TEK 5010 MAS

Lecture 6: Task allocation

Exercise: System dynamics

Question 1

a) Could you model and explain stimuli response function $T_{\theta_i}(s)$?

If we have one stimuli s and one type of worker with stimuli threshold θ, we can model the respons by using the threshold model $T_\theta(s)$, where T is the probability of the worker engaging in work for a stimuli level s.

* Model I: Biological model

$$T_\theta(s) = 1 - e^{-s/\theta}$$

Hard to work with analytically

* Model II: Approximation

$$T_\theta(s) = \frac{s^n}{s^n + \theta^n}$$

where s is stimuli
θ is threshold
n is steepness of threshold

$s \ll \theta$ low probability of doing task

$s \gg \theta$ high probability of doing task

$s \approx \theta$ 50/50% probability of doing or not doing task

⇒ $n=2$ gives nice differential equations possible to solve analytically.

b) Could you explain the variables used and describe the dynamics of the system?

We have two types of workers with different response thresholds θ_1 and θ_2 reacting to one type of stimulus s.

* This dynamic could be modeled by the coupled differential equations for $i = 1, 2$ describing the transition dynamics (in continuous-time)

$$\underbrace{\frac{dx_i}{dt}}_{\substack{\text{change in} \\ \text{active workers}}} = \underbrace{T_{\theta_i}(s)(1 - x_i)}_{\substack{\text{Inactive} \\ \text{workers} \\ \text{recruited}}} - \underbrace{p x_i}_{\substack{\text{Retired} \\ \text{workers}}}$$

where $x_i = \frac{N_i}{n_i}$ is fraction of workers type i doing task T

n_i is number of workers of type i, i.e. $N = \sum_i n_i$

N_i is number of workers of type i engaged in task T

p is probability of an agent of type i gives up task T

* The stimulus dynamics (cont-time)

$$\frac{ds}{dt} = \delta - \alpha \frac{N_1 + N_2}{N}$$

where s is stimuli at time t

δ is the increase in stimuli in time

α is a scale factor measuring the efficiency of task performance

$N_1 + N_2$ is number of active agents

c) Could you model the average fraction of workers x, as a function of the fraction f of workers of type 1 in the population using parameters
$\theta_1 = 2$, $\theta_2 = 6$, $p = 0{,}2$, $\delta = 1$ and $\alpha = 3$

We get that worker 1 is the specialist $\theta_1 << \theta_2$

d) What happens if $\alpha \approx \delta$?
And what happens when $\alpha >> \delta$?

* When $\alpha \approx \delta$ the efficiency of doing the task gets reduced for both workers, meaning that both types of workers have to devote more workers into task in order to keep stimuli low.

* When $\alpha \gg \delta$ the efficiency of doing task is high. Most of the work is done by workers of type 1, and when α is sufficiently high only a small fraction of the total population need to participate in the task

* What about the explicit modelling of worker type 2? (optional)

$$\text{I}: \frac{N_1 + N_2}{N} = f x_1 + (1-f) x_2$$

$$\text{II}: \partial s = \delta - \alpha \cdot \frac{N_1 + N_2}{N}$$

I and II gives in equilibrium

$$\partial s = \delta - \alpha f x_1 - \alpha (1-f) x_2 = 0$$

Giving

$$\Rightarrow \quad x_2 = \frac{\delta - \alpha f x_1}{\alpha(1-f)}$$