Program Verification in Elixir

Master's Degree in Formal Methods and Computer Engineering

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  - Verification conditions discharged by the Z3 theorem prover
  - Compiled also to other programming languages to be executed

The Elixir programming language

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- A functional programming language that runs on the Erlang Virtual Machine

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  - Both of them show the presence of errors rather than their absence

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Provide a system similar to that of Dafny but specialized for Elixir and implemented in Elixir itself

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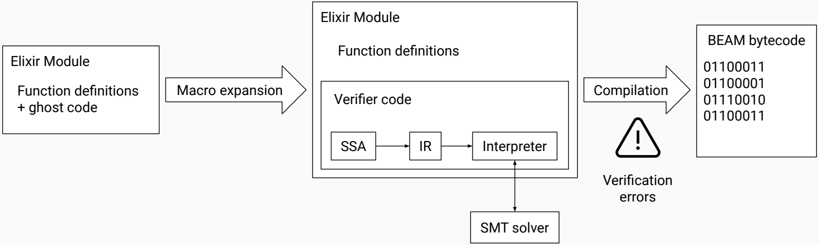

https://github.com/adrianen-ucm/verixir-project

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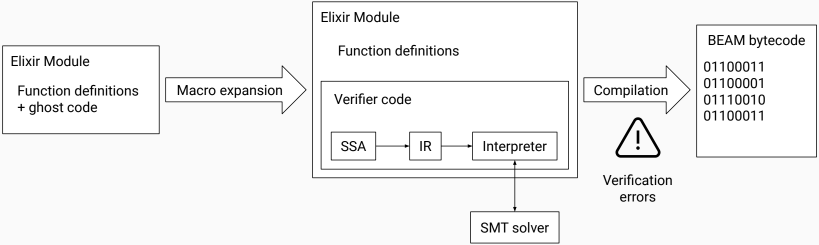

`https://github.com/adrianen-ucm/verixir-project`

Scope: only a subset of sequential Elixir for the moment, and partial verification (i.e. not verifying termination)

A valid Elixir program

```
result =
  if selector === 1 do
    1
  else
    false
  end

result =
  if selector === 1 do
    result + 1
  else
    not result
  end
```

Plan

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1. SMT solver integration in Elixir

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2. L0, a low level language close to the SMT solver

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4. L2, a high level language that models Elixir + verification code

SMT Solver Integration in Elixir

Elixir SMT-LIB binding

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

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- Different SMT solvers that implement SMT-LIB can be easily integrated

Elixir SMT-LIB binding

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

- An SMT-LIB (subset) DSL
- Different SMT solvers that implement SMT-LIB can be easily integrated
- Out-of-the-box support for Z3

Elixir SMT-LIB binding example

```
import SmtLib

with_local_conn do
  declare_const x: Int,
                y: Int

  assert !(
      (:x + 3 <= :y + 3) ~> (:x <= :y)
    )

  check_sat
end
```

The L0 language

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- The lowest level language of our verification stack

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- The lowest level language of our verification stack
- Close to the SMT solver
- Restricted SMT-LIB + control flow + failure

L0 expressions syntax

$$
\begin{array}{rcl}
\textbf{Exp}^0 \ni \epsilon & ::= & \textbf{skip} \\
& \mid & \textbf{fail} \\
& \mid & \epsilon_1 ; \epsilon_2 \\
& \mid & \textbf{local}\ \epsilon \\
& \mid & \textbf{add}\ \varphi \\
& \mid & \textbf{declare}\ x \\
& \mid & \textbf{when-unsat}\ \epsilon_1\ \textbf{do}\ \epsilon_2\ \textbf{else}\ \epsilon_3
\end{array}
$$

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& \mid & \textbf{when-unsat } \epsilon_1 \textbf{ do } \epsilon_2 \textbf{ else } \epsilon_3
\end{array}
$$

where $x \in V$ is a variable name and $\varphi \in \mathbb{F}$ is a formula with many-sorted terms $t \in \mathbb{T}$

L0 big-step operational semantics

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- (X, Φ) SMT solver state
- $\langle \epsilon,\ X,\ \Phi \rangle \Downarrow (X', \Phi')$ judgement

