LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

JANUARY 16, 2024

ADVENTURES IN DEMOCRATIC DECISION MAKING

APPORTIONMENT IN THE US

Adrian Haret
a.haret@lmu.de

NORTH AMERICA

1776


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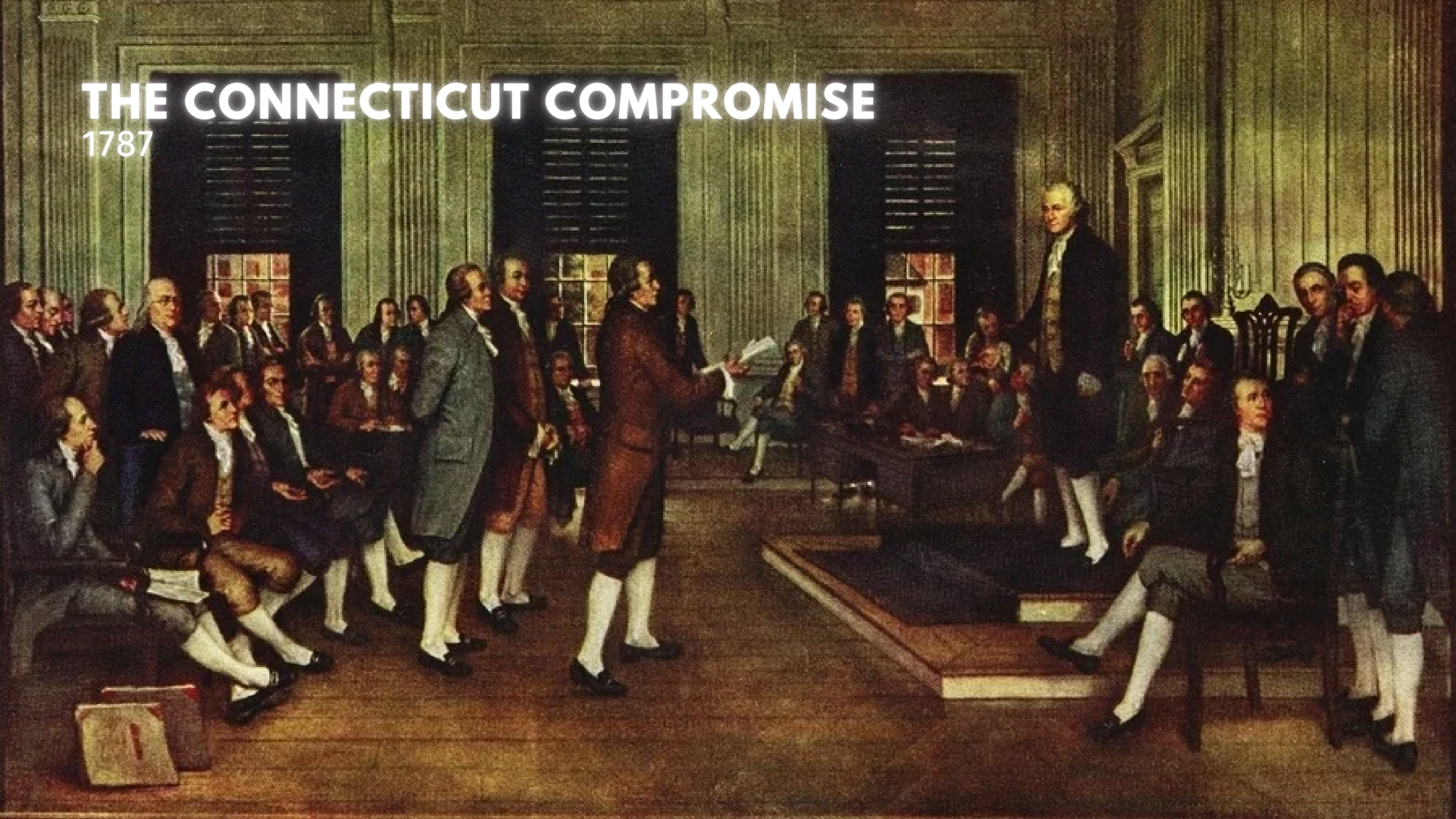
How will the constituent states be represented at the national level?

THE CONNECTICUT COMPROMISE

1787



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THE CONNECTICUT COMPROMISE

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The US legislature will have two chambers. Approval by both chambers is required to pass a bill. The Senate is composed of two representatives from every state.

As for the House of Representatives…

THE US CONSTITUTION
1789

Representatives [...] shall be apportioned among the several States [...] according to their respective Numbers.

US Constitution (1789), Article I, Section 2, Clause 3

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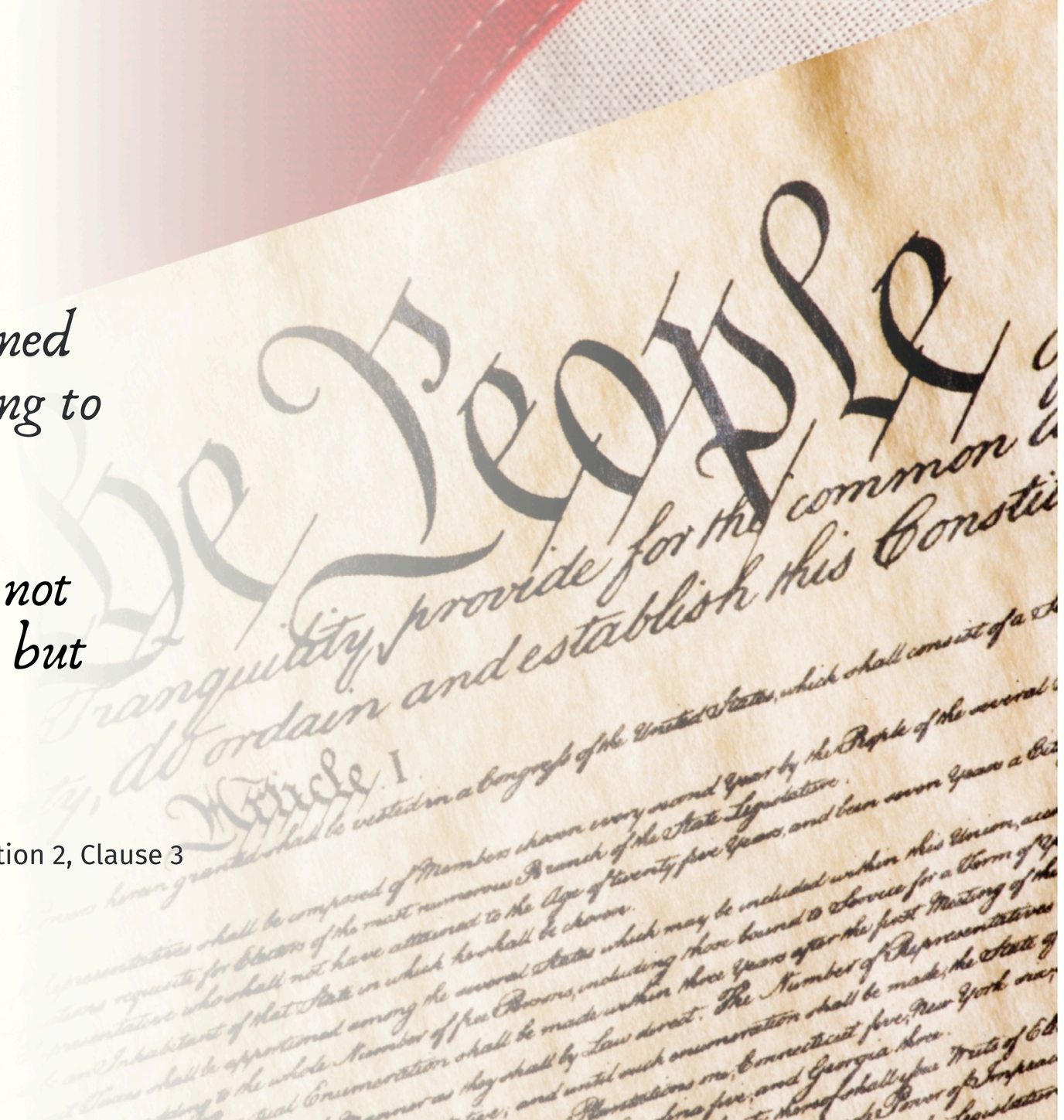
Representatives [...] shall be apportioned among the several States [...] according to their respective Numbers.

The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative...

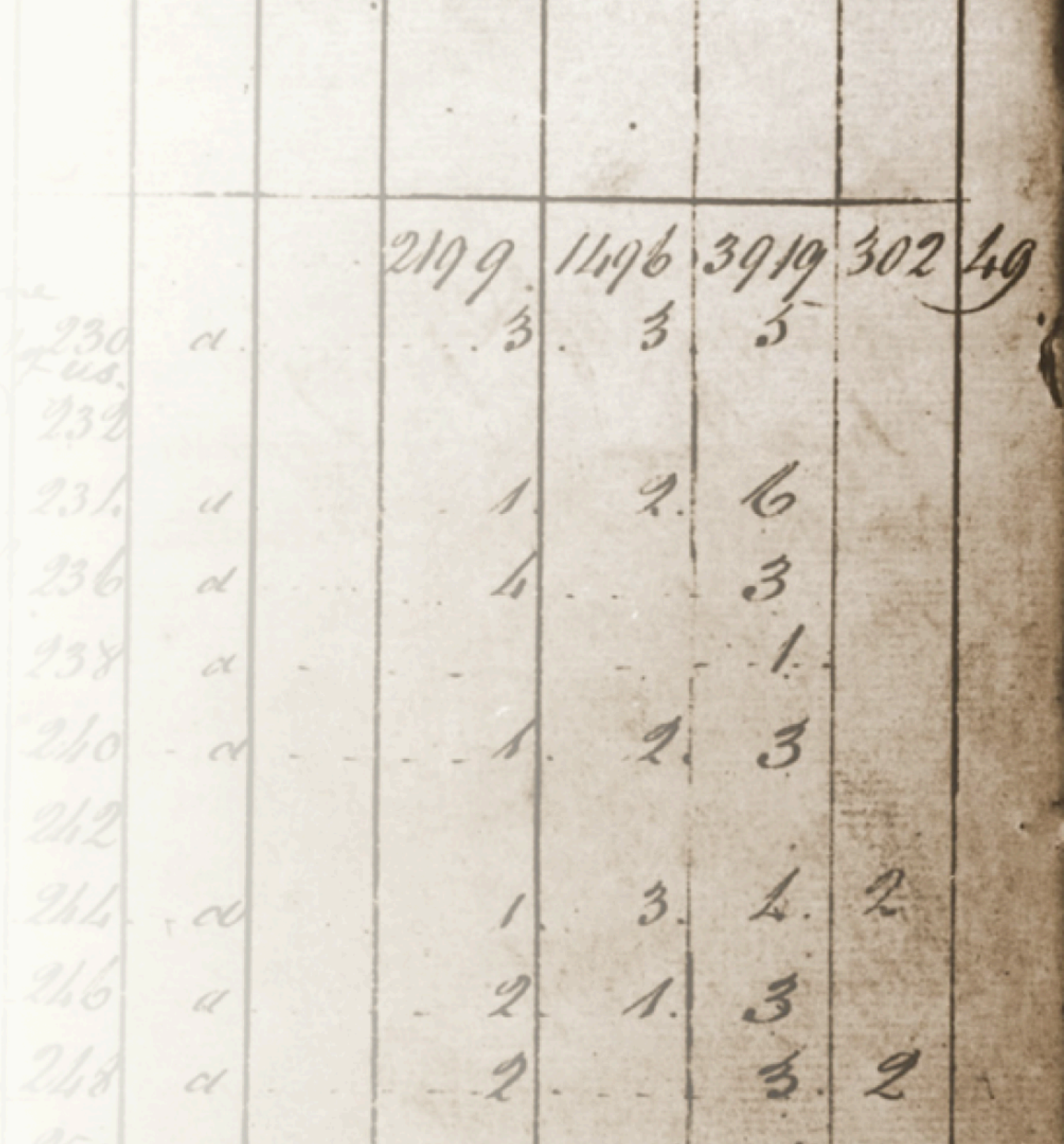
US Constitution (1789), Article I, Section 2, Clause 3



THE FIRST US CENSUS

1790

Fifteen states.



