

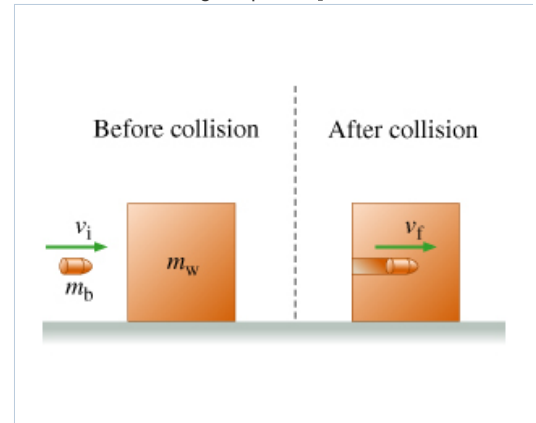
## HW06

Due: 11:59pm on Monday, November 18, 2024

You will receive no credit for items you complete after the assignment is due. [Grading Policy](#)

## A Bullet Is Fired into a Wooden Block

A bullet of mass  $m_b$  is fired horizontally with speed  $v_i$  at a wooden block of mass  $m_w$  resting on a frictionless table. The bullet hits the block and becomes completely embedded within it. After the bullet has come to rest relative to the block, the block, with the bullet in it, is traveling at speed  $v_f$ .



## Part A

Which of the following *best* describes this collision?**Hint 1.** Types of collisions

An inelastic collision is a collision in which kinetic energy is not conserved. In a *partially* inelastic collision, kinetic energy is lost, but the objects colliding do not stick together. From this information, you can infer what completely inelastic and elastic collisions are.

ANSWER:

- ☐ perfectly elastic
- ☐ partially inelastic
- ☒ perfectly inelastic

**Correct**

## Part B

Which of the following quantities, if any, are conserved during this collision?

**Hint 1.** When is kinetic energy conserved?

Kinetic energy is conserved only in perfectly elastic collisions.

ANSWER:

- ☐ kinetic energy only  
☒ momentum only  
☐ kinetic energy and momentum  
☐ neither momentum nor kinetic energy

Correct

### Part C

What is the speed of the block/bullet system after the collision?

Express your answer in terms of  $v_i$ ,  $m_w$ , and  $m_b$ .

**Hint 1.** Find the momentum after the collision

What is the total momentum  $p_{\text{total}}$  of the block/bullet system after the collision?

Express your answer in terms of  $v_f$  and other given quantities.

ANSWER:

$$p_{\text{total}} = (m_w + m_b) v_f$$

**Hint 2.** Use conservation of momentum

The momentum of the block/bullet system is conserved. Therefore, the momentum before the collision is the same as the momentum after the collision. Find a second expression for  $p_{\text{total}}$ , this time expressed as the total momentum of the system before the collision.

Express your answer in terms of  $v_i$  and other given quantities.

ANSWER:

$$p_{\text{total}} = m_b v_i$$

ANSWER:

$$v_f = \frac{m_b v_i}{m_b + m_w}$$

Correct

## A Game of Frictionless Catch

Chuck and Jackie stand on separate carts, both of which can slide without friction. The combined mass of Chuck and his cart,  $m_{\text{cart}}$ , is identical to the combined mass of Jackie and her cart. Initially, Chuck and Jackie and their carts are at rest.

Chuck then picks up a ball of mass  $m_{\text{ball}}$  and throws it to Jackie, who catches it. Assume that the ball travels in a straight line parallel to the ground (ignore the effect of gravity). After Chuck throws the ball, his speed relative to the ground is  $v_c$ . The speed of the thrown ball relative to the ground is  $v_b$ .

Jackie catches the ball when it reaches her, and she and her cart begin to move. Jackie's speed relative to the ground after she catches the ball is  $v_j$ .

When answering the questions in this problem, keep the following in mind:

1. The original mass  $m_{\text{cart}}$  of Chuck and his cart does not include the mass of the ball.
2. The speed of an object is the magnitude of its velocity. An object's speed will always be a nonnegative quantity.

### Part A

Find the relative speed  $u$  between Chuck and the ball after Chuck has thrown the ball.

Express the speed in terms of  $v_c$  and  $v_b$ .

#### Hint 1. How to approach the problem

All this question is asking is: "How fast are Chuck and the ball moving away from each other?" If two objects are moving at the same speed (with respect to the ground) in the same direction, their relative speed is zero. If they are moving at the same speed,  $v$ , in opposite directions, their relative speed is  $2v$ . In this problem, you are given variables for the speed of Chuck and the ball with respect to the ground, and you know that Chuck and the ball are moving directly away from each other.

ANSWER:

$$u = v_c + v_b$$

#### Correct

Make sure you understand this result; the concept of "relative speed" is important. In general, if two objects are moving in opposite directions (either toward each other or away from each other), the relative speed between them is equal to the sum of their speeds with respect to the ground. If two objects are moving in the same direction, then the relative speed between them is the absolute value of the difference of their two speeds with respect to the ground.

### Part B

What is the speed  $v_b$  of the ball (relative to the ground) while it is in the air?

Express your answer in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ , and  $u$ .

#### Hint 1. How to approach the problem

Apply conservation of momentum. Equate the initial (before the ball is thrown) and final (after the ball is thrown) momenta of the system consisting of Chuck, his cart, and the ball. Use the result from Part A to eliminate  $v_c$  from this equation and solve for  $v_b$ .

#### Hint 2. Initial momentum of Chuck, his cart, and the ball

Before the ball is thrown, Chuck, his cart, and the ball are all at rest. Therefore, their total initial momentum is zero.

#### Hint 3. Find the final momentum of Chuck, his cart, and the thrown ball

What is the total momentum  $p_{\text{final}}$  of Chuck, his cart, and the ball after the ball is thrown?

Express your answer in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ ,  $v_c$ , and  $v_b$ .

Remember that  $v_c$  and  $v_b$  are speeds, not velocities, and thus are positive scalars.

ANSWER:

$$p_{\text{final}} = -m_{\text{cart}}v_c + m_{\text{ball}}v_b$$

ANSWER:

$$v_b = \frac{m_{\text{cart}}u}{m_{\text{cart}} + m_{\text{ball}}}$$

#### Correct

### Part C

What is Chuck's speed  $v_c$  (relative to the ground) after he throws the ball?

