Experimentally Measuring the Speed of Light

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The speed of light is a highly important universal physical constant. We present an experimental way to measure the speed of light, c, using a laser beam, a rotating mirror and adequate space. This process involves a laser bouncing off a rotating mirror, returning back with a set of mirrors, and therefore being displaced. Measuring this displacement, the frequency of rotation of the mirror, and the length the laser traveled allow us to derive the speed of light. This was repeated for different lengths and frequencies of rotation. A speed of $c = (3.06 \pm 0.03) * 10^8 \frac{m}{s}$ was calculated using a weighted average, by errors, method and a value of $c = (3.08 \pm 0.02) * 10^8 \frac{m}{s}$ was calculated by using a method of spread between experiment values. Nearly all uncertainty in this experiment came from the uncertainty in the focal length of the microscope.

I. INTRODUCTION AND HISTORY

The speed of light has been experimentally measured since before the idea of light was even well understood. Even in 1638, Galileo tried to measure the speed of light by covering a lantern and calculating how long it would take the light to travel across a field to him. Though to his experiment was inconclusive, other methods for measuring the speed of light proved surprisingly successful by using ellipses in Jupiter's moons and other methods similar to the rotating mirror method used in this experiment. It was not until Maxwell discovered that a mass-less electromagnetic radiation was extremely close to the expected speed of light; that the idea that light is an electromagnetic wave emerged was more closely studied. In 1973 the speed of light was so accurately measured with lasers and atomic clocks, with most of the uncertainty coming from the definition of a meter, that the speed of light was defined to be 299,792,458 meters per second. This definition has implications what we are therefore measuring, which are discussed in the Theory Section of this paper. [1]

The speed of light is important for multitudes of reasons. The speed of light is a fundamental constant that defines the speed for which mass-less particles travel at in a vacuum. The speed of light is a constant by any choice of spacial coordinates and time as predicted by Einstein's Theory of Special Relativity. The speed of light also relates mass and energy through Einstein's infamous Theory of Mass Energy Equivalence. It defined other interesting things like gravity, gravitational waves, and the speed at which information can travel.

This fundamental constant has many applications in different fields of physics. It is both a tool and a limiting factor in studying astrophysics. The speed of light is highly important in particle physics and modern experiments, like those involving the Large Hadron Collider. The speed of light is also important in many everyday technologies, such as, GPS, radios, TVs, Bluetooth, fiber optics, and WiFi.

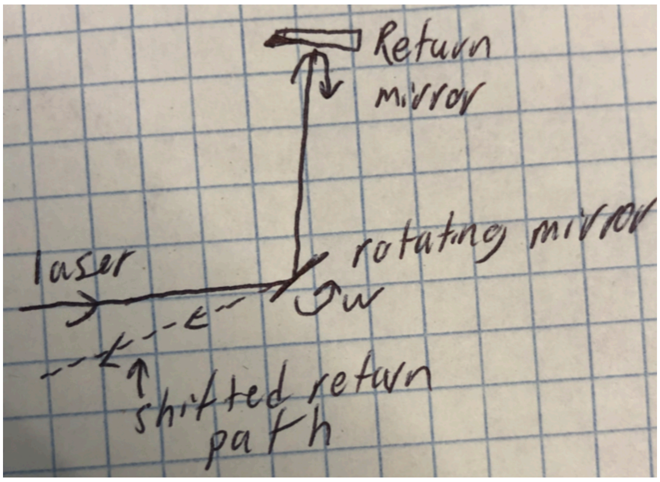

FIG. 1: A simplified version of the laboratory set-up used.

II. THEORY

Theoretical Note: As stated, the speed of light is well defined. In reality, what we were doing was measuring a meter stick using the speed of light; however, comparing these results through the experimental and true value of the speed of light. This implication is trivial and is not referenced again in this paper.[2]

Looking at Figure 1, an expression for the speed of the laser, or the speed of light, can be derived using the angular frequency of the rotating mirror, ω, the length the laser travels before between the rotating and return mirror, L, and the displacement angle of the laser (the optical angle), θ_o. ω is constant: $\omega = \frac{\theta_m}{t'}$, where θ_m is the mechanical angle, or the angel the mirror rotates before the laser returns to it and t is the time it takes for the laser to return to the rotating mirror. It is also clear from Figure 1 that $c = \frac{2L}{t'}$ or $t' = \frac{2L}{c}$. In accordance with

