



UPPSALA  
UNIVERSITET

# High Performance Computing: Speed Optimisation Using NumPy, Cython, MPI And GPU Acceleration

## Day 3

Advanced Scientific Programming with Python

# Class Inheritance

- Classes can inherit from other classes.
- A class can inherit attributes and behaviour methods from another class, called the superclass or parent class.
- A class which inherits from a superclass is called a subclass, also called heir class or child class.
- We could implement a vehicle class in Python, which might have methods like accelerate and brake. Cars, Buses and Trucks and Bikes can be implemented as subclasses which will inherit these methods from vehicle.

# Class Inheritance

```
class Pet(object):
    def __init__(self, name, species):
        self.name = name
        self.species = species
    def getName(self):
        return self.name
    def getSpecies(self):
        return self.species
    def __str__(self):
        return "%s is a %s" % (self.name, self.species)
```

```
class Dog(Pet):
    def __init__(self, name, chases_cats):
        Pet.__init__(self, name, "Dog")
        self.chases_cats = chases_cats
    def chasesCats(self):
        return self.chases_cats
```

```
class Cat(Pet):
    def __init__(self, name, hates_dogs):
        #Pet.__init__(self, name, "Cat")
        super(Cat, self).__init__(name, "Cat")
        self.hates_dogs = hates_dogs
    def hatesDogs(self):
        return self.hates_dogs
```

# Multiple Inheritance

- Python also support multiple inheritance

```
class Reader(object):
    def __init__(self):
        self._size = 0
    @property
    def size(self):
        return self._size
    def read(self):
        data = 'data read'
        return data
```

```
class Writer(object):
    def __init__(self):
        self._size = 0
    @property
    def size(self):
        return self._size
    def write(self, data):
        # write data somewhere
        pass
```

```
class ReaderWriter(Reader,Writer):
    def __init__(self):
        super(ReaderWriter, self).__init__()
```

# Why NumPy?

```
import numpy as np
```

```
a = np.random.random((1000000))
```

```
b = list(a)
```

```
%timeit a.sum()
```

```
%timeit sum(b)
```

```
In [4]: %timeit a.sum()
```

```
1000 loops, best of 3: 415 µs per loop
```

```
In [5]: %timeit sum(b)
```

```
10 loops, best of 3: 41.3 ms per loop
```

```
In [6]: 41.3/0.415
```

```
Out[2]: 99.51807228915662
```

How much faster is the NumPy version?

A – About 2x

B – About 5x

C – About 25x

D – About 100x

E – NumPy is slower



# What Is NumPy?

- The **numpy** package is used in almost all numerical computation using Python.
- It provide high-performance vector, matrix and higher-dimensional data structures for Python.
- It is implemented in C and Fortran so when calculations are vectorized (formulated with vectors and matrices), **performance is very good**.

# Numpy Data Types

- To achieve it's high performance NumPy arrays are much more restrictive about data types than Python lists
- All the elements have the same type
- The most common NumPy data types are:
  - `numpy.float64`, `numpy.float32` (also known as double and single precision. The default python float is also 64-bits long)
  - `numpy.int64`, `numpy.uint64`, and it's 32 and 16 bit equivalents
  - `numpy.complex128` and `numpy.complex64` (two `float64`s and two `float32`s)
- These are closely related to C types
- The suffix shows the size of the type in bits

# Numpy Data Types

- A floating point number **x** is generally represented as:  
 **$x = \text{sign} * \text{mantissa} * \text{base}^{\text{exponent}}$**
- They are stored in memory as a sign bit followed by the exponent bits and finally the mantissa or significand. For example for **float32**:



- The number of bits of the exponent defines the range of the type
- The size of the mantissa defines its precision, or the number of significant digits of the type
- Certain values are reserved for a special meaning. The example above typically means NaN (not a number, like 0/0)



# Numpy Data Types

- `np.uint64` corresponds to an unsigned 64-bit integer
- This is probably the simplest type, using 64bits for the number, it simply encodes integers between **0** and  **$2^{64}-1$** .
- Just be careful with negative numbers or overflow problems, e.g.:

```
In [142]: a = np.array([-1], dtype=np.uint64)
```

```
In [143]: a
```

```
Out[143]: array([18446744073709551615], dtype=uint64)
```

```
In [144]: a += 1
```

What's the value of `a[0]`  
now?

