TIØ4140 Project Evaluation and Financing

Exercise 6: The Black-Scholes Equation and Monte Carlo Valuation[1]

Posted: Monday, February 17, 2025, Morning.
Deadline: Tuesday, February 25, 2025, 23:59.
Grading: Approved / Not approved.

Overview of Tasks

- **Mandatory Tasks**:
  - **Ch. 26**: The Black-Scholes-Merton Equation 5
  - Exercises from other sources
- **Voluntary Tasks:**
  - Exercises from other sources

Mandatory Tasks

Chapter 26: The Black-Scholes-Merton Equation

26.5. Verify that $V(S, t) = Se^{-\delta(T-t)} N(d_1)$ satisfies the Black-Scholes equation (see Eq. (26.11), McDonald (2013, p.644)).

- ### Exercises from other sources

1. Assume that a non-dividend-paying stock has an expected return of μ and a volatility of σ. An innovative financial institution has just announced that it will trade a security that pays off a dollar amount equal to $\ln S_T$ at time T, where S_T denotes the value of the stock price at time T.
(a) Use risk-neutral valuation to calculate the price of the security at time t in terms of the stock price, S, at time t.
(b) Confirm that your price satisfies the Black-Scholes-Merton differential equation

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS\frac{\partial f}{\partial S} = rf$$

2. Monte-Carlo valuation of Asian options

In this task, you are going to write a program to price Asian calls and puts by simulation using real data.

To obtain real data, you can, for example, go to https://finance.yahoo.com/lookup and select a stock. You can select one of the ‘trending tickers’ visible in the link or use the search bar above to find a stock you want to study. When you have selected a stock, you can download the data in a CSV-file under the tab ‘Historical data’. Download daily data over a period of at least one year.

Use the closing price for calculating log returns and for the latest stock price (i.e. the column *Close*). The other columns are not relevant to this task.

a. What is the (annualized) volatility of your stock? How did you estimate it?

Now, consider an at-the-money Asian option you can buy on the last day of your data set, i.e. the strike price is equal to the last (most recent) close of your stock in your data set. Assume the option matures in one month and can only be exercised at maturity. The Asian option should be priced

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[*] In this exercise, we have some question related to the previous topic (Brownian motion and Itô’s lemma)

according to the discrete arithmetic average of simulated daily stock prices for the upcoming month.

For convenience, assume the risk-free rate to be (a constant) 2.5%. Furthermore, independent of the stock you consider, the stock has a dividend yield of 4%. Use the volatility as estimated in part (a).

b. Price the aforementioned at-the-money Asian call using simulation.

c. How does the option value change when changing the number of simulations? Briefly explain how the variation (standard deviation) of the option values change with the number of simulations.

Voluntary Tasks

- **Exercises from other sources**

1. Suppose that a stock price, S, follows geometric Brownian Motion with expected return and volatility:
$$dS = \mu S\, dt + \sigma S\, dz$$
What is the process followed by the variable S^n?
2. Suppose that *G* is a function of a stock price, *S*, and time. Suppose that σ_S and σ_G are the volatilities of S and G. Show that when the expected return of S increases by $\lambda\sigma_S$, the growth rate of G increases by $\lambda\sigma_G$, where λ is a constant.
3. Consider an option on a stock when the stock price is 30$, the exercise price is 29$, the risk-free interest rate is 5 %, the volatility is 25 % per annum, and the time to maturity is 4 months. Assume that the stock is due to go ex-dividend in $1\frac{1}{2}$ months. The expected dividend is 50 cents.

   (a) What is the price of the option if it is a European call?

   (b) What is the price of the option if it is a European put?

   (c) If the option is an American call, are there any circumstances under which it will be exercised early?

References

Esser, A. (2003). General valuation principles for arbitrary payoffs and applications to power options under stochastic volatility. *Financial Markets and Portfolio Management*, 17(3):351–372.

McDonald, R. L. (2014). *Derivatives markets*. 3rd. ed., New International Edition. Pearson Education.