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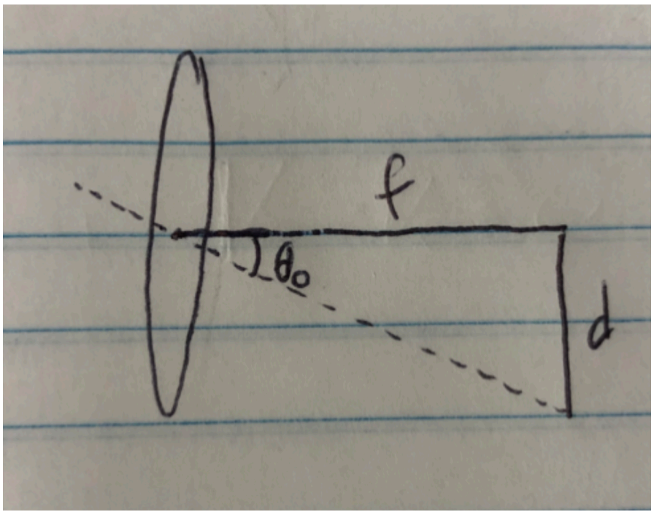

FIG. 2: A visual representation of the relationship between the optical angle, the focal length, and the displacement seen in the telescope, d.

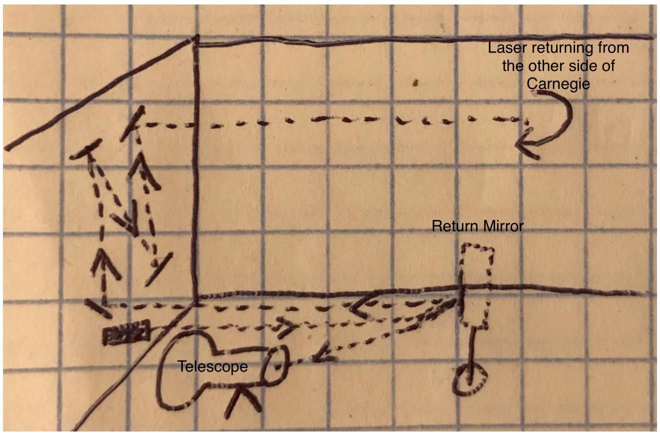

FIG. 3: The laboratory set up in the basement of Carnegie, Bates College.

theory, the optical angle, or the angle at which the light is displaced, is twice the mechanical angle: $\theta_o = 2\theta_m$. Combining these results in Eq. 1.

$$c = \frac{4\omega L}{\theta_o} \tag{1}$$

The small angle approximation and to get the lasers shifted return path in terms of a displacement some distance away from the rotating mirror. This becomes, $tan(\theta_o) \approx \theta_o = \frac{d}{f}$ as shown in Figure 2, where f is the focal length of our measurement device and d is the laser displacement.

As will be explained in the set-up, only changes in frequency and displacement will be measured. It is also frequency that is measured not angular frequency, $\omega = 2\pi F$. Where F denotes the frequency of the mirror rotation. These relationship give Eq 2.; however, many of these values are not direct measurement.

$$c = \frac{8\pi \Delta F f L}{\Delta d} \tag{2}$$

The focal length, is calculated using the following equation:

$$f = \frac{d_o M}{1 + M} \tag{3}$$

where M is the magnification of an arbitrary object and d_0 is its distance from the magnification device.

All weighted averages based on errors are calculated from the best values of each of its parameters. The error in this calculation is based of the following equation

$$\sigma_V = \sqrt{\sum_i \left(\frac{\partial V}{\partial p_i}\right)^2 \sigma_{p_i}^2} \tag{4}$$

where V is any value and p_i is each parameters.

All spread values are calculated my finding the mean of a given set of data. It's error is calculated using the equation for the typical standard error of means.

III. APPARATUS AND PROCEDURE

This experiment involved aiming a laser at a rotating mirror, which was sent some amount of passes across Carnegie Building, Bates College, before returning to the rotated mirror and reflected into a telescope. We measured displacements between different frequencies of rotation of the mirror for three different distances, or passes through Carnegie. Specifically, we took 10 observations of changes in frequency and displacement of the laser for 1, 2, and 3 passes through Carnegie.

All measurements of the lasers displacement were done using a telescope. The telescope gave a viewpoint where the reflected laser's displacement could be measured. The telescope would be aligned for a low frequency of rotation and again when the rotations frequency was increased. This measured the change in the displacement of the laser, d. To measure the uncertainty in each measurement we considered the difference between the lowest and highest possible points one might consider the mean.

The frequency of the rotation of the mirror was measured with an electronic device. The frequency could also be increased or decreased in accordance with the voltage we supplied to it via an adjustable knob. Frequencies were varied as much as possible as so to decrease relative uncertainty in the laser displacement. Uncertainties came from the oscillating readings of frequency.