THE FIRST US CENSUS

1790

Fifteen states.

State	Population
Connecticut	236,841
Delaware	55,540
Georgia	70,835
Kentucky	68,705
Maryland	278,514
Massachusetts	475,327
New Hampshire	141,822
New Jersey	179,570
New York	331,589
North Carolina	353,523
Pennsylvania	432,879
Rhode Island	68,446
South Carolina	206,236
Vermont	85,533
Virginia	630,560
US (total)	**3,615,920**

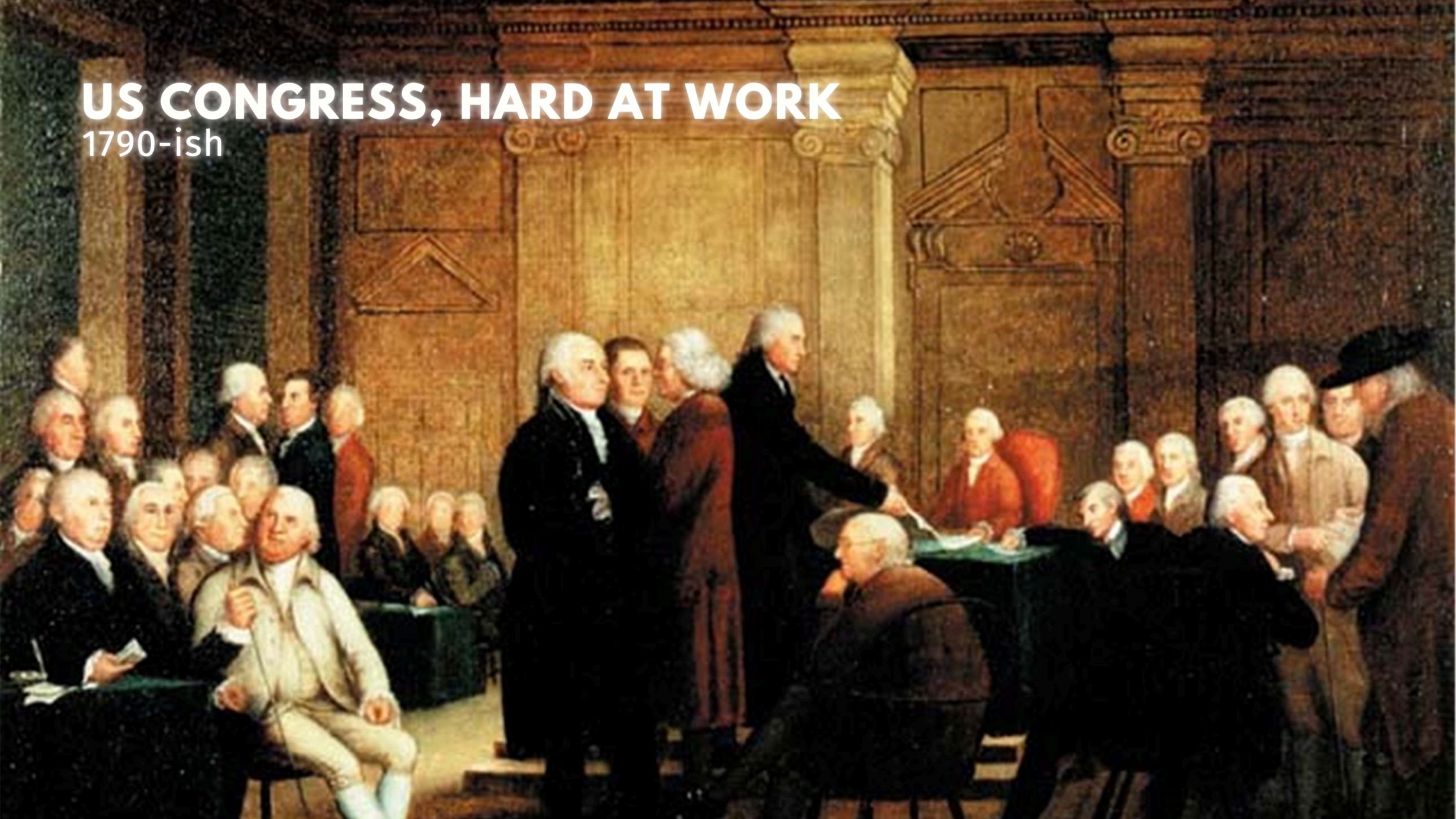
THE FIRST US CENSUS
1790

Fifteen states.

The Constitution sets some constraints, but does not specify how to apportion representatives *exactly*.

State	Population	Seats
Connecticut	236,841	?
Delaware	55,540	?
Georgia	70,835	?
Kentucky	68,705	?
Maryland	278,514	?
Massachusetts	475,327	?
New Hampshire	141,822	?
New Jersey	179,570	?
New York	331,589	?
North Carolina	353,523	?
Pennsylvania	432,879	?
Rhode Island	68,446	?
South Carolina	206,236	?
Vermont	85,533	?
Virginia	630,560	?
US (total)	**3,615,920**	?

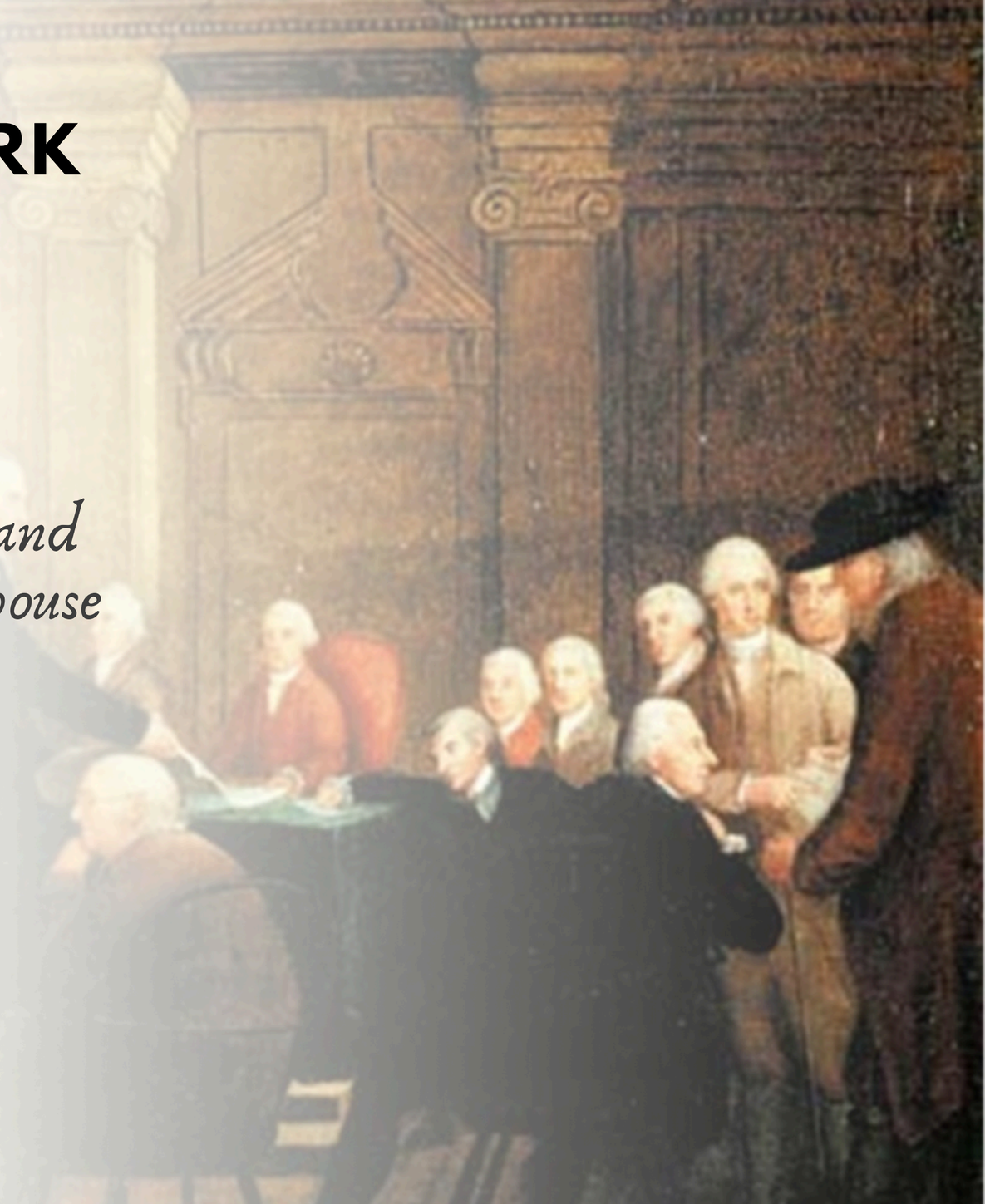
Things are getting tricky…


US CONGRESS, HARD AT WORK
1790-ish

US CONGRESS, HARD AT WORK

1790-ish

Take one representative for every d persons, and let the total number of representatives, i.e., house size, fall where it may.



A DIVIDED LAND

The preferred approach at the time was to start with a *divisor* d, which stands for the desired number of people per representative.

So if a state has population p_i, the number k_i of representatives it gets should be:

$$k_i \approx \frac{p_i}{d}.$$

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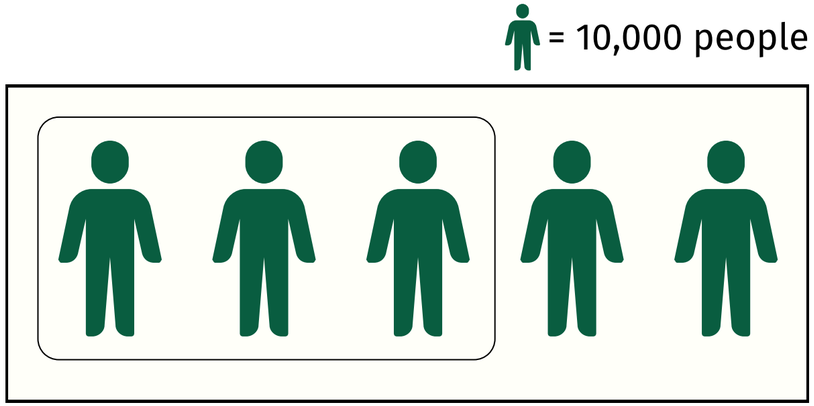

With a divisor of 30,000, a state with a population of 50K should get... one representative.

Note that the US Constitution requires at least one representative and does not allow two, as that would amount to 1.2 representatives per 30K people!

QUOTAS

Choose a divisor d.

If state i has population p_i, then i's *quota* is:

$$q_i = \frac{p_i}{d}.$$

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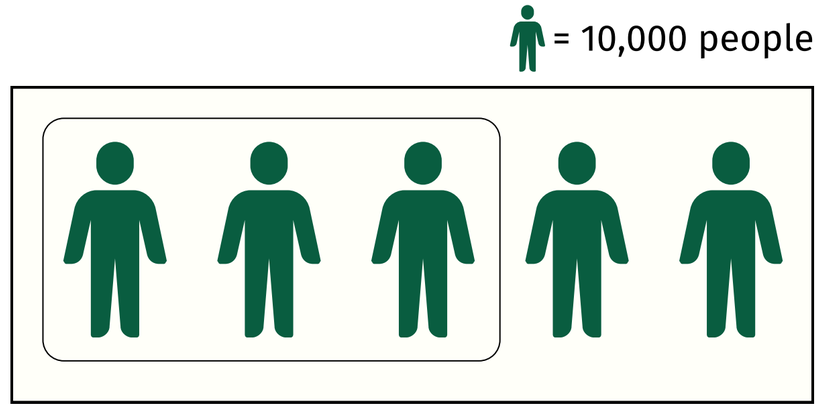

For divisor 30K, state's quota is 50K/30K = 1.67.

QUOTAS

Take the divisor to be 30,000, the minimum value allowed by the US Constitution.

State	Population	d = 30,000
Connecticut	236,841	
Delaware	55,540	
Georgia	70,835	
Kentucky	68,705	
Maryland	278,514	
Massachusetts	475,327	
New Hampshire	141,822	
New Jersey	179,570	
New York	331,589	
North Carolina	353,523	
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QUOTAS

Take the divisor to be 30,000, the minimum value allowed by the US Constitution.

Calculate quotas.

