

Program Verification in Elixir

Master's Degree in Formal Methods and Computer Engineering

Adrián Enríquez Ballester

Supervisor: Manuel Montenegro Montes

July 7, 2022

Complutense University of Madrid



Table of Contents

Introduction

SMT Solver Integration in Elixir

Intermediate Representation for Verification

Elixir Code Verification

Conclusions

Introduction

Motivation

- Light-weight program verification systems:

- Light-weight program verification systems:
 - Allow specifying and verifying code

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions
 - Help the system by providing ghost verification annotations

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions
 - Help the system by providing ghost verification annotations
- Dafny under the hood:

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions
 - Help the system by providing ghost verification annotations
- Dafny under the hood:
 - Compiled to Boogie, an IR for verification

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions
 - Help the system by providing ghost verification annotations
- Dafny under the hood:
 - Compiled to Boogie, an IR for verification
 - Verification conditions discharged by the Z3 theorem prover

Motivation

- Light-weight program verification systems:
 - Allow specifying and verifying code
 - Reduce human intervention
- Dafny:
 - Specify code with pre/postconditions
 - Help the system by providing ghost verification annotations
- Dafny under the hood:
 - Compiled to Boogie, an IR for verification
 - Verification conditions discharged by the Z3 theorem prover
 - Compiled also to other programming languages

The Elixir programming language

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed
- Suitable for developing DSLs through macros

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed
- Suitable for developing DSLs through macros
- Main current verification approaches:

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed
- Suitable for developing DSLs through macros
- Main current verification approaches:
 - Dialyzer (static)

The Elixir programming language

- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed
- Suitable for developing DSLs through macros
- Main current verification approaches:
 - Dialyzer (static)
 - Property-based testing (dynamic)

The Elixir programming language

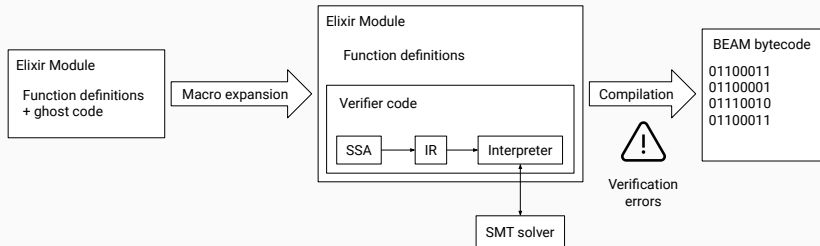
- A functional programming language that runs on the Erlang Virtual Machine
- Dynamically typed
- Suitable for developing DSLs through macros
- Main current verification approaches:
 - Dialyzer (static)
 - Property-based testing (dynamic)
 - Both of them show the presence of errors rather than their absence

Our aim

Provide a system similar to Dafny but specialized for Elixir and implemented in Elixir itself

Our aim

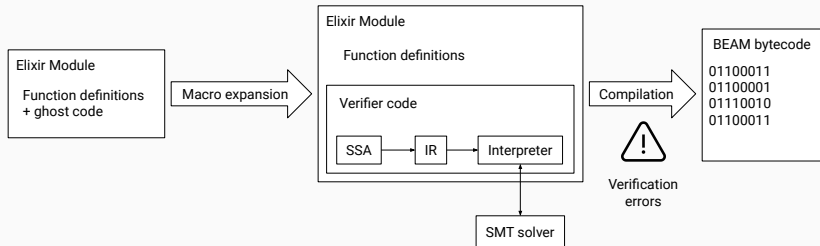
Provide a system similar to Dafny but specialized for Elixir and implemented in Elixir itself



<https://github.com/adrianen-ucm/verixir-project>

Our aim

Provide a system similar to Dafny but specialized for Elixir and implemented in Elixir itself



<https://github.com/adrianen-ucm/verixir-project>

Scope: only a subset of sequential Elixir for the moment, and partial verification (i.e. not verifying termination)

A valid Elixir program

```
result =  
  if selector === 1 do  
    1  
  else  
    false  
  end
```

```
result =  
  if selector === 1 do  
    result + 1  
  else  
    not result  
  end
```


1. SMT solver integration in Elixir

1. SMT solver integration in Elixir
2. L0, a low level language close to the SMT solver

1. SMT solver integration in Elixir
2. L0, a low level language close to the SMT solver
3. L1, a verification IR for dynamically typed Elixir expressions

