TIØ4140 Project Evaluation and Financing

Exercise 4: Binomial option pricing and lognormality

Posted: Monday, February 3, 2025.
Deadline: Tuesday, February 11, 2025, 23:59.
Grading: Approved / Not approved.

N.B.:

- *To get "Approved", you should attempt to solve all mandatory tasks and have **70%** correct.*
- *Remember to write down the **main solution steps**!*

Overview of Tasks

- **Mandatory Tasks**:
  - **Ch. 14:** Binomial Option Pricing: Selected Topics 4, 12
  - Exam problem 1
  - Exam problem 2
- **Voluntary Tasks:**
  - **Ch. 23:** The Lognormal Distribution 12
  - Exercises from other sources

Mandatory Tasks

Problems from McDonald (2013)

Chapter 14: Binomial Option Pricing: Selected Topics

14.4. Consider a one-period binomial model with $h = 1$, where $S = \$100$, $r = 0.08$, $\sigma = 30\%$, and $\delta = 0$. Compute American put option prices for $K = \$100$, \$110, \$120, and \$130.

a) At which strike(s) does early exercise occur?

b) Use put-call parity to explain why early exercise does not occur at the other strikes.

11.12. [Note: the text in this exercise is edited compared to the original exercise in McDonald (2013). The $n = 10$ has been changed to $n = 25$ to be in accordance with Figure 8, but the other parameter values as well as the question itself have essentially remained the same.]

Let $S_0 = \$100$, $\sigma = 0.30$, $r = 0.08$, $t = 1$ and $\delta = 0$. Compute the probability of reaching each terminal node i using the following equation (equation (12), page 349, in McDonald (2013)):

$$\text{Probability of reaching the } i \text{ th node} = (p^*)^{n-i} \times (1 - p^*)^i \times \frac{n!}{(n-i)!i!}$$

where n is the number of nodes and p^* the risk-neutral probability of an up-move in the tree. Use $S \cdot u^{n-i} \cdot d^i$, where u and d denote the up- and down-factor respectively, to compute the price at the node i. Plot the risk-neutral distribution of year-1 stock prices as Figures 7 and 8 (McDonald, 2013, p. 348-349) for $n = 3$ and $n = 25$.

Exam problems

Exam problem 1

Consider a stock with an initial price of 500 NOK and annual volatility of 10%. The stock pays 3% dividend, which is continuously compounded. The continuously compounded risk-free interest rate is 8%. You can buy an American call option that expires after three years and has a strike equal to 500 NOK.

a) Consider the binomial tree below, in which each step has a length of one year. Determine the correct stock prices according to the binomial tree below, using the Cox-Ross-Rubinstein (CRR) approach.

$S_{uuu} = \square$

$S_{uu} = \square$

$S_u = \square$ $S_{uud} = \square$

$S = 500$ $S_{ud} = \square$

$S_d = \square$ $S_{udd} = \square$

$S_{dd} = \square$

$S_{ddd} = \square$

b) Again consider the binomial tree in which each step has a length of one year. Fill in the correct values of the standard American call in the binomial tree below, using the Cox-Ross-Rubinstein (CRR) approach.

$C_{uuu} = \square$

$C_{uu} = \square$

$C_u = \square$ $C_{uud} = \square$

$C = \square$ $C_{ud} = \square$

$C_d = \square$ $C_{udd} = \square$

$C_{dd} = \square$

$C_{ddd} = \square$

Exam problem 2

Consider the standard lognormal model for the stock price *S*, with dynamics given as follows:

$$ln\left(\frac{S_t}{S_0}\right) = \left(\alpha - \delta - \frac{1}{2}\sigma^2\right) \times t + \sigma \times \sqrt{t} \times Z$$

where *Z* denotes a standard normal random variable. Note that this expression is identical to Chapter 23, Equation (19) in McDonald (2013, p.568).

Assume that $\alpha = 10\%$, $\delta = 1\%$, and $\sigma = 30\%$. If the stock price today is $S_0 = 50$, determine:

a. The expected stock price in 3 months.

b. A 95% confidence interval for the stock price in 3 months.

c. The probability that the stock price in 3 months is higher than 55.

Voluntary Tasks

- # Problems from McDonald (2013)

Chapter 23: The Lognormal Distribution

23.12. [Note: the text in this exercise is extended and edited compared to the original exercise in McDonald (2013). Unlike McDonald (2013), the values for T for the calculations have been specified here, but both the parameter values and the question itself have essentially remained the same.] Let S_t to be the price of a stock at time t, being lognormally distributed, i.e.

$$\ln\left(\frac{S_t}{S_0}\right) \sim \mathcal{N}\left((\alpha - \delta - 0.5\sigma^2)t, \sigma^2 t\right)$$

as stated in equation (18) in McDonald (2013). For this task, assume $S_0 = \$100$, $\alpha = 0.08$, $\sigma = 0.30$ and $\delta = 0$.

Let $K_T = S_0 \cdot e^{r \cdot T}$. Compute P $[S_T < K_T]$ and P $[S_T > K_T]$ for $T = 0.25$, $T = 1$, $T = 2.5$, $T = 10$ and $T = 25$ years. How do the probabilities behave? How do you reconcile your answer with the fact that *both* the call and put prices increase with time?

- # Exercises from other sources

1. Calculate the price of a three-month European put option on a stock with strike price of \$50; when the current stock price is \$50, risk-free interest rate is 10% per annum, the volatility is 30% per annum, with having an expected dividend of \$1.50 in two months.
2. Explain carefully why Black's approach to evaluating an American call option on a dividend-paying stock may give an approximate answer even when only one dividend is anticipated. Does the answer given by Black's approach understate or overstate the true option value? Explain your answer.
3. Consider an American call option on a stock. The stock price is \$50, the time to maturity is 15 months, the risk-free rate of interest is 8% per annum, the exercise price is \$55, and the volatility is 25%. Dividends of \$1.50 are expected in 4 months and 10 months. Show that it can never be optimal to exercise the option on either of two dividend dates. Calculate the price of the option.

References

McDonald, R. L. (2013). *Derivatives markets*. 3rd. ed., New International Edition. Pearson Education.