



JUNE 2, 2025

# NETWORKED MINDS: OPINION DYNAMICS AND COLLECTIVE INTELLIGENCE IN SOCIAL NETWORKS

# NETWORK MEASURES

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Florence, the early 1400s.

Florence, the early 1400s. In the shadow of church bells and towers, patrician old-money families jostle for power with upstart, ‘new men’ merchant families.



Rosselli, F. (c. 1470-82). Pianta della Catena, aka Rosselli's Chain Map.



Santa Maria del Fiore

The Arno

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Amid this jostling, one family, the Medicis, rises to prominence.

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# Cosimo de' Medici

1389 - 1464

Banker and politician.

Steers the Medicis to dominance in Florence, a position they go on to hold for three centuries to come.



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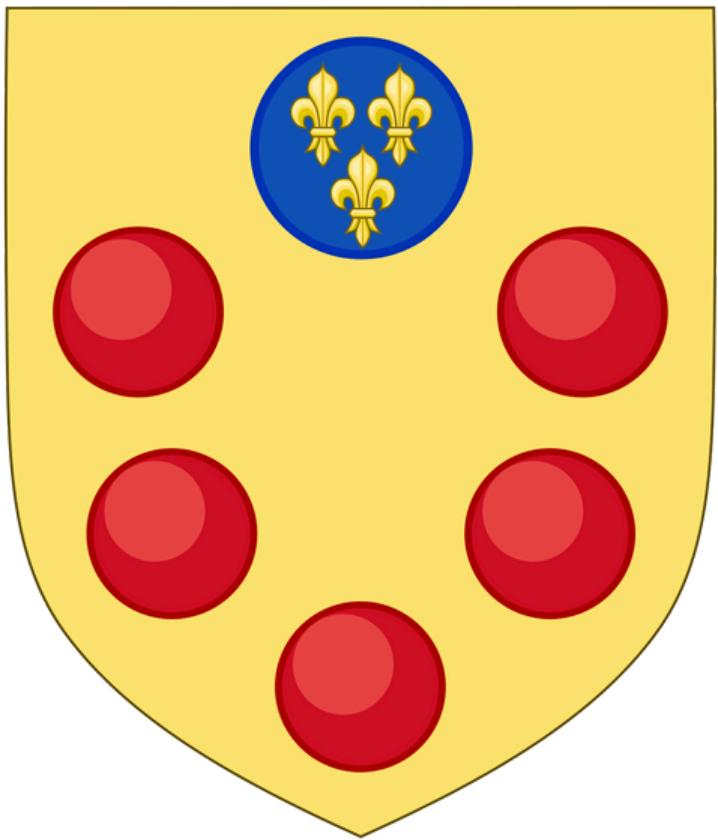
Steers the Medicis to dominance in Florence, a position they go on to hold for three centuries to come.

*Political questions are settled in [Cosimo's] house. The man he chooses holds office... He is who decides peace and war... He is king in all but name.*

Enea Silvio Piccolomini, Bishop of Siena, later Pope Pius II



# MEDICI



Originally from the farmlands  
north of Florence, in Mugello.

Become wealthy through banking.

Cosimo seen as 'champion of the  
new men'.

Yet, it seems Cosimo was not an obvious leader.



JOHN F. PADGETT

Cosimo was described by people who knew him as an  
indecipherable sphinx...

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...with little more commitment than “Yes my son, I  
shall look into that.”

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We have a puzzle on our hands.

We have a puzzle on our hands. If not his charisma, what *was* the secret to Cosimo's success?



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He engineered himself into a very important position in the network of early 15<sup>th</sup> century Florentine elites.



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*So what is a network?*

# MODEL

We represent a *social network* as a graph  $G = (V, E)$ , where  $V$  is the set of *vertices* (agents) and  $E$  is the set of *edges* (relationships).

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The graph can be *directed* or *undirected*, depending on whether relationships are one-way or two-way.



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We can start to gauge the influence of the Medicis by drawing some of their relationships.



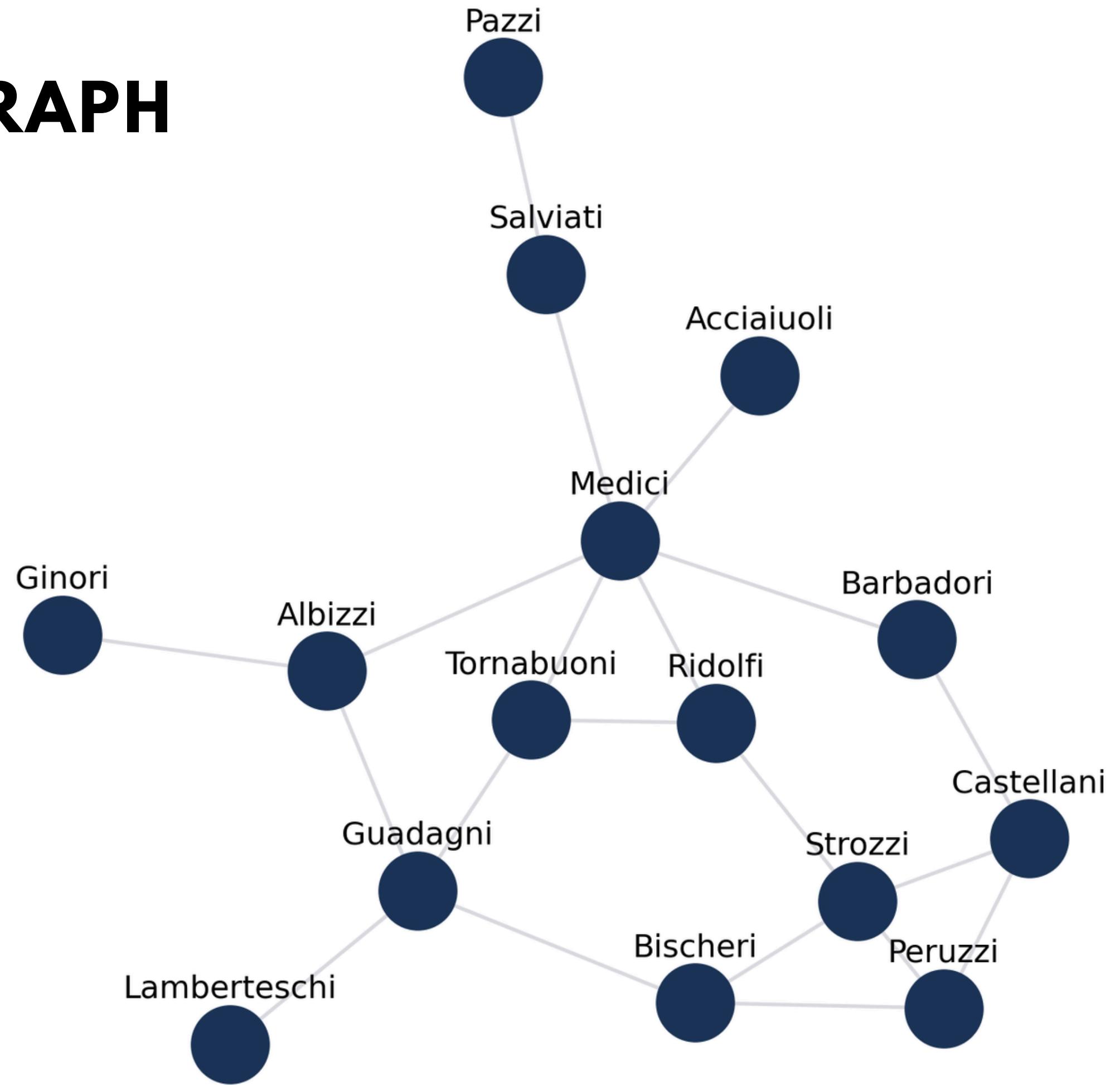
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Like, who married who.

# FLORENTINE FAMILIES GRAPH

A link represents a marriage between the two families.

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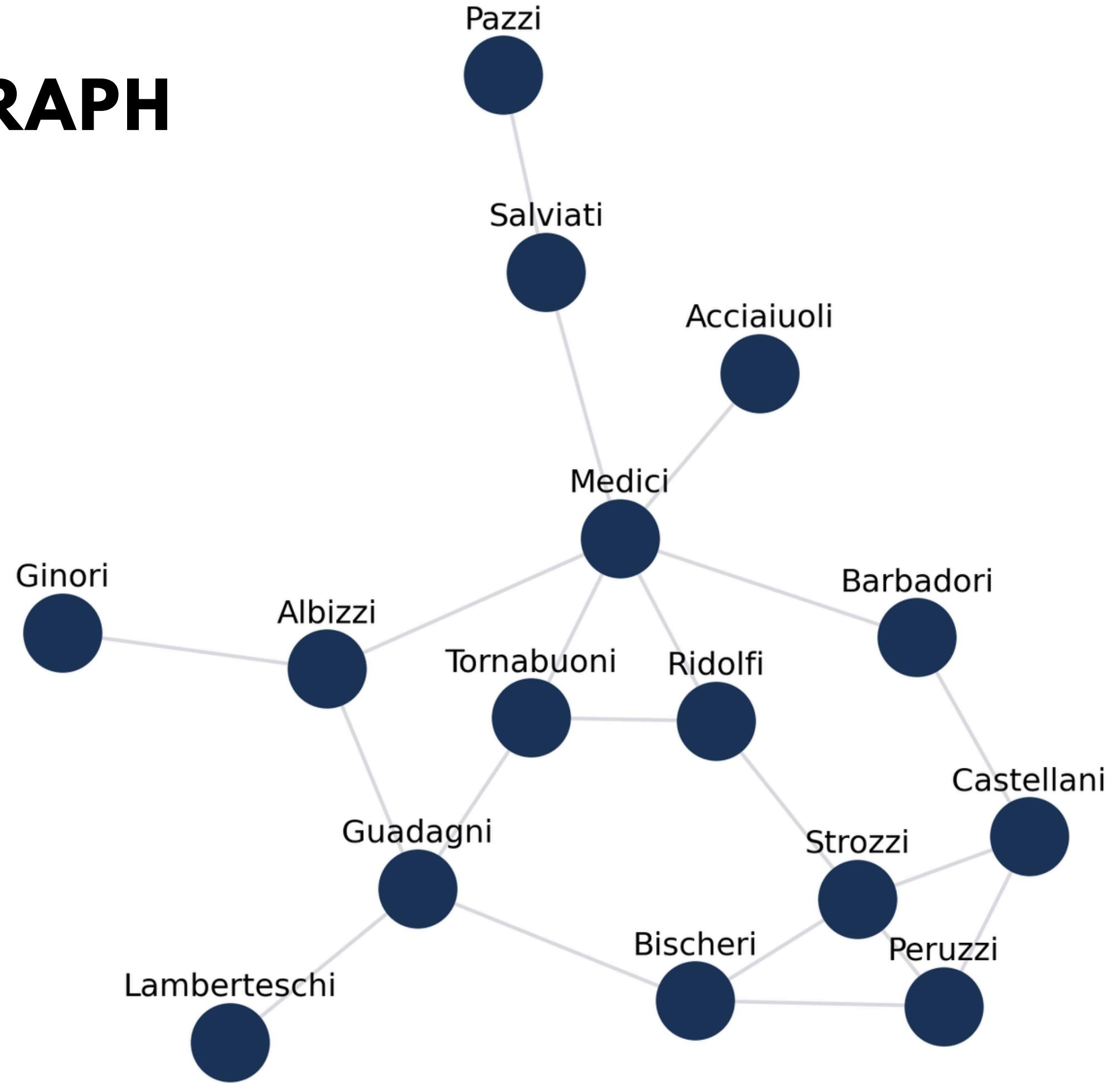


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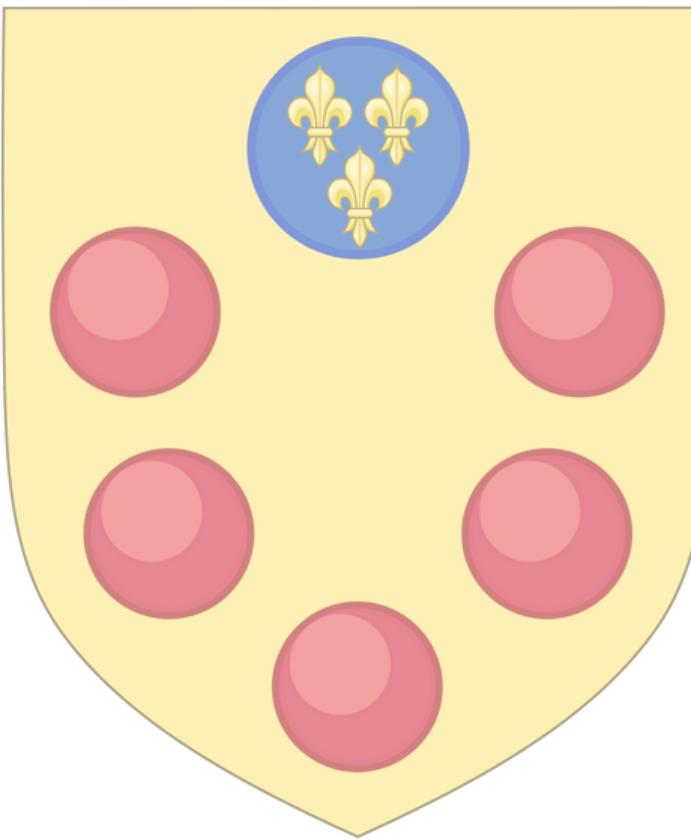
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Considered as *strong ties*, in Granovetter's sense.

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## MEDICI



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Become wealthy through banking.

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## ALBIZZI



Old-money wool-merchant dynasty.

Maso, and then Rinaldo degli Albizzi basically *were* the Florentine state until Cosimo showed them the door in 1434.



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Check out our paper!

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What do we want to keep track of with networks?

## **DEFINITION (NEIGHBORHOODS)**

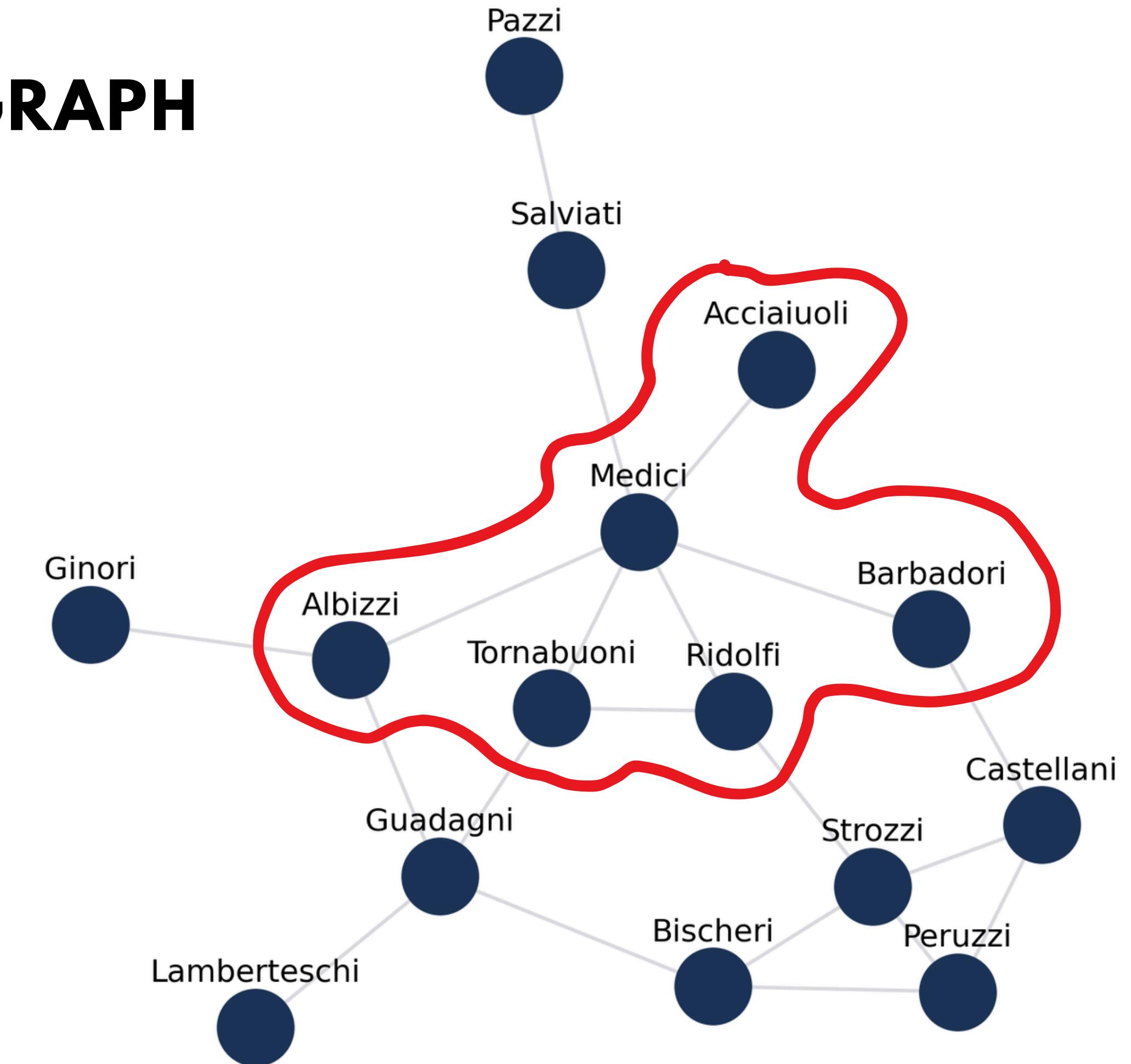
The *neighborhood*  $N(i)$  of a node  $i$  is the set of nodes that are directly connected to  $i$ :

$$N(i) = \{j \in V \mid (i, j) \in E\}.$$

# FLORENTINE FAMILIES GRAPH

Neighborhoods

The neighborhood of the Medici consists of the Salviati, Acciaiuoli, Barbadori, Ridolfi, Tornabuoni and Albizzi.



## **DEFINITION (DEGREE)**

The *degree*  $d_i$  of a node  $i$  is the size of  $i$ 's neighborhood:

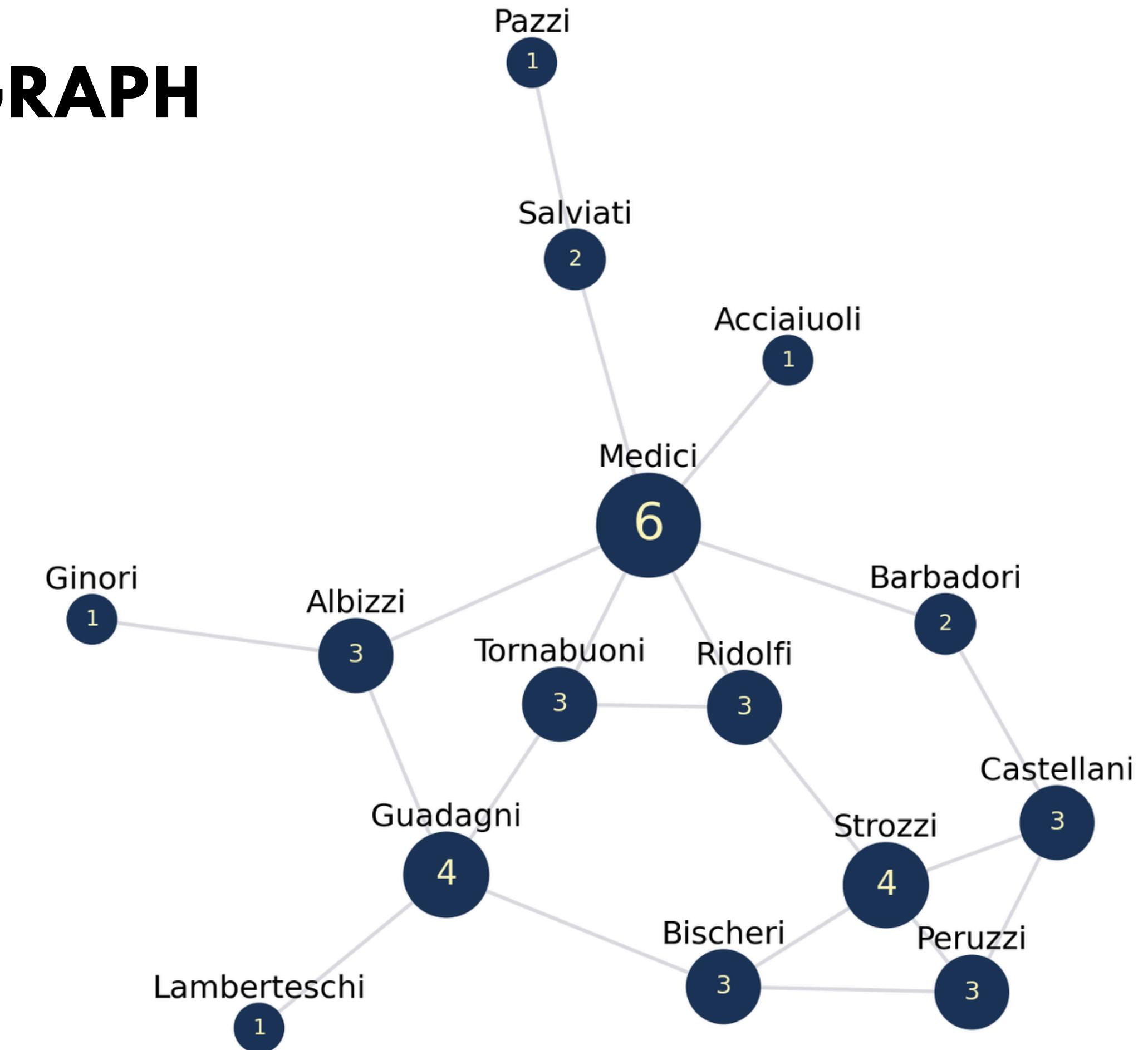
$$d_i = |N(i)|,$$

i.e., the number of nodes directly connected to  $i$ .

# FLORENTINE FAMILIES GRAPH

Degrees

Highest degrees: the Medicis (6),  
the Guadagnis (4), the Strozzi (4).



## **DEFINITION (DEGREE DISTRIBUTION)**

The *degree distribution*  $P(d)$  of a network is a description of the relative frequencies of nodes that have different degrees.

