Math 32

Lecture 2: Inclusion-Exclusion

Example 1: Consider rolling one six-sided die. For each of the following events, list their possible ways and find their probabilities.



For probability notation, *events* are usually denoted in capital letters. (This will make later descriptions and formulas easier to write.)

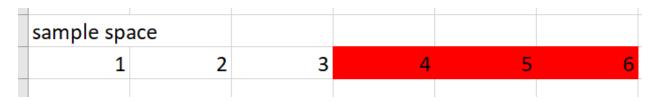
• Let A: rolling an even number

Α	В	С	D	Е	F	
sample spa	ace					
1	2	3	4	5	6	

The probability of A, the probability of observing an even number is 3/6, or ½

$$P(A) = \frac{3}{6}$$

- o (aside: no need to reduce fractions in Math 32)
- Let B: rolling a number greater than 3
 - \circ True if data x > 3



The probability of B, the probability of observing a number greater than 3 is 3/6

$$P(B) = \frac{3}{6}$$

• Let C: rolling a double-digit result

sample spa	ace				
1	2	3	4	5	6

Since there are no possible outcomes that meet that description, we say

$$P(C) = 0$$

We will now try to solve P(A or B). That is, what is the probability of observing an even number or a number greater than 3?

- $A = \{2, 4, 6\}$
- $B = \{4, 5, 6\}$

We start out with P(A or B) = P(A) + P(B) = 3/6 + 3/6 = 100 percent??

- But the answer is clearly not all 100 percent
- Chat query: what went wrong?
 - o Observe: a couple of the items were counted twice
 - o We need to be careful with the intersection
- Intersection (set of values that appear in both A and B)
 - \circ A and B = {4, 6}
- Union (set of values that appear in A, B, or both)
 - \circ A or B = {2, 4, 5, 6}
- Key idea: we need to subtract the overlap

Inclusion-Exclusion Principle

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 3/6 + 3/6 - 2/6 = 4/6$$



Consider rolling two, six-sided dice. Find the probability that their total is 8 or the second die shows a number greater or equal to 5.

Define the events

- A: total is 8
- B: second die shows a number >= 5

Compute

- P(A) = 5/36
- P(B) = 12/36

•	_	_		_	•	
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Inclusion-Exclusion

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 5/36 + 12/36 - 2/36 = 15/36$$

• Tip: when we encounter the signal word "or", consider using this Inclusion-Exclusion formula Definition: Two events A and B are *disjoint* if there is no intersection

$$P(A \cap B) = 0$$

Shiny app demonstration

https://dsollberger.shinyapps.io/inclusionExclusion/