TIØ4140 Project Evaluation and Financing	**Exercise 1**
26.01.2025	**Scribe: Adrian Fagerland**

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(a)

- For $S_T = \$38$: Payoff $= 48 - 38 = \$10$
- For $S_T = \$43$: Payoff $= 48 - 43 = \$5$
- For $S_T = \$48$: Payoff $= 48 - 48 = \$0$
- For $S_T = \$53$: Payoff $= 48 - 53 = -\$5$
- For $S_T = \$58$: Payoff $= 48 - 58 = -\$10$

The payoffs for the short forward position are $\boxed{\$10, \$5, \$0, -\$5, -\$10}$ for spot prices of $\boxed{\$38, \$43, \$48, \$53, \$58}$ respectively.

(b)

For a long put option with a strike price $K = \$48$, the payoff at maturity is given by $\max(K - S_T, 0)$.

- **If** $S_T = \$38$**:** Payoff $= \max(48 - 38, 0) = \max(10, 0) = \$10$
- **If** $S_T = \$43$**:** Payoff $= \max(48 - 43, 0) = \max(5, 0) = \$5$
- **If** $S_T = \$48$**:** Payoff $= \max(48 - 48, 0) = \max(0, 0) = \$0$
- **If** $S_T = \$53$**:** Payoff $= \max(48 - 53, 0) = \max(-5, 0) = \$0$
- **If** $S_T = \$58$**:** Payoff $= \max(48 - 58, 0) = \max(-10, 0) = \$0$

(c)

Comparing the payoffs, we see that for spot prices at or below \$48, the payoffs are the same for both the short forward and long put. However, for spot prices above \$48, the short forward position leads to negative payoffs (losses), while the long put option payoff is capped at \$0, preventing further losses below \$0.

The long put option should be more expensive than the short forward contract (considering the premium for the put option and zero initial cost for the forward). This is because the long put option provides downside protection that the short forward does not. The put option limits losses when the spot price increases above the strike price (\$48), whereas the short forward position has unlimited potential losses as the spot price rises. Investors are willing to pay a premium for the limited downside risk offered by the put option compared to the unlimited risk exposure of a short forward position beyond the agreed forward price.

Therefore, to acquire the downside protection inherent in the put option, one must pay a premium, making it "more expensive" in terms of upfront cost or value compared to entering a short forward contract at no initial cost but with unbounded potential loss.

In summary, the long put is more expensive because it offers limited downside risk, unlike the short forward position.

2

(a)

The forward curve shows an initial upward trend from December Year 0 to June Year 1, likely due to the cost of carry (interest and storage) increasing with time, and potentially expectations of rising spot prices. The significant drop in September Year 1 could be attributed to seasonality in widget supply or demand, or a specific market event expected to lower prices around that time. The curve then resumes an upward trend from September Year 1 to June Year 3, indicating that longer-term factors like cost of carry and general price appreciation expectations dominate again after the short-term anomaly in September.

(b)

1. Initial Investment (Year 0 December): Assume spot price $S_0 = \$3.00$.
2. Revenue at Close (March Year 1): Forward price $F_{0,0.25} = \$3.075$.
3. Storage Cost (at Year 1 March): \$0.03.
4. Net Cash Inflow at Year 1 March: $\$3.075 - \$0.03 = \$3.045$.
5. Return over 3 months: $\frac{\$3.045-\$3.00}{\$3.00} = 0.015 = 1.5\%$.
6. Annualized continuously compounded rate of return: $\frac{\ln(1.015)}{0.25} \approx 5.96\%$.

Yes, the annualized rate of return of approximately $\boxed{5.96\%}$ is sensible. It is very close to the risk-free interest rate of 6%, which is expected in an efficient market for a cash-and-carry strategy, after accounting for storage costs. The slight difference is likely due to rounding or minor market frictions not considered.

3

(a)

In this situation, the lease rate for widgets would likely be very close to zero. Because widgets don't deteriorate and storage is free, merchants can keep a large supply at no cost. Since they can adjust production, they can easily replace any lent widgets. Lending doesn't really cost them anything. With many merchants able to lend, competition would push the lease rate down to almost zero.

(b)

Even with seasonal demand, the lease rate will probably stay low on average, but it might change a little with the seasons. The basic reasons for a low rate still apply: cheap storage and flexible production. During high demand seasons, borrowing widgets to short sell might increase, so merchants could maybe charge a slightly higher lease rate then. But, because supply can adjust and storage is cheap, these changes would probably be small, and the average lease rate would be near zero.

(c)

This changes things a lot. The lease rate will now likely change with the seasons and be more up and down. When demand is high but production is fixed, widgets become less available. Merchants lose more by lending widgets when they could be selling them at higher prices. So, the lease rate will likely go up a lot during peak demand times. When demand is low, merchants might be more willing to lend at a lower rate.

The timing of your short sale matters. If you borrow when demand is high for a short time, the lease rate will be highest. If you borrow for a longer period that includes a high demand season, the average lease rate over that time will be higher too. If you borrow only during low demand times, you'll probably get a lower lease rate.

(d)

Here, the lease rate will relate to the production season, and how long you borrow will affect the rate. When production is high and demand is steady, there are lots of new widgets. Merchants will be happy to lend them out at a low lease rate, maybe even close to zero. When production is low (off-season), widgets become less common in terms of new supply. Even with steady demand, this limited production can make widgets more valuable. The lease rate will likely increase when production is low.