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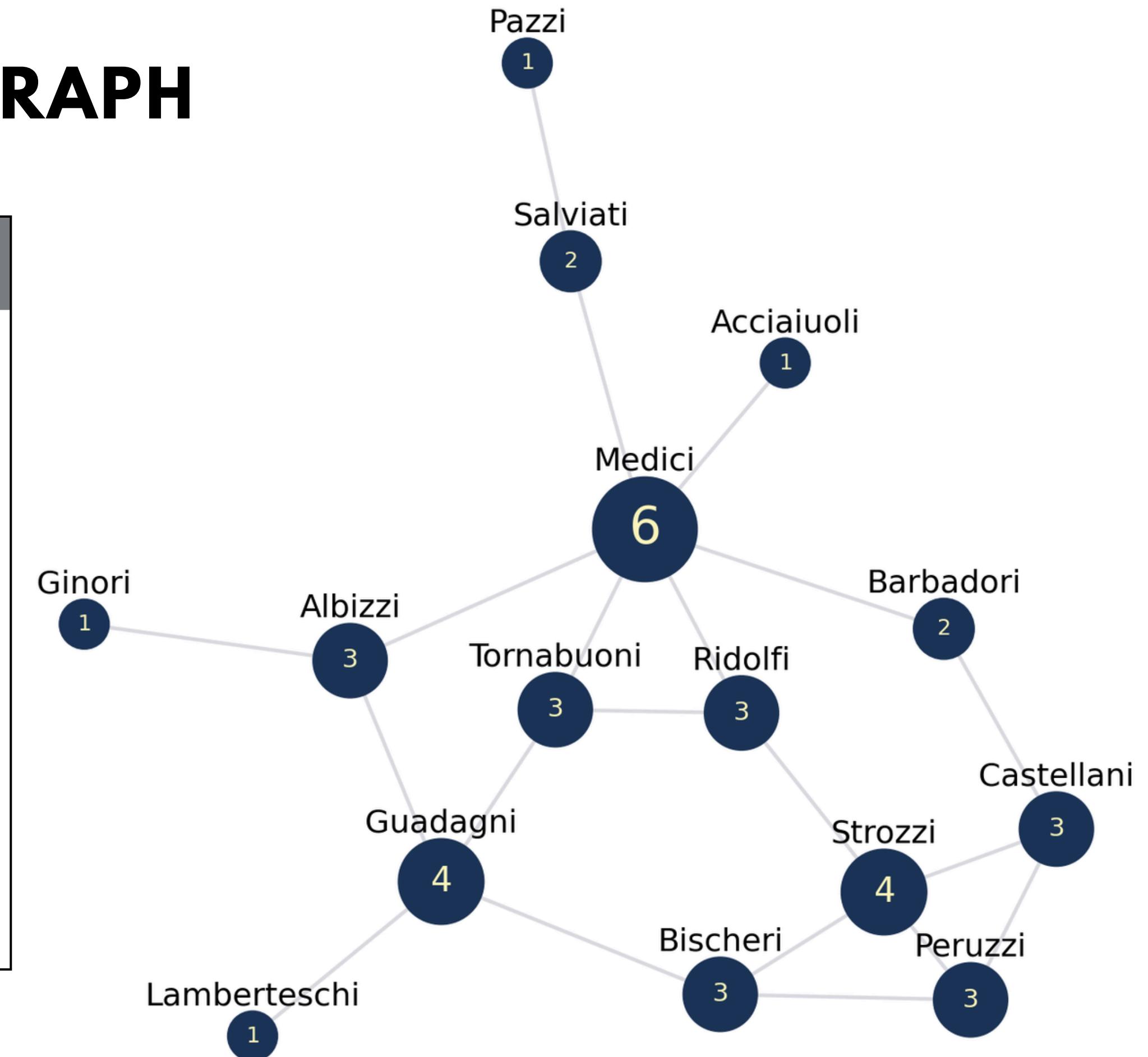
In other words,  $P(d)$  is the fraction of nodes that have degree  $d$ .

The function  $P$  can be a frequency distribution, if we are describing a specific network; or a probability distribution, if we are working with random networks.

# FLORENTINE FAMILIES GRAPH

Degree distribution

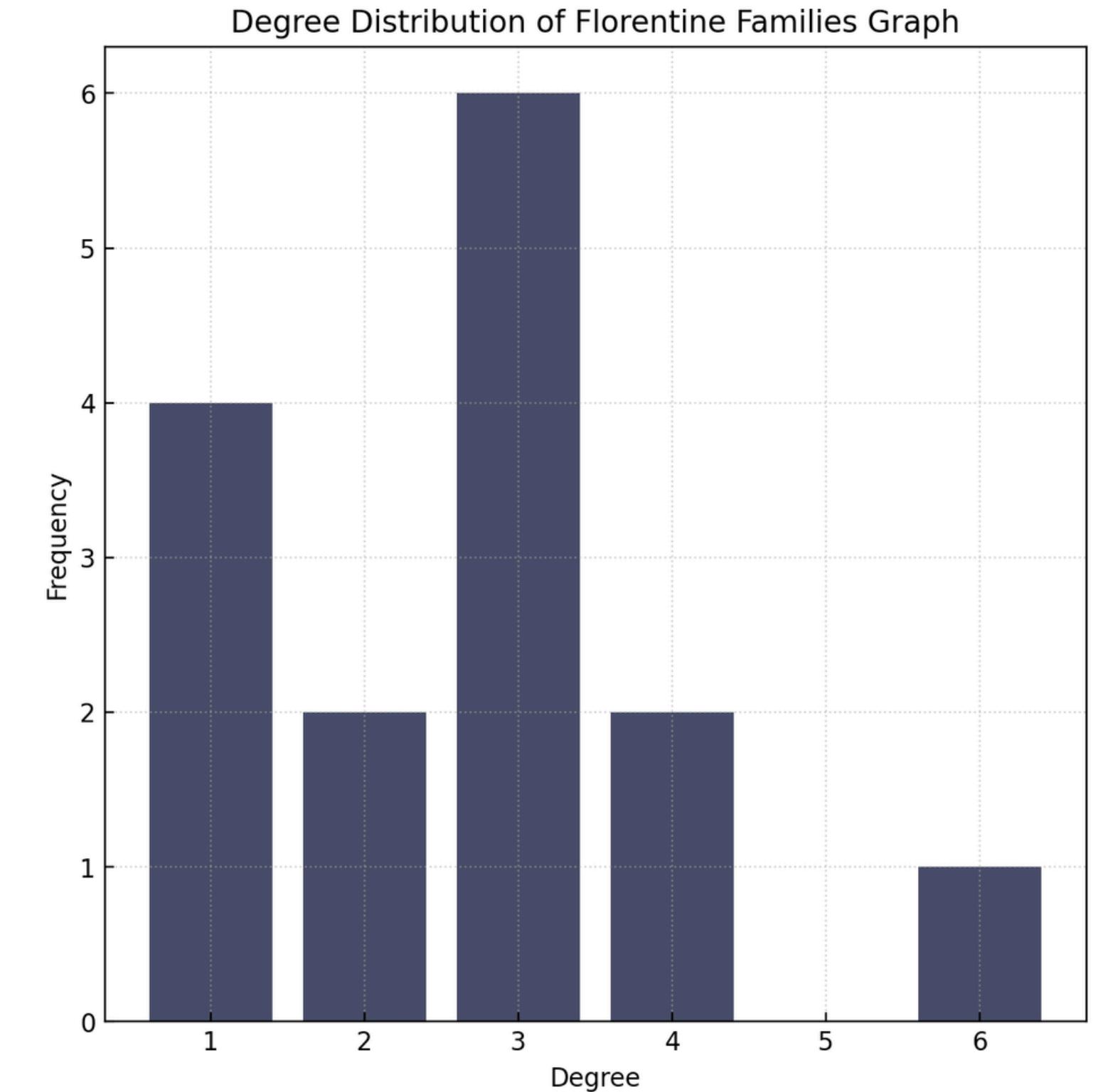
Degree	Family
6	Medici
5	-
4	Guadagni, Strozzi
3	Tornabuoni, Ridolfi, Albizzi, Bischeri, Peruzzi, Castellani
2	Salviati, Barbadori
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# FLORENTINE FAMILIES GRAPH

## Degree distribution

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What kind of degree distributions are there?

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Here's one popular model.

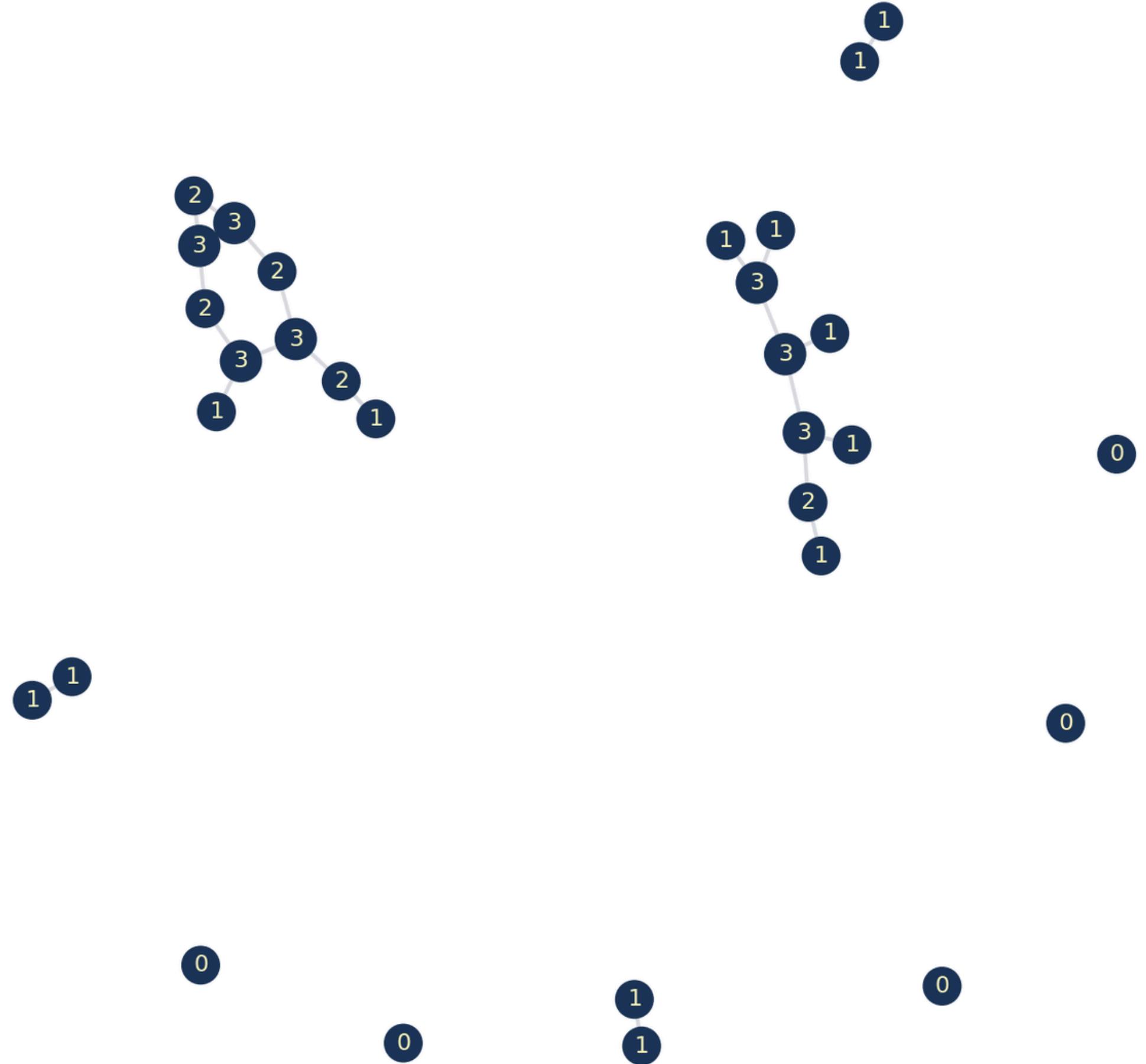
## **DEFINITION (ERDŐS–RÉNYI MODEL)**

For a set of  $n$  nodes and a probability  $p$ , the *Erdős-Rényi model* generates a random graph  $G(n, p)$  by going through all possible pairs of nodes and adding an edge between them with probability  $p$ .

# $G(n, p)$ EXAMPLE

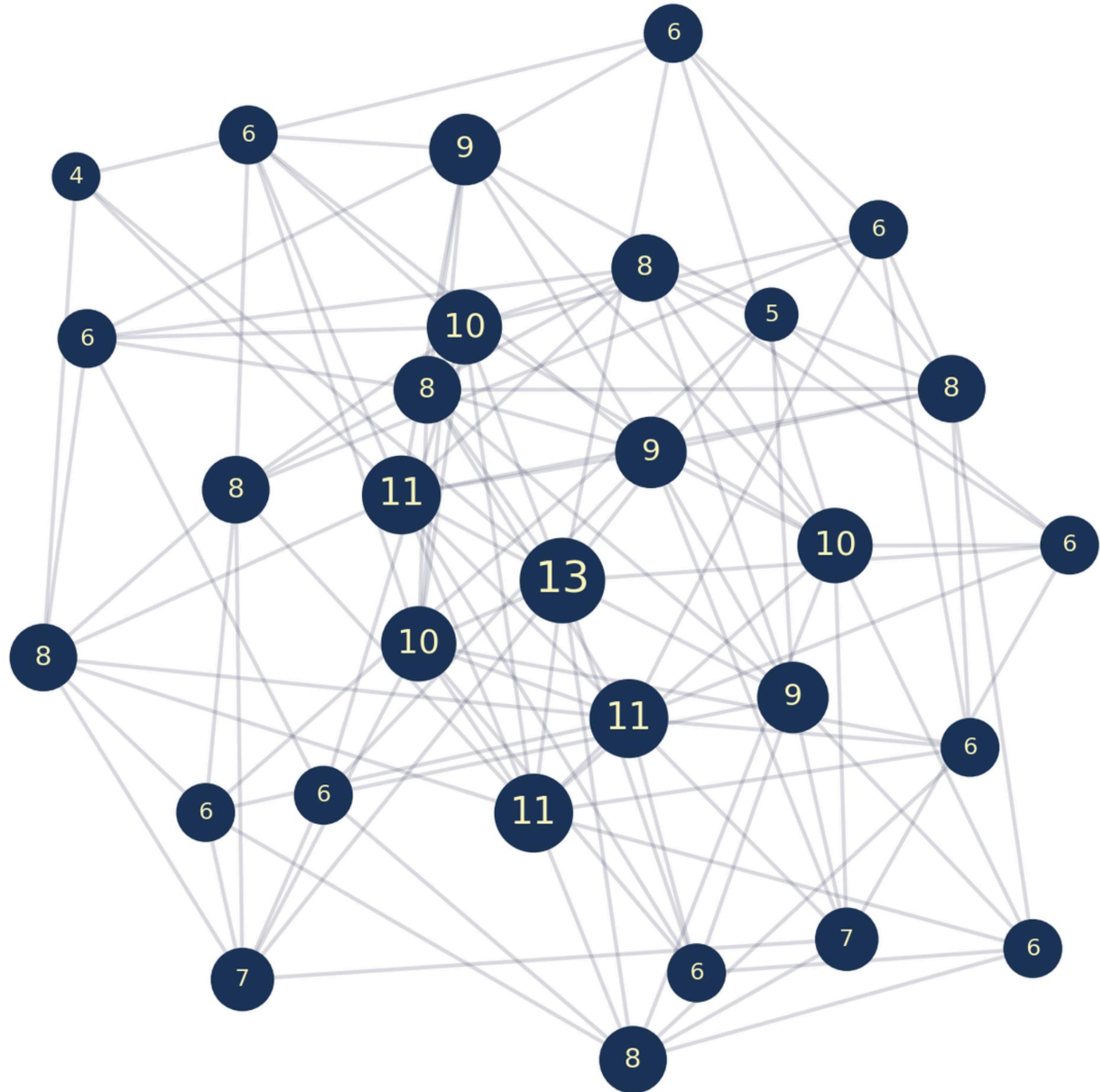
$n = 30, p = 0.05$

\*Node labels are the degrees.



# $G(n, p)$ EXAMPLE

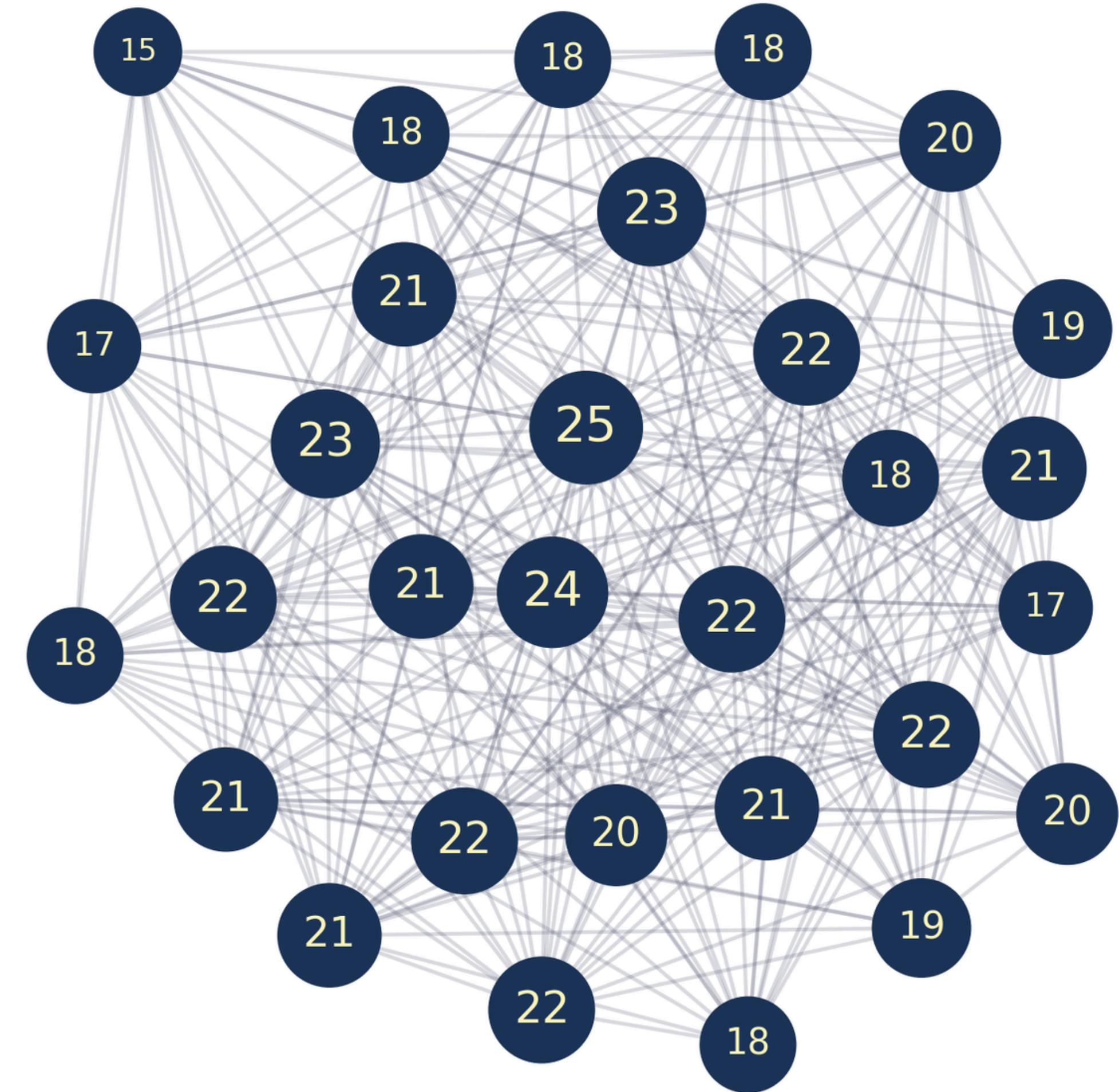
$n = 30, p = 0.25$



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# $G(n, p)$ EXAMPLE

$$n = 30, p = 0.7$$



What does the degree distribution for random graphs look like?

$G(n, p)$

Degree distribution

The degree distribution of an Erdős-Rényi random graph  $G(n, p)$  is given by the *binomial distribution*:

$$\Pr[d = k] = \binom{n-1}{k} p^k (1-p)^{n-1-k}.$$

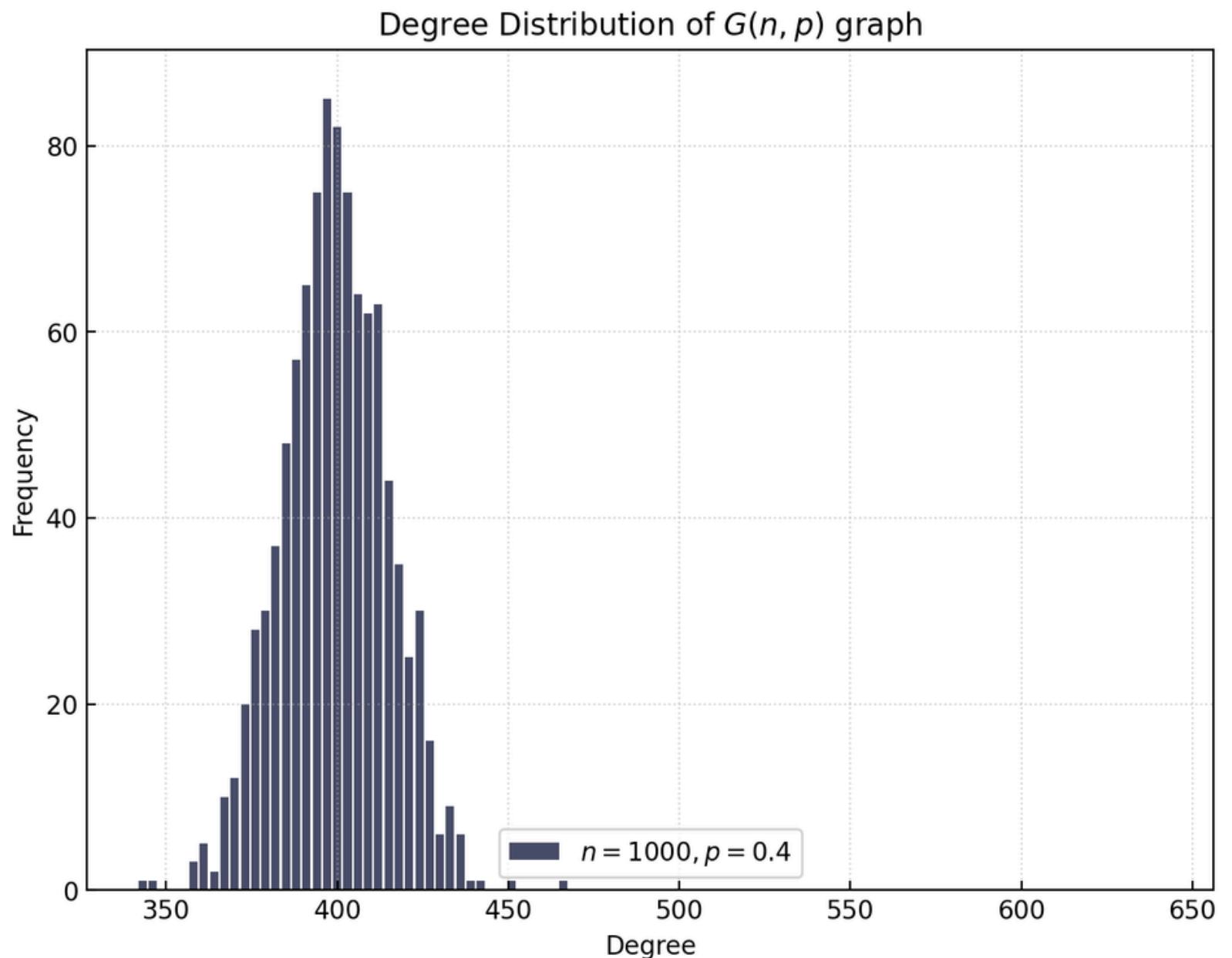
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The ‘typical’ node has degree  $(n-1)p$ .