C - 0

D - -1

A - 18446744073709551616

B - 18446744073709551615



# Numpy Data Types

- `np.uint64` corresponds to an unsigned 64-bit integer
- This is probably the simplest type, using 64bits for the number, it simply encodes integers between **0** and  **$2^{64}-1$** .
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```
In [142]: a = np.array([-1], dtype=np.uint64)
```

```
In [143]: a
```

```
Out[143]: array([18446744073709551615], dtype=uint64)
```

```
In [144]: a += 1
```

What's the value of `a[0]`  
now?

C - 0

D - -1

A - 18446744073709551616

B - 18446744073709551615



```
In [145]: a
```

```
Out[145]: array([0], dtype=uint64)
```

# Numpy Data Types

- `np.int64` corresponds to a signed 64-bit integer
- It encodes integers with 1 bit for the sign and 63 bits for the number, with a range between  $-2^{63}$  and  $2^{63}-1$ .
- You can find this with:  
In [100]: `np.iinfo(np.int64)`  
Out[100]: `iinfo(min=-9223372036854775808, max=9223372036854775807, dtype=int64)`
- The only thing to keep in mind is avoiding overflow problems, e.g.:

```
In [103]: a = np.int64(2**63-1)
```

```
In [104]: a
```

```
Out[104]: 9223372036854775807
```

```
In [105]: a += 1
```

```
<ipython-input-105-8076006b0952>:1: RuntimeWarning: overflow encountered in  
long_scalars
```

```
    a += 1
```

```
In [106]: a
```

```
Out[106]: -9223372036854775808
```

# Numpy Data Types

- This means floating point arithmetic does not always behave like you expect. Adding and subtracting different values can give 0:

```
In [88]: a = np.array([2**24+1],dtype=np.float32)
```

```
In [89]: a[0] -= 2**24
```

```
In [90]: a[0]
```

```
Out[90]: 0.0
```

- The result of a sum can depend on the order:

```
In [159]: a = np.array(np.random.rand(100000),dtype=np.float16) - 0.5
```

```
In [160]: a.sum()
```

```
Out[160]: -86.2
```

```
In [161]: np.sort(a).sum()
```

```
Out[161]: -93.44
```

- Numbers that should be the same are not:

```
In [162]: 1/3 == 1-2/3
```

```
Out[162]: False
```

# Numpy Data Types

```
In [159]: a = np.array(np.random.rand(100000), dtype=np.float16) - 0.5
```

```
In [160]: a.sum()
```

```
Out[160]: -86.2
```

```
In [161]: np.sort(a).sum()
```

```
Out[161]: -93.44
```

Which is the more accurate result sum in the previous example? Why?

A - The unsorted sum

B - The sorted sum



# Numpy Data Types

```
In [159]: a = np.array(np.random.rand(100000), dtype=np.float16) - 0.5
```

```
In [160]: a.sum()
```

```
Out[160]: -86.2
```

```
In [161]: np.sort(a).sum()
```

```
Out[161]: -93.44
```

Which is the more accurate result sum in the previous example? Why?

A – The unsorted sum

B – The sorted sum



```
In [165]: a.sum(dtype=np.float64)
```

```
Out[165]: -86.242919921875
```

```
In [166]: np.sort(a).sum(dtype=np.float64)
```

```
Out[166]: -86.242919921875
```

# Fancy Indexing

- NumPy offers more indexing facilities than regular Python sequences.
- In addition to indexing by integers and slices, arrays can be indexed by arrays of integers and arrays of booleans.

```
>>> a = np.arange(12)**2                                # the first 12 square numbers
>>> i = np.array( [ 1,1,3,8,5 ] )                       # an array of indices
>>> a[i]                                                  # the elements of a at the positions i
array([ 1,  1,  9, 64, 25])
>>>
>>> j = np.array( [ [ 3, 4], [ 9, 7 ] ] )               # a bidimensional array of indices
>>> a[j]                                                  # the same shape as j
array([[ 9, 16],
       [81, 49]])
```

- You can also use indexing with arrays as a target to assign to:

