



MAY 19, 2025

REAL LIFE GAMES:
HOW GAME THEORY SHAPES HUMAN
DECISIONS

GAME THEORY

MIXED NASH EQUILIBRIA

Adrian Haret
a.haret@lmu.de

Pure Nash equilibria always exist.

Pure Nash equilibria always exist.
Except when they don't.

Matching Pennies



Two players have a penny each.

They decide on a face and reveal it at the same time.

If the faces match, player 1 wins \$1, player 2 loses \$1.

If the faces do not match, player 2 wins \$1, player 1 loses \$1.



payoffs

	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

Pareto optimal strategies

all

pure Nash equilibria

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Pareto optimal strategies

all

pure Nash equilibria

none

There is, however, a different way
to play this game.



JOHN NASH

Sometimes the best thing to do is
to flip a coin.

MIXED STRATEGIES

DEFINITION

A *mixed strategy* s_i for player i is a probability distribution over actions, written $s_i = (p_1, \dots, p_j, \dots)$, where p_i is the probability with which player i plays action j .

Note that it needs to hold that $\sum_j p_j = 1$ and $p_j \geq 0$.

MIXED STRATEGIES: EXAMPLE

If Player 1 plays $s_1 = (0.9, 0.1)$, that means they play Heads with probability 0.9 and Tails with probability 0.1.

Note that the pure strategies we've been dealing with so far are special cases of mixed strategies, in which one action is played with probability 1 and the rest with probability 0.



		payoffs	
		Heads	Tails
Heads (0.9)	Heads (0.9)	1, -1	-1, 1
	Tails (0.1)	-1, 1	1, -1

Pareto optimal strategies
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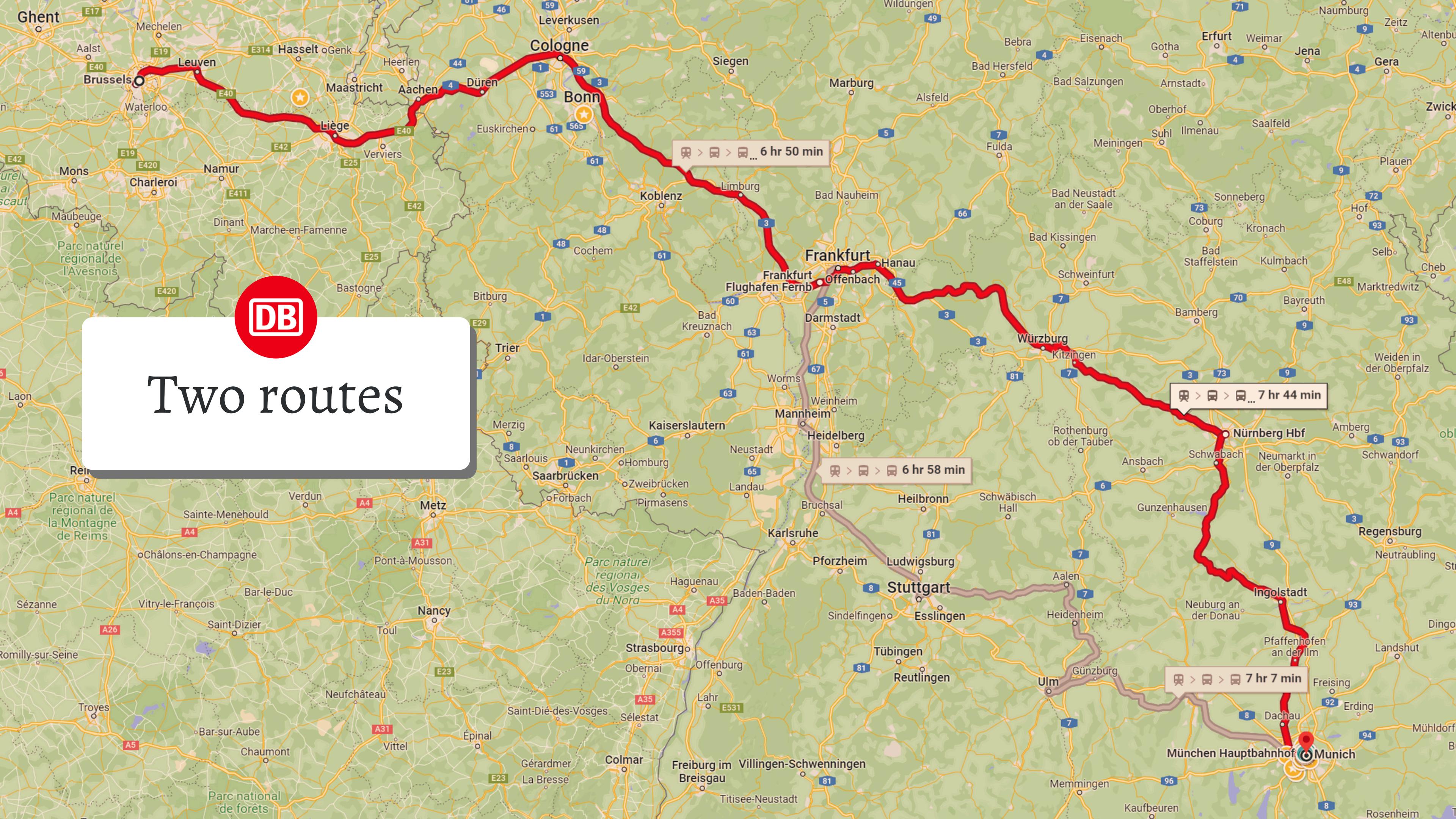


ADRIAN

I need to get from Brussels to Munich.



Two routes

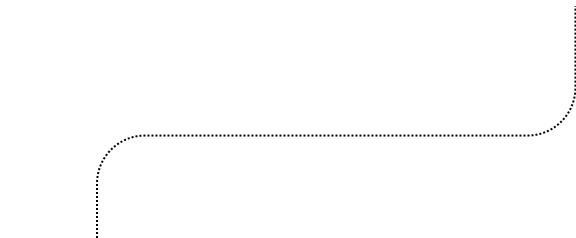


EXPECTED UTILITY: EXAMPLE

My utility is determined by the arrival time.

