LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

INFORMED CHOICES, INCLUSIVE VOICES: EPISTEMIC JOURNEYS IN DEMOCRATIC DECISION MAKING

LEARNING FROM OTHERS

SOCIAL LEARNING AND INFORMATION CASCADES

Adrian Haret
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May 6, 2024

Quiz time!

GDP per capita of Germany, according to recent estimates?

- ☐ 64,600
- ☐ 54,000
- ☐ 44,300

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GDP per capita, Purchasing Power Parity, 2022: The average for 2022 based on 19 countries was 34945 U.S. dollars. The highest value was in the USA: 64623 U.S. dollars and the lowest value was in India: 7112 U.S. dollars. The indicator is available from 1990 to 2022. Below is a chart for all countries where data are available.

Measure: U.S. dollars; Source: The World Bank

G20 Download data API

Countries ▲▼	GDP per capita, PPP, 2022 ▲▼	Global rank ▲▼	Available data ▲▼
USA	64623	1	1990 - 2022
Germany	53970	2	1990 - 2022
Australia	51090	3	1990 - 2022
Saudi Arabia	50188	4	1990 - 2022
Canada	49296	5	1990 - 2022
UK	47587	6	1990 - 2022
France	45904	7	1990 - 2022
South Korea	45560	8	1990 - 2022
Italy	44292	9	1990 - 2022
Japan	41838	10	1990 - 2022
Turkey	33150	11	1990 - 2022
Russia	27450	12	1990 - 2022
Argentina	22461	13	1990 - 2022
Mexico	20255	14	1990 - 2022
China	18188	15	1990 - 2022
Brazil	15093	16	1990 - 2022
South Africa	13479	17	1990 - 2022
Indonesia	12410	18	1990 - 2022
India	7112	19	1990 - 2022

Can social factors make things go awry with collective beliefs?

ODORIC OF PORDENONE

In a province of the Grand Can there grow gourds, which, when they are ripe, open, and within them is found a little beast like unto a young lamb…

Odoric of Pordenone [*trans.* Sir Henry Yule] (2002). *The Travels of Friar Odoric.* W.B. Eerdmans Publishing Company.

AD 1330

SIR JOHN MANDEVILLE

In Tartary groweth a manner of fruit, as though it were gourds. And when they be ripe, men cut them a-two, and men find within a little beast, in flesh, in bone, and blood, as though it were a little lamb without wool. And men eat both the fruit and the beast. And that is a great marvel.

Of that fruit I have eaten…

and found it wondirfulle.

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AD 1357 - 1371

AD 1515 - 1553

BARON SIGISMUND VON HERBERSTEIN

[…] a certain seed like that of a melon, but rather rounder and longer, from which, when it was set in the earth, grew a plant resembling a lamb, and attaining to a height of about two and a half feet…

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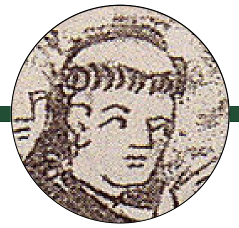
CLAUDE DURET

Duret, C. (1605). *Histoire Admirable des Plantes.*

AD 1605



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AD 1641

ATHANASIUS KIRCHER

[...] we assert that it is a plant. Though its form be that of a quadruped, and the juice beneath its woolly covering be blood which flows if an incision be made in its flesh, these things will not move us. It will be found to be a plant.

Kircher, A. (1641). Magnes; sive de arte magneticâ opus tripartitum.

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AD 1641

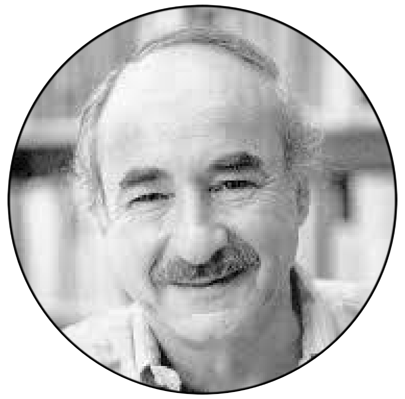
ENGELBERT KAEMPFER

I have searched ad risum et nauseam for this zoophyte feeding on grass, but have found nothing.

Kaempfer, E. (1712). Amœnitatum Exoticarum politico-physico-medicarum fascicul.

AD 1683

Let's model this.



MORRIS DEGROOT

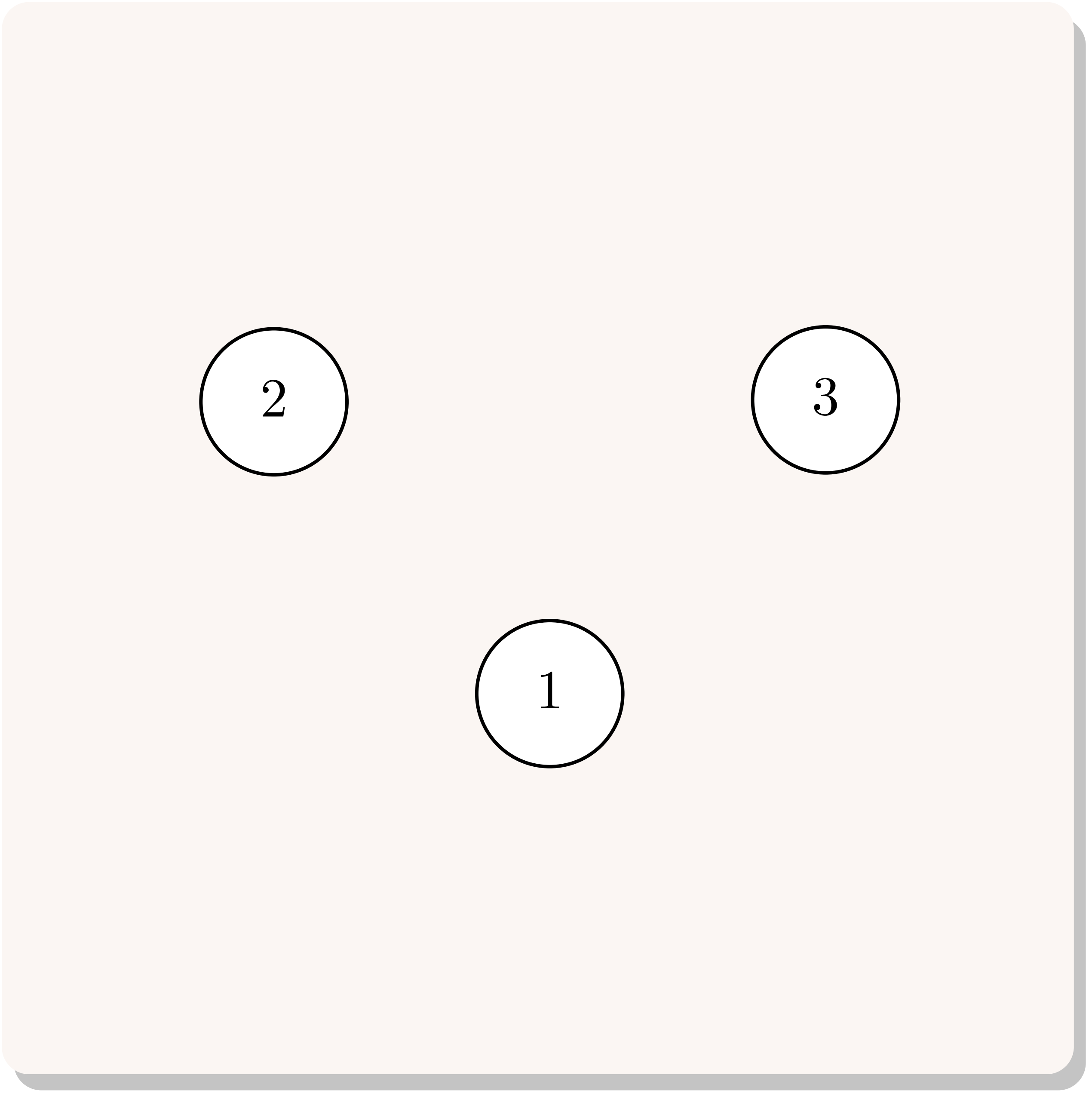
Agents are represented by nodes in a social network, and update their opinions depending on the opinions of their peers.

THE DEGROOT MODEL

agents	$1, 2, \ldots, n$
time	$t \in \{0, 1, 2, \ldots\}$
belief of agent i at t	number between 0 and 1
social network	directed graph with agents as vertices, and who-pays-attention-to-who as edges
agent i's neighborhood	agents that i pays attention to
weight on edge from i to j	number that indicates how much weight i places on j's opinion; we assume i distributes a total weight of 1 across i's neighborhood
update rule	at time $t + 1$ every agent updates their belief to a weighted average over the beliefs of neighbors

Agents

$1, 2, 3$

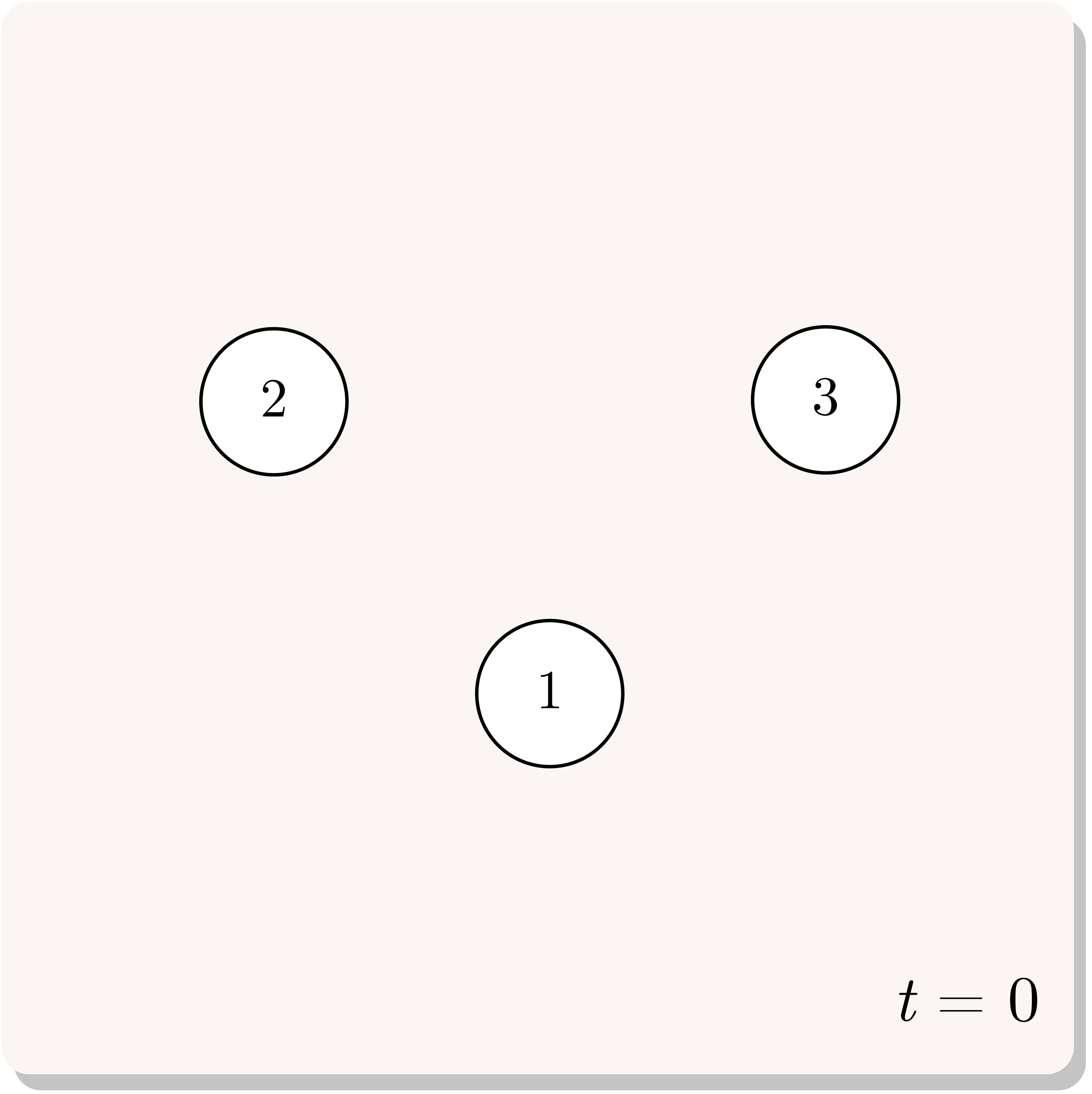


Agents

1, 2, 3

Time

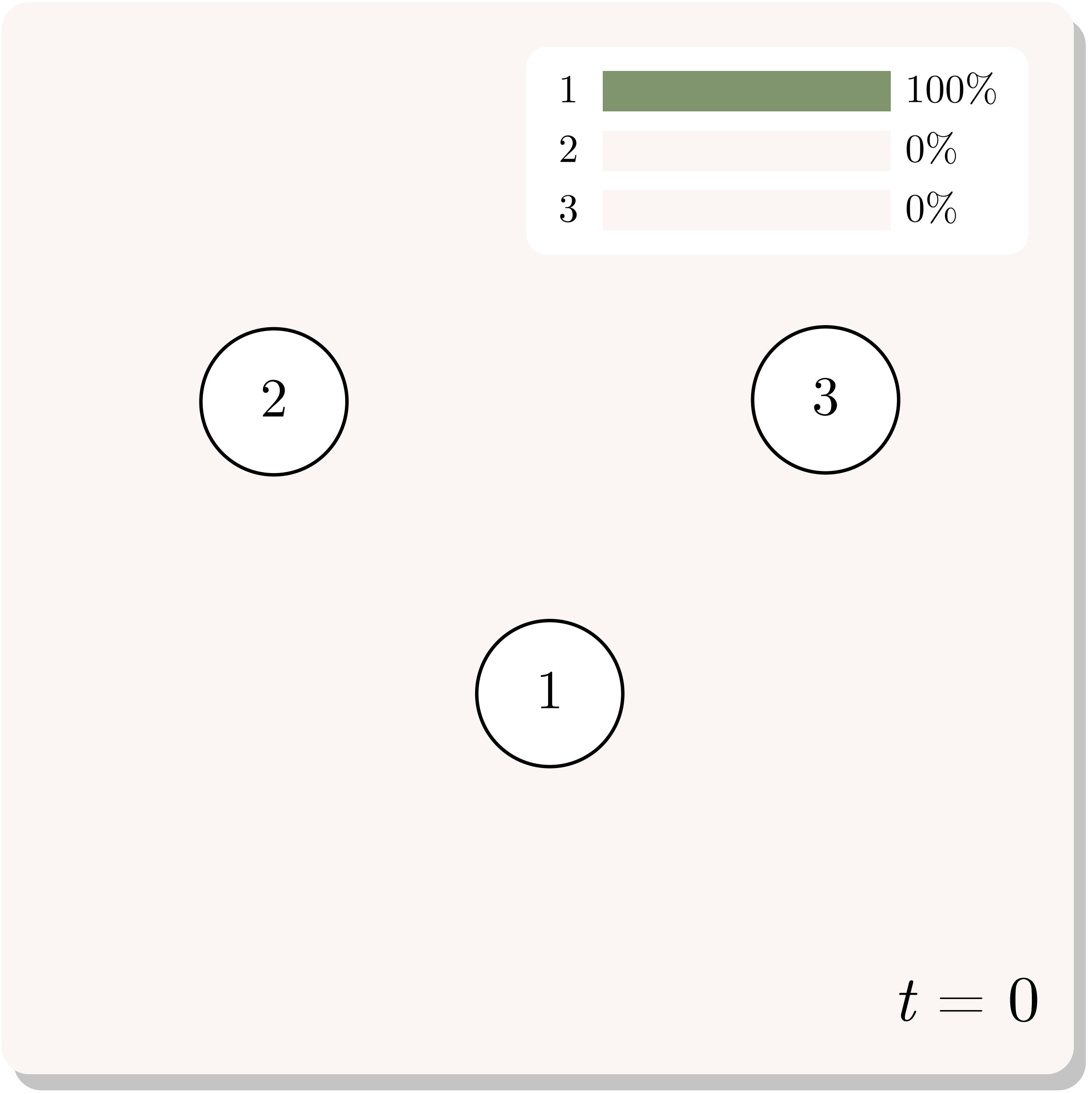
$t = 0, 1, 2, \ldots$



Agents
1, 2, 3

Time
$t = 0, 1, 2, \ldots$

Beliefs
agent 1 starts out at 0, agent 2 starts out at 0.6,
agent 3 starts out at 1