```
>>> a = np.arange(5)
>>> a
array([0, 1, 2, 3, 4])
>>> a[[1,3,4]] = 0
>>> a
array([0, 0, 2, 0, 0])
```

# Fancy Indexing

- When we index arrays with arrays of (integer) indices we are providing the list of indices to pick.
- With boolean indices the approach is different; we explicitly choose which items in the array we want and which ones we don't.
- The most natural way one can think of for boolean indexing is to use boolean arrays that have the same shape as the original array:

```
>>> a = np.arange(12).reshape(3,4)
>>> b = a > 4
>>> b                                     # b is a boolean with a's shape
array([[False, False, False, False],
       [False,  True,  True,  True],
       [ True,  True,  True,  True]], dtype=bool)
>>> a[b]                                 # 1d array with the selected elements
array([ 5,  6,  7,  8,  9, 10, 11])
```



# Fancy Indexing

- Fancy indexing is the most general selection method, but it is also the slowest.

```
# Let's create an array with a large number of rows.  
# We will select slices of this array along the first dimension.
```

```
n, d = 100000, 100
```

```
a = np.random.random_sample((n, d)); aid = id(a)
```

```
# Let's select one every ten rows, using two different methods  
# (array view and fancy indexing).
```

```
b1 = a[::10]
```

```
b2 = a[np.arange(0, n, 10)]
```

```
np.array_equal(b1, b2)
```

```
True
```

```
# Let's compare the performance of both methods.
```

```
%timeit a[::10]
```

```
1000000 loops, best of 3: 804 ns per loop
```

```
%timeit a[np.arange(0, n, 10)]
```

```
100 loops, best of 3: 14.1 ms per loop
```

- Fancy indexing is several orders of magnitude slower as it involves copying a large array.

# Fancy Indexing

```
>>> a = np.arange(5)
>>> a[[0,0,2]]=[1,2,3]
>>> a
>>> a
array([2, 1, 3, 3, 4])
```



What's the value of a[0]?

```
>>> a = np.arange(5)
>>> a[[0,0,2]]+=1
>>> a
array([1, 1, 3, 3, 4])
```



And in this case what's a[0]?

# Views

- A view is simply another way of viewing the data of the array.
- The data of both objects are shared.
- You can create views by selecting a slice of the original array or by changing the data type.
- Slice views are the most common.
- The rule of thumb for creating a slice view is that the viewed elements can be addressed with offsets, strides, and counts in the original array. For example:

# Slice Views

- Are the most common view.
- The viewed elements must be able to be addressed with offsets, strides, and counts in the original array.

```
>>> a = numpy.arange(10)
>>> a
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
>>> v1 = a[1:2]
>>> v1
array([1])
>>> a[1] = 2
>>> v1
array([2])
>>> v2 = a[1::3]
>>> v2
array([2, 4, 7])
>>> a[7] = 10
>>> v2
array([ 2,  4, 10])
```

- Any selection which involves only **slices** (e.g. **0:10**, **::-1**, etc...) returns a view.

# Data Type Views

- Sometimes you want to inspect memory with a different data type
- This is when data type views come in

```
In [12]: b = numpy.arange(10, dtype='int16')
```

```
In [13]: b
```

```
Out[13]: array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9], dtype=int16)
```

```
In [14]: v3 = b.view('int32')
```

```
In [15]: v3
```

```
Out[15]: array([ 65536, 196610, 327684, 458758, 589832], dtype=int32)
```

```
In [16]: v3 += 1
```

```
In [17]: b
```

```
Out[17]: array([1, 1, 3, 3, 5, 5, 7, 7, 9, 9], dtype=int16)
```

```
In [18]: v4 = b.view('int8')
```

```
In [19]: v4
```

```
Out[19]: array([1, 0, 1, 0, 3, 0, 3, 0, 5, 0, 5, 0, 7, 0, 7, 0, 9, 0, 9, 0],  
              dtype=int8)
```

- You get the raw byte stream casted to whatever data type you selected

# Data Ownership

- Now you see that multiple arrays can modify the same data
- Who owns the data?

In [20]: b.flags

Out[20]:

C\_CONTIGUOUS : True

F\_CONTIGUOUS : True

OWNDATA : True

WRITEABLE : True

ALIGNED : True

UPDATEIFCOPY : False



In [21]: v4.flags

Out[21]:

C\_CONTIGUOUS : True

F\_CONTIGUOUS : True

OWNDATA : False

WRITEABLE : True

ALIGNED : True

UPDATEIFCOPY : False



# View And Data Ownership

```
In [26]: a = numpy.array([1])
```

```
In [27]: b = a[:]
```

```
In [28]: b[0] = 0
```

```
In [29]: a[0] == 0
```

```
Out[29]: True
```



What is the result of the code?