Option 1

Brussels - Frankfurt - München



23:00

Option 2

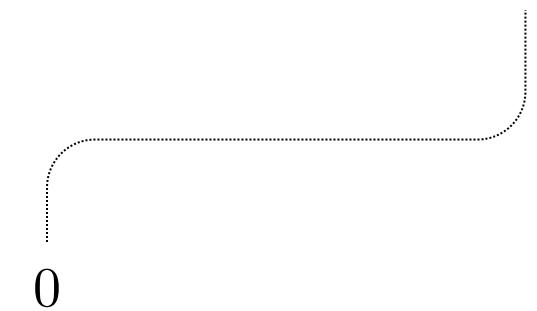
Brussels - Köln - München

23:20

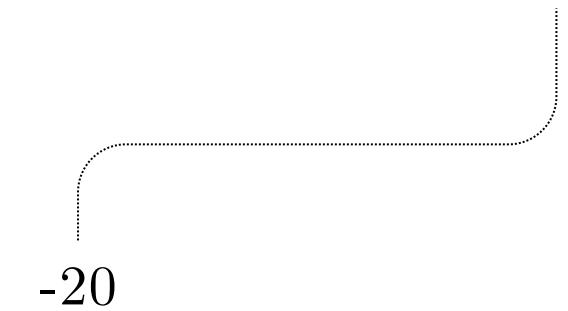
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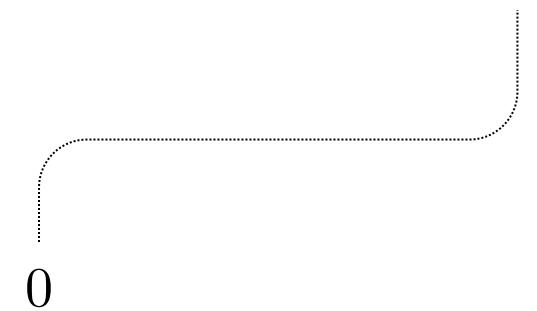


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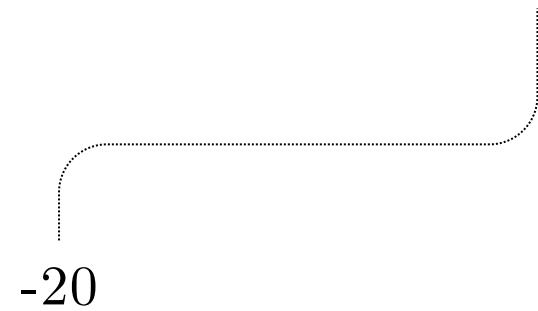
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Which option is best?

Option 1
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Option 2
Brussels - Köln - München

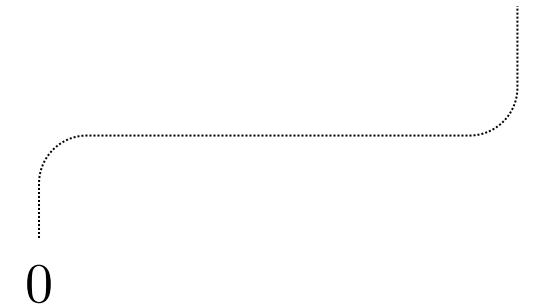


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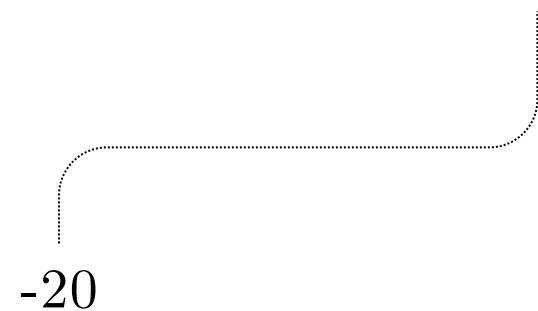
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Which option is best?

Option 1 ✓
Brussels - Frankfurt - München



Option 2
Brussels - Köln - München

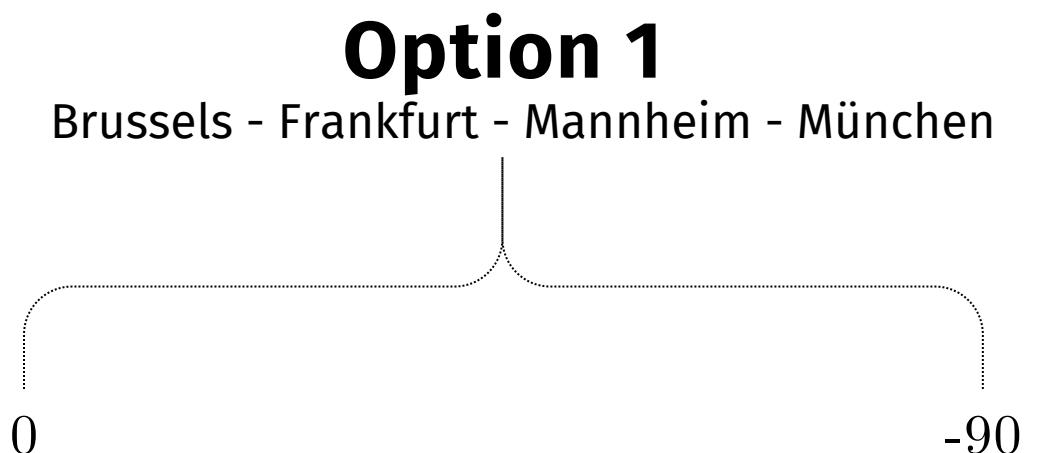


EXPECTED UTILITY: EXAMPLE

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Which option is best?

But with the first option I might miss the Frankfurt connection, meaning and will get home even later.