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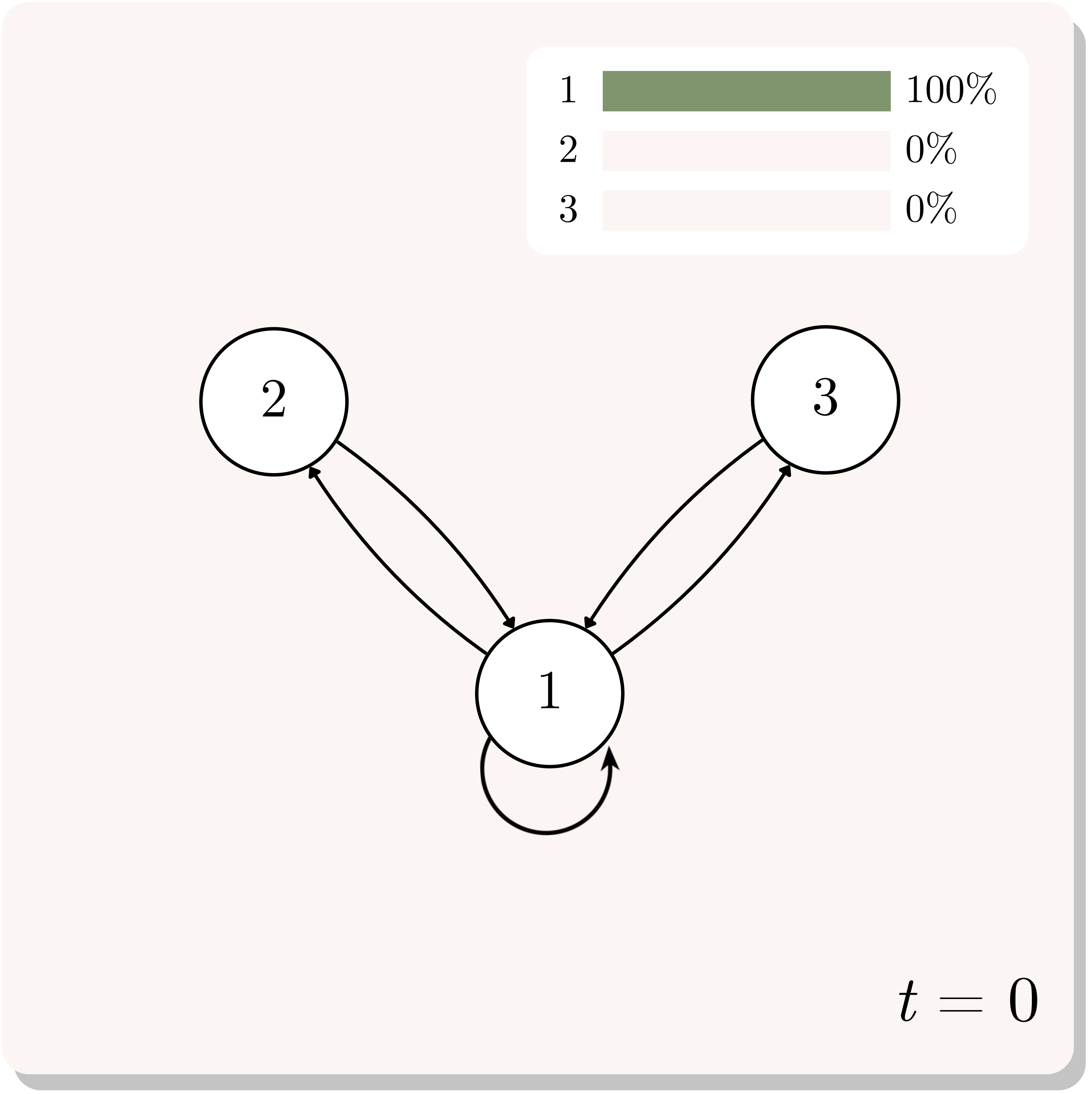
Social network
1 pays attention to everyone, 2 and 3 pay attention only to 1

Neighborhoods
1's neighborhood is {1, 2, 3}, etc.



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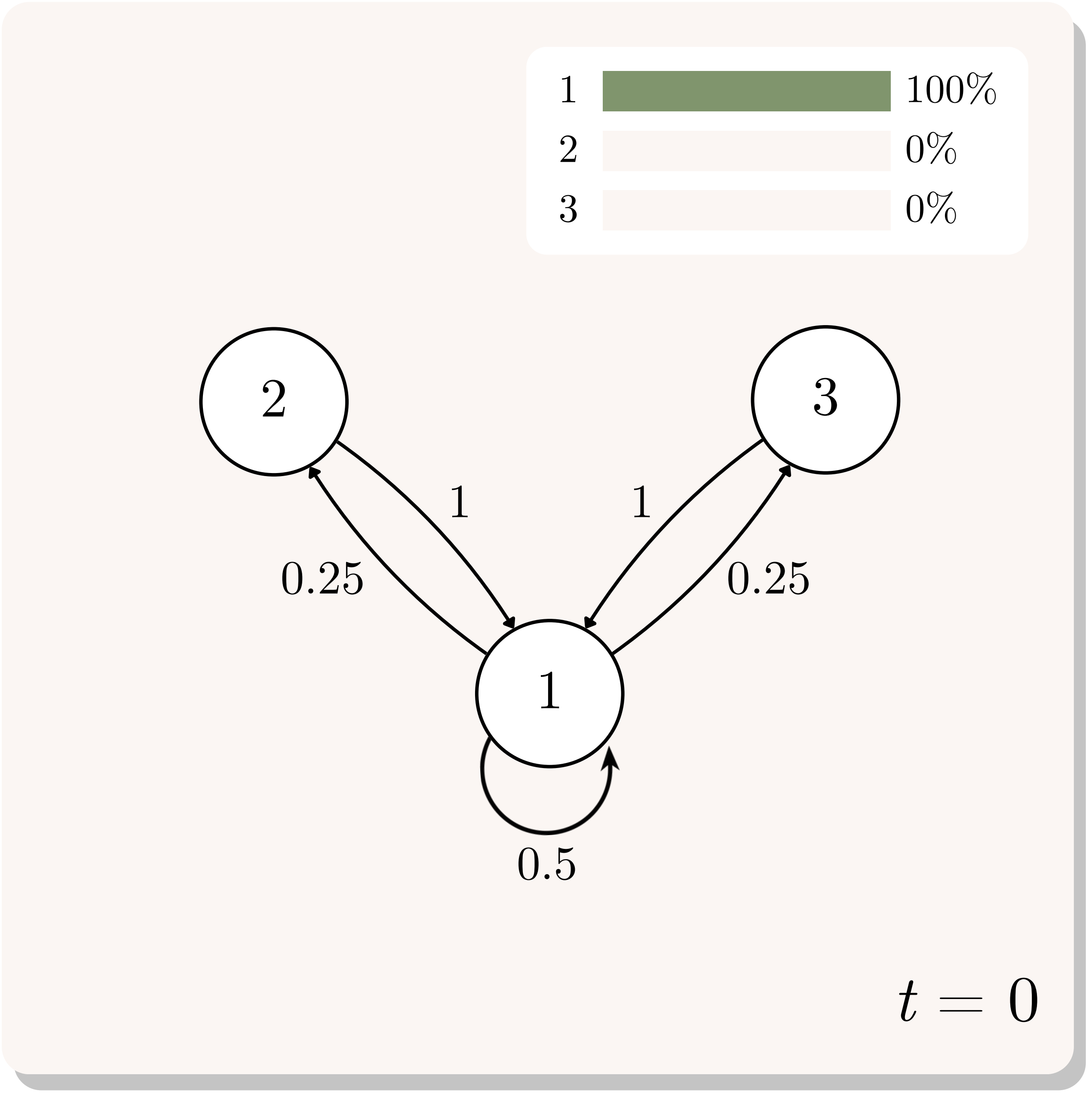
1 places 0.5 weight on themselves and 0.25 on 2 and 3 each, etc.



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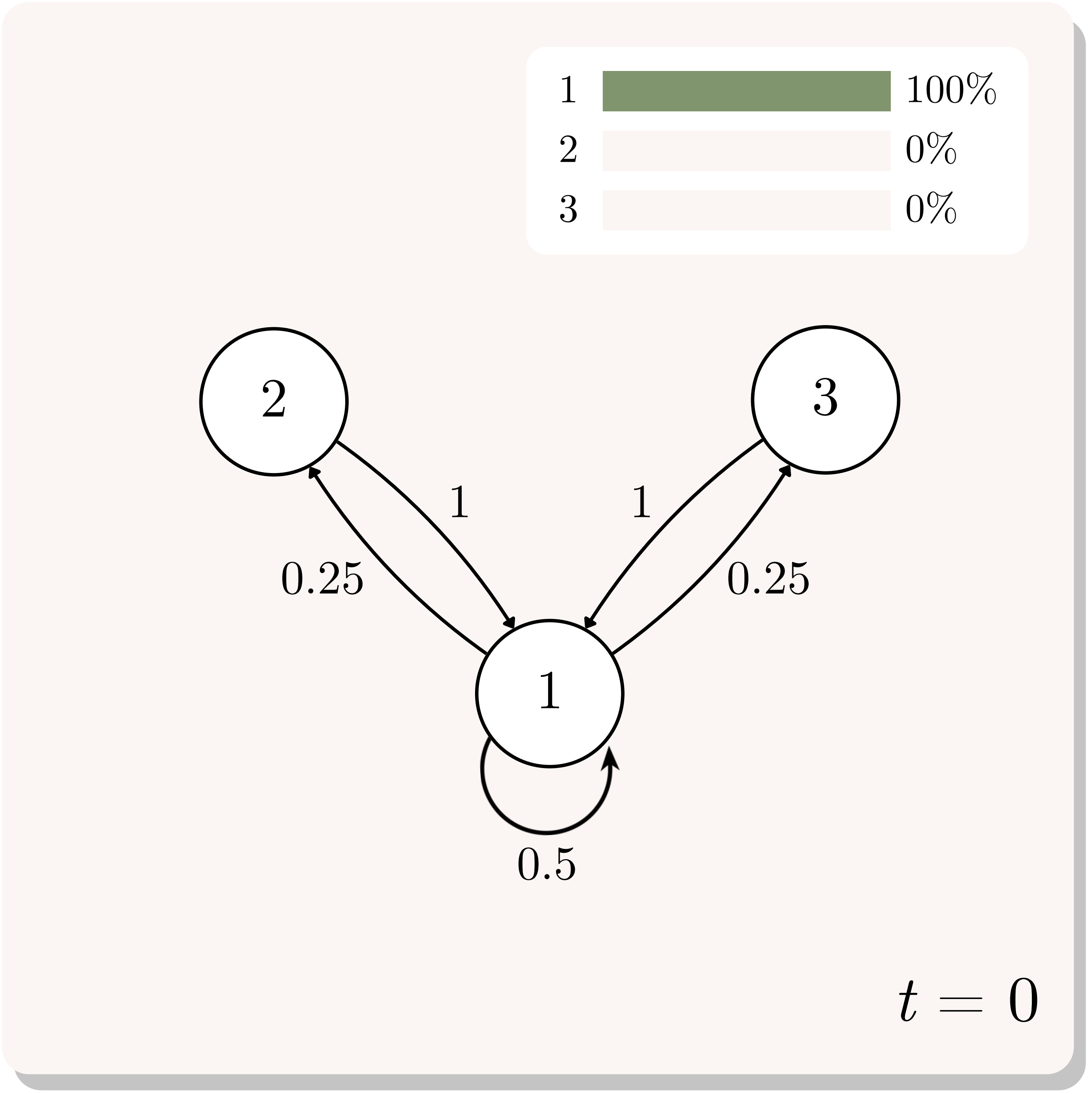
$$0.5 \cdot 1 + 0.25 \cdot 0 + 0.25 \cdot 0 = 0.5$$



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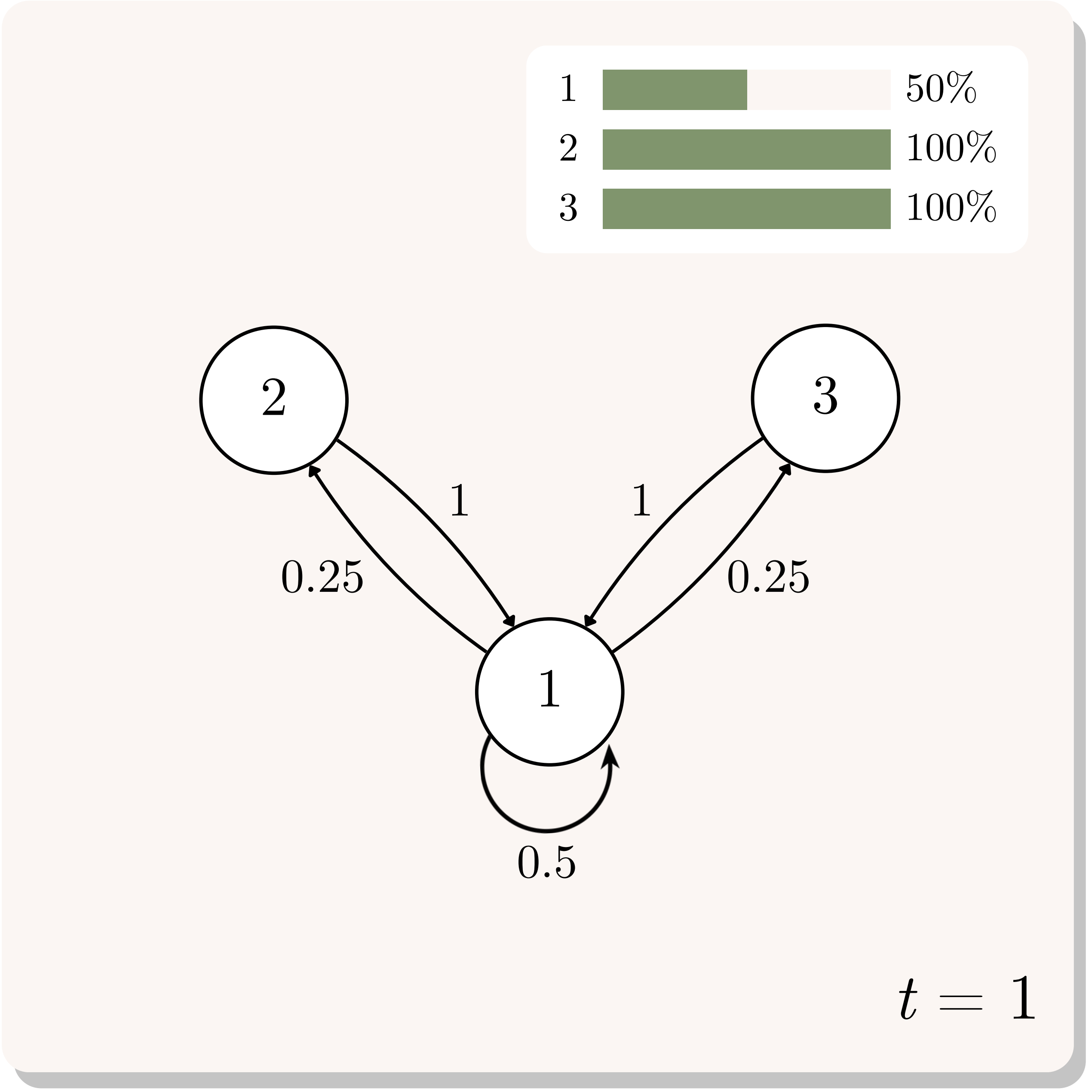

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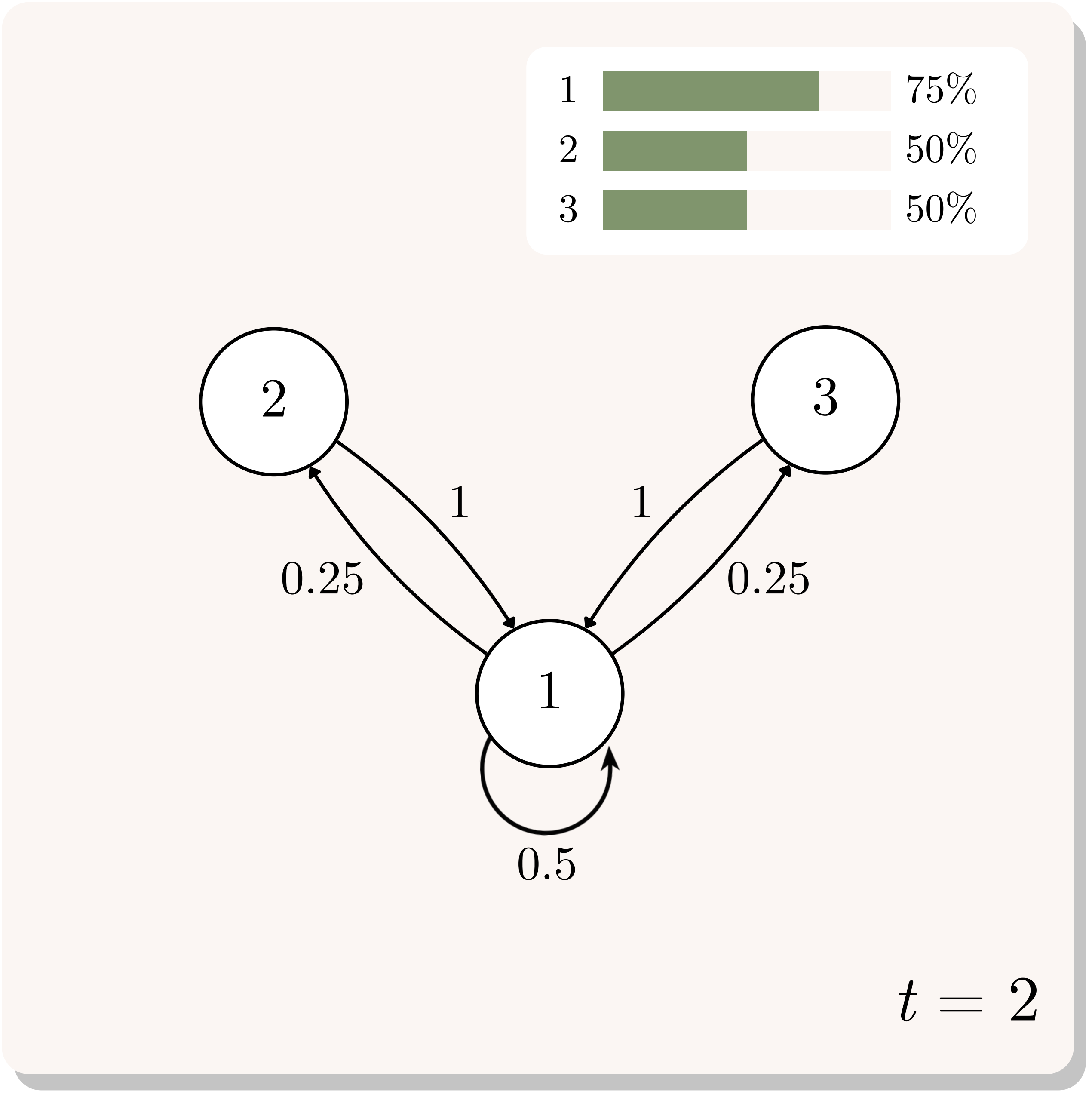

