5.)

$$-\frac{1}{Re} e_z \times \nabla \Delta V_\varphi(r) + \nabla \Pi = 0$$

$$\overset{2a}{\Rightarrow} -\frac{1}{Re} e_z \times \psi(r) e_r + \nabla \Pi = 0$$

$e_\varphi = e_z \times e_r$

$$\Rightarrow -\frac{1}{Re} \psi(r)\, e_\varphi + \nabla \Pi = 0$$

$\Pi = p + \frac{1}{2} \|v\|^2 - g$

Since $p = p(r)$ and $g = g(r)$

$\Rightarrow \Pi = \Pi(r)$

$\Rightarrow \nabla \Pi = \frac{d\Pi}{dr} e_r$

$$\Rightarrow -\frac{1}{Re} \psi(r)\, e_\varphi + \frac{d\Pi}{dr} e_r = 0$$

Since e_φ and e_r are linear independent

$$\Rightarrow -\frac{1}{Re} \psi(r) = 0 \quad \wedge \quad \frac{d\Pi}{dr} = 0$$

$$\Rightarrow \psi = 0 \quad \wedge \quad \Pi \equiv \text{const}$$