```
In [36]: a = numpy.random.random((3))
```

```
In [37]: b = a[::-1]
```

```
In [38]: b[0] = 1
```

```
In [39]: b[0] == a[-1]
```

```
Out[39]: True
```

And now?

# View And Data Ownership

```
In [40]: a = numpy.random.random((3))
```

```
In [41]: b = a
```

```
In [42]: b[0] = 1
```

```
In [43]: b[0] == a[0]
```

```
Out[43]: True
```



And in this one?

```
In [44]: a = numpy.random.random((3))
```

```
In [45]: b = a[[0,1,2]]
```

```
In [46]: b[0] = 1
```

```
In [47]: b[0] == a[0]
```

```
Out[47]: False
```

And finally?



# Some Tricky Cases

Both snippets on the right seem to do the same thing but they don't.

```
>>> a = numpy.arange(10)
>>> a[[1,2]] = 100
>>> a
array([ 0, 100, 100,  3,  4,  5,  6,  7,  8,  9])
```

```
>>> a = numpy.arange(10)
>>> c1 = a[[1,2]]
>>> c1[:] = 100
>>> a
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
>>> c1
array([100, 100])
```

# Some Tricky Cases

```
>>> a = numpy.arange(12).reshape(3,4)
>>> ifancy = [0,2]
>>> islice = slice(0,3,2)
>>> a[islice, :][:, ifancy] = 100
>>> a
array([[100, 1, 100, 3],
       [ 4, 5, 6, 7],
       [100, 9, 100, 11]])
```

```
>>> a = numpy.arange(12).reshape(3,4)
>>> ifancy = [0,2]
>>> islice = slice(0,3,2)
# note that ifancy and islice
# are interchanged here
>>> a[ifancy, :][:, islice] = 100
>>> a
array([[ 0, 1, 2, 3],
       [ 4, 5, 6, 7],
       [ 8, 9, 10, 11]])
```



Is `a[0,0] == 100`?

Is `a[0,0]` also 100 in this case?

# Broadcasting

- NumPy operations are usually done on pairs of arrays on an element-by-element basis.
- In the simplest case, the two arrays must have exactly the same shape, as in the following example:

```
>>> a = np.array([1.0, 2.0, 3.0])
>>> b = np.array([2.0, 2.0, 2.0])
>>> a * b
array([ 2.,  4.,  6.])
```

- NumPy's broadcasting rule relaxes this constraint when the arrays' shapes meet certain constraints.
- The simplest broadcasting example occurs when an array and a scalar value are combined in an operation:

