ii) according to i) $v_z = 0$

$$\mathrm{rot}\, v = \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix} = \begin{pmatrix} -\partial_z v_y \\ \partial_z v_x \\ \partial_x v_y - \partial_y v_x \end{pmatrix}$$

we furtha know

$$v = v_\varphi(r) \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} = -\sin\varphi\, v_\varphi(r)\, e_x + \cos\varphi \cdot v_\varphi(r)\, e_y + 0 e_z$$

$$= -\frac{y}{r} v_\varphi(r)\, e_x + \frac{x}{r} v_\varphi(r)\, e_y = v_x e_x + v_y e_y + 0 e_z$$

$$\Rightarrow \quad v_x = -\frac{y}{r} v_\varphi(r) \qquad v_y = \frac{x}{r} v_\varphi(r)$$

→ these components are independent of $z$, thus:

$$\mathrm{rot}(v) = \begin{pmatrix} 0 \\ 0 \\ \partial_x v_y - \partial_y v_x \end{pmatrix} = (\partial_x v_y - \partial_y v_x)\, e_z .$$

$$\to \partial_x v_y = \left(\frac{d}{dx} \frac{x}{r(x,y)}\right) v_\varphi(r) + \frac{x}{r} \partial_r v_\varphi(r) \frac{d}{dx} r(x,y)$$

$$= \frac{1 \cdot r - x \frac{d}{dx} r(x,y)}{r^2} v_\varphi(r) + \frac{x}{r} \partial_r v_\varphi(r) \frac{x}{r}$$

$$= \frac{r - \frac{x^2}{r}}{r^2} v_\varphi(r) + \frac{x^2}{r^2} \partial_r v_\varphi(r)$$

$$\to \partial_y v_x = -\frac{r - \frac{y^2}{r}}{r^2} v_\varphi(r) + \frac{y^2}{r^2} \partial_r v_\varphi(r)$$

$$\Rightarrow \partial_x v_y - \partial_y v_x = \left(\frac{r - \frac{x^2}{r}}{r^2} + \frac{r - \frac{y^2}{r}}{r^2}\right) v_\varphi(r) + \frac{x^2 + y^2}{r^2} \partial_r v_\varphi(r)$$

$$= \frac{2r - \frac{\overbrace{x^2 + y^2}^{= r}}{r}}{r^2} v_\varphi(r) + \partial_r v_\varphi(r) = \frac{1}{r} v_\varphi(r) + \partial_r v_\varphi(r)$$

→ with $\frac{1}{r} \partial_r (r\, v_\varphi(r))\, e_z = \left(\frac{1}{r} v_\varphi(r) + \partial_r v_\varphi(r)\right) e_z$ we conclude

$$\mathrm{rot}(v) = (\partial_x v_y - \partial_y v_x)\, e_z = \left(\frac{1}{r} v_\varphi(r) + \partial_r v_\varphi(r)\right) e_z$$ H

$\partial_r$