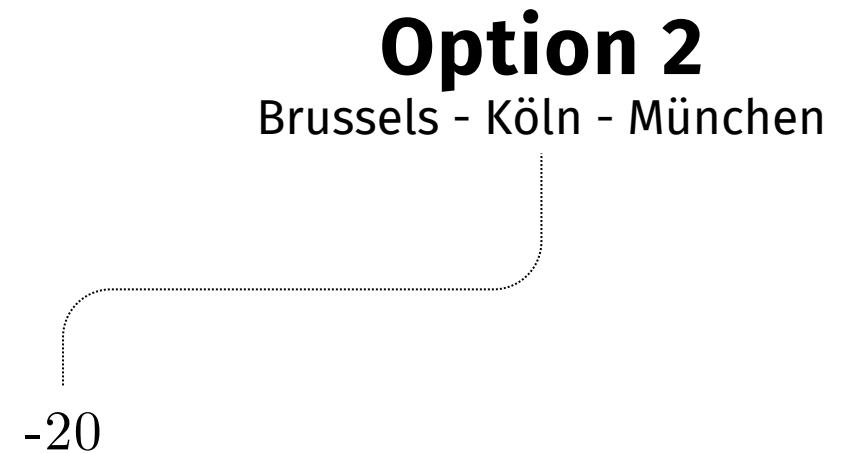
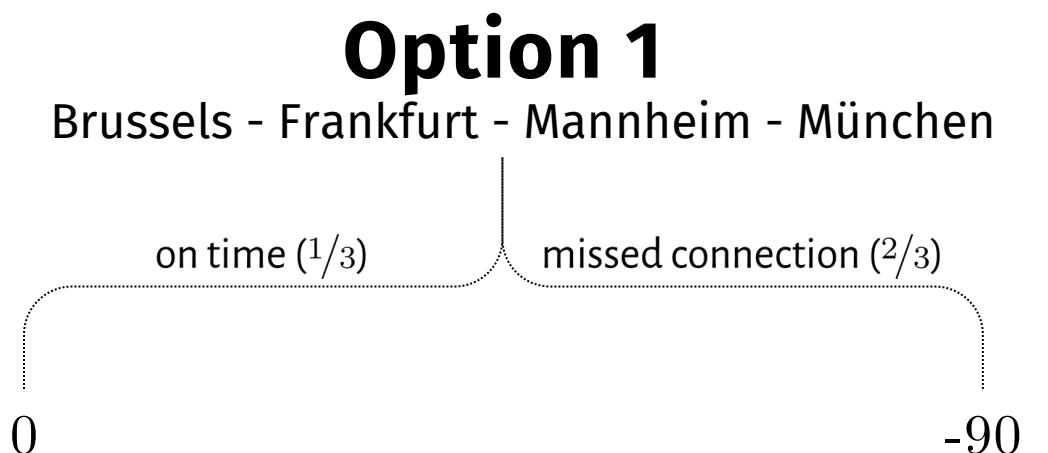
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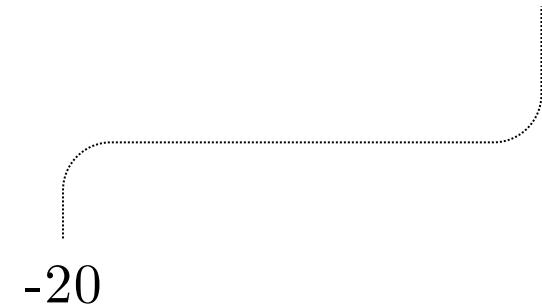
Brussels - Frankfurt - Mannheim - München



$$\begin{aligned}\mathbb{E}[\text{Route 1}] &= \Pr[\text{on time}] \cdot 0 + \Pr[\text{missed connection}] \cdot (-90) \\ &= \frac{1}{3} \cdot 0 + \frac{2}{3} \cdot (-90) \\ &= -60.\end{aligned}$$

Option 2

Brussels - Köln - München



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Brussels - Frankfurt - Mannheim - München



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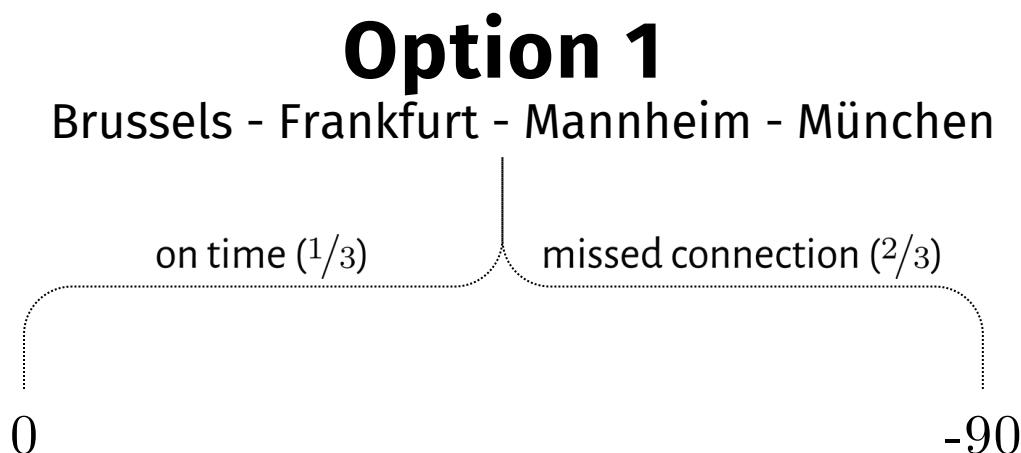
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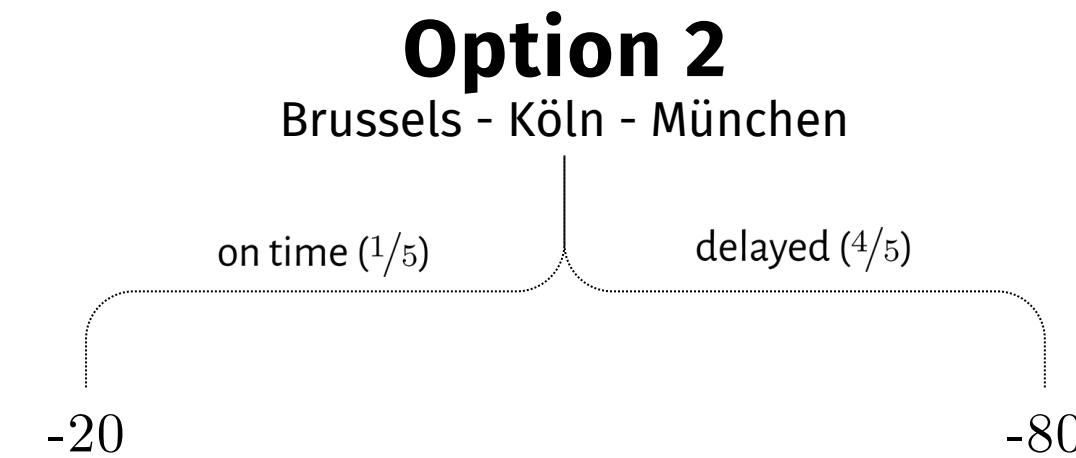
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But wait! The second option has some uncertainty too: past experience suggests a likely delay.



