Homework Assignment #3: Chapters 2 & 3

This Homework Assignment is based on Chapters 2 & 3 from your course textbook. Remember, this Homework Assignment is **not collected or graded**! But you are advised to do it anyway because the problems for Homework Quiz #3 will be chosen from these problems![1]

1. An elementary school is offering 3 language classes: Spanish, French and German. The classes are open to any of the 100 students in the school. Moreover, none of the classes overlap so it is possible for students to take 0, 1, 2, or all 3 language classes.

   There are 28 students in the Spanish class, 26 in the French class and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

   (a) If a student is chosen at random, what is the probability they are not in any of the language classes?

   (b) If a student is chosen randomly, what is the probability that they are taking **exactly 1** language class.

   (c) If 2 students are chosen randomly, what is the probability that **at least 1** is taking a language class?

2. Consider the following experiment with a *biased* coin. The probability of heads is p and tails $(1-p)$ where $0 < p < 1$. The outcome of the experiment we are interested in is the number of tosses it takes until a heads occurs for the *second* time.

   (a) What is the sample space?

   (b) What is the probability that this takes exactly 5 flips?

3. An urn contains n white balls and m black balls. (m and n are both positive numbers.)

   (a) If two balls are drawn **without replacement**, what is the probability that both balls are the same color?

   (b) If two balls are drawn **with replacement** (i.e., One ball is drawn, its color was recorded and then put back to the urn. Then the second ball is drawn.) What is the probability that both balls are the same color?

   (c) Show that the probability in part (b) is always larger than the one in part (a).

4. Section 3.3 in your textbook describes a test for BSE. Read that section and then answer the following question:

   A Dutch Cow is tested for BSE using Test A (as described in Section 3.3), suppose $P(T|B) = 0.70$ and $P(T|B^c) = 0.10$. Assume that the BSE risk for the Netherlands is the same as in 2003, when it was estimated to be $P(B) = 1.3 \times 10^{-5}$. Compute:

   (a) $P(B|T)$

   (b) $P(B|T^c)$

5. This question is similar to the last example on Lecture 5.

   Suppose that three jars contain colored balls as described in the table below.

[1] Course instructors reserve the right to *slightly* modify the questions from these when they make the Homework Quiz!

	Red	White	Blue
Jar 1	3	4	1
Jar 2	1	2	3
Jar 3	4	3	2

Note: This question has two parts, between the two parts assume that all balls are returned to their original jar. The jars in each case are chosen independently.

(a) One jar is chosen at random and a single ball is selected. If the ball is white, what is the probability that it came from the 2nd jar?

(b) One jar is chosen at random and **two balls** are selected. If both balls are blue, what is the probability it came from the 2nd jar?