

Homework Assignment #4: Chapters 4

This Homework Assignment is based on Chapter 4 from your course textbook. Remember, this Homework Assignment is **not collected or graded!** But you are advised to do it anyway because the problems for Homework Quiz #4 will be chosen from these problems!¹

1. A school class of 120 students is driven in 3 buses to a movie theatre to see a special screening of the original "The Karate Kid." There are 36 students in one of the buses, 40 in another and 44 in the third bus.

When the buses arrive, one of the 120 students is randomly chosen. Let X be the number of students that were on the bus taken by that randomly chosen student.

- (a) What is the state space of X ? (Hint: There are **only 3** values in the sample space!)
- (b) What is the probability mass function associated with X ?
- (c) What is the expected value of X : $E[X]$?

2. Let X be a random variable equal to the outcome of a 6 sided die roll squared. (For example, if you roll a 2 the outcome of X is $2^2 = 4$.)

- (a) What is the state space of X ?
- (b) What is the probability mass function associated with X ?
- (c) What is the expected value of X : $E[X]$?
- (d) What is the variance of X : $V(X) = E[X^2] - E[X]^2$?

3. Consider the following piecewise function: $p(a)$

$$p(a) = \begin{cases} 0.05 & a = -2 \\ 0.25 & a = 0 \\ 0.25 & a = 1 \\ 0.35 & a = 2 \\ 0.1 & a = 5 \\ 0 & \text{all other } a \end{cases}$$

- (a) Verify that $p(a)$ could be the probability mass function of some discrete random variable X .
 - (b) Compute $P(X < 2)$, $P(2 < X \leq 5)$, and $P(X \geq 5)$.
 - (c) Find the cumulative distribution function $F(a) = P(X \leq a)$ of X . (This is like a problem on the first discussion section worksheet).
 - (d) Let $Y = X^2$. Find the probability mass function of Y .
4. There are 7 red marbles and 3 blue ones in a jar. We randomly take out marbles from the jar without replacement, which means that we do not put any marbles back into the jar after they are taken out.
 - (a) We take out 3 marbles. Let X be the number of red marbles among the three. Does X have a binomial distribution? Why or why not?
 - (b) Let Y be the number of marbles we have to take out until we get the first blue one. Does Y have a geometric distribution? Why or why not?

¹Course instructors reserve the right to *slightly* modify the questions from these when they make the Homework Quiz!

5. A shop receives a shipment of 1000 cheaply made lamps. The probability that any individual lamp is defective is 0.001. Assume the defectiveness is independent for each lamp. Let X be the number of defective lamps in the batch of 1000.
- (a) What is the state space of X ?
 - (b) What kind of probability mass function does X have?
 - (c) What is the probability that none of the lamps are defective?
 - (d) What is the probability that more than 2 of the lamps are defective?
6. You have n coins each of which takes on the outcome heads with probability p and tails with probability $(1 - p)$. You conduct a coin tossing experiment in two phases. First, you independently toss each of the n coins. Then, for each coin that is showing tails you toss it again.
- Let X be the total number of heads you observe among your n coins at the end of your experiment.
- (a) What is the state space of X ?
 - (b) What is the probability mass function of X ? (Hint: X actually fits one of the distributions that was discussed in Lecture 7, but realizing it takes a bit of thought!)