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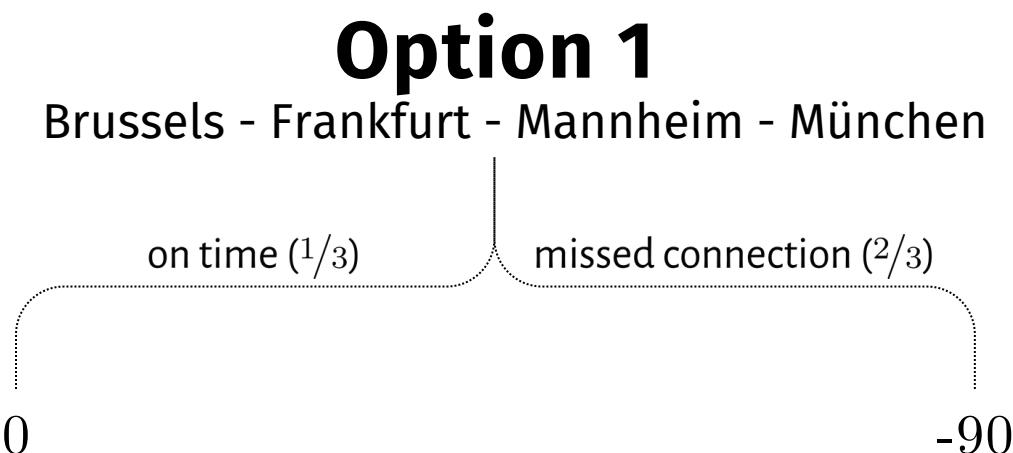
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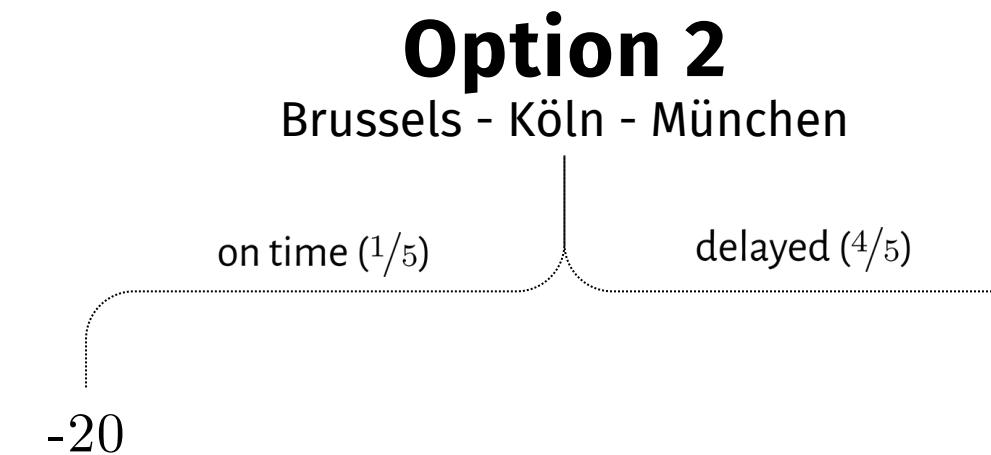
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$$\begin{aligned}\mathbb{E}[\text{Route 2}] &= \Pr[\text{on time}] \cdot (-20) + \Pr[\text{delayed}] \cdot (-80) \\ &= \frac{1}{5} \cdot (-20) + \frac{4}{5} \cdot (-80) \\ &= -68.\end{aligned}$$

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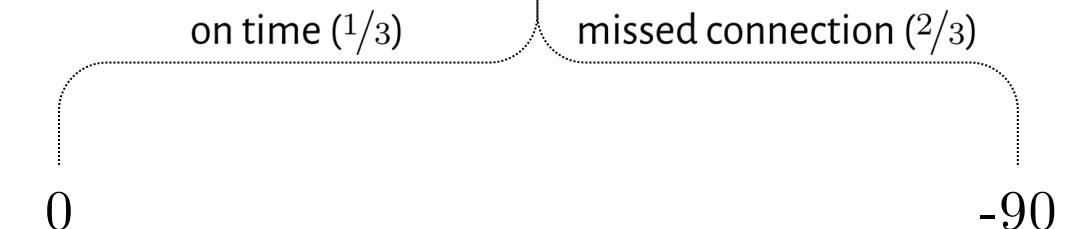
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Better to stick with the first option after all...

