Homework 3

Solution overview

NOTE

All parameter values and neg log likelihoods are in the file

Homework3solutiontables.xlsx

Part 1

Homework Project 3 - part 1

In this project, you will analyse data from one observer an experiment in which one observer was presented with 8 types of stimuli.

- Sound at an azimuthal angle of 0 or 30 degrees
- Light at an azimuthal angle of 0 or 30 degrees
- All 4 audiovisual combinations of the sound and light.

Each type of stimulus was presented 20 times. The order of the stimulus presentations was randomised.

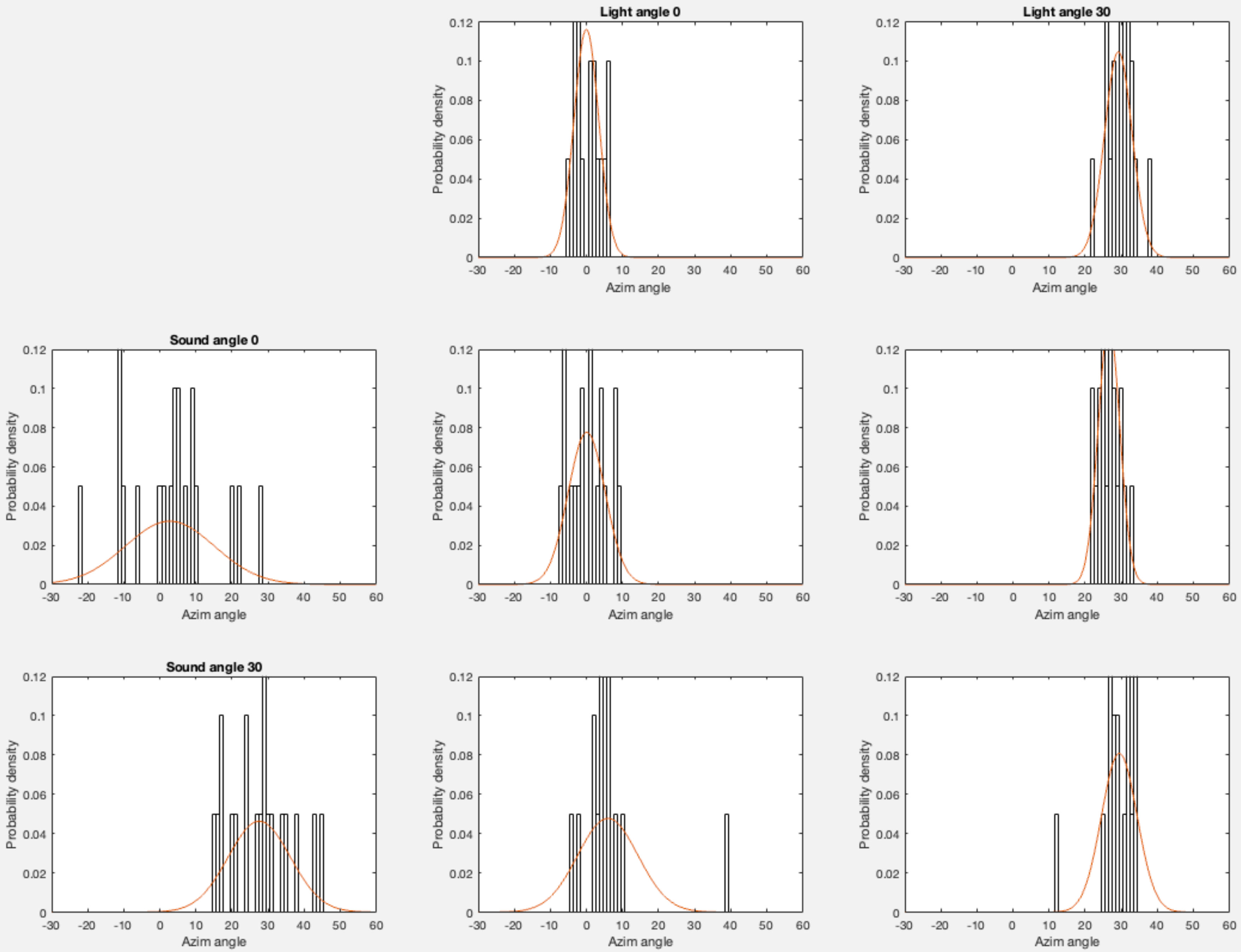
The task of the observer was to point a laser pointer towards the sound for audio and audiovisual stimuli and towards the light for visual stimuli. The azimuthal angle of the response was recorded in 1 degree intervals but you can assume that the responses are distributed on a continuum.

