Backwards algorithm for coverability exercise

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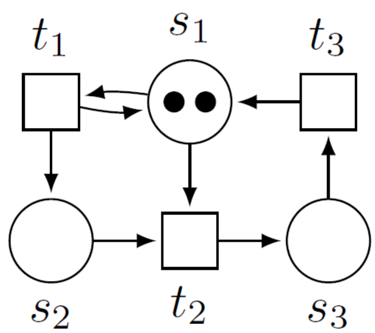
Apply the backwards reachability algorithm to the Petri net below to decide if the marking $M = (0, 0, 2)$ can be covered.



Record all intermediate steps and all the intermediate sets of markings with their finite representation of minimal elements.

Answer

Preliminaries

First of all, let us gather the minimal markings that allow us to fire "backwards" the transitions of the net:

$$R[t_1] = (1, 1, 0)$$
$$R[t_2] = (0, 0, 1)$$
$$R[t_3] = (1, 0, 0)$$

We are going to annotate with a subindex the variable m of the algorithm for specifying its corresponding iteration, so the initial one is

$$m_0 := \{(0, 0, 2)\}$$

and the sequence is computed iteratively with the following termination conditions, being i the current one in the algorithm execution:

1. Some marking from m_i is smaller or equal than M_0, which means that M can be covered.

2. The sequence stabilizes (i.e. $m_i = m_{i-1}$) and the previous condition does not hold, which means that M cannot be covered.

Note also that $M_0 = (2, 0, 0)$ is the initial marking.

Iteration i explanation

Each iteration computes

$$m_i = m_{i-1} \cup \Big(\bigcup_{t \in T} pre(R[t] \wedge m_{i-1}, t) \Big)$$

where $T = \{t_1, t_2, t_3\}$. For that, it also needs to compute $pre(R[t] \wedge m_{i-1}, t)$, which also requires $R[t] \wedge m_{i-1}$.

All these finite representations of marking sets are shown for each iteration, alongside with a small comment about its termination check.

Iteration 1

$$R[t_1] \wedge m_0 = \{(1, 1, 2)\}$$
$$R[t_2] \wedge m_0 = \{(0, 0, 2)\}$$
$$R[t_3] \wedge m_0 = \{(1, 0, 2)\}$$

$$pre(R[t_1] \wedge m_0, t_1) = \{(1, 0, 2)\}$$
$$pre(R[t_2] \wedge m_0, t_2) = \{(1, 1, 1)\}$$
$$pre(R[t_3] \wedge m_0, t_3) = \{(0, 0, 3)\}$$

$$m_1 \coloneqq \{(0, 0, 2), (1, 1, 1)\}$$

As none of $(0, 0, 2)$ and $(1, 1, 1)$ is smaller or equal than M_0 and $m_0 \neq m_1$, we proceed to the next iteration.

Iteration 2

$$R[t_1] \wedge m_1 = \{(1, 1, 1)\}$$
$$R[t_2] \wedge m_1 = \{(0, 0, 2), (1, 1, 1)\}$$
$$R[t_3] \wedge m_1 = \{(1, 0, 2), (1, 1, 1)\}$$

$$pre(R[t_1] \wedge m_1, t_1) = \{(1, 0, 1)\}$$
$$pre(R[t_2] \wedge m_1, t_2) = \{(1, 1, 1), (2, 2, 0)\}$$

$$pre(R[t_3] \wedge m_1, t_3) = \{(0,0,3),(0,1,2)\}$$

$$m_2 \coloneqq \{(0,0,2),(1,0,1),(2,2,0)\}$$

Again, as none of $(0,0,2),(1,0,1)$ and $(2,2,0)$ is smaller or equal than M_0 and $m_1 \neq m_2$, we proceed to the next iteration.

Iteration 3

$$R[t_1] \wedge m_2 = \{(1,1,1),(2,2,0)\}$$
$$R[t_2] \wedge m_2 = \{(0,0,2),(1,0,1)\}$$
$$R[t_3] \wedge m_2 = \{(1,0,1),(2,2,0)\}$$

$$pre(R[t_1] \wedge m_2, t_1) = \{(1,0,1),(2,1,0)\}$$
$$pre(R[t_2] \wedge m_2, t_2) = \{(1,1,1),(2,1,0)\}$$
$$pre(R[t_3] \wedge m_2, t_3) = \{(0,0,2),(1,2,1)\}$$

$$m_3 \coloneqq \{(0,0,2),(1,0,1),(2,1,0)\}$$

Again, as none of $(0,0,2),(1,0,1)$ and $(2,1,0)$ is smaller or equal than M_0 and $m_2 \neq m_3$, we proceed to the next iteration.

Iteration 4

$$R[t_1] \wedge m_3 = \{(1,1,1),(2,1,0)\}$$
$$R[t_2] \wedge m_3 = \{(0,0,2),(1,0,1)\}$$
$$R[t_3] \wedge m_3 = \{(1,0,1),(2,1,0)\}$$

$$pre(R[t_1] \wedge m_3, t_1) = \{(1,0,1),(2,0,0)\}$$
$$pre(R[t_2] \wedge m_3, t_2) = \{(1,1,1),(2,1,0)\}$$
$$pre(R[t_3] \wedge m_3, t_3) = \{(0,0,2),(1,1,1)\}$$

$$m_4 \coloneqq \{(0,0,2),(1,0,1),(2,0,0)\}$$

Finally, as $(2,0,0)$ in m_4 is precisely M_0, we have that M_0 belongs to $pre^*(M\uparrow)$, thus M can be covered.