b.) Prove: If solution has assumed symmetry, then:

$$-\frac{1}{Re}\Delta v + \nabla\Pi = 0 \qquad \text{with } \Pi = p + \frac{1}{2}\|v\|^2 - g$$

Proof:

We use Homework 1b:

$$(v\cdot\nabla)v = \frac{1}{2}\nabla\|v\|^2 - v \times \operatorname{rot} v$$

and put in the results from Ex 1iii:

$$(v\cdot\nabla)v = \frac{1}{2}\nabla\|v\|^2 - \nabla g \qquad \text{with } \frac{dg}{dr} = -V_\varphi(r)\,\lambda(r)$$

The Navier stokes equation with 0 on the rightside,

can be written as

$$-\frac{1}{Re}\Delta v + (v\cdot\nabla)v + \nabla p = 0 \text{ where } p \text{ is pressure.}$$

Therefore we get

$$-\frac{1}{Re}\Delta v + \left(\frac{1}{2}\nabla\|v\|^2 - \nabla g\right) + \nabla p = 0$$

This can be rewritten

as:

$$-\frac{1}{Re}\Delta v + \nabla\left(\frac{1}{2}\|v\|^2 - g + p\right) = 0$$

~~and~~ which is

$$-\frac{1}{Re}\Delta v + \nabla\Pi = 0 \qquad \text{where } \Pi = p + \frac{1}{2}\|v\|^2 - g$$

$$\text{and } \frac{dg}{dr} = -V_\varphi(r)\,\lambda(r)$$