State	Population	Quota d = 30,000
Connecticut	236,841	7.895
Delaware	55,540	1.851
Georgia	70,835	2.361
Kentucky	68,705	2.29
Maryland	278,514	9.284
Massachusetts	475,327	15.844
New Hampshire	141,822	4.727
New Jersey	179,570	5.986
New York	331,589	11.053
North Carolina	353,523	11.784
Pennsylvania	432,879	14.429
Rhode Island	68,446	2.282
South Carolina	206,236	6.875
Vermont	85,533	2.851
Virginia	630,560	21.019
US (total)	**3,615,920**	**120.531**

QUOTAS

Take the divisor to be 30,000, the minimum value allowed by the US Constitution.

Calculate quotas.

Assign seats as per quotas.

State	Population	Quota d = 30,000	Seats
Connecticut	236,841	7.895	
Delaware	55,540	1.851	
Georgia	70,835	2.361	
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QUOTAS

Take the divisor to be 30,000, the minimum value allowed by the US Constitution.

Calculate quotas.

Assign seats as per quotas.

But, um... what do we do about the fractions?

State	Population	Quota d = 30,000	Seats
Connecticut	236,841	7.895	?
Delaware	55,540	1.851	?
Georgia	70,835	2.361	?
Kentucky	68,705	2.29	?
Maryland	278,514	9.284	?
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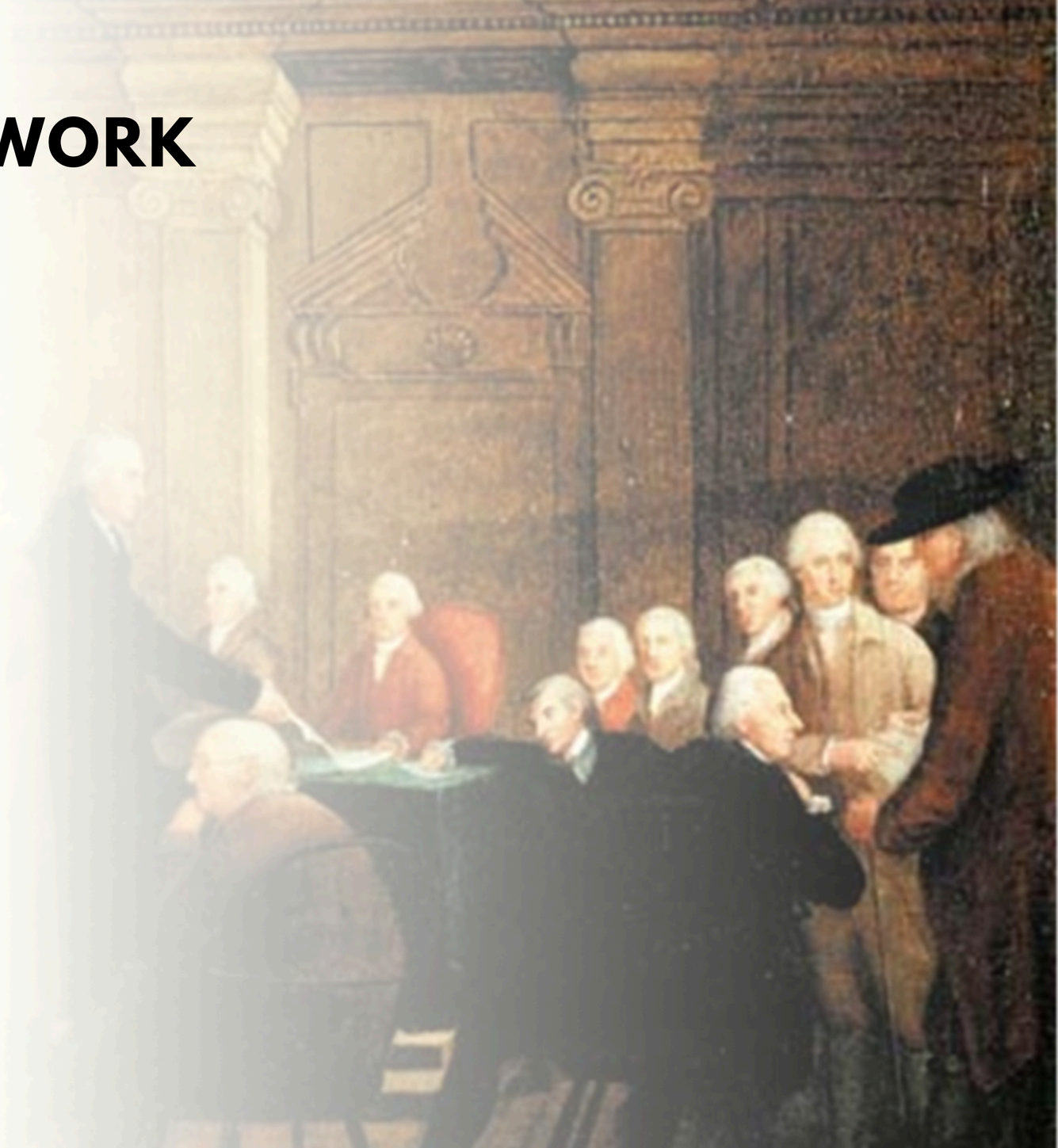
New Hampshire	141,822	4.727	?
New Jersey	179,570	5.986	?
New York	331,589	11.053	?
North Carolina	353,523	11.784	?
Pennsylvania	432,879	14.429	?
Rhode Island	68,446	2.282	?
South Carolina	206,236	6.875	?
Vermont	85,533	2.851	?
Virginia	630,560	21.019	?
US (total)	**3,615,920**	**120.531**	**?**

US CONGRESS, HARD AT WORK

1790-ish

Let's just drop the fractions!



HOUSE APPORTIONMENT BILL
1792

Seats assigned by dropping fractions from the quotas.

Leads to a house size of 112.

State	Population	Quota *d* = 30,000	Seats
Connecticut	236,841	7.895	7
Delaware	55,540	1.851	1
Georgia	70,835	2.361	2
Kentucky	68,705	2.29	2
Maryland	278,514	9.284	9
Massachusetts	475,327	15.844	15
New Hampshire	141,822	4.727	4
New Jersey	179,570	5.986	5
New York	331,589	11.053	11
North Carolina	353,523	11.784	11
Pennsylvania	432,879	14.429	14
Rhode Island	68,446	2.282	2
South Carolina	206,236	6.875	6
Vermont	85,533	2.851	2
Virginia	630,560	21.019	21
US (total)	**3,615,920**	**120.531**	**112**

Were the states happy with this?

REPRESENTATION RATIOS

If a state i with population p_i gets k_i representatives,
then the *representation ratio* of i is:

$$\frac{p_i}{k_i},$$

i.e., the number of people per representative.

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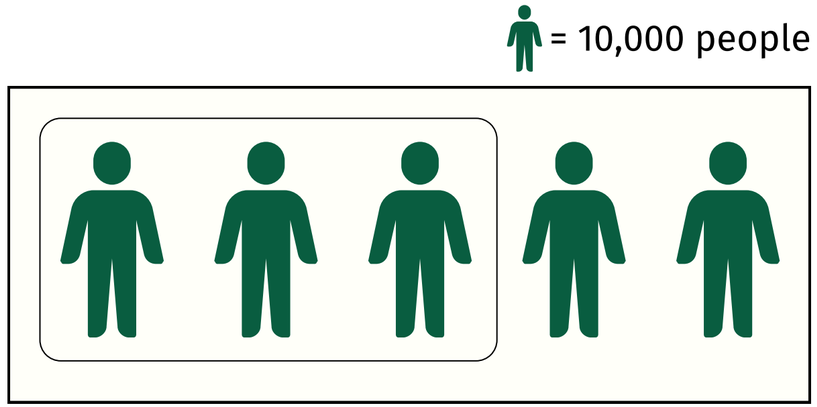

If the state gets one representative, its representation ratio is 50K/1 = 50K.

REPRESENTATION RATIOS

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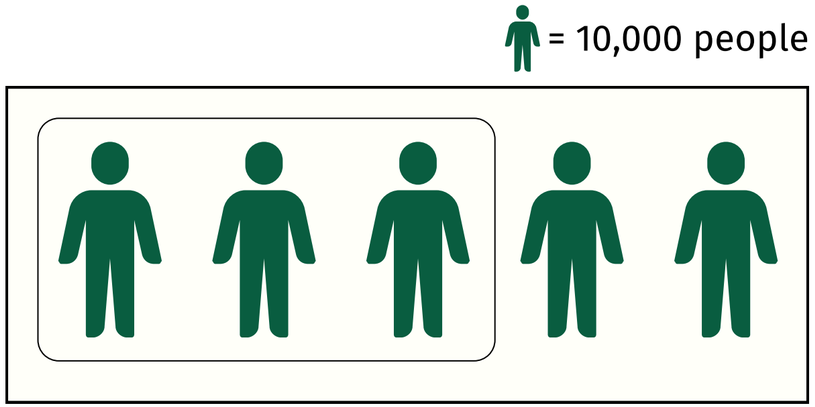
Smaller is better.



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REPRESENTATION RATIOS

In the 1792 House bill Delaware gets one seat per 55,540 people, while Massachusetts gets one seat per 31,688 people.

Every resident of Delaware has a 43% smaller share of representation in the House than a resident of Massachusetts.

HOUSE APPORTIONMENT BILL (1792)

State	Population	Quota *d* = 30,000	Seats	Repr. Ratio
Connecticut	236,841	7.895	7	33,834.43
Delaware	55,540	1.851	1	55,540
Georgia	70,835	2.361	2	35,417
Kentucky	68,705	2.29	2	34,352.5
Maryland	278,514	9.284	9	30,946
Massachusetts	475,327	15.844	15	31,688.47
New Hampshire	141,822	4.727	4	35,455.5
New Jersey	179,570	5.986	5	35,914
New York	331,589	11.053	11	30,144.45
North Carolina	353,523	11.784	11	32,138.45
Pennsylvania	432,879	14.429	14	30,919.93
Rhode Island	68,446	2.282	2	34,223
South Carolina	206,236	6.875	6	34,372.67
Vermont	85,533	2.851	2	42,766.5
Virginia	630,560	21.019	21	30,026.67
US (total)	**3,615,920**	**120.531**	**112**	**32,285**

SIZE BIAS

Dropping fractions hits states of different sizes differently.

And, fractions being equal, it favors larger states!

State	Population	Quota $d = 5$	Seats	Repr. Ratio
State 1	9	1.8	1	9
State 2	99	19.8	19	5.21
State 3	999	199.8	199	5.02
State 4	9999	1999.8	1999	5.002

The Senate disagreed with the House bill. They proposed a different apportionment, by raising the divisor to 33,000.

SENATE APPORTIONMENT BILL
1792

Choose divisor d = 33,000.

Assign seats by ignoring the fractions from the quotas.

Leads to a house size of 105.

State	Population	Quota d = 33,000	Seats	Repr. Ratio
Connecticut	236,841	7.177	7	33,834.43
Delaware	55,540	1.683	1	55,540
Georgia	70,835	2.147	2	35,417
Kentucky	68,705	2.082	2	34,352.5
Maryland	278,514	8.44	8	34,814.25
Massachusetts	475,327	14.404	14	33,951.93
New Hampshire	141,822	4.298	4	35,455.5
New Jersey	179,570	5.442	5	35,914
New York	331,589	10.048	10	33,158.9
North Carolina	353,523	10.713	10	35,352.3
Pennsylvania	432,879	13.118	13	33,298.38
Rhode Island	68,446	2.074	2	34,223
South Carolina	206,236	6.25	6	34,372.67
Vermont	85,533	2.592	2	42,766.5
Virginia	630,560	19.108	19	33,187.37
US (total)	**3,615,920**	**109.573**	**105**	**34,437.3**

All the wrangling over divisors came across as silly.

