Problem 2: Signal detection theory and the psychometric function (20 points)

What are the parameter values for the parameters of the unequal variance receiver operating characteristics (ROC) for this experiment?

We have 5 response categories so our SDT model will have 4 criteria.

For the criterion, c_45 that lies between response category 4 and 5, the true positive is the response counts for exciting images for category 5. The false positive is the Response counts for not exciting images for category 5.

For the criterion, c_34 that lies between response category 3 and 4, the true positive is the response counts for exciting images summed over category 4 and 5. The false positive is the response counts for not exciting images summed over category 4 and 5.

The true and false positives for the other criteria are similarly calculated as sums over response counts. The true and false positive counts are listed in the table below

Response criteron	C_12	C_23	C_34	C_45
False positives	46	31	9	2
True positives	48	44	36	26

We can now estimate the probability of a true positive / false positive by dividing the response counts in the table by the total number of trials N=50.

We probit transform the estimated probabilies of a true positive / false positive using the inverse standard normal cumulative function

We the fit a straight line to the true positive probabilities as a function of the false positive probabilities. I get a slope of 0.5307 and an intercept of 1.0164.

According to eq 1.19 in the lecture notes, sigma = 1/slope = 1/0.5307=1.8842
According to eq 1.19 in the lecture notes, mu = intercept * sigma = 1.0164 * 1.8842 = 1.9152

Rebin the data by pooling responses in response categories 2-5 into a single response category. Calculate the sensitivity d' for the rebinned data. Rebin the data by pooling responses in response categories 1-4 into a single response category. Calculate the sensitivity d' for the rebinned data. Interpret your results.

Rebinning responses in response categories 2-5 corresponds to using criterion c_12 in the table above so the number of true positive responses = 48 and the number of false positive responses = 46 can be read off the table above. Again, dividing by the total number of trials = 50 gives an estimate of the underlying probabilities. Using equation 1.10 in the lecture notes I can calculate d’ = 0.3456

Rebinning responses in response categories 1-4 corresponds to using criterion c_45 in the table above so the number of true positive responses = 26 and the number of false positive responses = 2 can be read off the table above. Again, dividing by the total number of trials = 50 gives an estimate of the underlying probabilities. Using equation 1.10 in the lecture notes I can calculate d' = 1.8008

I get very different estimates of d' depending on which critera I use. This should NOT be the case. It indicates that the equal variance model does NOT hold. In agreement with this the estimate for sigma when fitting the roc curve was sigma =1.8842 > 1 which also indicates that the equal variance model does NOT hold

Problem 4: Linear encoding of faces (30 points)

The data set that comes with this exercise contains data from three selected images. What the model's predicted smile index of for each of the three faces? Explain how you calculated this. How do they compare to the true values?

Denote the regression parameters as a 20-dimensional vector w. Denote the principal score for image 1 as i_1. I get these from the first row of the PCA_Scores file. Denote the intercept of the regression model as delta. I get this from the file RegressionIntercept. According to equation 2.1 in the lecture notes I can calculate the smile index, x, as the dot product of the PCA score and the regression parameters, w, plus the intercept (setting the error term, epsilon to zero). This gives me the following values for the smile index,

x: 0.2409; 1.0879; 0.6533

for image 1, 2, and 3 respectively.

The model is not based on the actual images but on the PCA scores. Visualise the three faces reconstructed from the 20 principal components. Visualise the reconstructed faces and the corresponding actual images. Try to find an explanation for the discrepancies in predicted and true smile intensities from this visualisation

For each image I can make the reconstruction by multiplying the principla components by the principal component scores for that image

Reconstruction = PCAcomponents x scores^T

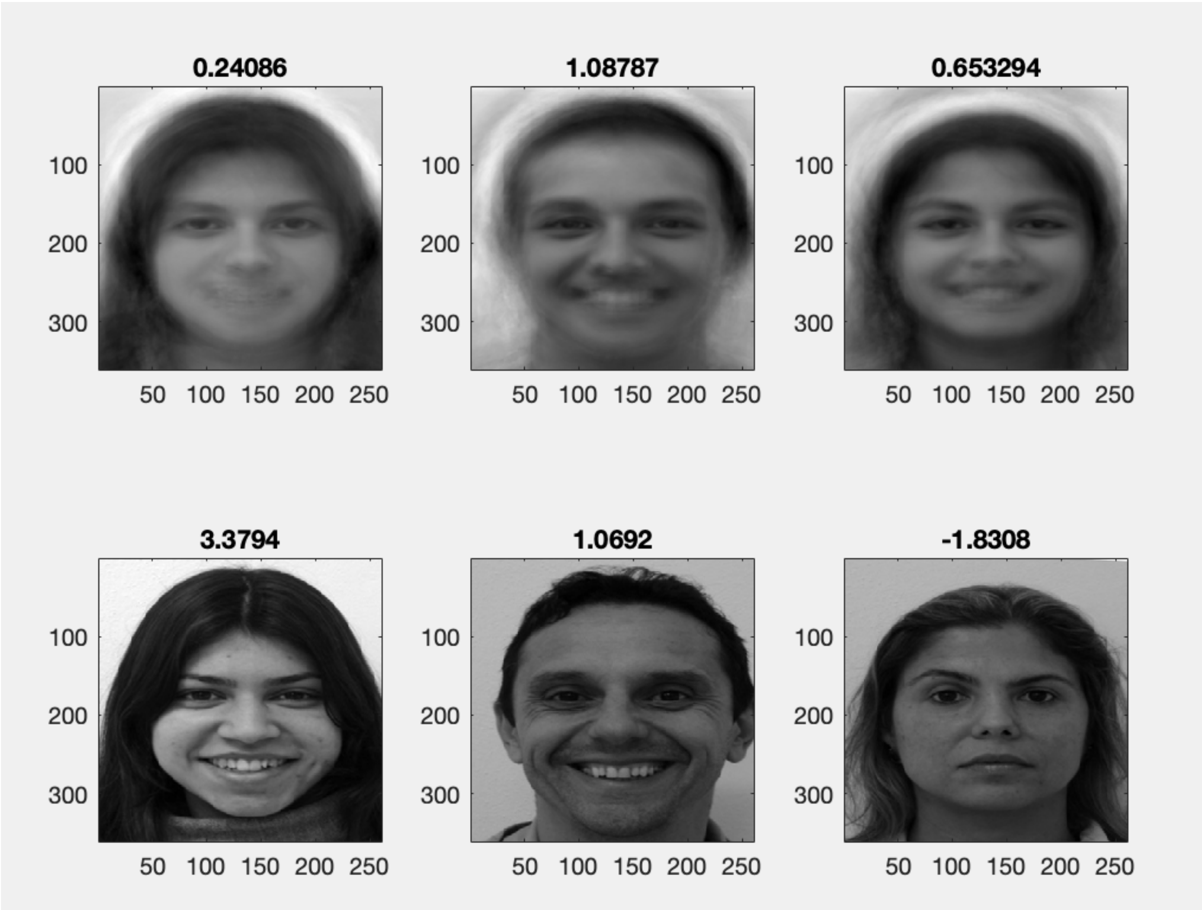
This gives me the following figure. The top row shows the reconstructed images and the smile index from the model. The lower row shows the original images and the true smile index from the SmileIndx file.

