LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MAY 19, 2025

NETWORKED MINDS:
OPINION DYNAMICS AND COLLECTIVE INTELLIGENCE IN SOCIAL NETWORKS

WEIGHTED VOTING RULES

Adrian Haret
a.haret@lmu.de

Recall that when competences are different a voter can outperform the majority opinion.

Recall that when competences are different a voter can outperform the majority opinion. So is majority the most efficient decision rule?

WEIGHTED VOTING RULES

Recall that votes are $v_i \in \{0, 1\}$. The *sum of the votes* is $S_n = v_1 + \cdots + v_n$, and the majority outcome is correct when $S_n > n/2$, which is equivalent to:

$$\frac{1}{n} \cdot v_1 + \cdots + \frac{1}{n} \cdot v_n > \frac{1}{2}.$$

WEIGHTED VOTING RULES

Recall that votes are $v_i \in \{0, 1\}$. The *sum of the votes* is $S_n = v_1 + \cdots + v_n$, and the majority outcome is correct when $S_n > n/2$, which is equivalent to:

$$\frac{1}{n} \cdot v_1 + \cdots + \frac{1}{n} \cdot v_n > \frac{1}{2}.$$

We can interpret $1/n$ as a weight, such that each vote v_i is weighted equally.

WEIGHTED VOTING RULES

Recall that votes are $v_i \in \{0, 1\}$. The *sum of the votes* is $S_n = v_1 + \cdots + v_n$, and the majority outcome is correct when $S_n > n/2$, which is equivalent to:

$$\frac{1}{n} \cdot v_1 + \cdots + \frac{1}{n} \cdot v_n > \frac{1}{2}.$$

We can interpret $1/n$ as a weight, such that each vote v_i is weighted equally.

Relaxing this assumption, we assume that each voter i has a *weight* $w_i \in \mathbb{R}$.

WEIGHTED VOTING RULES

Recall that votes are $v_i \in \{0, 1\}$. The *sum of the votes* is $S_n = v_1 + \cdots + v_n$, and the majority outcome is correct when $S_n > {}^{n}/_{2}$, which is equivalent to:

$$\frac{1}{n} \cdot v_1 + \cdots + \frac{1}{n} \cdot v_n > \frac{1}{2}.$$

We can interpret ${}^{1}/_{n}$ as a weight, such that each vote v_i is weighted equally.

Relaxing this assumption, we assume that each voter i has a *weight* $w_i \in \mathbb{R}$.

The *weighted sum of the votes* is:

$$W_n = w_1 v_1 + \cdots + w_n v_n,$$

WEIGHTED VOTING RULES

Recall that votes are $v_i \in \{0, 1\}$. The *sum of the votes* is $S_n = v_1 + \cdots + v_n$, and the majority outcome is correct when $S_n > n/2$, which is equivalent to:

$$\frac{1}{n} \cdot v_1 + \cdots + \frac{1}{n} \cdot v_n > \frac{1}{2}.$$

We can interpret $1/n$ as a weight, such that each vote v_i is weighted equally.

Relaxing this assumption, we assume that each voter i has a *weight* $w_i \in \mathbb{R}$.

The *weighted sum of the votes* is:

$$W_n = w_1 v_1 + \cdots + w_n v_n,$$

We define the *group vote* to be a (the correct alternative) if:

$$W_n > \frac{w_1 + \cdots + w_n}{2}.$$

WEIGHTED VOTING RULES: EXAMPLE

Take three voters, with weights $w_1 = 4$, $w_2 = 1$, $w_3 = 1$. The weighted sum of the votes is:

$$W_3 = 4v_1 + v_2 + v_3,$$

and the threshold for voting for a is 3.

WEIGHTED VOTING RULES: EXAMPLE

Take three voters, with weights $w_1 = 4$, $w_2 = 1$, $w_3 = 1$. The weighted sum of the votes is:

$$W_3 = 4v_1 + v_2 + v_3,$$

and the threshold for voting for a is 3.

Suppose the votes are $v_1 = 1$, $v_2 = 0$ and $v_3 = 0$. The weighted sum is:

$$W_3 = 4 > 3,$$

which means the group vote is a. The group votes correctly, despite a wrong vote from v_2 and v_3.

WEIGHTED VOTING RULES: EXAMPLE

Take three voters, with weights $w_1 = 4$, $w_2 = 1$, $w_3 = 1$. The weighted sum of the votes is:

$$W_3 = 4v_1 + v_2 + v_3,$$

and the threshold for voting for a is 3.