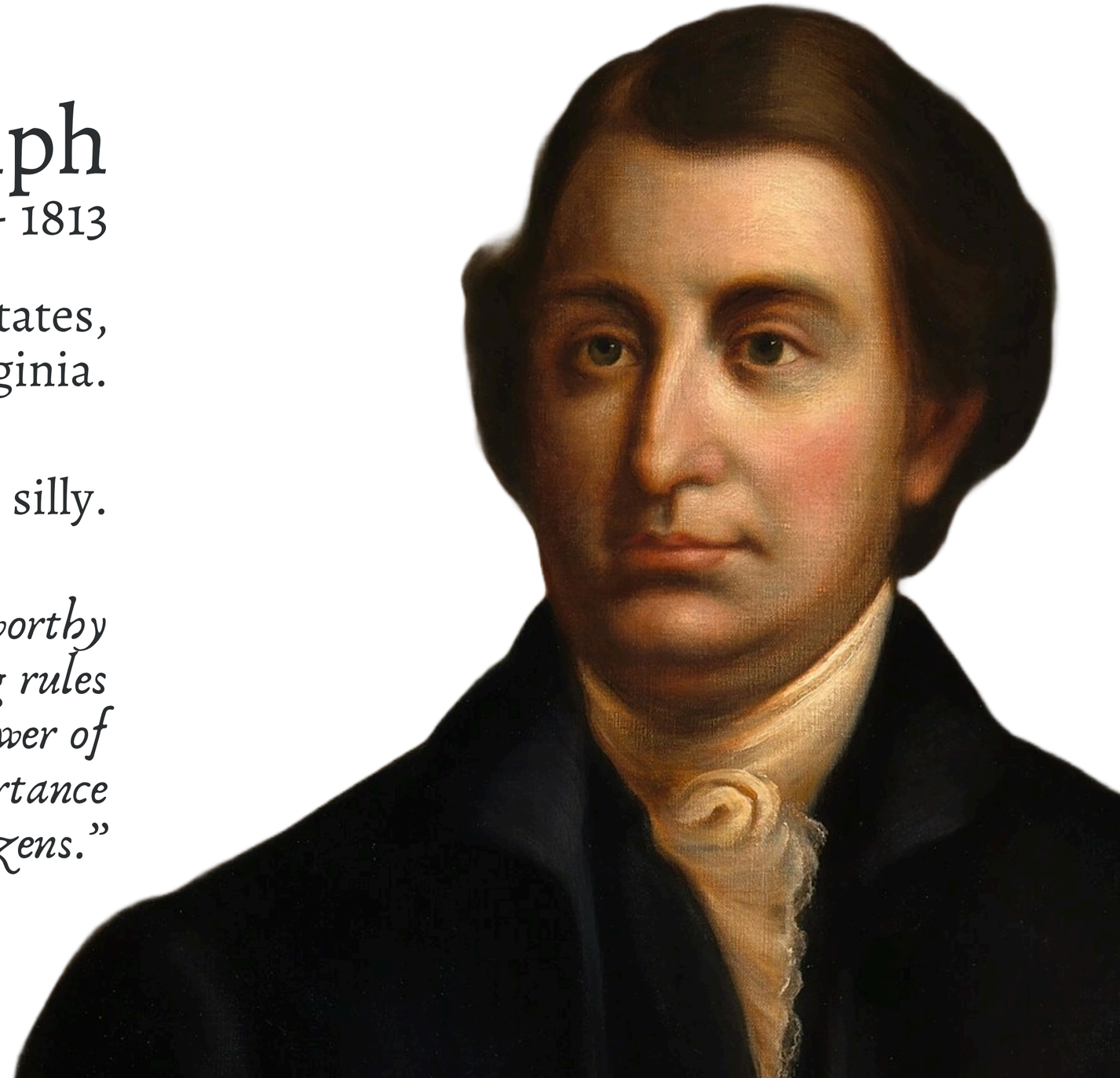
Edmund Ranolph

1753 - 1813

Founding father of the United States, attorney, seventh governor of Virginia.

Thought the hunt for divisors was silly.

“Sir, it gave me pain to find these worthy members calculating and coldly applying rules of arithmetic to a subject beyond the power of numbers to express the degree of its importance to their fellow citizens.”



But it was difficult to avoid this ‘silly’ hunt for divisors.

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With the growing tensions between North and South,
every seat mattered…

Enter Hamilton.

Alexander Hamilton

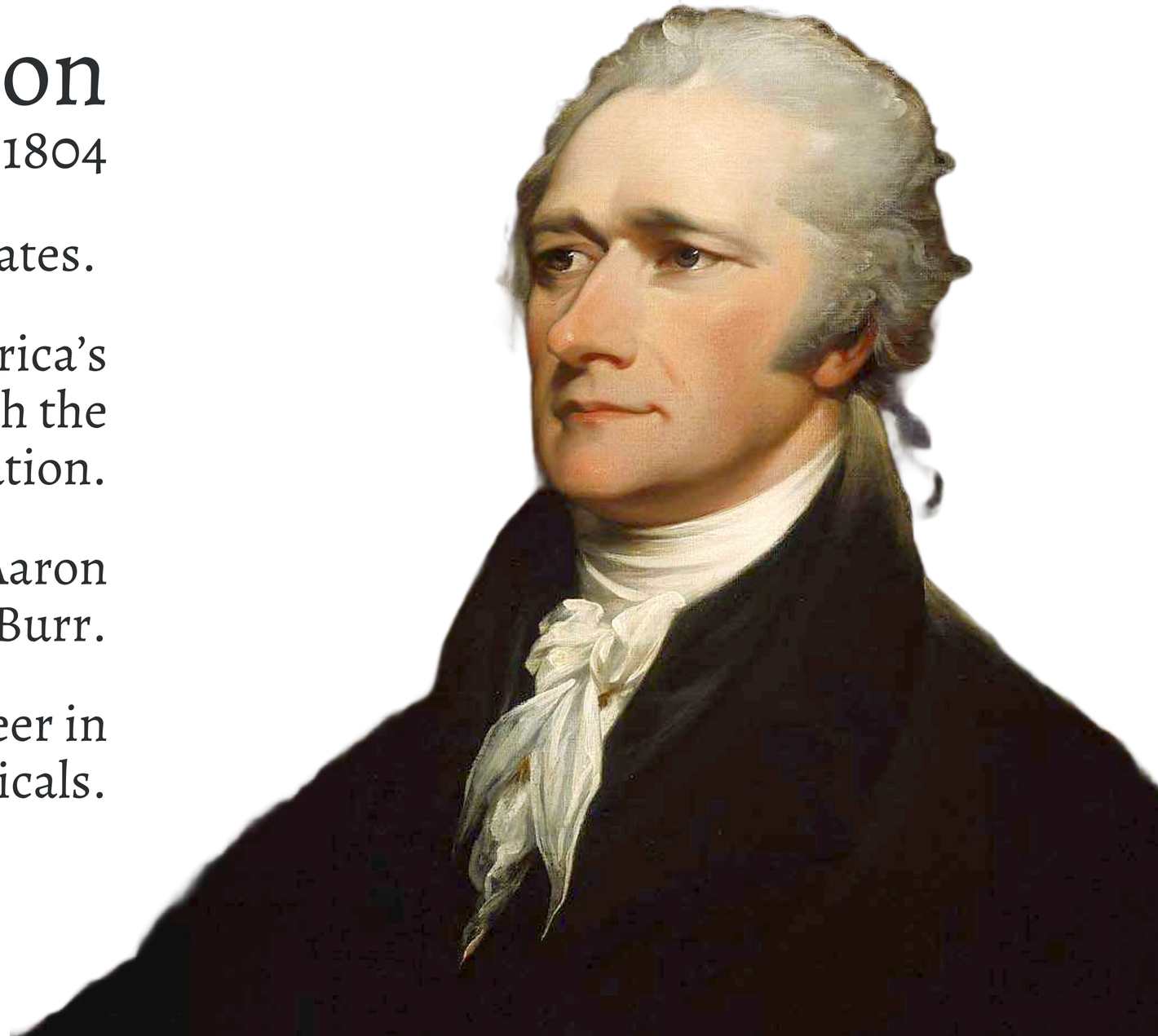
1757 - 1804

Founding father of the United States.

Played a key role in securing America's independence, and pushing through the Constitution.

Died in a duel with political rival Aaron Burr.

Went on to have a successful career in musicals.



ALEXANDER HAMILTON

The whole number of Representatives being first fixed, they shall be apportioned to any state according to its census...

...the Rule of Three will show what part of the representation any State shall have...

Hamilton believed that the number of representatives should be proportional to the state's share of the total population.

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STANDARD DIVISORS AND STANDARD QUOTAS

If the total population (across all states) is p, and we want to assign k seats, the *standard divisor* is:

$$\hat{d} = \frac{p}{k}.$$

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If a state i has population p_i, i's *standard quota* is:

$$\hat{q}_i = \frac{p_i}{\hat{d}}.$$

Hamilton reasoned that to get 30,000 people per representative, the US population of 3,615,920 would require 3,615,920/30,000 ≈ 120 seats.

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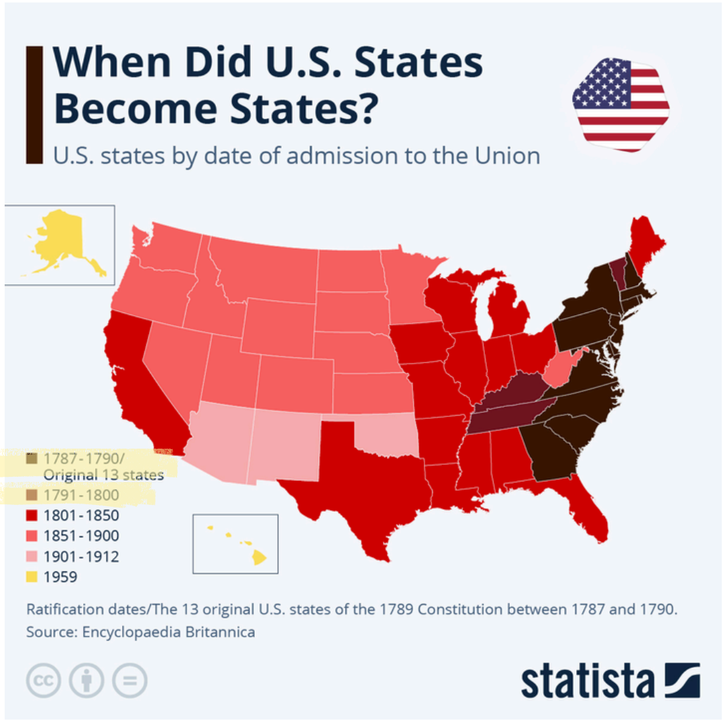

At a population of 3,615,920 (in 1790) and a desired House size of 120, the exact value for the standard divisor comes down to 30,132.67.

What about the fractions though?

ALEXANDER HAMILTON

Fix the number of seats to be allocated.

Start by giving each state its standard quota, rounded down.

If there are seats that remain to be allocated, look at the residue of each state, i.e., the fractional part.

Distribute the remaining seats (one each) to the states with the largest residues.

HAMILTON'S METHOD

Fix the number of seats to 120.

State	Population	Quota
Connecticut	236,841	
Delaware	55,540	
Georgia	70,835	
Kentucky	68,705	
Maryland	278,514	
Massachusetts	475,327	
New Hampshire	141,822	
New Jersey	179,570	
New York	331,589	
North Carolina	353,523	
Pennsylvania	432,879	
Rhode Island	68,446	
South Carolina	206,236	
Vermont	85,533	
Virginia	630,560	
US (total)	**3,615,920**	**120**

HAMILTON'S METHOD

Fix the number of seats to 120.

The standard divisor is d = 30,132.67.

State	Population	Quota d = 30,132.67
Connecticut	236,841	7.86
Delaware	55,540	1.843
Georgia	70,835	2.351
Kentucky	68,705	2.28
Maryland	278,514	9.243
Massachusetts	475,327	15.774
New Hampshire	141,822	4.707
New Jersey	179,570	5.959
New York	331,589	11.004
North Carolina	353,523	11.732
Pennsylvania	432,879	14.366
Rhode Island	68,446	2.271
South Carolina	206,236	6.844
Vermont	85,533	2.839
Virginia	630,560	20.926
US (total)	**3,615,920**	**120**

HAMILTON'S METHOD

Fix the number of seats to 120.

The standard divisor is d = 30,132.67.

Give each state its rounded down standard quota, leaving 9 empty seats.