If you borrow for a short time during the production off-season, you will likely pay the highest lease rate. Borrowing for a period that includes the off-season will mean a higher average rate. Borrowing only during peak production will get you the lowest rates.

(e)

If widgets can't be stored, the lease rate becomes much more unstable and depends heavily on the immediate balance of supply and demand. Seasonal effects in demand or production become much stronger. If demand is higher than current production, lease rates can jump up a lot because borrowing is the only way to meet needs. If production is higher than demand, lease rates can drop sharply, maybe even become negative to encourage widgets to move out. The lease rate will change a lot with any small changes in demand or production because there's no stored inventory to smooth things out. It's less likely the average lease rate will be near zero; it will be more of a key factor in balancing supply and demand day to day.

4

(a)

To find the no-arbitrage forward price for delivery in 9 months, we can use the formula $F_0 = S_0 e^{rT}$. Here, the current spot price S_0 is \$1100, the continuously compounding risk-free rate r is 5% or 0.05, and the time to delivery T is 9 months, or 0.75 years. Plugging these values in, we calculate $F_0 = 1100 \times e^{(0.05 \times 0.75)}$, which is approximately $1100 \times e^{0.0375}$. Evaluating this, we get about 1100×1.038208, resulting in a forward price of approximately \$1142.03. Therefore, the no-arbitrage forward price for delivery in 9 months is about \$1142.03.

(b)

If a customer wishes to enter a short index futures position and you, as the market-maker, take the opposite long position, you can hedge this risk by creating a synthetic short forward position. To do this, you would first short the S&R Index, effectively selling \$1100 worth of it. Then, you would take the \$1100 proceeds from this short sale and invest them at the risk-free rate of 5% per annum, compounded continuously, for the 9-month duration. Let's see how this works as a hedge. At time 0, you enter into a long forward contract, short sell the S&R index and receive \$1100, and invest that \$1100 at the risk-free rate. Now consider the value at delivery in 9 months. Let S_T be the spot price at that time. The payoff from your long forward contract will be $S_T - 1142.03$. The investment of \$1100 will have grown to $1100 \times e^{0.05 \times 0.75}$, which is approximately \$1142.03. Finally, you need to cover your short index position, which will cost you S_T. The net value at delivery is then the sum of these: $(S_T - 1142.03) + 1142.03 - S_T$, which simplifies to 0. This shows that regardless of the spot price S_T at delivery, your net position will have a value of zero, demonstrating a risk-free hedge.

(c)

If a customer wishes to enter a long index futures position, and you take the opposite short position as the market-maker, you can hedge this short forward position by creating a synthetic long forward. The strategy here is to buy the S&R Index, purchasing \$1100 worth, and finance this purchase by borrowing \$1100 at the risk-free rate of 5% per annum, continuously compounded, for 9 months. Let's examine how this hedges. At time 0, you enter into a short forward contract, buy the S&R index costing \$1100, and borrow \$1100 to pay for it. At delivery in 9 months, again let S_T be the spot price. The payoff from your short forward contract will be $1142.03 - S_T$. The value of the S&R index you bought is now S_T. And you will need to repay the borrowed amount, which has grown to $1100 \times e^{0.05 \times 0.75}$, or approximately

\$1142.03. The net value at delivery is the sum of these: $(1142.03 - S_T) + S_T - 1142.03$, which also simplifies to 0. This demonstrates that this strategy also creates a risk-free hedge, with a net value of zero at delivery regardless of the spot price S_T.

5

(a)

A forward curve for oil where the forward prices decrease as the maturity increases and are below the spot price is called an inverted forward curve or a curve in backwardation.

To estimate the lease rate for different maturities, we can use the cost of carry model and the given forward prices and interest rates. We can calculate the approximate lease rate for each maturity using the formula: $l = r - \frac{1}{T}\ln(F_0/S_0)$.

For the 3-month maturity, the lease rate is approximately 9.69% per annum. For the 6-month maturity, it is about 8.40% per annum. For the 1-year maturity, it's around 6.42% per annum. For the 3-year maturity, approximately 3.45% per annum. And for the 5-year maturity, it is about 3.77% per annum.

The lease rate is positive and generally decreases as the maturity extends, suggesting the market values immediate oil availability more highly, and this premium diminishes over time.

(b)

Several factors could explain this inverted forward curve for oil. One key reason is a high convenience yield driven by short-term supply concerns. If there are worries about immediate oil availability due to geopolitical events or production issues, market participants might value having physical oil right now, leading to a higher spot price compared to future delivery prices.

Another reason could be expectations that oil prices will fall in the future. If the market anticipates increased future supply or decreased demand, perhaps due to new energy technologies or economic shifts, then forward prices for later delivery would be lower to reflect these expectations.

Storage constraints could also play a role. If storing oil for longer periods becomes more costly or difficult, it might push down longer-dated forward prices relative to the spot price.

Market sentiment and risk aversion can also contribute. In uncertain times, holding physical oil might be seen as less risky than holding longer-term contracts, again pushing spot prices up and forward prices down. It's likely a combination of these factors that results in the observed inverted forward curve.

(c)

Even if oil forward prices seem out of line, arbitrage can be difficult. Transaction costs like fees and bid-ask spreads reduce profits. Storage for physical oil is expensive and complex. Transportation and logistics add further costs. Arbitrage takes time, and prices can move unfavorably before it's complete due to market volatility. Counterparty risk in forward contracts also exists.