DTU Compute
Department of Applied Mathematics and Computer Science

Cognitive Modeling Lecture Notes

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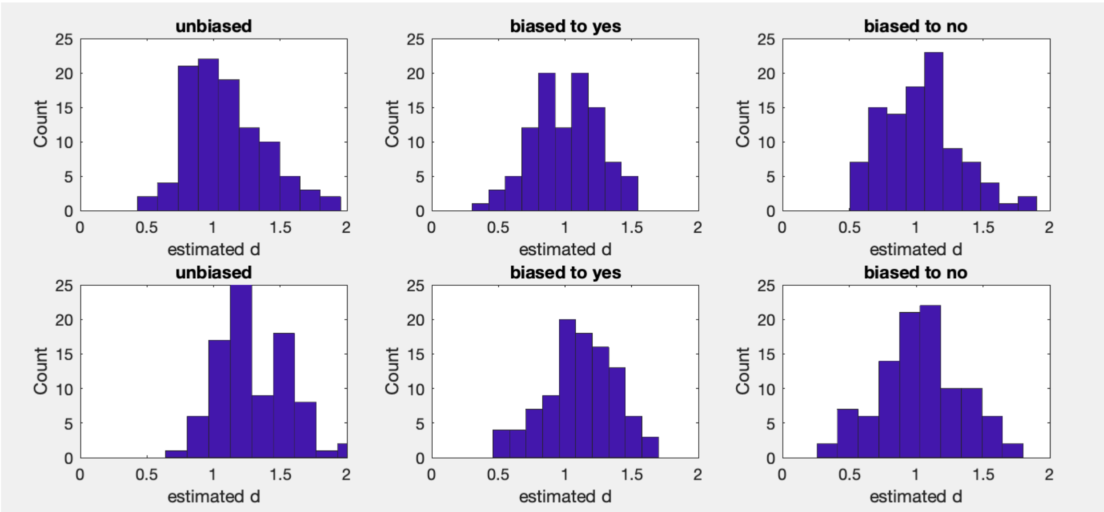
Contents

0.1 Equal variance model

We have 3 observers with criteria $c_u = 0.5$, $c_y = 0.0$ and $c_u = 1.1$ that are unbiased, biased towards yes-responses, and biased towards no-responses respectively. They all have sensitivity $d' = 1$. To sample the internal representation value, x, for $N = 50$ trials with no stimulus we draw 50 samples from $x \sim \mathcal{N}(\mu = 0, \sigma = 1)$ for each of the three observers. To obtain the number of false positives, n_{fp} we count the trials for which $x > c$ for each observer. Likewise, to sample the internal representation value, x, for $N = 50$ trials with stimulus we draw 50 samples from $x \sim \mathcal{N}(\mu = 1, \sigma = 1)$. To obtain the number of true positives, n_{tp} we count the trials for which $x > c$ for each observer. This allows us to estimate the perceptual sensitivity, d' as $d' = \Phi^{-1}\left(\frac{n_{tp}}{N}\right) - \Phi^{-1}\left(\frac{n_{fp}}{N}\right)$.

We repeat the process described above 100 times for each criterion to simulate 100 experiments. A histogram of the obtained values for the estimate of d' for each observer are shown in the top plots in the figure below. Taking the average across experiments we found that the estimate for d' was 1.09, 1.01 and 1.03 for $c_u = 0.5$, $c_y = 0.0$ and $c_u = 1.1$ respectively. This shows that the estimated values for d' do not vary much from the true value $d' = 1$ regardless of the criterion, which is what we would expect.

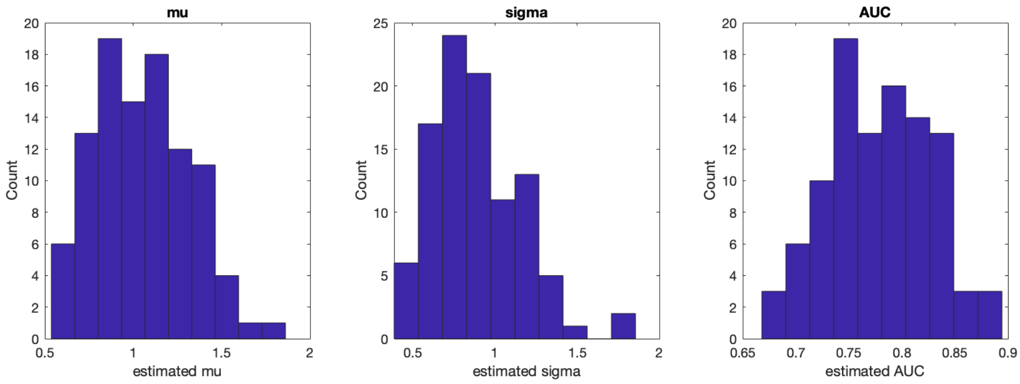


We then repeat the process but for unequal variance observers for which $\sigma = 0.8$. The process is the same as before except that we sample the internal representation value, x, for $N = 50$ trials with stimulus by drawing 50 samples from $x \sim \mathcal{N}(\mu = 1, \sigma = 0.8)$. A histogram of the obtained values for the estimate of d' for each observer are shown in the bottom plots in the figure above. Taking the average across experiments we found that the estimate for d' was 1.30, 1.12 and 1.03 for $c_u = 0.5$, $c_y = 0.0$ and $c_u = 1.1$ respectively. This shows that the estimated values for d' varies quite a bit from the true value $d' = 1$ depending on the criterion, which is not what we would expect. This is because d' is not a correct measure of sensitivity for an unequal variance observer.