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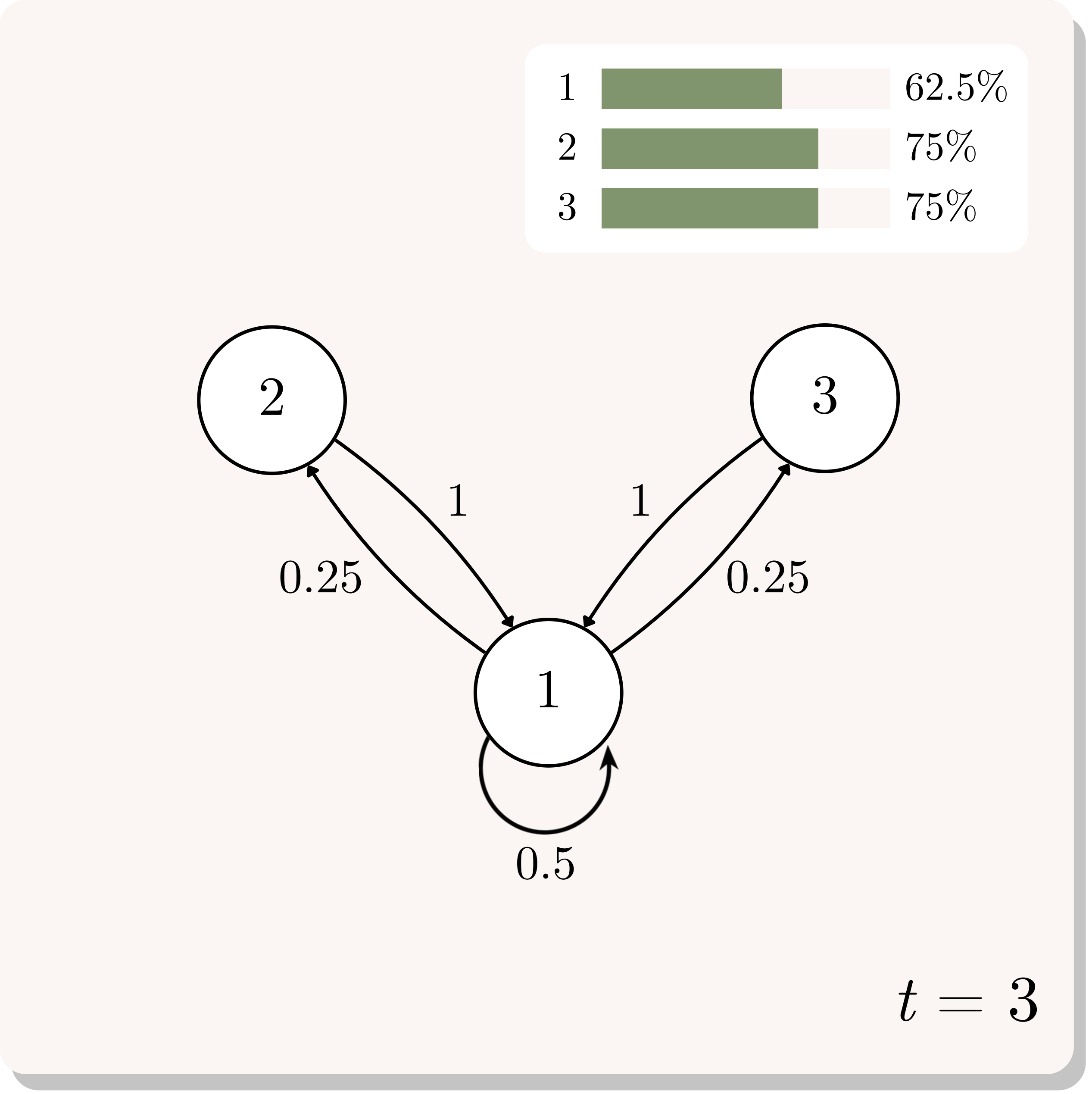

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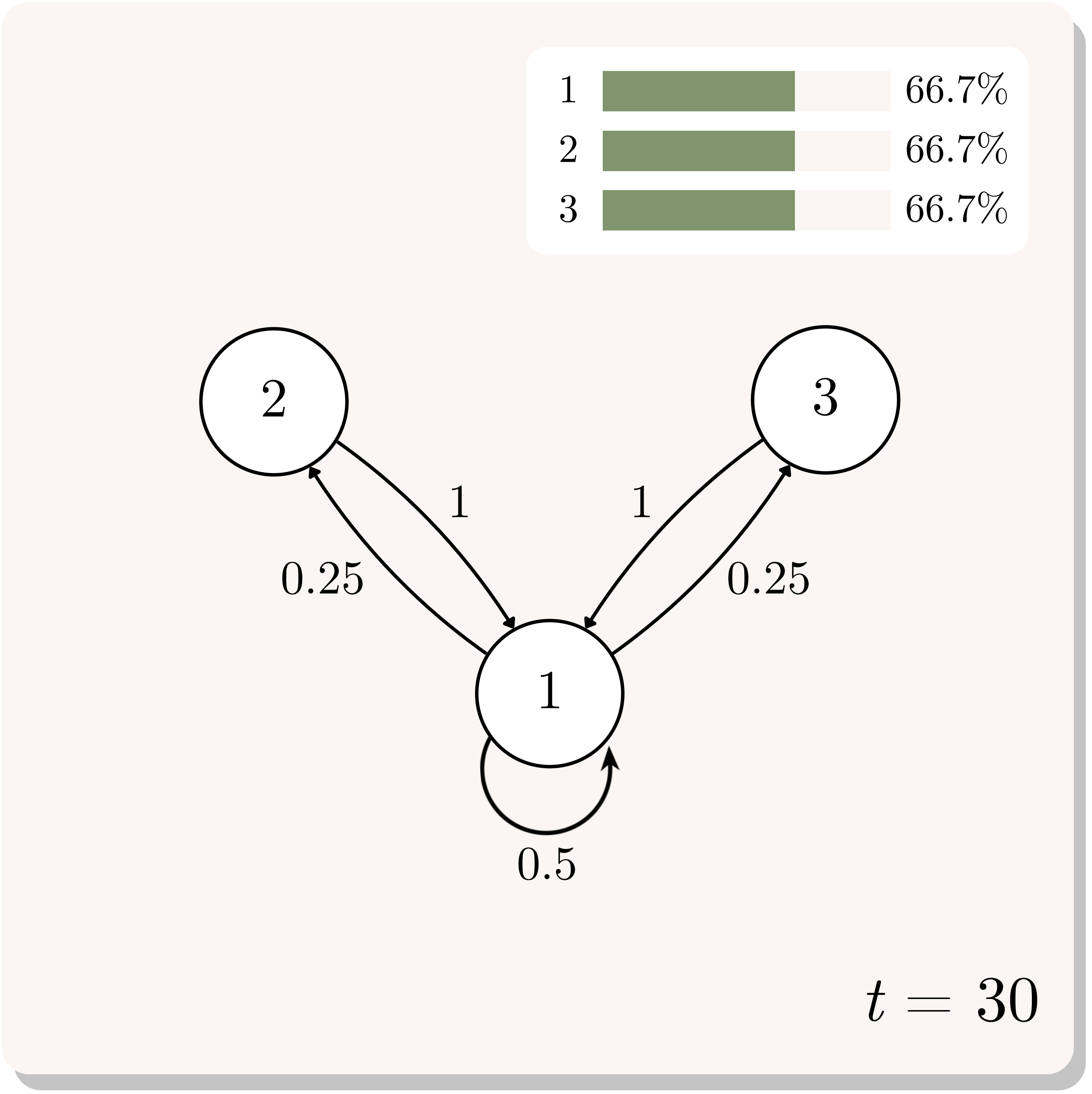

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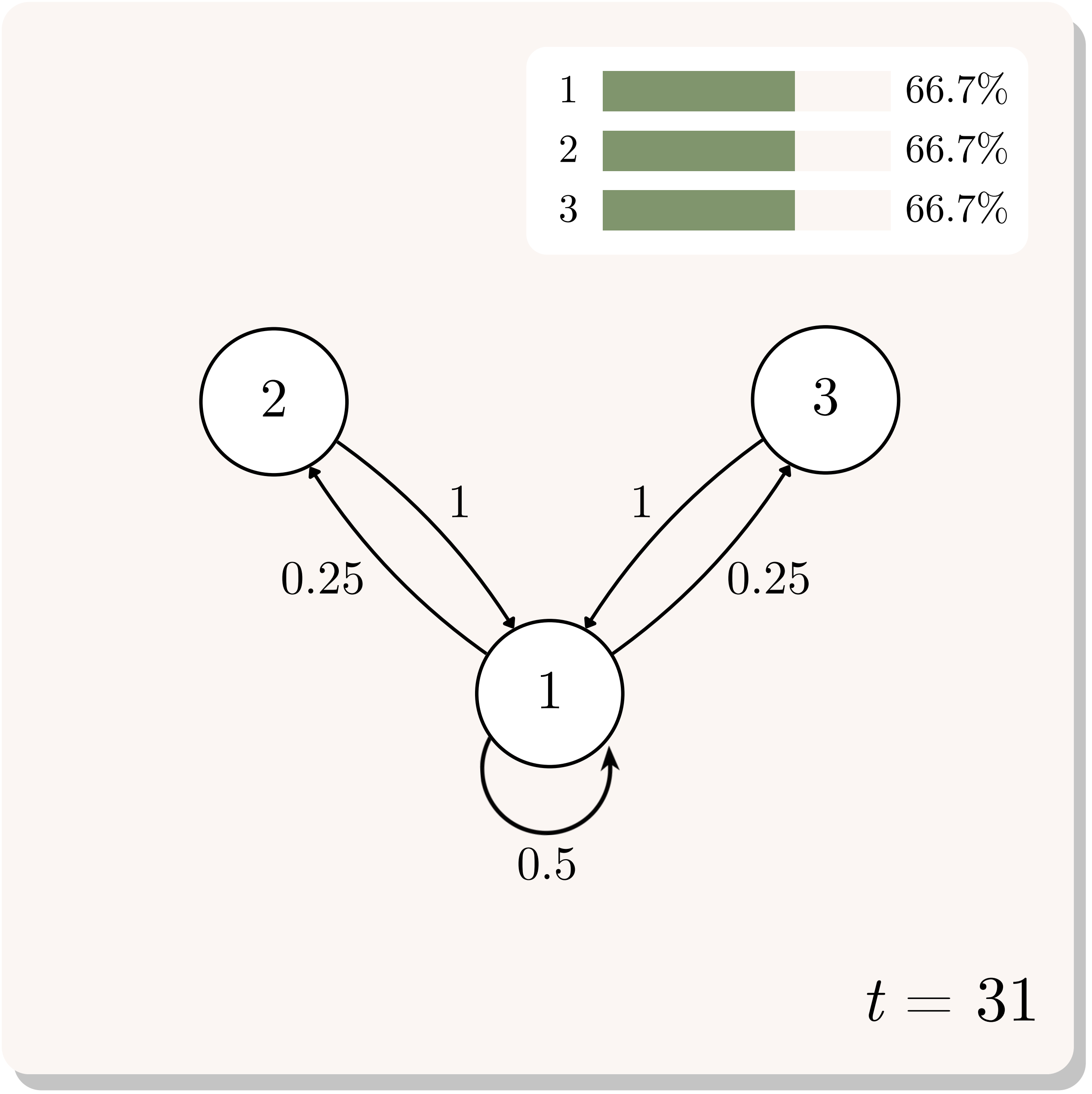

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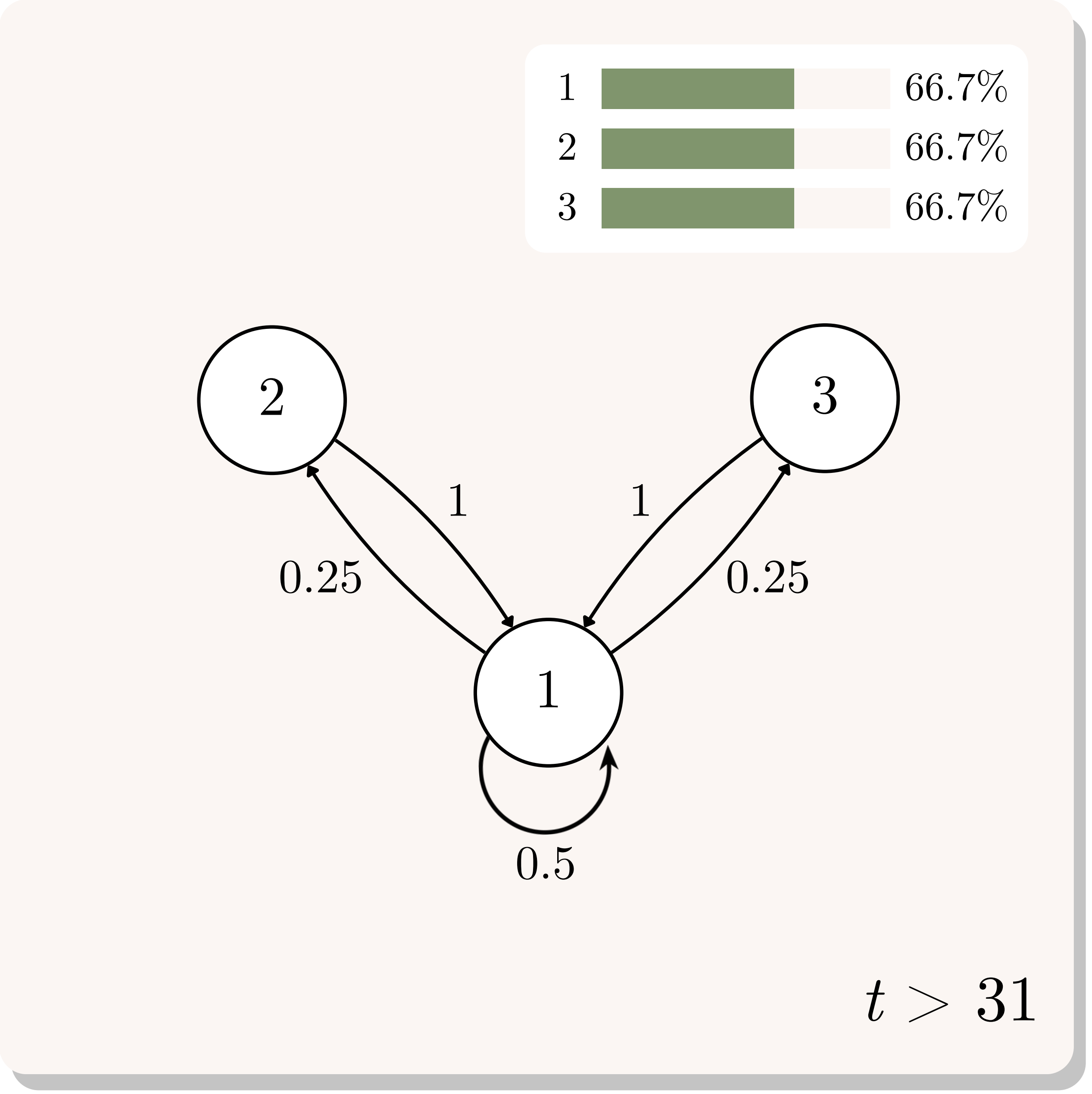
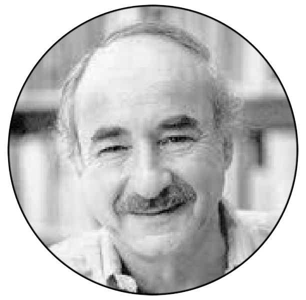


Do the beliefs of each agent *converge*, i.e., reach a point t after which they do not change anymore?