Image 1 (leftmost) has a very strong smile in the original image and in the true smile index. The model smile index is much lower. Accordingly, the smile is not very visible in the reconstruction.

Image 2 (center) has a moderate smile in the original image and in the true smile index. The model smile index is similar. Accordingly, the smile is also of similar strength in the reconstruction.

Image 3 (rightmost) has no smile in the original image and in the true smile index. The model smile index is higher. Accordingly, the smile is also of higher strength in the reconstruction.

We see that the model seems to be right according to the reconstructions, which is the data that was used to fit the model. The smiles of images 1 and 3 has been altered by dimension reduction.



Homework 2 we created synthetic images with specific levels of smile, gender or some other attribute. In this problem we want to make synthetic face in a different way: take a specific face and change it so that it gets a smile index of 0.2 without changing other attributes of the face. Do this for each of the three faces that are included in the data that comes with this exam. Plot the images.

To change the smile of the face I must add a vector, delta-I, to the image principal component scores so that

$$x = (\boldsymbol{i} + \Delta\boldsymbol{i})^T \boldsymbol{w} + \delta$$

Where x = 0.2 is the desired smile intensity.

To do this without changing other attributes of the face, delta-I, must be proportional to the weight vector

$$\Delta\boldsymbol{i} = \alpha\boldsymbol{w}$$

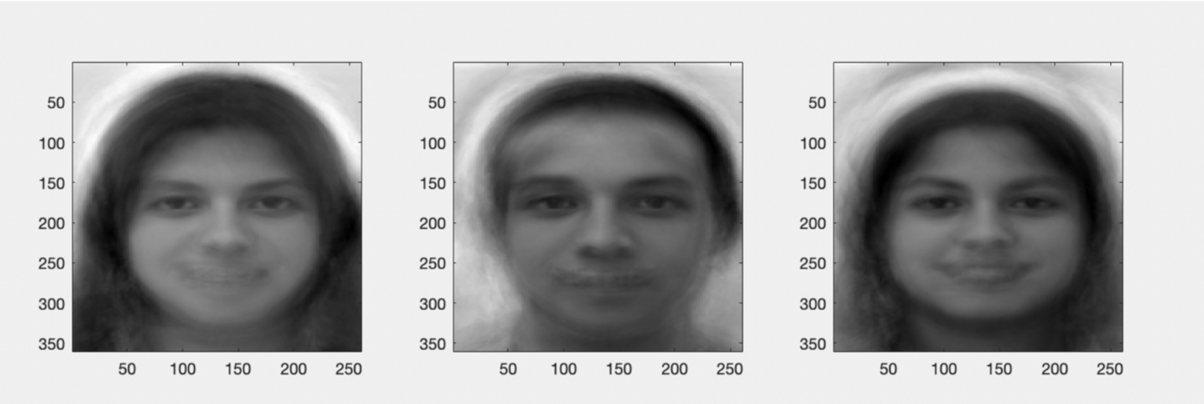
so that

$$x = (\boldsymbol{i} + \alpha\boldsymbol{w})^T \boldsymbol{w} + \delta$$

The only unknown is alpha so I can solve for alpha and calculate the scores for the new images as

$$(\boldsymbol{i} + \alpha\boldsymbol{w})$$

The resulting images are shown in the figure below



The principal component scores for the three images are listed below. Note that the matrix has been transposed for readability. The first column lists the values for image 1 and so forth.

−54.8643	7.4063	−37.7646
−5.6324	9.6285	−3.0627
−17.1888	−14.7135	−31.1497
7.8695	−3.4313	−20.1819
−1.0024	−13.8824	0.1457
21.0567	6.9361	6.5439
1.4660	17.1460	7.4842
−6.9052	15.9625	−2.4328
6.6983	−0.4162	−2.2417
−0.5919	−2.6202	−0.7957
−4.0305	6.1148	2.6272
4.6553	0.7249	−3.9462
−7.9285	−2.6772	2.6987
0.0499	1.7973	0.4454
0.2009	−5.4978	−1.8028
−1.6682	−6.7500	−6.1235
0.7095	−0.0992	−3.1468
−0.7068	−6.4932	−2.7551
−2.4219	−3.2927	−4.0333
−16.5432	−8.2624	5.1273