0.2 ROC curves

We have one observer responding in four ordered categories, so we need three response criteria, which we chose to be the same as in the previous problem. The sampling of internal representation values, x, is done as in the unequal variance model above. Trials for which $x > 1.1$ are counted as 'yes - high confidence' responses. Trials for which $0.5 < x < 1.1$ are counted as 'yes - low confidence' responses. Trials for which $0.0 < x < 0.5$ are counted as 'no - low confidence' responses. Remaining trials are counted as 'no - high confidence responses.

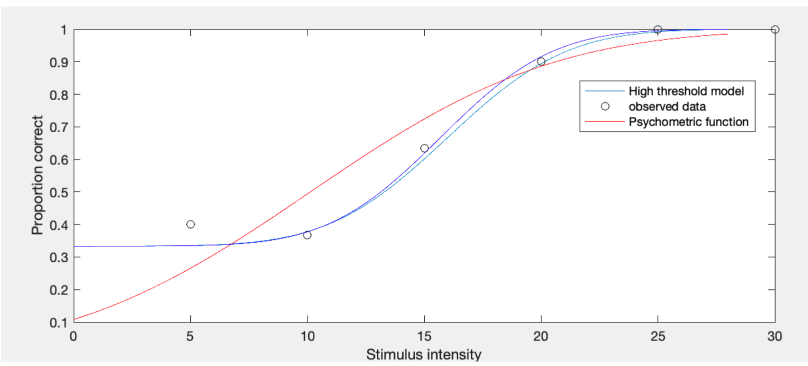
We then count the number positive responses for each response criterion. For $c = 1.1$ only 'yes - high confidence' responses count as positive responses. For $c = 0.5$ all 'yes'-responses counts as positive responses. For $c = 0.0$ all 'yes'-responses *and* 'no - low confidence'-responses count as positive responses. True positive responses are responses to a stimulus trial and false positive responses are responses to a no-stimulus trials. This give us three points on the probit transformed ROC curve $\Phi^{-1}(P_{tp}) = \frac{1}{\sigma}\Phi^{-1}(P_{fp}) + \frac{\mu_s}{\sigma}$ by fitting a line to these points we can estimate the slope, a and the intercept, b, and calculate $\sigma = \frac{1}{a}$ and $\mu_s = b\sigma$. The process is repeated 100 times to simulate 100 experiments. For each experiment we also calculate the AUC using Eq. 1.18 in the lecture notes. The distribution of the estimated parameter values and the AUC are shown in the histograms below. Averaging the estimates of μ_s and σ across experiments produced the values 1.08 and 0.88 respectively showing that the distributions are approximately centered around the true underlying values. The estimated values for AUC are centered around 0.78. We calculate the true underlying value for AUC by inserting the true underlying values for μ_s and σ into Equation 1.18, which yields a value of 0.78. Hence the estimated AUC values are centered around the true underlying AUC as we would expect.



0.3 The psychometric function

We find the optimal parameter values for c and σ using an optimisation routine that maximises the log likelihood (Lecture notes Eq. 1.26) summed over stimulus intensities, I_s, where $P_s = \Psi(I_s)$ is given by the psychometric functions in Eq. 1.19, 1.23 and 1.24, n_s is the number of correct listed in the table and $N_s = 30$.

The three psychometric functions are plotted with the observed response proportions below. From visual inspection we find that uncorrected psychometric gives a poor fit whereas the high threshold fit the data better. Correcting also for lapsing does not seem to influence the fit very much.



For the uncorrected psychometric function (Eq. 1.19) we find that $\sigma = 8.19$ and $c = 10.13$. The negative log likelihood was found to be 11.32. The number of free parameters is 2. The AIC is therefore $2(2 + 11.32) = 26.64$.