1. SMT solver integration in Elixir
2. L0, a low level language close to the SMT solver
3. L1, a verification IR for dynamically typed Elixir expressions
4. L2, a high level language that models Elixir + verification annotations

SMT Solver Integration in Elixir

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

- A DSL for a subset of SMT-LIB

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

- A DSL for a subset of SMT-LIB
- Different SMT solvers that implement SMT-LIB can be easily integrated

We have developed an SMT-LIB (an SMT standard) binding for Elixir with the following features:

- A DSL for a subset of SMT-LIB
- Different SMT solvers that implement SMT-LIB can be easily integrated
- Out-of-the-box support for Z3

Elixir SMT-LIB binding example

```
import SmtLib

with_local_conn do
  declare_const x: Int,
               y: Int

  assert !(
    (:x + 3 <= :y + 3) ~> (:x <= :y)
  )

  check_sat
end
```

The L0 language

The L0 language

- The lowest level language of our verification stack

The L0 language

- The lowest level language of our verification stack
- Close to the SMT solver

The L0 language

- The lowest level language of our verification stack
- Close to the SMT solver
- Restricted SMT-LIB + control flow + failure

$\text{Exp}^0 \ni \epsilon ::=$

- skip**
- fail**
- $\epsilon_1; \epsilon_2$
- local** ϵ
- add** φ
- declare** x
- when-unsat** ϵ_1 **do** ϵ_2 **else** ϵ_3

$$\begin{array}{lcl} \mathbf{Exp}^0 \ni \epsilon & ::= & \mathbf{skip} \\ & | & \mathbf{fail} \\ & | & \epsilon_1; \epsilon_2 \\ & | & \mathbf{local} \ \epsilon \\ & | & \mathbf{add} \ \varphi \\ & | & \mathbf{declare} \ x \\ & | & \mathbf{when-unsat} \ \epsilon_1 \ \mathbf{do} \ \epsilon_2 \ \mathbf{else} \ \epsilon_3 \end{array}$$

where $x \in V$ is a variable name and $\varphi \in \mathbb{F}$ is a formula with many-sorted terms $t \in \mathbb{T}$

L0 Elixir example

```
eval conn do
  declare_const :x

  when_unsat add :x != :x do
    skip # Does not reach fail
  else
    fail
  end
end
```

L0 Elixir example

```
eval conn do
  declare_const :x

  when_unsat add :x == :x do
    skip
  else
    fail # Reaches fail
  end
end
```

Intermediate Representation for Verification

The L1 language

- IR for Verification

- IR for Verification
- It models dynamically typed Elixir expressions

- IR for Verification
- It models dynamically typed Elixir expressions
- Statements for writing verification annotations (ghost code)

$$\begin{array}{lcl} \mathbf{Exp}^1 \ni e & ::= & c \\ & | & x \\ & | & e_1 \textbf{ and } e_2 \\ & | & e_1 \textbf{ or } e_2 \\ & | & [] \\ & | & [e_1 \mid e_2] \\ & | & \{e_1, \dots, e_n\} \\ & | & f(e_1, \dots, e_n) \end{array}$$

$$\begin{array}{lcl} \mathbf{Exp}^1 \ni e & ::= & c \\ & | & x \\ & | & e_1 \mathbf{and} e_2 \\ & | & e_1 \mathbf{or} e_2 \\ & | & [] \\ & | & [e_1 \mid e_2] \\ & | & \{e_1, \dots, e_n\} \\ & | & f(e_1, \dots, e_n) \end{array}$$

where c is a constant literal of a simple type, currently integer or boolean, and $f \in \Sigma^1$ a function name

Stm $\ni S$::= **skip**
 | **block** S
 | **havoc** x
 | $S_1; S_2$
 | **assume** e
 | **assert** e
 | **unfold** $f(e_1, \dots, e_n)$

Built-in SMT-LIB declarations

Prelude to represent L1 expressions in the underlying many-sorted logic (all of them have sort *Term* and can be associated to a *Type*):

Built-in SMT-LIB declarations

Prelude to represent L1 expressions in the underlying many-sorted logic (all of them have sort *Term* and can be associated to a *Type*):

```
(declare-sort Term 0)
(declare-sort Type 0)
...
(declare-const int Type)
(declare-const bool Type)
(assert (distinct int bool))
...
(declare-fun type (Term) Type)
(define-fun is_integer ((x Term)) Bool
  (= (type x) int)
)
...
```