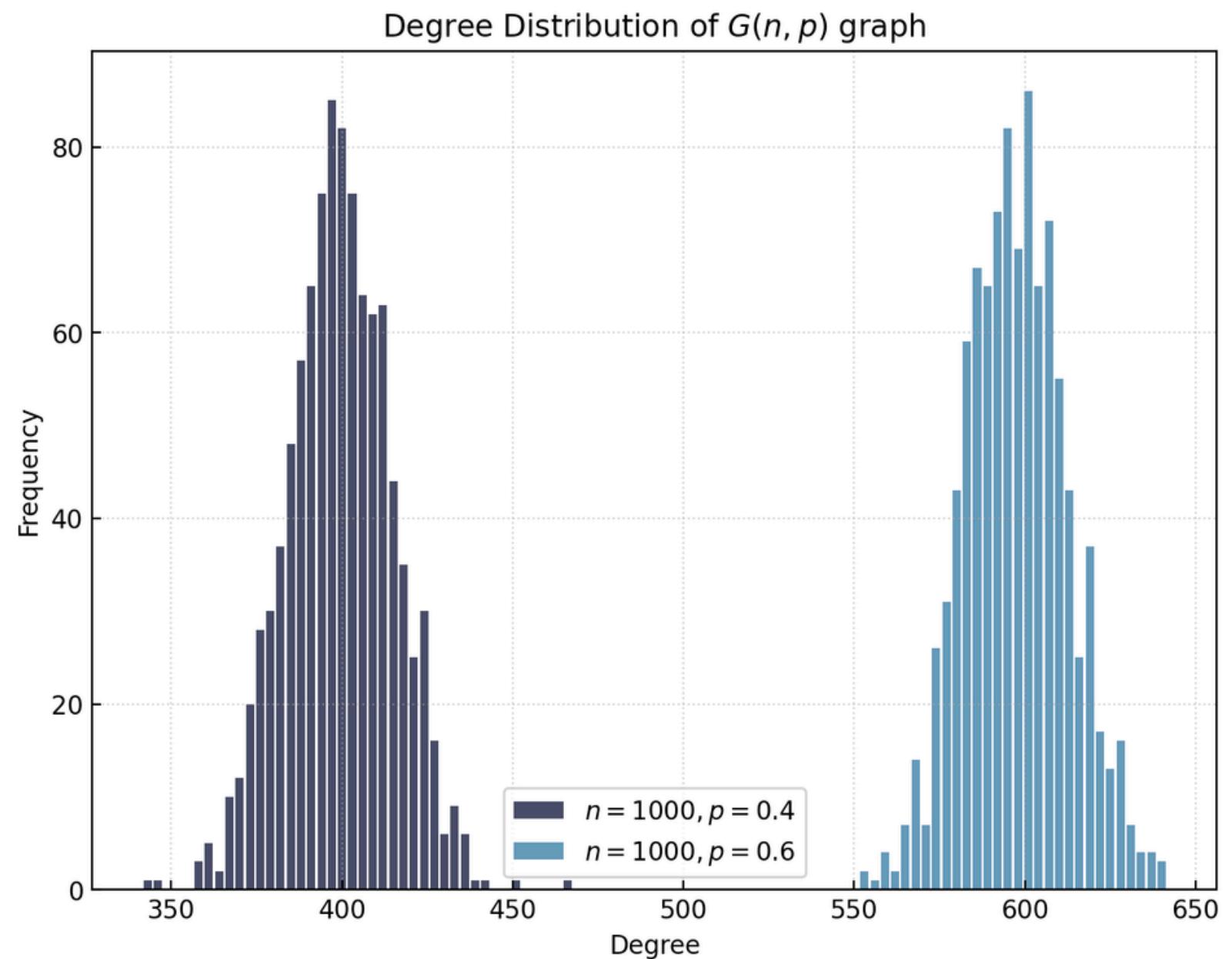
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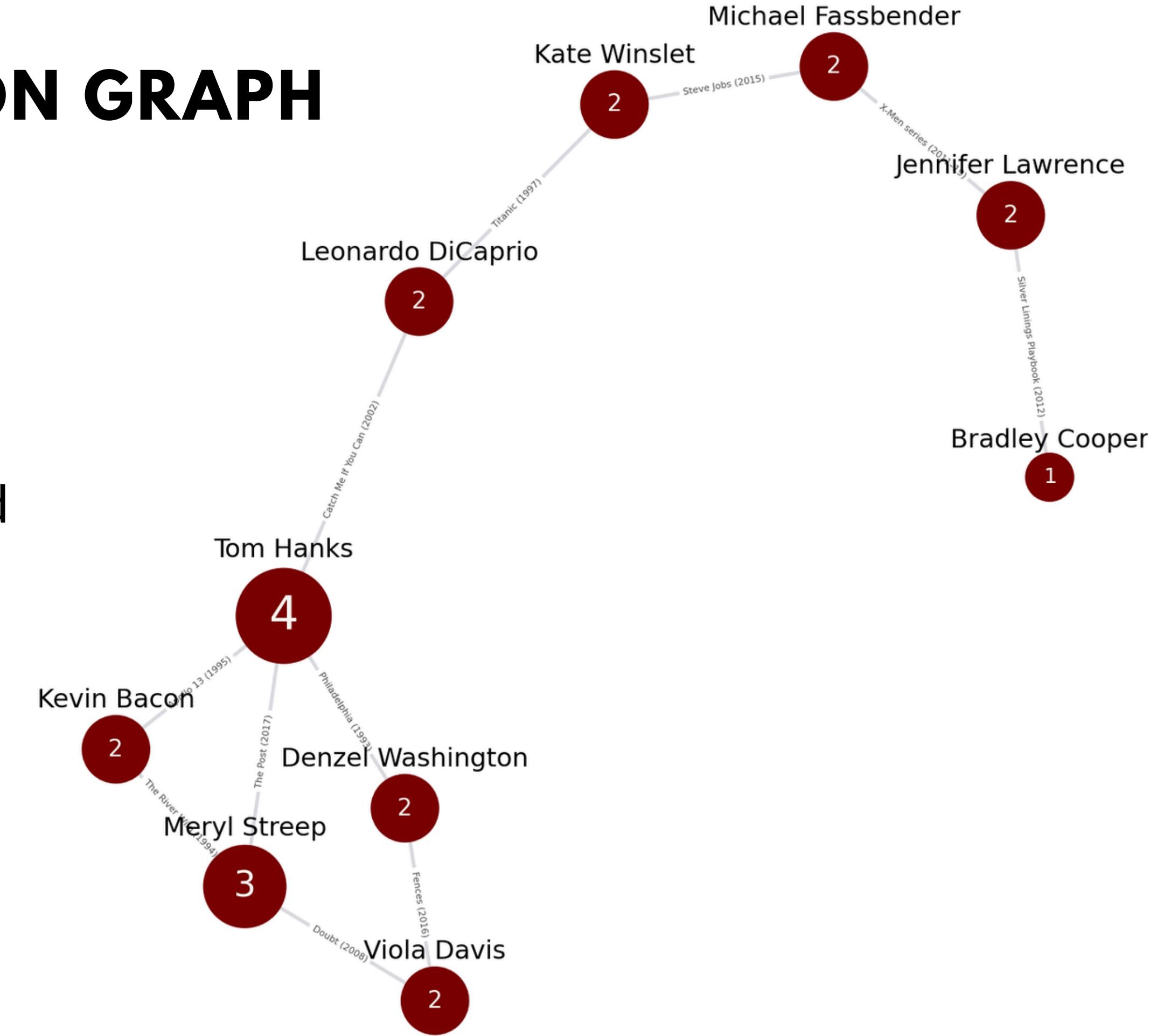
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Unfortunately, many real world social networks do not look like random graphs.

# ACTOR COLLABORATION GRAPH

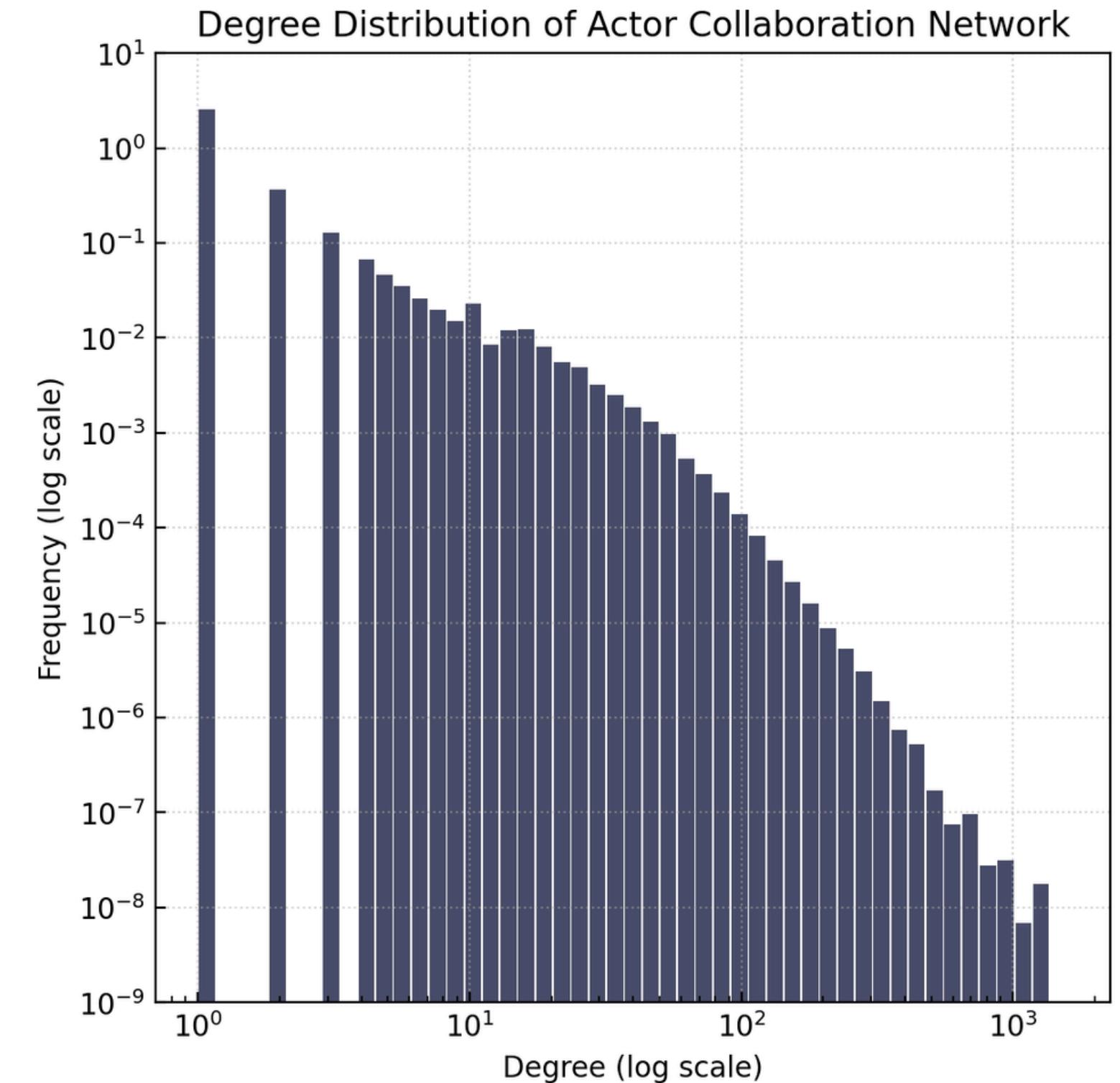
Actors are connected if they acted together in a movie.



# ACTOR COLLABORATION GRAPH

Degree distribution

Many nodes of low degree, some of middling degree, a couple with high degree.



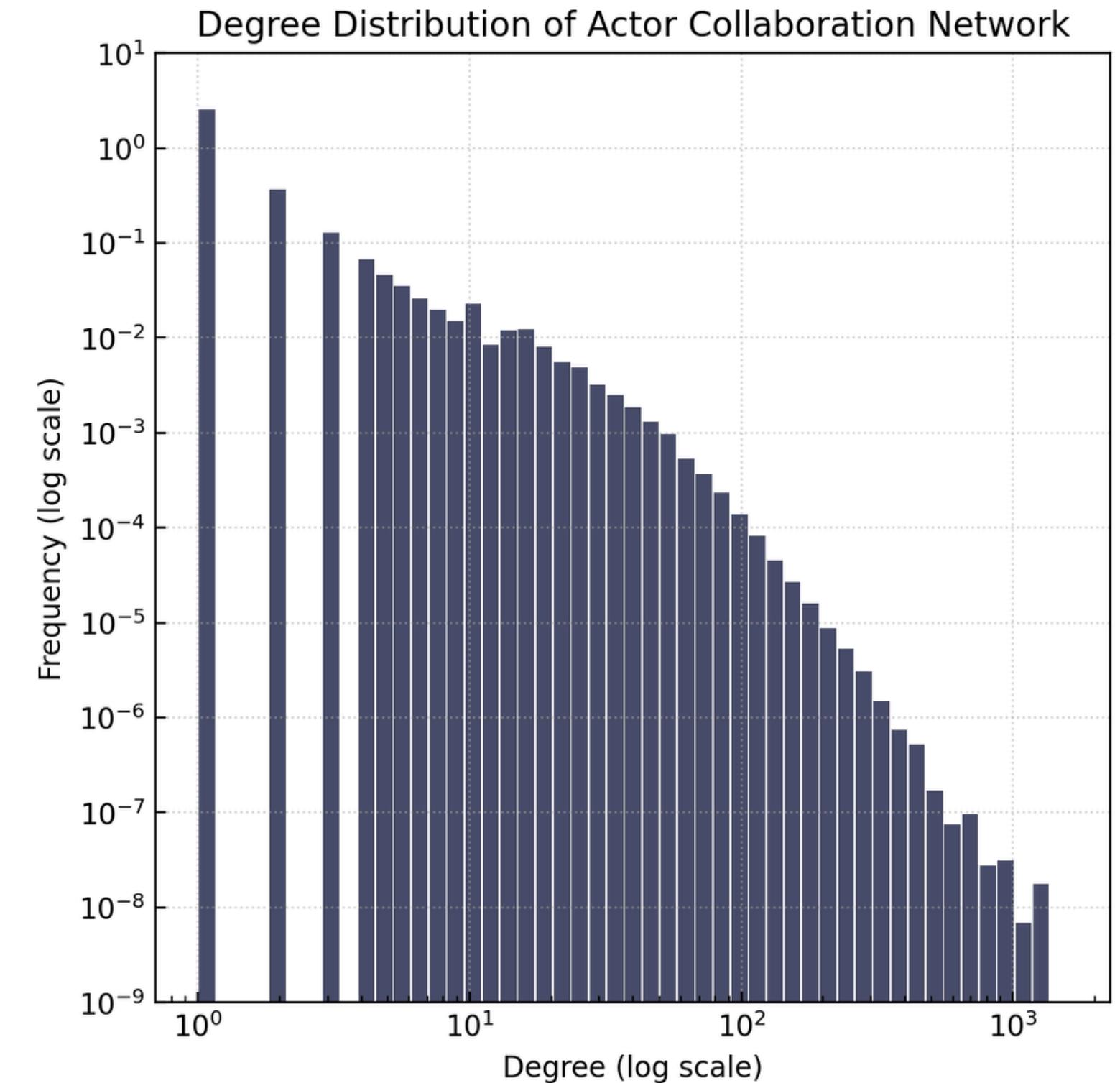
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# ACTOR COLLABORATION GRAPH

Degree distribution

Many nodes of low degree, some of middling degree, a couple with high degree.

No ‘typical’ node.



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## **DEFINITION (SCALE FREE NETWORK)**

A *scale-free network* is a type of network whose degree distribution follows a power law:

$$P(d) \sim c \cdot d^{-\gamma},$$

where  $c > 0$  and  $\gamma$  is a constant typically in the range  $2 < \gamma < 3$ .

# SCALE FREE NETWORKS

## Intuition

The relative frequencies stay constant as the degree grows from  $d$  to  $kd$ :

$$\begin{aligned}\frac{P(kd)}{P(d)} &= \frac{c \cdot k^{-\gamma} d^{-\gamma}}{c \cdot d^{-\gamma}} \\ &= k^{-\gamma}.\end{aligned}$$

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This is true regardless of the degree  $d$  we start with:

$$\frac{P(2)}{P(1)} = \frac{P(20)}{P(10)} = \frac{P(200)}{P(100)} = \dots$$

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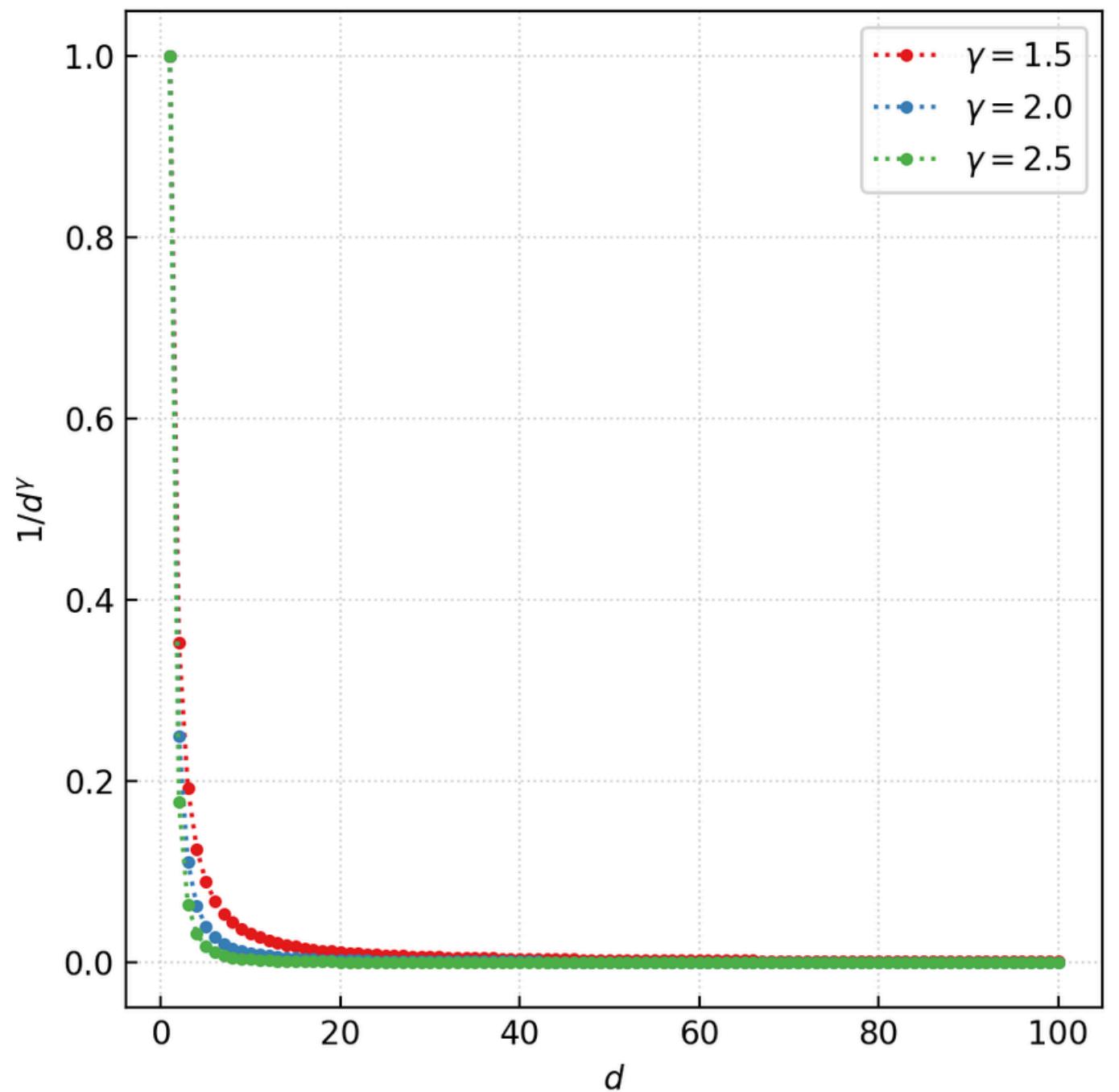
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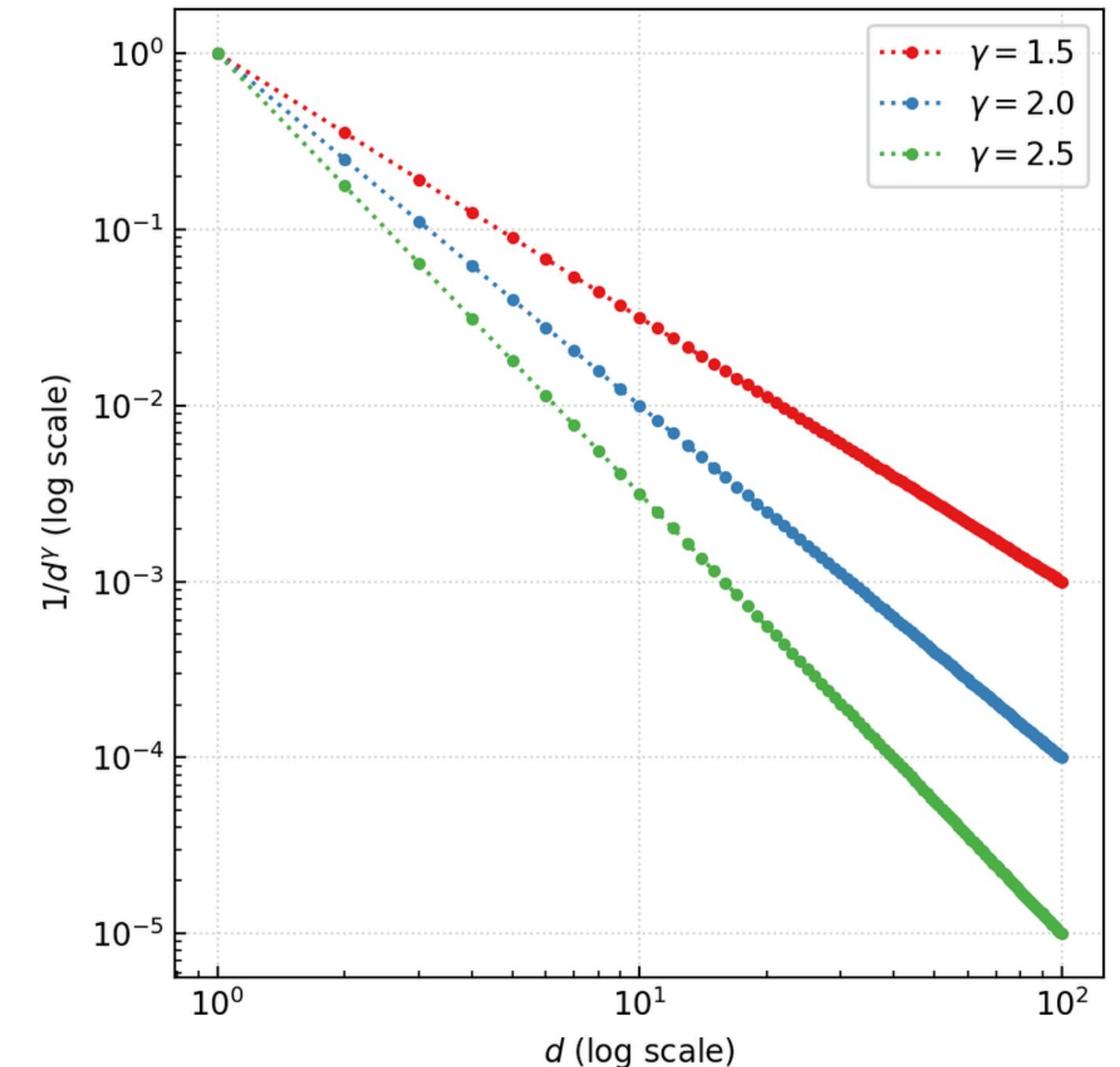
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Best seen when graphing the degree distribution...