```
>>> a = np.array([1.0, 2.0, 3.0])
>>> b = 2.0
>>> a * b
array([ 2.,  4.,  6.])
```

# General Broadcasting Rules

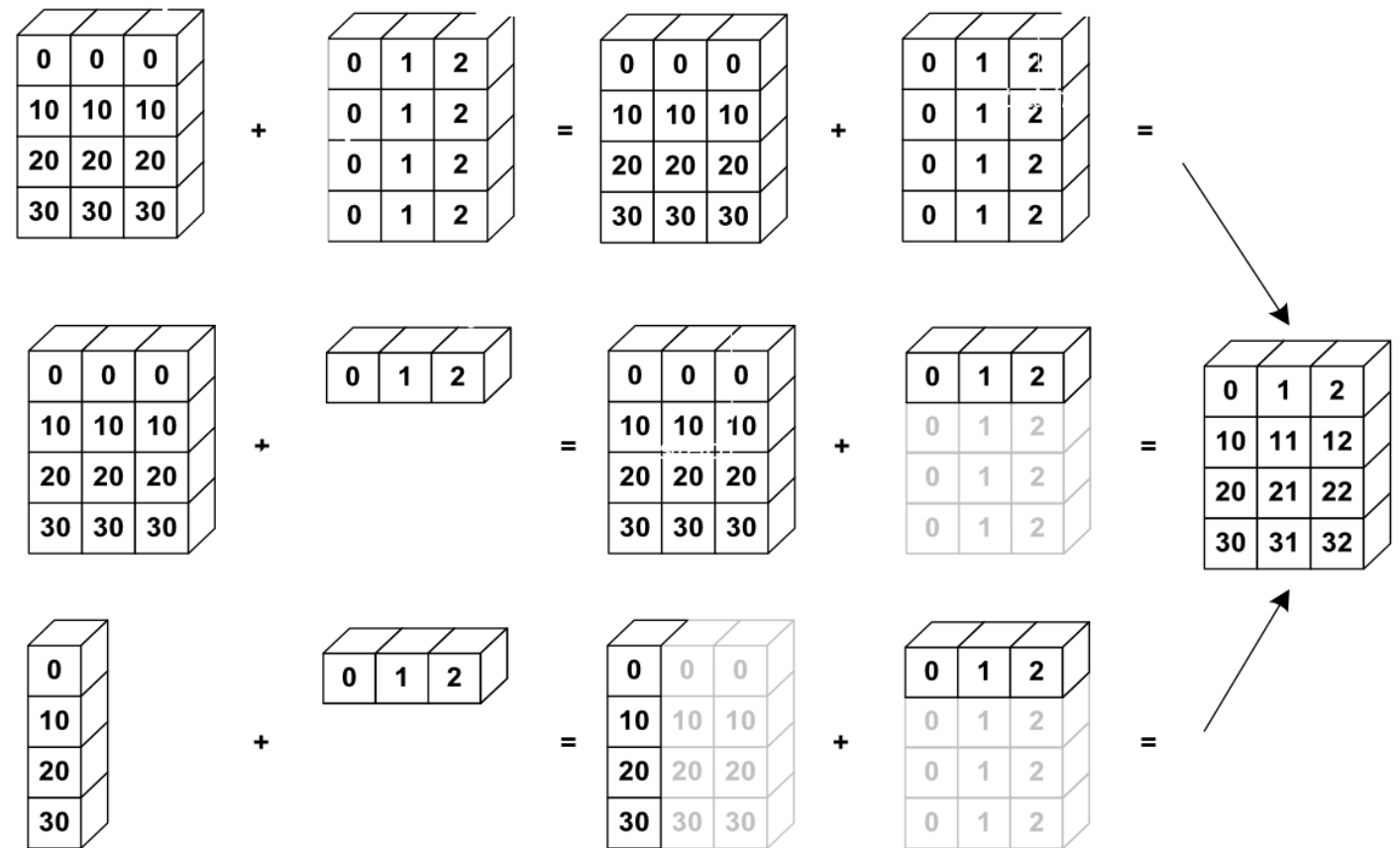
- When operating on two arrays, NumPy compares their shapes element-wise.
- It starts with the trailing dimensions, and works its way forward.
- Two dimensions are compatible when:
  1. they are equal, or
  2. one of them is 1
- The size of the resulting array is the maximum size along each dimension of the input arrays.
- Arrays do not need to have the same **number** of dimensions. For example:

```
Image  (3d array): 256 x 256 x 3
Scale  (1d array):           3
Result (3d array): 256 x 256 x 3
```

# Broadcasting

- Broadcasting provides a convenient way of taking the outer product (or any other outer operation) of two arrays.
- The following example shows an outer addition operation of two 1-d arrays:

```
>>> a = np.array([0.0, 10.0, 20.0, 30.0])
>>> b = np.array([1.0, 2.0, 3.0])
>>> a[:, np.newaxis] + b
array([[ 1.,  2.,  3.],
       [11., 12., 13.],
       [21., 22., 23.],
       [31., 32., 33.]])
```



# Broadcasting

- Here's a comparison between broadcasting and explicit tiling for the outer product:

```
n = 1000
a = np.arange(n)
ac = a[:, np.newaxis]
ar = a[np.newaxis, :]
%timeit np.tile(ac, (1, n)) * np.tile(ar, (n, 1))
100 loops, best of 3: 10 ms per loop
```

- And now using broadcasting