The length the laser traveled, L, was measured with a combination of using a meter stick for measuring the distance between close mirrors, and counting tiles for measuring the distance of the basement of Carnegie, Bates

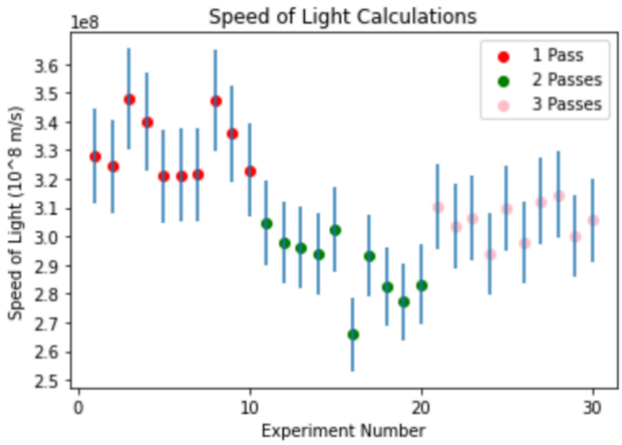

FIG. 4: The calculated speed of light calculations from each experiment.

College. Uncertainties came from measurements using a meter stick and a non-integer amount of tiles.

The length of an object of a known size was measured through the telescope used from the the other side of the hallway of Carnegie. M, the magnification, was measured by dividing the object's size in the telescope by its measured size. S_0. The distance from the telescope's lens to the object, was measured by counting tiles. The object measured was a small mirror. Uncertainties in these measurements also came from using the meter stick, measuring the image size in the microscope and the non-integer amount of tiles across the hallway we measured in Carnegie.

IV. OBSERVATIONS AND ANALYSIS

The experiment gave 30 different speed of light calculations. These are shown in Figure 4.

This data was used to calculate a weighted average of these values based on the errors in each observation, in accordance with Eq 4 and using the best value of each parameter. All intermediate calculations of values in this experiment use the weighted average method as well. In this experiment, the spread calculated value for the speed of light is done by taking the spread between each pass group, groups where each speed of light calculation uses the same number of passes through Carnegie Basement in Bates College, and then getting a weighted average of those values.

For robustness, these values were checked for outliers, as it could be easy to miscount the number of rotations. This was done with calculating z-scores and eliminating observations associated with z-scores above those allowed in accordance with Chauvenet's Criterion Table, which was 2.394. Chauvenet's criterion was applied to the data set as a whole, not each pass group.

In addition, when calculating the spread value for the speed of light, it became clear that each of these spreads between passes should be corrected as they are t-statistics. This t-statistic correction for the error was 5.9%.

To give more insight on this experiment, specifically what our largest sources of error were, the proportion of error coming from our estimated focal length, traveled length, displacement, and frequency were all calculated. The largest proportion of error came from the error in the focal length with an average proportion of 0.9132.

V. CONCLUSIONS AND OUTLOOK

The weighted average value, weighted by errors, for the speed of light is $c = (3.06 \pm 0.03) * 10^8 \frac{m}{s}$. The spread calculated value for the speed of light is $c = (3.08 \pm 0.02) * 10^8 \frac{m}{s}$, which was the same value whether accounting for a t-statistic correction or not. Using Chauvenet's Criterion, there were no outliers in the data. These values are pretty close to each other and the true value of $c = 3.00 * 10^8 m/s$. In addition, the t-statistic correction was not significant enough to change the value in our error for the spread calculated speed of light.

It appears that the same natural phenomenon is occurring within pass groups and in the whole experiment. There were however, some clear differences between the pass groups. Specifically the first pass group had values higher than the rend and the second pass group had values lower than the rest. This could be due to the fact that our measurements for the lengths between mirrors or the length of the Carnegie Basement could have be slightly biased. This could also be due to the fact that whoever measured the second pass might have been more conservative with turning the microscope knob, and therefore the displacement values, than the person who measured the first pass displacements.

The error in each c value is nearly the same. The average proportion of error came from the focal length uncertainty, with an average fractional error of 0.9132, across all experiments. Though this might seem surprising at first glance, our calculated value for the focal length was $0.410 \pm .02m$, which is about one part in 20, which is relatively large. This explains why errors did not change much between pass groups, as the focal length uncertainty remained the same throughout all experiments. If we had the opportunity to repeat this experiment, we would measure a larger object at a farther distance to minimize the errors of our measurements used to calculate the focal length.

Acknowledgments

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[1] H. Kushwaha and P. B. Adhikaria, Measurement **5** (2020).
[2] D. de la Seconde, Metrologia **33**, 99 (1996).