Built-in L1 specifications

Built-in **sets** of pair/postconditions for functions to model their behavior in Elixir

Built-in L1 specifications

Built-in **sets** of pair/postconditions for functions to model their behavior in Elixir

$$\{is_integer(x) \wedge is_integer(y)\}$$
$$x + y$$
$$\{$$
$$is_integer(\hat{+}(x, y)) \wedge$$
$$integer_value(\hat{+}(x, y)) = integer_value(x) + integer_value(y)$$
$$\}$$

Built-in L1 specifications

Built-in **sets** of pair/postconditions for functions to model their behavior in Elixir

$$\{is_integer(x) \wedge is_integer(y)\}$$
$$x + y$$
$$\{$$
$$is_integer(\hat{+}(x, y)) \wedge$$
$$integer_value(\hat{+}(x, y)) = integer_value(x) + integer_value(y)$$
$$\}$$

There could be more for other types (e.g. float)

$$\begin{aligned} trExp \llbracket - \rrbracket &: \mathbf{Exp}^0 \times \mathbf{Exp}^1 \rightarrow \mathbf{Exp}^0 \times \mathbb{T} \\ trStm \llbracket - \rrbracket &: \mathbf{Stm} \rightarrow \mathbf{Exp}^0 \end{aligned}$$

$$\begin{aligned} trExp \llbracket _ \rrbracket &: \mathbf{Exp}^0 \times \mathbf{Exp}^1 \rightarrow \mathbf{Exp}^0 \times \mathbb{T} \\ trStm \llbracket _ \rrbracket &: \mathbf{Stm} \rightarrow \mathbf{Exp}^0 \end{aligned}$$

$trExp \gamma \llbracket e \rrbracket$ returns a tuple (ϵ, t) where

$$\begin{aligned} trExp \llbracket _ \rrbracket &: \mathbf{Exp}^0 \times \mathbf{Exp}^1 \rightarrow \mathbf{Exp}^0 \times \mathbb{T} \\ trStm \llbracket _ \rrbracket &: \mathbf{Stm} \rightarrow \mathbf{Exp}^0 \end{aligned}$$

$trExp \gamma \llbracket e \rrbracket$ returns a tuple (ϵ, t) where

- ϵ is an L0 expression that models the semantics of e

Translation from L1 into L0

$$\begin{aligned} trExp \llbracket _ \rrbracket &: \mathbf{Exp}^0 \times \mathbf{Exp}^1 \rightarrow \mathbf{Exp}^0 \times \mathbb{T} \\ trStm \llbracket _ \rrbracket &: \mathbf{Stm} \rightarrow \mathbf{Exp}^0 \end{aligned}$$

$trExp \gamma \llbracket e \rrbracket$ returns a tuple (ϵ, t) where

- ϵ is an L0 expression that models the semantics of e
- t is a term in the underlying logic to refer to the result of e

Translation from L1 into L0

$$\begin{aligned} trExp \llbracket _ \rrbracket &: \mathbf{Exp}^0 \times \mathbf{Exp}^1 \rightarrow \mathbf{Exp}^0 \times \mathbb{T} \\ trStm \llbracket _ \rrbracket &: \mathbf{Stm} \rightarrow \mathbf{Exp}^0 \end{aligned}$$

$trExp \ \gamma \llbracket e \rrbracket$ returns a tuple (ϵ, t) where

- ϵ is an L0 expression that models the semantics of e
- t is a term in the underlying logic to refer to the result of e
- γ models known facts by the time e is evaluated

