that we have recorded a 4.

   What is $P(A)$? (Your answer should be entered numerically and correct to 3 decimal places.)

[1] Course instructors reserve the right to *slightly* modify the questions from these when they make the Homework Quiz!

7. You have a pair of dice, each of which has its sides numbered from 1 to 6.

   You perform an experiment where you roll the pair of dice and stop when the sum of the numbers is 10.

   Your experiment is to count the number of rolls you carried out until this happened. Answer the following in complete sentences:

   - Describe your sample space (i.e., the set of all possible outcomes).
   - Compute the probability you roll 10 either before or on your second roll.