...on a *log-log scale*, for readability: the degree distribution of a scale-free network appears as a straight line with slope  $-\gamma$ .



How can we generate scale-free networks?

## **DEFINITION (BARABÁSI–ALBERT MODEL)**

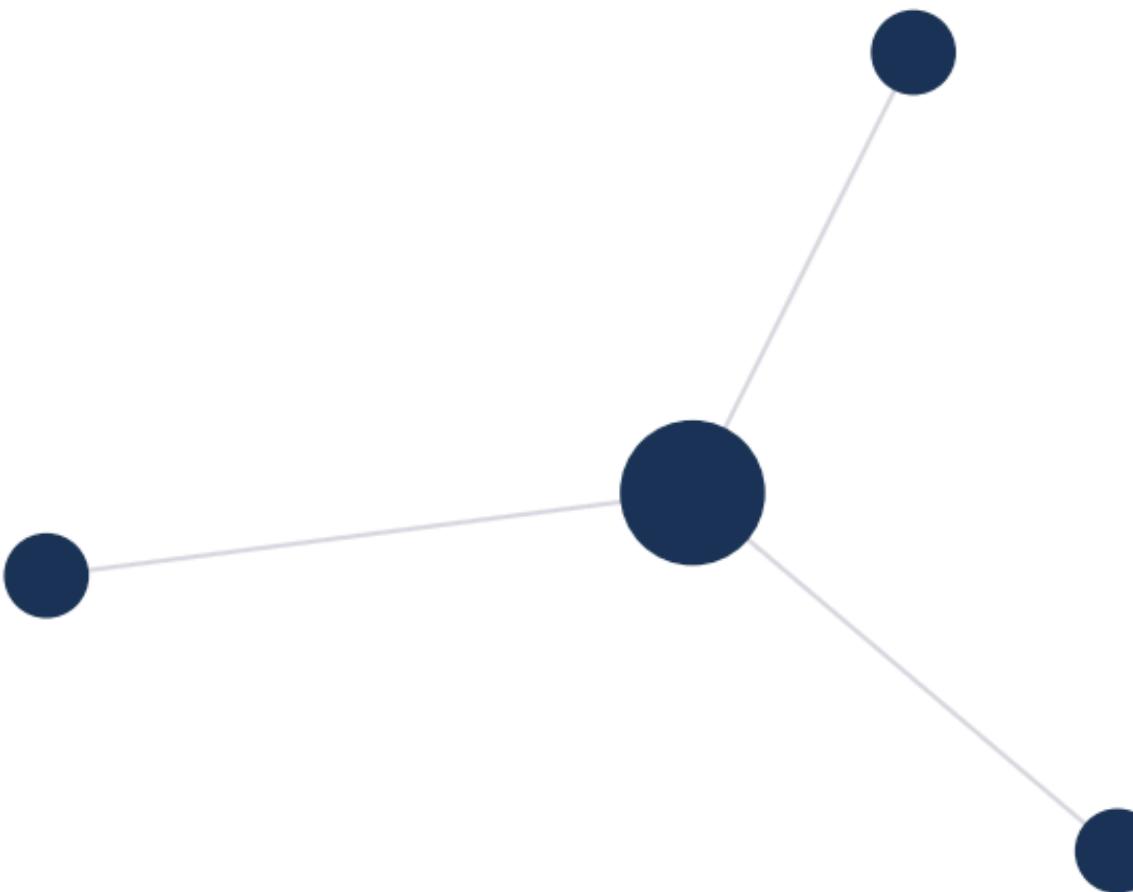
Start with  $m_0$  nodes. New nodes are added, one at a time. Each new node connects to an existing node  $i$  with a probability  $p_i$  proportional to  $i$ 's degree:

$$p_i = \frac{d_i}{\sum_j d_j}.$$

# BARABÁSI-ALBERT MODEL

## Intuition

In this process, nodes that are already well-connected get more connections.

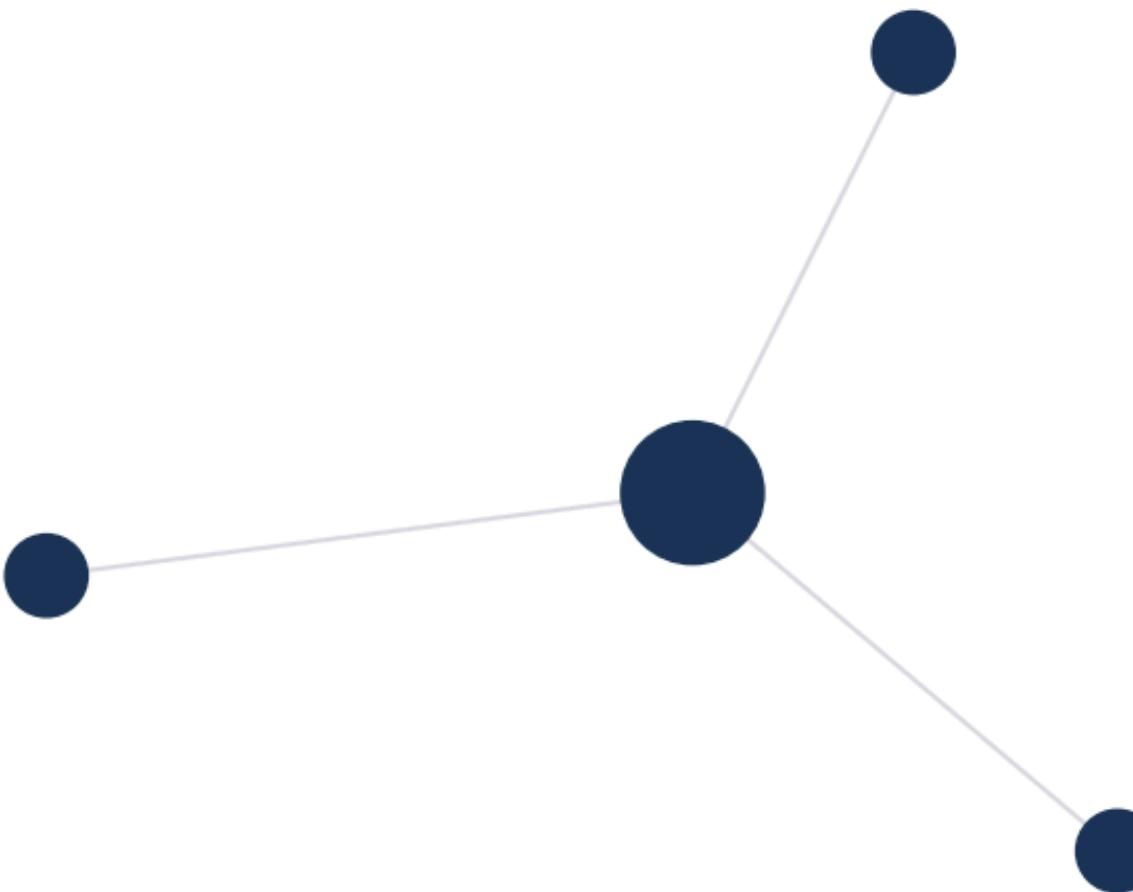


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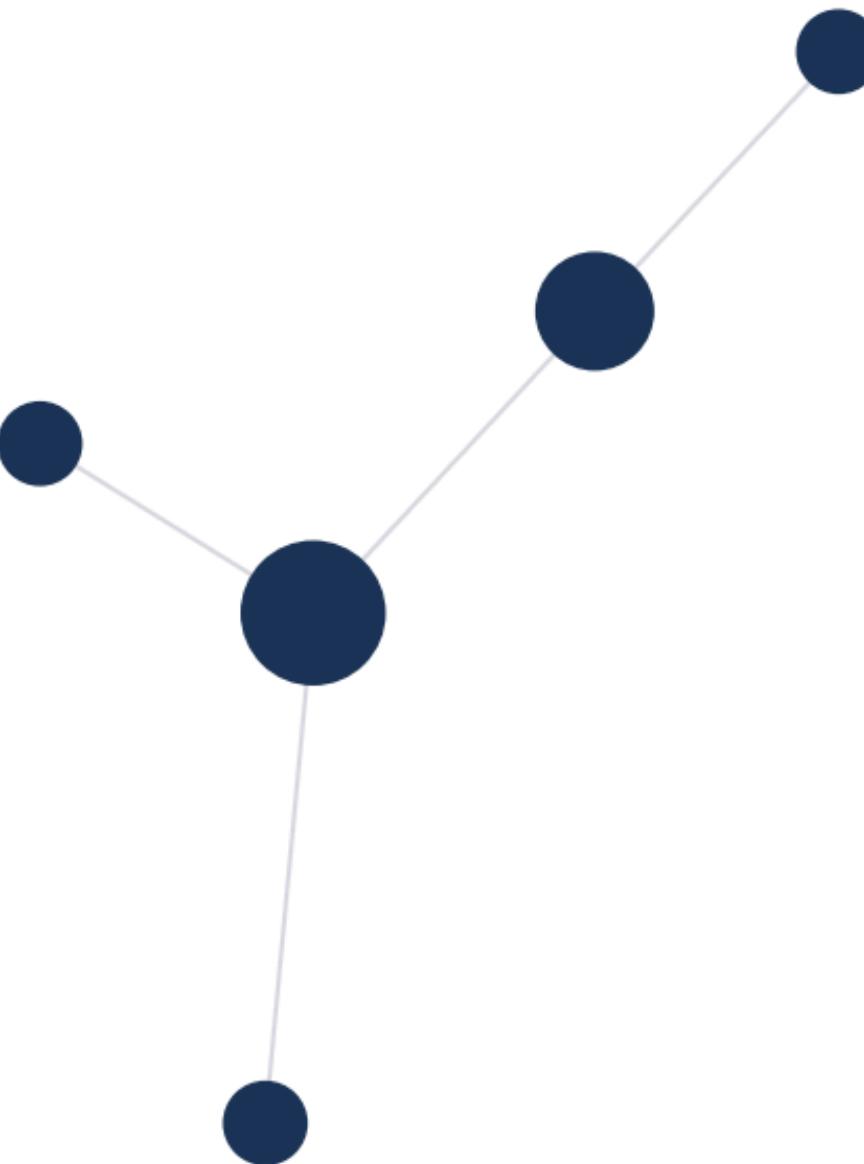


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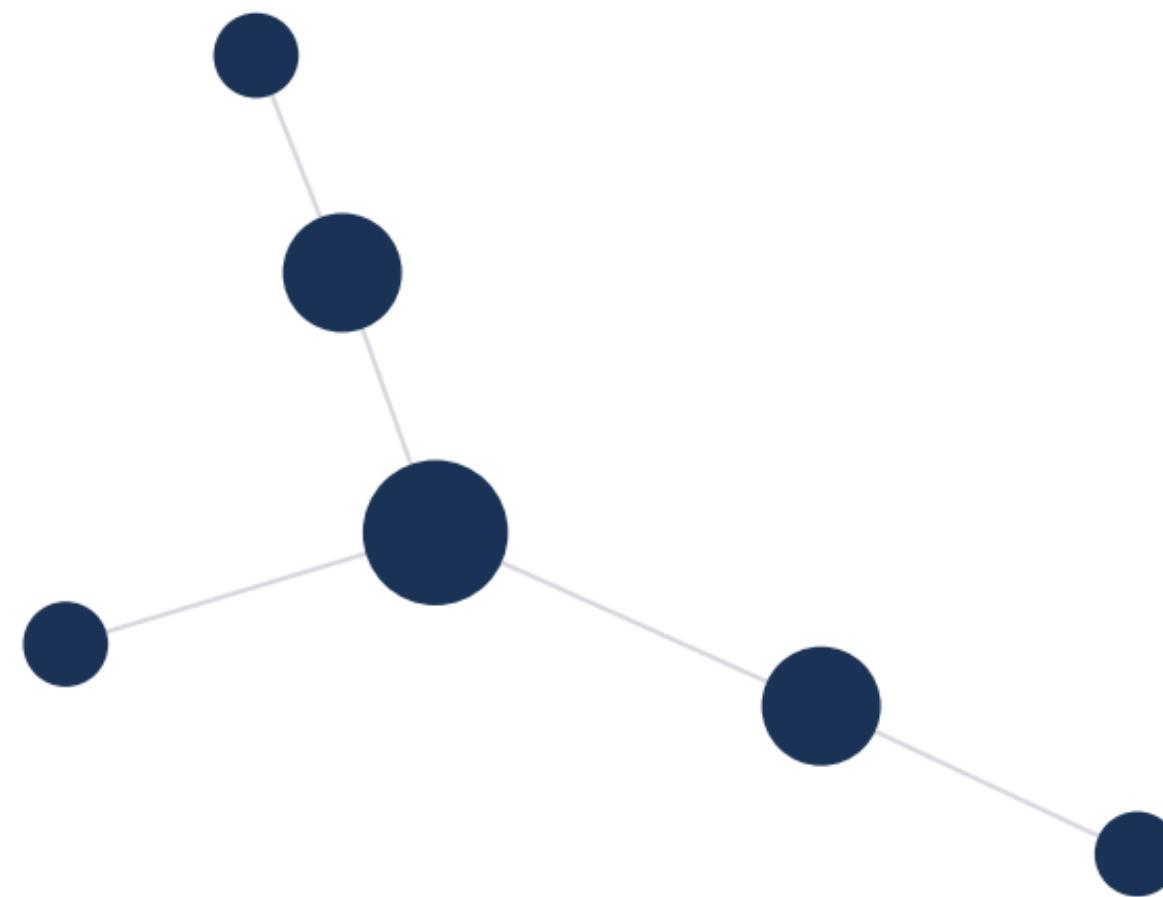
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Leads to the emergence of hubs.

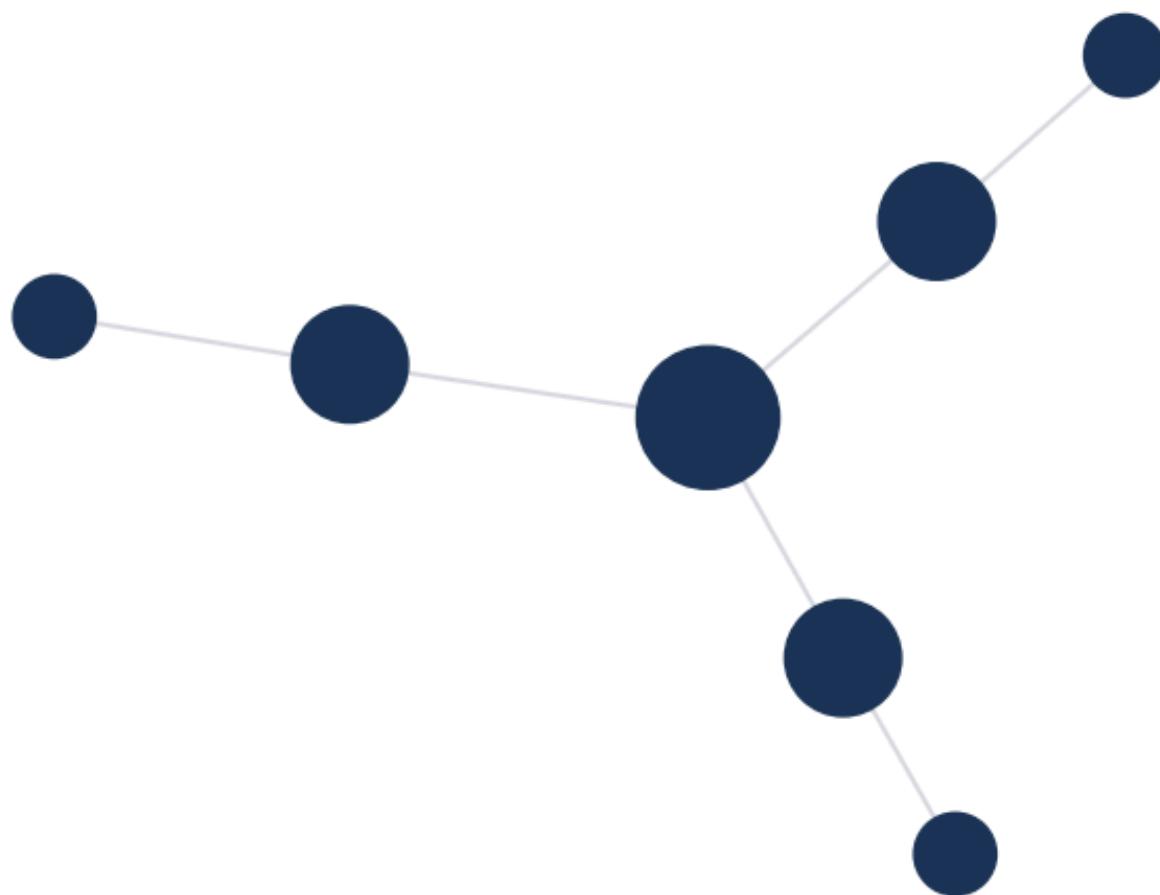


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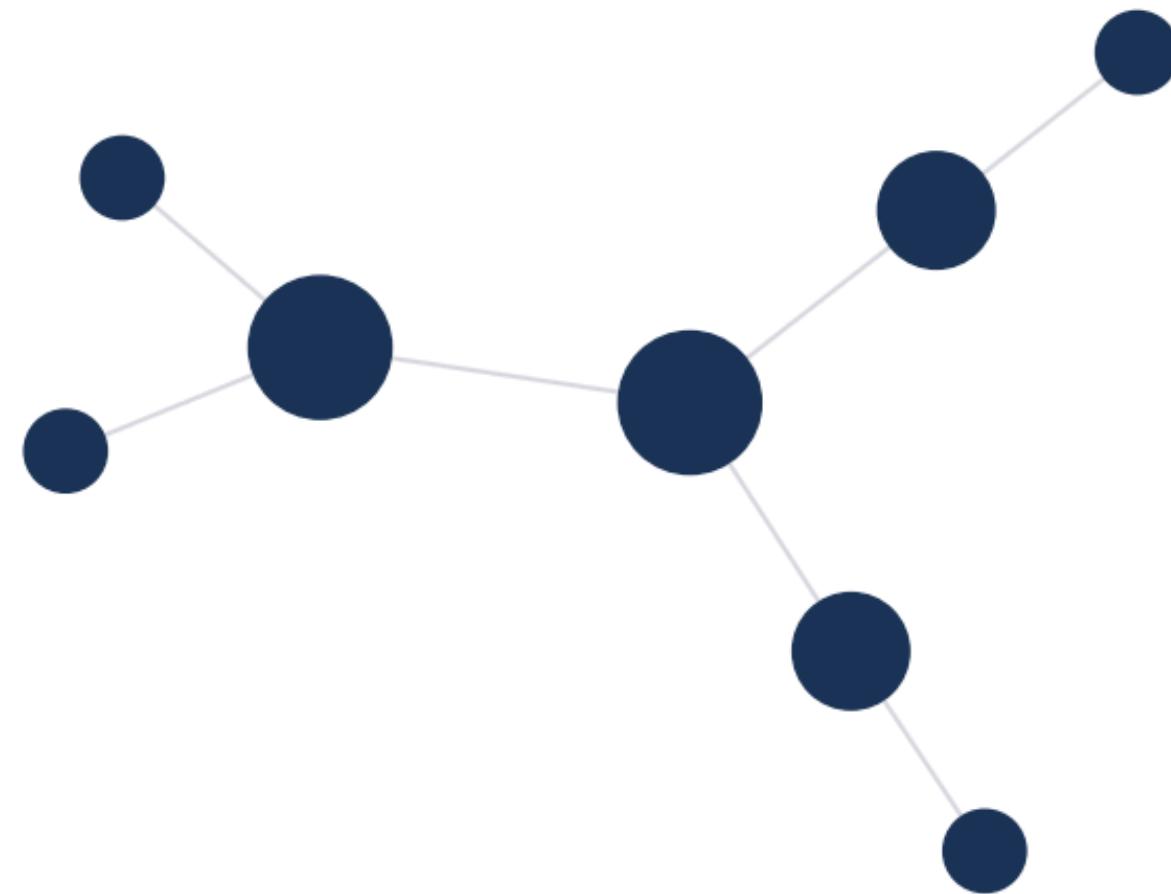


