Problem 1 (10 points)

When two flashes of light are presented in rapid sequence it can be difficult to perceive whether there were two flashes or just a single flash. To test this, we have conducted an experiment in which observers had to respond whether they saw 1 or 2 flashes. We have recorded the responses as the number of trials in which the observer responded to have seen two flashes. The data is in the table below. Each type of stimulus was presented 20 times.

1 flash	2 flashes
3	12

Calculate the perceptual sensitivity, *d*', for the observer.

Problem 2 - 25 points

We have extended the experiment in Problem 1 by presenting 1-3 flashes to the observer. The response options are '1', '2' and '3'. The results in terms of response counts for each stimulus type are in the table below. Each type of stimulus was presented 20 times.

	Response 2	Response 2	response 3
Stimulus 1 flash	15	3	2
Stimulus 2 flashes	5	12	3
Stimulus 3 flashes	4	2	14

Fit a psychometric function to responses from response category 3. Fit another psychometric function to the sum of responses from category 2 and 3. List the parameters of the two psychometric functions. Plot the functions and the data that you fitted them to. Explain your approach.

Fit an unequal variance receiver operating characteristics (ROC) curve to stimulus 1 and 2. Fit an unequal variance receiver operating characteristics (ROC) curve to stimulus 1 and 3. List the parameters of the ROC curves. Plot the ROC curves and the data you have fitted them. Explain your approach.

Problem 3 - 30 points

In another experiment the observer was presented with beeps, flashes or both beeps and flashes. Observers had to respond whether they saw 1 or 2 flashes. We have recorded the responses as the number of trials in which the observer responded to have seen two flashes. The data are in the tables below. Each type of stimulus was presented 20 times. For audiovisual stimuli, the observer reported the number of perceived flashes, but this will not be important for your analysis.

1 beep	2 beeps
1	20

1 flash	2 flashes
6	12

	1 beep	2 beeps
1 flash	6	18
2 flashes	5	20

Fit the strong fusion probability matching model (also known as the Fuzzy Logical Model of Perception) and the early strong fusion model to the data. Write the exact expression for the likelihood function you have used to fit the model. List the parameter values and the negative log likelihood of the data given the model. Explain you approach.

Problem 4 - 35 points

In Homework 2 we built a linear regression model of the encoding of a facial feature from the principal components of images. Here we will use similar methods on a similar data set.

The data are provided to you in two .csv files

- In the images.txt file you will find a 400-by-3744 matrix. Each column of the matrix contains a 72-by-52 pixel greyscale image with values ranging from 0 (black) to 1 (white). The first 52 entries of each column contain the first column of pixels of an image. The next 52 entries contain the second column of pixels in an image and so forth.
- In the smile_index file you will find a 400-dimensional vector. Each entry corresponds to the perceived strength of the smile in the image in the corresponding column of the matrix in the file images.txt

4.1 Perform Principal Component Analysis on the images and determine how many principal components are needed to contain 90% of the total variance in the image set. Reconstruct the image in row 23 using only those components. Compare it to the original image and evaluate the reconstruction.

4.2 Using forward selection, the first principal component has not been selected but the 16th principal component has been selected. Each of these two components account for a proportion of the total variance in the set of images. Calculate this proportion for each of the two components and compare the values.

4.3 Find the image that has the lowest principal component score for the first principal component. Find the image that has the highest principal component score for the first principal component. Reconstruct the two images using only the first principal component (and the average image). Do the same for the 16th principal component. Describe the image features that each of the two components seem to capture.

4.4 Calculate the covariance between the smile intensity and the scores for the first principal component. Calculate the covariance between the smile intensity and the scores for the 16th principal component. Compare the two values.

4.5 (5 points) Discuss why forward selection would select the 16th principal component and not the first in relation to your answers to 4.2-4.4

4.6 Calculate the covariance between the smile intensity and the scores for each principal component. (This is the same as you did in question 4.4 but now you should do it for all the principal components). Find the three principal component for which this covariance has the highest *absolute* value. (The absolute value of -3 is greater than 2).

4.7 Fit two linear regression models to the data. In both models, the dependent variable (*y*) is the smile intensity, and the independent variable (*x*) is the scores of principal components. Use the scores of principal components 1, 2 and 3 in one model. Use the scores of principal components that you found in question 4.6 in the other model.

List the parameter values for both models.

Create a set of 5 synthetic images from each model. These should have a smile intensity of -0.5, 0, 0.5, 1.0 and 1.5. Use the same method as you used in Homework 2. Explain your approach.

Describe the images you have created. Which of the two models capture a smile better? Which set is more influenced by other visual features? Discuss your result in relation to questions 4.2 - 4.5.

End of exam