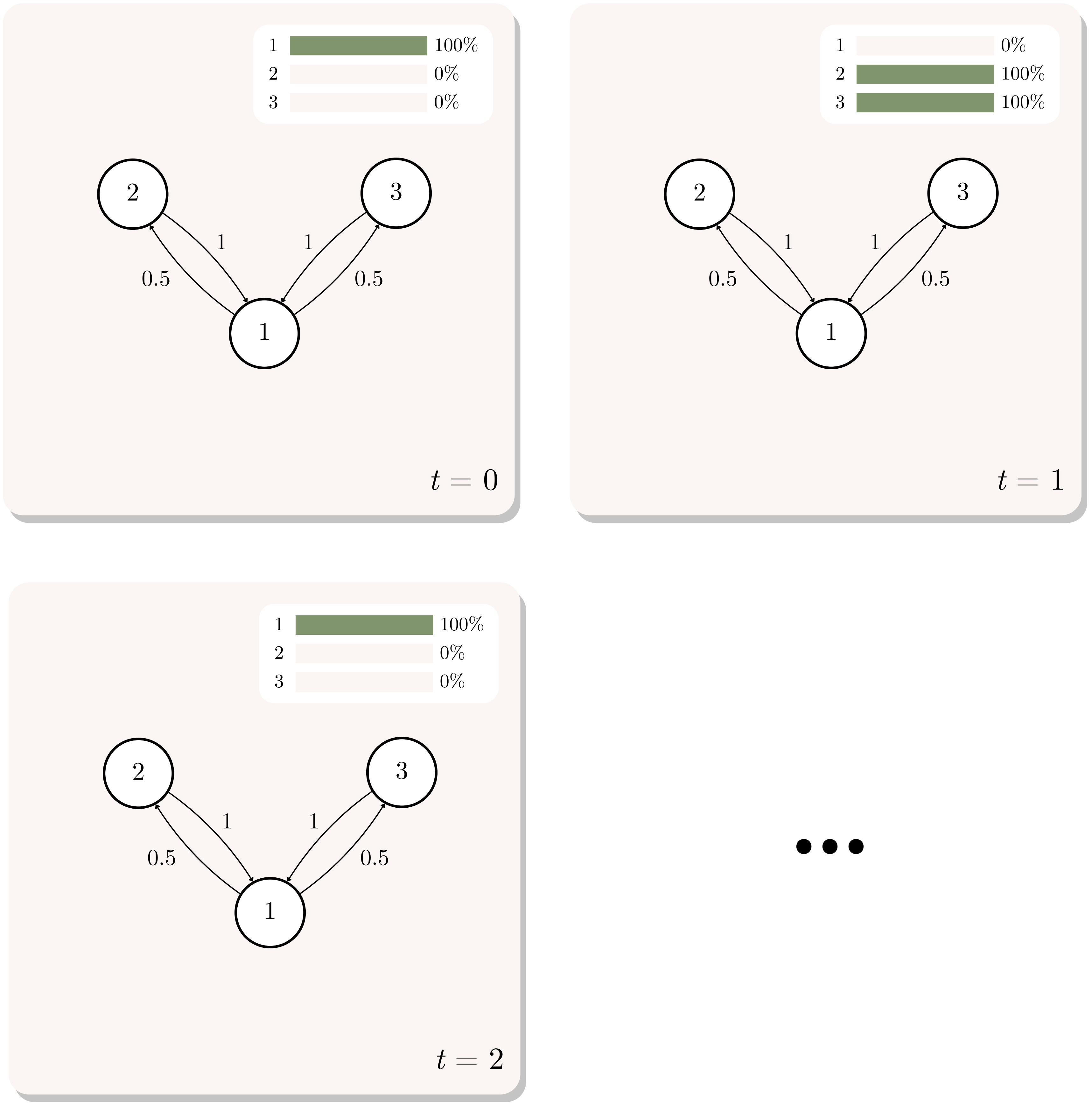
If yes, then is there *consensus*, i.e., do beliefs converge to the same value?

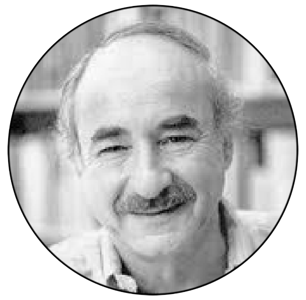


MORRIS DEGROOT

Yes, under certain conditions!

Cycles are bad news.



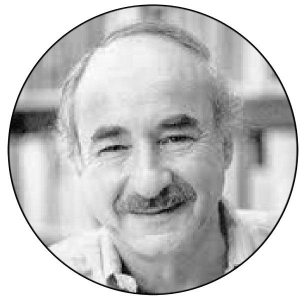


MORRIS DEGROOT

Ok, then let’s just assume there aren’t any bad cycles.

DEFINITION

A network is *aperiodic* if the greatest common divisor of any two cycle lengths is 1.

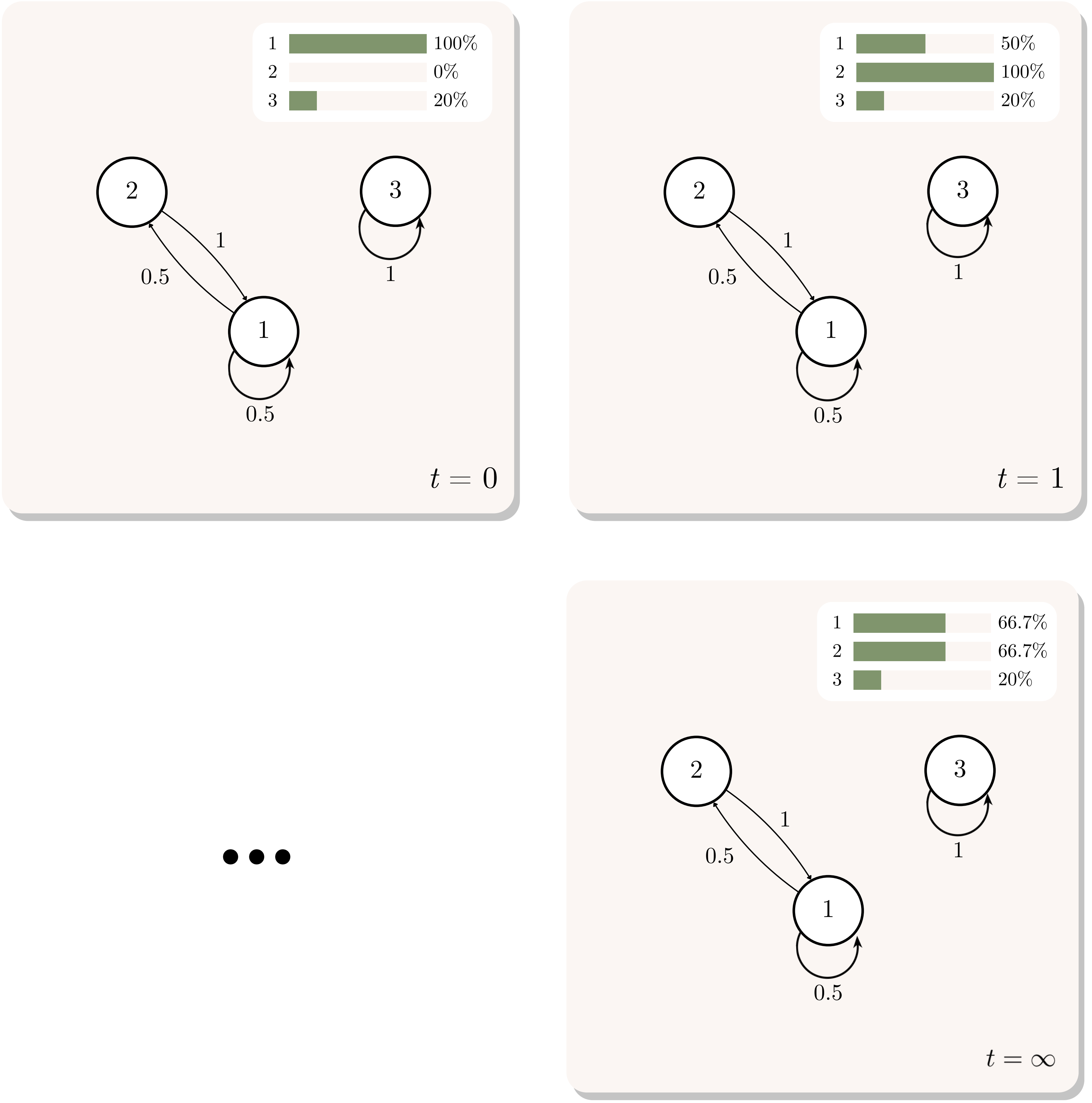


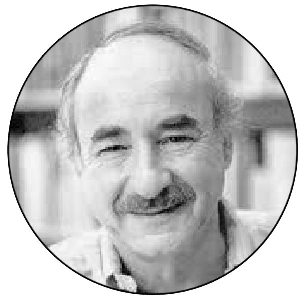
MORRIS DEGROOT

Cycles of lengths 2, 3, 4 are fine.

Cycles of length 2 and 4, or 3 and 6, are not.

Isolated nodes are also bad news.



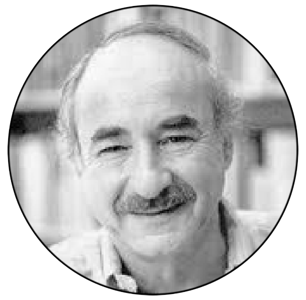


MORRIS DEGROOT

Ok, let’s assume there aren’t any isolated nodes.

DEFINITION

A network is *strongly connected* if there is a path from any node to any other node.



MORRIS DEGROOT

Aperiodicity and strong connectedness do the trick.

THEOREM (DEGROOT, 1974)

If the social network is strongly connected and aperiodic, then, for any initial beliefs, agents converge in the limit to the same belief.

DeGroot, M. H. (1974). Reaching a Consensus. *Journal of the American Statistical Association*, 69(345), 118–121.

Nice! But what needs to happen for agents in the DeGroot model to arrive at a consensus that is also *correct*?

THE DEGROOT MODEL WITH TRUTH

agents	$1, 2, \ldots, n$
time	$t \in \{0, 1, 2, \ldots\}$
true state	$\mu \in (0, 1)$
belief of agent i at t	number between 0 and 1 drawn from a distribution with mean μ and finite variance above a threshold $\delta > 0$
social network	aperiodic, strongly connected directed graph with agents as vertices, and who-pays-attention-to-who as edges
agent i's neighborhood	agents that i pays attention to
weight on edge from i to j	number that indicates how much weight i places on j's opinion; we assume i distributes a total weight of 1 across i's neighborhood
update rule	at time $t + 1$ every agent updates their belief to a weighted average over the beliefs of neighbors

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BENJAMIN GOLUB

We want to speak now of wise *networks*.

MATTHEW O. JACKSON

As with the Condorcet Jury Theorem, this is a limit condition as the network grows larger and larger.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

DEFINITION

We write G_n for a network with n vertices.

A sequence $G_1, G_2, \dots, G_n, \dots$ of (strongly connected and aperiodic) networks of increasing size is *wise* if the consensus belief approaches the true state μ asymptotically, as n goes to infinity.

BENJAMIN GOLUB

There's a really cool way of thinking about the consensus belief.

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BENJAMIN GOLUB

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MATTHEW O. JACKSON

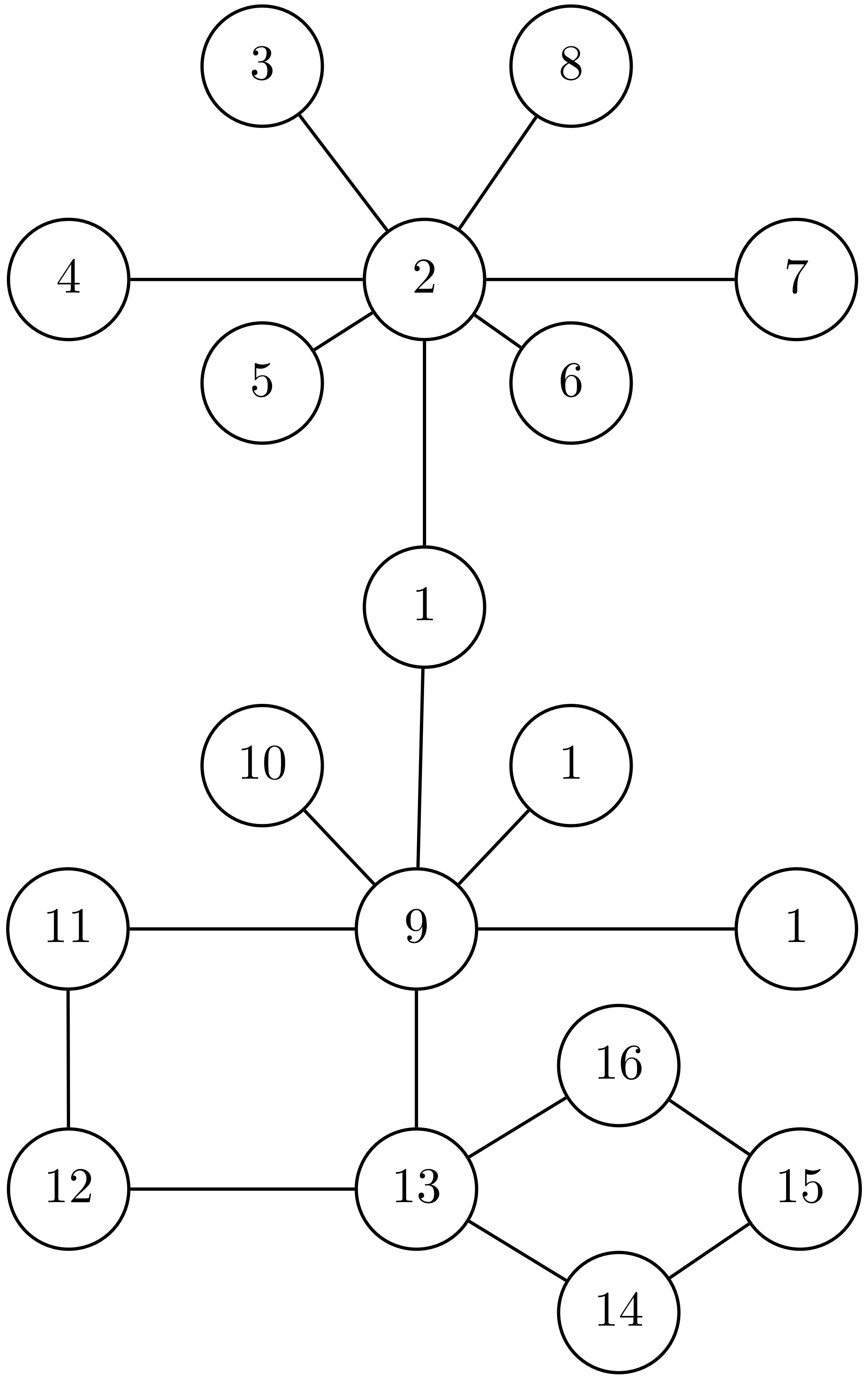
The consensus belief is a linear combination of the initial beliefs and the *eigenvector centralities* of the nodes.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

The *centrality* of a node in a network is a measure of how influential that node is.

EIGENVECTOR CENTRALITY

A node is influential if it is connected to an influential node.



EIGENVECTOR CENTRALITY

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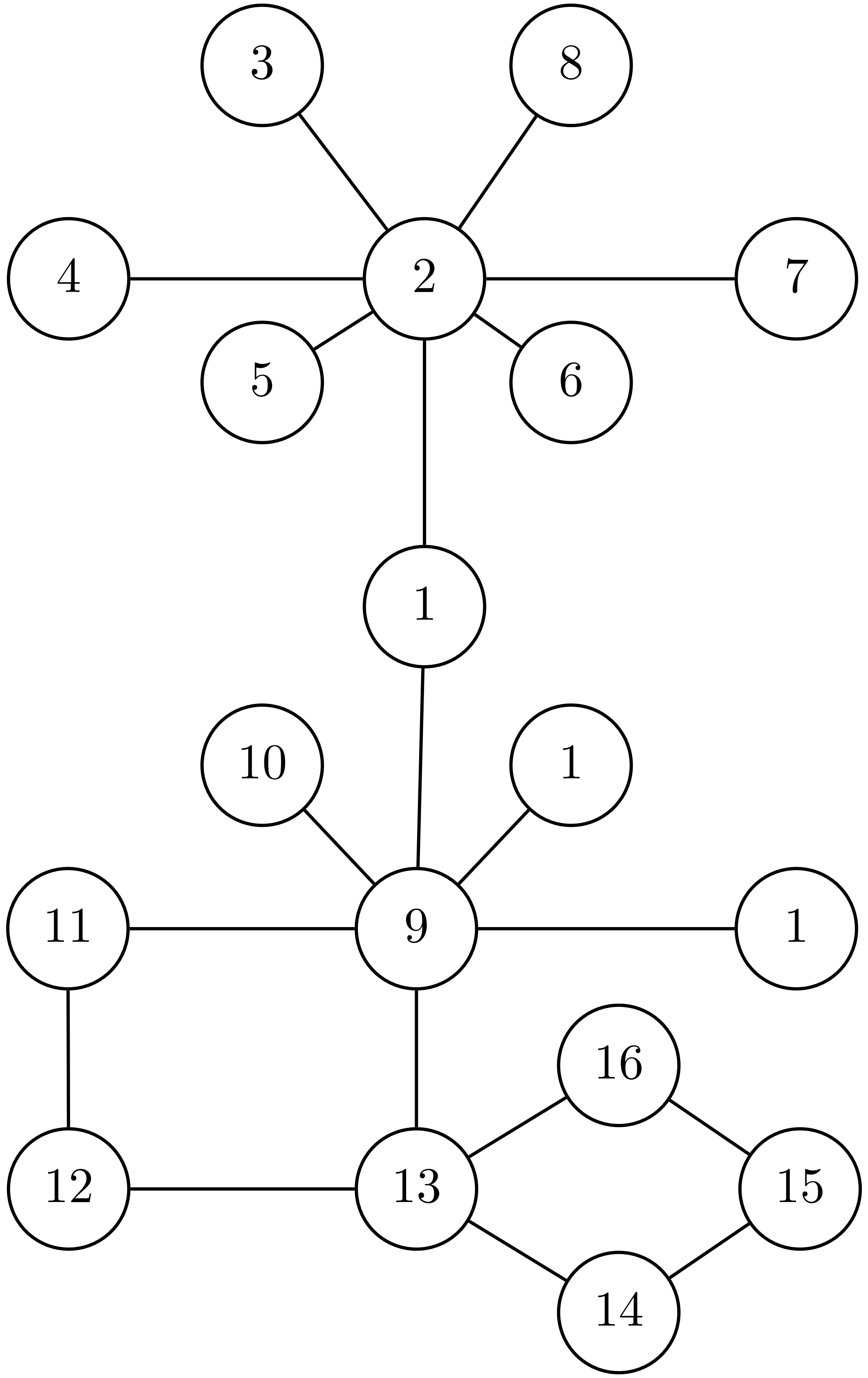
The centrality of a neighbor is proportional to the sum of neighbors’ centralities.



