Extrema

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Example

Let X_1 and X_2 be independently generated uniformly distributed random variables on the interval $[0, 1]$. You will consider the PDF and CDF of $M = \min\{X_1, X_2\}$. We can determine the CDF and PDF of M

$$\begin{aligned} F_M(x) &= P(M \leq x) \\ &= 1 - P(M > x) \\ &= 1 - P(\min\{X_1, X_2\} > x) \\ &= 1 - P(X_1 > x \text{ and } X_2 > x) \\ &= 1 - P(X_1 > x)P(X_2 > x) \\ &= 1 - (1 - F_X(x))^2 \end{aligned}$$

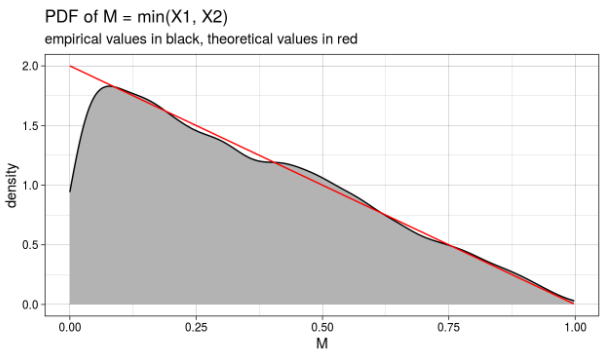
and

$$F_M(x) = \begin{cases} 0, & x < 0 \\ 1 - (1-x)^2, & 0 < x < 1 \\ 1, & 1 < x \end{cases}$$

We take the derivative to compute the PDF

$$\frac{d}{dm} F_M(x) = f_M(x) = \begin{cases} 0, & x < 0 \\ 2(1-x), & 0 < x < 1 \\ 0, & 1 < x \end{cases}$$

Then we can graph the empirical and theoretical PDFs of M



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Maximum

Let X_1, X_2, and X_3 be independently generated uniformly distributed random variables on the interval $[0, 1]$. You will consider the PDF and CDF of $M = \max\{X_1, X_2, X_3\}$. Determine the CDF and PDF of M, then graph the empirical and theoretical PDFs of M

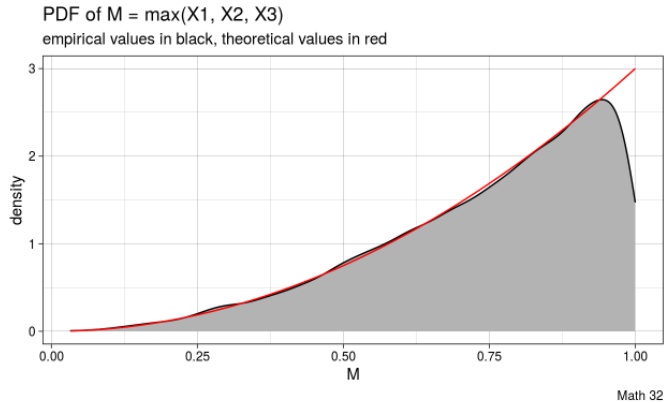
Code | Start Over | Run Code

```
X2 <- runif(N, 0, 1)
X3 <- runif(N, 0, 1)

M <- pmax(X1, X2, X3) #hint: you need X3
empirical_data <- data.frame(M)

true_PDF <- function(x){ 3*((x)*(x)) } #hint: you need to change this PDF

ggplot(empirical_data, aes(x = M)) +
  geom_density(color = "black", fill = "gray70") +
  geom_function(fun = true_PDF, color = "red", geom = "line") +
  labs(title = "PDF of M = max(X1, X2, X3)",
       subtitle = "empirical values in black, theoretical values in red",
       caption = "Math 32") +
  theme_linedraw()
```



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Mimimum

Let X_1, X_2, and X_3 be independently generated exponentially distributed random variables with rate parameter $\lambda = 1/32$. You will consider the PDF and CDF of $M = \min\{X_1, X_2, X_3\}$. Determine the CDF and PDF of M, then graph the empirical and theoretical PDFs of M

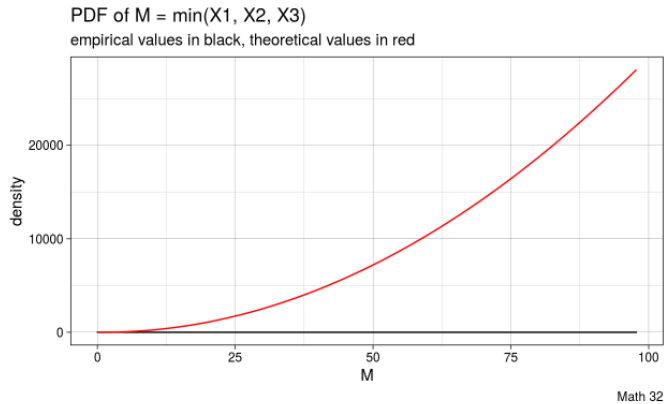
Code | Start Over | Run Code

```
N <- 12345
X1 <- rexp(N, 1/32)
X2 <- rexp(N, 1/32)
X3 <- rexp(N, 1/32)

M <- pmin(X1, X2, X3) #hint: you need X3
empirical_data <- data.frame(M)

true_PDF <- function(x){ 3*((1 - x)*(1 - x)) } #hint: you need to change this PDF

ggplot(empirical_data, aes(x = M)) +
  geom_density(color = "black", fill = "gray70") +
  geom_function(fun = true_PDF, color = "red", geom = "line") +
  labs(title = "PDF of M = min(X1, X2, X3)",
       subtitle = "empirical values in black, theoretical values in red",
```



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Submission

- take a screenshot of each page of this assigment (try not to include the left-hand menu to “zoom in” on the content)
- copy and paste the screenshots onto a Word document (or Google Doc or equivalent)
- be sure that your name appears on the document
- save as a PDF
- upload the PDF back to our CatCourses page

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