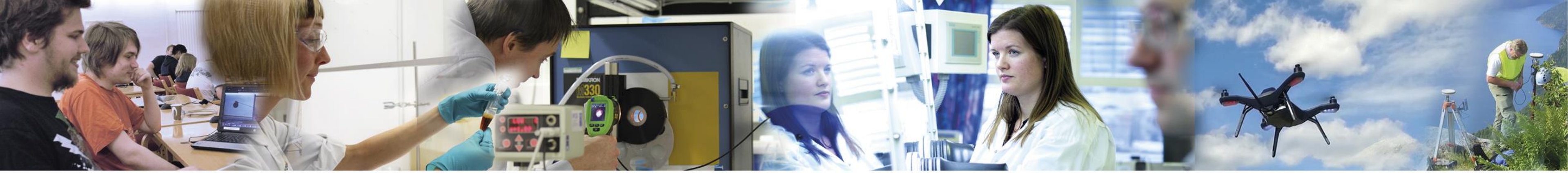

UiO : Department of Technology Systems
University of Oslo

TEK5010/9010 - Multiagent systems 2023
Lecture 12

Bargaining

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Highlights lecture 12 – Bargaining*

- Bargaining terminology
- Bargaining for resource division
  - Ultimatum/dictator game
- Bargaining for task allocation
  - Risk
- Bargaining for resource allocation

*Wooldridge, 2009: chapter 15

Bargaining

Bargaining, or negotiation, is to reach agreements, especially in the case of conflicting goals or preferences.

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Bargaining, or negotiation, is to reach agreements, especially in the case of conflicting goals or preferences.

Today:

- Experiments in the first part of the lecture
- Last part of lecture with discussion of results

Next week: Arguing and review of exercises on auctions

Bargaining for resource division

So we are going to arrange a set of experiments…

1. First, ultimatum games (20 min)
2. Then, dictator games (10 min)
3. Finally, risk profile (15 min)

⇒ Prepare results in break 1 and discuss findings in lecture.

Bargaining

How can we formalize and implement bargaining and negotiation in multi-agent systems?

This is the domain of classic game theory.

How well is empirical results described by theory?

This is the domain of behavioural game theory.

Ultimatum game

Dividing a pie in $x \cdot pie$ and $(1 - x) \cdot pie$ between two players:

1. Player 1 makes an offer $\{(x, 1 - x); 0 \leq x \leq 1\}$.
2. Player 2 can decide to accept or reject (both players get 0 in payoff).

Dictator game

Dividing a pie in $x \cdot pie$ and $(1 - x) \cdot pie$ between two players:

1. Player 1 makes an offer $\{(x, 1 - x); 0 \leq x \leq 1\}$.
2. Player 2 must accept offer.

Bargaining terminology

Components in a negotiation setting:

1. A negotiation set
   The possible proposals that agents can make.
2. A protocol
   The legal proposals that agents can make, as a function of prior negotiation history.

Bargaining terminology

Components in a negotiation setting:

3. A collection of strategies
   Strategies will determine what proposals agents will make.
   One strategy for each agent, possibly ‘private’ information.
4. Rules for determining when a deal has been struck and what this agreement deal is.

Bargaining terminology

Properties of bargaining:

1. Single-issue negotiation scenario
   e.g. two agents, buyer and seller, negotiating the price of one good.
   – In this case the preferences of the agents are symmetric, meaning that a preferred deal for one of the agents is less preferred by the other agent, and conversely.
   – Also, in this case, it is easy to see what represents a concession: buyer must concede by increasing proposed price and seller must concede by decreasing price.

Bargaining terminology

Properties of bargaining:

2. Multi-issue negotiation scenario
   e.g. two agents, buyer and seller, negotiating the price of one good based on multiple attributes of the goods.
   - In this case, it is not so clear what constitutes a concession.
   - If there are $v_1, \ldots, v_N$ variables of M values, then there are M^N possible deals.
   - Also, if it is unclear what the issues are, it can be very hard to calculate the possible deals.

