TIØ4140 Project Evaluation and Financing	**Exercise 1**
26.01.2025	**Scribe: Adrian Fagerland**

1

(a)

1. The selling price: $400 \times \$25.31 = \$10,124$
2. The buying price: $400 \times \$22.87 = \$9,148$
3. The profit: $\$10,124 - \$9,148 = \$976$

The profit earned is $\boxed{\$976}$.

(b)

$$\text{Selling Comm.} = 0.3\% \times \text{Selling Price} = 0.003 \times \$10,124 = \$30.372$$

$$\text{Buying Comm.} = 0.3\% \times \text{Buying Price} = 0.003 \times \$9,148 = \$27.444$$

$$\text{Total Comm.} = \text{Selling Comm.} + \text{Buying Comm.} = \$30.372 + \$27.444 = \$57.816$$

$$\text{Profit with Comm.} = \text{Profit without Comm.} - \text{Total Comm.} = \$976 - \$57.816 = \$918.184$$

Rounding to two decimal places, the profit is $\boxed{918.18}$

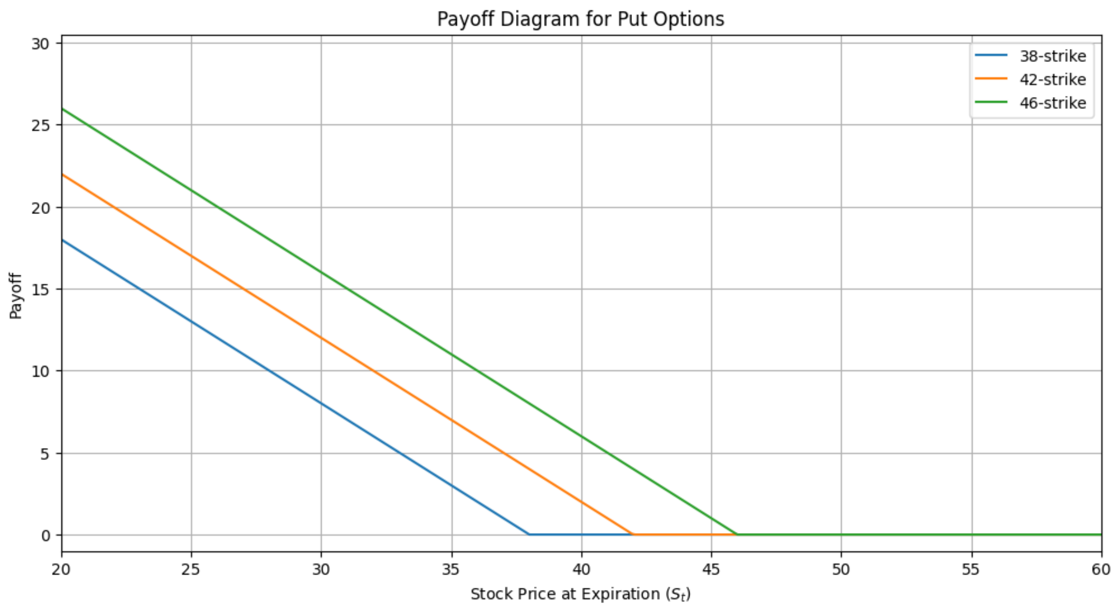
(c)

The initial proceeds from the short sale is $\$10,124$ (from part a) The interest lost using the 6-month interest rate of 3%: $3\% \times \$10,124 = 0.03 \times \$10,124 = \$303.72$

The interest lost during the 6 months is $\boxed{\$303.72}$.

2

(a)


Payoff Diagram for Put Options
38-strike
42-strike
46-strike
Payoff
Stock Price at Expiration (S_t)


Profit Diagram for Put Options
38-strike
42-strike
46-strike
Profit
Stock Price at Expiration (S_t)

(b)

We observe that as the strike price of the put options increases from \$38 to \$46, the premium also rises, from \$1.72 to \$6.10. This is because higher strike put options offer greater potential payoff. For any stock price below the strike, a put with a higher strike will always yield a larger payoff. Consequently, the breakeven stock price, which is the strike price minus the premium, also increases with the strike price, indicating a wider range of stock prices where the option can be profitable. While the maximum possible loss is limited to the premium paid for all options, the potential profit increases with higher strike prices when the stock price falls significantly.

The reason for the increasing premiums is primarily due to the higher intrinsic value and greater probability of higher strike put options finishing in-the-money. For a given current stock price, a higher strike put has a higher or equal intrinsic value. Furthermore, it is more probable that the stock price will fall below a higher strike price like \$46, compared to a lower one like \$38. This increased likelihood of a positive payoff, along with the greater potential for profit in a significant downturn, justifies the higher premium for put options with higher strike prices. Essentially, investors pay more for increased downside protection and a higher chance of the option being valuable at expiration.