```
%timeit ar * ac
100 loops, best of 3: 2.36 ms per loop
```

- It's also much shorter to write!

# Broadcasting Exercises

**A**    **A**    (4d array): 8 x 1 x 6 x 1  
      **B**    (3d array): 7 x 1 x 5

**B**    **A**    (2d array): 5 x 4  
      **B**    (1d array): 1

**C**    **A**    (1d array): 3  
      **B**    (1d array): 4

**D**    **A**    (2d array): 2 x 1  
      **B**    (3d array): 8 x 4 x 3

**E**    **A**    (3d array): 15 x 3 x 5  
      **B**    (3d array): 15 x 1 x 5



Which of these arrays pairs  
broadcasts correctly?

# Boolean Operations

```
>>> import numpy as np

>>> a = np.array([1, 1, 0, 0], dtype=bool)
>>> b = np.array([1, 0, 1, 0], dtype=bool)

>>> np.logical_or(a, b)
array([ True,  True,  True, False], dtype=bool)
>>> a + b
array([ True,  True,  True, False], dtype=bool)
>>> a | b
array([ True,  True,  True, False], dtype=bool)

>>> np.logical_and(a, b)
array([ True, False, False, False], dtype=bool)
>>> a * b
array([ True, False, False, False], dtype=bool)
>>> a & b
array([ True, False, False, False], dtype=bool)

>>> np.logical_not(a)
array([False, False,  True,  True], dtype=bool)
>>> -a
array([False, False,  True,  True], dtype=bool)
>>> ~a
array([False, False,  True,  True], dtype=bool)
```

There are many ways to express boolean operation in NumPy.



# Avoid Unnecessary Array Copies

The following function can be used to check if two arrays share the same data:

```
def id(x):  
    # This function returns the memory  
    # block address of an array.  
    return x.__array_interface__['data'][0]
```

```
a = np.zeros(10);  
aid = id(a);
```

```
b = a.copy();  
id(b) == aid  
>>> False
```

# Avoid Unnecessary Array Copies

Array computations can involve in-place operations (the array is modified):

```
a *= 2; id(a) == aid  
>>> True
```

or implicit-copy operations (a new array is created):

```
c = a * 2; id(c) == aid  
>>> False
```

Be sure to choose the type of operation you actually need.

Implicit-copy operations are significantly slower, as shown here:

```
%timeit a = np.zeros(10000000)  
a *= 2  
>>> 10 loops, best of 3: 19.2 ms per loop
```

```
%timeit a = np.zeros(10000000)  
b = a * 2  
>>> 10 loops, best of 3: 42.6 ms per loop
```

# How It Works?

- A NumPy array is basically described by metadata (number of dimensions, shape, data type, and so on) and the actual data.
- The data is stored in a homogeneous and contiguous block of memory, at a particular address in system memory (Random Access Memory, or RAM).
- This block of memory is called the data buffer. This is the main difference with a pure Python structure, like a list, where the items are scattered across the system memory.
- This aspect is the critical feature that makes NumPy arrays so efficient.

# How It Works?

Why is this so important? Here are the main reasons:

1. Array computations can be written very efficiently in a low-level language like C (and a large part of NumPy is actually written in C). Knowing the address of the memory block and the data type, it is just simple arithmetic to loop over all items, for example. There would be a significant overhead to do that in Python with a list.
2. Spatial locality in memory access patterns results in significant performance gains, notably thanks to the CPU cache. Indeed, the cache loads bytes in chunks from RAM to the CPU registers. Adjacent items are then loaded very efficiently (sequential locality, or locality of reference).
3. Data elements are stored contiguously in memory, so that NumPy can take advantage of vectorized instructions on modern CPUs, like Intel's SSE and AVX, AMD's XOP, and so on. For example, multiple consecutive floating point numbers can be loaded in 128, 256 or 512 bits registers for vectorized arithmetical computations implemented as CPU instructions.

# What Is The Difference Between In-Place And Implicit-Copy Operations?

- An expression like `a *= 2` corresponds to an in-place operation, where all values of the array are multiplied by two.
- By contrast, `a = a * 2` means that a new array containing the values of `a * 2` is created, and the variable `a` now points to this new array.
- The old array becomes unreferenced and will be deleted by the garbage collector.
- No memory allocation happens in the first case, contrary to the second case.
- More generally, expressions like `a[i:j]` are views to parts of an array: they point to the memory buffer containing the data.
- Modifying them with in-place operations changes the original array. Hence, `a[:] = a * 2` results in an in-place operation, unlike `a = a * 2`.

# What Is The Difference Between In-Place And Implicit-Copy Operations?

Knowing this subtlety of NumPy can help you fix some bugs (where an array is implicitly and unintentionally modified because of an operation on a view), and optimize the speed and memory consumption of your code by reducing the number of unnecessary copies.

