Worksheet 1

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12.1 - 3

1 Which two of the three points $P1(1,2,3)$, $P2(3,2,1)$ and $P3(1,1,0)$ are closest to each other?
$P1P2:$
$d = \sqrt{(3-1)^2 + (2-2)^2 + (1-3)^2}$
$d = \sqrt{4+4} = \sqrt{8}$
$P2P3:$
$d = \sqrt{(1-3)^2 + (1-2)^2 + (0-1)^2}$
$d = \sqrt{4+1+1} = \sqrt{6}$
$P1P3:$
$d = \sqrt{(1-1)^2 + (1-2)^2 + (0-3)^2}$
$d = \sqrt{1+9} = \sqrt{10}$
$\therefore P2P3$ has the shortest distance.

2 Describe the set of points
a. whose distance from the x axis is 2.
$-2 \geq y \geq 2$
b. located a distance 3 from the point $(0,1,1)$ and find what equation they satisfy.
$3 = \sqrt{(x-0)^2 + (y-1)^2 + (z-1)^2}$
$9 = x^2 + (y-1)^2 + (z-1)^2$

3 Find a formula for the shortest distance between a point (a,b,c) and the y axis.

4 Describe the set of all points equidistant from the points $A(-1,5,3)$ and $B(6,2,-2)$. (Challenge: find an equation for this set)

5 A car goes clockwise on an elliptic track $\frac{x^2}{4} + y^2 = 1$, decelerating at the curves and accelerating along the straighter portions. Sketch the track and possible velocity vectors at $(-2,0)$, $(0,1)$ and $(\sqrt{3}, \frac{1}{2})$.

6 Perform the indicated operations with: $\overrightarrow{a} = \overrightarrow{j} + 3\overrightarrow{k}$, $\overrightarrow{b} = 4\overrightarrow{i} - 5\overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{c} = 3\overrightarrow{i} + 5\overrightarrow{j}$
i. $5\overrightarrow{b}$
ii. $\overrightarrow{a} + \overrightarrow{c}$
iii. $2\overrightarrow{c} + \overrightarrow{b}$
iv. $||\overrightarrow{b}||$
v. $\frac{\overrightarrow{b}}{||\overrightarrow{b}||}$
vi. $2\overrightarrow{a} + 7\overrightarrow{b} - 5\overrightarrow{c}$

7 Find the components of the vector $\overrightarrow{v}$ in the xy plane if $||\overrightarrow{v}|| = 10$ and $\overrightarrow{v}$ is at an angle of $35°$ below the positive x-axis.

8 Find a unit vector from $P = (-1,3)$ and toward $Q = (2,5)$. Find a vector of length 12 pointing in the same direction.

9 An airplane is heading due east and climbing at the rate of 85 km/hr. If its air speed indicator reads 490 km/hr. After some time, there is a (horizontal) wind blowing 90 km/hr toward the northeast, which the plane does not adjust for. what is the horizontal rate of speed of the plane in the wind as detected by a stationary radar unit on the ground? (Hint: Write everything down as vectors, finding their components.)

10 A 100-meter dash is run in the direction of the vector $\overrightarrow{v} = 2\overrightarrow{i} + 6\overrightarrow{j}$. The wind velocity $\overrightarrow{w}$ is $5\overrightarrow{i} + \overrightarrow{j}$ km/hr. A legal wind speed measured in the direction of the dash may not exceed 5 km/hr. Will the race results be disqualified due to excessive wind?

11 The Parallelogram Law states that: $|\overrightarrow{a} + \overrightarrow{b}|^2 + |\overrightarrow{a} - \overrightarrow{b}|^2 = 2|\overrightarrow{a}|^2 + 2|\overrightarrow{b}|^2$
a. Give a geometric interpretation of the Parallelogram Law.
b. Prove the this Law using the properties of the dot product. (Don't introduce coordinates.)

12 Complete the following
a. Draw the vectors $\overrightarrow{a} =< 3, 2 >$, $\overrightarrow{b} =< 2, -1 >$, and $\overrightarrow{c} =< 7, 1 >$.
b. Show, by means of a sketch, that there are scalars s and t such that $\overrightarrow{c} = s\overrightarrow{a} + t\overrightarrow{b}$.
c. Use the sketch to estimate the values of $\overrightarrow{s} and \overrightarrow{t}$.
d. Compute the exact values of s and t.

13 For any given vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, consider the following function of $t : q(t) = (\overrightarrow{a} + t\overrightarrow{b})x(\overrightarrow{a} + t\overrightarrow{b})$.
i. Explain why $q(t) \geq 0$ for any real t.
ii. Expand $q(t)$ as a quadratic polynomial in t using the properties of the dot product. (Do not introduce coordinates.) iii. Using the discriminant of the quadratic, show that $|\overrightarrow{a} x \overrightarrow{b}| \leq |\overrightarrow{a}||\overrightarrow{b}|$. This result is called the Cauchy–Schwarz Inequality.