Bargaining terminology

Properties of bargaining:

1. One-to-one negotiations
   e.g. two agents negotation the price of a good.
2. Many-to-one negotiations
   Often treated as a number of concurrent one-to-one negotiations.
3. Many-to-many negotiations
   Many agents negotiate with many other agents simultaneously. If there are N agents then there might be $N(N-1)/2$ negotiation threads.

Bargaining for resource division

The alternating offer bargaining model [Kraus, 2001; Osborne and Rubinstein, 1990].

For simplicity of analysis we assume negotations to be:

1. single-issue,
2. symmetric
3. one-to-one

Bargaining for resource division

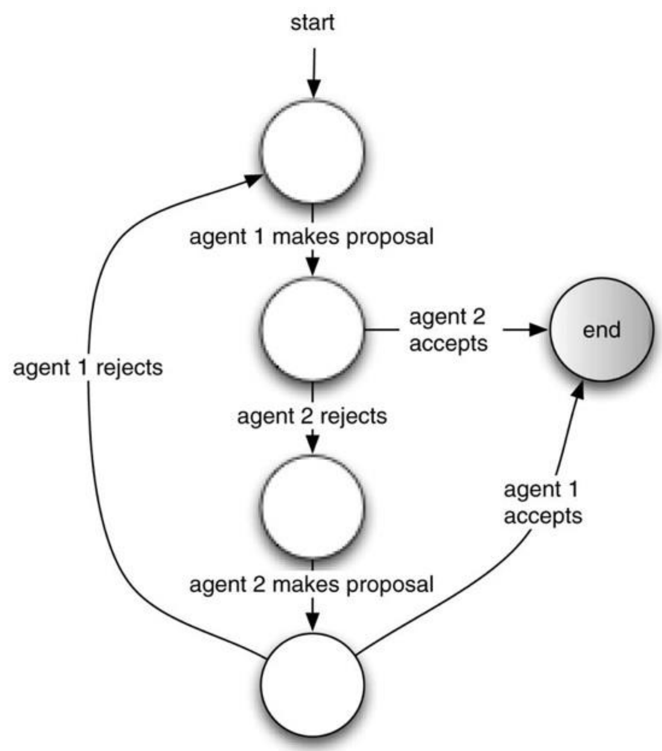


Image: Figure 15.1, Wooldridge 2009

Bargaining for resource division

1. One-to-one negotiations, $Ag = \{1,2\}$
2. Sequences of rounds indexed by $t = \{1,2, \dots, m\}$
3. Alternating proposals $x_i^{t=0}, x_j^{t=1}, \dots, x_j^{t=m}$ where i, j are agents and t is time.
4. Proposals can be rejected (R) or accepted (A) by opponent.
5. Unless proposal is accepted the bargaining could go on forever.

Bargaining for resource division

Conflict deal:

A negotiation without an agreement is assigned the conflict deal Θ.

Bargaining for resource division

Assumptions in the alternating offer protocol:

1. The conflict deal Θ is the worst outcome.
2. Agents seek to maximize utility.

Bargaining for resource division

Ultimatum game

Dividing a pie in $x \cdot pie$ and $(1 - x) \cdot pie$ between two players:

1. Player 1 makes an offer $\{(x, 1 - x); 0 \leq x \leq 1\}$
2. Player 2 can decide to accept or reject (Θ is implemented)

Bargaining for resource division

Ultimatum game:

1. One-shot ultimatum game
   Nash equilibrium is $(1,0)$. A rational player 2 would accept all proposals over 0.

Bargaining for resource division

Ultimatum game:

2. Two-shot ulimatum game
   Nash equilibrium is $(0,1)$. Player 2 is indifferent to player 1's offer and a rational player 1 would accept all player 2's proposals over Θ.

Bargaining for resource division

Ultimatum game:

3. Unlimited rounds ulimatum game
   Nash equilibrium is $(1{,}0)$, conditional that player 1 can convince player 2 that he will reject all other offers.