Part 1

Visualising the data

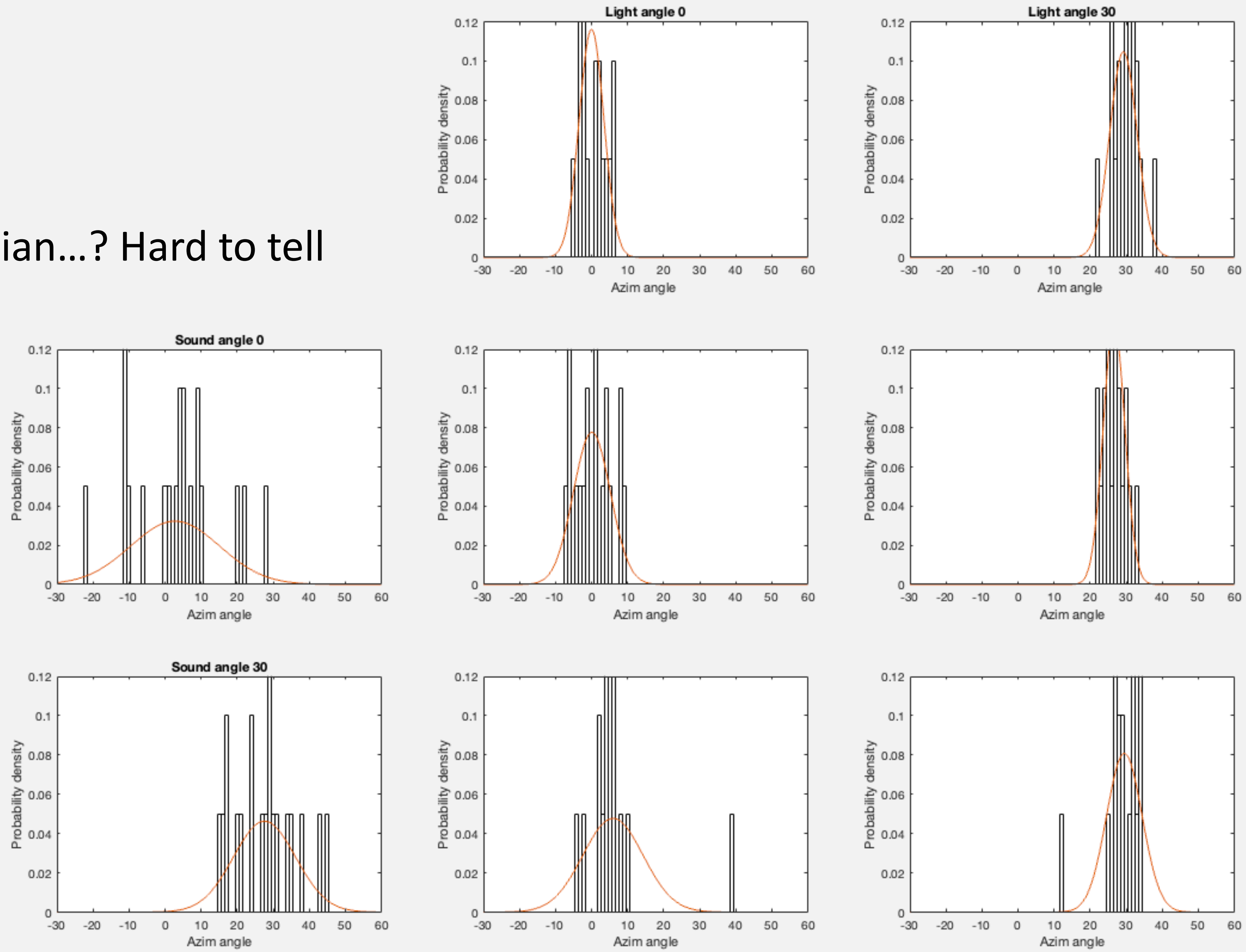
- Plot a probability density histogram for each type of stimulus. This height of each bar should be the proportion of responses divided by the width of the column.
- Calculate the mean and standard deviation of the responses for each type of stimulus. Plot the corresponding Gaussian probability density function.
- Based on the above, estimate whether the observer experiences the enhancement effects and the ventriloquist illusion.
- Check by visual inspection if the responses for each stimulus type follows a Gaussian distribution.