3

(a)

Given that the premiums on the call and put options are approximately the same and cancel each other out, and they have the same strike price and time to expiration, the strike price must be close to the at-the-money level. More precisely, the strike price is likely near the forward price of the stock for the expiration date of the options. This is implied by put-call parity: if call and put premiums are equal, $C \approx P$, then $S_0 - Ke^{-rT} \approx 0$, so $K \approx F_0$.

(b)

The position created by purchasing a call and selling a put option with the same strike and expiration is a synthetic long stock position, or a synthetic purchased stock.

(c)

For a synthetic purchased stock with zero net premium inclusive of the bid-ask spread, the strike price will likely be very close to the forward price. To achieve a zero net premium, the ask price of the call (cost) must be approximately equal to the bid price of the put (revenue). This condition is generally met when the strike price is near the forward price.

(d)

Similarly, for a synthetic short stock with zero net premium inclusive of the bid-ask spread, the strike price will also be very close to the forward price. To achieve zero net premium, the bid price of the call (revenue) must be approximately equal to the ask price of the put (cost), which again occurs when the strike price is near the forward price.

(e)

No, the "transaction fees" are not really "a wash". The quote is an oversimplification. "Transaction fees" here primarily refer to the bid-ask spread, which is a real cost. When creating a synthetic stock, you buy at the ask price and sell at the bid price, inherently incurring the bid-ask spread. Even if the net premium is zero inclusive of the bid-ask spread, it just means the initial cash flow is minimal, but the cost of the bid-ask spread is still present. Furthermore, this statement ignores other transaction fees like brokerage commissions and exchange fees, which are additional costs not "washed away".

4

(a)

After constructing the binomial tree for the stock price, we get that the stock prices at 6 months are: $S_{uu} = \$56.18$, $S_{ud} = S_{du} = \$50.35$, $S_{dd} = \$45.125$.

We use the formula for risk-neutral probabilities: $p = \frac{e^{r\Delta t} - d}{u - d}$, where $r = 5\%$, $\Delta t = 0.25$, $u = 1.06$, $d = 0.95$. Calculating, we get $p \approx 0.56889$ and $1 - p \approx 0.43111$.

For the call option payoffs at expiration, we have: $C_{uu} = \max(S_{uu} - K, 0) = \max(56.18 - 51, 0) = \$5.18$, $C_{ud} = C_{du} = \max(S_{ud} - K, 0) = \max(50.35 - 51, 0) = \$0$, $C_{dd} = \max(S_{dd} - K, 0) = \max(45.125 - 51, 0) = \$0$.

Working backwards through the tree, we calculate the call option values at Time 3 months: $C_u = e^{-r\Delta t}[pC_{uu} + (1-p)C_{ud}] \approx \$2.9103$, $C_d = e^{-r\Delta t}[pC_{ud} + (1-p)C_{dd}] = \$0$.

Finally, the value of the 6-month European call option at Time 0 is: $C_0 = e^{-r\Delta t}[pC_u + (1-p)C_d] \approx \$1.6359$.

The value of the 6-month European call option is approximately $\boxed{\$1.64}$.

(b)

For the put option payoffs at expiration, we have: $P_{uu} = \max(K - S_{uu}, 0) = \max(51 - 56.18, 0) = \$0$, $P_{ud} = P_{du} = \max(K - S_{ud}, 0) = \max(51 - 50.35, 0) = \$0.65$, $P_{dd} = \max(K - S_{dd}, 0) = \max(51 - 45.125, 0) = \$5.875$.

Working backwards through the tree, we calculate the put option values at Time 3 months: $P_u = e^{-r\Delta t}[pP_{uu} + (1-p)P_{ud}] \approx \$0.2767$, $P_d = e^{-r\Delta t}[pP_{ud} + (1-p)P_{dd}] \approx \$2.8676$.

Finally, the value of the 6-month European put option at Time 0 is: $P_0 = e^{-r\Delta t}[pP_u + (1-p)P_d] \approx \$1.3766$.

The value of the 6-month European put option is approximately $\boxed{\$1.38}$.

(c)

Using the call-put parity formula: $C + Ke^{-rT} = P + S_0$, we calculate $Ke^{-rT} = 51 \times e^{-0.05\times 0.5} \approx \$49.7408$.

Then, we check both sides of the equation: $C + Ke^{-rT} = \$1.6359 + \$49.7408 = \$51.3767$
$P + S_0 = \$1.3766 + \$50 = \$51.3766$

Since $\$51.3767 \approx \$51.3766$, call-put parity is verified.