TIØ4317 Empirical and Quantitative Methods in Finance	Exercise 5
28.02.2025	Scribe: Adrian Fagerland

1

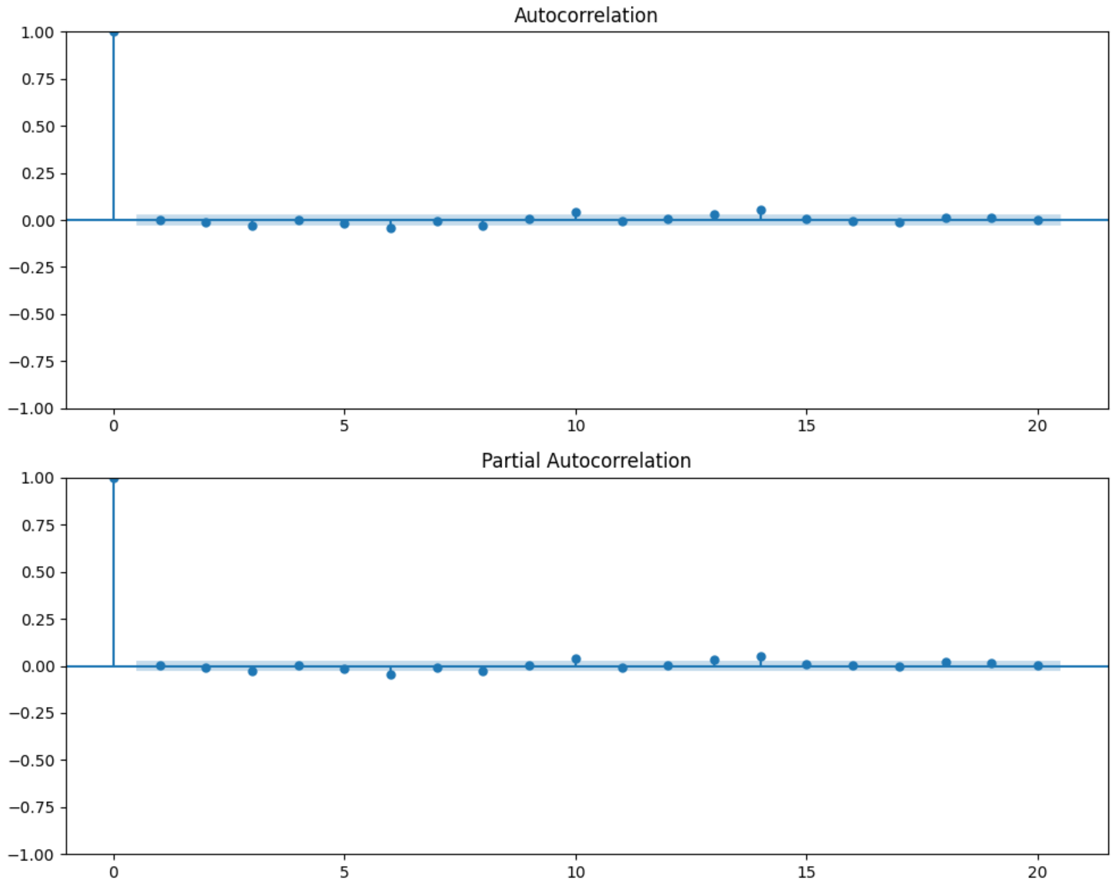
Best ARMA model: ARMA(0, 0, 3) with AIC: -19605.529408445313

Dep. Variable:	Return	**No. Observations:**	4507
Model:	ARIMA(0, 0, 3)	**Log Likelihood**	9807.765
Date:	Fri, 28 Feb 2025	**AIC**	-19605.529
Time:	12:39:46	**BIC**	-19573.462
Sample:	0 - 4507	**HQIC**	-19594.231
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	0.0002	0.000	0.544	0.586	-0.001	0.001
ma.L1	0.0013	0.009	0.152	0.880	-0.016	0.018
ma.L2	-0.0118	0.011	-1.096	0.273	-0.033	0.009
ma.L3	-0.0311	0.009	-3.495	0.000	-0.049	-0.014
sigma2	0.0008	8.04e-06	93.788	0.000	0.001	0.001

Ljung-Box (L1) (Q):	0.00	**Jarque-Bera (JB):**	7512.73
Prob(Q):	0.95	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.54	**Skew:**	-0.08
Prob(H) (two-sided):	0.00	**Kurtosis:**	9.32

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).