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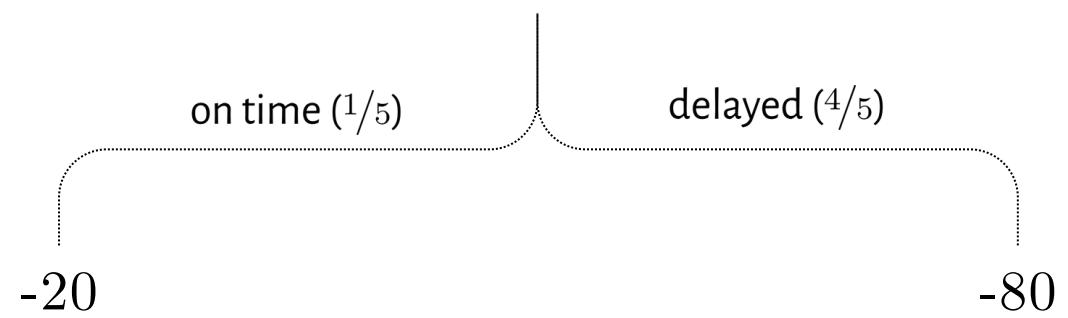
Brussels - Frankfurt - Mannheim - München



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Brussels - Köln - München



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	state of nature				
	on time	missed connection in Frankfurt	delay on Köln - München line	...	expected utility*
Option 1	0	-90	0	...	-60 
Option 2	-20	0	-80 	...	-68

Diagram annotations:

- A curved arrow labeled "action" points from the "on time" outcome of Option 1 to the "missed connection in Frankfurt" outcome of Option 2.
- A curved arrow labeled "utility for taking this action in this state" points from the "expected utility" column of Option 1 to the "expected utility" column of Option 2.

*Table isn't 100% correct: in general, states need to be mutually exclusive

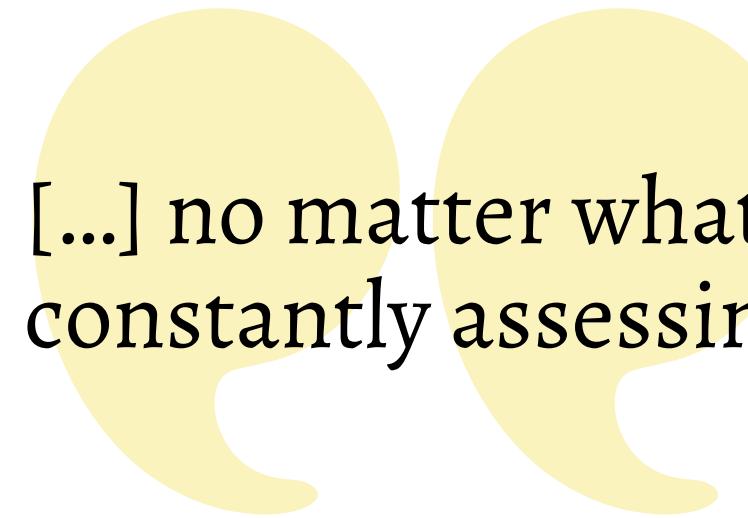
In general, rational agents
(aim to) maximize
expected utility.

$$\mathbb{E}[u(\text{action})] = \sum_{\text{state}} \left(u(\text{action}, \text{state}) \cdot \Pr[\text{state}] \right)$$



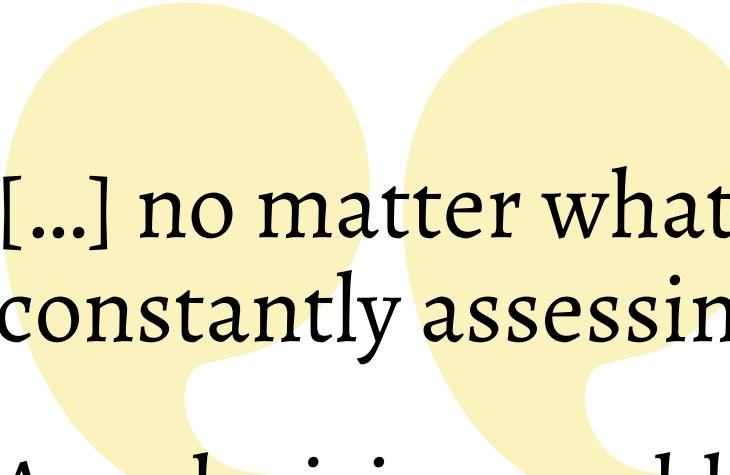
SAM BANKMAN-FRIED
We should be maximising expected
value in everything.

And I mean *everything*.



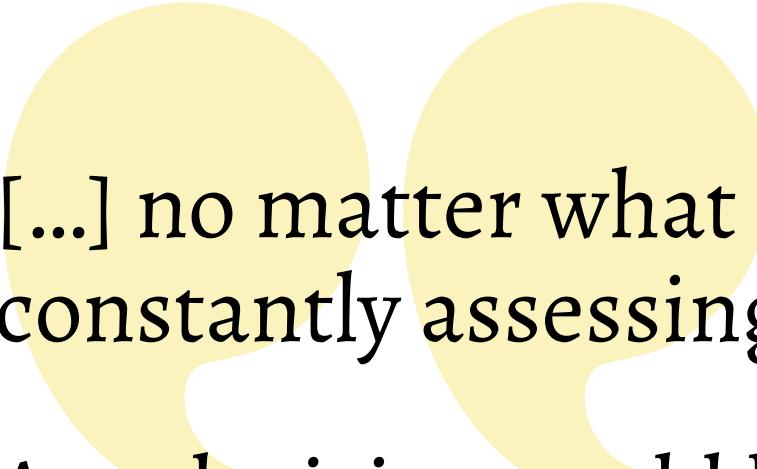
[...] no matter what Bankman-Fried was doing, he was constantly assessing the odds, costs, and benefits.

Faux, Z. (2023). *Number Go Up: Inside Crypto's Wild Rise and Staggering Fall*. Crown Currency.