   Player 1 benefits by explicitly showing commitment to his strategy.

Bargaining for resource division

Impatient players, i.e. time is valuable:

For any outcome x and times $t_1 < t_2$ agents prefer outcome x at t_1 over outcome x at t_2.

This is modelled by a discount factor$0 \leq \delta \leq 1$ meaning that

$\delta_{i,t} = 0$ is impatient player

$\delta_{i,t} = 1$ is patient player

$\delta_{i,0} = 1$ by default

Bargaining for resource division

Impatient players, i.e. time is valuable:

$$t_0 : \left(x, (1 - x)\right)$$
$$t_1 : \left(\delta_1 \ (1 - x), \delta_2 \ x\right)$$
$$\vdots$$
$$t_m : \left(\delta_1 \ \delta_1 \ \ldots \delta_1 \ x, \delta_2 \ \delta_2 \ \ldots \delta_2 \ (1 - x)\right)$$

Bargaining for resource division

Impatient players, i.e. time is valuable:

$$
\begin{gathered}
t_0\colon \left(\delta_1^0 x, \delta_2^0(1-x)\right) \\
t_1\colon \left(\delta_1^1(1-x), \delta_2^1 x\right) \\
t_2\colon \left(\delta_1^2 x, \delta_2^2(1-x)\right) \\
\vdots \\
t_m\colon \left(\delta_1^m x, \delta_2^m(1-x)\right)
\end{gathered}
$$

Bargaining for resource division

Ultimatum game with time discount:

1. One-shot ultimatum game
   Nash equilibrium is $(1,0)$, since $\delta_i^0 = 1$ always.

$$t_0 \colon \left(\delta_1^0 x, \delta_2^0 (1-x)\right) = \left(x, (1-x)\right)$$

Bargaining for resource division

Ultimatum game with time discount:

2. Two-shot ultimatum game
   Nash equilibrium is $(1-\delta_2, \delta_2)$

$$t_0\colon \left(x_{t0}, (1-x_{t0})\right)$$

$$t_1\colon \left(\delta_1\ (1-x_{t1}), \delta_2\ x_{t1}\right)$$

Player 1 must make an offer that player 2 accepts in first round, otherwise player 2 gets nothing.

Bargaining for resource division

Ultimatum game with time discount:

2. Two-shot ultimatum game
   Nash equilibrium is $(1 - \delta_2, \delta_2)$

   $(1 - x_{t0}) \geq \delta_2^1 x_{t1}$, assuming $x_{t1} = 1$

   $1 - \delta_2 \geq x_{t0}$

Bargaining for resource division

Ultimatum game with time discount:

3. Unlimited rounds ultimatum game
   Nash equilibrium is reached in first time step giving

$$\left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$

Bargaining for resource division

Negotiation decision function

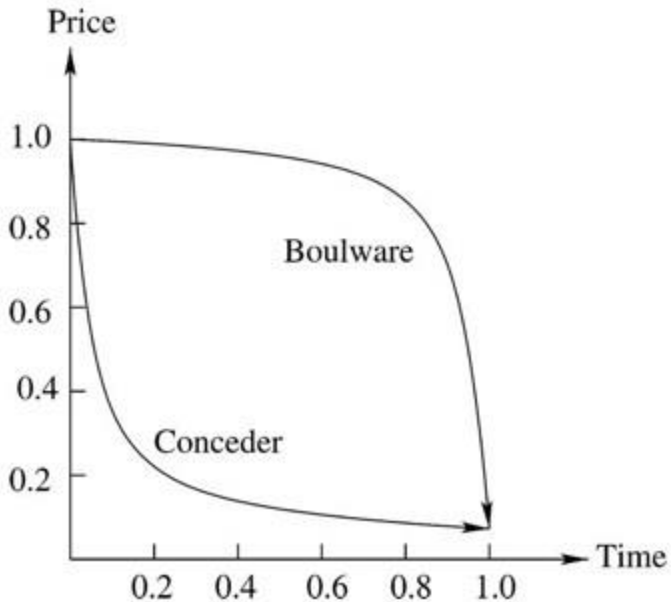
Non-strategic approach where agent's strategy is a preplanned time-dependent function [Faratin et *al.*, 1998].