$trExp \text{ - } \llbracket [] \rrbracket \equiv (\mathbf{skip}, nil)$

$trExp \gamma \llbracket [e_1 \mid e_2] \rrbracket \equiv (\epsilon_1; \epsilon_2; \epsilon, t)$

where $(\epsilon_1, t_1) = trExp \gamma \llbracket e_1 \rrbracket$

$(\epsilon_2, t_2) = trExp \gamma \llbracket e_2 \rrbracket$

$t = cons(t_1, t_2)$

$\epsilon = \left[\begin{array}{l} \mathbf{add} \text{ is-nonempty-list}(t); \\ \mathbf{add} \text{ hd}(t) = t_1; \\ \mathbf{add} \text{ tl}(t) = t_2 \end{array} \right]$

$$\begin{aligned} trExp \text{ - } \llbracket [] \rrbracket &\equiv (\mathbf{skip}, nil) \\ trExp \gamma \llbracket [e_1 \mid e_2] \rrbracket &\equiv (\epsilon_1; \epsilon_2; \epsilon, t) \\ &\quad \mathbf{where} \ (\epsilon_1, t_1) = trExp \gamma \llbracket e_1 \rrbracket \\ &\quad \quad (\epsilon_2, t_2) = trExp \gamma \llbracket e_2 \rrbracket \\ &\quad \quad t = cons(t_1, t_2) \\ \epsilon &= \left[\begin{array}{l} \mathbf{add} \ is\text{-}nonempty\text{-}list(t); \\ \mathbf{add} \ hd(t) = t_1; \\ \mathbf{add} \ tl(t) = t_2 \end{array} \right] \end{aligned}$$

Remember: the constants and functions involved in the result of the translation must be defined in the SMT prelude. For example:

```
(declare-const nil Term)
(declare-fun cons (Term Term) Term)
```

L1 Elixir example

```
import Boogiex

with_local_env do
  assert (false or 2) === 2
  assert elem({1, 2, 3}, 0) === 1
  assert true or true + true

  havoc x
  assert x === x
  assert not (x !== x)
end
```

Elixir Code Verification

- The highest level language of our verification stack

- The highest level language of our verification stack
- A subset of Elixir + ghost verification annotations

```
Exp2  $\ni$  E ::= e
      | P = E
      | empty
      | E1; E2
      | case E do
          P1 when f1 → E1
          ⋮
          Pn when fn → En
      | end
      | ghost do S end
```

$$\begin{array}{lcl} \mathbf{Exp}^2 \ni E & ::= & e \\ & | & P = E \\ & | & \mathbf{empty} \\ & | & E_1; E_2 \\ & | & \mathbf{case } E \mathbf{ do} \\ & & \quad P_1 \mathbf{ when } f_1 \rightarrow E_1 \\ & & \quad \vdots \\ & & \quad P_n \mathbf{ when } f_n \rightarrow E_n \\ & & \mathbf{end} \\ & | & \mathbf{ghost do } S \mathbf{ end} \end{array}$$

where $e, f_1, \dots, f_n \in \mathbf{Exp}^1$ and P, P_1, \dots, P_n are patterns:

$$\mathbf{Pat} \ni P ::= c \mid x \mid [] \mid [P_1 \mid P_2] \mid \{P_1, \dots, P_n\}$$

$$\begin{aligned} trEXP \llbracket - \rrbracket &: \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1] \\ trMatch \llbracket - \rrbracket \llbracket - \rrbracket &: \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1 \end{aligned}$$

$$trEXP \llbracket - \rrbracket : \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1]$$

$$trMatch \llbracket - \rrbracket \llbracket - \rrbracket : \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1$$

$trEXP \llbracket E \rrbracket$ generates a sequence of pairs (S, e) where

$$\begin{aligned} trEXP \llbracket - \rrbracket &: \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1] \\ trMatch \llbracket - \rrbracket \llbracket - \rrbracket &: \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1 \end{aligned}$$

$trEXP \llbracket E \rrbracket$ generates a sequence of pairs (S, e) where

- S is an L1 statement that models the semantics of E

$$\begin{aligned} trEXP \llbracket - \rrbracket &: \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1] \\ trMatch \llbracket - \rrbracket \llbracket - \rrbracket &: \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1 \end{aligned}$$

$trEXP \llbracket E \rrbracket$ generates a sequence of pairs (S, e) where

- S is an L1 statement that models the semantics of E
- e is an L1 expression that represents the result to which E is evaluated

$$\begin{aligned} trEXP \llbracket - \rrbracket &: \mathbf{Exp}^2 \rightarrow [\mathbf{Stm} \times \mathbf{Exp}^1] \\ trMatch \llbracket - \rrbracket \llbracket - \rrbracket &: \mathbf{Exp}^1 \times \mathbf{Pat} \rightarrow \mathbf{Exp}^1 \end{aligned}$$