```
import numpy as np
```

```
a = np.array([1])
```

```
b = np.array([0])
```

```
print(a[0]+1.5)
```

```
b = a[:]
```

```
a += 1
```

```
print(a[0])
```

```
print(b[0])
```



What are the results of the  
print statements?

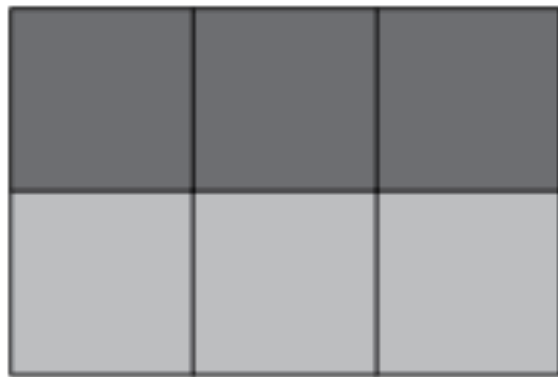
# Why Some Arrays Cannot Be Reshaped Without A Copy?

- A 2D matrix contains items indexed by two numbers (row and column), but it is stored internally as a 1D contiguous block of memory, accessible with a single number.
- There is more than one way of storing matrix items in a 1D block of memory: we can put the elements of the first row first, the second row then, and so on, or the elements of the first column first, the second column then, and so on.
- The first method is called row-major order, whereas the latter is called column-major order.
- Choosing between the two methods is only a matter of internal convention: NumPy uses the row-major order, like C, but unlike FORTRAN.

# Why Some Arrays Cannot Be Reshaped Without A Copy?

How the array is represented in Numpy

Row Major  
Order (C)  
(default in Numpy)



How the array is stored in memory



Column Major  
Order (Fortran)





# Why Some Arrays Cannot Be Reshaped Without A Copy?

- More generally, NumPy uses the notion of strides to convert between a multidimensional index and the memory location of the underlying (1D) sequence of elements.
- The specific mapping between `array[i1, i2]` and the relevant byte address of the internal data is given by

$$\text{offset} = \text{array.strides}[0] * i1 + \text{array.strides}[1] * i2$$

- When reshaping an array, NumPy avoids copies when possible by modifying the strides attribute.
- For example, when transposing a matrix, the order of strides is reversed, but the underlying data remains identical.
- However, flattening a transposed array cannot be accomplished simply by modifying strides (try it!), so a copy is needed.

# Why Some Arrays Cannot Be Reshaped Without A Copy?

- Internal array layout can also explain some unexpected performance discrepancies between very similar NumPy operations.
- As a small exercise, can you explain the following benchmarks?

```
a = np.random.rand(5000, 5000)
%timeit a[0,:].sum()
%timeit a[:,0].sum()
100000 loops, best of 3: 9.57 µs per loop
10000 loops, best of 3: 68.3 µs per loop
```

# References

code examples have been taken and adapted from

- <https://github.com/jrjohansson/scientific-python-lectures/blob/master/Lecture-2-Numpy.ipynb>
- <https://docs.scipy.org/doc/numpy-dev/user/quickstart.html>
- <http://scipy-cookbook.readthedocs.io/items/ViewsVsCopies.html>
- <https://docs.scipy.org/doc/numpy/user/basics.broadcasting.html>
- <http://ipython-books.github.io/featured-01/>
- [http://cython.readthedocs.io/en/latest/src/tutorial/cython\\_tutorial.html](http://cython.readthedocs.io/en/latest/src/tutorial/cython_tutorial.html)
- [materials.jeremybejarano.com/MPIwithPython/introMPI.html](http://materials.jeremybejarano.com/MPIwithPython/introMPI.html)
- <https://mpi4py.readthedocs.io/en/stable/intro.html#what-is-mpi>
- <http://arrayfire.org/arrayfire-python/>
- <http://www.jesshamrick.com/2011/05/18/an-introduction-to-classes-and-inheritance-in-python/>