Suppose the votes are $v_1 = 1$, $v_2 = 0$ and $v_3 = 0$. The weighted sum is:

$$W_3 = 4 > 3,$$

which means the group vote is a. The group votes correctly, despite a wrong vote from v_2 and v_3.

If the votes are $v_1 = 0$, $v_2 = 1$ and $v_3 = 1$, the weighted sum is:

$$W_3 = 2 < 3,$$

which means the group vote is b. The group votes incorrectly, despite correct votes from v_2 and v_3.

Note that weights can be negative!

WEIGHTED VOTING RULES: ANOTHER EXAMPLE

Take three voters, with weights $w_1 = 2$, $w_2 = -3$, $w_3 = -3$. The weighted sum of the votes is:

$$W_3 = 2v_1 - 3v_2 - 3v_3,$$

and the threshold for voting for a is -2.

WEIGHTED VOTING RULES: ANOTHER EXAMPLE

Take three voters, with weights $w_1 = 2$, $w_2 = -3$, $w_3 = -3$. The weighted sum of the votes is:

$$W_3 = 2v_1 - 3v_2 - 3v_3,$$

and the threshold for voting for a is -2.

Suppose the votes are $v_1 = 1$, $v_2 = 1$ and $v_3 = 1$. The weighted sum is:

$$W_3 = -4 < -2,$$

which means the group vote is b.

WEIGHTED VOTING RULES: ANOTHER EXAMPLE

Take three voters, with weights $w_1 = 2$, $w_2 = -3$, $w_3 = -3$. The weighted sum of the votes is:

$$W_3 = 2v_1 - 3v_2 - 3v_3,$$

and the threshold for voting for a is -2.

Suppose the votes are $v_1 = 1$, $v_2 = 1$ and $v_3 = 1$. The weighted sum is:

$$W_3 = -4 < -2,$$

which means the group vote is b.

The group votes incorrectly, even though everyone votes correctly (!!!).

WEIGHTED VOTING RULES: ANOTHER EXAMPLE

Take three voters, with weights $w_1 = 2$, $w_2 = -3$, $w_3 = -3$. The weighted sum of the votes is:

$$W_3 = 2v_1 - 3v_2 - 3v_3,$$

and the threshold for voting for a is -2.

Suppose the votes are $v_1 = 1$, $v_2 = 1$ and $v_3 = 1$. The weighted sum is:

$$W_3 = -4 < -2,$$

which means the group vote is b.

The group votes incorrectly, even though everyone votes correctly (!!!).

Wut?

The collective decision is geared towards the *opposite* of what voters 2 and 3 think.

The collective decision is geared towards the *opposite* of what voters 2 and 3 think. This would make sense if these voters tend to get it wrong most of the time.

The collective decision is geared towards the *opposite* of what voters 2 and 3 think. This would make sense if these voters tend to get it wrong most of the time. That is, if their competence is low.

The collective decision is geared towards the *opposite* of what voters 2 and 3 think. This would make sense if these voters tend to get it wrong most of the time. That is, if their competence is low.

If we knew voters' accuracies, how should we choose the weights to maximize group competence?

$$\Pr\left[W_n > \frac{w_1 + \cdots + w_n}{2}\right]$$

THE OPTIMAL VOTING RULE

THEOREM (GROFMAN, 1978)
For n voters with competences $p_1, \ldots, p_n$, the neutral* decision rule that maximizes the probability of a correct decision is a weighted voting rule with weights $w_1, \ldots, w_n$ such that:

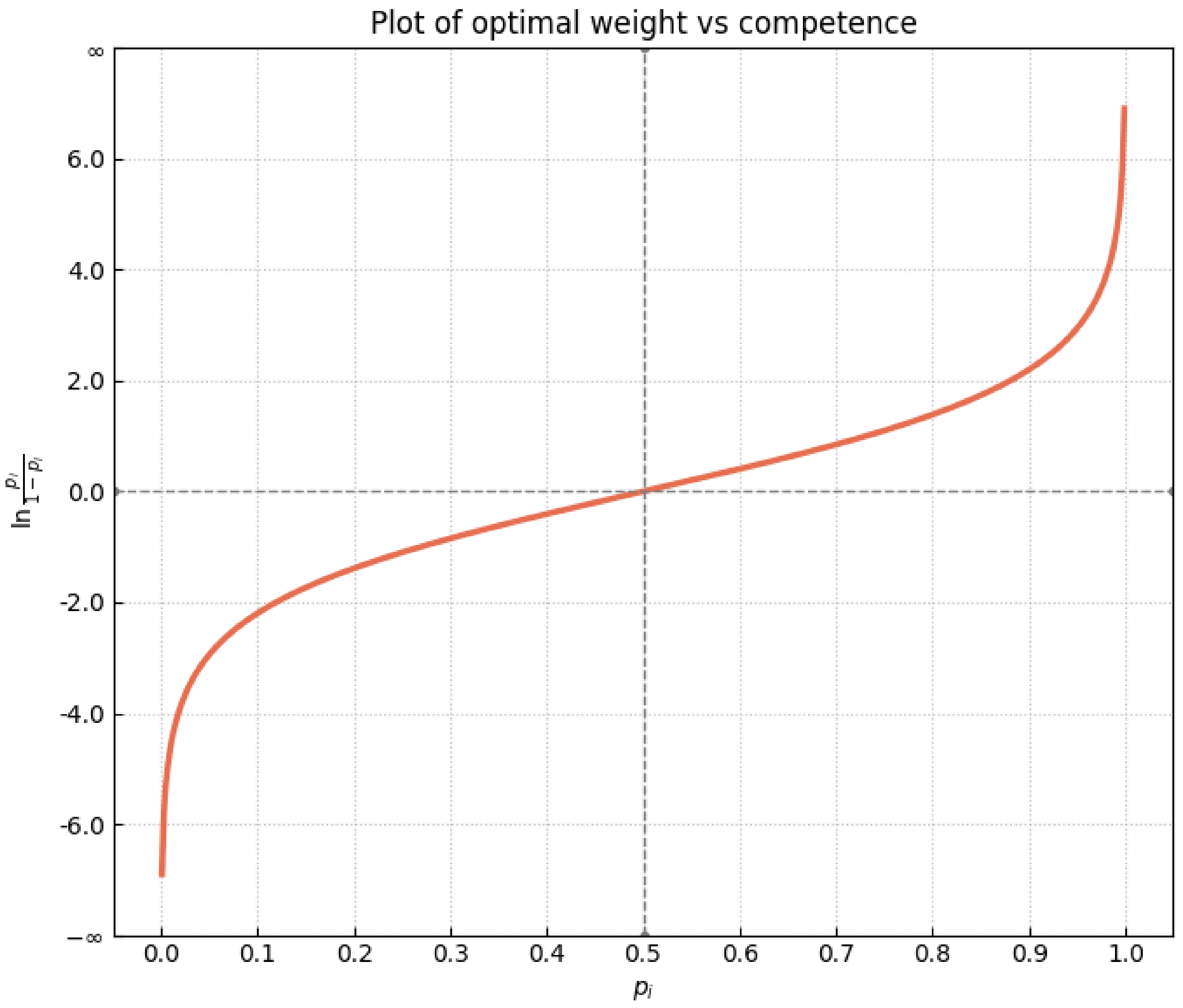
$$w_i \propto \ln \frac{p_i}{1 - p_i}, \text{ for every voter } i.$$

Nitzan, S., & Paroush, J. (1982). Optimal Decision Rules in Uncertain Dichotomous Choice Situations. *International Economic Review*, 23(2), 289–297.
Shapley, L., & Grofman, B. (1984). Optimizing group judgmental accuracy in the presence of interdependencies. *Public Choice*, 43(3), 329–343.

*Neutral means that if the votes are flipped, the group decision is also flipped.

