

We need to show that:

$$\lim_{n \rightarrow \infty} \Pr [S_n > \lfloor n/2 \rfloor] = 1.$$

Now, the expected value (i.e., mean μ) of the voter random variables X_i is:

$$\begin{aligned}\mathbb{E}[X_i] &= 1 \cdot p + 0 \cdot (1 - p) \\ &= p,\end{aligned}$$

and the Weak Law of Large Numbers gives us that, for any $\varepsilon > 0$:

$$\lim_{n \rightarrow \infty} \Pr \left[\left| \frac{S_n}{n} - p \right| > \varepsilon \right] = 0.$$

Choosing ε appropriately and massaging this expression we obtain the desired conclusion.