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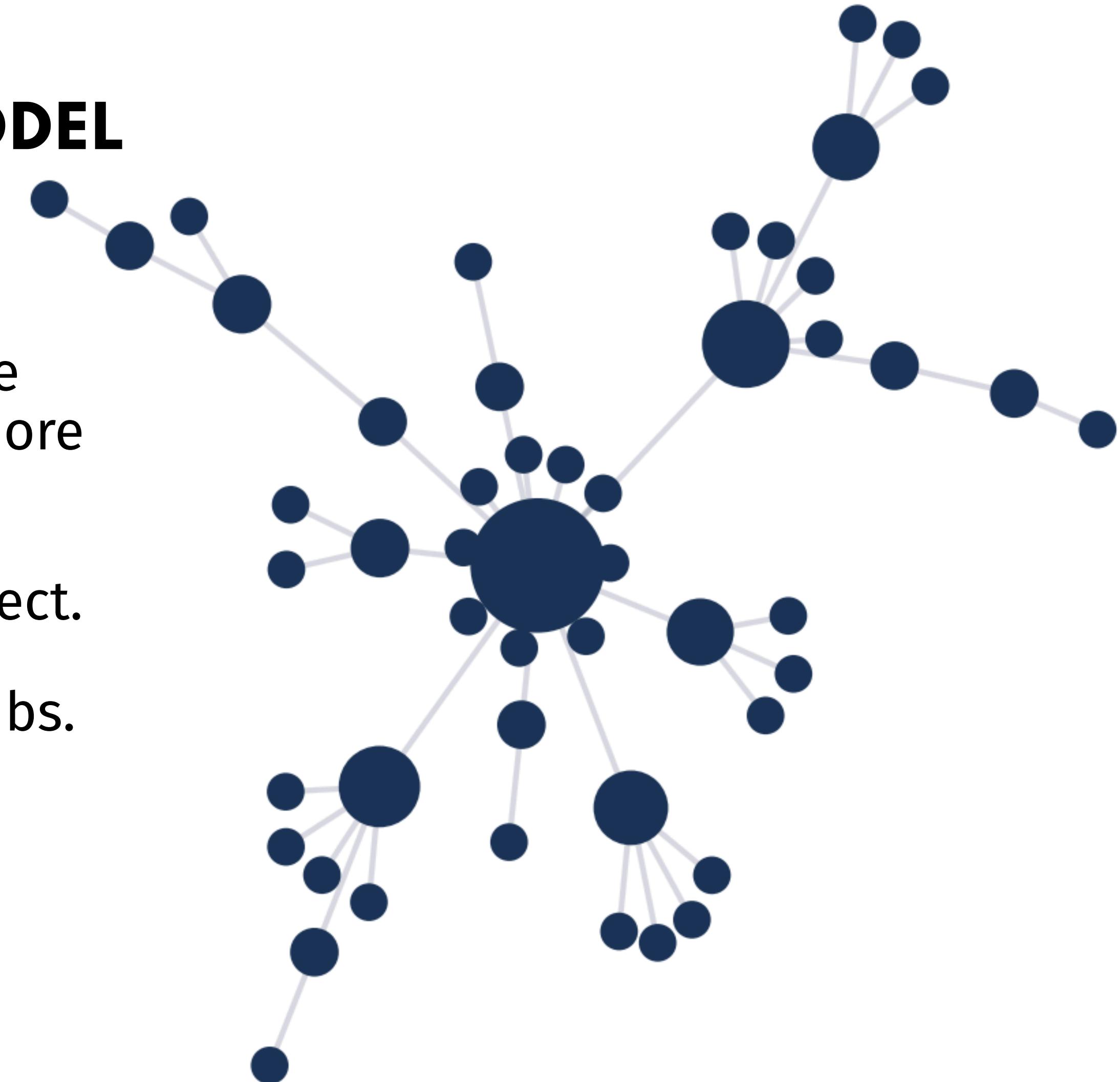
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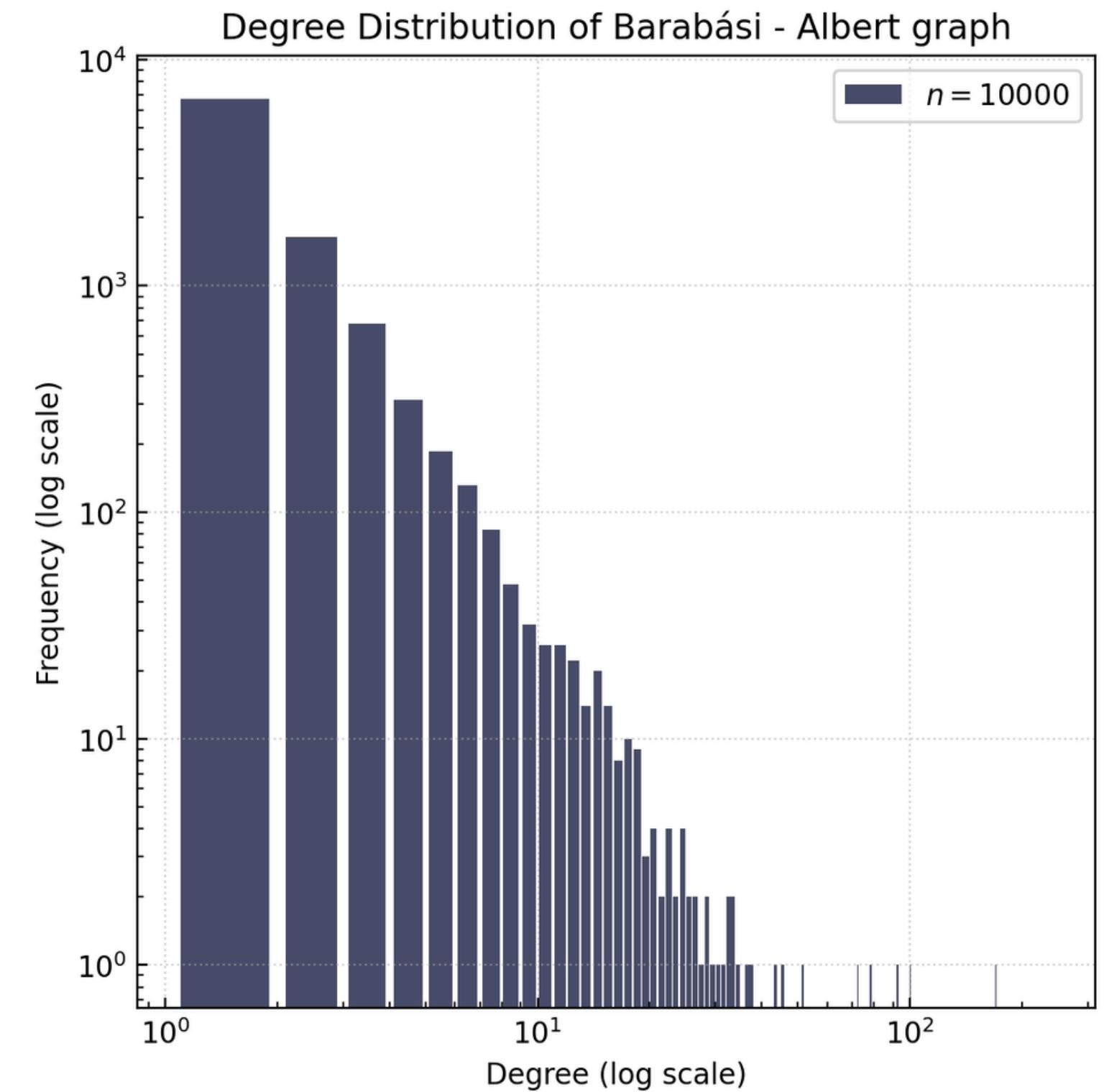
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# BARABÁSI-ALBERT MODEL

Degree distribution

We see the expected pattern in the degree distribution.



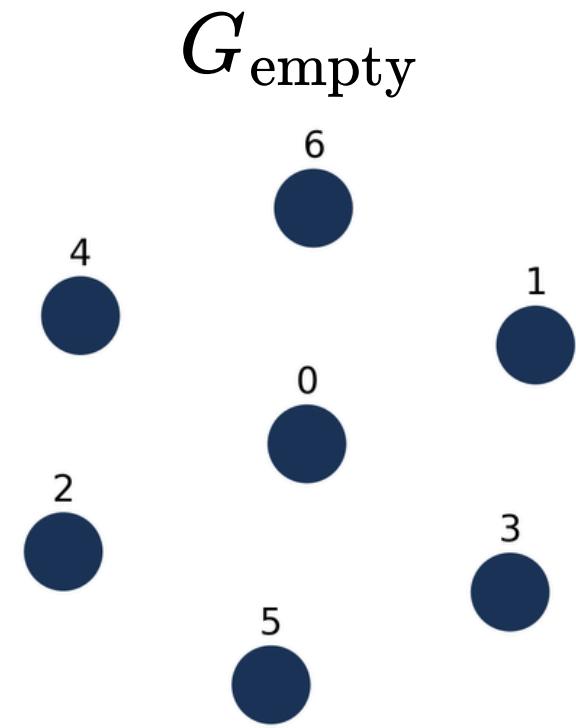
Another thing we want to look at are the distances between the nodes.

## **DEFINITION (DISTANCE BETWEEN NODES)**

The *distance*  $d(i, j)$  between two nodes  $i$  and  $j$  in a network is the length of the shortest path between  $i$  and  $j$ .

# PATHS AND STARS AND TREES

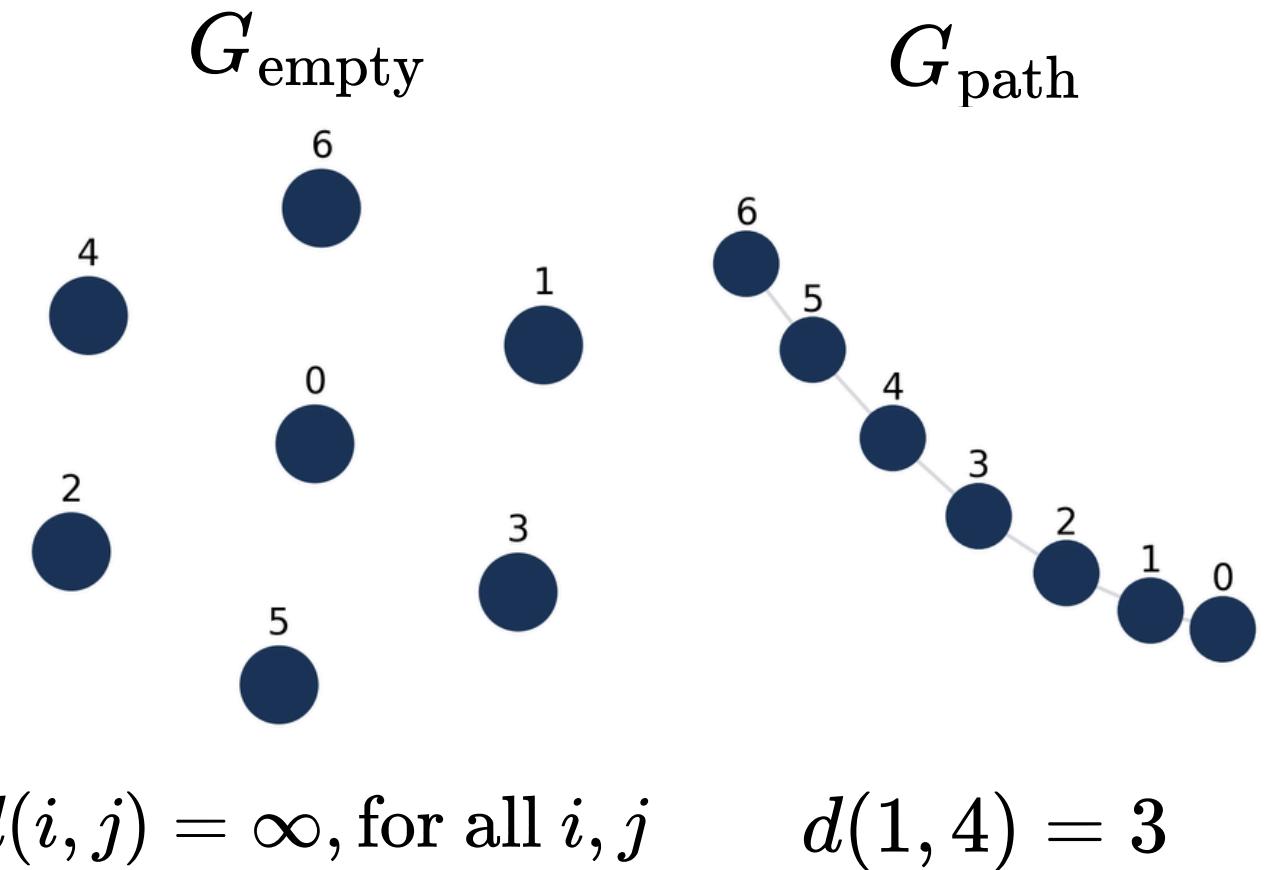
Distances



$$d(i, j) = \infty, \text{ for all } i, j$$

# PATHS AND STARS AND TREES

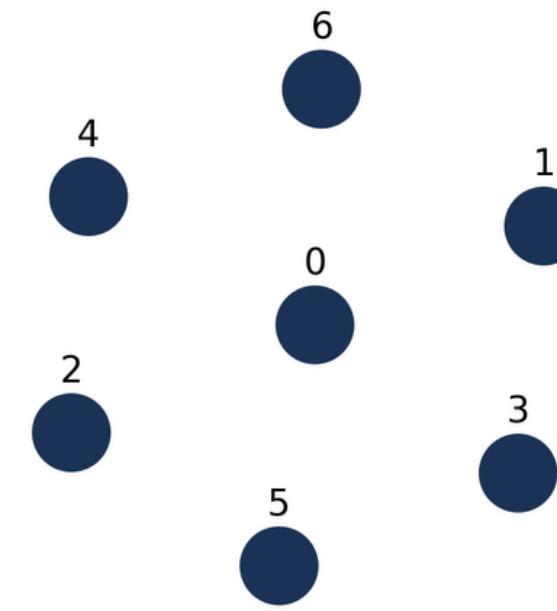
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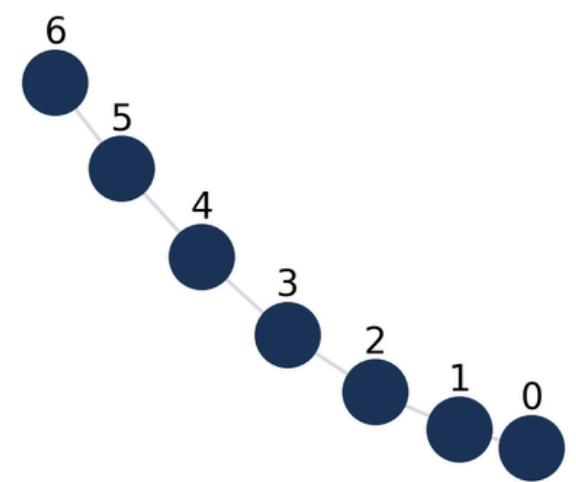
Distances

$G_{\text{empty}}$



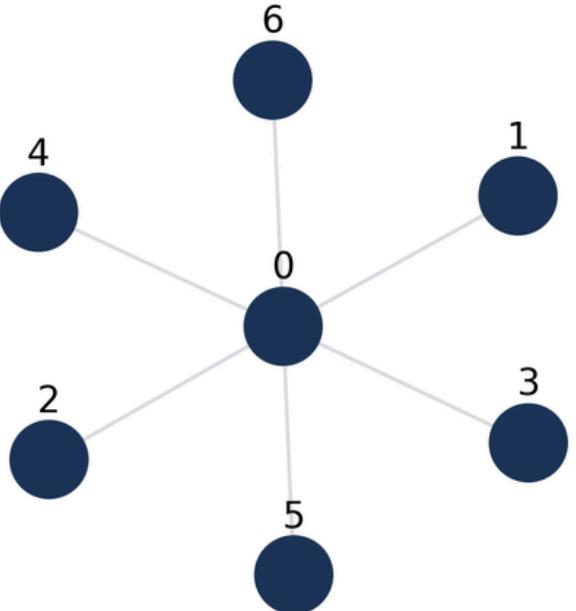
$$d(i, j) = \infty, \text{ for all } i, j$$

$G_{\text{path}}$



$$d(1, 4) = 3$$

$G_{\text{star}}$

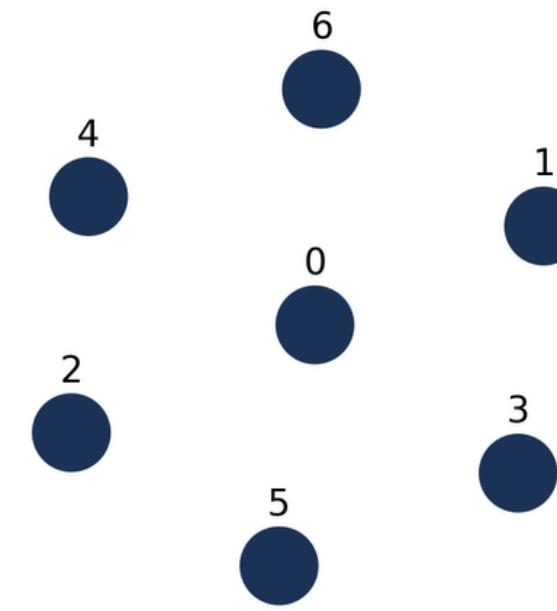


$$d(1, 2) = 2$$

# PATHS AND STARS AND TREES

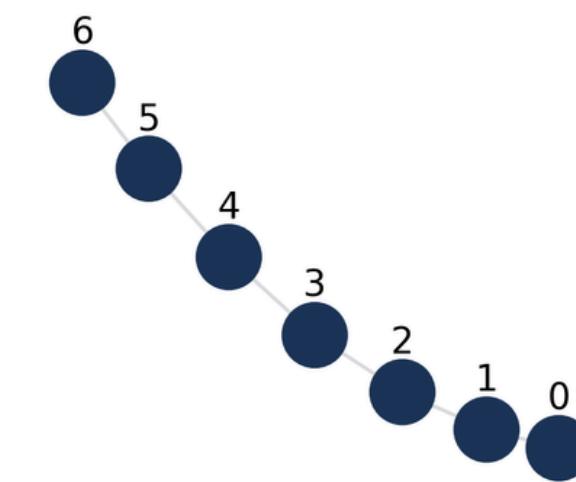
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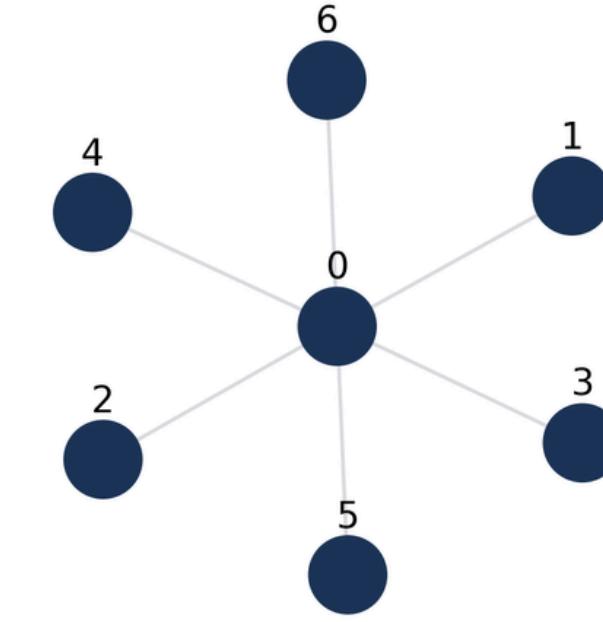
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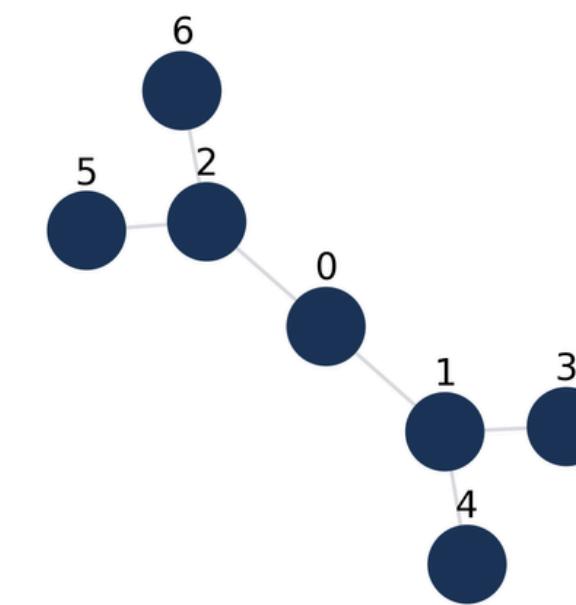
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$G_{\text{star}}$



