

Suppose Players 1 and 2 play mixed strategies  $s_1 = (p, 1 - p)$  and  $s_2 = (q, 1 - q)$ , respectively, for  $p, q > 0$ .

Note that Player 2's expected payoff with these strategies is:

$$\mathbb{E}[s_2 \mid s_1] = \mathbb{E}[\text{Heads} \mid s_1] \cdot q + \mathbb{E}[\text{Tails} \mid s_1] \cdot (1 - q).$$

Suppose, now, that Player 1's strategy makes Heads more attractive for Player 2:

$$\mathbb{E}[\text{Heads} \mid s_1] > \mathbb{E}[\text{Tails} \mid s_1].$$

In this case, Player 2 would want to deviate to  $s_2' = (1, 0)$ :

$$\begin{aligned}\mathbb{E}[s_2' \mid s_1] &= \mathbb{E}[\text{Heads} \mid s_1] \cdot 1 + \mathbb{E}[\text{Tails} \mid s_1] \cdot 0 \\ &> \mathbb{E}[\text{Heads} \mid s_1] \cdot q + \mathbb{E}[\text{Tails} \mid s_1] \cdot (1 - q).\end{aligned}$$

So  $s = (s_1, s_2)$  cannot be a Nash equilibrium. Same if  $\mathbb{E}[\text{Tails} \mid s_1] > \mathbb{E}[\text{Heads} \mid s_1]$ .

The only way to avoid this is for Player 1 to play a strategy  $s_1^* = (p, 1 - p)$  that makes Player 2 indifferent between their actions:

$$\begin{aligned}\mathbb{E}[\text{Heads} \mid s_1] &= \mathbb{E}[\text{Tails} \mid s_2] \text{ iff } (-1) \cdot p + 1 \cdot (1 - p) = 1 \cdot p + (-1) \cdot (1 - p) \\ &\text{iff } p = 1/2.\end{aligned}$$

So Player 1 wants to play  $s_1^* = (1/2, 1/2)$ . Similarly, Player 2 wants to play  $s_2^* = (1/2, 1/2)$ . This is the mixed Nash equilibrium.