TEK5010 MAS

Lecture 8: Non-cooperative game theory

Exercise: Symmetric games 1

Question 1

a) For each of these payoff matrices identify
   i) Pure strategy Nash eq.
   ii) P to optimal outcomes
   iii) Maximal social outcome

* Pure strategy Nash eq:
  Two strategies s_i and s_j of agent i and j are Nash eq
  1) if player i plays s_i, player j can do no better than s_j
  2) if player j plays s_j, player i can do no better than s_i.

PD

i \ j	D	C
D	2,2*	4,1
C	1,4	3,3

⇒ (D,D) is Nash in PD

MP

i \ j	H	T
H	1,-1	-1,1
T	-1,1	1,-1

⇒ No Nash in MP

GC

i \ j	D	C
D	1,1	4,2*
C	2,4*	3,3

⇒ (D,C) and (C,D) are Nash eq in GC

PD

i \ j	D	C
D	2,2*	4,1
C	1,4	3,3

⇒ (D,D) is Nash in PD

MP

i \ j	H	T
H	1,-1	-1,1
T	-1,1	1,-1

⇒ No Nash in MP

GC

i \ j	D	C
D	1,1	4,2*
C	2,4*	3,3

⇒ (D,C) and (C,D) are Nash eq in G

* Pareto optimal outcome is the solution where no improvement is possible without making someone else worse off.

PD

i \ j	D	C
D	2,2	4,1+
C	1,4+	3,3+

⇒ (D,C), (C,D) and (C,C) are PO in PD

MP

i \ j	H	T
H	1,-1+	-1,1+
T	-1,1+	1,-1+

⇒ All outcomes are PO (in the sense of strong dominance) in MP

GC

i \ j	D	C
D	1,1	4,2+
C	2,4+	3,3+

⇒ (C,D), (D,C) and (C,C) are PO in GC

* Social welfare optimum

$$sw(w_i) = \sum_{j \in A_j} u_j(w_i)$$

PD

i \ j	D	C
D	2,2 4	4,1 5
C	1,4 5	3,3 6

⇒ (C,C) is SO in PO

MP

	H	T
H	1, -1 ⊙	-1, 1 ⊙
T	-1, 1 ⊙	1, -1 ⊙

⇒ All out comes are SO in MP

GC

	D	C
D	1, 1 ②	4, 2 ⑤
C	2, 4 ⑤	3, 3 ⑤

⇒ (D,C), (C,D) and (C,C) are SO in GC

Let's sum up

PD

i \ j	D	C
D	*	+
C	+	+□

* Nash
\+ PO
□ SO

⇒ Rational outcome is not PO or SO, why is SO stable in many real world PDs?

MP

	H	T
H	+□	+□
T	+□	+□

⇒ No rational outcome, all outcomes are PO and SO. Only solution is in mixed strategy Nash eq, guaranteed by Nash's theorem.

(GC)

i \ j	D	C
D		✱ + □
C	✱ + □	+ □

⇒ There are multiple rational outcomes that are also SO and PO. Which one to pick?

b) "Program equilibria make cooperation possible in one-shot PD". Explain and critically assess this statement.

Program equilibrium:

Basic idea is to compare strategies (programs) before conditional action is taken by moderator

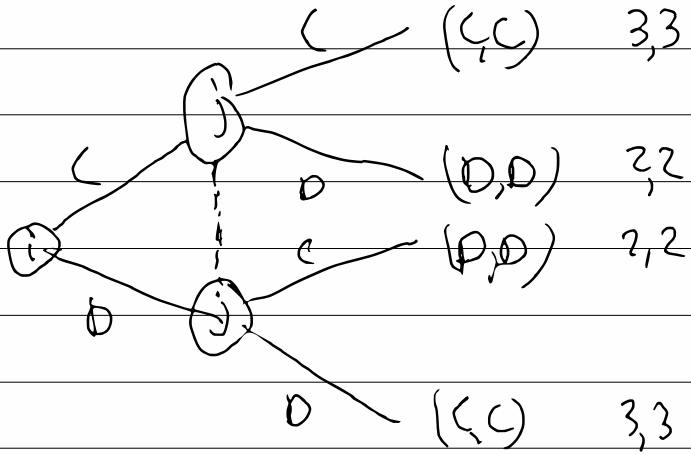
"I will cooperate if you will"

if $p_1 = = p_2$ p_1 = program 1 (string 1)
do (C, C) p_2 = program 2 (string 2)
else
do (D, D)
end

i \ j	D	C
D	2, 2	4, 1
C	1, 4	3, 3

Max Π_i and Π_j

Lets view the possible payoff Π_i using extensive form



⇒ (C, C) or (D, D) are rational strategies