Problem 1

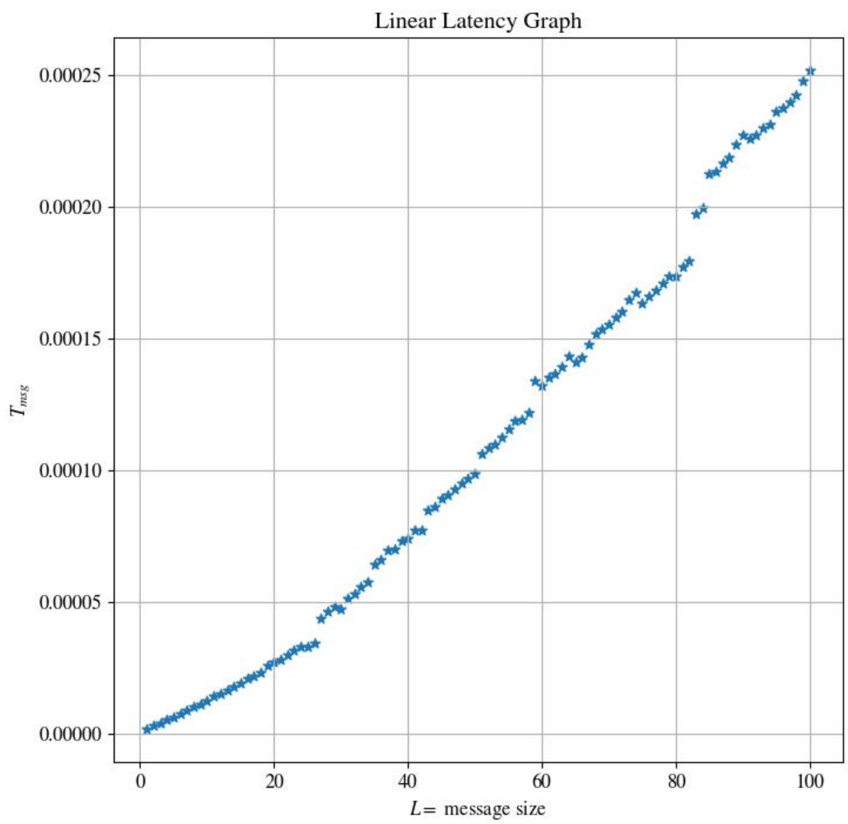

Figure 1: Experimental data illustrating the linear model $T_{msg} = t_s + t_w L$

Problem 2

Solution. Let $N_x \times N_y \times N_z$ denote a 3D grid of data points. Then, in assuming $N_x = N_y = N$ (for simplicity) and using a 9-point stencil, we have that each P task is responsible for $(N/\sqrt{P}) \times (N/\sqrt{P}) \times N_z$ points and must exchange $2(N/\sqrt{P})N_z$ points with 4 neighboring

P tasks. Hence,

$$\begin{aligned}
T_{comp} &= t_c N^2 N_z. && \text{(No replicated computation)}\\
T_{comm} &= 4P(t_s + 2t_w(N/\sqrt{P})N_z).\\
\implies T_{2D} &= \frac{t_c N^2 N_z}{P} + 4t_s + 8t_w(N/\sqrt{P})N_z.\\
\implies E &= \frac{t_c N^2 N_z}{t_c N^2 N_z + 4Pt_s + 8\sqrt{P}t_w N N_z}.
\end{aligned}$$

Subsequently, for constant efficiency, we require

$$\begin{aligned}
t_c N^2 N_z &= E\Big(t_c N^2 N_z + 4Pt_s + 8\sqrt{P}t_w N N_z\Big),\\
\implies t_c N_z &= E\Big(t_c N_z + 4t_s + 8t_w N_z\Big). && (N = \sqrt{P})
\end{aligned}$$

Thus, the isoefficiency of this algorithm is $\sim O(P)$. Therefore, in comparing this to the results found in class for the 1D decomposition, we may conclude that a 2D decomposition is more scalable than a 1D decomposition. ■