Express your answer in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ , and  $u$ .

#### Hint 1. How to approach the problem

Use the answer to Part B to eliminate  $v_b$  from the equation derived in Part A. Then solve for  $v_c$ .

ANSWER:

$$v_c = \frac{m_{\text{ball}} u}{m_{\text{cart}} + m_{\text{ball}}}$$

Correct

**Part D**Find Jackie's speed  $v_j$  (relative to the ground) after she catches the ball, in terms of  $v_b$ .Express  $v_j$  in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ , and  $v_b$ .**Hint 1. How to approach the problem**

Apply conservation of momentum. Equate the initial (before Jackie catches the ball) and final (after the ball is caught) momenta of the system consisting of Jackie, her cart, and the ball, and solve for  $v_j$ .

**Hint 2. Initial momentum**

Just before Jackie catches the ball, the momentum of the system consisting of Jackie, her cart, and the ball is equal to the momentum of the ball as it flies through the air:  $p_{\text{initial}} = m_{\text{ball}} v_b$ .

**Hint 3. Find the final momentum**

What is the final momentum  $p_{\text{final}}$  of the system after Jackie catches the ball?

Express your answer in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ , and  $v_j$ .

ANSWER:

$$p_{\text{final}} = (m_{\text{cart}} + m_{\text{ball}}) v_j$$

ANSWER:

$$v_j = \frac{v_b m_{\text{ball}}}{m_{\text{ball}} + m_{\text{cart}}}$$

Correct

**Part E**Find Jackie's speed  $v_j$  (relative to the ground) after she catches the ball, in terms of  $u$ .Express  $v_j$  in terms of  $m_{\text{ball}}$ ,  $m_{\text{cart}}$ , and  $u$ .**Hint 1. How to approach the problem**

In Part B, you found an expression for  $v_b$  in terms of  $u$ . You can substitute this expression for  $v_b$  into the equation you found in Part D, which will give you an expression for  $v_j$  in terms of the desired quantities.

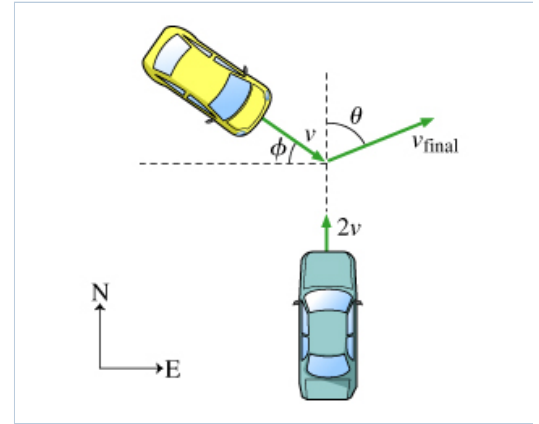
ANSWER:

$$v_j = \frac{m_{\text{ball}} m_{\text{cart}} u}{(m_{\text{cart}} + m_{\text{ball}})^2}$$

Correct

**Collision at an Angle**

Two cars, both of mass  $m$ , collide and stick together. Prior to the collision, one car had been traveling north at speed  $2v$ , while the second was traveling at speed  $v$  at an angle  $\phi$  south of east (as indicated in the figure). After the collision, the two-car system travels at speed  $v_{\text{final}}$  at an angle  $\theta$  east of north.



### Part A

Find the speed  $v_{\text{final}}$  of the joined cars after the collision.

Express your answer in terms of  $v$  and  $\phi$ .

#### Hint 1. Determine the conserved quantities

Which of the following statements is true for the collision described?

ANSWER:

- ☒ Momentum is conserved but kinetic energy is not conserved.
- ☐ Kinetic energy is conserved but momentum is not conserved.
- ☐ Both kinetic energy and momentum are conserved.
- ☐ Neither kinetic energy nor momentum is conserved.

#### Hint 2. The component of the final velocity in the east-west direction

Find the component of  $\vec{v}_{\text{final}}$  in the east-west direction.

Express your answer in terms of  $v$  and  $\phi$ .

#### Hint 1. Find the east-west component of the initial momentum

What is  $p_e$ , the magnitude of the total momentum  $\vec{p}_e$  of the two cars in the east-west direction? (Take eastward to be positive, westward negative.)

Express your answer in terms of  $m$ ,  $v$ , and  $\phi$ .

ANSWER:

$$p_e = mv \cos(\phi)$$

ANSWER:

$$v_{\text{final}} (\text{east-west}) = \frac{v \cos(\phi)}{2}$$

#### Hint 3. Find the north-south component of the final momentum

Find the component of  $v_{\text{final}}$  in the north-south direction.

Express your answer in terms of  $v$  and  $\phi$ .

**Hint 1.** Find the north-south component of the initial momentum

What is the magnitude  $p_n$  of the total momentum  $\vec{p}_n$  of the two cars in the north-south direction? (Take northward to be positive, southward negative).

Express your answer in terms of  $m$ ,  $v$ , and  $\phi$ .

ANSWER:

$$p_n = 2mv - mv \sin(\phi)$$

ANSWER:

$$v_{\text{final}} (\text{north-south}) = v - \frac{v \sin(\phi)}{2}$$

**Hint 4.** Math help

Let  $v_{\text{final},x}$  be the east-west component of  $v_{\text{final}}$ , and  $v_{\text{final},y}$  the north-south component. Then

$$v_{\text{final}} = \sqrt{v_{\text{final},x}^2 + v_{\text{final},y}^2}.$$

You will also need to use the following trigonometric identity when you evaluate the right-hand side of the above equation in terms of  $v$  and  $\phi$ :

$$\cos^2 \phi + \sin^2 \phi = 1.$$

ANSWER:

$$v_{\text{final}} = \sqrt{\frac{5v^2}{4} - v^2 \sin \phi}$$

**Correct**

**Part B**

What is the angle  $\theta$  with respect to north made by the velocity vector of the two cars after the collision?

Express your answer in terms of  $\phi$ . Your answer should contain an inverse trigonometric function using the notation  $\text{asin}$ ,  $\text{atan}$  etc. and *not*  $\text{arcsin}$ ,  $\text{arctan}$  etc.