State	Population	Quota d = 30,132.67	Seats
Connecticut	236,841	7.86	7
Delaware	55,540	1.843	1
Georgia	70,835	2.351	2
Kentucky	68,705	2.28	2
Maryland	278,514	9.243	9
Massachusetts	475,327	15.774	15
New Hampshire	141,822	4.707	4
New Jersey	179,570	5.959	5
New York	331,589	11.004	11
North Carolina	353,523	11.732	11
Pennsylvania	432,879	14.366	14
Rhode Island	68,446	2.271	2
South Carolina	206,236	6.844	6
Vermont	85,533	2.839	2
Virginia	630,560	20.926	20
US (total)	**3,615,920**	**120**	**111**

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The nine states with the largest remainders get an extra seat each.

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New Hampshire	141,822	4.707	4
New Jersey	179,570	5.959	5
New York	331,589	11.004	11
North Carolina	353,523	11.732	11
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State	Population	Quota d = 30,132.67	Seats
Connecticut	236,841	7.86	7 + 1
Delaware	55,540	1.843	1 + 1
Georgia	70,835	2.351	2
Kentucky	68,705	2.28	2
Maryland	278,514	9.243	9
Massachusetts	475,327	15.774	15 + 1
New Hampshire	141,822	4.707	4 + 1
New Jersey	179,570	5.959	5 + 1
New York	331,589	11.004	11
North Carolina	353,523	11.732	11 + 1
Pennsylvania	432,879	14.366	14
Rhode Island	68,446	2.271	2
South Carolina	206,236	6.844	6 + 1
Vermont	85,533	2.839	2 + 1
Virginia	630,560	20.926	20 + 1
US (total)	**3,615,920**	**120**	**120**

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The standard divisor is d = 30,132.67.

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State	Population	Quota d = 30,132.67	Seats	Repr. Ratio
Connecticut	236,841	7.86	7 + 1	29,605.13
Delaware	55,540	1.843	1 + 1	27,770
Georgia	70,835	2.351	2	35,417.5
Kentucky	68,705	2.28	2	34,352.5
Maryland	278,514	9.243	9	30,946
Massachusetts	475,327	15.774	15 + 1	29,707.94
New Hampshire	141,822	4.707	4 + 1	28,364.4
New Jersey	179,570	5.959	5 + 1	29,928.33
New York	331,589	11.004	11	30,144.45
North Carolina	353,523	11.732	11 + 1	29,460.25
Pennsylvania	432,879	14.366	14	30,919.93
Rhode Island	68,446	2.271	2	34,223
South Carolina	206,236	6.844	6 + 1	29,462.29
Vermont	85,533	2.839	2 + 1	28,511
Virginia	630,560	20.926	20 + 1	30,026.67
US (total)	**3,615,920**	**120**	**120**	**30,132.667**

A compromise bill with this exact apportionment was passed by narrow majorities on March 26, 1792.

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A compromise bill with this exact apportionment was passed by narrow majorities on March 26, 1792. All that remained was for President George Washington to sign it. He had until April 5 to make a decision…

Enter Washington.

George Washington

1732 - 1799

Founding father of the United States, general, first president.

Defeated the pesky British, ensuring the independence of the US.

Refused the title of king, stayed on for two spells as president.

Father of the nation.

GEORGE WASHINGTON

So I guess I should sign the compromise bill...

GEORGE WASHINGTON

So I *guess* I *should sign the compromise bill...*

ALEXANDER HAMILTON

Yup!

It results from a logical method, that works for any situation...

GEORGE WASHINGTON

So I guess I should sign the compromise bill...

ALEXANDER HAMILTON

Yup!

It results from a logical method, that works for any situation...

THOMAS JEFFERSON

Not so fast!

Enter Jefferson.

Thomas Jefferson

1743 - 1826

Founding father, primary author of the Declaration of Independence, secretary of state under George Washington.

Third president of the US.

During his tenure the US doubles in size.

Lives on as a champion of freedom and democracy.

Thomas Jefferson

1743 - 1826

Founding father, primary author of the Declaration of Independence, secretary of state under George Washington.

Third president of the US.

During his tenure the US doubles in size.

Lives on as a champion of freedom and democracy.

(Who also owned slaves.)

THOMAS JEFFERSON

Hamilton's doctrine of fractions is difficult and unobvious.

THOMAS JEFFERSON

Hamilton's doctrine of fractions is difficult and unobvious.

EDMUND RANDOLPH

I *agree!*

In fact, by Hamilton's method, all states whose delegation is rounded up get more than one representative for 30,000 residents.

For instance, New Hampshire gets one representative per 28,364 citizens:

New Hampshire	141,822	4.707	4 + 1	28,364.4

THOMAS JEFFERSON

Hamilton's doctrine of fractions is difficult and unobvious.

EDMUND RANDOLPH

I *agree!*

In fact, by Hamilton's method, all states whose delegation is rounded up get more than one representative for 30,000 residents.

For instance, New Hampshire gets one representative per 28,364 citizens:

New Hampshire	141,822	4.707	4 + 1	28,364.4

This is unconstitutional!

Side note.

Side note. Hamilton's *actual* proposal in letters to Washington used a divisor of 30,000 (rather than 30,132.67), in which case Virginia does not get an extra seat at redistribution.

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Side note. Hamilton's *actual* proposal in letters to Washington used a divisor of 30,000 (rather than 30,132.67), in which case Virginia does not get an extra seat at redistribution. Both Jefferson and Randolph, who were opposed to Hamilton, hailed from Virginia. But surely that was a coincidence…

GEORGE WASHINGTON

What a nuisance!

This apportionment issue is pitching Northern states versus Southern states.

But I do not want to take a side.

April 5 arrives and Washington is yet to make a decision…

GEORGE WASHINGTON

Jefferson! In my office! Now!

GEORGE WASHINGTON

Jefferson! In my office! Now!

THOMAS JEFFERSON

But I have not even had breakfast yet...

GEORGE WASHINGTON

Jefferson! In my office! Now!

THOMAS JEFFERSON

But I have not even had breakfast yet...

GEORGE WASHINGTON

What shall we do?

GEORGE WASHINGTON

Jefferson! In my office! Now!

THOMAS JEFFERSON

But I have not even had breakfast yet...

GEORGE WASHINGTON

What shall we do?

THOMAS JEFFERSON

You should negative the bill...

Washington vetoes the bill (!).

GEORGE WASHINGTON

What now?

GEORGE WASHINGTON

What now?

THOMAS JEFFERSON

Here's what I propose.

Start with the desired number of seats k.

Find a divisor d such that:

$$\left\lfloor \frac{p_1}{d} \right\rfloor + \ldots + \left\lfloor \frac{p_n}{d} \right\rfloor = k$$

State i gets $\lfloor p_i/d \rfloor$ seats.

floor, or value rounded down

JEFFERSON'S METHOD

Choose the house size, in this case 120.

Find an appropriate divisor.

State	Population	Quota d = ?
Connecticut	236,841	
Delaware	55,540	
Georgia	70,835	
Kentucky	68,705	
Maryland	278,514	
Massachusetts	475,327	
New Hampshire	141,822	
New Jersey	179,570	
New York	331,589	
North Carolina	353,523	
Pennsylvania	432,879	
Rhode Island	68,446	
South Carolina	206,236	
Vermont	85,533	
Virginia	630,560	
US (total)	**3,615,920**	**120**

JEFFERSON'S METHOD

Choose the house size, in this case 120.

Find an appropriate divisor.

In this case 30,000 does not work. But 28,500 does.*

Calculate quota and assign seats by rounding down.

*Here any divisor between 28,356 and 28,511 works.

State	Population	Quota d = 28,500	Seats
Connecticut	236,841	8.31	8
Delaware	55,540	1.949	1
Georgia	70,835	2.485	2
Kentucky	68,705	2.411	2
Maryland	278,514	9.772	9
Massachusetts	475,327	16.678	16
New Hampshire	141,822	4.976	4
New Jersey	179,570	6.301	6
New York	331,589	11.635	11
North Carolina	353,523	12.404	12
Pennsylvania	432,879	15.189	15
Rhode Island	68,446	2.402	2
South Carolina	206,236	7.236	7
Vermont	85,533	3.001	3
Virginia	630,560	22.125	22
US (total)	**3,615,920**	**120**	**120**

JEFFERSON'S METHOD

Choose the house size, in this case 120.

Find an appropriate divisor.

In this case 30,000 does not work. But 28,500 does.*

Calculate quota and assign seats by rounding down.

*Here any divisor between 28,356 and 28,511 works.

State	Population	Quota d = 28,500	Seats	Repr. Ratio
Connecticut	236,841	8.31	8	29,605.13
Delaware	55,540	1.949	1	55,540
Georgia	70,835	2.485	2	35,417.5
Kentucky	68,705	2.411	2	34,352.5
Maryland	278,514	9.772	9	30,946
Massachusetts	475,327	16.678	16	29,707.94
New Hampshire	141,822	4.976	4	35,455.5
New Jersey	179,570	6.301	6	29,928.33
New York	331,589	11.635	11	30,144.45
North Carolina	353,523	12.404	12	29,460.25
Pennsylvania	432,879	15.189	15	28,858.6
Rhode Island	68,446	2.402	2	34,223
South Carolina	206,236	7.236	7	29,462.29
Vermont	85,533	3.001	3	28,511
Virginia	630,560	22.125	22	28,661.82
US (total)	**3,615,920**	**120**	**120**	**30,132.667**

GEORGE WASHINGTON

No bueno! A representation ratio smaller than 30,000 landed us in this mess in the first place!

GEORGE WASHINGTON

No bueno! A representation ratio smaller than 30,000 landed us in this mess in the first place!

THOMAS JEFFERSON

My bad!

To get better representation ratios we'll need to raise the divisor.

A bigger divisor leads to a smaller house though...

Two days later a new bill was proposed, using Jefferson's method with a divisor of 33,000 and a house size of 105.

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Jefferson had triumphed.

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Jefferson had triumphed. His method was used until the 1830s.

Two days later a new bill was proposed, using Jefferson's method with a divisor of 33,000 and a house size of 105. The Senate voted for it on the same day, and Washington signed the bill into law on April 14, 1792.

Jefferson had triumphed. His method was used until the 1830s. Until some states noticed something fishy…

LARGE STATE BIAS OF JEFFERSON'S METHOD

We have 100 seats for a population of 10,000,000.

However, the divisor 100,000 does not deliver enough seats.

State	Population	Quota d = 100,000	Seats
New York	2,620,000	26.2	26
Delaware	168,000	1.68	1
...	...	...	...
US (total)	**10,000,000**	**100**	**99**

LARGE STATE BIAS OF JEFFERSON'S METHOD

We have 100 seats for a population of 10,000,000.

However, the divisor 100,000 does not deliver enough seats.

Decreasing the divisor to 97,000 does the trick, but the additional seat goes to the larger state, i.e., New York.

State	Population	Quota d = 100,000	Seats
New York	2,620,000	26.2	26
Delaware	168,000	1.68	1
...	...	...	...
US (total)	**10,000,000**	**100**	**99**

State	Population	Quota d' = 97,000	Seats
New York	2,620,000	27.01	27
Delaware	168,000	1.732	1
...	...	...	...
US (total)	**10,000,000**	**100**	**100**

LARGE STATE BIAS OF JEFFERSON'S METHOD

We have 100 seats for a population of 10,000,000.

However, the divisor 100,000 does not deliver enough seats.

Decreasing the divisor to 97,000 does the trick, but the additional seat goes to the larger state, i.e., New York.

Larger states arrive 'earlier' at the additional seat.

State	Population	Quota d = 100,000	Seats
New York	2,620,000	26.2	26
Delaware	168,000	1.68	1
...	...	...	...
US (total)	**10,000,000**	**100**	**99**

State	Population	Quota d' = 97,000	Seats
New York	2,620,000	27.01	27
Delaware	168,000	1.732	1
...	...	...	...
US (total)	**10,000,000**	**100**	**100**

Jefferson's method disenfranchises voters in the left out fractions of small states.

Enter Lowndes.

William Jones Lowndes

1782 - 1822

Congressman from South Carolina.

Involved in negotiations around the Missouri compromise.

Proposed a new apportionment method.

WILLIAM J. LOWNDES

Start with the desired number of seats.

Calculate the standard quota of each seat and round down, like with Hamilton's method.

Now divide the quotas by the initial number of seats given.

Assign remaining seats in order of this new quantity.

LOWNDES' METHOD

Start by giving each state its lower standard quota.

State	Population	Standard Quota	Initial Seats
Pennsylvania	1,049,313	24.971	24
Illinois	54,843	1.302	1
...	...	...	...
US (total)	**...**	**...**	**...**

LOWNDES' METHOD

Start by giving each state its lower standard quota.

