Transfer nets exercise

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A transfer net is a a Petri net extended with a set Tr of transfer arcs of the form (p, t, q). In a transfer net, the enabling condition is exactly the same as in plain Petri Nets. However, the firing rule of transitions is changed so that when a transition t is fired:

- First, tokens are removed from the preconditions of t.
- Then, if there is a transfer arc $(p, t, q) \in Tr$, then all the tokens in p are transferred to q.
- Finally, tokens are added to the postconditions of t.

For the sake of simplicity, we assume that for each transition $t \in T$ there is at most one transfer arc in t (i.e. one arc of the form $(p, t, q) \in Tr$).

1 Formal definition

A *net with transfer arcs* $N = (S, T, F, Tr)$ consists of two disjoint sets of places and transitions S and T, a set $F \subseteq (S \times T) \cup (T \times S)$ of arcs, and a set $Tr \subseteq S \times T \times S$ of transfer arcs where, if $(p, t, q) \in Tr$ is a transfer arc then:

1. $(p, t) \notin F$ and $(t, q) \notin F$ (i.e. transfer arcs do not overlap with regular arcs).
2. If $(p', t, q') \in Tr$, then $p' = p$ and $q' = q$ (i.e. one single transfer arc per transition at most).

A transition $t \in T$ is enabled at marking M of N if $M(s) > 0\ \forall s \in {}^\bullet t$. If t is enabled, then it can $fire$ leading to a marking M' defined as:

$$M'(s) = \begin{cases} M(s) - 1 & \text{if } s \in {}^\bullet t \setminus t^\bullet \\ M(s) + 1 & \text{if } s \in t^\bullet \setminus {}^\bullet t \\ 0 & \text{if } \exists s' \text{ s.t } (s, t, s') \in Tr \land s \neq s' \\ M(s') & \text{if } \exists s' \text{ s.t } (s', t, s) \in Tr \land s \neq s' \\ M(s) & \text{otherwise} \end{cases}$$

2 Informal discussion about its monotonicity

This Petri net variant seems to satisfy the strict version of monotonicity (i.e. if $M_1 \xrightarrow{t} M_2$ for a transition t and some markings M_1 and M_2 of the net, then t is enabled for any marking $M'_1 > M_1$ and, when fired, results in a marking $M'_2 > M_2$).

Petri nets with reset arcs fail to satisfy this monotonicity version because we could add tokens to a reset precondition of a transition and, when fired, the resulting marking would not be modified.

When it is a transfer arc instead of a reset one, these tokens would be moved to another place instead of being completely removed. It allows to satisfy $M'_2 > M_2$ but, as the added tokens to M_1 do not need to be kept there after the firing, the strongest version of monotonicity is not satisfied.