Bargaining for resource division

Negotiation decision function

1. Boulware strategy
   The agent initially does not decrease price much, but as deadline of negotiations approaches concessions increases until reservation price is met.
2. Conceder strategy
   The agent makes most of its concessions early and does not concede much as deadline approaches.

Bargaining for resource division



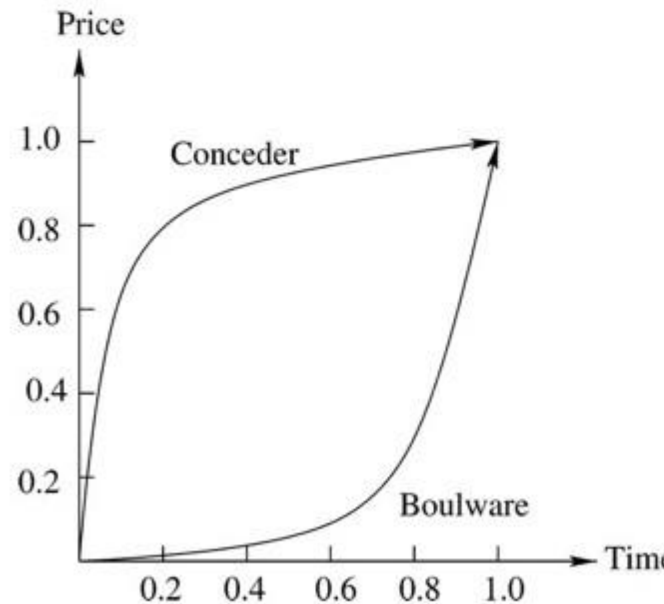


Image: Figure 15.2 and 15.3, Wooldridge 2009

Bargaining for resource division

Boulware strategy might give you a better deal but run the risk of delaying the deal being struck at the same price as conceder-conceder.

Bargaining for task allocation

Negotiations in task-oriented domains (TOD) [Rosenschein and Zlotkin, 1994].

Agents may benefit from reorganizing the distribution of tasks among themselves.

Bargaining for task allocation

Task-oriented domains (TOD) [Rosenschein and Zlotkin, 1994]

$$\langle T, Ag, c\rangle$$

where T is the set of all possible tasks (finite)

$Ag = \{1, \ldots, N\}$ are agents in the negotiation

$c: 2^T \rightarrow \mathbb{R}_+$ is the cost function defining the cost of executing each subset of tasks

a. Monotonic. If $T_1 \subseteq T_2 \subseteq C$ then $\mathrm{c}(T_1) \leq c(T_2)$

b. Cost of doing nothing is zero $c(\emptyset) = 0$

Bargaining for task allocation

An encounter

A TOD where agents are assigned tasks to perform the set $T = \langle T_1, T_2, \ldots, T_N \rangle$ over $i \in Ag$ and $T_i \subseteq T$.

Bargaining for task allocation

A deal

$$\delta = \langle D_1, D_2 \rangle$$

where $D_1 \cup D_2 = T_1 \cup T_2$ is a reorganization of original tasks
$utility_i(\delta) = c(T_i) - cost_i(\delta)$ is the marginal utility of the deal to agent i

Bargaining for task allocation

The negotiation set:

1. The conflict deal
   $\Theta = \langle T_1, T_2 \rangle$ is the original task allocation
2. Individual rationality
   $\delta \succcurlyeq \Theta$
3. Pareto optimal
   $\delta' \succ \delta$