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$G_{\text{tree}}$

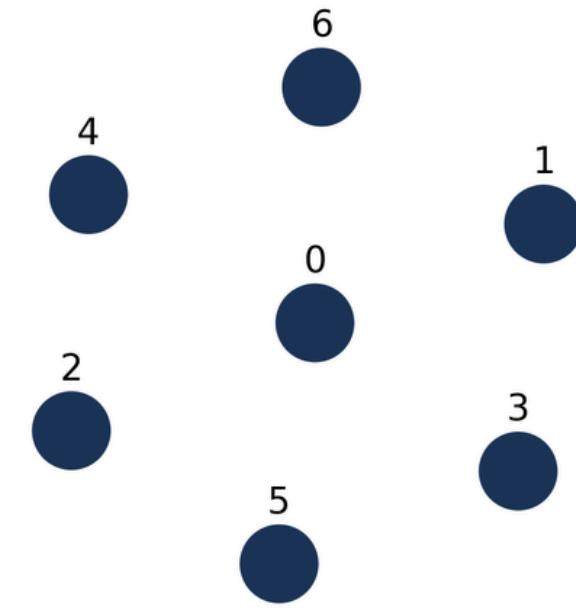


$$d(4, 6) = 4$$

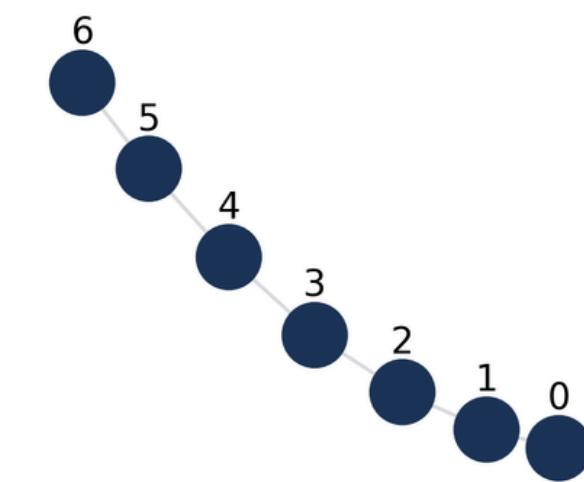
# PATHS AND STARS AND TREES

Distances

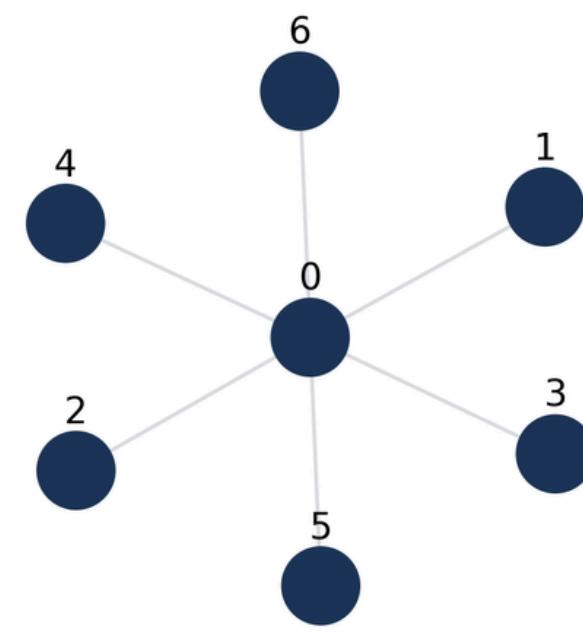
$G_{\text{empty}}$



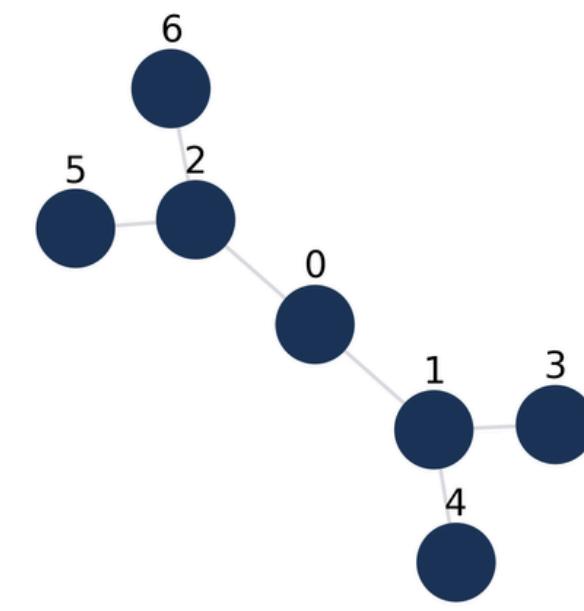
$G_{\text{path}}$



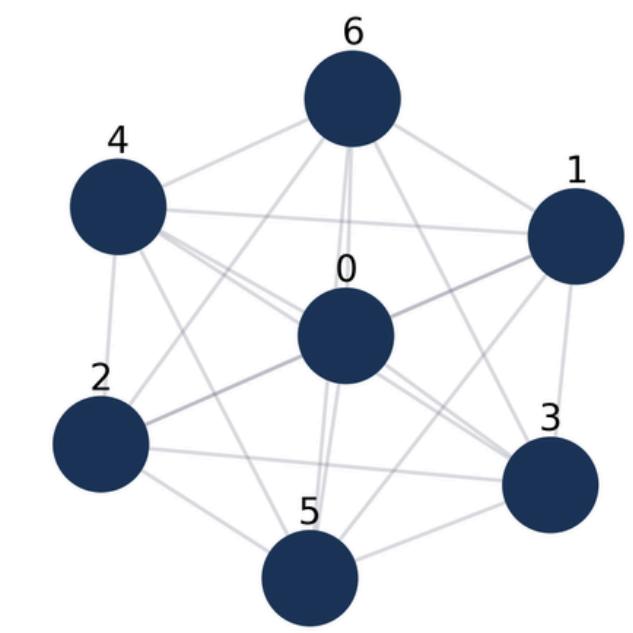
$G_{\text{star}}$



$G_{\text{tree}}$



$G_{\text{complete}}$



$$d(i, j) = \infty, \text{ for all } i, j$$

$$d(1, 4) = 3$$

$$d(1, 2) = 2$$

$$d(4, 6) = 4$$

$$d(i, j) = 1, \text{ for any } i, j$$

## **DEFINITION (DIAMETER)**

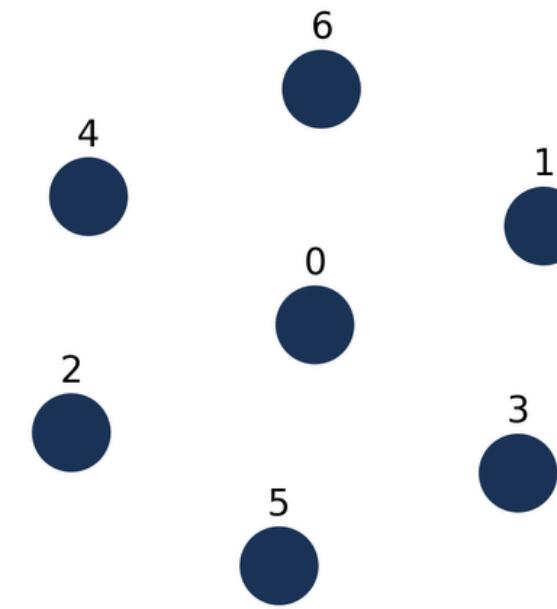
The *diameter* of a network  $G$  is the maximum distance between any two nodes:

$$\text{diam}(G) = \max_{i,j \in V} d(i, j).$$

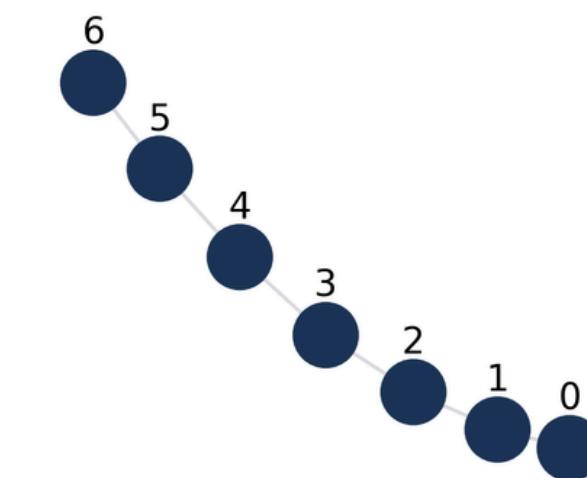
# PATHS AND STARS AND TREES

## Diameters

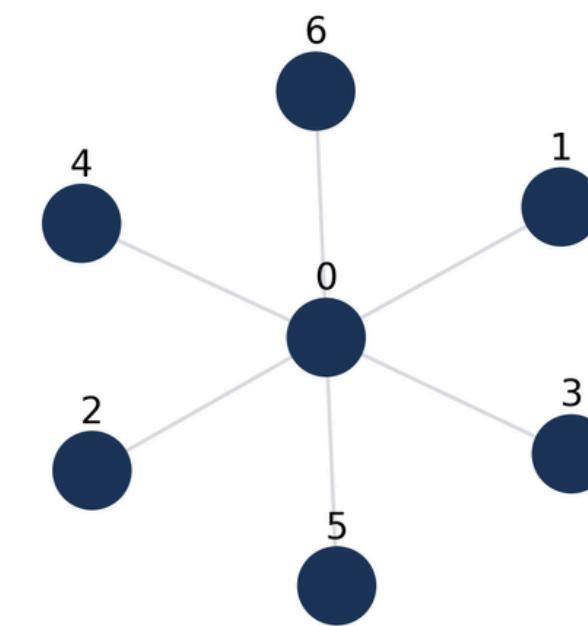
$G_{\text{empty}}$



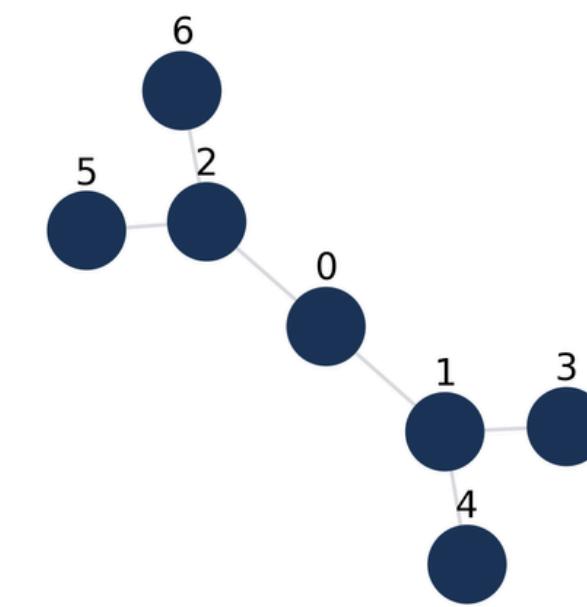
$G_{\text{path}}$



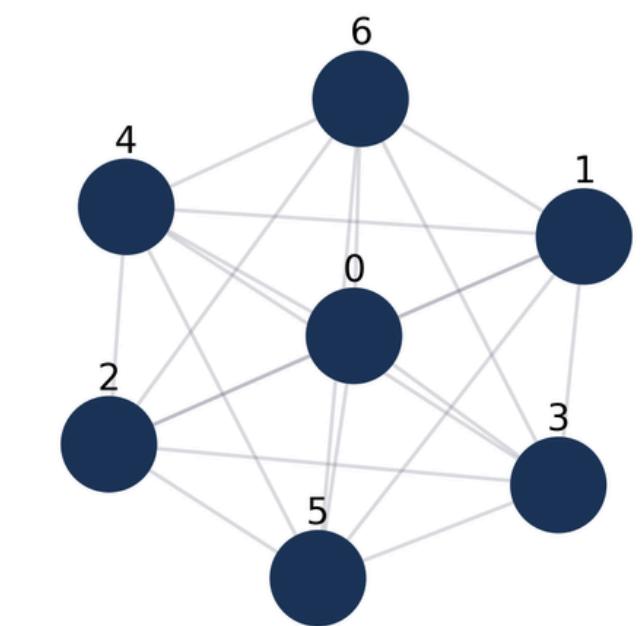
$G_{\text{star}}$



$G_{\text{tree}}$



$G_{\text{complete}}$



$$d(i, j) = \infty, \text{ for all } i, j$$

$$\text{diam}(G_{\text{empty}}) = \infty$$

$$d(1, 4) = 3$$

$$\text{diam}(G_{\text{path}}) = 5$$

$$d(1, 2) = 2$$

$$\text{diam}(G_{\text{star}}) = 2$$

$$d(4, 6) = 4$$

$$\text{diam}(G_{\text{tree}}) = 4$$

$$d(i, j) = 1, \text{ for any } i, j$$

$$\text{diam}(G_{\text{complete}}) = 1$$

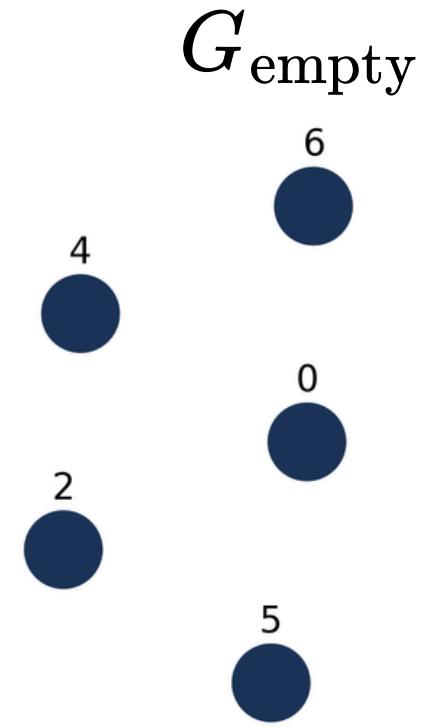
## **DEFINITION (AVERAGE SHORTEST PATH LENGTH)**

The *average shortest path length*  $L(G)$  of a network  $G$  is the average distance between all pairs of nodes:

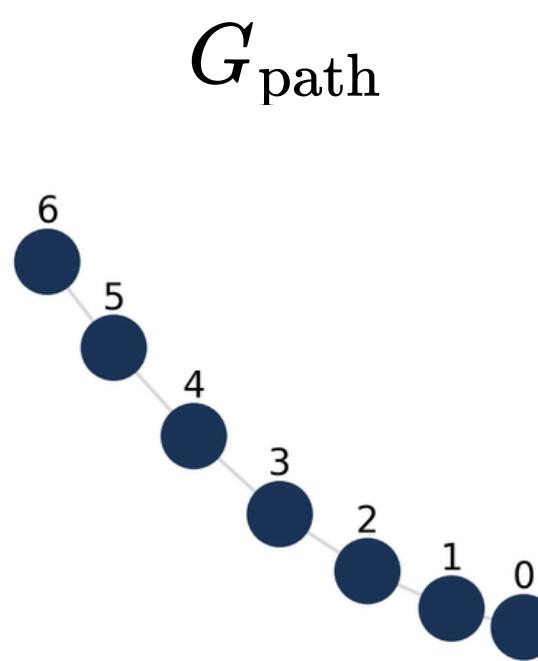
$$L(G) = \frac{1}{|V|(|V| - 1)} \sum_{i,j \in V} d(i, j).$$

# PATHS AND STARS AND TREES

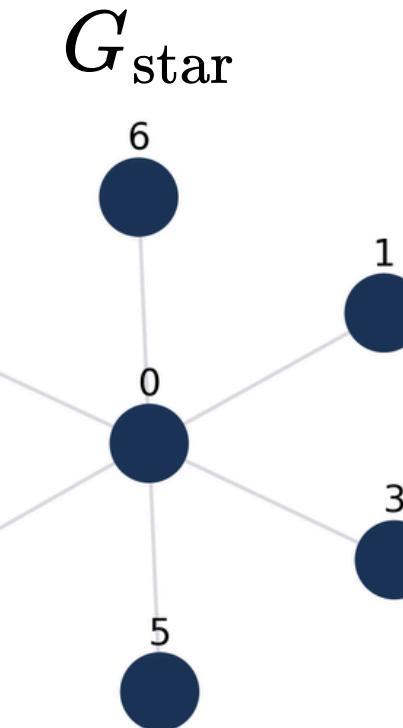
Average shortest path lengths



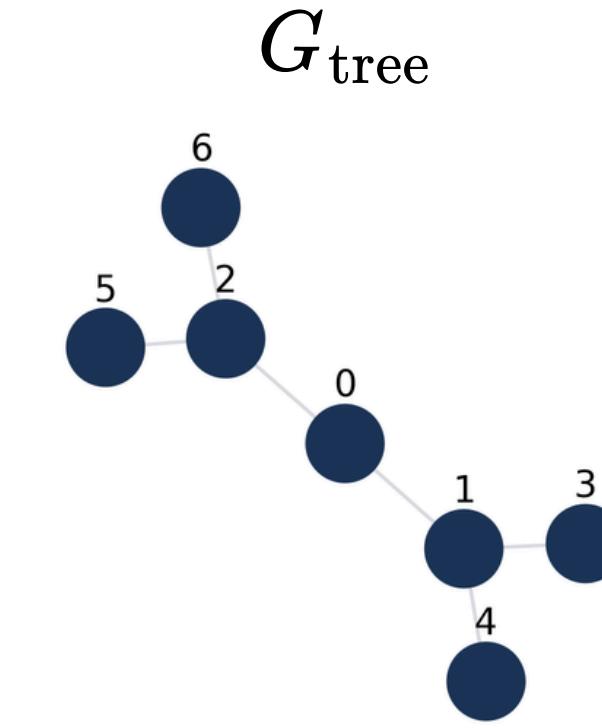
$$d(i, j) = \infty, \text{ for all } i, j$$
$$\text{diam}(G_{\text{empty}}) = \infty$$
$$L(G_{\text{empty}}) = \infty$$



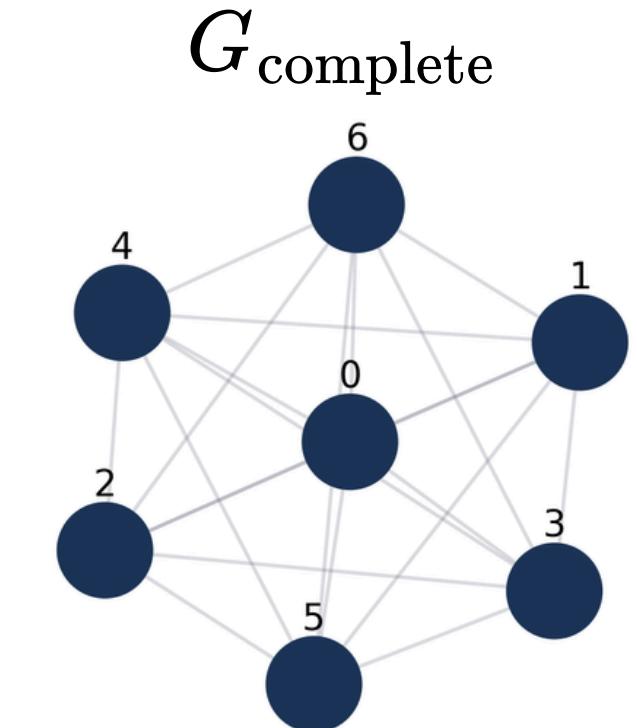
$$d(1, 4) = 3$$
$$\text{diam}(G_{\text{path}}) = 5$$
$$L(G_{\text{path}}) = 2.33$$



$$d(1, 2) = 2$$
$$\text{diam}(G_{\text{star}}) = 2$$
$$L(G_{\text{star}}) = 1.71$$



$$d(4, 6) = 4$$
$$\text{diam}(G_{\text{tree}}) = 4$$
$$L(G_{\text{tree}}) = 2.28$$

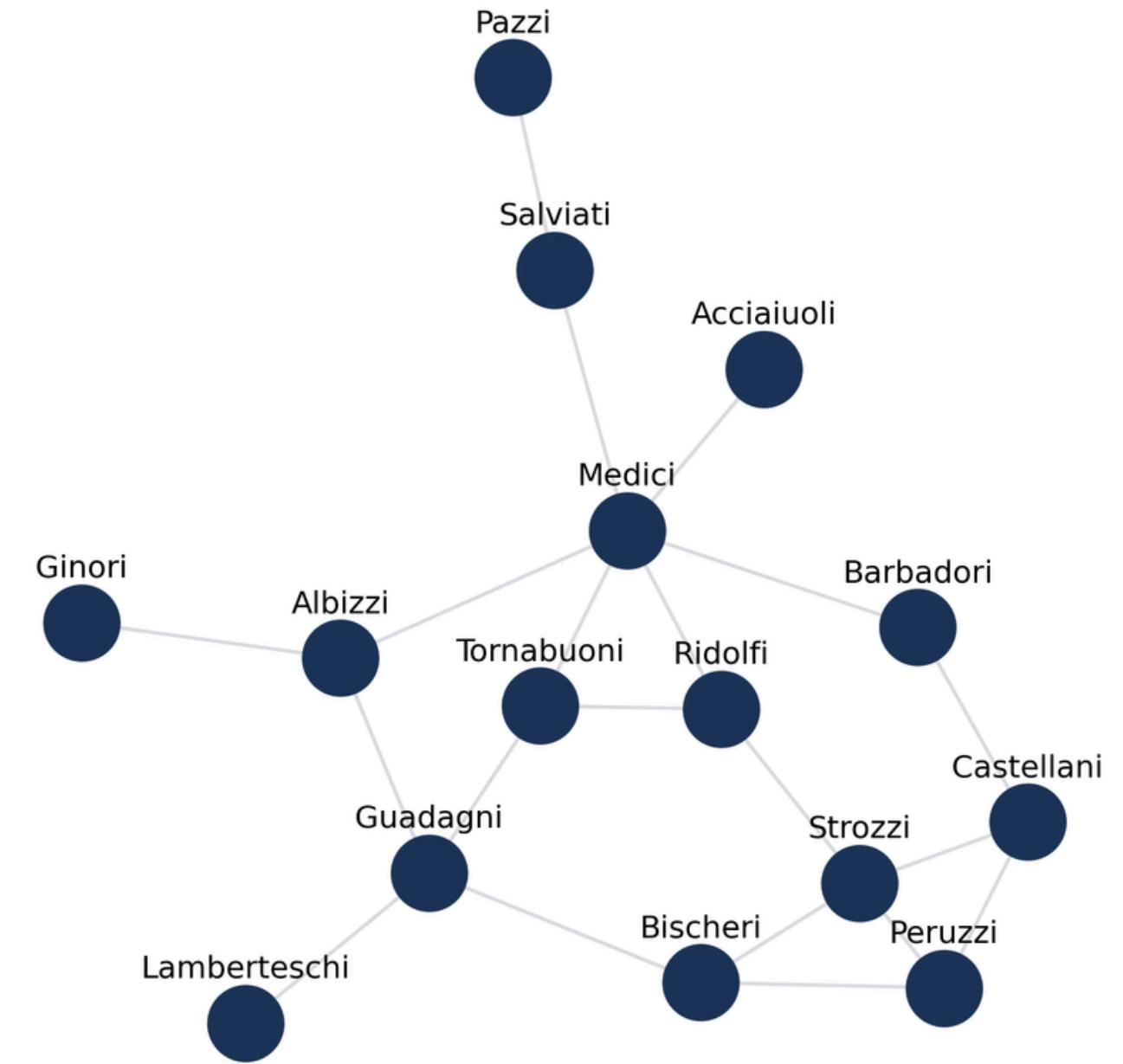


$$d(i, j) = 1, \text{ for any } i, j$$
$$\text{diam}(G_{\text{complete}}) = 1$$
$$L(G_{\text{complete}}) = 1$$

# FLORENTINE FAMILIES GRAPH

Diameter & average shortest path length

If the graph was a line with 15 elements, the diameter would be 14 and the average shortest path length would be 5.3.



$$\text{diam}(G_{\text{Florentine Families}}) = 5$$
$$L(G_{\text{Florentine Families}}) = 2.49$$

Another important factor is the clustering,  
i.e., how close a graph is to being a clique.

