TIØ4317
Empirical and Quantitative Methods in Finance
Exercise 1

Instructions

Solutions to the problems will be posted on BlackBoard after the deadline. You can use either Excel or a high-level programming language, e.g., R or Python, to solve the programming exercises. We suggest that you write your solutions using MS Word or LaTeX. Also, hand in all code and/or Excel files.

Deadline: Monday January 27th, 2025, 23:59. **Grading:** Passed/Failed.

Tasks

1. The following table gives annual, end-of-year prices of a bond and the consumer price index:

Year	Bond value	CPI value
2006	36.9	108.0
2007	39.8	110.3
2008	42.4	113.6
2009	38.1	116.1
2010	36.4	118.4
2011	39.2	120.9
2012	44.6	123.2
2013	45.1	125.4

Table 1: Bond and CPI values.

   (a) Calculate the simple returns.

   (b) Calculate the continuously compounded returns.

   (c) Calculate the prices of the bond each year in 2013 terms.

   (d) Calculate the real returns.

2. Consider the normal linear regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, where $u_i \sim N(0, \sigma^2)$ are i.i.d. normal, and X_i and u_i are independent.

   (a) Derive the distribution of Y.

(b) Derive the likelihood function for β_0 and β_1.

(c) Show that $\hat{\beta}_{0,ML} = \hat{\beta}_{0,OLS}$ and $\hat{\beta}_{1,ML} = \hat{\beta}_{1,OLS}$.

3. Explain, with the use of equations, the difference between the sample regression function and the population regression function.

4. Do we make assumptions concerning the unobservable error terms or about their sample counterparts, the estimated residuals? Explain your answer.

5. "Linear regression assumes that the conditional mean of the dependent variable is linear in the independent variable." Explain this statement using equations. Given these formulations, how should we interpret the estimates of β_0 and β_1? Do these estimates always have meaning?

6. The capital asset pricing model (CAPM) can be written as

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

using standard notation. The first step in using the CAPM is to estimate the stock's beta using the market model. The market model can be written as

$$R_{it} = \alpha_i + \beta_i R_{mt} + u_{it}$$

where R_{it} is the excess return for security i at time t, R_{mt} is the excess return on a proxy for the market portfolio at time t, and u_t is an i.i.d. random disturbance term. Select one of the stock series from the 'capm.xls' file and:

(a) Make a scatter plot and a distribution plot of the data. Is there an obvious relationship between variables that you see?

(b) Present the table with summary statistics for that series and comment on those.

- Explain the differences between the mean, mode, and median. Which is the most useful measure of an average and why?
- Which is a more useful measure of central tendency for stock returns—the arithmetic mean or the geometric mean? Explain your answer.

(c) Find the estimates for α and β and their standard error. How can your estimates be interpreted?

(d) Test the null hypothesis that $\beta = 0$. Comment on the result.

7. There are mainly two types of errors when doing hypothesis testing: Type I error and Type II error. Explain in your own words what these errors represent. What is the relationship between the significance level and Type I error? Can you find an example where for a researcher:

(a) Type I error is more important.

(b) Type II error is more important.

8. Are hypotheses tested concerning the actual values of the coefficients or their estimated values? Explain your answer.

Voluntary task

9. You want to investigate the relation between the size of initial public offerings (IPOs) and the fees that investment banks charge for executing them. IPO size is measured in terms of 10,000 shares placed, whereas the fee is measured as a percentage of the total IPO value. For 10 IPOs, you have data as described in Table 2.

   (a) Formulate an appropriate normal linear regression model and find the OLS estimates for the intercept and slope.

   (b) Draw a scatter plot of the data and draw the graph of $FEE = \frac{1}{5+\gamma_1 \cdot SIZE^{\gamma_2}}$ for $\gamma_1 = 6$ and $\gamma_2 = 3$.

   (c) Rewrite the equation as a linear model. How do you transform the data? How are the intercept and slope linked to γ_1 and γ_2?

   (d) Compute the OLS estimates for the intercept and slope and use these to find estimates for γ_1 and γ_2.

Size	**Fee (%)**	**Size**	**Fee (%)**
1.6136	2.00	0.5214	15.11
1.7672	2.00	3.0714	2.00
1.0190	5.62	0.8088	12.45
2.0209	3.41	2.2771	2.00
1.5179	4.03	3.0002	2.00

Table 2: Data on 10 IPOs, recording size (in 10,000 shares) and Fee (as a % of IPO value).