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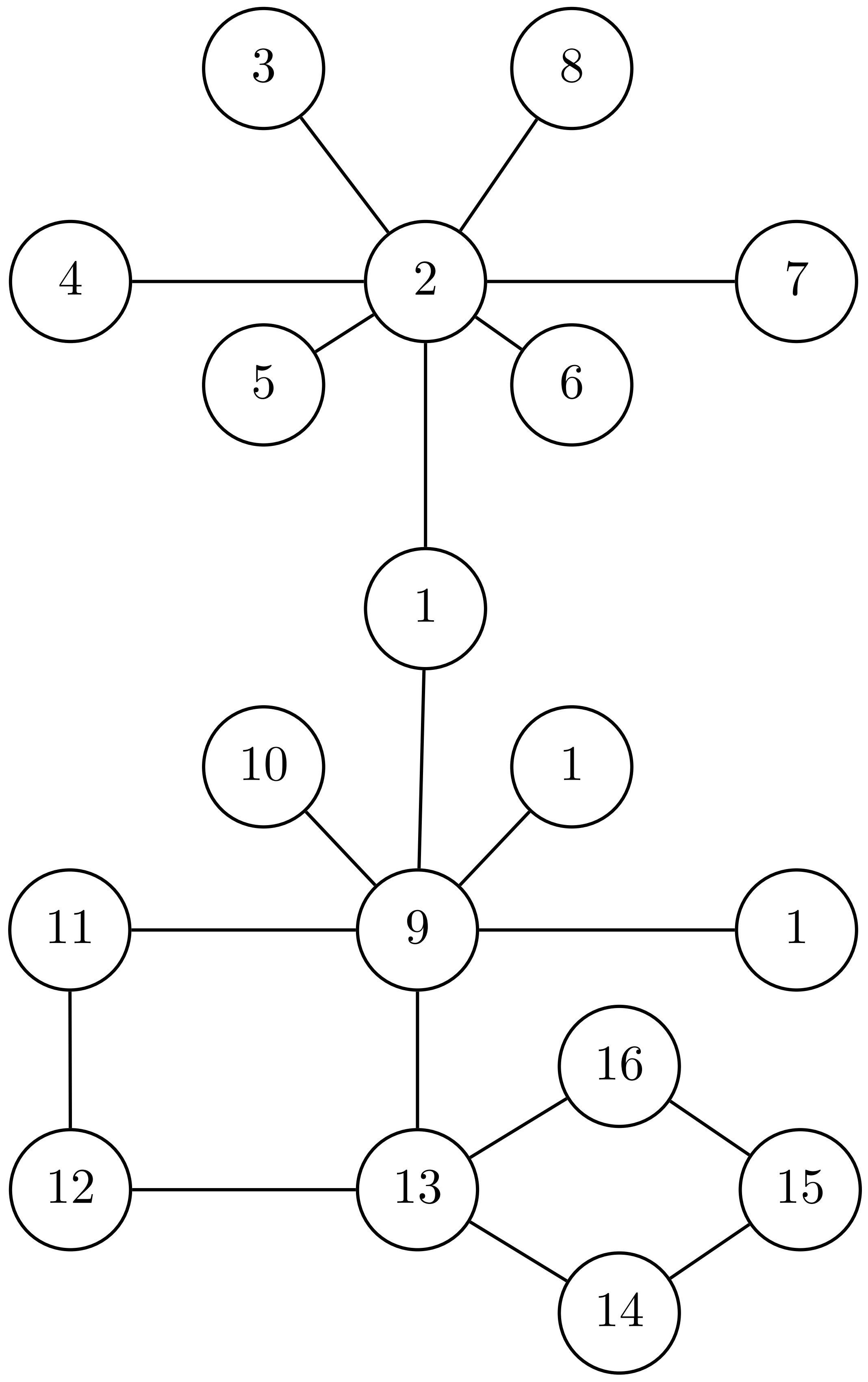
Obtained by solving a system of linear equations.



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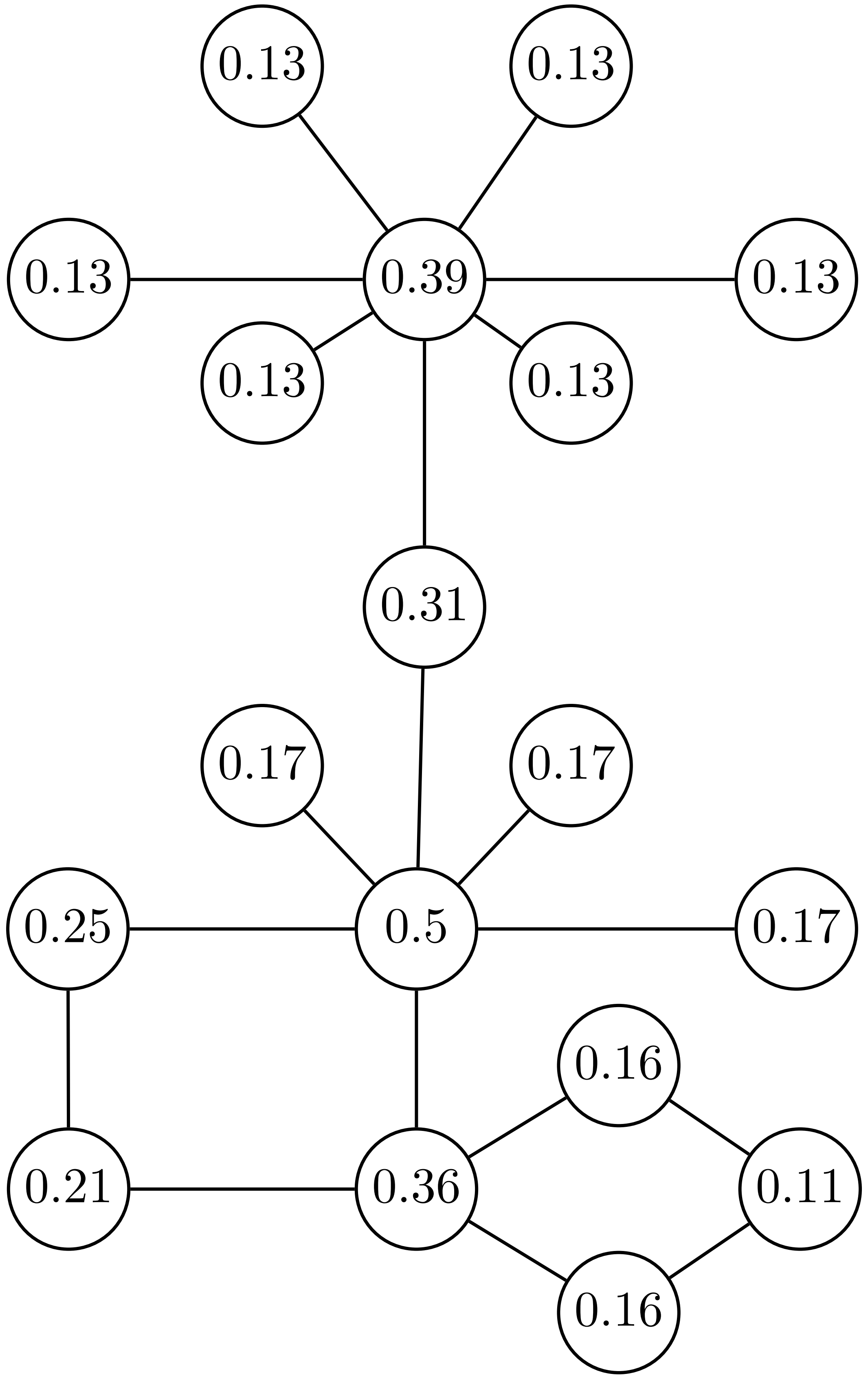


LARRY PAGE

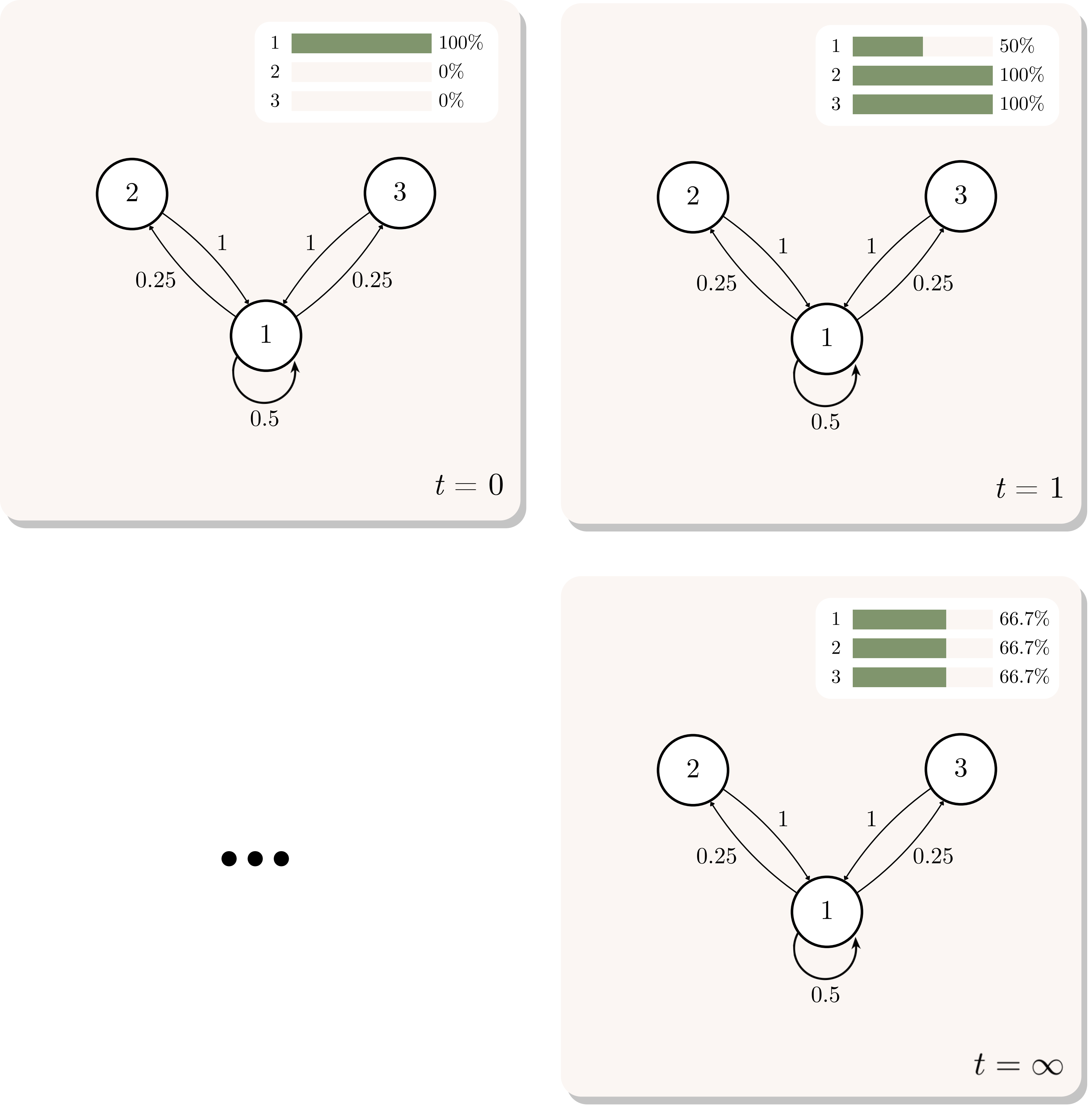
SERGEY BRIN

Eigenvector centrality behind the (original) Google algorithm to rank webpages.

The eigenvector centralities are $\boldsymbol{c} = (2/3, 1/6, 1/6)$.

Centralities indicate the importance of the nodes for the limit consensus belief:

$$\left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right) \cdot (1, 0, 0) = \frac{2}{3} \cdot 1 + \frac{1}{6} \cdot 0 + \frac{1}{6} \cdot 0$$
$$= \frac{2}{3}.$$



THEOREM (GOLUB & JACKSON, 2010)

A sequence $G_1, G_2, \ldots, G_n, \ldots$ of (strongly connected and aperiodic) networks of increasing size is *wise* if and only if the eigenvector centrality of every agent i approaches 0 asymptotically, as n goes to infinity.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

BENJAMIN GOLUB

For a network to be wise, there can't be a node that, in the long run, retains positive influence.

MATTHEW O. JACKSON

As the network grows and grows, the influence of every node should go to 0.

Golub, B., & Jackson, M. O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds. *American Economic Journal: Microeconomics*, 2(1), 112–149.

NETWORKS THAT ARE NOT WISE

The network grows by adding agents that listen to the central agent 1.

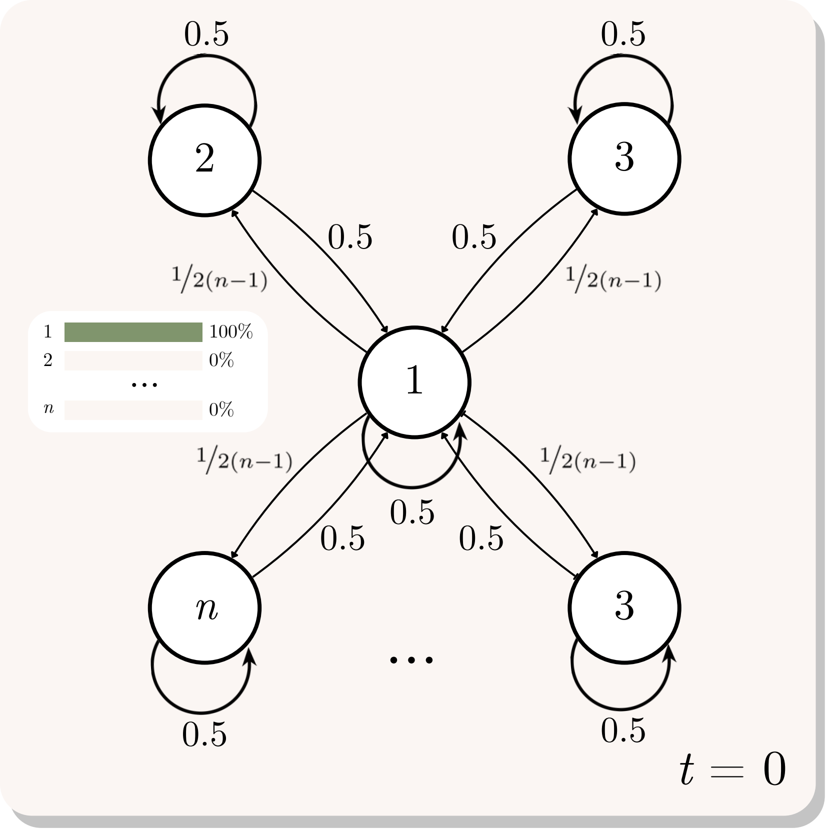
The eigenvector centralities are:

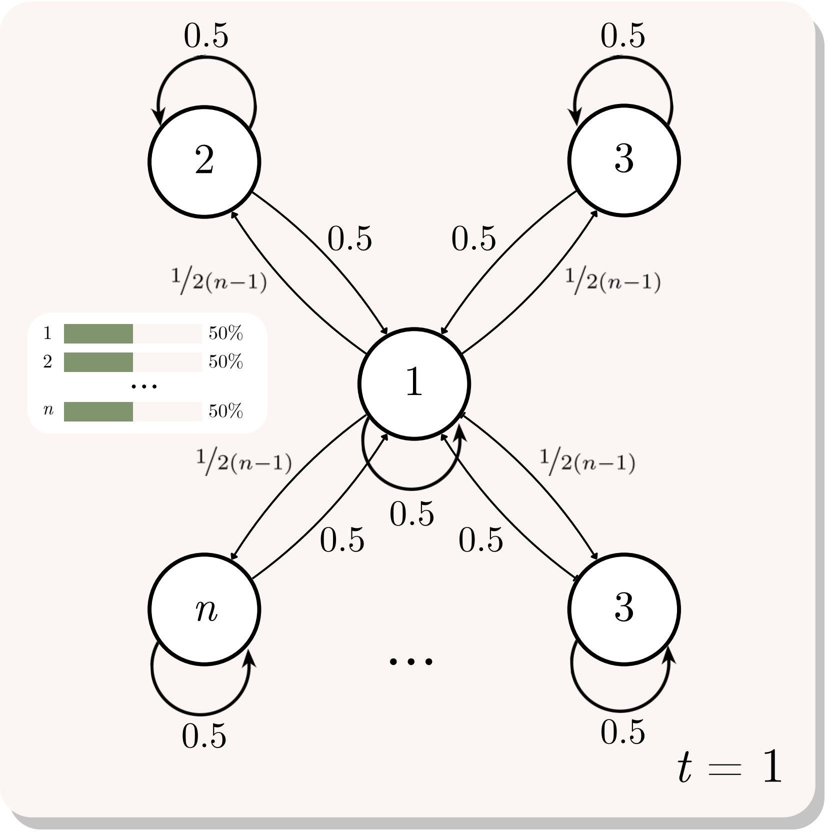
$$\boldsymbol{c} = \left(\frac{1}{2}, \frac{1}{2(n-1)}, \dots, \frac{1}{2(n-1)}\right)$$

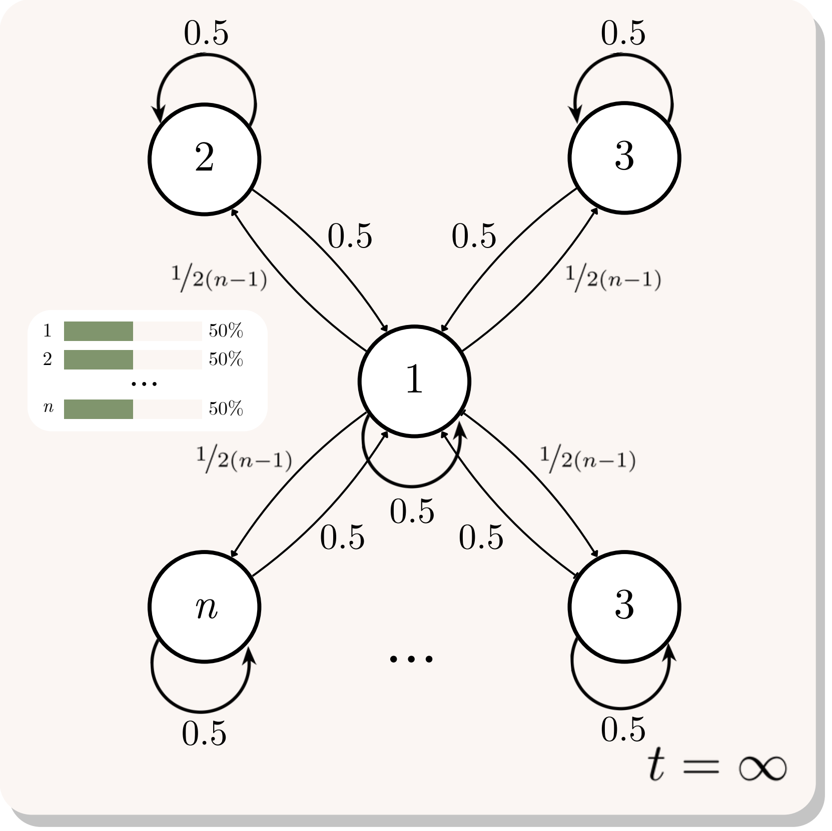
Agent 1 retains a constant share of (network) influence as n grows.