## **DEFINITION (CLUSTERING COEFFICIENT OF INDIVIDUAL NODES)**

The *clustering coefficient*  $Cl(i)$  of a node  $i$  is the fraction of pairs of its neighbors connected to each other:

$$Cl(i) = \frac{\text{number of edges between neighbors of } i}{\text{number of pairs of neighbors of } i} = \frac{\left| \left\{ \{j, k\} \subseteq N(i) \mid (j, k) \in E \right\} \right|}{\left| \left\{ \{j, k\} \subseteq N(i) \right\} \right|}$$

## DEFINITION (CLUSTERING COEFFICIENT OF INDIVIDUAL NODES)

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$$= \frac{\left| \left\{ \{j, k\} \subseteq N(i) \mid (j, k) \in E \right\} \right|}{\binom{d_i}{2}}$$

## DEFINITION (CLUSTERING COEFFICIENT OF INDIVIDUAL NODES)

The *clustering coefficient*  $Cl(i)$  of a node  $i$  is the fraction of pairs of its neighbors connected to each other:

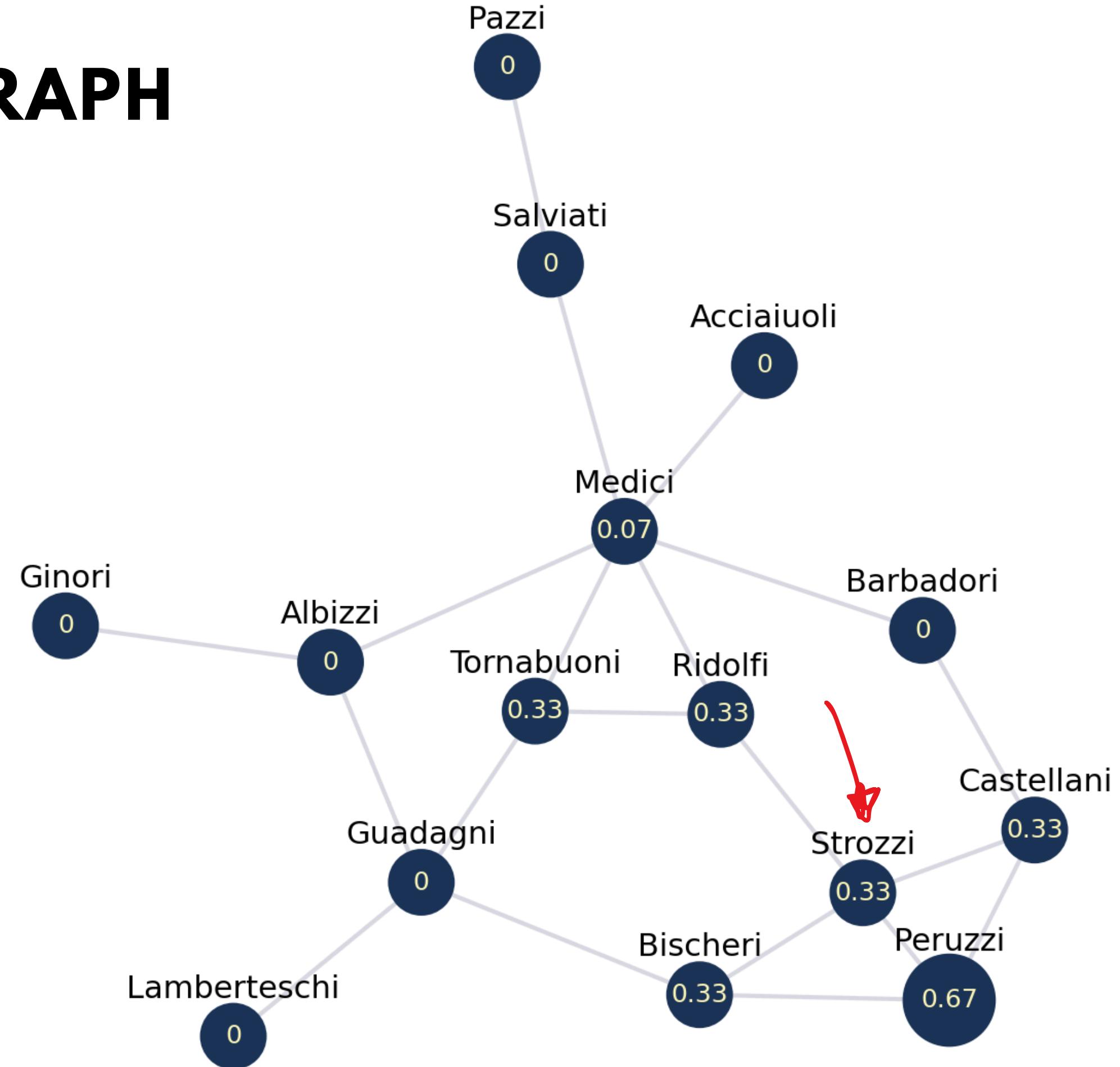
$$\begin{aligned} Cl(i) &= \frac{\text{number of edges between neighbors of } i}{\text{number of pairs of neighbors of } i} = \frac{\left| \left\{ \{j, k\} \subseteq N(i) \mid (j, k) \in E \right\} \right|}{\left| \left\{ \{j, k\} \subseteq N(i) \right\} \right|} \\ &= \frac{\left| \left\{ \{j, k\} \subseteq N(i) \mid (j, k) \in E \right\} \right|}{\binom{d_i}{2}} \\ &= 2 \cdot \frac{\left| \left\{ \{j, k\} \subseteq N(i) \mid (j, k) \in E \right\} \right|}{d_i(d_i - 1)}. \end{aligned}$$

# FLORENTINE FAMILIES GRAPH

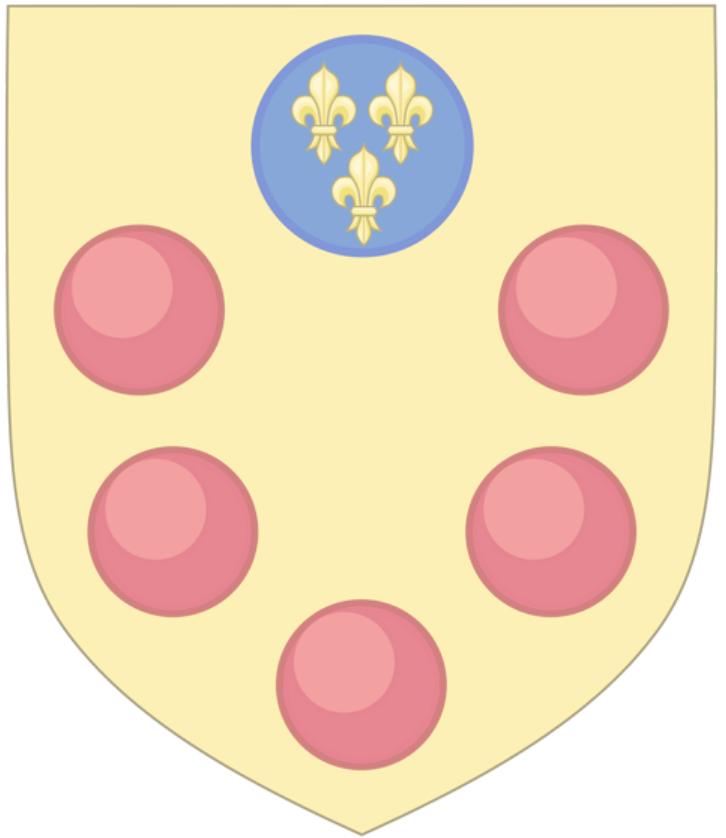
Clustering coefficients of nodes

The Strozzi have six pairs of neighbors, two of which are connected.

Their clustering coefficient is thus  $\frac{1}{3}$ .



## MEDICI

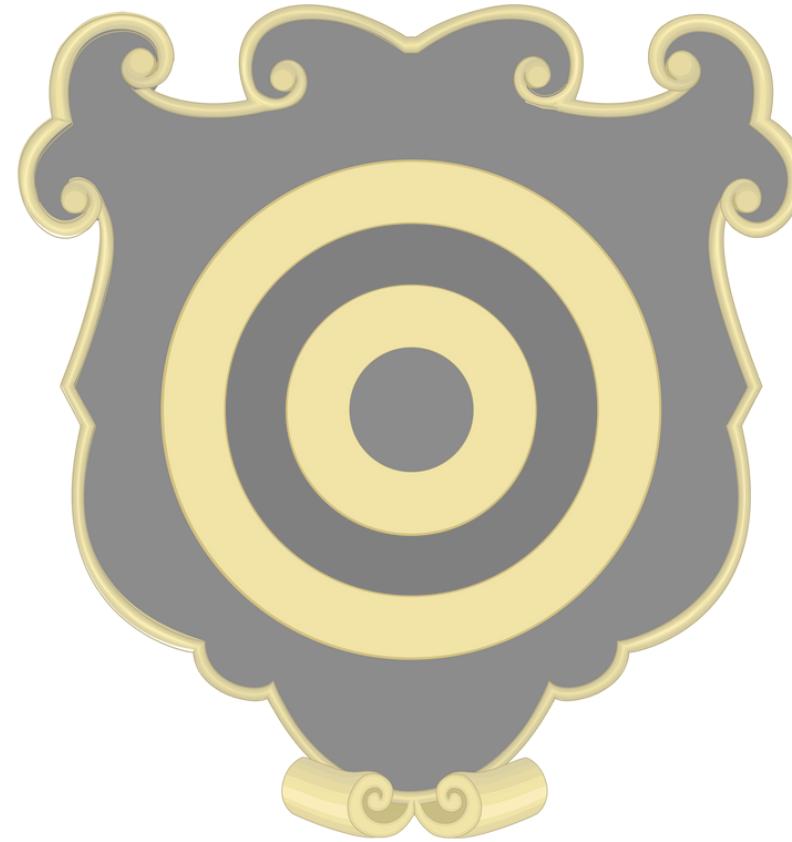


Originally from the farmlands north of Florence, in Mugello.

Become wealthy through banking.

Cosimo seen as 'champion of the new men'.

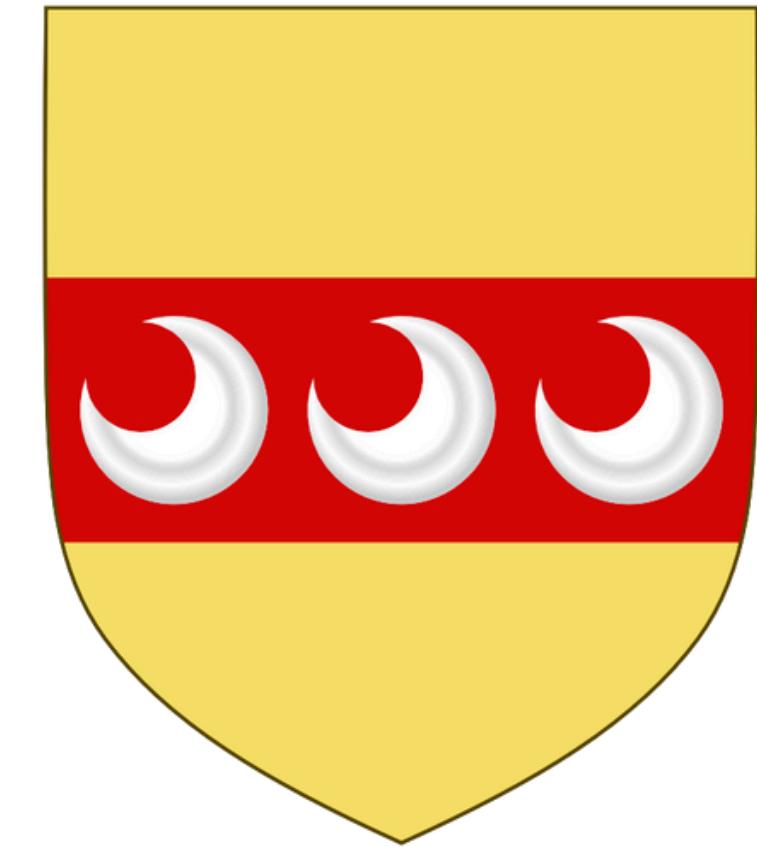
## ALBIZZI



Old-money wool-merchant dynasty.

Maso, and then Rinaldo degli Albizzi basically were the Florentine state until Cosimo showed them the door in 1434.

## STROZZI



Biggest balance-sheet in Florence until 1434.

Bankers, art patrons, ringleaders of one-year exile for Cosimo.

Line goes on, with living descendants to this day.



NATALIA GUICCIARDINI STROZZI

**The Strozzi's were enemies of the Medici family.**

The Medici's kept us in exile, and throughout the Renaissance the two clans were constantly involved in a power struggle.

Yet today we have portraits of the Medici in our house.

As a reminder that we're still here and they're not...

An interview with Natalia Guicciardini Strozzi. (2007, May 16). The Florentine.

## **DEFINITION (CLUSTERING COEFFICIENT OF NETWORK)**

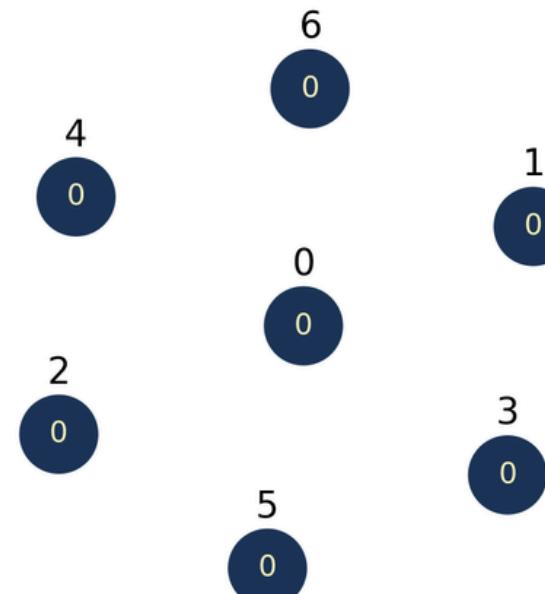
The *clustering coefficient*  $Cl(G)$  of network  $G$  is the average clustering coefficient of its nodes:

$$Cl(G) = \frac{1}{|V|} \sum_{i \in V} Cl(i).$$

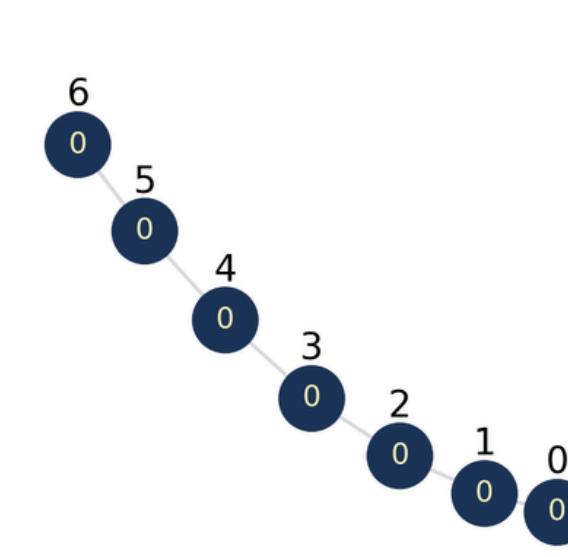
Note that the clustering coefficient is a number between 0 and 1.

# PATHS AND STARS AND TREES

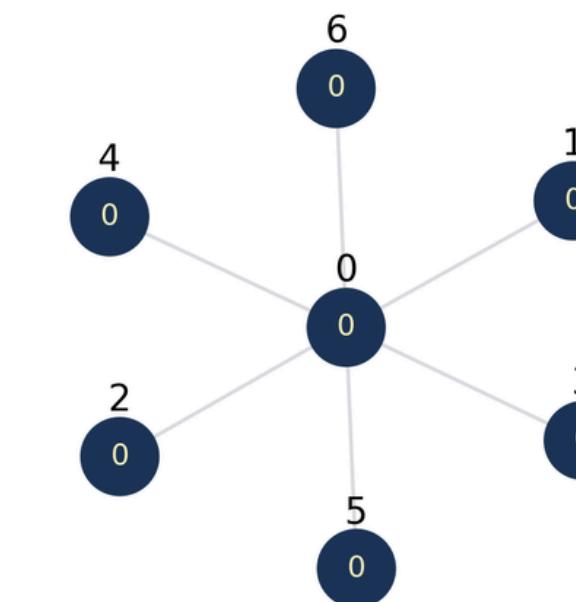
Average clustering



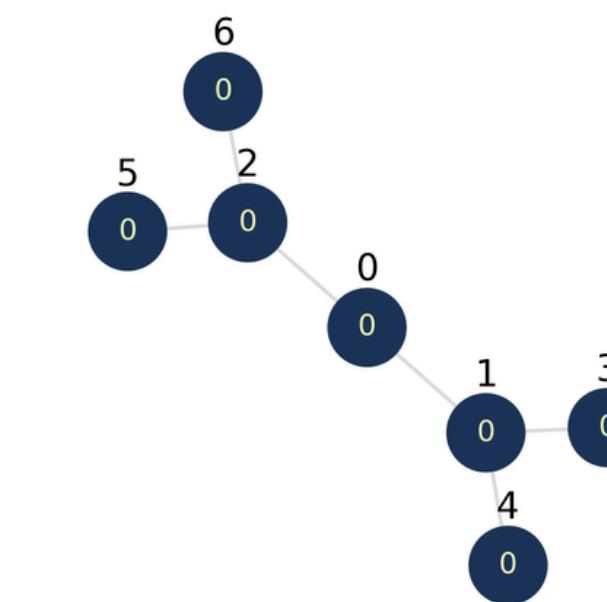
$$Cl(G_{\text{empty}}) = 0$$



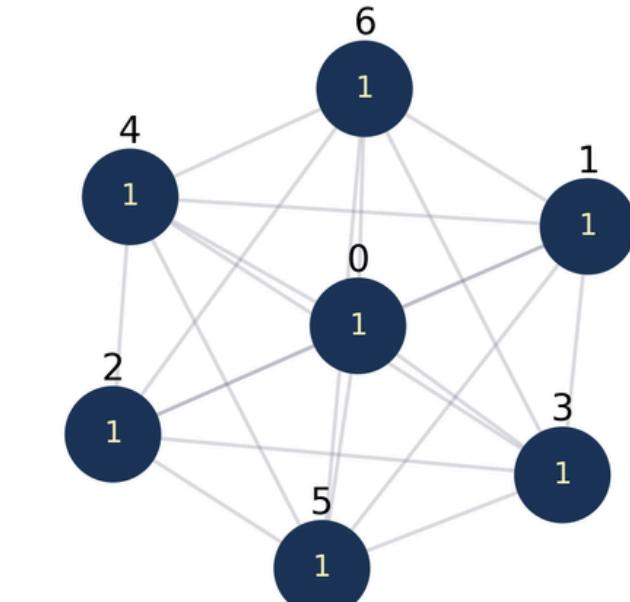
$$Cl(G_{\text{path}}) = 0$$



$$Cl(G_{\text{star}}) = 0$$



$$Cl(G_{\text{tree}}) = 0$$

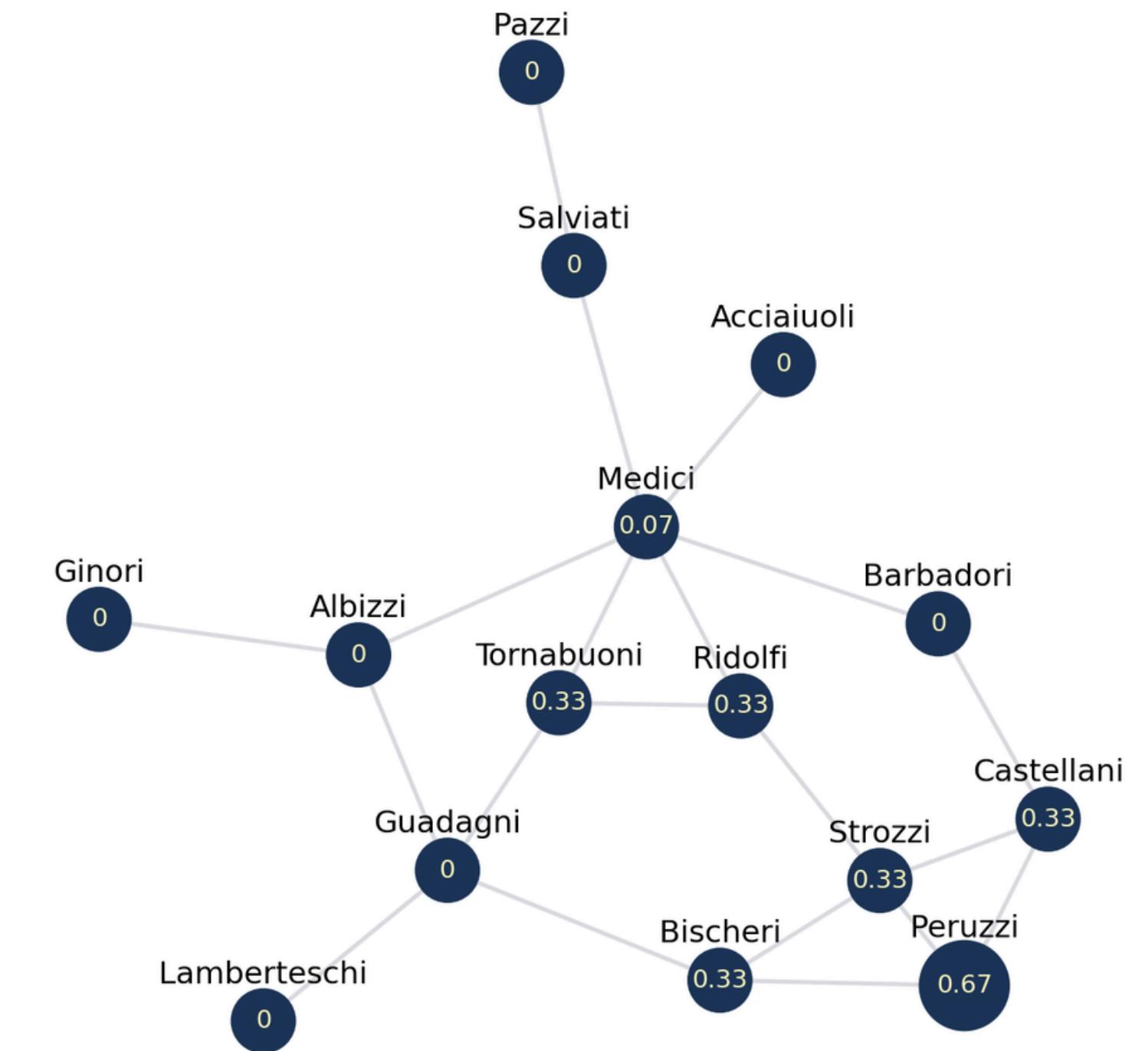


$$Cl(G_{\text{complete}}) = 1$$

# FLORENTINE FAMILIES GRAPH

Clustering coefficient

Average clustering for Florentine families.



$$Cl(G_{\text{Florentine-Families}}) = 0.16$$

An important class of networks are *small-world* networks.

## **DEFINITION (SMALL WORLD NETWORK)**

A *small-world network* is a type of network characterized by a *high clustering coefficient* and *short average path length*:

$$L(G) \sim \log(|V|),$$

i.e., the average distance between any two nodes is proportional to the logarithm of the number of nodes.