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$$\frac{x \notin X}{\langle \textbf{declare}\ x,\ X,\ \Phi\rangle \Downarrow (X \cup \{x\}, \Phi)}$$

$$\frac{\langle \epsilon_1,\ X,\ \Phi\rangle \Downarrow (X', \Phi') \qquad \langle \epsilon_2,\ X',\ \Phi'\rangle \Downarrow (X'', \Phi'')}{\langle \epsilon_1; \epsilon_2,\ X,\ \Phi\rangle \Downarrow (X'', \Phi'')}$$

L0 big-step operational semantics

$$\frac{\langle \epsilon,\ X,\ \Phi \rangle \Downarrow (X', \Phi')}{\langle \textbf{local}\ \epsilon,\ X,\ \Phi \rangle \Downarrow (X, \Phi)}$$

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$$\frac{\langle \epsilon,\ X,\ \Phi \rangle \Downarrow (X', \Phi')}{\langle \textbf{local}\ \epsilon,\ X,\ \Phi \rangle \Downarrow (X, \Phi)}$$

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$$\frac{\langle \epsilon_1,\ X,\ \Phi \rangle \Downarrow (X', \Phi') \qquad \neg\mathit{unsat}(\Phi') \qquad \langle \epsilon_3,\ X,\ \Phi \rangle \Downarrow (X'', \Phi'')}{\langle \textbf{when-unsat}\ \epsilon_1\ \textbf{do}\ \epsilon_2\ \textbf{else}\ \epsilon_3,\ X,\ \Phi \rangle \Downarrow (X'', \Phi'')}$$

L0 Elixir implementation

A simple implementation in Elixir is straightforward by using our SMT-LIB binding

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```
defmacro eval(conn, {:local, _, [e]}) do
  quote do
    conn = unquote(conn)
    :ok = push conn
    eval conn, unquote(e)
    :ok = pop conn
  end
end
```

L0 Elixir example

```
eval conn do
  declare_const :x

  when_unsat add :x != :x do
    skip # Does not reach fail
  else
    fail
  end
end
```

L0 Elixir example

```
eval conn do
  declare_const :x

  when_unsat add :x == :x do
    skip
  else
    fail # Reaches fail
  end
end
```

Verification Intermediate Representation

The L1 language

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- Verification IR

The L1 language

- Verification IR
- It models Elixir expressions dynamically typed

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- Verification IR
- It models Elixir expressions dynamically typed
- Statements for writing verification code

L1 expressions syntax

$$
\begin{array}{rcl}
\mathbf{Exp}^1 \ni e & ::= & c \\
& | & x \\
& | & e_1 \ \mathbf{and} \ e_2 \\
& | & e_1 \ \mathbf{or} \ e_2 \\
& | & [] \\
& | & [e_1 \mid e_2] \\
& | & \{e_1, \ldots, e_n\} \\
& | & f(e_1, \ldots, e_n)
\end{array}
$$

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& | & \{e_1, \ldots, e_n\} \\
& | & f(e_1, \ldots, e_n)
\end{array}
$$

where c is a constant literal of a simple type, currently integer or boolean, and $f \in \Sigma^1$ a function name

L1 statements syntax

$$
\begin{array}{rcl}
\textbf{Stm} \ni S & ::= & \textbf{skip} \\
& \mid & \textbf{block}\ S \\
& \mid & \textbf{havoc}\ x \\
& \mid & S_1 ; S_2 \\
& \mid & \textbf{assume}\ e \\
& \mid & \textbf{assert}\ e \\
& \mid & \textbf{unfold}\ f(e_1, \ldots, e_n)
\end{array}
$$

Built-in SMT-LIB declarations

Foundation to represent L1 expressions in the underlying many-sorted logic (all of them have sort *Term* and can be associated to a *Type*):

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```
(declare-sort Term 0)
(declare-sort Type 0)
...
(declare-const int Type)
(declare-const bool Type)
(assert (distinct int bool))
...
(declare-fun type (Term) Type)
(define-fun is_integer ((x Term)) Bool
  (= (type x) int)
)
...
```

Built-in L1 specifications

Built-in **sets** of pair/postconditions for functions to model their behavior in Elixir

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$\{\textit{is-integer}(x) \wedge \textit{is-integer}(y)\}$

x + y

$\{$

$\quad \textit{is-integer}(\widehat{+}(x, y)) \wedge$

$\quad \textit{integer-value}(\widehat{+}(x, y)) = \textit{integer-value}(x) + \textit{integer-value}(y)$