**Hint 1.** A formula for  $\tan \theta$ 

Let  $v_{\text{final},x}$  be the east-west component of  $\vec{v}_{\text{final}}$ , and  $v_{\text{final},y}$  the north-south component. Then

$$\tan \theta = \frac{v_{\text{final},x}}{v_{\text{final},y}},$$

since the angle asked for is the angle east of north.

ANSWER:

$$\theta = \text{atan} \left( \frac{v \cos \phi}{2v - v \sin \phi} \right)$$

**Correct**

## Impulse and Change in Velocity

A glob of very soft clay is dropped from above onto a digital scale. The clay sticks to the scale on impact. A graph of the clay's velocity vs. time,  $v_{\text{clay}}(t)$ , is given, with the upward direction defined as positive.

The experiment is then repeated, but instead of using the clay glob, a superball (an extremely elastic bouncy ball) with identical mass is dropped from the same height onto the scale.

Both the clay and the superball hit the scale 2.9 s after they are dropped. Assume that the duration of the collision is the same in both cases and the force exerted by the scale on the clay and the force exerted by the scale on the superball are constant.

### Part A

Sketch the graph of the superball's velocity vs. time,  $v_{\text{ball}}(t)$ , from the instant it is dropped ( $t = 0$  s) until it bounces to its maximum height ( $t = 6$  s). Assume that the superball undergoes an elastic collision with the scale, and that the scale's recoil velocity is negligible. The green-colored graph already present in the answer window is  $v_{\text{clay}}(t)$ .

#### Hint 1. Velocity vs. time before the collision

The clay ball and superball have exactly the same mass and are dropped from exactly the same point above exactly the same scale. Therefore, the velocity vs. time graphs for the superball and the clay ball should coincide before the collision.

#### Hint 2. Find the speed of the ball as it leaves the scale

Since the superball experiences an elastic collision with the scale, what is the speed of the superball as it leaves the scale's surface?

ANSWER:

- ☐ greater than 28.4 m/s
- ☐ less than 28.4 m/s
- ☒ approximately 28.4 m/s

#### Hint 3. Determine the sign of the ball's velocity after the collision

Keeping in mind the sign convention given in the problem introduction, after the collision with the scale, is the superball's velocity positive or negative?

ANSWER:

- ☒ positive
- ☐ negative

#### Hint 4. Velocity vs. time graph after the collision

The initial velocity of the superball after the collision can be determined based on the statement that the collision with the scale is elastic. After the ball rebounds off of the scale, the superball is in free-fall and should be represented by a velocity vs. time graph illustrating a constant negative acceleration. (Remember that since the positive direction is upwards, the acceleration due to gravity is negative.)

#### Hint 5. Find the time of the ball's collision with the scale

During the collision, how long is the superball in contact with the scale?

Enter your answer in seconds using one significant figure.

#### Hint 1. Identify the time it takes the ball to reach its maximum height

It takes the ball 2.9 s to fall from its maximum height to the scale. How long does it take for the ball to rise from the scale back to its maximum height?

Enter your answer in seconds using two significant figures.

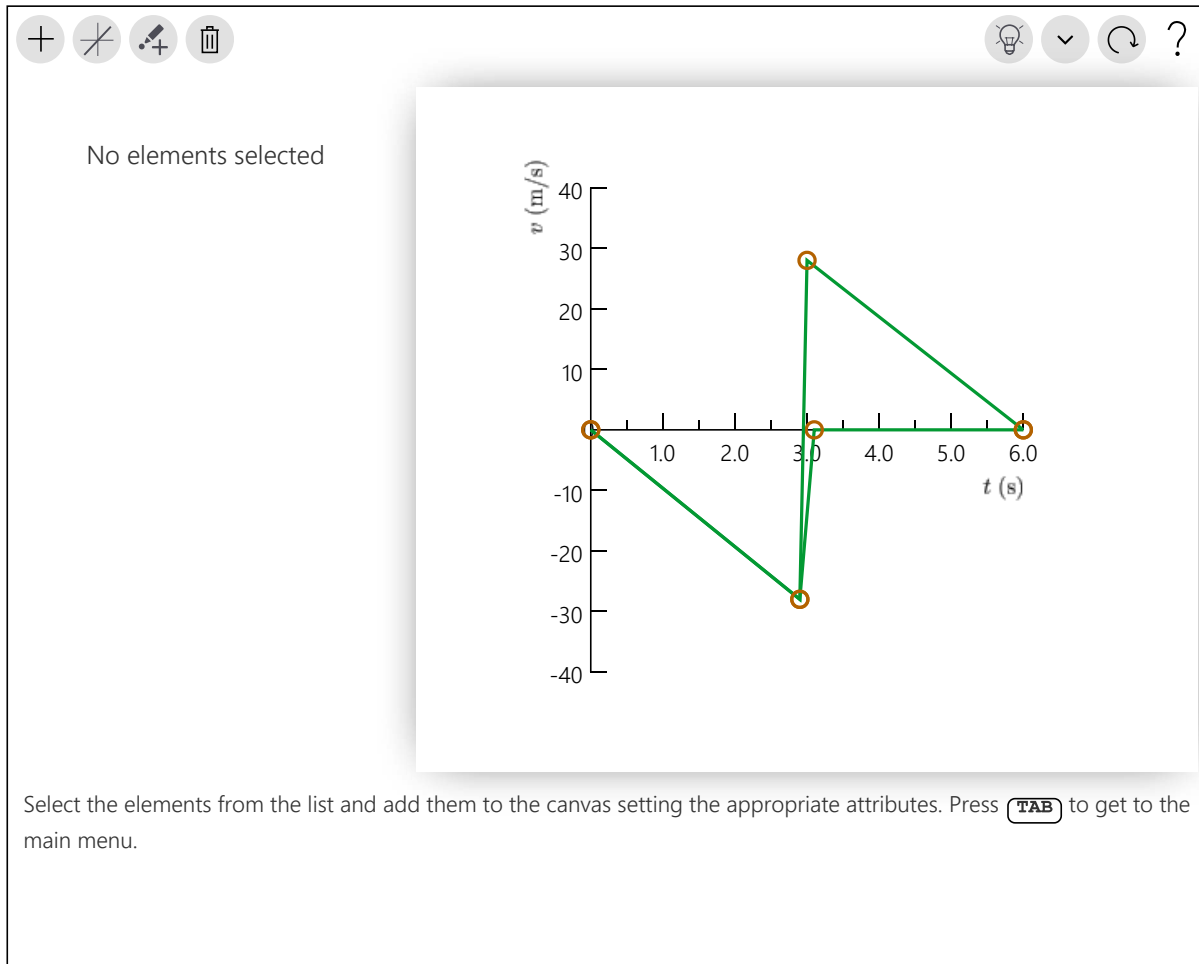
ANSWER:

time to reach maximum height = 2.9 s

ANSWER:

time in contact with scale = 0.2 s

ANSWER:



Correct

### Part B

Based on your graph, is the change in momentum of the superball during its collision with the scale greater than, less than, or equal to the change in momentum of the clay during its collision with the scale?

