The probability that an infectious agent meets a susceptible agent at time t is:

$$\frac{I_t}{N} \cdot \frac{N - I_t}{N},$$

which leads to infection with probability τ. Over N possible agents, the average number of new infections at round t is:

$$N \cdot \tau \cdot \frac{I_t}{N} \cdot \left(\frac{N - I_t}{N} \right).$$

Hence, the recursion relation for the number of infectious agents at time $t + 1$ is:

$$I_{t+1} = I_t + N \cdot \tau \cdot \frac{I_t}{N} \cdot \left(\frac{N - I_t}{N} \right)$$
$$= I_t + \tau I_t \left(1 - \frac{I_t}{N} \right).$$