WEIGHT AS FUNCTION OF COMPETENCE

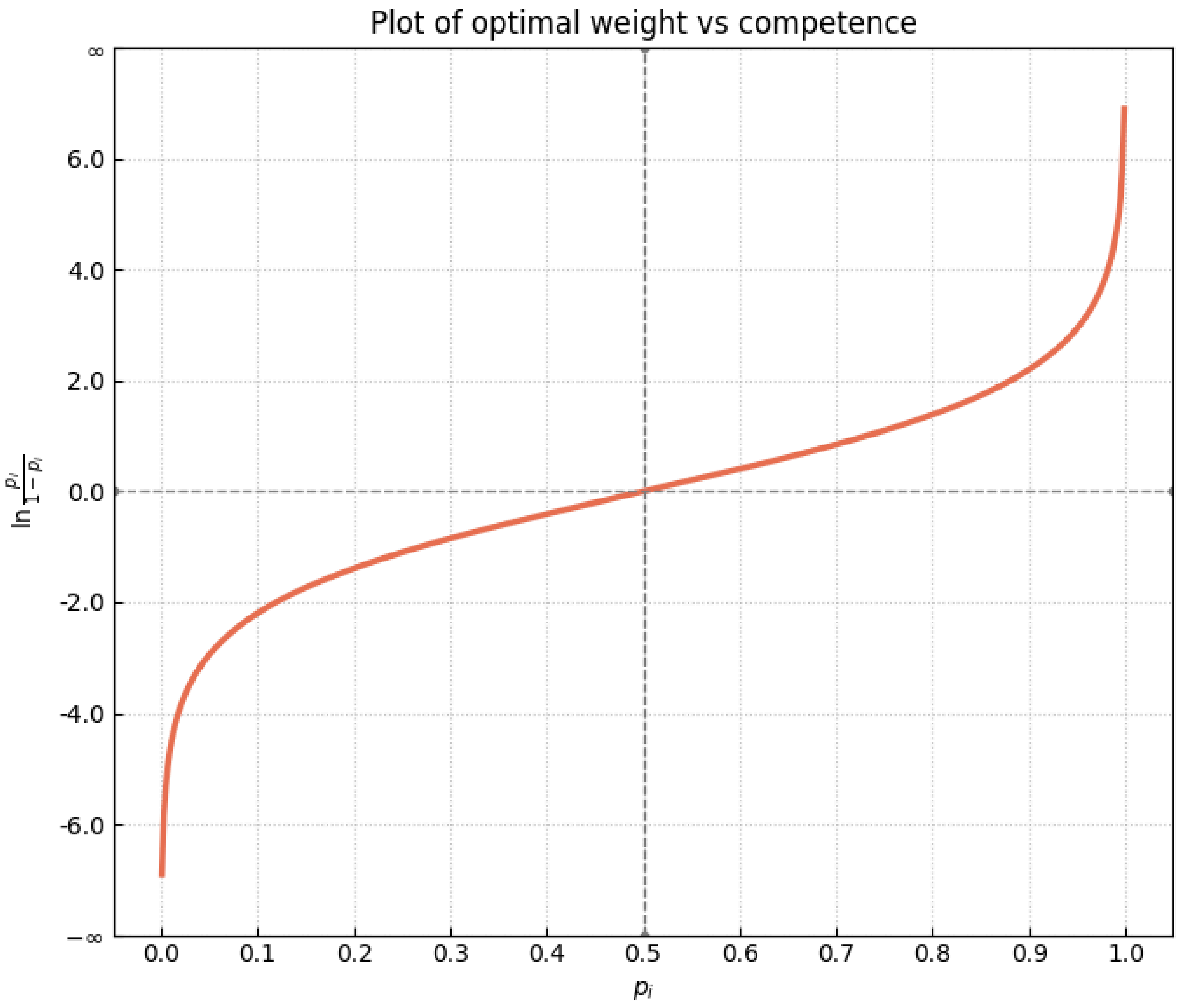
Note that if competence dips below ½, weight becomes negative.



WEIGHT AS FUNCTION OF COMPETENCE

Note that if competence dips below ½, weight becomes negative.

The voter pushes the decision towards the opposite of their choice.