ANSWER:

- ☒ The change in momentum of the superball is greater than the change in momentum of the clay.
- ☐ The change in momentum of the superball is less than the change in momentum of the clay.
- ☐ The change in momentum of the superball is equal to the change in momentum of the clay.

Correct

### Part C

Is the impulse delivered to the superball during its collision with the scale greater than, less than, or equal to the impulse delivered to the clay during its collision with the scale?

**Hint 1. Relating change in momentum to impulse**

The impulse-momentum theorem tells us that the change in momentum of an object is exactly equal to the impulse that object experiences.

ANSWER:

- ☒ The impulse delivered to superball is greater than the impulse delivered to the clay.
- ☐ The impulse delivered to superball is equal to the impulse delivered to the clay.
- ☐ The impulse delivered to superball is less than the impulse delivered to the clay.

**Correct**

**Part D**

Is the force exerted by the scale on the superball greater than, less than, or equal to the force exerted by the scale on the clay?

**Hint 1. Relating force to impulse for a constant force**

For a constant force the impulse can be calculated as the product of the force applied and the time interval over which the force acts.

ANSWER:

- ☒ The force exerted by the scale on the superball is greater than the force exerted by the scale on the clay.
- ☐ The force exerted by the scale on the superball is less than the force exerted by the scale on the clay.
- ☐ The force exerted by the scale on the superball is equal to the force exerted by the scale on the clay.

**Correct**

**Problem 11.24 - Enhanced - with Expanded Hints**

A package of mass  $m$  is released from rest at a warehouse loading dock and slides down the 5.5-m-high, frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass  $2m$ , from the bottom of the chute.



**Part A**

Suppose the packages stick together. What is their common speed after the collision?

Express your answer with the appropriate units.

**Hint 1. How to approach the problem**

Use the energy conservation to determine the speed of the upper package just before the collision, then use the momentum conservation to determine the speed of the packages after the inelastic collision.

**Hint 2. Simplify: the upper package before the collision**

Determine the velocity of the upper package just before the collision, at the bottom of the chute.

Express your answer with the appropriate units. Enter positive value for the motion down the chute and negative value for the motion up the chute.

ANSWER:

ANSWER:

$v$  =

**Part B**

Suppose the collision between the packages is perfectly elastic. To what height does the package of mass  $m$  rebound?

Express your answer with the appropriate units.

You did not open hints for this part.

ANSWER:

$h$  =

**Video Tutor: Happy/Sad Pendulums**

First, [launch the video](#) below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the questions on the right. You can watch the video again at any point.

**Part A**

Imagine that you replace the block in the video with a happy or sad ball identical to the one used as a pendulum, so that the sad ball strikes a sad ball and the happy ball strikes a happy ball. The target balls are free to move, and all the balls have the same mass. In the collision between the sad balls, how much of the balls' kinetic energy is *dissipated*?

**Hint 1.** How to identify the nature of the collision

In the video, the sad ball doesn't rebound after striking the block—it hangs motionless, meaning that this collision is essentially completely inelastic. We can predict that a collision between two sad balls will also be completely inelastic.

Just before the collision, the kinetic energy of the two balls resides entirely in the fast-moving pendulum ball.

Given that momentum is conserved in the collision, how will the two balls move after the collision? What does that say about their combined kinetic energy?

ANSWER:

- ☐ All of it
- ☒ Half of it
- ☐ None of it

**Correct**

The collision between the sad balls is completely inelastic, and due to the conservation of momentum, the velocity of the balls after the collision will be half the velocity of the incoming ball before the collision. The expression for kinetic energy is  $\frac{1}{2}mv^2$ , and while there is double the mass moving after the collision as before, the decrease in velocity by a factor of two results in the overall kinetic energy being *half* its initial value.

**Part B**

Now, consider the collision between two happy balls described in Part A. How much of the balls' kinetic energy is *dissipated*?

**Hint 1.** How to approach the problem

In the video, when the happy ball collides with the block it rebounds with essentially the same speed, meaning that the collision is elastic. We can predict that a collision between two happy balls will also be elastic.

Just before the collision, the kinetic energy of the two balls resides entirely in the fast-moving pendulum ball.

Given that momentum is conserved in the collision, how will the two balls move after the collision? What does that say about their combined kinetic energy?

ANSWER:

- ☒ None of it
- ☐ Half of it
- ☐ All of it

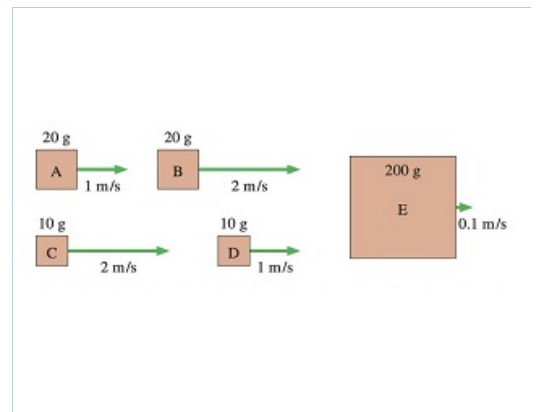
**Correct**

In this scenario, since the collision between the happy balls is elastic, the kinetic energy will be conserved, and thus none will be dissipated.

**Conceptual Question 11.1****Part A**

Rank in order, from largest to smallest, the momenta  $(p_x)_a$  to  $(p_x)_e$  of the objects in .