Part 1


Light angle 0
Light angle 30
Sound angle 0
Sound angle 30
Probability density
Azim angle

Part 1

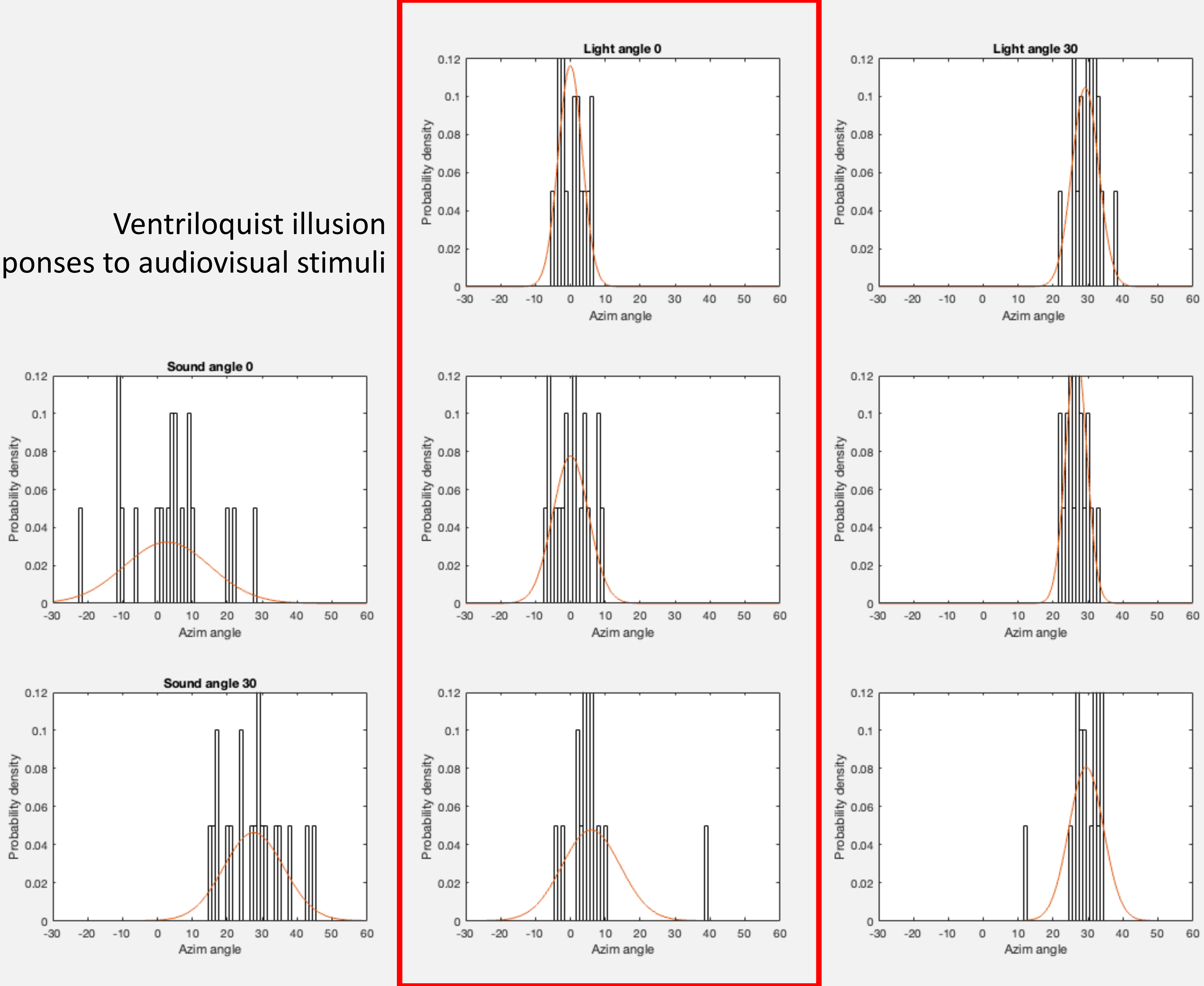
Gaussian…? Hard to tell



Part 1

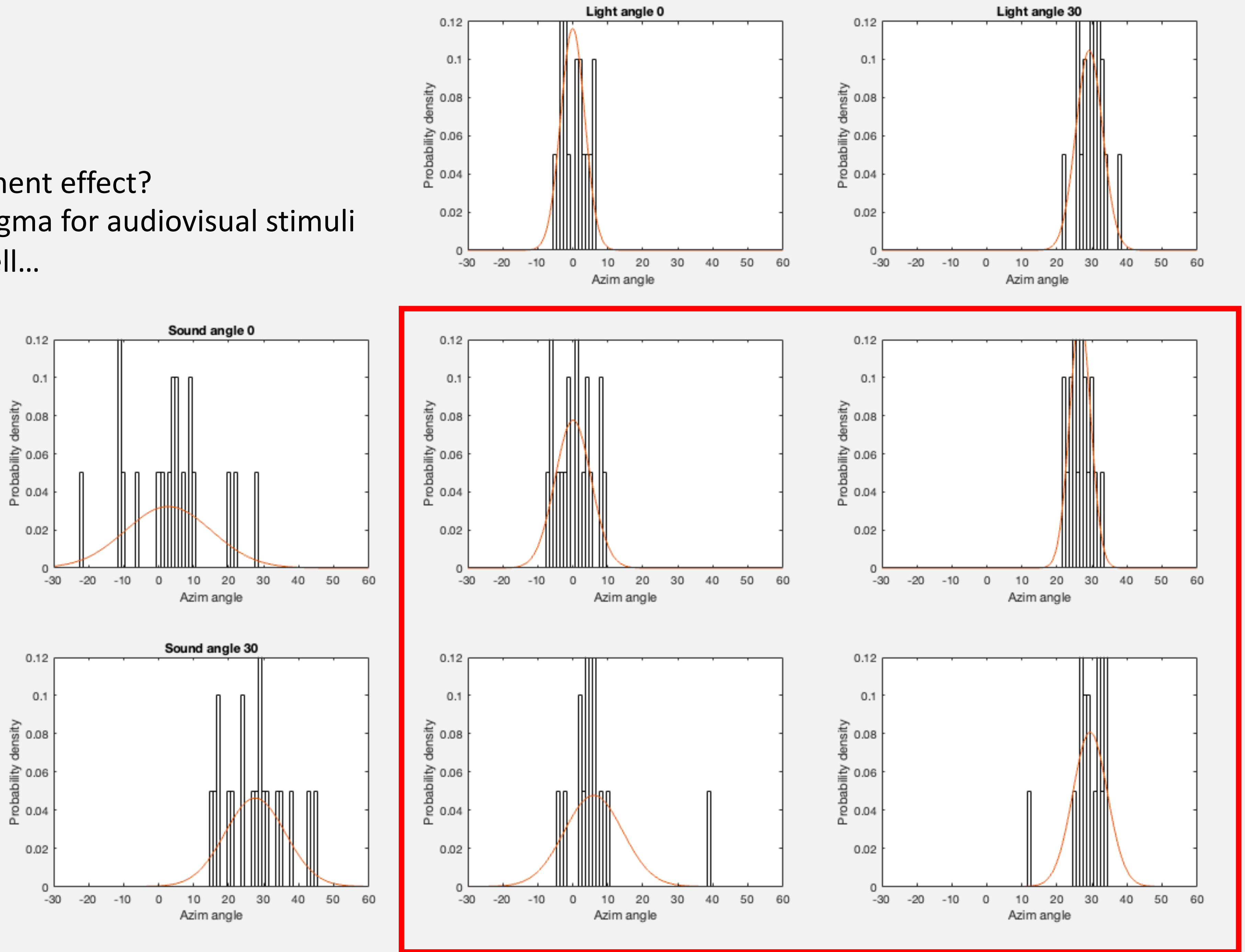
Ventriloquist illusion

Visual stimulus dominates responses to audiovisual stimuli



Part 1

Enhancement effect?
Smaller sigma for audiovisual stimuli
Hard to tell…



Part 1

Fit the Strong Fusion (MLE) model to the data

- Use all the data to estimate the free parameters (the mean and standard deviation for each of the two auditory stimuli and each of the two visual stimuli).
- List the values of the parameter estimates and the negative log likelihood of the fits.
- Plot the models' probability density and the corresponding histogram for each stimulus type. Evaluate the model fits by visual inspection.