Say this leaves us with some unallocated seats.

Order states by *priority number*, defined as the standard quota divided by the number of initial seats.

Allocate leftover seats in order of priority number.

State	Population	Standard Quota	Initial Seats
Pennsylvania	1,049,313	24.971	24
Illinois	54,843	1.302	1
...	...	...	...
US (total)	**...**	**...**	**...**

LOWNDES' METHOD

Start by giving each state its lower standard quota.

Say this leaves us with some unallocated seats.

Order states by *priority number*, defined as the standard quota divided by the number of initial seats.

State	Population	Standard Quota	Initial Seats	Standard Quota/Initial Seats
Pennsylvania	1,049,313	24.971	24	1.04
Illinois	54,843	1.302	1	1.30
...	...	...	...	...
US (total)	**...**	**...**	**...**	**...**

LOWNDES' METHOD

Start by giving each state its lower standard quota.

Say this leaves us with some unallocated seats.

Order states by *priority number*, defined as the standard quota divided by the number of initial seats.

Allocate leftover seats in order of priority number.

State	Population	Standard Quota	Initial Seats	Standard Quota/Initial Seats	Final Seats
Pennsylvania	1,049,313	24.971	24	1.04	24
Illinois	54,843	1.302	1	1.30	1 + 1
...	...	...	...	...	...
US (total)	**...**	**...**	**...**	**...**	**...**

In 1820, Lowndes' method would have given all the extra seats to the smallest states.

In 1820, Lowndes' method would have given all the extra seats to the smallest states. It was promptly rejected by Congress.

Enter Adams.

John Adams

1735 - 1826

Founding father and second president of the US.

While president, waged an unofficial naval war with France.

> *He means well for his country, is always an honest man, often a wise one, but sometimes, and in some things, absolutely out of his senses.*
>
> Benjamin Franklin

JOHN ADAMS

Start with the desired number of seats k.

Find a divisor d such that:

$$\left\lceil \frac{p_1}{d} \right\rceil + \ldots + \left\lceil \frac{p_n}{d} \right\rceil = k$$

State i gets $\left\lceil p_i / d \right\rceil$ seats.

ceiling, or value rounded up

Unsurprisingly, Adams' method favors small states.

SMALL STATE BIAS OF ADAMS' METHOD

We have to fill 100 seats for a population of 10,000,000.

The divisor 100,000 results in too many seats.

State	Population	Quota d = 100,000	Seats
New York	2,668,000	26.68	27
Delaware	120,000	1.2	2
...	...	...	...
US (total)	**10,000,000**	**100**	**101**

SMALL STATE BIAS OF ADAMS' METHOD

We have to fill 100 seats for a population of 10,000,000.

The divisor 100,000 results in too many seats.

We need to *increase* the divisor to 104,000 to get the desired number of seats.

But now the small states get an advantage.

State	Population	Quota d = 100,000	Seats
New York	2,668,000	26.68	27
Delaware	120,000	1.2	2
...	...	...	...
US (total)	**10,000,000**	**100**	**101**

State	Population	Quota d' = 104,000	Seats
New York	2,668,000	25.654	26
Delaware	120,000	1.154	2
...	...	...	...
US (total)	**10,000,000**	**100**	**100**

Adams' method was considered by Congress, but never enacted.

Adams' method was considered by Congress, but never enacted. The larger states, having the upper hand, would have none of it.

JOHN ADAMS

I *hung my harp upon my willows, and gave up.*

Enter Webster.

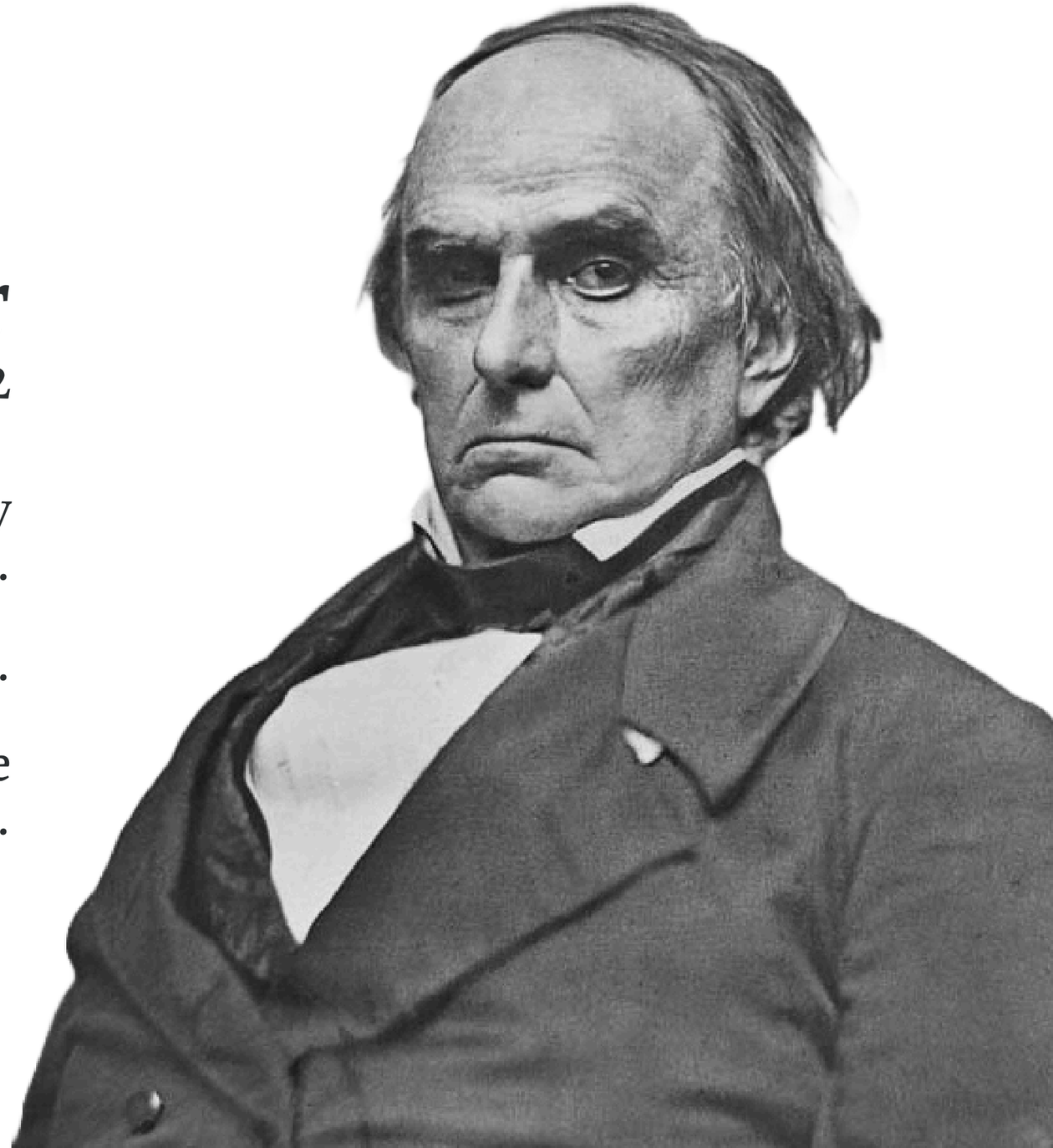
Daniel Webster

1782 - 1852

Lawyer, congressman, and US secretary of state under three presidents.

Famous for his oratory.

His speeches were reported to move even the most stone-hearted to tears.



DANIEL WEBSTER

Start with the desired number of seats k.

Find a divisor d such that:

$$\left[\frac{p_1}{d}\right] + \ldots + \left[\frac{p_n}{d}\right] = k$$

State i gets $\left[p_i/d\right]$ seats.

nearest integer

WEBSTER'S METHOD SEEMS IMPARTIAL

We want to distribute 33 seats among a population of 330,000.

The divisor d = 10,000, together with Webster's method, delivers the right number of seats.

State	Population	Quota d = 10,000	Seats
Colorado	304,000	30.4	30
Nebraska	26,000	2.6	3
...	...	...	...
US (total)	**330,000**	**33**	**33**

WEBSTER'S METHOD SEEMS IMPARTIAL

We want to distribute 33 seats among a population of 330,000.

The divisor d = 10,000, together with Webster's method, delivers the right number of seats.

Rounding to the nearest integer sometimes favors the smaller state, sometimes the larger state.

State	Population	Quota d = 10,000	Seats
Colorado	304,000	30.4	30
Nebraska	26,000	2.6	3
...	...	...	...
US (total)	**330,000**	**33**	**33**

State	Population	Quota d = 10,000	Seats
Colorado	296,000	29.6	30
Nebraska	34,000	3.4	3
...	...	...	...
US (total)	**330,000**	**33**	**33**

Webster’s method was adopted in 1842.

Webster’s method was adopted in 1842. Not ten years passed until it was challenged.

Enter Vinton.

Samuel Finley Vinton

1792 - 1862

Member of the House of Representatives, hailing from Ohio.

Helped create the US Department of the Interior.

SAMUEL F. VINTON

Fix the number of seats to be allocated.

Start by giving each state its standard quota, rounded down.

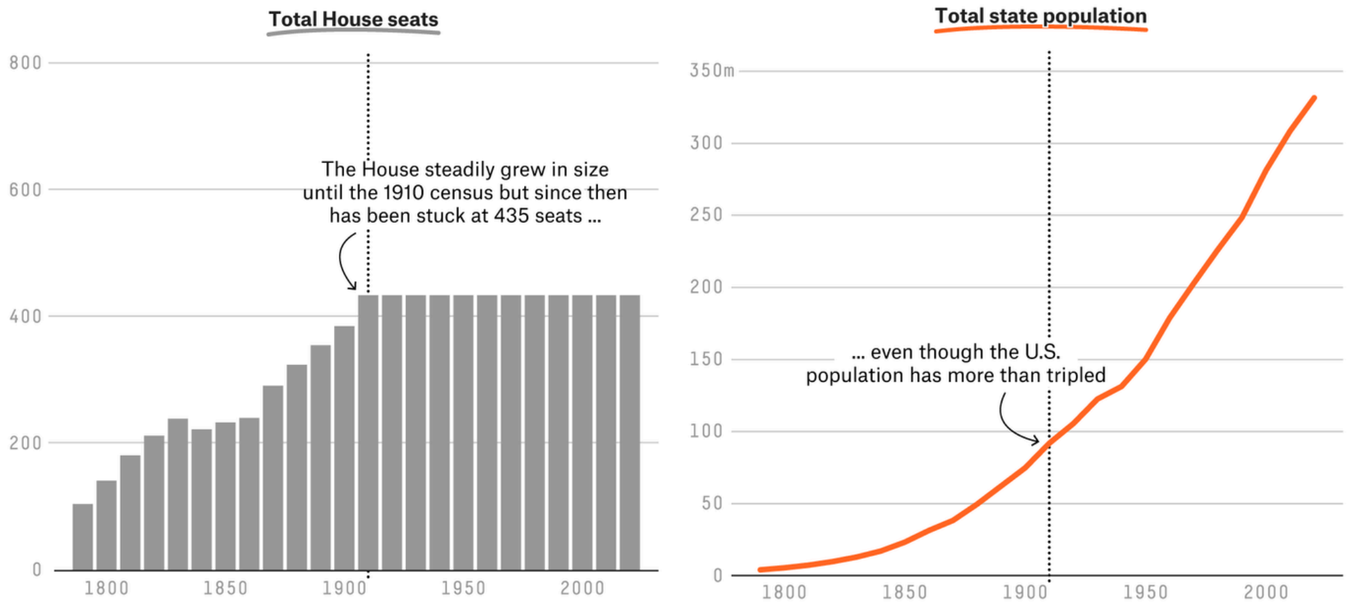
If there are seats that remain to be allocated, look at the residue of each state, i.e., the fractional part.

Distribute the remaining seats (one each) to the states with the largest residues.

Vinton’s method was, of course, the same as Hamilton’s, and which was vetoed by Washington in 1792.