**Rank from largest to smallest. To rank items as equivalent, overlap them.**



ANSWER:

largest		smallest
B	C E A	D

☐ The correct ranking cannot be determined.

Correct

## Conceptual Question 11.4

A 0.2 kg plastic cart and a 20 kg lead cart can both roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a time of 1 s, starting from rest.

### Part A

After the force is removed at  $t = 1$  s, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart?

**Match the words in the left column to the appropriate blanks in the sentences on the right.**

ANSWER:

Reset

Help

the net force applied, the  
force action time, and the  
object's mass

the object's mass and the net  
force applied

-----

inversely proportional to

directly proportional to

-----

greater than

lesser than

Because both carts start from rest, the change in momentum of each cart equals its final momentum. According to the momentum principle, the change in momentum of an object equals the impulse, which depends on **the net force applied and the force action time**. Because equal forces are exerted over equal times and impulse is **independent of** the object's mass, the change in the momentum, as well as the final momentum of the plastic cart is **equal to** that of the lead cart.

Correct

## Conceptual Question 11.9

### Part A

A golf club continues forward after hitting the golf ball. Is momentum conserved in the collision? Explain, making sure you are careful to identify "the system."

ANSWER:

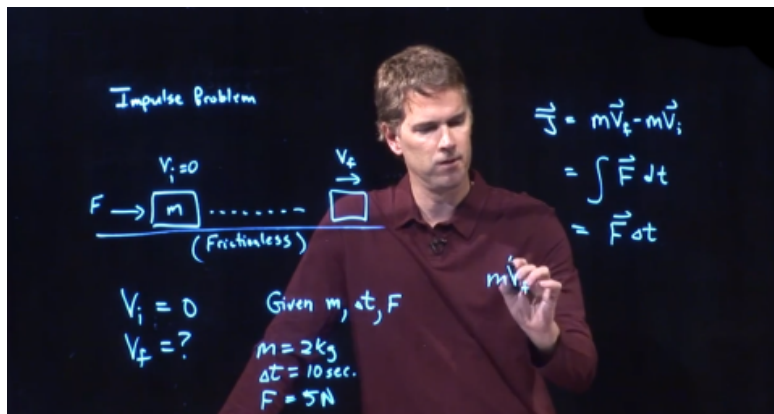
Essay answers are limited to about 500 words (3800 characters maximum, including spaces).

3785 Character(s) remaining

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## Anderson Video - Impulse Problem

First, launch the video [Impulse Problem](#). After watching the video, answer the follow-up question below.

**Part A**

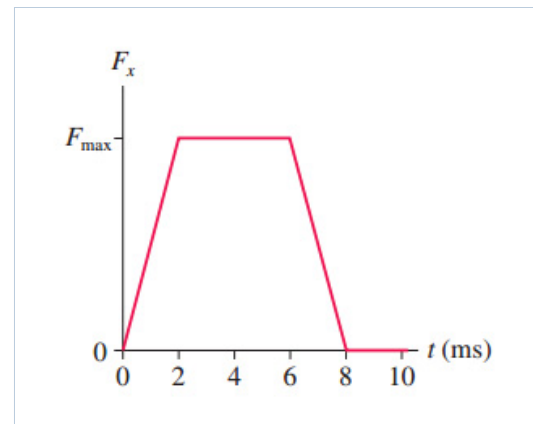
A 0.90 kg puck is being pushed across a table with a horizontal force of 2.0 N. It starts from rest and is pushed for 13 seconds, ending with a speed of 1 m/s. Calculate the coefficient of friction  $\mu_k$  between the puck and the table.

ANSWER:

$\mu_k =$

### Problem 11.5 - Enhanced - with Hints and Feedback

Force-versus-time graph is shown in .

**Part A**

What value of  $F_{\max}$  gives an impulse of 9.6 N s?

Express your answer with the appropriate units.

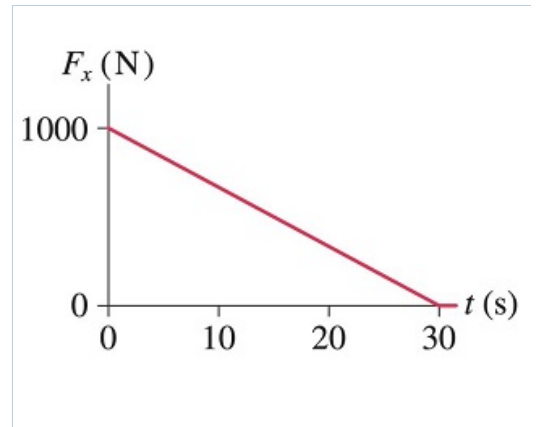
You did not open hints for this part.

ANSWER:

$F_{\max} =$

### Problem 11.10 - Enhanced - with Hints and Feedback

Far in space, where gravity is negligible, a 425 kg rocket traveling at 65 m/s fires its engines. shows the thrust force as a function of time. The mass lost by the rocket during these 30 s is negligible.



### Part A

What impulse does the engine impart to the rocket?

**Express your answer in newton-seconds.**

You did not open hints for this part.

ANSWER:

$J_x =$   N · s

### Part B

At what time does the rocket reach its maximum speed?

**Express your answer with the appropriate units.**

You did not open hints for this part.

ANSWER:

$t =$

### Part C

What is the maximum speed of the rocket?

**Express your answer with the appropriate units.**

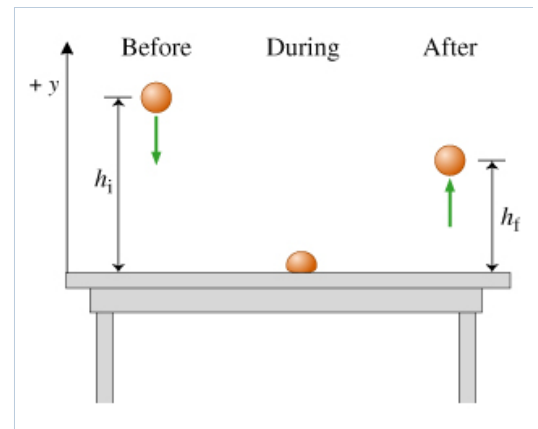
You did not open hints for this part.