$\}$

Built-in L1 specifications

Built-in **sets** of pair/postconditions for functions to model their behavior in Elixir

$$\{\textit{is-integer}(x) \land \textit{is-integer}(y)\}$$

x + y

$$\{$$

$$\textit{is-integer}(\widehat{+}(x, y)) \land$$

$$\textit{integer-value}(\widehat{+}(x, y)) = \textit{integer-value}(x) + \textit{integer-value}(y)$$

$$\}$$

There could be more for other types (e.g. float)

Translation from L1 into L0

$$
\begin{aligned}
&\mathit{trExp}\ \_\ [\![\_]\!] : \quad && \mathbf{Exp}^0 \times \mathbf{Exp}^1 \to \mathbf{Exp}^0 \times \mathbb{T} \\
&\mathit{trStm}\ [\![\_]\!] : && \mathbf{Stm} \to \mathbf{Exp}^0
\end{aligned}
$$

Translation from L1 into L0

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$\mathit{trExp}\ \gamma\ [\![e]\!]$ returns a tuple (ϵ, t) where

- ϵ is an L0 expression that models the semantics of e
- t is a term in the underlying logic to refer to the result of e

Translation from L1 into L0

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$\mathit{trExp}\ \gamma\ [\![e]\!]$ returns a tuple (ϵ, t) where

- ϵ is an L0 expression that models the semantics of e
- t is a term in the underlying logic to refer to the result of e
- γ models known facts by the time e is evaluated

Translation of L1 lists

$$
\begin{aligned}
&\mathit{trExp}\ \_\ [\![[]]\!] \equiv (\mathbf{skip}, \mathit{nil})\\
&\mathit{trExp}\ \gamma\ [\![[e_1 \mid e_2]]\!] \equiv (\epsilon_1; \epsilon_2; \epsilon, t)\\
&\qquad \mathbf{where}\ (\epsilon_1, t_1) = \mathit{trExp}\ \gamma\ [\![e_1]\!]\\
&\qquad\qquad\quad (\epsilon_2, t_2) = \mathit{trExp}\ \gamma\ [\![e_2]\!]\\
&\qquad\qquad\quad t = \mathit{cons}(t_1, t_2)\\
&\qquad\qquad\quad \epsilon = \begin{bmatrix} \mathbf{add}\ \mathit{is\text{-}nonempty\text{-}list}(t);\\ \mathbf{add}\ \mathit{hd}(t) = t_1;\\ \mathbf{add}\ \mathit{tl}(t) = t_2 \end{bmatrix}
\end{aligned}
$$

Translation of L1 lists example

$$trExp\ \gamma\ [\![[2, x]]\!] \equiv (\epsilon, cons(2, cons(\hat{x}, nil)))$$

$$\textbf{where}\ \epsilon = \left[\begin{array}{l}
\textbf{add}\ \textit{is-integer}(\textit{integer-lit}(2));\\
\textbf{add}\ \textit{integer-value}(\textit{integer-lit}(2)) = 2;\\
\textbf{add}\ \textit{is-nonempty-list}(cons(\hat{x}, nil));\\
\textbf{add}\ hd(cons(\hat{x}, nil)) = \hat{x};\\
\textbf{add}\ tl(cons(\hat{x}, nil)) = nil;\\
\textbf{add}\ \textit{is-nonempty-list}(cons(2, cons(\hat{x}, nil)));\\
\textbf{add}\ hd(cons(2, cons(\hat{x}, nil))) = 2;\\
\textbf{add}\ tl(cons(2, cons(\hat{x}, nil))) = cons(\hat{x}, nil);
\end{array}\right]$$

L1 Elixir implementation

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```
def tr_exp(_, [{:|, _, [h, t]}]) do
  {h, h_sem} = tr_exp(_, h)
  {y, t_sem} = tr_exp(_, t)
  t =
    quote(do: :cons.(unquote(h), unquote(t)))

  { t, quote do
    unquote(h_sem)
    unquote(t_sem)
    add :is_nonempty_list.(unquote(t))
    add :hd.(unquote(t)) == unquote(h)
    add :tl.(unquote(t)) == unquote(t)
  end }
end
```

L1 Elixir example