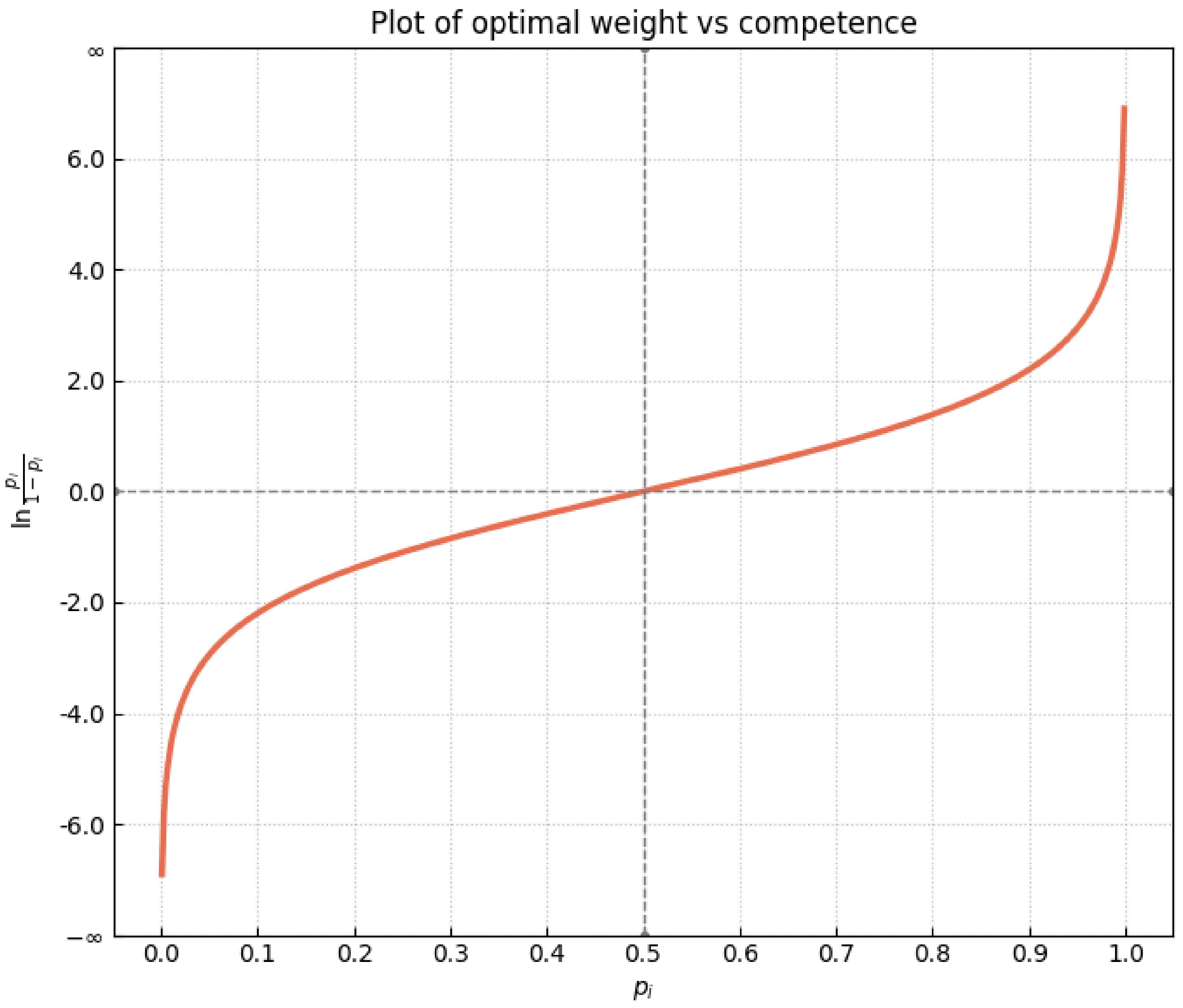


WEIGHT AS FUNCTION OF COMPETENCE

Note that if competence dips below ½, weight becomes negative.

The voter pushes the decision towards the opposite of their choice.

This is because the decision rule takes a vote for alternative *x* to be a signal that the *other* alternative is the correct one.



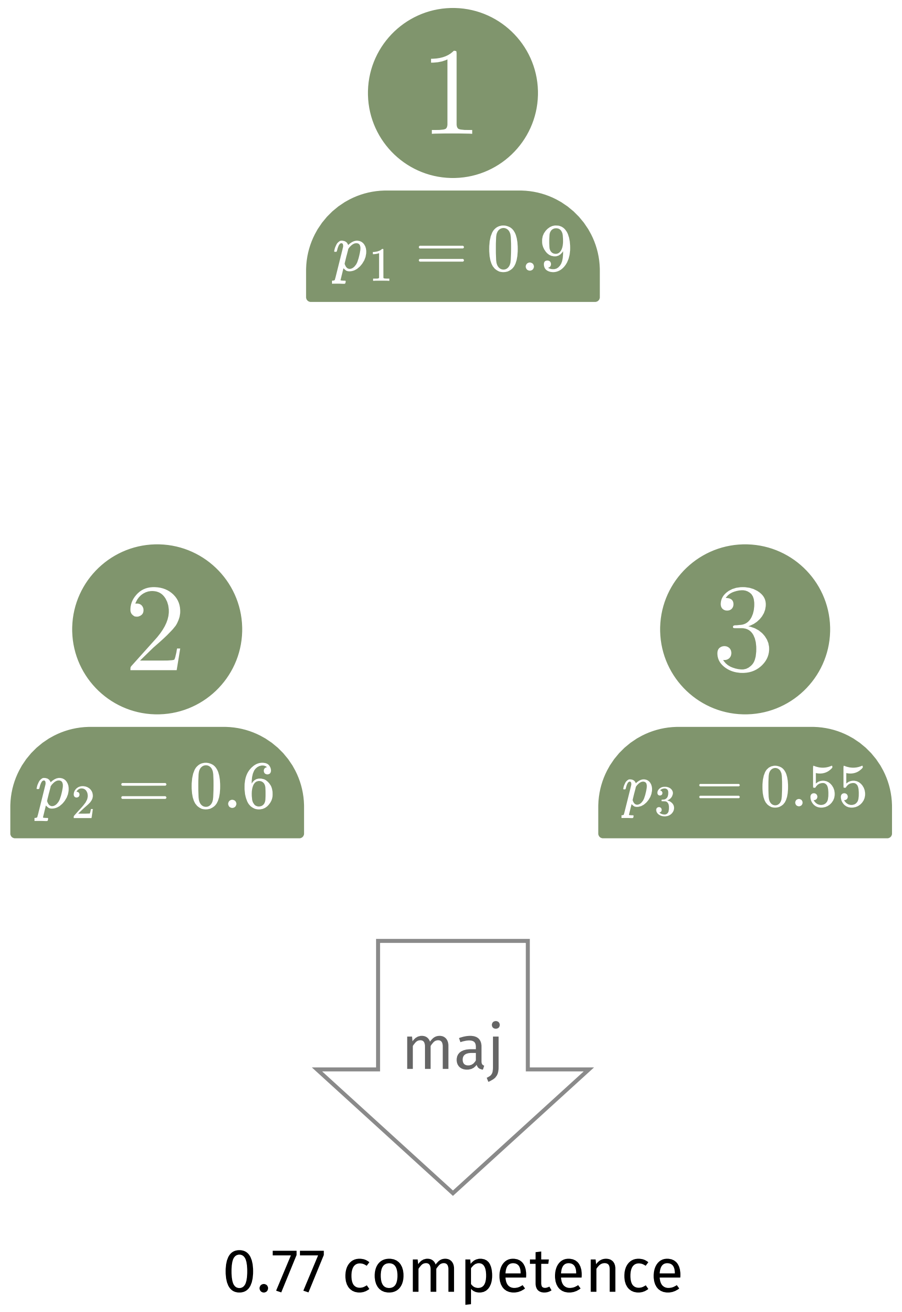
Good luck selling a voting rule that might do the opposite of what the voters want…

