

Math 32

Lecture 2: Inclusion-Exclusion

Example 1: Consider rolling one six-sided die. For each of the following events, list their possible ways and find their probabilities.



For probability notation, **events** are usually denoted in capital letters. (This will make later descriptions and formulas easier to write.)

- Let A: rolling an even number

	A	B	C	D	E	F
sample space						
	1	2	3	4	5	6

The probability of A, the probability of observing an even number is $3/6$, or $\frac{1}{2}$

$$P(A) = 3/6$$

- (aside: no need to reduce fractions in Math 32)

- Let B: rolling a number greater than 3

- True if data $x > 3$

sample space						
	1	2	3	4	5	6

The probability of B, the probability of observing a number greater than 3 is $3/6$

$$P(B) = 3/6$$

- Let C: rolling a double-digit result

sample space					
1	2	3	4	5	6

Since there are no possible outcomes that meet that description, we say

$$P(C) = 0$$

We will now try to solve $P(A \text{ or } B)$. That is, what is the probability of observing an even number or a number greater than 3?

- $A = \{2, 4, 6\}$
- $B = \{4, 5, 6\}$

We start out with $P(A \text{ or } B) = P(A) + P(B) = 3/6 + 3/6 = 100 \text{ percent??}$

- But the answer is clearly not all 100 percent
- Chat query: what went wrong?
 - Observe: a couple of the items were counted twice
 - We need to be careful with the intersection
- Intersection (set of values that appear in both A and B)
 - $A \text{ and } B = \{4, 6\}$
- Union (set of values that appear in A, B, or both)
 - $A \text{ or } B = \{2, 4, 5, 6\}$
- Key idea: we need to subtract the overlap

Inclusion-Exclusion Principle

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 3/6 + 3/6 - 2/6 = 4/6$$

Example 2:



Consider rolling two, six-sided dice. Find the probability that their total is 8 or the second die shows a number greater or equal to 5.

Define the events

- A: total is 8
- B: second die shows a number ≥ 5

Compute

- $P(A) = 5/36$
- $P(B) = 12/36$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Inclusion-Exclusion

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 5/36 + 12/36 - 2/36 = 15/36$$

- Tip: when we encounter the signal word “or”, consider using this Inclusion-Exclusion formula

Definition: Two events A and B are ***disjoint*** if there is no intersection

$$P(A \cap B) = 0$$

Shiny app demonstration

<https://dsollberger.shinyapps.io/inclusionExclusion/>