Bargaining for task allocation

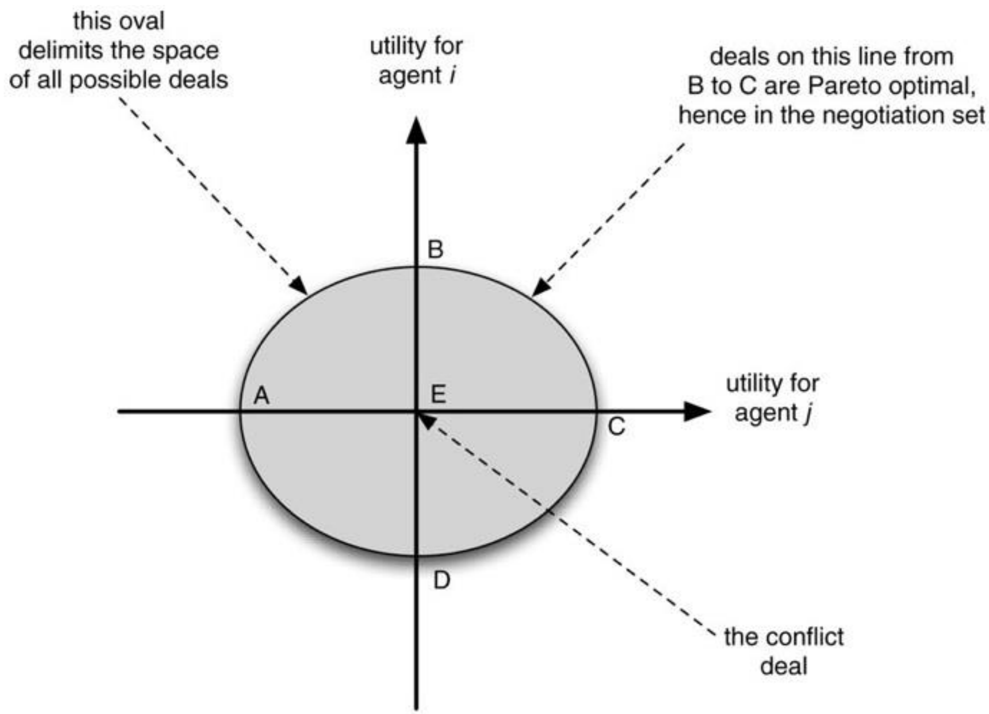


Image: Figure 15.4, Wooldridge 2009

Bargaining for task allocation

How should the agents reach an agreement? [Rosenschein and Zlotkin, 1994]

- The monotonic concession protocol
- The Zeuthen strategy

Bargaining for task allocation

The monotonic concession protocol :

1. Agents simultaneously propose deals from negotiation set.
2. A deal is struck when
   $$utility_i(\delta_j) \geq utility_i(\delta_i) \text{ or } utility_j(\delta_i) \geq utility_j(\delta_j)$$
   Chose one at random if both proposals are accepted
3. If no deal then agents propose new deal with requirement
   $utility_i(\delta_j') \geq utility_i(\delta_j)$ and vice versa
4. If no concessions are made the conflict deal is employed.

Bargaining for task allocation

The monotonic concession protocol :

Properties

1. Verifiable
2. Guaranteed to end
3. Computational complexity of deals are $\mathcal{O}\left(2^{|T|}\right)$

Bargaining for task allocation

The Zeuthen strategy

How to determine:

1. The first proposal of the Agents?
2. Which agent should concede?
3. How much should the agents concede?

Bargaining for task allocation

The Zeuthen strategy

1. Agents first proposal should be the optimal deal.

Bargaining for task allocation

The Zeuthen strategy

2. Which agent should concede is dependent on risk:

$$risk_i^t = \frac{\text{utility } i \text{ loses by conceding and accepting } j'\text{s offer}}{\text{utility } i \text{ loses by not conceding and causing conflict}}$$

Measures an agents willingness to risk a conflict.
Low values of risk should mean high willingness to concede (more to lose from conflict).

Bargaining for task allocation

The Zeuthen strategy

3. How much should be conceded?

   Agent of least risk should concede enough to make the other agent concede in next round (too much waste utility and too little is inefficient).