2

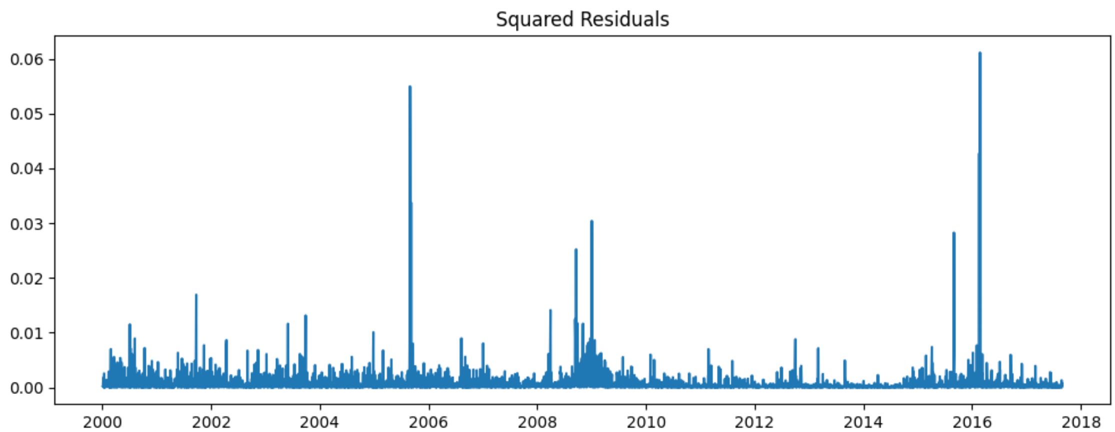
Ljung-Box Test on Residuals:

	$\text{lb}_s tat$	$\text{lb}_p value$
1	0.004035	0.949348
2	0.023130	0.988502
3	0.023171	0.999068
4	0.023171	0.999933
5	1.142103	0.950310
6	8.894291	0.179611
7	9.079874	0.246972
8	12.551090	0.128257
9	12.620791	0.180529
10	20.580949	0.024213

Ljung-Box Test on Squared Residuals:

	$\text{lb}_{s}tat$	$\text{lb}_{p}value$
1	319.352371	0.000000
2	392.727698	0.000000
3	649.803186	0.000000
4	709.771928	0.000000
5	812.289201	0.000000
6	841.152911	0.000000
7	857.797019	0.000000
8	867.738854	0.000000
9	879.774545	0.000000
10	883.774629	0.000000

ARCH-LM Test p-value: 4.1329892296863164e-116



We can therefore conclude that heteroskedasticity is present, and that there are significant non-linearities in the data.

3

Best GARCH model: GARCH(1, 2) with AIC: -20405.169282378793

Dep. Variable:	Return	**R-squared:**	-0.000
Mean Model:	AR	**Adj. R-squared:**	-0.000
Vol Model:	GARCH	**Log-Likelihood:**	10208.6
Distribution:	Normal	**AIC:**	-20405.2
Method:	Maximum Likelihood	**BIC:**	-20366.7
		No. Observations:	4506
Date:	Fri, Feb 28 2025	**Df Residuals:**	4504
Time:	12:39:46	**Df Model:**	2

	coef	**std err**	**t**	**P> |t|**	**95.0% Conf. Int.**
Const	4.5335e-04	3.472e-04	1.306	0.192	[-2.271e-04,1.134e-03]
Return[1]	2.9568e-03	1.689e-02	0.175	0.861	[-3.016e-02,3.607e-02]

	coef	**std err**	**t**	**P> |t|**	**95.0% Conf. Int.**
omega	1.3995e-05	1.278e-11	1.095e+06	0.000	[1.399e-05,1.399e-05]
alpha[1]	0.1088	1.838e-02	5.920	3.217e-09	[7.279e-02, 0.145]
beta[1]	0.3263	0.333	0.981	0.327	[-0.326, 0.979]
beta[2]	0.5487	0.327	1.675	9.385e-02	[-9.318e-02, 1.191]

Covariance estimator: robust

4

GARCH persistence (+): 0.43514650063945454

Is the model covariance stationary (persistence < 1)? Yes

A GARCH process is covariance stationary if $\alpha + \beta < 1$. If the sum is very close to 1, it suggests high persistence in volatility and potential integration