Part 1

- How to fit a
  - What is the distribution? Gaussian
  - What are the parameters of the distribution? μ and σ
  - Are all the model parameters free? No, the MLE model is a constraint

$$\mu_{av} = w_a\mu_a + (1 - w_a)\mu_v \tag{3.21}$$

$$\sigma_{av}^2 = w_a^2\sigma_a^2 + (1 - w_a)^2\sigma_v^2 = \frac{\sigma_a^2\sigma_v^2}{\sigma_a^2 + \sigma_v^2} \tag{3.22}$$

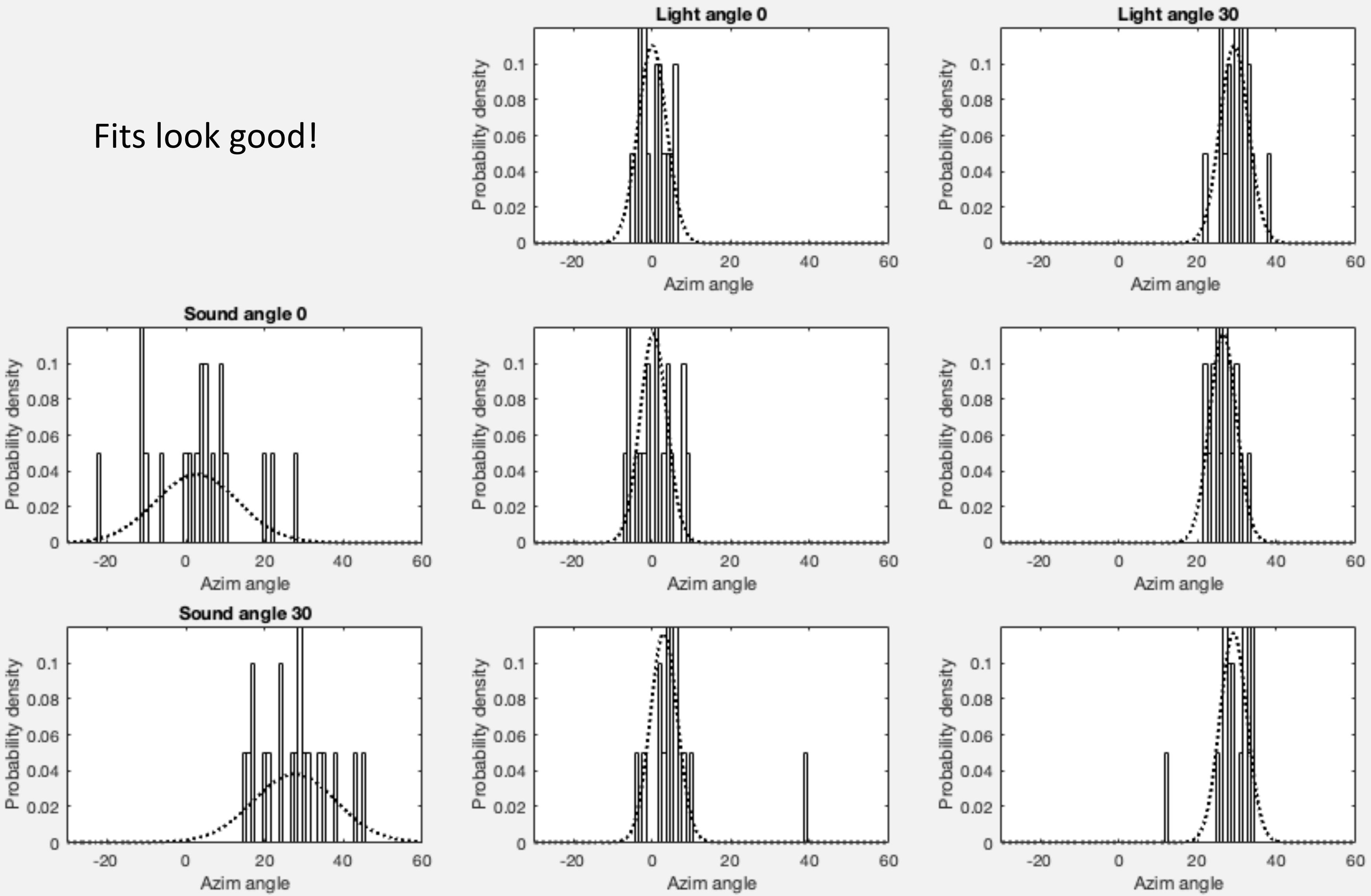
$$w_a = \frac{\sigma_v^2}{\sigma_a^2 + \sigma_v^2} \tag{3.19}$$

Part 1

- Free parameters: μ and σ for auditory and visual stimuli
- Constrained parameters μ and σ for audiovisual stimuli
- From this we can calculate the Gaussian log likelihood
- Optimisation routine can find the optimal parameters that minimise the negative log likelihood

Part 1

Fits look good!