PLATO

We should just go with what the expert says.

EXPERT RULE

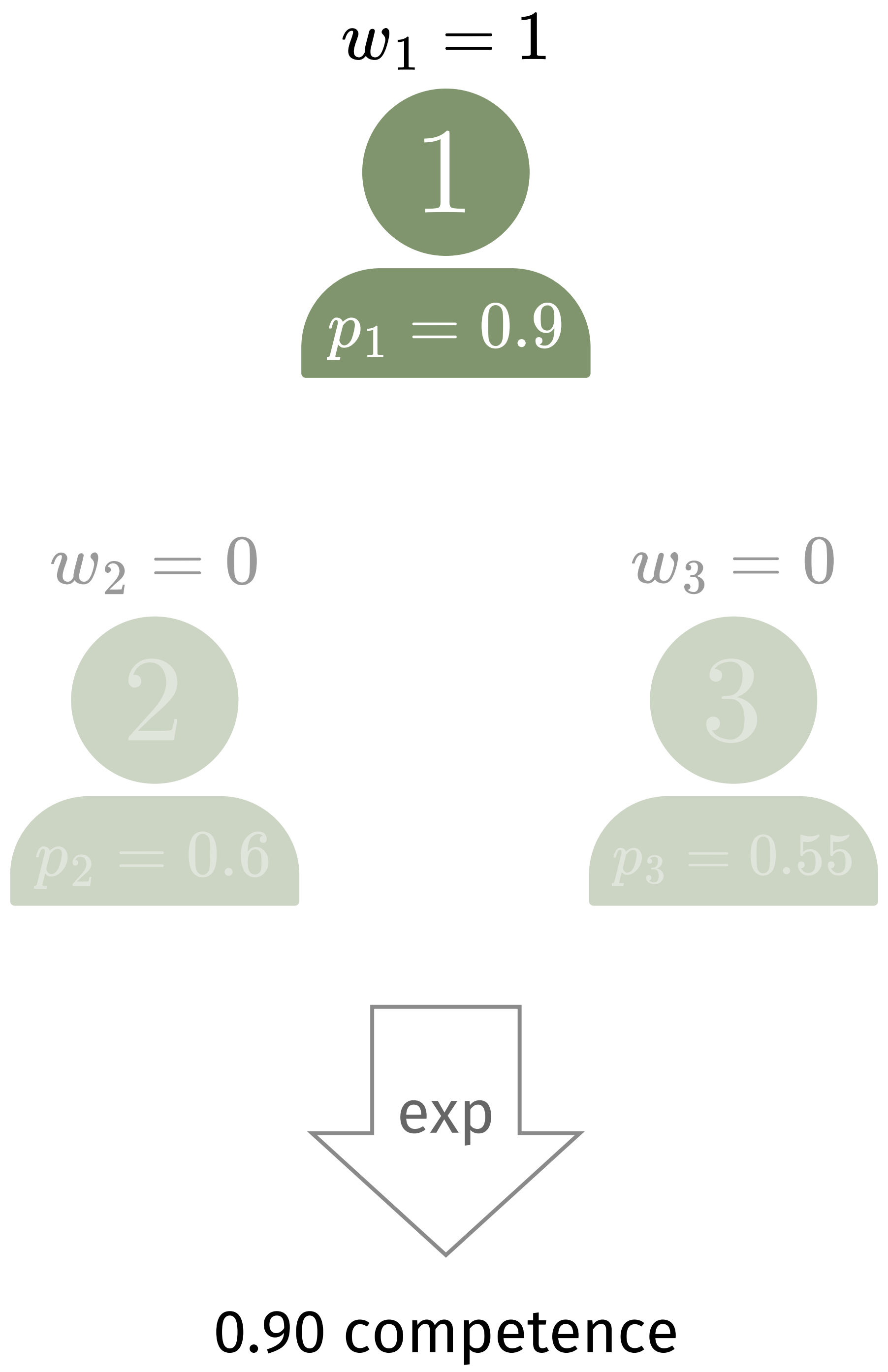
The *expert rule* says: do what the max-competence voter says.