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Dep. Variable:	Return	**R-squared:**	-0.000
Mean Model:	AR	**Adj. R-squared:**	-0.000
Vol Model:	FIGARCH	**Log-Likelihood:**	10214.4
Distribution:	Normal	**AIC:**	-20416.9
Method:	Maximum Likelihood	**BIC:**	-20378.4
		No. Observations:	4506
Date:	Fri, Feb 28 2025	**Df Residuals:**	4504
Time:	12:41:32	**Df Model:**	2

	coef	std err	t	P> |t|	95.0% Conf. Int.
Const	4.2991e-04	3.594e-04	1.196	0.232	[-2.746e-04,1.134e-03]
Return[1]	2.5414e-03	1.774e-02	0.143	0.886	[-3.223e-02,3.732e-02]

	coef	std err	t	P> |t|	95.0% Conf. Int.
omega	1.0771e-05	4.010e-06	2.686	7.225e-03	[2.912e-06,1.863e-05]
phi	0.1998	5.376e-02	3.716	2.023e-04	[9.442e-02, 0.305]
d	0.5005	0.110	4.559	5.150e-06	[0.285, 0.716]
beta	0.6294	9.758e-02	6.450	1.118e-10	[0.438, 0.821]

Covariance estimator: robust Fractional integration parameter (δ): 0.5005

p-value: 0.0000

Is δ significant at 5% (d$_p value < 0.05$)?Yes

The significance of the δ parameter indicates whether there is long memory in the volatility process. If significant, it suggests that shocks to volatility persist for a long time.

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Best TARCH model: TARCH(1, 2) with AIC: -20401.206588425353

Dep. Variable:	Return	**R-squared:**	-0.000
Mean Model:	AR	**Adj. R-squared:**	-0.000
Vol Model:	GJR-GARCH	**Log-Likelihood:**	10207.6
Distribution:	Normal	**AIC:**	-20401.2
Method:	Maximum Likelihood	**BIC:**	-20356.3
		No. Observations:	4506
Date:	Fri, Feb 28 2025	**Df Residuals:**	4504
Time:	12:39:47	**Df Model:**	2

	coef	std err	t	P> |t|	95.0% Conf. Int.
Const	3.8341e-04	3.542e-04	1.083	0.279	[-3.107e-04,1.078e-03]
Return[1]	2.6565e-03	1.708e-02	0.156	0.876	[-3.082e-02,3.614e-02]

	coef	std err	t	P> |t|	95.0% Conf. Int.
omega	1.5392e-05	2.376e-11	6.478e+05	0.000	[1.539e-05,1.539e-05]
alpha[1]	0.1000	3.966e-02	2.523	1.165e-02	[2.231e-02, 0.178]
gamma[1]	9.9603e-03	2.107e-02	0.473	0.636	[-3.134e-02,5.126e-02]
beta[1]	0.4375	1.213	0.361	0.718	[-1.939, 2.814]
beta[2]	0.4375	1.178	0.371	0.710	[-1.871, 2.746]

Covariance estimator: robust

7 & 8

Question 7 and 8 seem to be duplicates.

Dep. Variable:	Return_gas	**R-squared:**	-0.000
Mean Model:	AR	**Adj. R-squared:**	-0.000
Vol Model:	GARCH	**Log-Likelihood:**	10202.9
Distribution:	Normal	**AIC:**	-20395.8
Method:	Maximum Likelihood	**BIC:**	-20363.8
		No. Observations:	4506
Date:	Fri, Feb 28 2025	**Df Residuals:**	4504
Time:	12:39:47	**Df Model:**	2

	coef	**std err**	**t**	**P> |t|**	**95.0% Conf. Int.**
Const	5.2288e-04	3.535e-04	1.479	0.139	[-1.699e-04,1.216e-03]
Return_gas[1]	2.5457e-03	1.731e-02	0.147	0.883	[-3.137e-02,3.646e-02]

	coef	**std err**	**t**	**P> |t|**	**95.0% Conf. Int.**
omega	1.5098e-05	1.150e-11	1.313e+06	0.000	[1.510e-05,1.510e-05]
alpha[1]	0.1000	1.426e-02	7.014	2.319e-12	[7.206e-02, 0.128]
beta[1]	0.8800	1.229e-02	71.591	0.000	[0.856, 0.904]

Covariance estimator: robust

This approach models how Brent crude oil returns impact the volatility of Gasoline returns by including the absolute Brent returns as an external regressor in the volatility equation. Alternatively, I could model them jointly using a multivariate GARCH framework, but that's more complex and would require additional code.