TEK 5010 MAS

Lecture 6: Task allocation

Exercise: Deriving response threshold models

Question 1

a) What is the probability (observed over one time interval) of agent doing the task, $P_{task,1}$? What is the corresponding probability of agent not doing the task, $P_{not\ task,1}$?

$$P_{task} + P_{not\ task} = 1$$
$$P_{task,1} = S$$
$$P_{not\ task,1} = 1 - P_{task,1} = 1 - S$$

b) What is the probability of not doing the task if we sample two consecutive time intervals (which are assumed to be statistically indipendent)?. What is the corresponding probability of the agent doing the task when observed over 2 intervals?

$$P_{not\,task,2} = P_{not\,task,1} \cdot P_{not\,task\,1}$$

$$= (1-g)(1-g) = (1-g)^2$$

$$P_{task,2} = 1 - P_{not\,task\,2} = 1-(1-g)^2$$

$$= 1-(1-g)(1-g)$$
$$= 1-[1(1-g)-g(1-g)]$$
$$= 1-[1-g-g+g^2]$$
$$= 1-1+g+g-g^2$$
$$= 2g-g^2$$
$$= g(2-g) > 0 \quad \text{since } 0<g<1$$

c) Why is $P_{task,2} \neq P_{task,1}^2$ observed over 2 time intervals?

There are only one way an agent could end up not doing task over several consecutive intervals. That is to not do task in every interval, and this is fairly easy to calculate.

But there are many ways an agent could end up doing task, either in first interval or in second or in both, which is more complex to calculate. However, all these ways of ending up in doing task is given by

$$P_{task} = 1 - P_{not\,task}$$

d) What is the probability of not doing task over 3 intervals? And what is the probability of doing the task over 3 intervals?

$$P_{not\ task_3} = P_{not\ task,1} \cdot P_{not\ task,1} \cdot P_{not\ task,1}$$

$$= (1-g)(1-g)(1-g)$$

$$= (1-g)^3$$

$$P_{task,3} = 1 - P_{not\ task\ 3}$$

$$= 1 - (1-g)^3$$

$$= 1 - [(1-g)^2(1-g)]$$

$$= 1 - [(1-g)^2 - g(1-g)^2]$$

$$= 1 - [1 - 2g + g^2 - g(1 - 2g + g^2)]$$

$$= 1 - [1 - 2g + g^2 - g + 2g^2 - g^3]$$

$$= \cancel{1} - \cancel{1} + 2g - g^2 + g - 2g^2 + g^3$$

$$= 3g - 3g + g^3$$

e) What does $P_{task,N}$ and $P_{not\ task,N}$ look like for $N$ intervals?

Hint, use the binomial expansion.

$$(1+x)^n = \sum_{k=0}^{n} \frac{k}{(n-k)!\,k!} x^k \qquad 0! = 1$$

$$= \sum_{k=0} \binom{n}{k} x^k$$

$$= 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

which gives for $P_{\text{not task},N}$

$$P_{\text{not task}\,N} = P_{\text{not task},1}^{\;N}$$

$$= (1-g)^N$$

and for $P_{\text{task}\,N}$

$$P_{\text{task}\,N} = 1 - (1-g)^N$$

$$= 1 - \left[1 + N(-g) + \frac{N(N-1)}{2!}(-g)^2 + \frac{N(N-1)(N-2)}{3!}(-g)^3 + \dots\right.$$

$$P_{task,N} = Ng - \frac{N(N-1)}{2!}g^2 + \frac{N(N-1)(N-2)}{3!}g^3 + \ldots$$

If $g$ is small and/or $N$ is a low number, then

$$P_{task,N} \approx Ng$$

b) Could you reformulate $P_{task,N}$ into an exponential form?

Using the expression $x = e^{\ln x}$ we can rewrite $P_{task}$

$$P_{task,N} = 1-(1-g)^N = 1 - e^{\ln(1-g)^N}$$

$$= 1 - e^{N\ln(1-g)}$$

$$= 1 - e^{-s/\theta}$$

where $s = N$ is stimuli signal

$\theta = -\frac{1}{\ln(1-g)}$ is threshold

Now the stimuli signal is given by how long (N intervals) the agent is exposed to the signal as well as the threshold given by $\xi$.

g) Plot $P_{\text{task}}$ as a function of N (or s) for the 3 different models given in the lectures, using $\xi = 0.01$ $N \in [1, 1000]$

Model 1 $\quad T_\theta(s) = \dfrac{s^2}{s^2 + \theta^2}$

Model 2a $\quad T_\theta(s) = 1 - e^{-s/\theta}$ $\quad s = N$ $\quad \theta = -\dfrac{1}{\ln(1-\xi)}$

Model 3a $\quad T_\theta(s) = 1 - (1-\xi)^N$