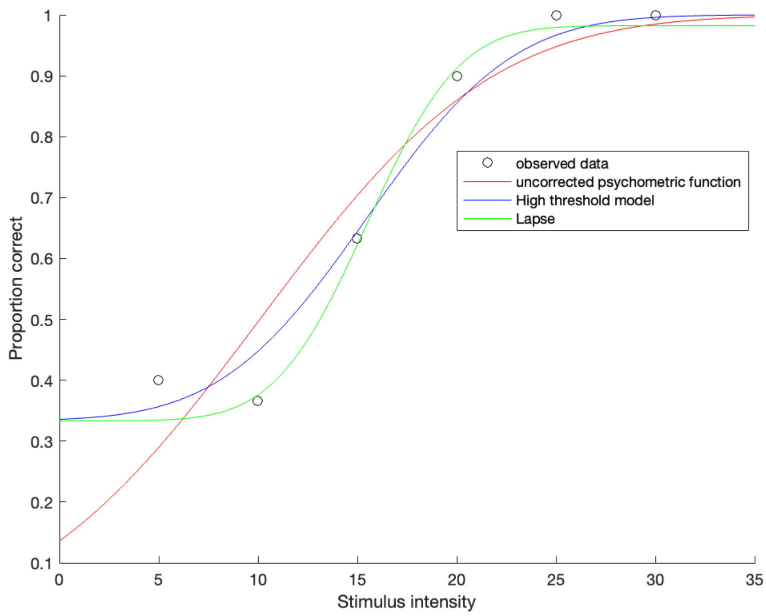
For the high-threshold (Eq. 1.23), $P_{guess} = \frac{1}{3}$ because $N_r = 3$. We find that $\sigma = 3.78$ and $c = 15.69$. The negative log likelihood was found to be 7.64. The number of free parameters is 2. The AIC is therefore $2(2 + 7.64) = 19.28$.

For the lapse rate corrected psychometric function in Equation 1.24 with the lapse rate as a free parameter. We find that $\sigma = 3.78$, $c = 15.69$ and $P_{lapse} = 0.00$. The negative log likelihood was found to be 7.64. The number of free parameters is 3. The AIC is therefore $2(3 + 7.64) = 21.28$.

We notice that the parameter estimates and the log likelihood changed a lot when we correct for guessing but not when we corrected also for lapsing. Also the log likelihood was lower when correcting for guessing but was not influenced by correcting also for lapsing. This is an agreement with the parameter estimate for $P_{lapse} = 0.00$ indicating that the observer did not lapse. The AIC was lowest for the model correcting for guessing but not for lapsing because it fit the data better than the uncorrected psychometric function and because the additional free parameter when correcting for lapsing did not improve the fit very much.

Now we do the same after modifying the number of correct responses is to $N_c = 29$ for the highest stimulus intensity level.

The three psychometric functions are plotted with the observed response proportions below. From visual inspection we find that uncorrected psychometric gives a poor fit whereas the high threshold fit the data better. Correcting also for lapsing provides an even better fit.



For the uncorrected psychometric function (Eq. 1.19) we find that $\sigma = 9.19$ and $c = 10.09$. The negative log likelihood was found to be 12.41. The number of free parameters is 2. The AIC is therefore $2(2 + 12.41) = 28.82$.

For the high-threshold (Eq. 1.23), $P_{guess} = \frac{1}{3}$ because $N_r = 3$. We find that $\sigma = 5.79$ and $c = 15.48$. The negative log likelihood was found to be 11.21. The number of free parameters is 2. The AIC is therefore $2(2 + 11.21) = 26.42$.

For the lapse rate corrected psychometric function in Equation 1.24 with the lapse rate as a free parameter. We find that $\sigma = 3.64$, $c = 15.49$ and $P_{lapse} = 0.02$. The negative log likelihood was found to be 9.25. The number of free parameters is 3. The AIC is therefore $2(3 + 9.25) = 22.50$.

We notice that the parameter estimates changed a lot when we correct for guessing and also we corrected also for lapsing. Also the log likelihood was lower when correcting for guessing but even lower when correcting also for lapsing. This is an agreement with the parameter estimate for $P_{lapse} = 0.02$ indicating that the observer did lapse. Accordingly, the AIC was lowest for the model correcting for lapsing.

Guessing and lapsing did influence the parameter estimates. Note that the model corrected for lapsing provided similar estimates of parameters c and σ regardless of whether the observer lapsed or not whereas the parameter estimates of the other models were very much influenced by the observer lapsing. This shows that this model provide more robust parameter estimates even though it may have a higher AIC when the observer does not lapse.

0.4 Magnitude estimation

We first calculate the perceived intensity, I_p for stimulus intensities, $I_s = 1, 2, \ldots, 10$, using Stevens' law $I_p = 10 I_s^a$ with $a = 0.33$. We then fit Fechner's law to these simulated data noticing that Fechner's law can be written as $I_p = \frac{1}{k_w} \ln I_s - \frac{1}{k_w} \ln I_0$, so that we can fit Fechner's law by linear regression with respect to $\ln I_s$ and with slope, $a = \frac{1}{k_w}$, and intercept, $b = -\frac{1}{k_w} \ln I_0$. This fit is plotted in the left plot below along with the simulated data. We observe that the fit is very good. The parameter values obtained from the fit are $k = 0.20$ and $I_0 = 0.16$. We then repeat the process but for $a = 3.3$ and fit Fechner's law again. This fit is plotted in the right plot below along with the simulated data. We observe that the fit is very poor. The parameter values obtained from the fit are $k = 0.00014$ and $I_0 = 2.04$. We conclude that Fechner's law can mimic Stevens' law for $a < 1$ but not when $a > 1$.