   Flip a coin when equal risk.

Bargaining for task allocation

The Zeuthen strategy

Properties:

1. Will terminate, though not guaranteed to avoid conflict.
2. Outcome is PO though not guaranteed to be SO.
3. Individual rational if agreement is reached.
4. No arbiter required to monitor negotiations.
5. Strategy outcome is Nash, i.e. if one player plays strategy then all other players should play Zeuthen as well.

Bargaining for task allocation

The Zeuthen strategy

Conclusion:

This strategy is widely used. A natural and simple approach, though with high computational complexity limiting implementations to one-to-one negotiations.

Bargaining for resource allocation

Reallocate resources among agents in order to increase mutual benefit [Sandholm, 1998]

This is many-to-many negotiations of exchanging already endowed goods (in contrast to the one-to-many auctions in chapter 14).

Bargaining for resource allocation

A resource allocation setting

$$\langle Ag, \mathcal{Z}, v_1, \ldots, v_N \rangle$$

where $Ag = \{1, \ldots, N\}$ are agents
$\mathcal{Z} = \{z_1, z_2, \ldots, z_M\}$ is the set of M possible resources
$v_i: 2^{\mathcal{Z}} \rightarrow \mathbb{R}$ is the valuation function for each agent i

Bargaining for resource allocation

A deal is a triplet

$$\langle Z, Z', \bar{p} \rangle$$

where $Z = \{Z_1, Z_2, \ldots, Z_N\}$ is current allocation of resources

$Z' = \{Z'_1, Z'_2, \ldots, Z'_N\}$ is proposed allocation of resources

$\bar{p} = \{p_1, p_2, \ldots, p_N\}$ is side payment with the requirement

$$\sum_{i=1}^{N} p_i = 0$$

Bargaining for resource allocation

Individual rationality

$$v_i(Z_i') - p_i > v_i\left(Z_i\right)$$

Meaning that agent i only accept deals strictly increasing utility.

Bargaining for resource allocation

Protocol for resource allocation:

1. Initial allocation Z^0 is implemented and defined as current.
2. Any agent (randomly selected) can make a proposal $\langle Z, Z', \bar{p} \rangle$ where Z is the current allocation.
3. If all agents agree to this deal (individual rationality) and termination criteria is met (PO or SO), then negotiations terminate and Z' is implemented with a side payment $\bar{p}$.

Bargaining for resource allocation

Protocol for resource allocation:

4. If all agents agree to this deal (individual rationality) but termination criteria is not met, then current allocation is set to Z' and step 2 is repeated.
5. If any agent object to this deal (individual rationality), then current allocation remains the same and step 2 is repeated.

Bargaining for resource allocation

Protocol for resource allocation:

A Pareto optimal outcome is guaranteed to be obtained if each deal is individually rational [Sandholm, 1994].

However, the time complexity of reaching PO is exponential in agents and resources, making it NP-hard.

Bargaining for resource allocation

Protocol for resource allocation:

One-contracts

Involves moving only one resource and only one side payment. Possible to reach maximal social welfare but not without risk of violating individual rationality.

Bargaining for resource allocation

Protocol for resource allocation:

Cluster-contracts

Transfer of any number of resources greater than 1, but with the restriction of only from one agent to another agent. There are optimal allocations not reachable in this type of contracts.

Bargaining for resource allocation

Protocol for resource allocation:

Swap-contracts

Resources are swapped between agents with side payments. Also, there are optimal allocations not reachable in this type of contracts.

Bargaining for resource allocation

Protocol for resource allocation:

Multiagent-contracts

Involves at least 3 agents transferring a single resource each. Also, there are optimal allocations not reachable in this type of contracts.

Summary lecture 12 – Bargaining*

- Bargaining terminology
- Bargaining for resource division
  - Ultimatum/dictator game
- Bargaining for task allocation
  - Zeuthen strategy and risk
- Bargaining for resource allocation

*Wooldridge, 2009: chapter 15