TEK 5010 MA3

Lecture 10: Cooperative game theory

Exercise: Cooperative games 1

Question 1

a) Calculate the values of the
following coalitions...

Marginal contribution net is given

$$b \rightarrow 4 \qquad \text{rule 1}$$
$$c \rightarrow 6 \qquad \text{rule 2}$$
$$a \wedge b \rightarrow 7 \qquad \text{rule 3}$$
$$v \wedge c \rightarrow 3 \qquad \text{rule 4}$$

Find the v's (the characteristic function)

$v(\emptyset) = 0$ (empty coalition)
$v(\{a\}) = 0$ (no rule apply)
$v(\{b\}) = 4$ (rule 1)
$v(\{c\}) > 6$ (rule 2)
$v(\{a,b\}) = 4+7 = 11$ (Rule 1 & 3)
$v(\{a,c\}) = 6$ (Rule 2)
$v(\{b,c\}) = 4+6+3 = 13$ (Rule 1, 2 & 4)
$v(\{a,b,c\}) = 4+6+7+3 = 20$ (Rule 1,2,3,4)

b) Draw the weighted graph
representing this game
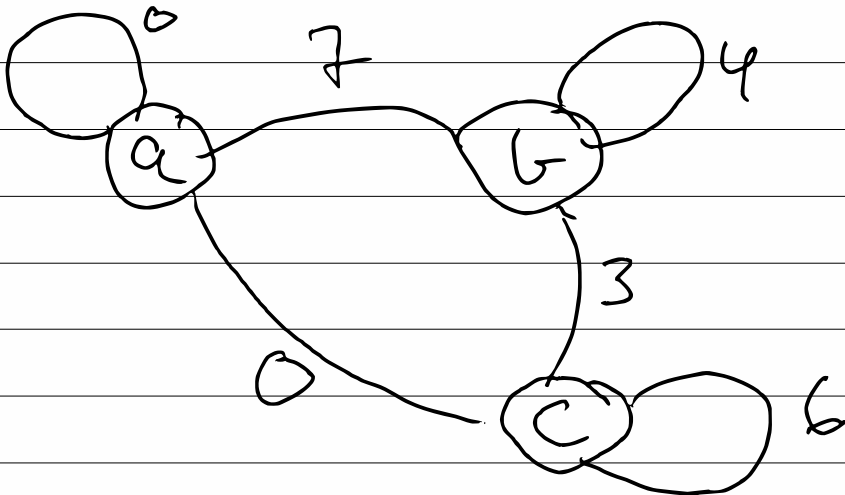
c) Is this game stable?

This game is stable if
we have a non-empty core

$$a \quad b \quad c$$

| level 1 | 0 | 4 | 6 | singelton C |

level 2   11   13

level 3   20   Grand C

lets look at the grand
coalition first

	a	b	c	
a=0	O	20	O	
	.	.	.	} c objects
	O	15	5	
	O	14	6	
	.	.	.	
	.	.	.	one is
	1	.	.	non-empty
	O	11	9	
	O	10	10	
	.	.	.	} ab objects
	O	O	20	

	a	b	c	
a=1	1	19	O	c objects
	1	13	6	} core
	1	10	9	
	1	O	19	ab objects

There exist some non-empty
core, i.e. the game is stable

d) Calculate the Shapley value
for each player in this game.

$$sh_i = \frac{1}{|Ag|!} \sum_{o \in \Pi(Ag)} M_i(G_i(o))$$

where $\Pi(Ag)$ is the set of all possible
orderings of coalition C
o is any ordering of a
coalition
$$M_i(C) = v(C \cup \{i\}) - v(C)$$
given that $C \subseteq Ag \setminus \{i\}$,
i.e. the marginal contribution
of agent i to C

Two ways of calculating Shapley
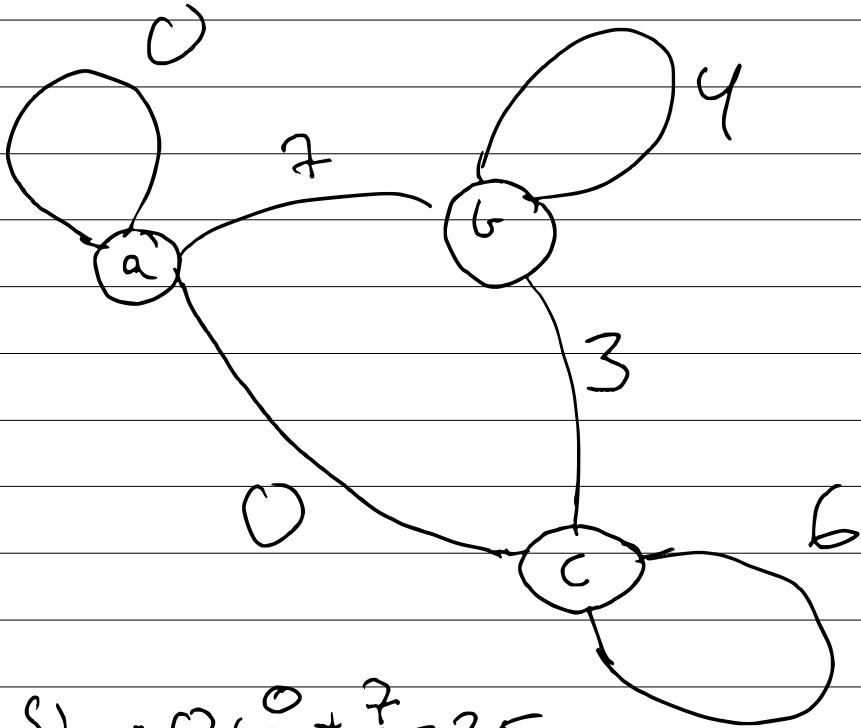1) Enumeration
2) from the induced subgraph

✳ Enumeration

Different ordering (permutations)	Marginal contribution of players		
	a	b	c
a b c	0	11	9
a c b	0	14	6
b a c	7	4	9
b c a	7	4	9
c a b	0	14	6
c b a	7	7	6
$Sh_i \Rightarrow$	$21/6 = 3.5$	$54/6 = 9$	$45/6 = 7.5$

* from the induced subgraph

$$Sh_a = 0 + \frac{0}{2} + \frac{7}{2} = 3.5$$

$$Sh_b = 4 + \frac{7}{2} + \frac{3}{2} = 9$$

$$Sh_c = 6 + \frac{0}{2} + \frac{3}{2} = 7.5$$