ANSWER:

$v_{fx} =$

## ± A Superball Collides Inelastically with a Table

As shown in the figure, a superball with mass  $m$  equal to 50 grams is dropped from a height of  $h_i = 1.5$  m. It collides with a table, then bounces up to a height of  $h_f = 1.0$  m. The duration of the collision (the time during which the superball is in contact with the table) is  $t_c = 15$  ms. In this problem, take the positive  $y$  direction to be upward, and use  $g = 9.8$  m/s<sup>2</sup> for the magnitude of the acceleration due to gravity. Neglect air resistance.

**Part A**

Find the  $y$  component of the momentum,  $p_{\text{before},y}$ , of the ball immediately before the collision.

**Express your answer numerically, to two significant figures.**

You did not open hints for this part.

ANSWER:

$$p_{\text{before},y} = \text{[input box]} \text{ kg} \cdot \text{m/s}$$

**Part B**

Find the  $y$  component of the momentum of the ball immediately after the collision, that is, just as it is leaving the table.

**Express your answer numerically, to two significant figures.**

You did not open hints for this part.

ANSWER:

$$p_{\text{after},y} = \text{[input box]} \text{ kg} \cdot \text{m/s}$$

**Part C**

Find  $J_y$ , the  $y$  component of the impulse imparted to the ball during the collision.

**Express your answer numerically, to two significant figures.**

You did not open hints for this part.

ANSWER:

$$J_y = \text{[input box]} \text{ kg} \cdot \text{m/s}$$

**Part D**

Find the  $y$  component of the time-averaged force  $F_{\text{avg},y}$ , in newtons, that the table exerts on the ball.

**Express your answer numerically, to two significant figures.**

You did not open hints for this part.

ANSWER:

$$F_{\text{avg},y} = \text{[input box]} \text{ N}$$

**Part E**

Find  $K_{\text{after}} - K_{\text{before}}$ , the change in the kinetic energy of the ball during the collision, in joules.

**Express your answer numerically, to two significant figures.**

You did not open hints for this part.

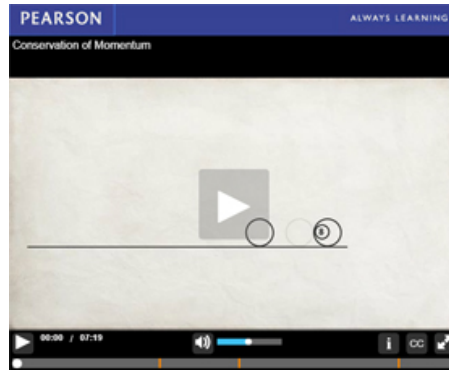
ANSWER:

$K_{\text{after}} - K_{\text{before}} =$   J

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## Prelecture Video: Conservation of Momentum

[Click Play](#) to watch the video. Answer the ungraded questions in the video and the graded follow-up questions at right.



### Part A

The video shows an animated billiards experiment in which a cue ball strikes a glued-in-place eight-ball. Which of the following explains why the momentum of the eight-ball is conserved?

ANSWER:

- ☐ The eight-ball is an isolated system.
- ☒ The "glue force" cancels the collision force.
- ☐ The cue ball rebounds with the same speed it had coming in.
- ☐ The collision is elastic.

#### Correct

During the collision, the force exerted by the glue (on the eight-ball) cancels the force exerted by the cue ball (on the eight-ball). Thus, the momentum of the eight-ball system is conserved because the net external force acting on it is zero.

### Part B

Which statement must be true for the momentum of a system to be conserved?

ANSWER:

- ☒ The net external force on the system is zero.
- ☐ The net external force on the system is non-zero.
- ☐ There are no external forces acting on the system.
- ☐ The internal forces sum to zero.

**Correct**

When the net external force on a system is zero, the system has zero acceleration and constant momentum.

**Part C**

Bumper cars A and B undergo a collision during which the momentum of the combined system is conserved. Which equation(s) correctly states the principle of conservation of momentum?

Check all that apply.

ANSWER:

- ☒  $\vec{P}_{A,i} + \vec{P}_{B,i} = \vec{P}_{A,f} + \vec{P}_{B,f}$
- ☐  $\vec{P}_{A,i} + \vec{P}_{A,f} = \vec{P}_{B,i} + \vec{P}_{B,f}$
- ☐  $\Delta\vec{P}_A = \Delta\vec{P}_B$
- ☒  $\Delta\vec{P}_A + \Delta\vec{P}_B = 0$

**Correct**

Two of the equations express the conservation principle in a different way. The equation  $\vec{P}_{A,i} + \vec{P}_{B,i} = \vec{P}_{A,f} + \vec{P}_{B,f}$  states that the total momentum of the system is the same before and after the collision; the equation  $\Delta\vec{P}_A + \Delta\vec{P}_B = 0$  states that the change in the total momentum of the system is zero.

**Part D**

While goofing off at the ice skating rink, a student takes off her shoes and places each of them on the ice. Her friend, a hockey player, then shoots a hockey puck at each shoe. The first puck immediately comes to rest after it collides with the left shoe. The second puck rebounds after it collides with the right shoe. If each hockey puck has the same incoming speed, which shoe has greater speed after the collision?

ANSWER:

- ☐ The left shoe
- ☐ Both shoes have the same speed.
- ☒ The right shoe

**Correct**

In each collision, the change in momentum of the puck has the same magnitude as the change in momentum of the shoe. The puck has a greater magnitude of change in momentum when it rebounds, so the right shoe must undergo a greater change in momentum than the left shoe!

**Problem 11.17 - Enhanced - with Hints and Feedback**

Three identical train cars, coupled together, are rolling east at speed  $v_0$ . A fourth identical car traveling east at  $2v_0$  catches up with the three and couples to make a four-car train. A moment later, the train cars hit a fifth identical car that was at rest on the tracks, and it couples to make a five-car train.

**Part A**

What is the speed of the five-car train?

Express your answer in terms of  $v_0$ .

**Hint 1.** How to approach the problem

Since the train cars couple together, this is a case of perfectly inelastic collisions. Momentum is conserved in these collisions in the impulse approximation, in which the external forces during the time of the collision are ignored.