Part 1

Check for over-fitting using cross-validation

- Check that the results of the model fits are not due to over-fitting using 8-fold leave-one-stimulus out cross-validation. In each fold, the validation set consist of the responses to one type of stimulus. The training set is the responses to the other 7 types of stimuli.
- List the validation negative log likelihood for the two models. Compare your finding to the model fits.
- Visualise the results of the cross-validation. For each fold/stimulus type, calculate the probability density for the validation stimulus using the parameters estimated from the training set. Plot the corresponding Gaussian probability density function for each stimulus type and the corresponding histogram. Evaluate the cross-validation results by visual inspection.

Part 1

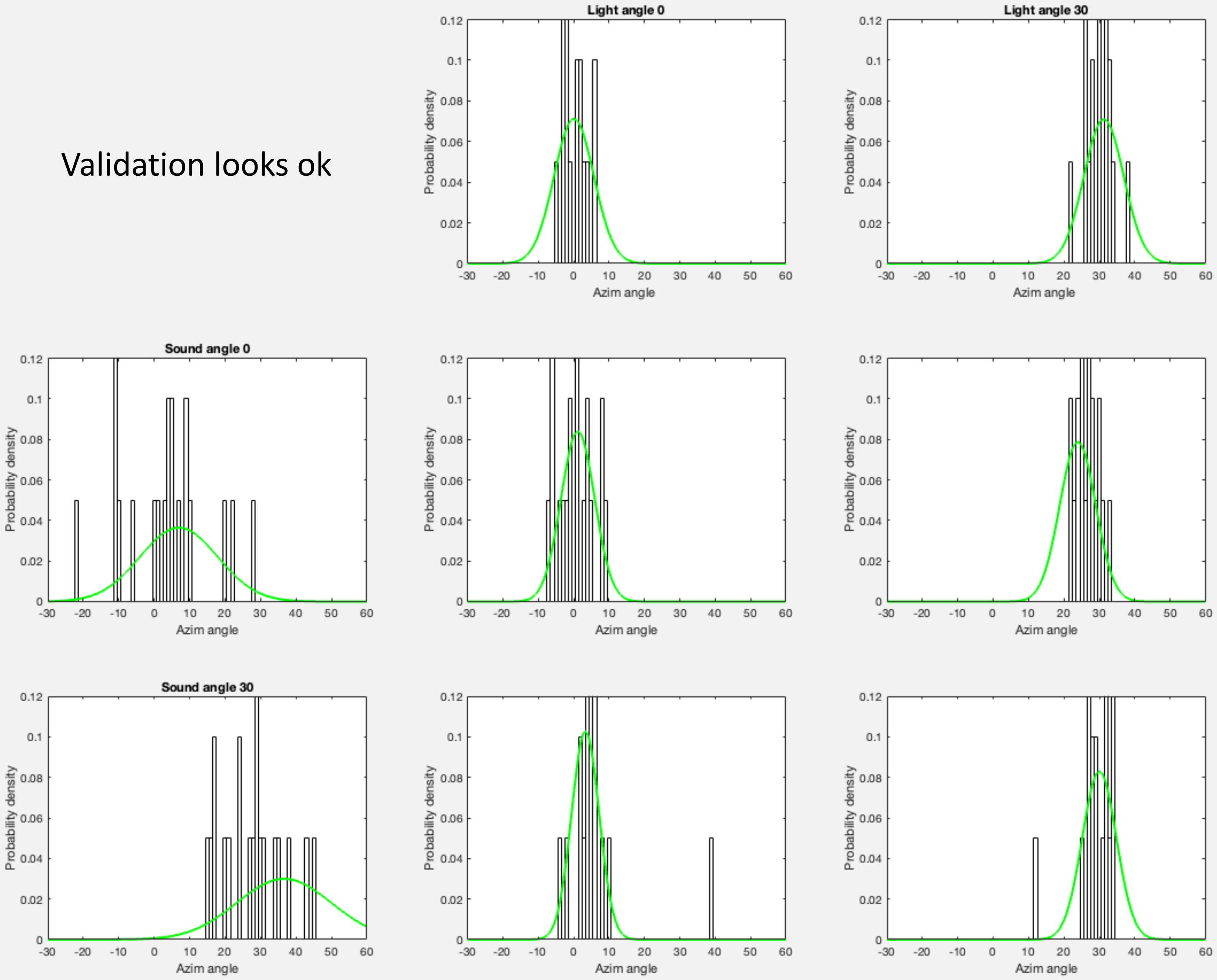
- Fit the model as before but leave the data from one stimulus out
- Calculate the validation negative log likelihood for the stimulus left out
- Repeat for all stimuli
- Validation negative log likelihood can be summed over stimuli
- Comparable to the negative log likelihood from fitting the model to all the data
- Higher validation negative log likelihood indicates over-fitting