```elixir
import Boogiex

with_local_env do
  assert (false or 2) === 2
  assert elem({1, 2, 3}, 0) === 1
  assert true or true + true

  havoc x
  assert x === x
  assert not (x !== x)
end
```

Elixir Code Verification

The L2 language

The L2 language

- The highest level language of our verification stack

The L2 language

- The highest level language of our verification stack
- Elixir (subset) + ghost verification code

L2 expressions syntax

$$
\begin{array}{rcl}
\mathbf{Exp}^2 \ni E & ::= & e \\
& | & P = E \\
& | & \textbf{empty} \\
& | & E_1; E_2 \\
& | & \textbf{case}\ E\ \textbf{do} \\
& & \quad P_1\ \textbf{when}\ f_1 \rightarrow E_1 \\
& & \quad \vdots \\
& & \quad P_n\ \textbf{when}\ f_n \rightarrow E_n \\
& & \textbf{end} \\
& | & \textbf{ghost do}\ S\ \textbf{end}
\end{array}
$$

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& & \quad \vdots \\
& & \quad P_n\ \mathbf{when}\ f_n \rightarrow E_n \\
& & \mathbf{end} \\
& | & \mathbf{ghost\ do}\ S\ \mathbf{end}
\end{array}
$$

where $P, P_1, \ldots P_n$ are patterns:

$$
\mathbf{Pat} \ni P ::= c \mid x \mid [\,] \mid [P_1 \mid P_2] \mid \{P_1, \ldots, P_n\}
$$

Translation from L2 into L1

$$trEXP\ [\![\_]\!] : \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1]$$
$$trMatch\ [\![\_]\!]\ [\![\_]\!] : \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1$$

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$trEXP\ [\![E]\!]$ generates a sequence of pairs (S, e) where

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$trEXP\ [\![E]\!]$ generates a sequence of pairs (S, e) where

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$trEXP\ [\![E]\!]$ generates a sequence of pairs (S, e) where

- S is an L1 statement that models the semantics of E
- e is an L1 expression that represents the result to which E is evaluated
- Each pair corresponds to an execution path

Translation of L2 lists pattern matching

trMatch $[\![e]\!]$ $[\![P]\!]$ returns an L1 expression that is a *boolean* term and is evaluated to *true* if and only if e matches P

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$$
\begin{aligned}
&\mathit{trMatch}\ [\![e]\!]\ [\![[P_1 \mid P_2]]\!] = \\
&\qquad \mathit{is\text{-}nelist}(e)\ \textbf{and} \\
&\qquad \mathit{trMatch}\ [\![\mathit{hd}(e)]\!]\ [\![P_1]\!]\ \textbf{and} \\
&\qquad \mathit{trMatch}\ [\![\mathit{tl}(e)]\!]\ [\![P_2]\!]
\end{aligned}
$$

Translation of L2 pattern matching expressions

$$
\begin{aligned}
&\mathit{trEXP}\ [\![P = E]\!] = [(S_1; S_1', e_1), \ldots, (S_n; S_n', e_n)] \\
&\qquad \textbf{where} \quad [(S_1, e_1), \ldots, (S_n, e_n)] = \mathit{trEXP}\ [\![E]\!] \\
&\qquad\qquad\qquad \{y_1, \ldots, y_m\} = \mathit{vars}(P) \\
&\qquad\qquad\qquad \forall i \in \{1..n\} : S_i' = \begin{pmatrix} \textbf{assert}\ \mathit{trMatch}\ [\![e_i]\!]\ [\![P]\!]; \\ \textbf{havoc}\ y_1; \\ \vdots \\ \textbf{havoc}\ y_m; \\ \textbf{assume}\ e_i \texttt{ === } P \end{pmatrix}
\end{aligned}
$$

Verifying user-defined functions

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Clauses of a function f with arity n:

$$Defs(f/n) = (def_1, \ldots, def_k)$$

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Note: our formalization does not address currently the verification of user-defined function invocations (i.e. their specifications and body unfolding), but our implementation does it by automatically generating ghost code

L2 Elixir implementation

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```
def tr_match({:|, _, [p1, p2]}, e) do
  tr_1 =
    tr_match(p1, quote(do: hd(unquote(e))))
  tr_2 =
    tr_match(p2, quote(do: tl(unquote(e))))

  quote(do:
    is_list(unquote(e)) and
    unquote(e) !== [] and
    unquote(tr_1) and unquote(tr_2)
  )
end
```

L2 Elixir example

Live demo

Conclusions

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  - The current implementation is in an early proof of concept stage

Program Verification in Elixir

Master's Degree in Formal Methods and Computer Engineering

Adrián Enríquez Ballester
Supervisor: Manuel Montenegro Montes

July 5, 2022

Complutense University of Madrid