$trEXP \llbracket E \rrbracket$ generates a sequence of pairs (S, e) where

- S is an L1 statement that models the semantics of E
- e is an L1 expression that represents the result to which E is evaluated
- Each pair corresponds to an execution path

Translation of L2 pattern matching expressions

$trEXP \llbracket P = E \rrbracket = [(S_1; S'_1, e_1), \dots, (S_n; S'_n, e_n)]$

where $[(S_1, e_1), \dots, (S_n, e_n)] = trEXP \llbracket E \rrbracket$

$\{y_1, \dots, y_m\} = vars(P)$

$\forall i \in \{1..n\} : S'_i = \left(\begin{array}{l} \mathbf{assert} \ trMatch \llbracket e_i \rrbracket \llbracket P \rrbracket; \\ \mathbf{havoc} \ y_1; \\ \vdots \\ \mathbf{havoc} \ y_m; \\ \mathbf{assume} \ e_i == P \end{array} \right)$

Verifying user-defined functions

A single clause of a function with arity n :

$$\text{def} \equiv (\{p\} \quad (P_1, \dots, P_n) \text{ B} \quad \{q\})$$

Verifying user-defined functions

A single clause of a function with arity n :

$$def \equiv (\{p\} \quad (P_1, \dots, P_n) \ B \quad \{q\})$$

where $p \in \mathbf{Exp}^1$ and $q \in \mathbf{Exp}^1$ denote a specified precondition and a postcondition, P_1, \dots, P_n are the parameter patterns and $B \in \mathbf{Exp}^2$ is its defined body

Verifying user-defined functions

A single clause of a function with arity n :

$$def \equiv (\{p\} \quad (P_1, \dots, P_n) \ B \quad \{q\})$$

where $p \in \mathbf{Exp}^1$ and $q \in \mathbf{Exp}^1$ denote a specified precondition and a postcondition, P_1, \dots, P_n are the parameter patterns and $B \in \mathbf{Exp}^2$ is its defined body

Clauses of a function f with arity n :

$$Defs(f/n) = (def_1, \dots, def_k)$$

Verifying user-defined functions

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches
2. Apply Static Single-Assignment to allow rebinding of variables

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches
2. Apply Static Single-Assignment to allow rebinding of variables
3. Translate into the corresponding execution paths in our IR

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches
2. Apply Static Single-Assignment to allow rebinding of variables
3. Translate into the corresponding execution paths in our IR
4. Translate each path into an L0 expression

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches
2. Apply Static Single-Assignment to allow rebinding of variables
3. Translate into the corresponding execution paths in our IR
4. Translate each path into an L0 expression
5. Verify paths independently of each other

Verifying user-defined functions

1. Transform the function definition clauses into an L2 case expression with the parameter variables free and a branch that trivially matches
2. Apply Static Single-Assignment to allow rebinding of variables
3. Translate into the corresponding execution paths in our IR
4. Translate each path into an L0 expression
5. Verify paths independently of each other

Note: our formalization does not address currently the verification of user-defined function invocations (i.e. their specifications and body unfolding), but our implementation does it by automatically generating ghost code

Live demo

Conclusions

Conclusions

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification
- Also, we have left several improvements on the way:

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification
- Also, we have left several improvements on the way:
 - More of the SMT-LIB standard and support for other SMT solvers

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification
- Also, we have left several improvements on the way:
 - More of the SMT-LIB standard and support for other SMT solvers
 - Extend our IR to model more Elixir value types and built-in functions

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification
- Also, we have left several improvements on the way:
 - More of the SMT-LIB standard and support for other SMT solvers
 - Extend our IR to model more Elixir value types and built-in functions
 - Extend the Elixir subset to verify (e.g. pin operator and higher-order)

Conclusions

- We have developed a framework for Elixir code verification across several areas (i.e. SMT solver integration, verification IR and code verification)
- Future work may address concurrent code and total verification
- Also, we have left several improvements on the way:
 - More of the SMT-LIB standard and support for other SMT solvers
 - Extend our IR to model more Elixir value types and built-in functions
 - Extend the Elixir subset to verify (e.g. pin operator and higher-order)
 - The current implementation is in an early proof of concept stage

Program Verification in Elixir

Master's Degree in Formal Methods and Computer Engineering

Adrián Enríquez Ballester

Supervisor: Manuel Montenegro Montes

July 7, 2022

Complutense University of Madrid