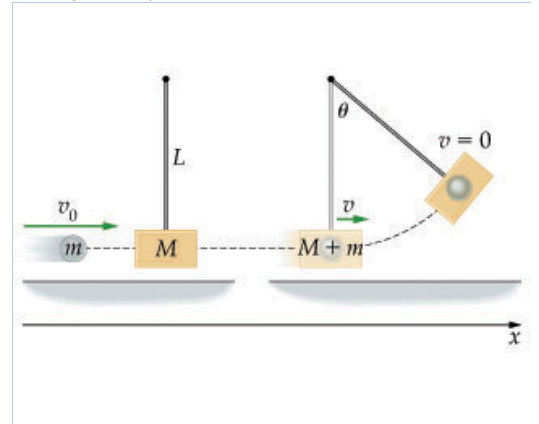
ANSWER:

$$v_5 = v_0$$

Correct

## Ballistic Pendulum

In a ballistic pendulum an object of mass  $m$  is fired with an initial speed  $v_0$  at a pendulum bob. The bob has a mass  $M$ , which is suspended by a rod of length  $L$  and negligible mass. After the collision, the pendulum and object stick together and swing to a maximum angular displacement  $\theta$  as shown.



### Part A

Find an expression for  $v_0$ , the initial speed of the fired object.

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

#### Hint 1. How to approach the problem

There are two distinct physical processes at work in the ballistic pendulum. You must treat the collision and the following swing as two separate events. Identify which physical law or principle applies to each event, write an expression to describe the collision, write an expression to describe the swing, and then relate the two expressions to find  $v_0$ .

#### Hint 2. Determine which physical laws and principles apply

Which of the following physical laws or principles can best be used to analyze the collision between the object and the pendulum bob? Which can best be used to analyze the resulting swing?

- A. Newton's first law
- B. Newton's second law
- C. Newton's third law
- D. Conservation of mechanical energy
- E. Conservation of momentum

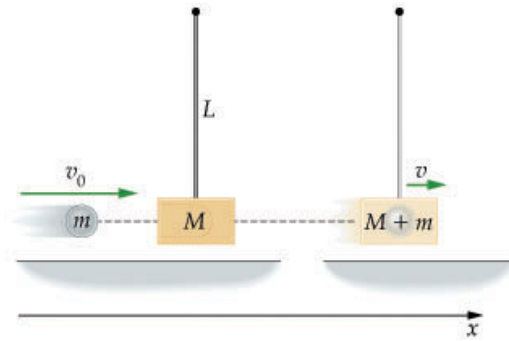
Enter the letters corresponding to the correct answer, with a letter first for the collision and then a second letter for the swing separated by a comma.

ANSWER:

E,D

#### Hint 3. Describe the collision

Compose an expression that describes the collision between the object and the pendulum bob. Put this expression in the form  $v_0 = \dots$ .



Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

#### Hint 1. Identify the type of collision

Is the collision between the object and the pendulum bob an elastic or inelastic collision?

ANSWER:

- ☐ elastic
- ☒ inelastic

#### Hint 2. Find the momentum before the collision

Compose an expression for  $p_{\text{before}}$ , the momentum of the object and pendulum bob before the collision when the object moves with speed  $v_0$ .

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

#### Hint 1. Momentum

The momentum of an object of mass  $m$  moving with speed  $v$  is given by  $mv$ .

ANSWER:

$$p_{\text{before}} = mv_0$$

#### Hint 3. Find the momentum after the collision

Compose an expression for  $p_{\text{after}}$ , the momentum of the object and pendulum bob after the collision when they move with speed  $v$ .

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

ANSWER:

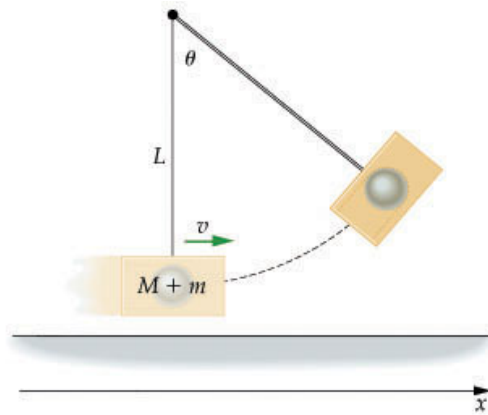
$$p_{\text{after}} = (m + M)v$$

ANSWER:

$$v_0 = \frac{m+M}{m}v$$

#### Hint 4. Describe the swing

Compose an expression that describes the motion of the object and the pendulum bob after the collision. Put this expression in the form  $v = \dots$ .



Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity  $g$ .

**Hint 1. Identify the energy at the bottom of the swing**

What is the mechanical energy  $E_{\text{bottom}}$  of the object and pendulum bob just after the collision but while they are still located at the bottom of the swing? Assume that the height of the pendulum bob and object is zero at this location.

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

**Hint 1. Mechanical energy**

The mechanical energy of a system is the total kinetic and gravitational potential energy of the system. The kinetic energy  $K$  of an object of mass  $m$  moving with speed  $v$  is

$$K = \frac{1}{2}mv^2.$$

The gravitational potential energy  $U$  of an object of mass  $m$  a height  $y$  above some reference point is

$$U = mgy.$$

**Hint 2. Find the gravitational potential energy at the bottom of the swing**

What is the gravitational potential energy  $U_{\text{bottom}}$  of the object and pendulum bob at the bottom of the swing? Keep in mind that the height of the pendulum bob and object is zero at this location.

ANSWER:

- ☐  $U_{\text{bottom}} = mgL$   
☒  $U_{\text{bottom}} = 0$   
☐  $U_{\text{bottom}} = -mgL$

**Hint 3. Find the kinetic energy at the bottom of the swing**

What is the kinetic energy  $K_{\text{bottom}}$  of the object and pendulum bob at the bottom of the swing?