And thus decides the consensus belief.

No bueno.

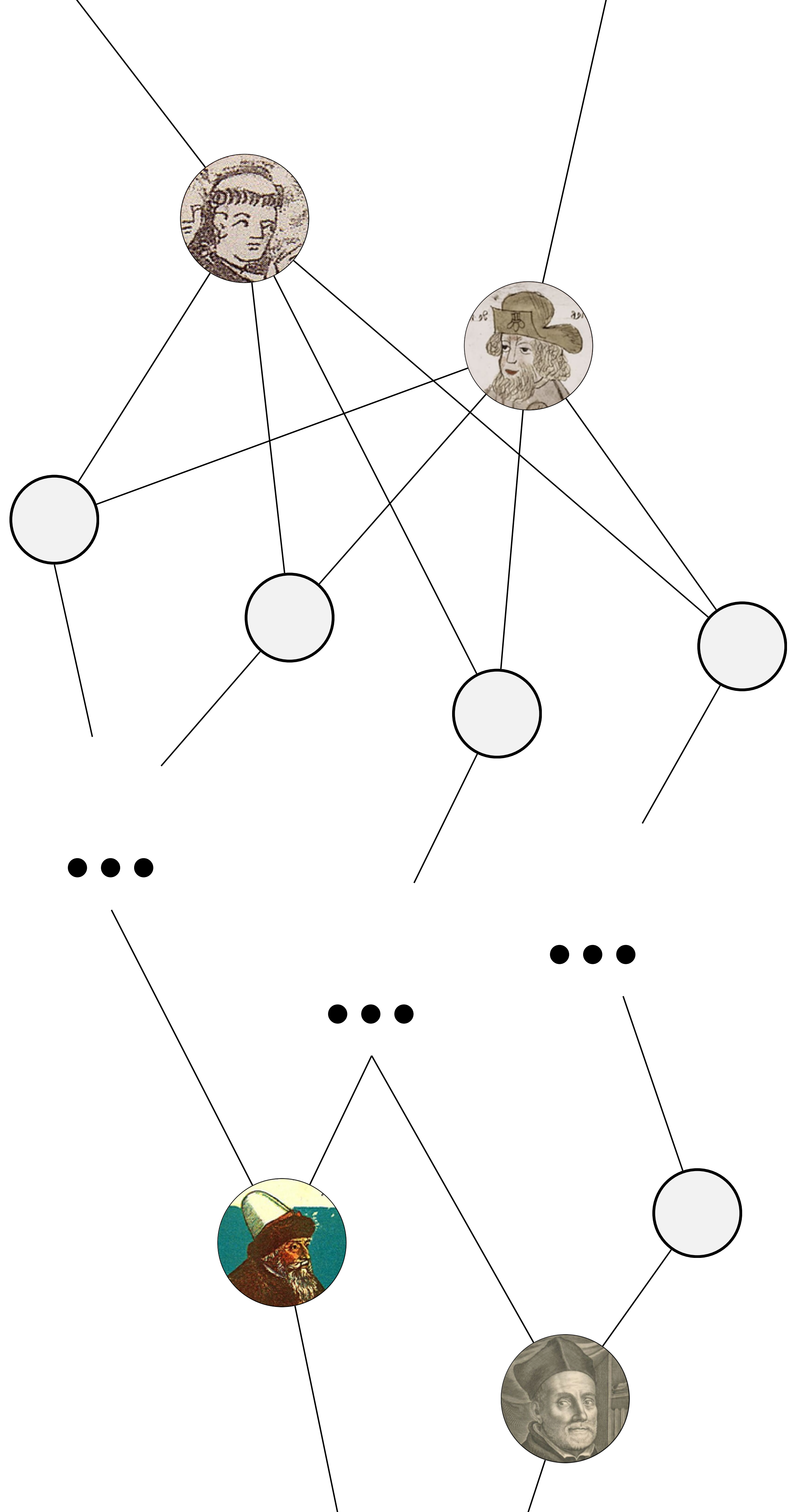






Influential nodes draw the collective opinion towards their own opinion, rather than the truth.

Maybe what happened with the vegetable lamb… 🔎



ELON MUSK

Free speech is the bedrock of a functioning democracy.

And [whatever it’s called now] is the digital town square where matters vital to the future of humanity are debated.

But the shape of the social network means that some agents have an outsized influence on collective opinion.

Is this still in line with democratic ideals?

🔎

This can happen even more dramatically in *information cascades*.

Let’s try this out!

There’s this restaurant you’ve been dying to try.

But you don’t know if it’s *good* or *bad*.

You try it once, and also see previous people’s reviews.

Thumbs up or down?

SUSHIL BIKHCHANDANI

One of the most striking regularities of human society is *localized conformity*.

DAVID HIRSHLEIFER

Americans act American, Germans act German...

IVO WELCH

We want an explanation of why people conform. Also of why convergence of behavior can be idiosyncratic and fragile.

Bikhchandani, S., Hirshleifer, D., & Welch, I. (1992). A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. *The Journal of Political Economy*, 100(5), 992–1026.

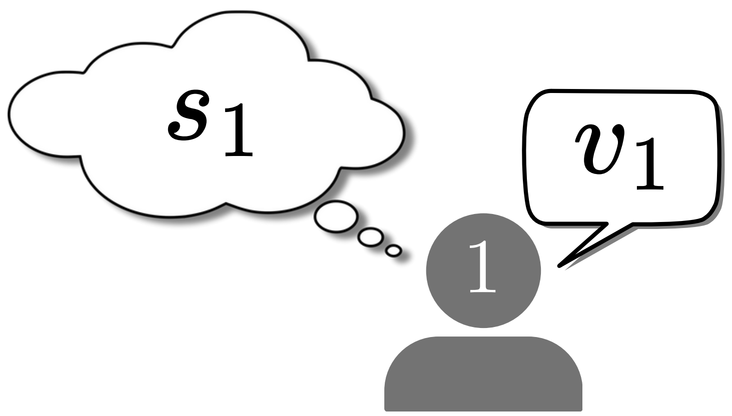
ABHIJIT BANERJEE

A common real world example concerns the choice of restaurants.

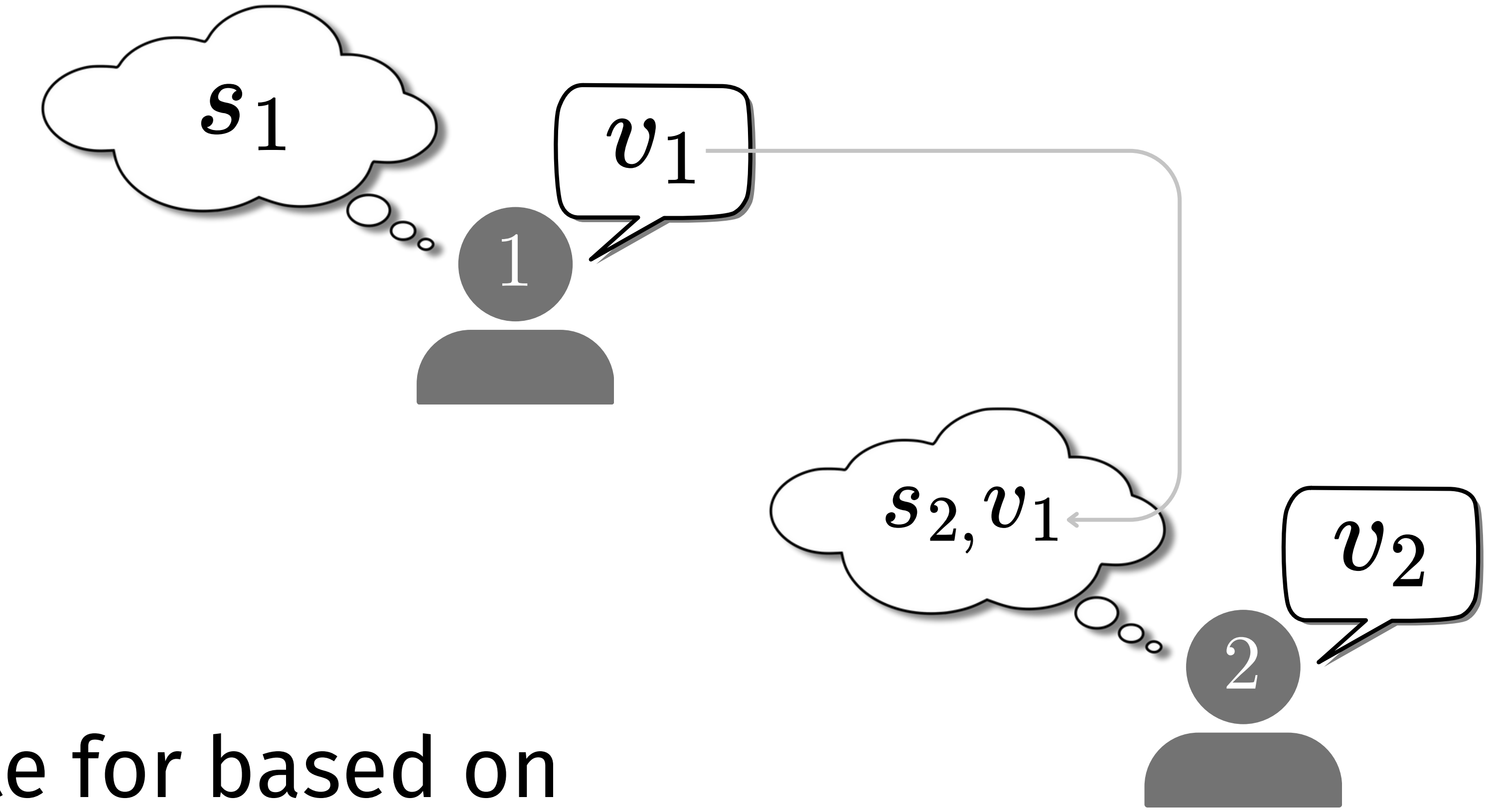
Banerjee, A. (1992). A Simple Model of Herd Behavior. *The Quarterly Journal of Economics*, 107(3), 797–817.

THE MODEL

agents	$N = \{1, \ldots, n\}$
alternatives	$A = \{a, b\}$
true alternative	$\theta \in A$, we usually assume $\theta = a$
voter i's signal	$s_i \in A$
probability of a correct signal i's	$\Pr[s_i = \theta] = p$, with $p > 1/2$
agents' prior probabilities	$\Pr[\theta = a] = \Pr[\theta = b] = 1/2$
agent i's verdict	$v_i \in A$
	agents speak out in sequence, and see previous verdicts



Agents decide what to vote for based on their private signal and the votes of their predecessors.



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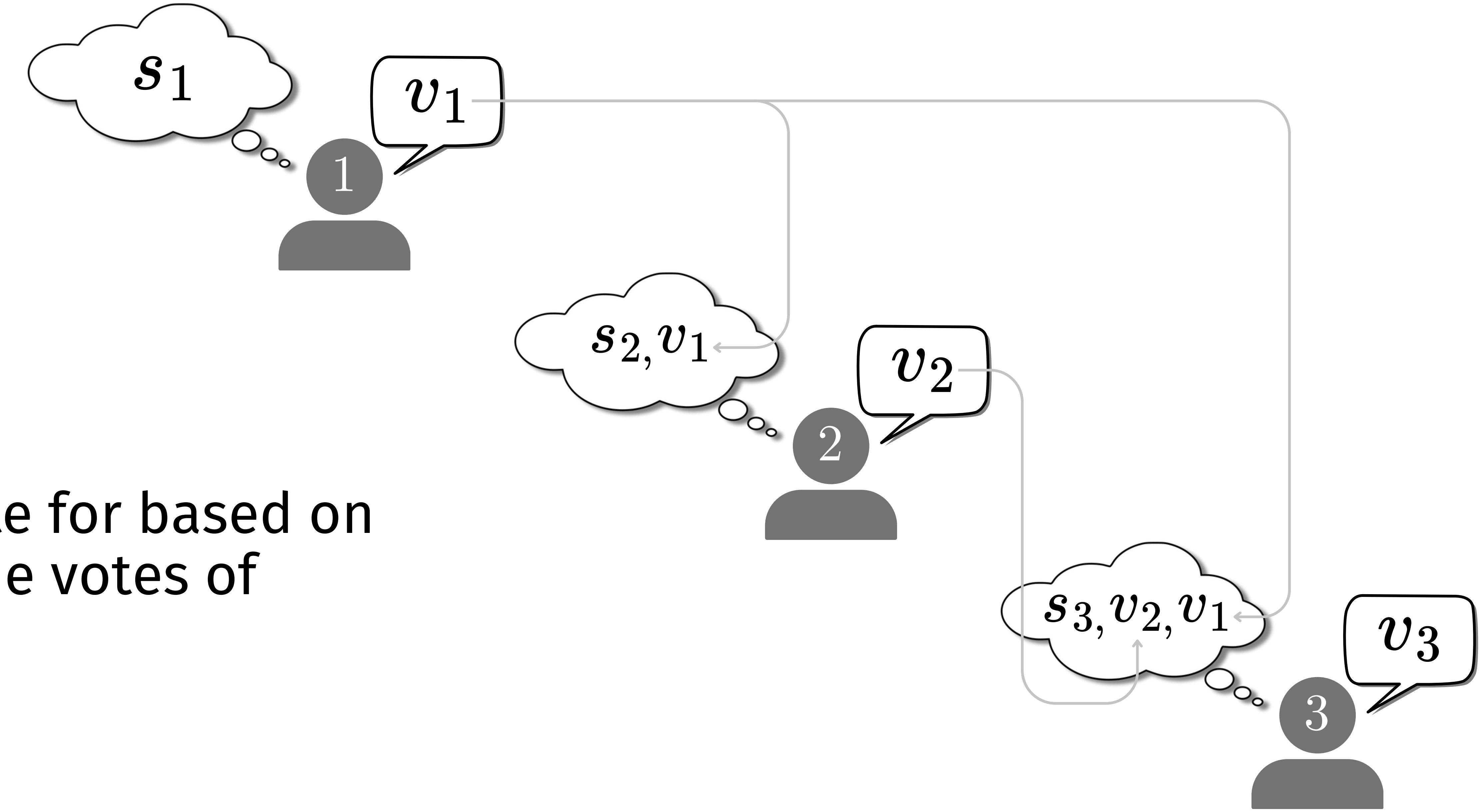