EXPERT RULE

The *expert rule* says: do what the max-competence voter says.

It can be implemented as a rule where the expert has full weight, and everyone else has 0 weight.



The problem with this is that we might not always know who the top expert is.

The problem with this is that we might not always know who the top expert is. That is, there might be uncertainty about the competences.

The problem with this is that we might not always know who the top expert is. That is, there might be uncertainty about the competences.

Regardless, the tension between majority rule and expert rule is a real one.

TOO MUCH DEMOCRACY?

Mexico will become the first country in the world where every judge on every court is chosen by popular vote.

The Americas | The end of independence

Mexico will be the only country that elects all its judges

The last meaningful check on Morena, the powerful ruling party, will fade away

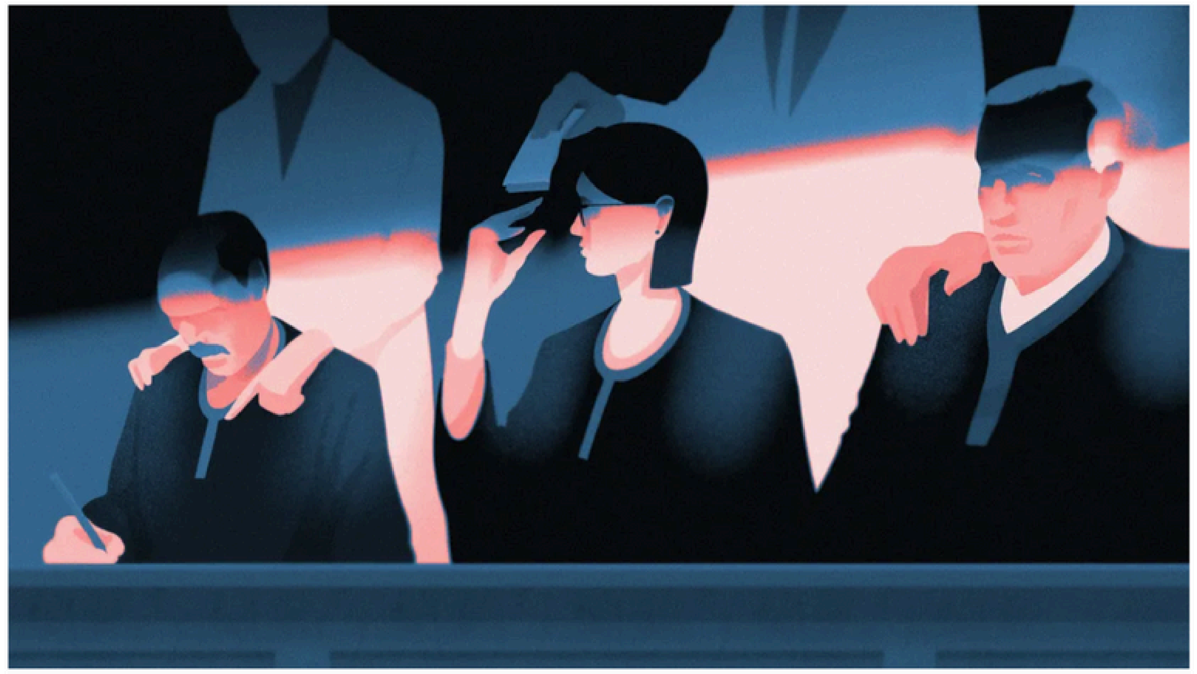


ILLUSTRATION: DANIEL STOLLE

Mexico will be the only country that elects all its judges. (2025, May 15). *The Economist*.