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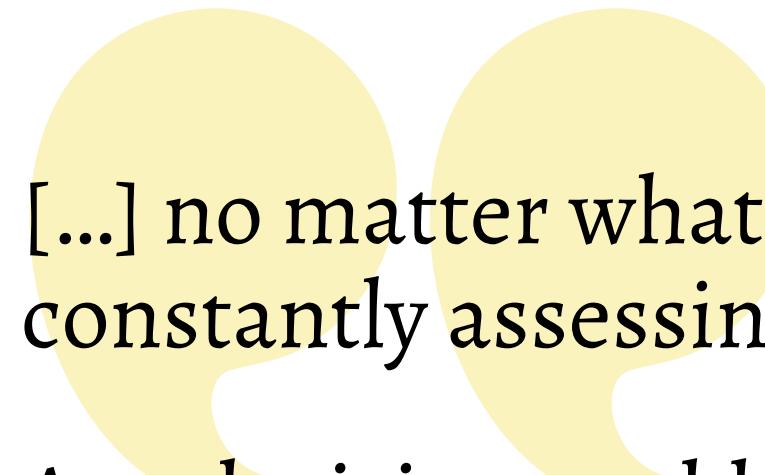
Any decision could be boiled down to an “expected value,” [...] whether that was a move in a board-game marathon, a billion-dollar trade, or whether to chat with Bezos at a party.



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Bankman-Fried’s goal was always to make as much money as possible, so that he could give it to charity.



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Bankman-Fried’s goal was always to make as much money as possible, so that he could give it to charity.

By this metric, even sleep was an unjustifiable luxury. The expected value of staying awake to trade was too high.



SAM BANKMAN-FRIED

Every minute you spend sleeping is costing
you \$x dollars, which means you can save
fewer lives.

This is not investment advice. Use with
caution.*

*Also keep in mind that SBF is in jail today for fraud.

Back to Matching Pennies. Let's try out some strategies.

TRYING OUT SOME STRATEGIES

Suppose Player 1 uses strategy $s_1 = (0.9, 0.1)$. What should Player 2 do?



payoffs

		Heads	Tails
		Heads (0.9)	-1, 1
		Tails (0.1)	1, -1
Heads	1, -1	-1, 1	
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Pareto optimal strategies

all

pure Nash equilibria

none

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$$\begin{aligned}\mathbb{E}[\text{Heads} \mid s_1] &= (-1) \cdot 0.9 + 1 \cdot 0.1 \\ &= -0.8.\end{aligned}$$



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If Player 2 always plays Heads, i.e., $s_2 = (1, 0)$, they get an average payoff of -0.8 . If they always play Tails, i.e., $s'_2 = (0, 1)$, they get an average payoff of 0.8 .



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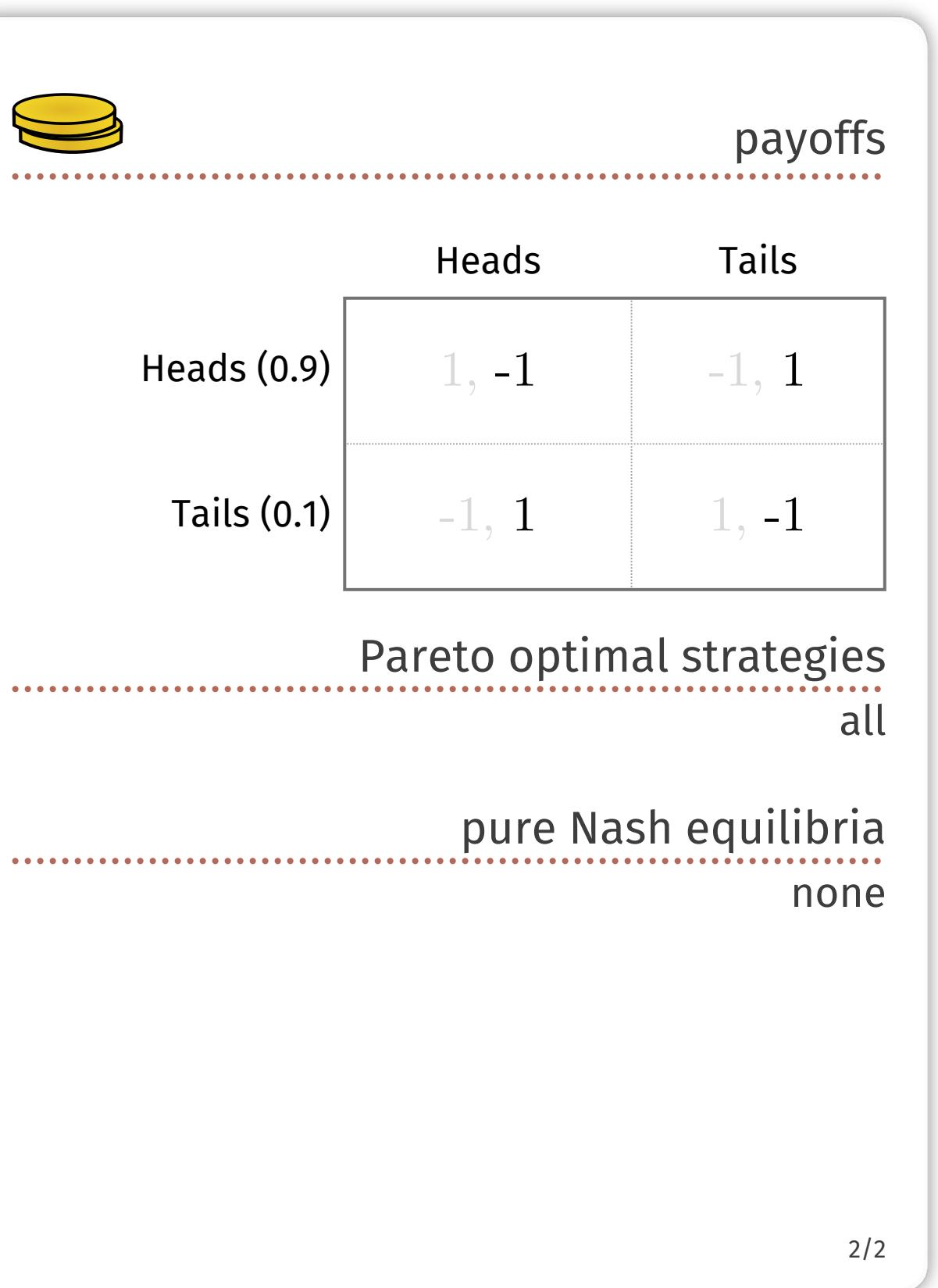
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$$\begin{aligned}\mathbb{E}[s''_2 \mid s_1] &= \mathbb{E}[\text{Heads}] \cdot 0.3 + \mathbb{E}[\text{Tails}] \cdot 0.7 \\ &= 0.32 \\ &< 0.8.\end{aligned}$$