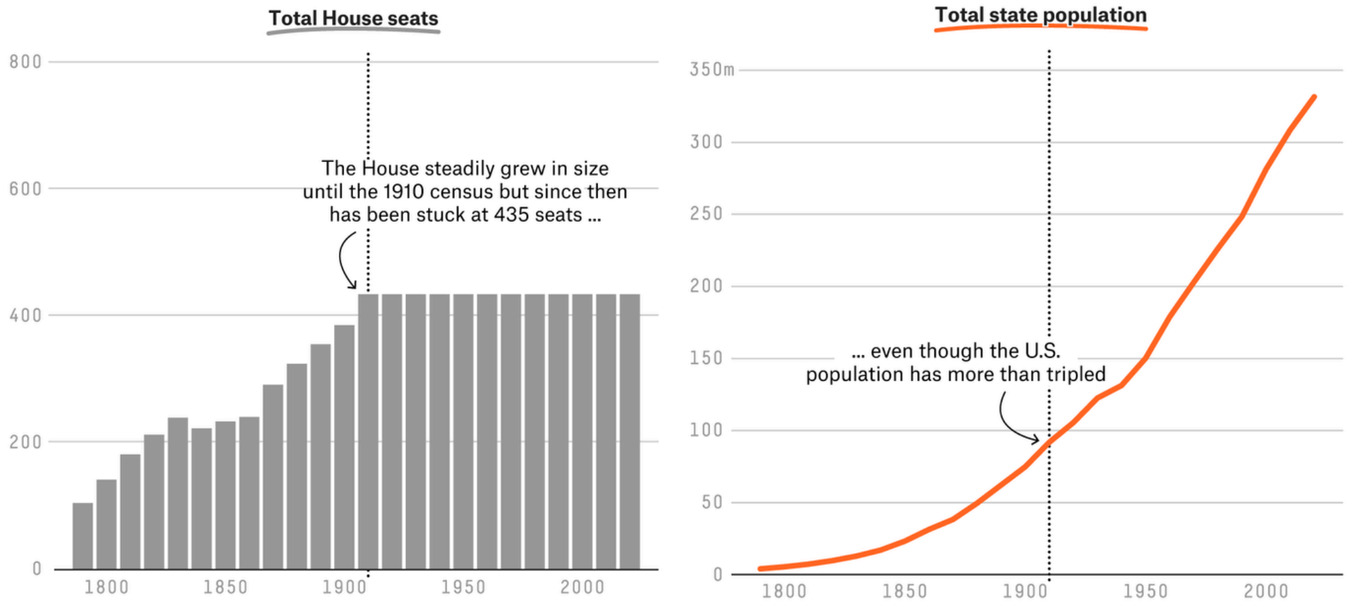
Vinton's method was, of course, the same as Hamilton's, and which was vetoed by Washington in 1792. Congress adopted it in 1850.

Meanwhile, the population of the US keeps growing…



Skelley, G. (2021). How The House Got Stuck At 435 Seats. *FiveThirtyEight*.

Meanwhile, the population of the US keeps growing… with the House struggling to keep up.



Skelley, G. (2021). How The House Got Stuck At 435 Seats. *FiveThirtyEight*.

After the 1880 census, the House was expected to grow again.

After the 1880 census, the House was expected to grow again. But when the seats were computed, something extraordinary happened…

The Alabama Paradox

Start with 299 seats, to be distributed among a population of ~50 mil.

Hamilton-Vinton gives Alabama 8 seats.

State	Population	Quota *d*	Seats
Alabama	1,262,505	7.646	7 + 1
Texas	1,591,749	9.646	9
Illinois	3,077,871	18.640	18
...	...	...	...
US (total)	**~50,000,000**	**299**	**299**

The Alabama Paradox

Start with 299 seats, to be distributed among a population of ~50 mil.

Hamilton-Vinton gives Alabama 8 seats.

Increasing the House size to 300 results in Alabama *losing* a seat!

State	Population	Quota *d*	Seats
Alabama	1,262,505	7.646	7 + 1
Texas	1,591,749	9.646	9
Illinois	3,077,871	18.640	18
…	…	…	…
US (total)	**~50,000,000**	**299**	**299**

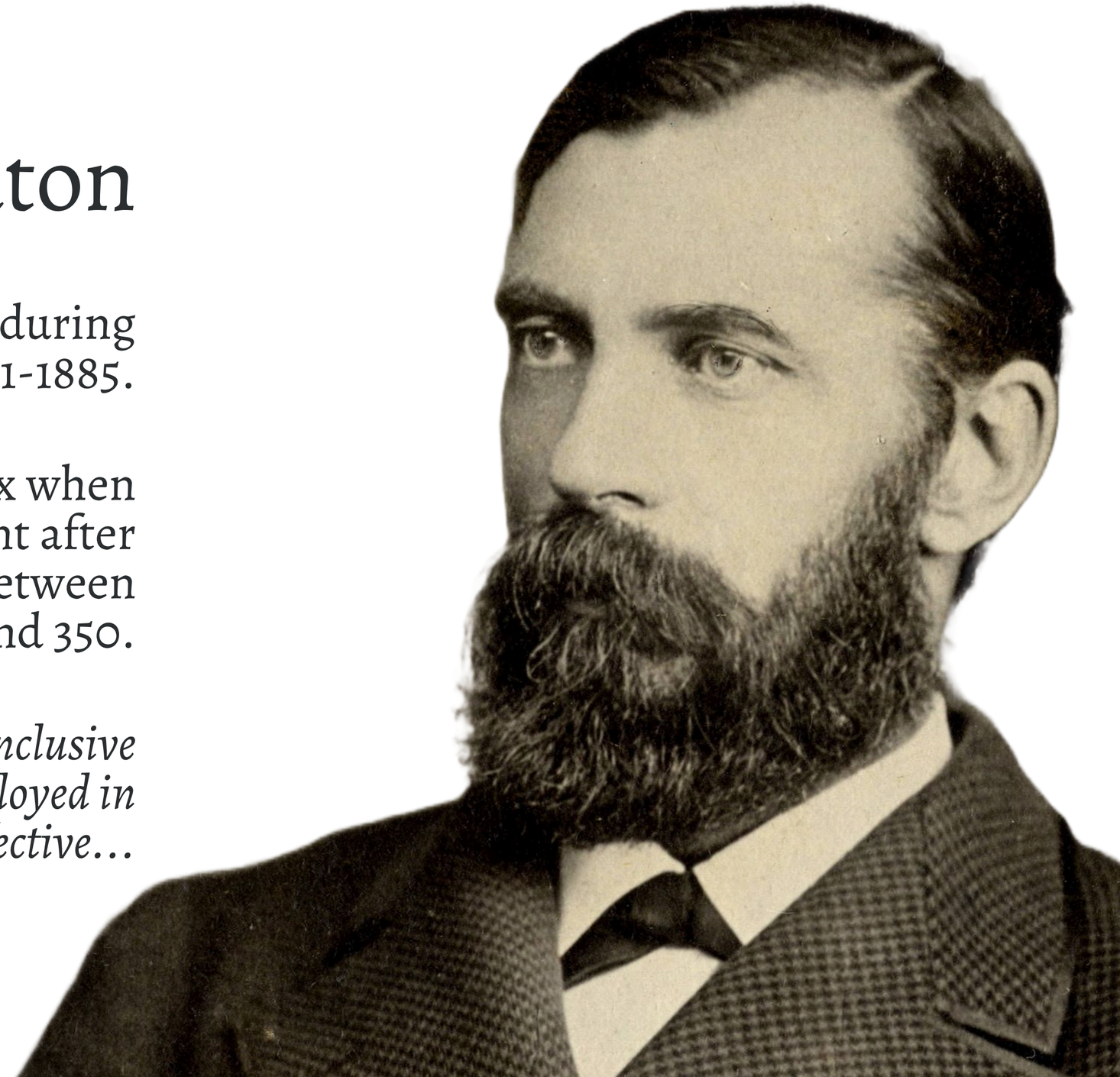
State	Population	Quota *d'*	Seats
Alabama	1,262,505	7.671	7
Texas	1,591,749	9.672	9 + 1
Illinois	3,077,871	18.702	18 + 1
…	…	…	…
US (total)	**49,713,370**	**300**	**300**

Charles W. Seaton

U.S. Census Bureau Director during 1881-1885.

Struck upon Alabama paradox when computing the apportionment after 1880 Census, for House sizes between 275 and 350.

Such a result as this is to me conclusive proof that the process employed in obtaining it is defective…



Seaton proposed a modified version of Jefferson's method to prevent the Alabama paradox.

Seaton proposed a modified version of Jefferson's method to prevent the Alabama paradox. Was met with derision from Congress.

US CONGRESS
1880

I have been told that while in astronomy and philosophy and geometry and all other sciences, there was something left for speculation, that mathematics, like the voice of Revelation, said when it spoke, "Thus saith the Lord."

But here is a new system of mathematics that demonstrates the truth to be false.

All the old arguments against Jefferson’s method were resurrected.

All the old arguments against Jefferson's method were resurrected. The compromise solution was to enlarge the House to 325 seats, on which Webster's and Hamilton's methods agreed.

All the old arguments against Jefferson's method were resurrected. The compromise solution was to enlarge the House to 325 seats, on which Webster's and Hamilton's methods agreed.

But soon enough, another problem emerged…

THE POPULATION PARADOX

In 1900 the size of the house had risen to 386 seats, to be distributed among a population of ~74.5 mil.

Hamilton-Vinton gives Virginia 8 seats.

State	Population	Quota *d* ~ 193,164	Seats
Virginia	1,854,184	9.599	10
Maine	694,466	3.595	3
...	...	...	...
US (total)	**74,562,608**	**386**	**386**

THE POPULATION PARADOX

In 1900 the size of the house had risen to 386 seats, to be distributed among a population of ~74.5 mil.

Hamilton-Vinton gives Virginia 8 seats.

A year later, Virginia's population grew by 1.06%, while Maine's grew by 0.7%.

But the extra seat goes to Maine!

State	Population	Quota *d* ~ 193,164	Seats
Virginia	1,854,184	9.599	10
Maine	694,466	3.595	3
...	...	...	...
US (total)	**74,562,608**	**386**	**386**

State	Population	Quota *d* ~ 197,164	Seats
Virginia	1,873,951	9.509	9
Maine	699,114	3.548	4
...	...	...	...
US (total)	**76,069,522**	**386**	**386**

And then another problem.

THE NEW STATE PARADOX

In 1907, Oklahoma joined the union.

At around 1 million people, Oklahoma deserved five seats in the House.

State	Population	Quota *d* ~ 193,167	Seats
New York	7,264,183	37.606	38
Maine	694,466	3.595	3
Oklahoma	~1,000,000	~5.175	~5
…	…	…	…
US (total)	**74,562,608**	**386**	**386**

THE NEW STATE PARADOX

In 1907, Oklahoma joined the union.

At around 1 million people, Oklahoma deserved five seats in the House.

Congress added five seats and used Hamilton-Vinton to recalculate the apportionment.

All extra seats went to Oklahoma.

State	Population	Quota *d* ~ 193,167	Seats
New York	7,264,183	37.606	38
Maine	694,466	3.595	3
Oklahoma	~1,000,000	~5.175	~5
…	…	…	…
US (total)	**74,562,608**	**386**	**386**

State	Population	Quota *d* ~ 193,167	Seats
New York	7,264,183	37.606	37
Maine	694,466	3.595	4
Oklahoma	1,000,000	5.175	5
…	…	…	…
US (total)	**75,562,608**	**391**	**391**

THE NEW STATE PARADOX

In 1907, Oklahoma joined the union.

At around 1 million people, Oklahoma deserved five seats in the House.

Congress added five seats and used Hamilton-Vinton to recalculate the apportionment.

All extra seats went to Oklahoma.

But New York lost a seat to Maine!

State	Population	Quota *d* ~ 193,167	Seats
New York	7,264,183	37.606	38
Maine	694,466	3.595	3
Oklahoma	~1,000,000	~5.175	~5
...	...	...	...
US (total)	**74,562,608**	**386**	**386**

State	Population	Quota *d* ~ 193,167	Seats
New York	7,264,183	37.606	37
Maine	694,466	3.595	4
Oklahoma	1,000,000	5.175	5
...	...	...	...
US (total)	**75,562,608**	**391**	**391**

In response to these paradoxes Congress switched back to Webster’s method.

In response to these paradoxes Congress switched back to Webster's method. Webster's method is more impartial, but Hamilton's method was preferred by the large states.