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

ANSWER:

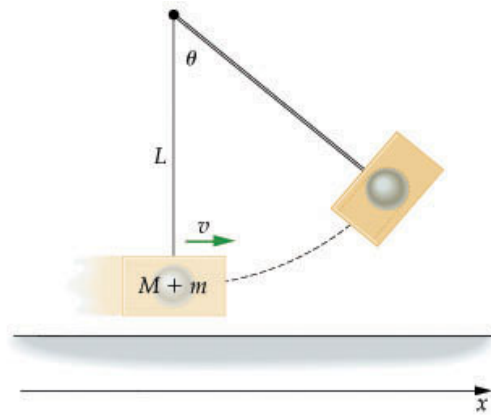
$$K_{\text{bottom}} = \frac{1}{2}(m + M)v^2$$

ANSWER:

$$E_{\text{bottom}} = \frac{1}{2}(m + M)v^2$$

**Hint 2. Identify the energy at the top of the swing**

What is the mechanical energy  $E_{\text{top}}$  of the object and pendulum bob at the top of the swing when it has reached its maximum angular displacement?



Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

#### Hint 1. Mechanical energy

The mechanical energy of a system is the total kinetic and gravitational potential energy of the system. The kinetic energy  $K$  of an object of mass  $m$  moving with speed  $v$  is

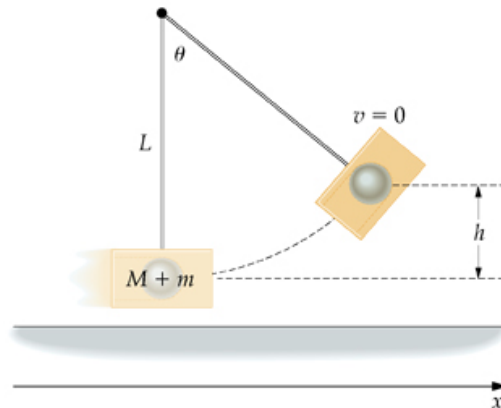
$$K = \frac{1}{2}mv^2.$$

The gravitational potential energy  $U$  of an object of mass  $m$  a height  $y$  above some reference point is

$$U = mgy.$$

#### Hint 2. Find the height at the top of the swing

What is the height  $h$  of the object and pendulum bob at the top of the swing, when they have reached their maximum displacement? Keep in mind that the pendulum has a length  $L$  and swings through an angle  $\theta$ .



Express your answer in terms of some or all of the variables  $L$  and  $\theta$ .

ANSWER:

$$h = L(1 - \cos(\theta))$$

#### Hint 3. Find the gravitational potential energy at the top of the swing

What is the gravitational potential energy  $U_{\text{top}}$  of the object and pendulum bob at the top of the swing?

Express your answer in terms of some or all of the variables  $m$ ,  $M$ ,  $v_0$ ,  $v$ ,  $L$ , and  $\theta$  and the acceleration due to gravity,  $g$ .

ANSWER:

$$U_{\text{top}} = (m + M)gL(1 - \cos(\theta))$$

#### Hint 4. Find the kinetic energy at the top of the swing

What is the kinetic energy  $K_{\text{top}}$  of the object and pendulum bob at the top of the swing?

ANSWER:

- ☐  $K_{\text{top}} = \frac{1}{2}mv_0^2$   
☐  $K_{\text{top}} = \frac{1}{2}mv^2$   
☐  $K_{\text{top}} = \frac{1}{2}(m + M)v^2$   
☒  $K_{\text{top}} = 0$

ANSWER:

$$E_{\text{top}} = (m + M)gL(1 - \cos(\theta))$$

ANSWER:

$$v = \sqrt{2gL(1 - \cos(\theta))}$$

**Hint 5. Relating the two physical processes**

By applying conservation of momentum to the collision, you found an expression for  $v_0$ :

$$v_0 = \left(\frac{m+M}{m}\right)v.$$

By applying conservation of mechanical energy to the swing, you found an expression for  $v$ :

$$v = \sqrt{2gL[1 - \cos(\theta)]}.$$

Combine these two expression into just one expression for  $v_0$ .

ANSWER:

$$v_0 = \frac{\sqrt{2g(L - L\cos\theta)}(m+M)}{m}$$

**Correct**

The ballistic pendulum was invented during the Napoleonic Wars to aide the British Navy in making better cannons. It has since been used by ballisticians to measure the velocity of a bullet as it leaves the barrel of a gun. In Part B you will use your expression for  $v_0$  to compare the initial speeds of bullets fired from 9.0-mm and .44-caliber handguns.

**Part B**

An experiment is done to compare the initial speed of bullets fired from different handguns: a 9.0 mm and a .44 caliber. The guns are fired into a 10-kg pendulum bob of length  $L$ . Assume that the 9.0-mm bullet has a mass of 6.0 g and the .44-caliber bullet has a mass of 12 g. If the 9.0-mm bullet causes the pendulum to swing to a maximum angular displacement of  $4.3^\circ$  and the .44-caliber bullet causes a displacement of  $10.1^\circ$ , find the ratio of the initial speed of the 9.0-mm bullet to the speed of the .44-caliber bullet,  $(v_0)_{9.0}/(v_0)_{44}$ .

Express your answer numerically.

**Hint 1. How to approach the problem**

Use your expression from Part A to set up the ratio  $(v_0)_{9.0}/(v_0)_{44}$ . Try to cancel as many terms as possible before plugging in your numbers to solve for a numeric answer.

ANSWER:

$$(v_0)_{9.0}/(v_0)_{44} = 0.85$$

**Correct**

Police officers in the United States commonly carry 9.0-mm handguns because they are easier to handle, having a shorter barrel than typical .44-caliber guns. Not only does the .44-caliber bullet have more mass than the 9.0-mm one, its passage through a longer gun barrel means that it also moves faster as it leaves the barrel, which makes the .44-caliber Magnum a particularly powerful handgun. A .44-caliber bullet can travel at speeds over 1000 miles per hour (1600 kilometers per hour).

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**Problem 11.26 - Enhanced - with Hints and Feedback**

A 68.5 kg football player is gliding across very smooth ice at 2.20 m/s. He throws a 0.420 kg football straight forward.

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**Part A**

What is the player's speed afterward if the ball is thrown at 16.5 m/s relative to the ground?

**Express your answer with the appropriate units.**

You did not open hints for this part.

ANSWER:

$v =$

---

**Part B**

What is the player's speed afterward if the ball is thrown at 16.5 m/s relative to the player?

**Express your answer with the appropriate units.**

You did not open hints for this part.

ANSWER:

$v =$

**Score Summary:**

Your score on this assignment is 58.8%.

You received 10 out of a possible total of 17 points.