Part 1

- Fit the model as before but leave the data from one stimulus out
- Estimate parameters for the stimulus left out
- Plot the probability distribution on top of the histograms as before
- Repeat for all stimuli

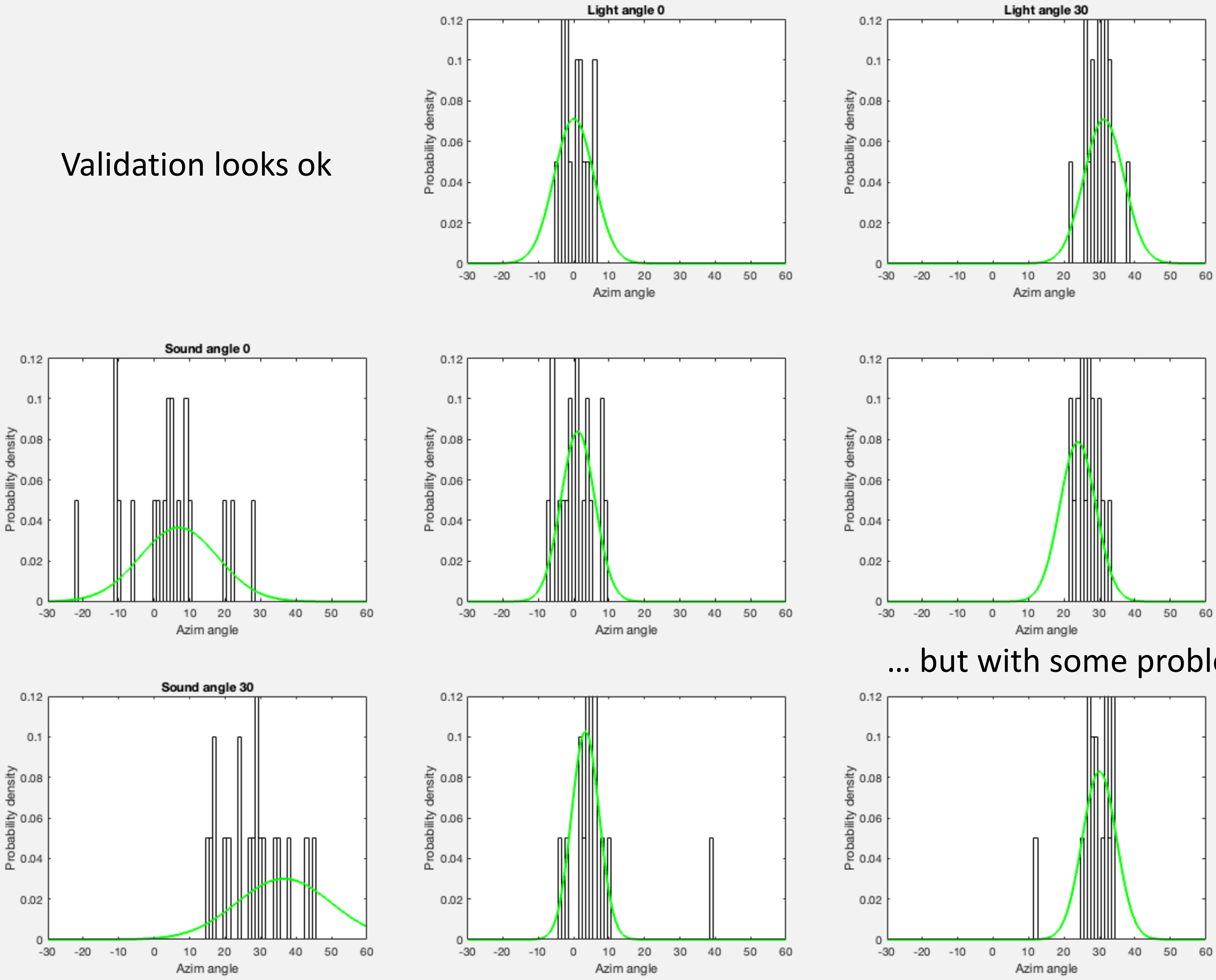
Part 1

Validation looks ok



Part 1

Validation looks ok



… but with some problems

Part 2

Homework Project 3 - part 2

In this part of the homework project we will work with the Bayesian model of signal detection under the unequal variance assumption. Calculate the probability of a true positive response and the probability of a false positive response for an observer, which behaves according to a Bayesian model with perceptual sensitivity parameters $\mu = 1.2$ and $\sigma = 1.8$ for prior probability *P*(*s*)=1/3 and for prior probability *P*(*s*)=2/3.

Fit the unequal variance signal detection theory model to the response probabilities that you calculated above. List the parameter values for perceptual sensitivity parameters μ and σ. Compare them to the true underlying values and discuss your result.

Part 2

Homework Project 3 - part 2

In this part of the homework project we will work with the Bayesian model of signal detection under the unequal variance assumption. Calculate the probability of a true positive response and the probability of a false positive response for an observer, which behaves according to a Bayesian model with perceptual sensitivity parameters $\mu = 1.2$ and $\sigma = 1.8$ for prior probability $P(s)$=1/3 and for prior probability $P(s)$=2/3.

Fit the unequal variance signal detection theory model to the response probabilities that you calculated above. List the parameter values for perceptual sensitivity parameters μ and σ. Compare them to the true underlying values and discuss your result.

Part 2

- Take the MAP rule and 'convert' it to criteria using Eq. 3.8

$$\frac{1}{2}\left(1-\frac{1}{\sigma^2}\right)x^2+\frac{\mu}{\sigma^2}x-\frac{\mu^2}{2\sigma^2}-\log(\sigma)>\log\left(\frac{(1-P(s))}{P(s)}\right) \tag{3.8}$$

- Solving this quadratic equation for *x* is straightforward
- Gives me two criteria, c_{lo} and c_{hi}, for each prior probability
