TIØ4140 Project Evaluation and Financing

Exercise 7: Delta Hedging and Exotic Options

Posted: Monday, March 10, 2025, Morning.
Deadline: Tuesday, March 18, 2025, 23:59.
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Overview of Tasks

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  - **Suggested reading:** Valuation of powered options – Esser, 2003.

Mandatory Tasks

- ## Problems from McDonald (2013)

Chapter 16: Market-Making and Delta-Hedging

In the following problems assume, unless otherwise stated, that S = \$40, σ = 30%, r = 8%, and $\delta = 0$.

16.7. Consider a 40-strike 180-day call with S = \$40. Compute a delta-gamma-theta approximation for the value of the call after 1, 5, and 25 days. For each day, consider stock prices of \$36 to \$44.00 in \$0.25 increments and compare the actual option premium at each stock price with the predicted premium. Where are the two the same?

Chapter 17: Exotic Options

17.11. Consider a stock with its price today equal to S_0 = \$40, its volatility given by $\sigma = 0.30$ and no dividend yield (i.e. $\delta = 0$). Consider an option with its strike price K = \$40, and let the annual risk-free rate be $r = 0.08$.

- **a.** What is the price of a standard European call with 2 years to expiration?
- **b.** Suppose you have a compound call giving you the right to pay \$2 one year from today to buy the option in part (a). For what stock prices in one year will you exercise this option?
- **c.** What is the price of this compound call?
- **d.** What is the price of a compound option giving you the right to *sell* the option in part (a) in 1 year for \$2?

Chapter 21: Real Options

21.13. A project has certain cash flows today of \$1, growing at 5% per year for 10 years, after which the cash flow is constant. The risk-free rate is 5%. The project costs \$20 and cash flows begin 1 year after the project is started. When should you invest and what is the value of the option to invest?

21.14. Consider the oil project with a single barrel, in which S = \$15, r = 5%, δ = 4%, and X = \$13.60. Suppose that, in addition, the land can be sold for the residual value of R = \$1 after the barrel of oil is extracted. What is the value of the land?

Exam problem

A market-maker sells 100 2-year European gap call options, and delta-hedges the position with shares. Each gap call option is written on 1 share of a non-dividend-paying stock. The current price of the stock is 100. The stock's volatility is 80% and the risk-free rate is 2%. Each gap call option has a strike price of 150. Each gap call option has a payment trigger of 100.

a) Determine the current price of one European gap call option.
b) Express the price of the European gap call option in terms of the current price of the otherwise equivalent plain vanilla call option.

Voluntary Tasks

- ## Problems from McDonald (2013)

Chapter 17: Exotic Options

17.20. A **chooser option** (also known as an **as-you-like-it option**) becomes a put or call at the discretion of the owner. For example, consider a chooser on the S&R index for which both the call, with value $C\ (S_t, K, T - t)$, and the put, with value $P\ (S_t, K, T - t)$, have a strike price of K. The index pays no dividends. At the choice date, t_1, the payoff of the chooser is:

$$\text{Max}\ [C\ (S_{t1}, K, T - t_1), P\ (S_{t1}, K, T - t_1)]$$

a. If the chooser option and the underlying options expire simultaneously, what ordinary option position is this equivalent to?

b. Suppose that the chooser must be exercised at t_1 and that the underlying options expire at T. Show that the chooser is equivalent to a call option with strike price K and maturity T plus $e^{-\delta(T-t1)}$ put options with strike price $Ke^{-(r-\delta)\ (T-t1)}$ and expiration t_1.

Chapter 25: Brownian Motion and Itô's Lemma

25.14. Assume that one stock follows the process,

$$\frac{dS}{S} = \alpha dt + \sigma dZ$$

Another stock follows the process,

$$\frac{dQ}{Q} = \alpha_Q dt + \sigma dZ + dq_1 + dq_2$$

(Note that the σdZ terms for S and Q are identical.)

Neither stock pays dividends. dq_1 and dq_2 are both Poisson jump processes with Poisson parameters λ_1 and λ_2. Conditional on either jump occurring, the percentage change in the stock price is $Y_1 - 1$ or $Y_2 - 1$. Consider the two stock price processes given above.

a. If there no jump terms (i.e., $\lambda_1 = \lambda_2 = 0$), what would be the relation between α and α_Q?

b. Suppose there is just one jump term ($\lambda_2 = 0$) and that $Y_1 > 1$. In words, what does it mean to have $Y_1 > 1$? What can you say about the relation between α and α_Q?

c. Write an expression for α_Q when both terms are nonzero. Explain intuitively why α_Q might be greater or less than α.

- ## Exam Problem

Stock options traded at Oslo Stock Exchange are specified as follows:

Contract type:	Option contract with delivery of underlying instrument.
Option type:	American.
Contract size:	100 of the underlying stocks.
Exercise price:	Price in NOK per underlying stock.

For vanilla options, option Greeks for American options are very close to those of European options. In this task, we assume they are identical.

An option portfolio is to be analysed. There is a long position in 90 put options with maturity in June (in 0.12 years), and a short position of 40 call options with maturity in September (in 0.37 years). Strike prices are 215 NOK for the June options and 220 NOK for the September options. There are no dividends, and the risk-free interest rate is 3.00%. The price of the underlying stock is 214 NOK. Volatility is 27%.

1. What is delta, gamma, and vega for the current portfolio? Use the definition of vega that is usual in the derivatives market. (Note that the contract size affects the answers.)
2. In order to make the portfolio both delta- and vega-neutral, which positions should the trader make in a September put option (strike 220) and in the underlying stock? Long or short, and how many, etc.?

References

McDonald, R. L. (2013). *Derivatives markets*. 3rd. ed., New International Edition. Pearson Education.

Esser, A. (2003). General valuation principles for arbitrary payoffs and applications to power options under stochastic volatility. *Financial Markets and Portfolio Management*, 17(3):351–372.