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Given that Player 1 plays $s_1 = (0.9, 0.1)$, then, between s_2 , s'_2 and s''_2 , Player 2 would rather play $s'_2 = (0, 1)$.



Have we found an equilibrium?

AN EQUILIBRIUM?

If Player 1 plays $s_1 = (0.9, 0.1)$, Player 2's maximizes expected utility by playing $s'_2 = (0, 1)$ (easy to check!).



payoffs

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Heads	Tails		

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If Player 1 plays $s_1 = (0.9, 0.1)$, Player 2's maximizes expected utility by playing $s'_2 = (0, 1)$ (easy to check!).

In other words, $s'_2 = (0, 1)$ is Player 2's *best response* to $s_1 = (0.9, 0.1)$.



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So is $s = (s_1, s'_2)$ a Nash equilibrium?



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So is $s = (s_1, s'_2)$ a Nash equilibrium?

No! If Player 2 plays $s'_2 = (0, 1)$, Player 1 wants to deviate to $s'_1 = (0, 1)$:

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$$\begin{aligned}\mathbb{E}[s'_1 | s'_2] &= \mathbb{E}[\text{Heads} | s'_2] \cdot 0 + \mathbb{E}[\text{Tails} | s'_2] \cdot 1 \\ &= (-1) \cdot 0 + 1 \cdot 1 \\ &= 1 \\ &> \mathbb{E}[s_1 | s'_2].\end{aligned}$$



payoffs

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pure Nash equilibria
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Let's find a mixed equilibrium.

FINDING MIXED EQUILIBRIA

Suppose Players 1 and 2 play mixed strategies $s_1 = (p, 1 - p)$ and $s_2 = (q, 1 - q)$, respectively, for $p, q > 0$.



payoffs

		Heads (q)	Tails ($1 - q$)
Heads (p)	Heads (p)	1, -1	-1, 1
	Tails ($1 - p$)	-1, 1	1, -1

Pareto optimal strategies

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pure Nash equilibria

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mixed Nash equilibria

$$s^* = \left(\left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right)$$

FINDING MIXED EQUILIBRIA

Suppose Players 1 and 2 play mixed strategies $s_1 = (p, 1 - p)$ and $s_2 = (q, 1 - q)$, respectively, for $p, q > 0$.

Note that Player 2's expected payoff with these strategies is:

$$\mathbb{E}[s_2 | s_1] = \mathbb{E}[\text{Heads} | s_1] \cdot q + \mathbb{E}[\text{Tails} | s_1] \cdot (1 - q).$$



payoffs

	Heads (q)	Tails (1 - q)
Heads (p)	1, -1	-1, 1
Tails (1 - p)	-1, 1	1, -1

Pareto optimal strategies

all

pure Nash equilibria

none

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payoffs

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Pareto optimal strategies
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So $s = (s_1, s_2)$ cannot be a Nash equilibrium. Same if $\mathbb{E}[\text{Tails} | s_1] > \mathbb{E}[\text{Heads} | s_1]$.



payoffs

		Heads (q)	Tails ($1 - q$)
Heads (p)	1, -1	-1, 1	
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Pareto optimal strategies
all

pure Nash equilibria
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mixed Nash equilibria

$$s^* = \left(\left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right)$$

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So $s = (s_1, s_2)$ cannot be a Nash equilibrium. Same if $\mathbb{E}[\text{Tails} | s_1] > \mathbb{E}[\text{Heads} | s_1]$.

The only way to avoid this is for Player 1 to play a strategy $s_1^* = (p, 1 - p)$ that makes Player 2 indifferent between their actions:

$$\begin{aligned} \mathbb{E}[\text{Heads} | s_1] &= \mathbb{E}[\text{Tails} | s_2] \text{ iff } (-1) \cdot p + 1 \cdot (1 - p) = 1 \cdot p + (-1) \cdot (1 - p) \\ &\text{iff } p = 1/2. \end{aligned}$$



payoffs

	Heads (q)	Tails (1 - q)
Heads (p)	1, -1	-1, 1
Tails (1 - p)	-1, 1	1, -1

Pareto optimal strategies
all

pure Nash equilibria
none

mixed Nash equilibria

$$s^* = \left((1/2, 1/2), (1/2, 1/2) \right)$$

FINDING MIXED EQUILIBRIA

Suppose Players 1 and 2 play mixed strategies $s_1 = (p, 1 - p)$ and $s_2 = (q, 1 - q)$, respectively, for $p, q > 0$.

Note that Player 2's expected payoff with these strategies is:

$$\mathbb{E}[s_2 | s_1] = \mathbb{E}[\text{Heads} | s_1] \cdot q + \mathbb{E}[\text{Tails} | s_1] \cdot (1 - q).$$

Suppose, now, that Player 1's strategy makes Heads more attractive for Player 2:

$$\mathbb{E}[\text{Heads} | s_1] > \mathbb{E}[\text{Tails} | s_1].$$

In this case, Player 2 would want to deviate to $s'_2 = (1, 0)$:

$$\begin{aligned} \mathbb{E}[s'_2 | s_1] &= \mathbb{E}[\text{Heads} | s_1] \cdot 1 + \mathbb{E}[\text{Tails} | s_1] \cdot 0 \\ &> \mathbb{E}[\text{Heads} | s_1] \cdot q + \mathbb{E}[\text{Tails} | s_1] \cdot (1 - q). \end{aligned}$$

So $s = (s_1, s_2)$ cannot be a Nash equilibrium. Same if $\mathbb{E}[\text{Tails} | s_1] > \mathbb{E}[\text{Heads} | s_1]$.