TOO MUCH DEMOCRACY?

Mexico will become the first country in the world where every judge on every court is chosen by popular vote.

Electing all judges is a bad idea “full stop”, says Julio Ríos, a political scientist at ITAM, a university in Mexico City.


The Americas | The end of independence

Mexico will be the only country that elects all its judges

The last meaningful check on Morena, the powerful ruling party, will fade away

ILLUSTRATION: DANIEL STOLLE

Mexico will be the only country that elects all its judges. (2025, May 15). *The Economist*.

TOO MUCH DEMOCRACY?

Mexico will become the first country in the world where every judge on every court is chosen by popular vote.

Electing all judges is a bad idea “full stop”, says Julio Ríos, a political scientist at ITAM, a university in Mexico City.

Having to seek election subjects judges to the warping power of public opinion.

The Americas | The end of independence

Mexico will be the only country that elects all its judges

The last meaningful check on Morena, the powerful ruling party, will fade away

ILLUSTRATION: DANIEL STOLLE

Mexico will be the only country that elects all its judges. (2025, May 15). *The Economist.*

TOO MUCH DEMOCRACY?

Mexico will become the first country in the world where every judge on every court is chosen by popular vote.

Electing all judges is a bad idea "full stop", says Julio Ríos, a political scientist at ITAM, a university in Mexico City.

Having to seek election subjects judges to the warping power of public opinion.

In Mexico, judicial elections pose a graver danger than mere chaos: control of the justice system by drug gangs.

The Americas | The end of independence

Mexico will be the only country that elects all its judges

The last meaningful check on Morena, the powerful ruling party, will fade away

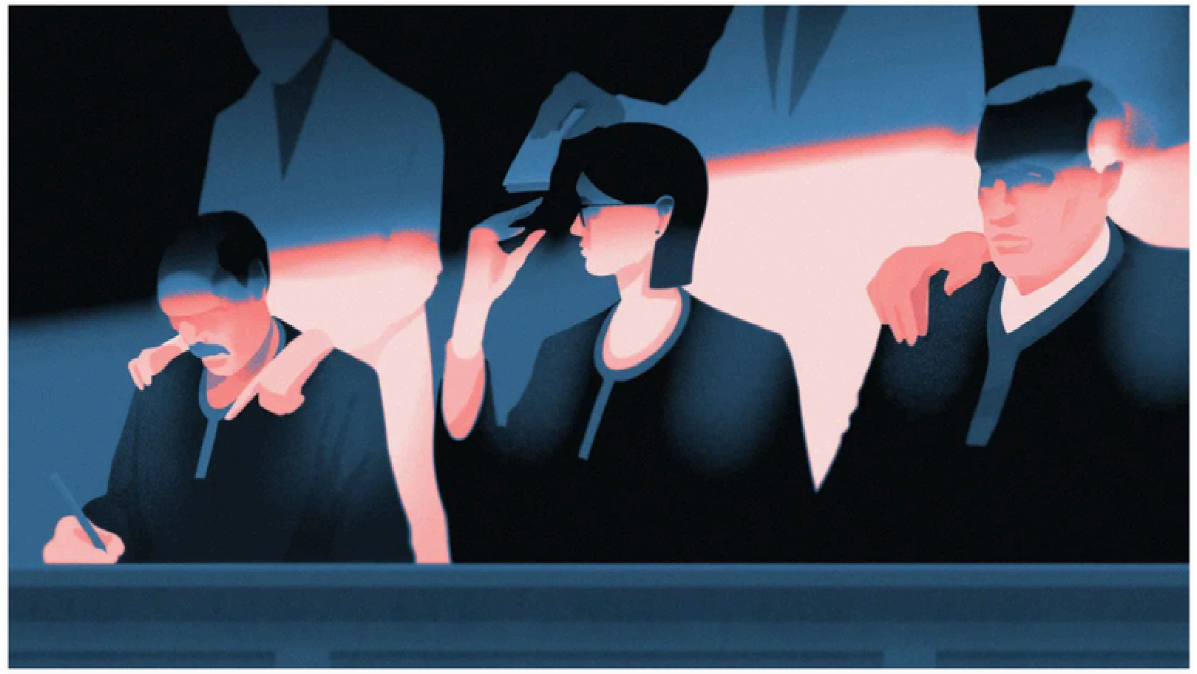


ILLUSTRATION: DANIEL STOLLE

Mexico will be the only country that elects all its judges. (2025, May 15). *The Economist*.