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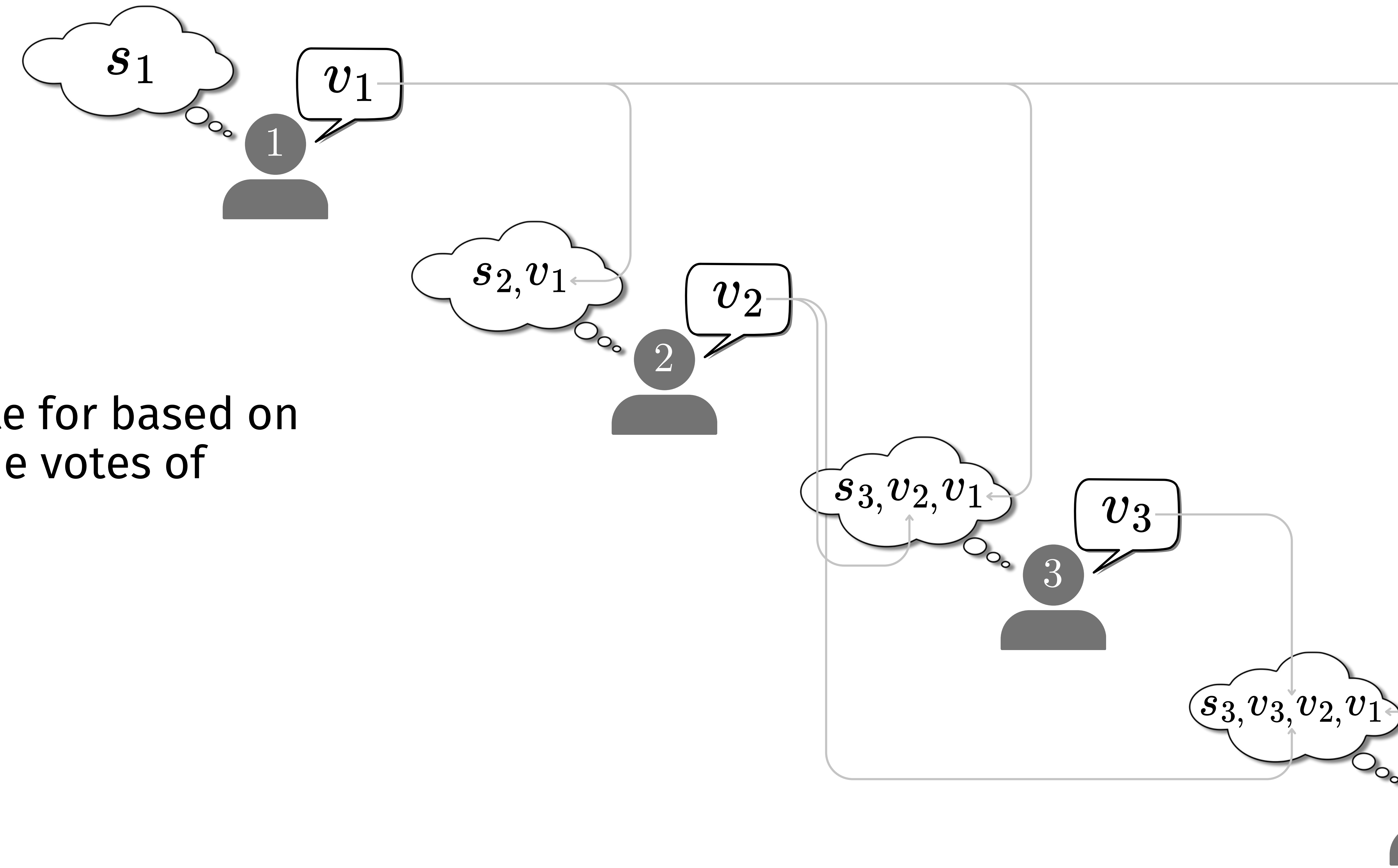


in what sense optimal?

DEFINITION

An *informational cascade* occurs when it is optimal for an agent, having observed the actions of previous agents, to follow the behavior of the preceding agent without regard to their own information.

THOMAS BAYES

Perforce, they must follow my rule.

BAYES' RULE

We can infer the probability of A given B from the probability of B given A, and the prior probabilities of A and B.

$$\Pr\left[A|B\right] = \frac{\Pr\left[B|A\right] \cdot \Pr\left[A\right]}{\Pr\left[B\right]}$$

How do agents decide?

The (Bayesian) first agent's verdict is determined by their private signal.

$$\Pr\left[\theta = a \mid s_1 = a\right] = \frac{\Pr\left[s_1 = a \mid \theta = a\right] \cdot \Pr\left[\theta = a\right]}{\Pr\left[s_1 = a\right]}$$

$$\Pr\left[\theta = b \mid s_1 = a\right] = \frac{\Pr\left[s_1 = a \mid \theta = b\right] \cdot \Pr\left[\theta = b\right]}{\Pr\left[s_1 = a\right]}$$

With $p > 1/2$, we have that $\Pr\left[\theta = a \mid s_1 = a\right] > \Pr\left[\theta = b \mid s_1 = a\right]$.

The (Bayesian) second agent will vote according to their signal and the previous voter's action.

If there is a majority for one alternative, the decision reflects that.

If there is a tie, the agent flips a coin.

$$\Pr\left[\theta = a \mid s_2 = a, v_1 = a\right] > \Pr\left[\theta = b \mid s_2 = a, v_1 = a\right]$$
$$\Pr\left[\theta = b \mid s_2 = b, v_1 = b\right] > \Pr\left[\theta = a \mid s_2 = b, v_1 = b\right]$$
$$\Pr\left[\theta = a \mid s_2 = a, v_1 = b\right] = \Pr\left[\theta = b \mid s_2 = a, v_1 = b\right]$$
$$\Pr\left[\theta = a \mid s_2 = b, v_1 = a\right] = \Pr\left[\theta = b \mid s_2 = b, v_1 = a\right].$$

The (Bayesian) third agent will go with the trend defined by the first two agents, if there is one.

$$\Pr\left[\theta = b \mid s_3 = b, v_2 = v_1 = b\right] > \Pr\left[\theta = a \mid s_3 = b, v_2 = v_1 = b\right]$$

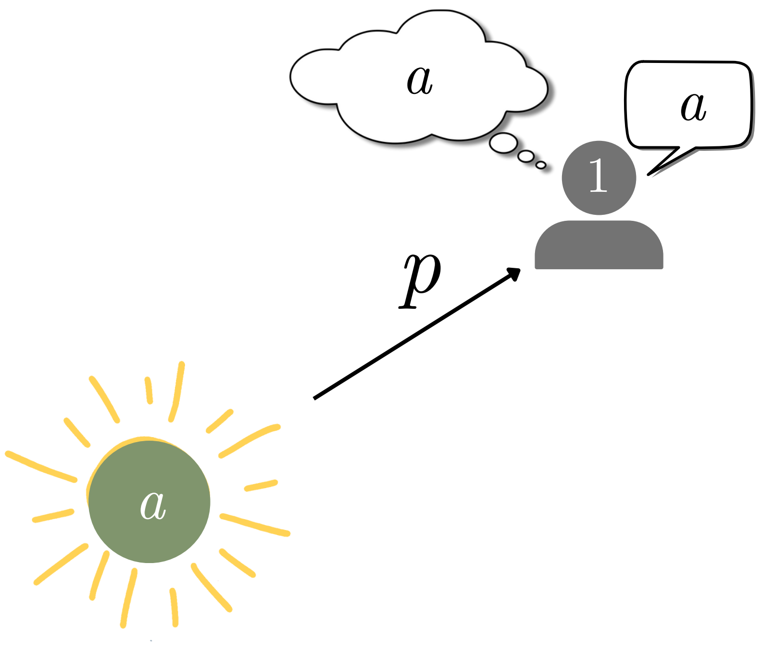
The (Bayesian) third agent will go with the trend defined by the first two agents, if there is one.

$$\Pr[\theta = b \mid s_3 = b, v_2 = v_1 = b] > \Pr[\theta = a \mid s_3 = b, v_2 = v_1 = b]$$

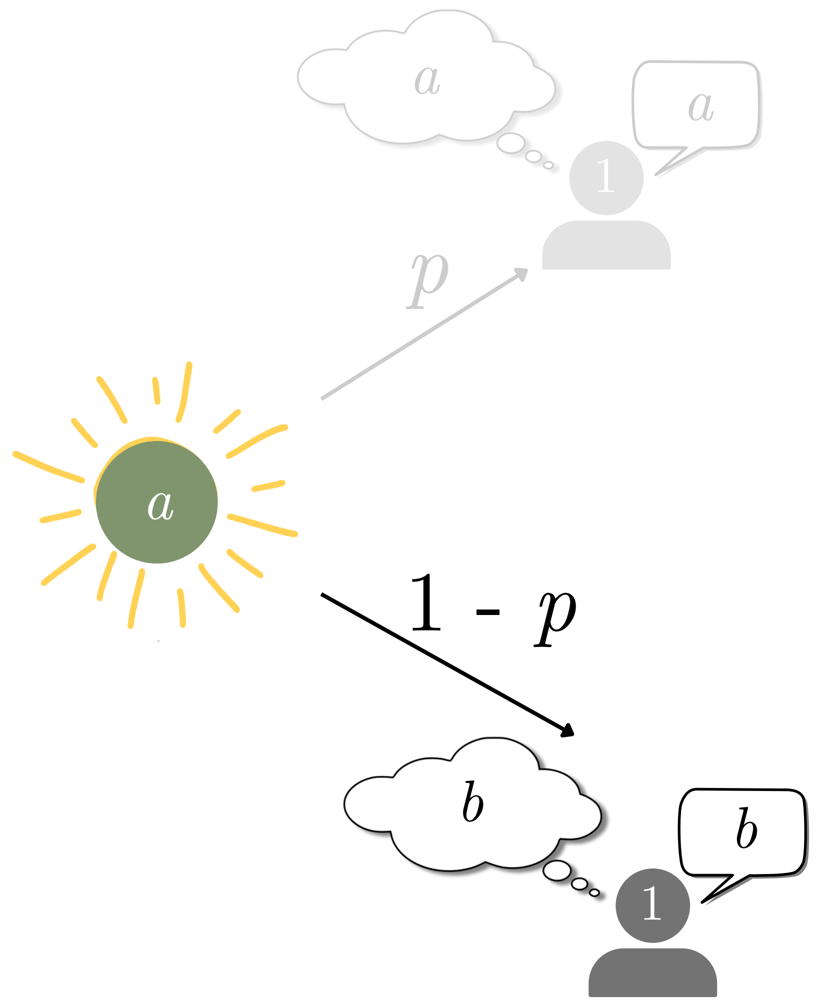
$$\Pr[\theta = b \mid s_3 = a, v_2 = v_1 = b] > \Pr[\theta = a \mid s_3 = a, v_2 = v_1 = b]$$

Regardless of their own private signal!

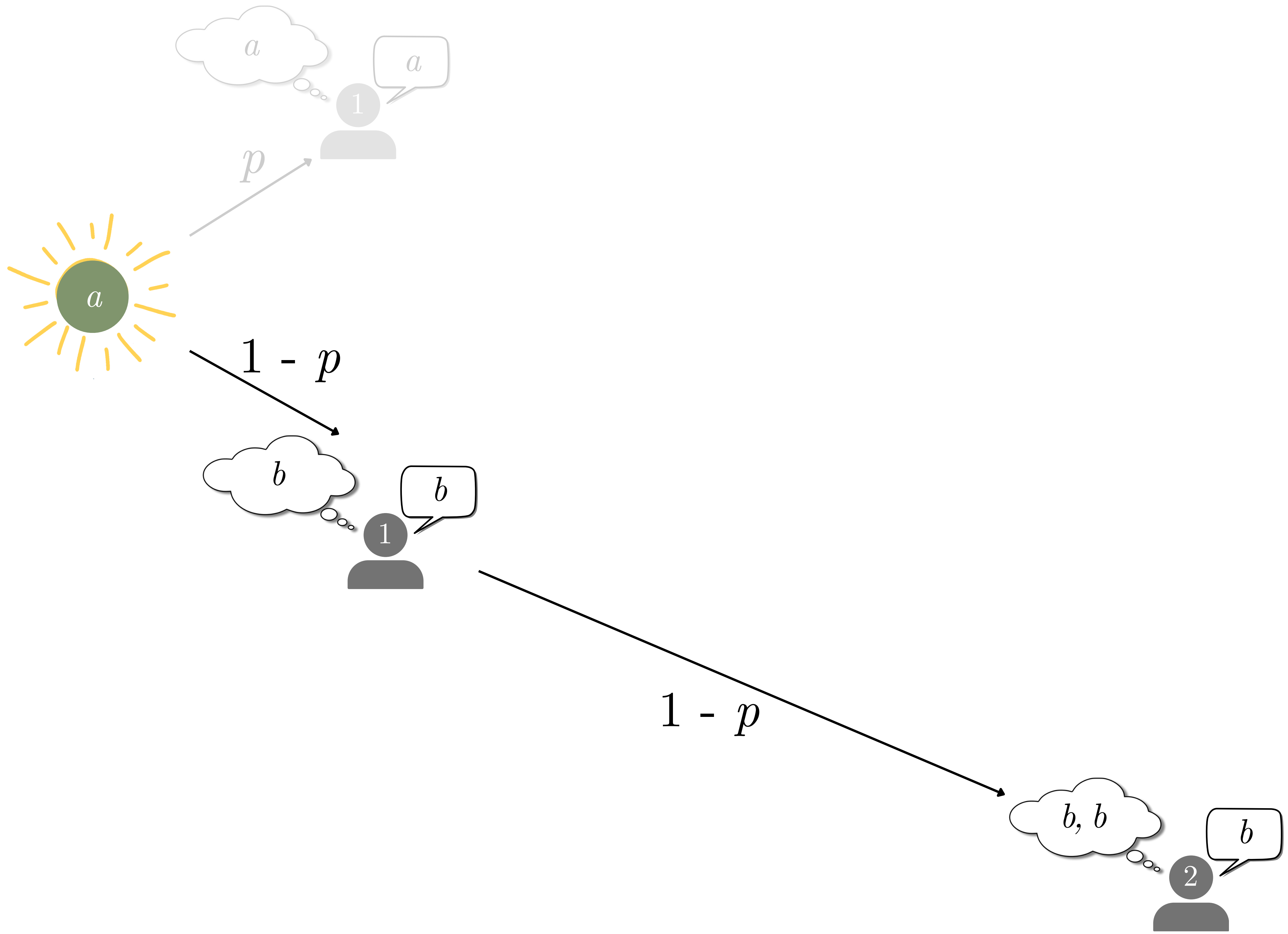
A cascade!