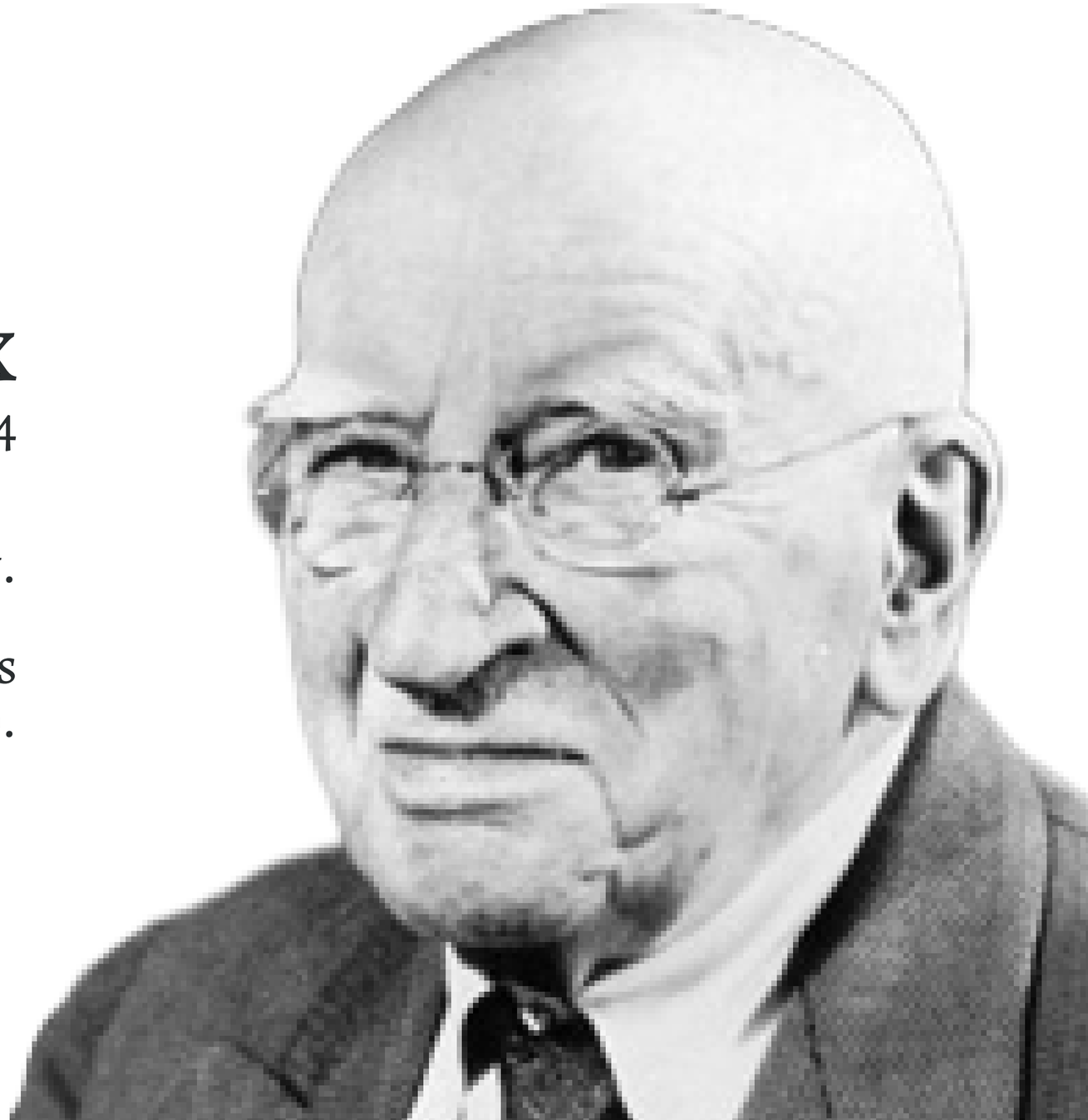
Enter Willcox.

Walter Francis Willcox

1861 - 1964

Statistician at Cornell University.

Served as one of five chief statisticians
for the US Census of 1900.



WALTER F. WILLCOX

After studying all the various apportionment methods, I am convinced Webster's method is best.

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This number is still in place today.

But with arguments erupting at every new apportionment, new ideas were needed...

Enter Hill.

Joseph Adna Hill

1860 - 1938

Statistician.

One of the authors of the *Method of Equal Proportions*, used to apportion representatives to states.

JOSEPH A. HILL

What we should be looking at is the number of people needed to get one representative.

What we called the *representation ratio*.

It doesn't seem fair to give state a representative per 50,000 people, and another state one per 70,000 people.

We should try to minimize the relative difference between these quantities.

MINIMIZING RELATIVE DIFFERENCES

There are 20 seats for a population of 4 million and two states.

An allocation of 16 and 4 seats leads to a relative difference (i.e., ratio) of 1.18.

State	Population	Quota d = 200,000	Seats	Repr. Ratio
State 1	3,300,000	16.5	16	206,250
State 2	700,000	3.5	4	175,000
US (total)	**4,000,000**	**20**	**20**	**200,000**

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An allocation of 17 and 3 seats leads to a relative difference of 1.20.

The first allocation is more equal, since 1.18 < 1.20, and thus preferred.

State	Population	Quota d = 200,000	Seats	Repr. Ratio	
State 1	3,300,000	16.5	16	206,250	ratio of 1.18
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State	Population	Quota d = 200,000	Seats	Repr. Ratio	
State 1	3,300,000	16.5	17	194,117.65	ratio of 1.20
State 2	700,000	3.5	3	233,333.33	
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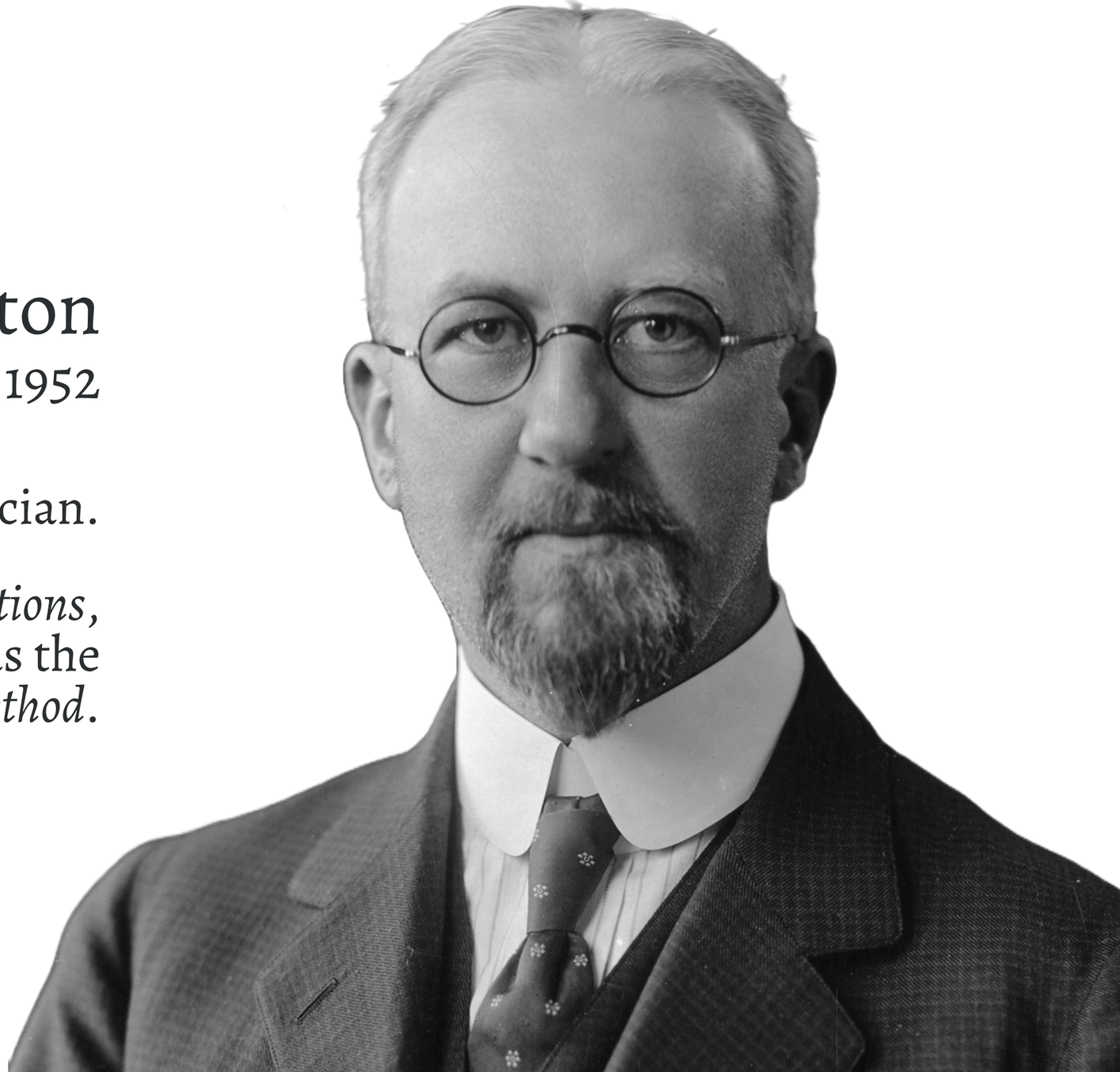
Enter Huntington.

Edward Vermilye Huntington

1874 - 1952

Mathematician.

Big fan of Hill's *Method of Equal Proportions*, which would go on to be known as the *Huntington-Hill method*.



EDWARD V. HUNTINGTON

There's a simpler way of thinking about Hill's procedure.

Consider first the following rounding function:

$$f(x) = \begin{cases} \lfloor x \rfloor, \text{ if } x < \sqrt{\lfloor x \rfloor \cdot \lceil x \rceil}, \\ \lceil x \rceil, \text{ if } x \geq \sqrt{\lfloor x \rfloor \cdot \lceil x \rceil}. \end{cases}$$

That is, we are rounding at the geometrical mean.

Now fix a number k of seats.

Find a divisor d such that:

$$f\left(\frac{p_1}{d}\right) + \cdots + f\left(\frac{p_n}{d}\right) = k.$$

State i gets $f\left(p_i/d\right)$ seats.

ROUNDING FUNCTIONS

More generally, we can think of f as a *rounding function* $f : \mathbb{R} \to \mathbb{Z}$.

It can be anything, as long as it satisfies these properties:

(*i*) $f(x) = x$, if x is an integer,

(*ii*) if $x \geq y$, then $f(x) \geq f(y)$.

NOTATION

states	$N = \{1, \ldots, n\}$
population of state i	p_i
total population	$p = p_1 + \cdots + p_n$
number of seats to be allocated	k
seats allocated to state i	k_i
divisor	d
quota of state i, for divisor d	$q_i = {p_i}/{d}$
standard (true) quota of state i	$\hat{q}_i = {p_i}/{p} \cdot k$
upper quota of state i	$\lceil \hat{q}_i \rceil$, i.e., $\hat{q}_i$ rounded up to the nearest integer
lower quota of state i	$\lfloor \hat{q}_i \rfloor$, i.e., $\hat{q}_i$ rounded down to the nearest integer

corresponds to a divisor of $\frac{p}{k}$

THEOREM (HUNTINGTON, 1928)

A divisor method is the Huntington-Hill method if and only if for all states $i, j \in N$ such that $p_i/k_i \geq p_j/k_j$, it holds that:

$$\frac{p_i/k_i}{p_j/k_j} < \frac{p_j/(k_j-1)}{p_i/(k_i+1)}.$$

representation ratio

Huntington, E. V. (1928). The Apportionment of Representatives in Congress. *Transactions of the American Mathematical Society*, 30(1), 85–110.

Consider states 1 and 2 from before, renamed as *i* and *j*.

Huntington's method requires thinking of the ratios when we re-distribute one seat from State *i* to State *j*.

State	Population	Seats	Repr. Ratio	
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State *j*	700,000	4	175,000	
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WALTER F. WILLCOX

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EDWARD V. HUNTINGTON

Willcox's false description, supported by impressive charts and diagrams, is misleading.

Our method of equal proportions, with its simplicity, directness and intelligibility, leaves nothing to be desired.

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After much acrimonious debate, both in Congress and scientific journals, the Huntington-Hill method prevailed. And stays on as the method used. For now…