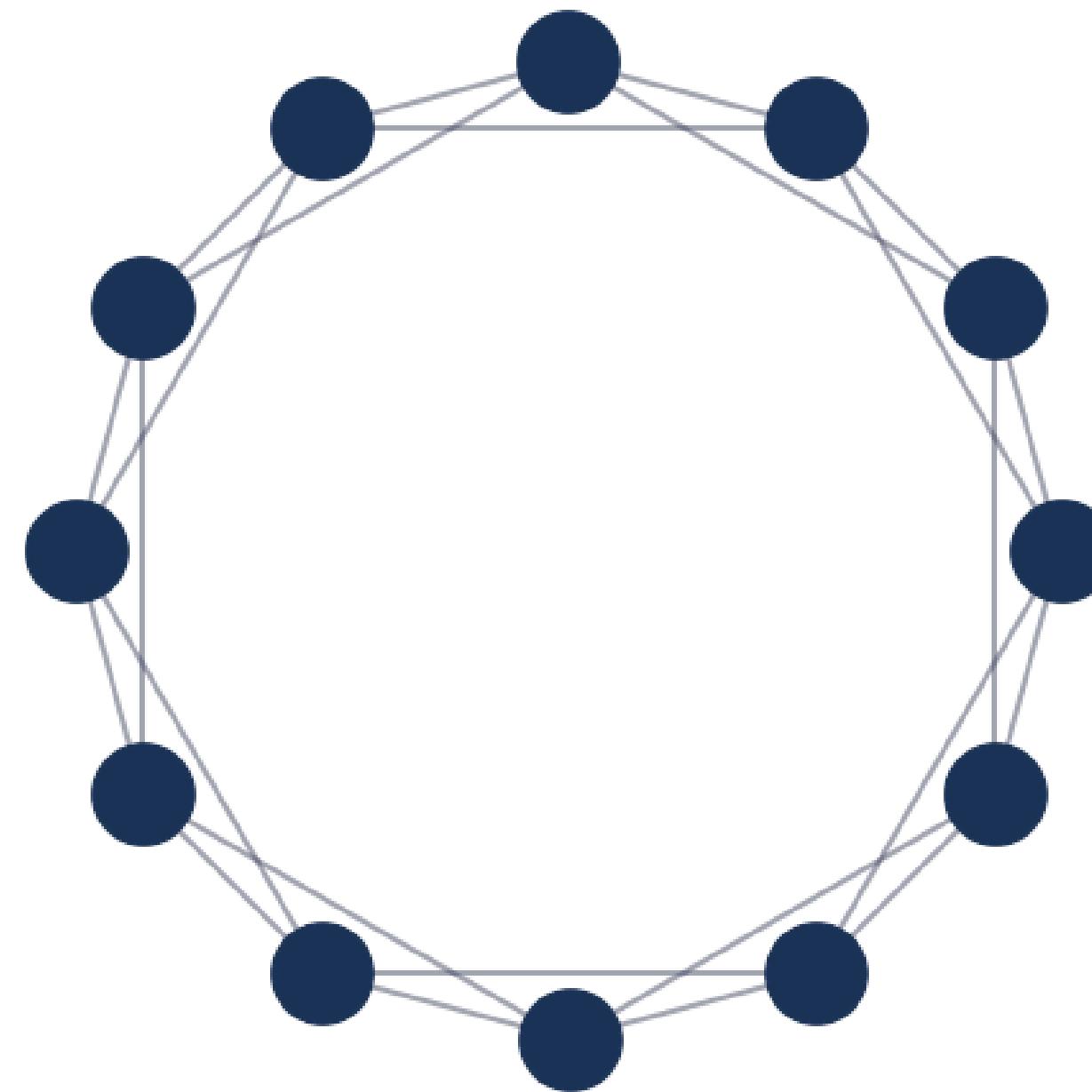
## **DEFINITION (WATTS-STROGATZ MODEL)**

Start with a graph where each node is connected to exactly  $k$  other nodes. Then, at every time step, randomly rewire each edge with probability  $p$  to an unattached node.

# WATTS-STROGATZ MODEL

Intuition

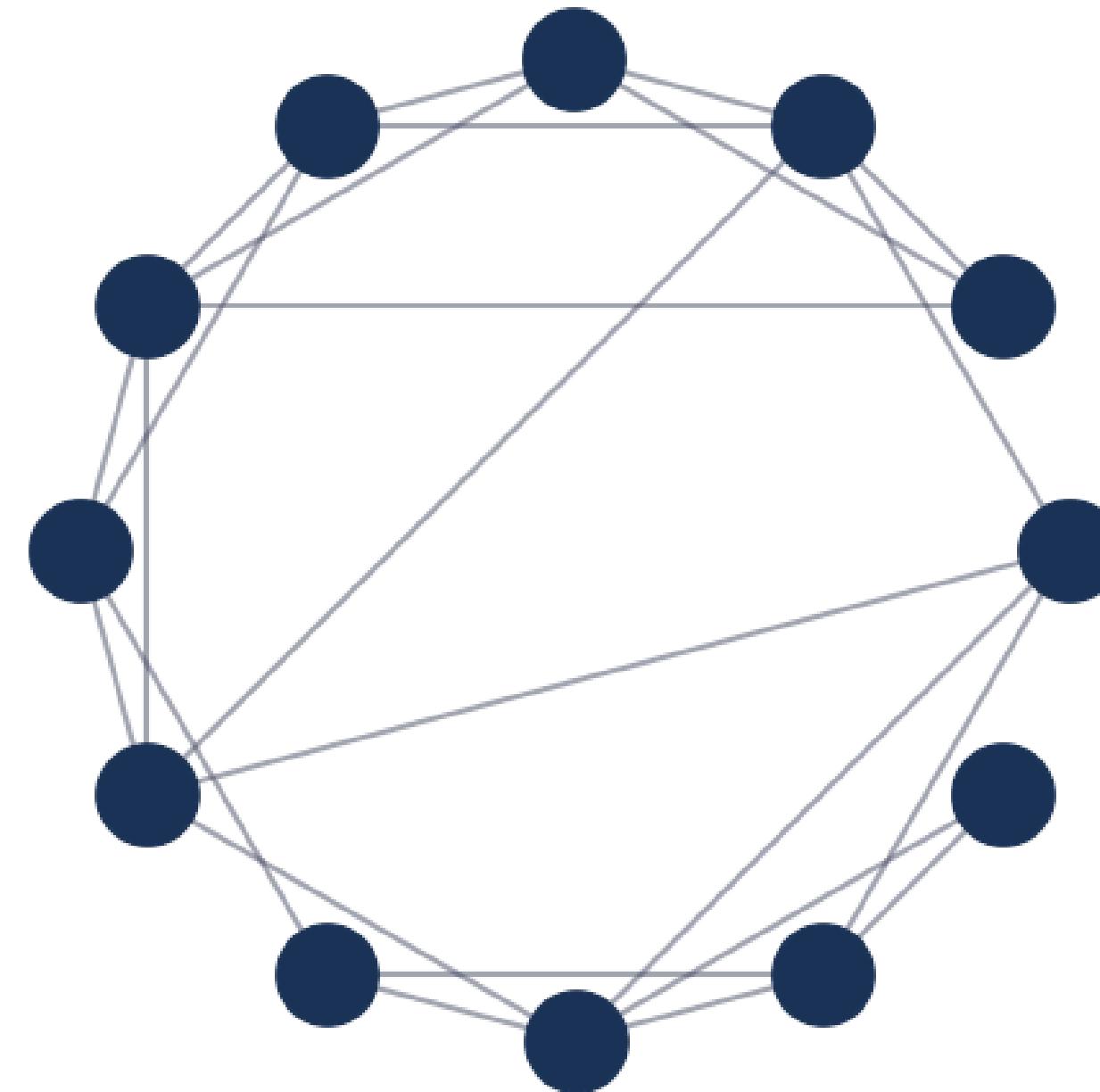
Take  $k = 4$  and  $p = 0.2$ .



# WATTS-STROGATZ MODEL

Intuition

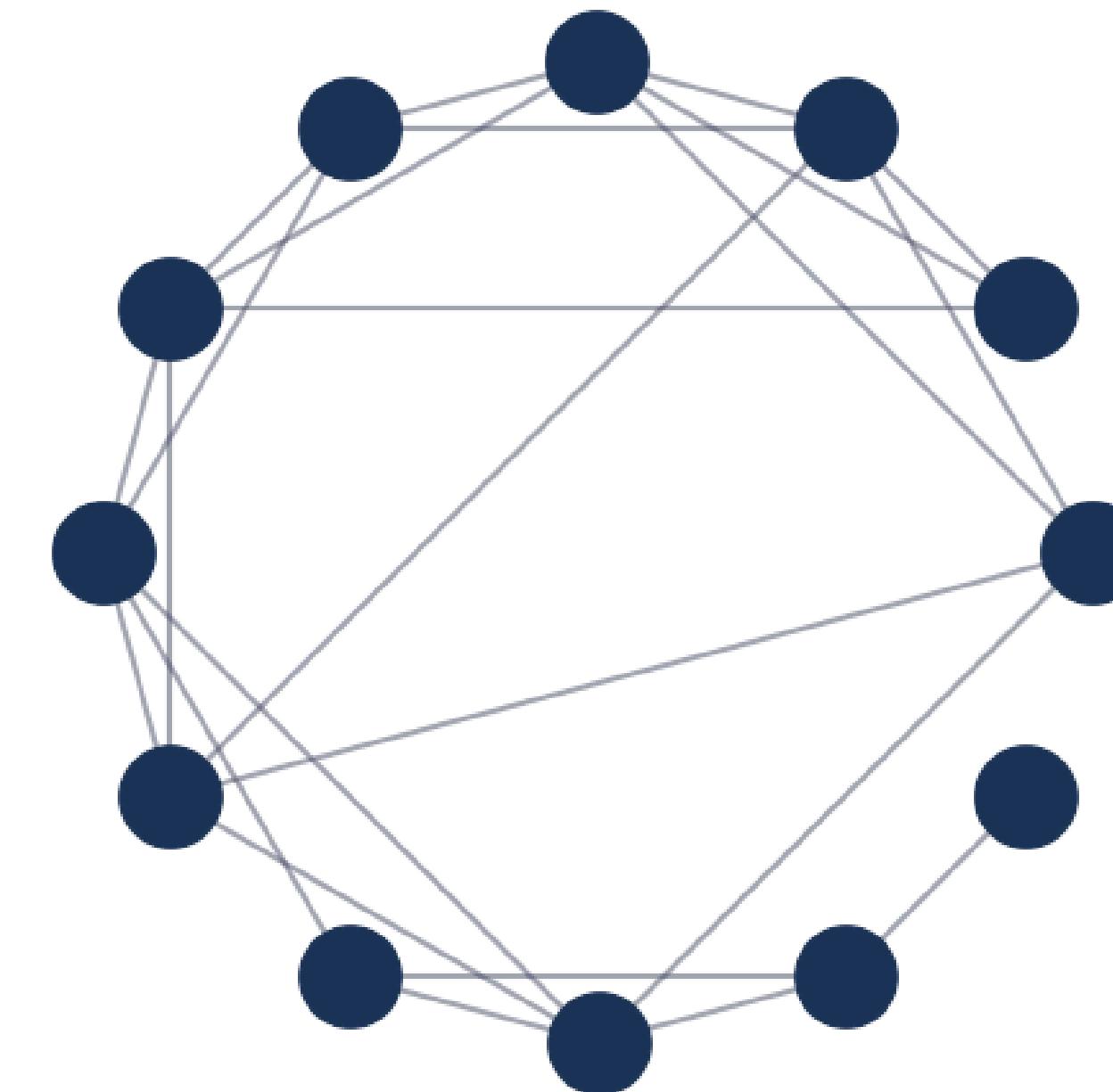
Take  $k = 4$  and  $p = 0.2$ .



# WATTS-STROGATZ MODEL

Intuition

Take  $k = 4$  and  $p = 0.2$ .



# WATTS-STROGATZ MODEL

## Intuition

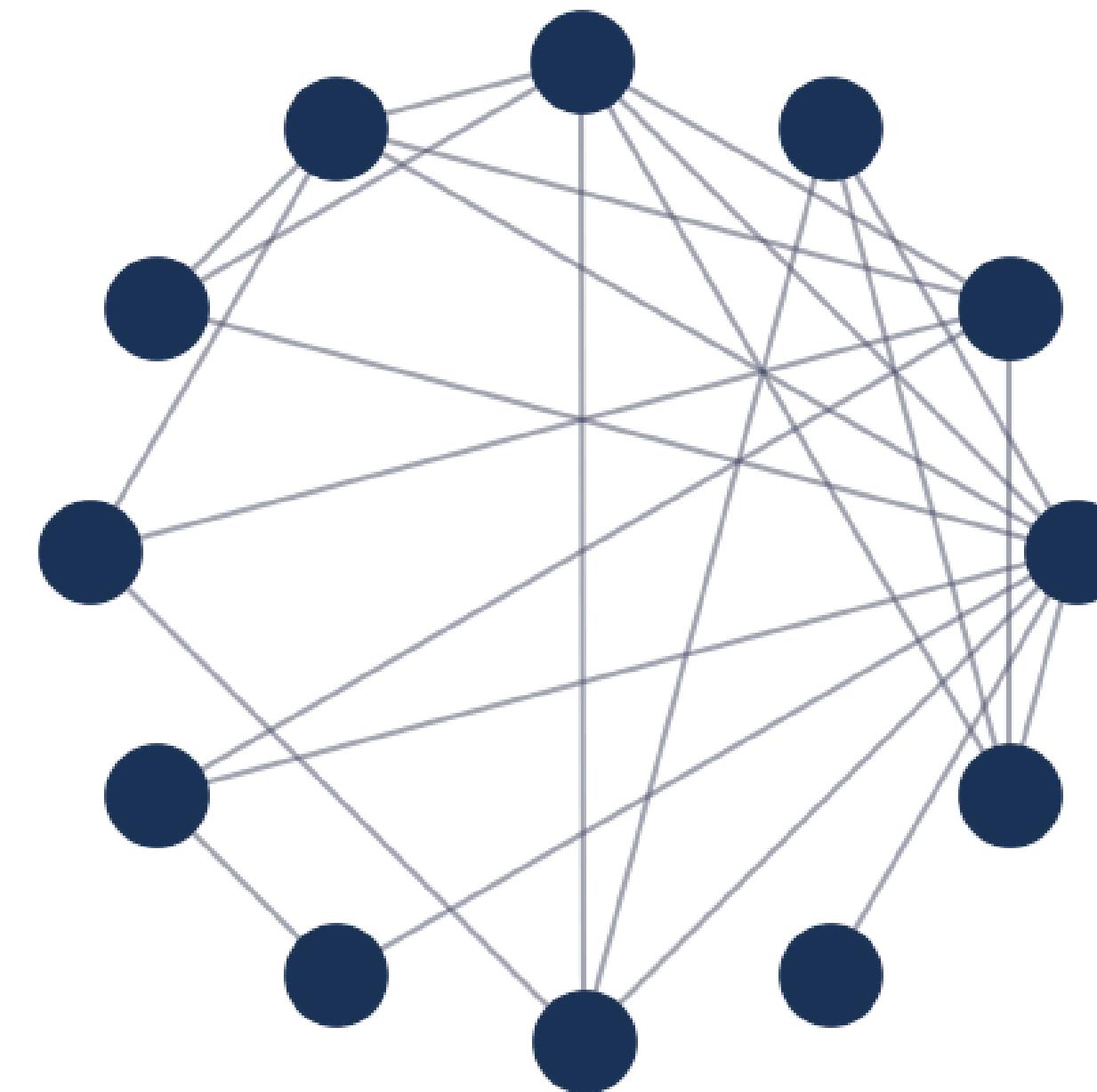
Take  $k = 4$  and  $p = 0.2$ .



# WATTS-STROGATZ MODEL

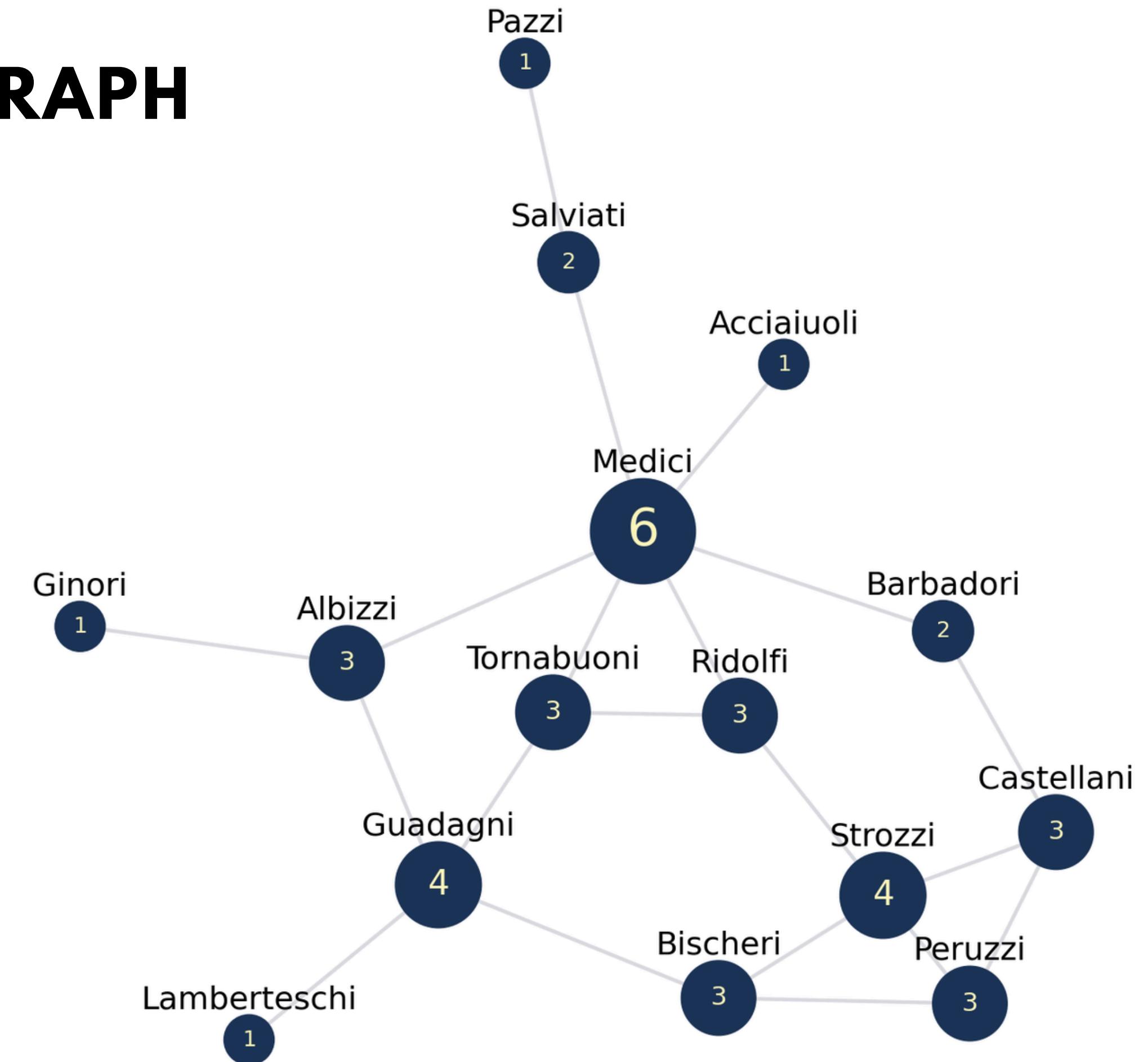
Intuition

Take  $k = 4$  and  $p = 0.2$ .



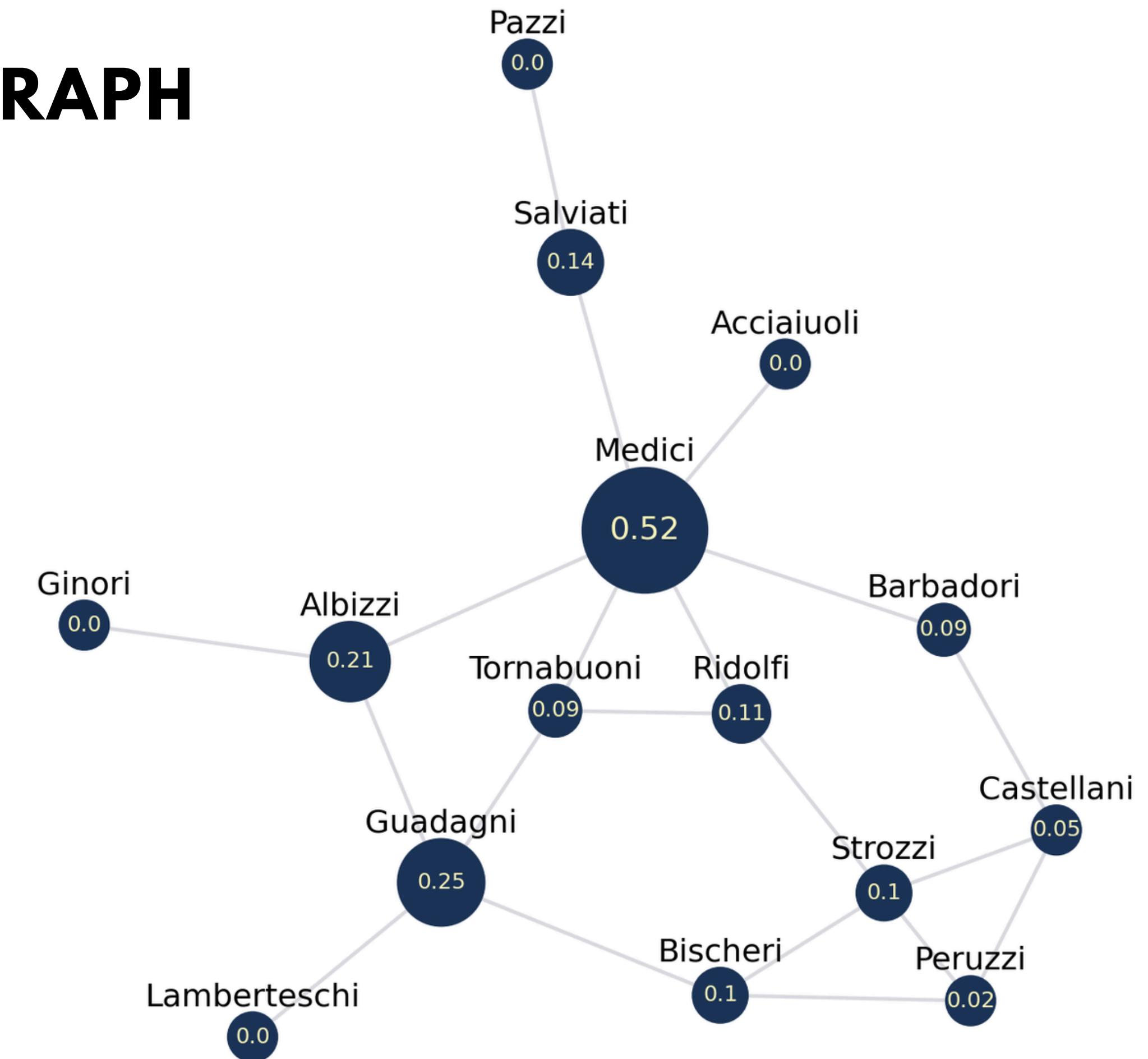
# FLORENTINE FAMILIES GRAPH

Degree centralities



# FLORENTINE FAMILIES GRAPH

Betweenness centralities



# FLORENTINE FAMILIES GRAPH

Eigenvector centralities