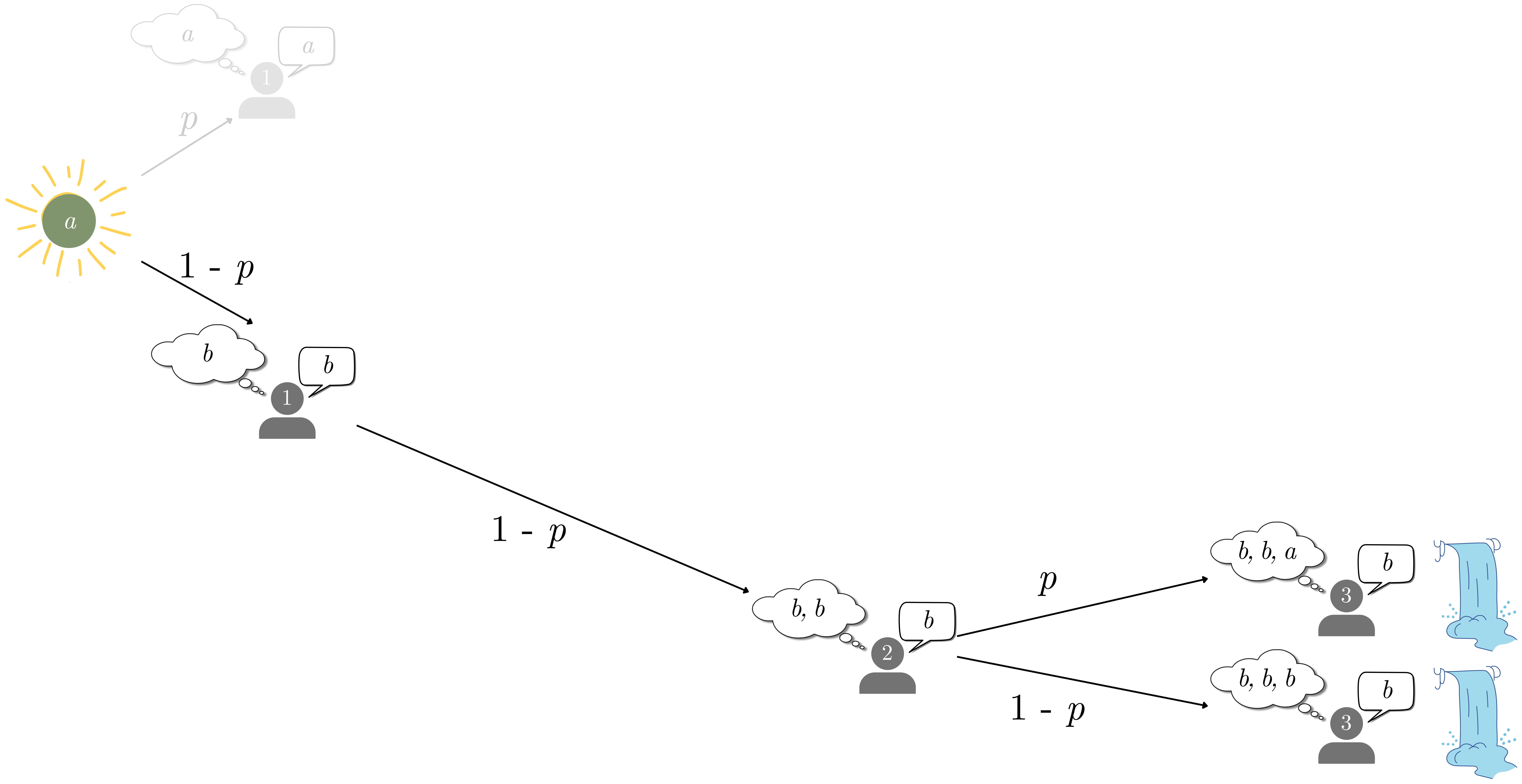


a
a
1
p
a

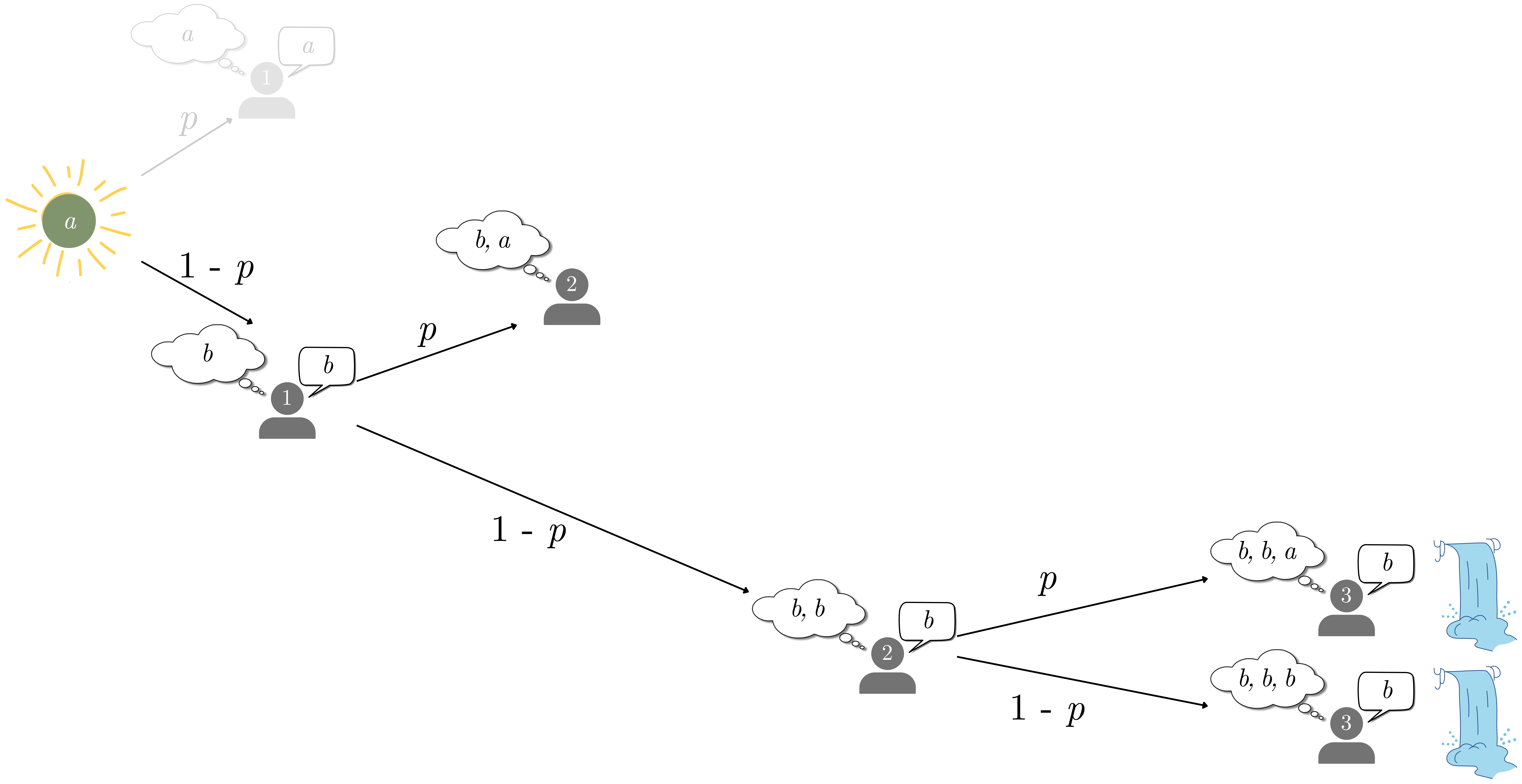


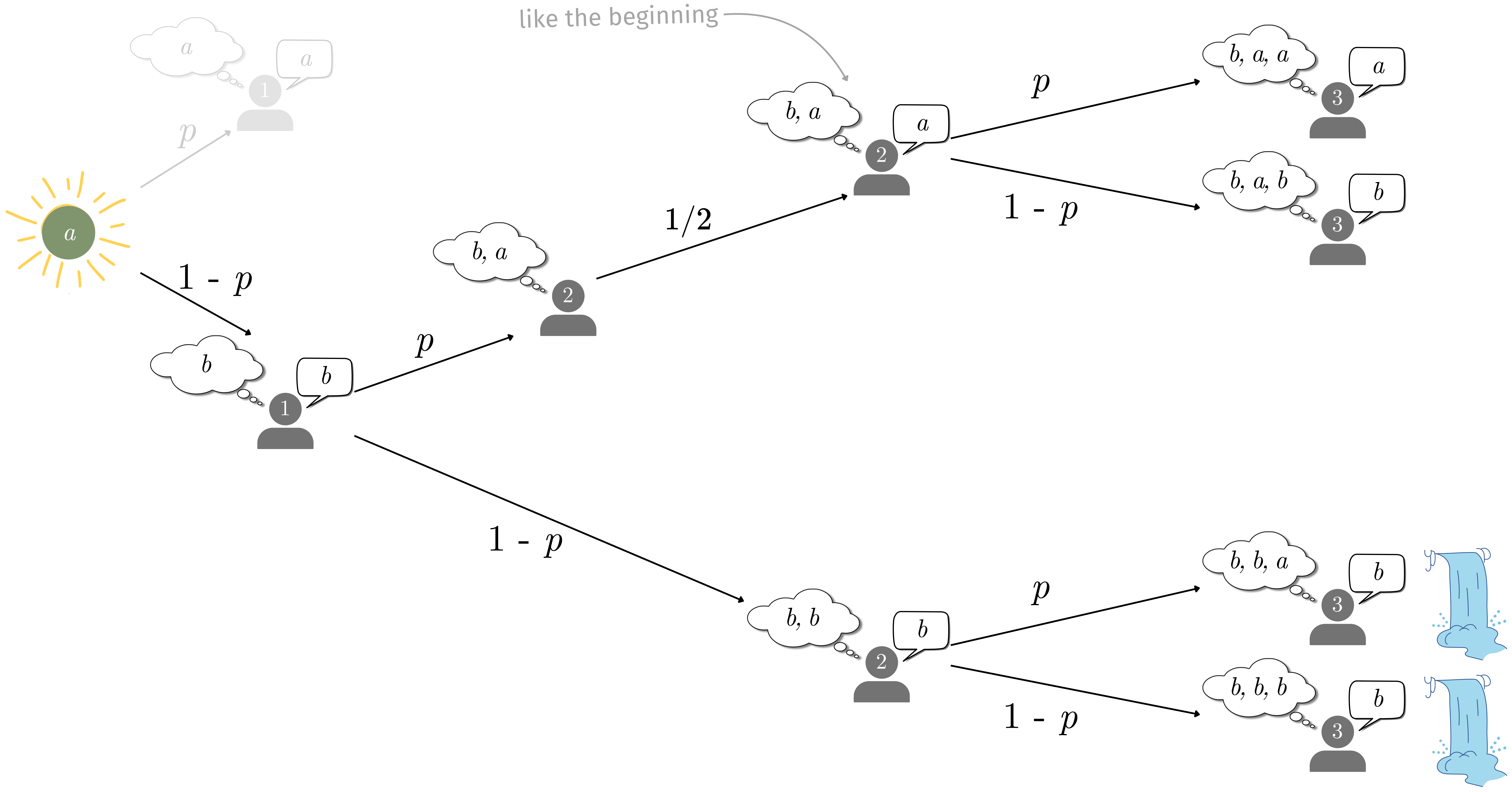
a
a
1
p
a
1 - p
b
b
1

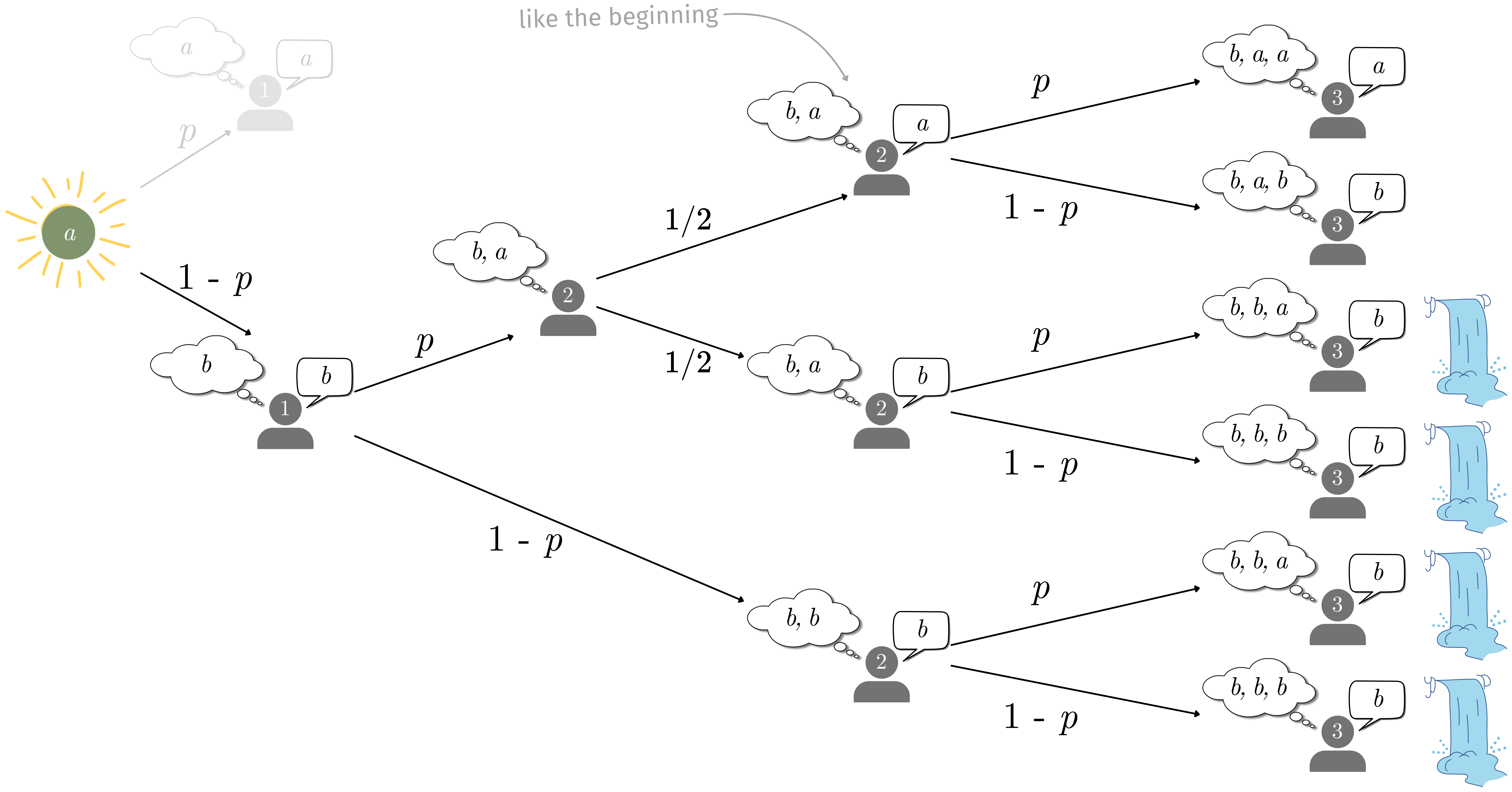

a
a
1
p
a
1 - p
b
b
1
1 - p
b, b
b
2



a
a
1
p
a
1 - p
b
b
1
1 - p
b, b
b
2
p
b, b, a
b
3
1 - p
b, b, b
b
3

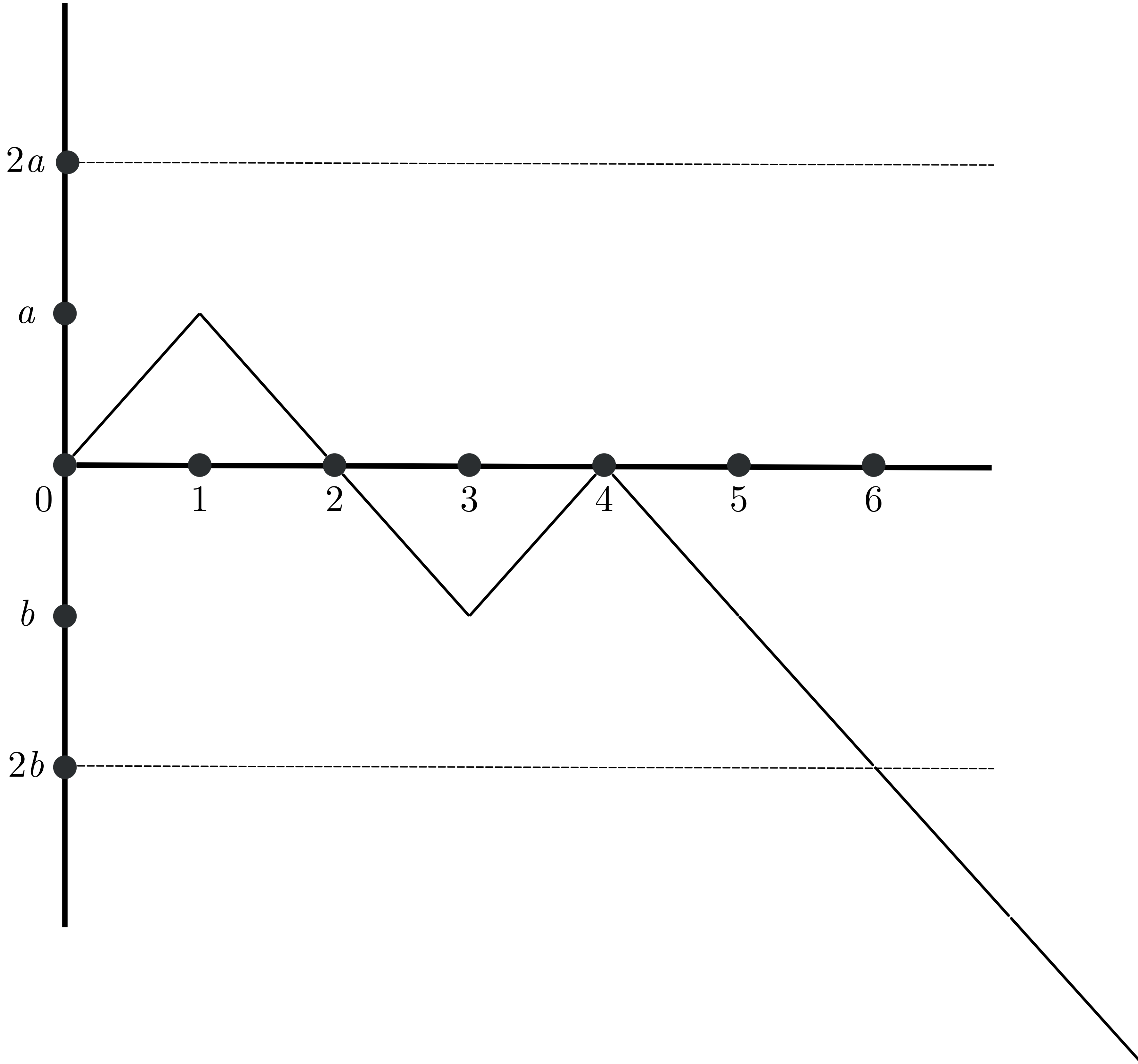

a
a
1
p
a
1 - p
b, a
2
p
b
b
1
1 - p
b, b
b
2
p
b, b, a
b
3
1 - p
b, b, b
b
3


like the beginning
a
a
a
1
p
a
1 - p
b
b
1
p
b, a
2
1/2
b, a
a
2
p
b, a, a
a
3
1 - p
b, a, b
b
3
1 - p
b, b
b
2
p
b, b, a
b
3
1 - p
b, b, b
b
3



like the beginning
a
a
a
1
p
a
1 - p
b
b
1
p
b, a
2
1 - p
1/2
1/2
b, a
a
2
b, a
b
2
b, b
b
2
p
1 - p
p
1 - p
p
1 - p
b, a, a
a
3
b, a, b
b
3
b, b, a
b
3
b, b, b
b
3
b, b, a
b
3
b, b, b
b
3

In general, a cascade occurs as soon as two consecutive agents take the same action.



SUSHIL BIKHCHANDANI

For large *n*, we are sure to fall into a cascade.

DAVID HIRSHLEIFER

The cascade can be for the correct alternative, or for the wrong one.

IVO WELCH

Cascades are also fragile, i.e., sensitive to the release of public information.

Bikhchandani, S., Hirshleifer, D., & Welch, I. (1992). A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. *The Journal of Political Economy*, 100(5), 992–1026.

Let’s sum up.

CONDORCET

Wisdom of crowds is a fragile phenomenon, and it can break down in the presence of strong correlation between voters.

HÉLÈNE LANDEMORE

At the same time, there is more and more evidence that certain forms of communication, e.g., deliberation, are good for decision making.

Landemore, H. (2020). *Open Democracy: Reinventing Popular Rule for the Twenty-First Century*. Princeton University Press.

Can we find better insights, for more realistic scenarios?