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So Player 1 wants to play $s_1^* = (1/2, 1/2)$. Similarly, Player 2 wants to play $s_2^* = (1/2, 1/2)$. This is the mixed Nash equilibrium.



payoffs

	Heads (q)	Tails (1 - q)
Heads (p)	1, -1	-1, 1
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Pareto optimal strategies
all

pure Nash equilibria
none

mixed Nash equilibria

$$s^* = \left((1/2, 1/2), (1/2, 1/2) \right)$$



JOHN NASH

This can be generalized to any
game with finitely many actions.

NASH'S THEOREM

THEOREM (NASH, 1951)

Any game with a finite number of players and finite actions has a Nash equilibrium in mixed strategies.

Nash, J. (1951). Non-Cooperative Games. *Annals of Mathematics*, 54(2), 286–295.



JOHN NASH

They gave me the Nobel prize for
this result!

Fun fact: humans are not that good at randomizing.



ARIEL RUBINSTEIN

In experiments, they keep trying to detect patterns, are susceptible to stories and framing effects.

Mookherjee, D., & Sopher, B. (1994). Learning Behavior in an Experimental Matching Pennies Game. *Games and Economic Behavior*, 7(1), 62–91.

Eliaz, K., & Rubinstein, A. (2011). Edgar Allan Poe's riddle: Framing effects in repeated matching pennies games. *Games and Economic Behavior*, 71(1), 88–99.

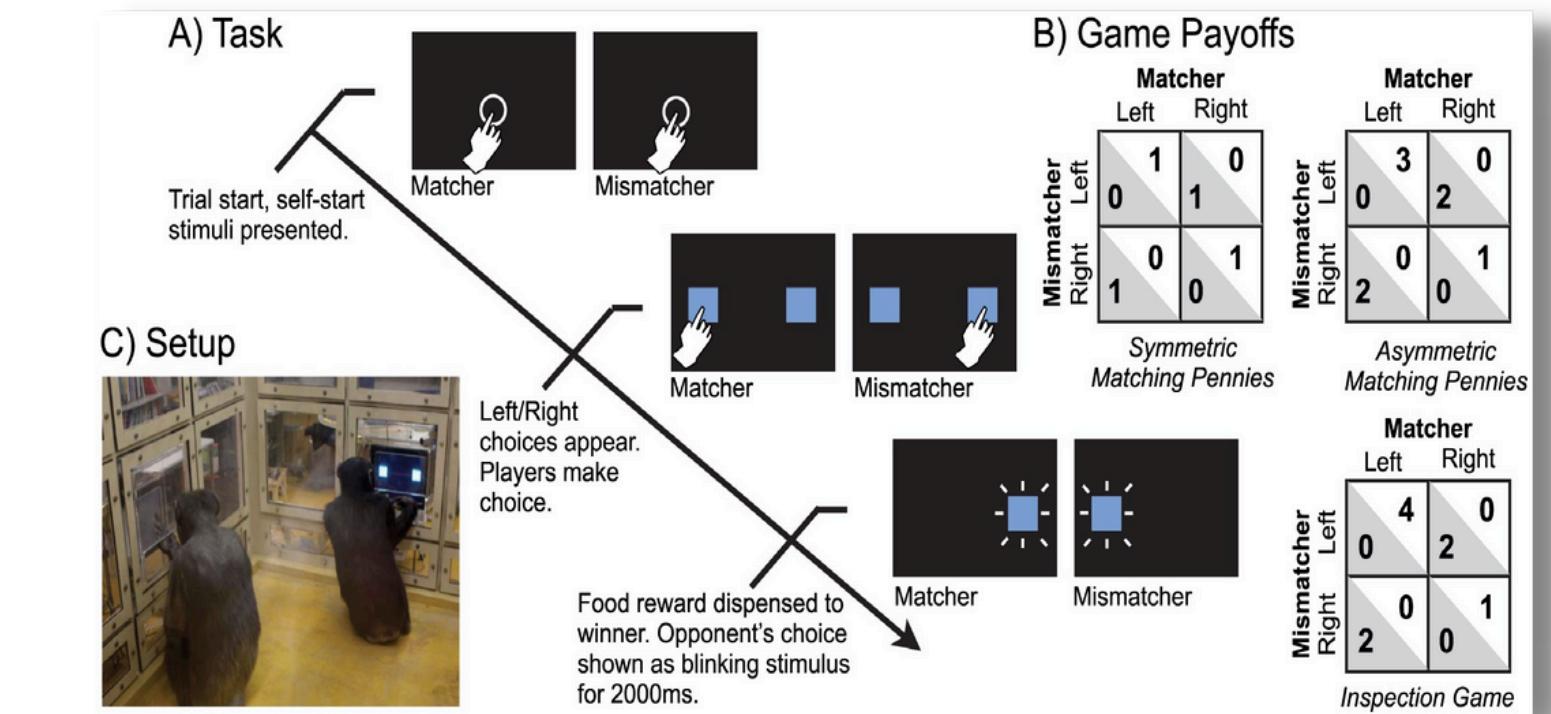
But chimpanzees seem pretty good at it.

MATCHING PENNIES WITH CHIMPANZEES



COLIN CAMERER

In a matching pennies experiment, chimpanzees were quite good at approximating the Nash equilibrium.



Martin, C. F., Bhui, R., Bossaerts, P., Matsuzawa, T., & Camerer, C. (2014). Chimpanzee choice rates in competitive games match equilibrium game theory predictions. *Nature: Scientific Reports*, 4, 5182.