- Since $\sigma > 1$ the solution is $x < c_{lo} \wedge x > c_{hi}$
- Therefore the probability, P_{tp}, of a true positive is
  - $P_{tp} = \Phi\left(\frac{c_{lo}-\mu}{\sigma}\right) + \left(1-\Phi\left(\frac{c_{hi}-\mu}{\sigma}\right)\right)$
- Same for the probability, P_{fp}, of a false positive except $\sigma = 1$ and $\mu = 0$, so
  - $P_{fp} = \Phi(c_{lo}) + \left(1-\Phi(c_{hi})\right)$

Part 2

Homework Project 3 - part 2

In this part of the homework project we will work with the Bayesian model of signal detection under the unequal variance assumption. Calculate the probability of a true positive response and the probability of a false positive response for an observer, which behaves according to a Bayesian model with perceptual sensitivity parameters $\mu = 1.2$ and $\sigma = 1.8$ for prior probability $P(s)$=1/3 and for prior probability $P(s)$=2/3.

Fit the unequal variance signal detection theory model to the response probabilities that you calculated above. List the parameter values for perceptual sensitivity parameters μ and σ. Compare them to the true underlying values and discuss your result.

Part 2

- I now have two points on my ROC curve (Eq. 1.13)

$$\Phi^{-1}\left(P_{TP}\right) = \frac{1}{\sigma}\Phi^{-1}\left(P_{FP}\right) + \frac{\mu_s}{\sigma} \tag{1.13}$$

- Probit transform
- Fit straight line
- Derive the parameters, μ and σ, from the slope and intercept

Part 3

Homework Project 3 - part 3

In this part of the homework project we will fit three Bayesian models of multisensory integration to data from an experiment on audiovisual speech perception. The three models are described in Section 3.3 of the lecture notes: The early strong fusion model, the probability matching model and the late strong fusion model. The data has been used for testing Bayesian models of audiovisual integration in the literature and is described in the lecture notes and in the paper by Andersen (JASA, 2015).

Your report

Your answer to this part of the homework assignment should contain the following

- The exact formula for the negative log likelihood function you used to fit each of the three model to the data. Specify the free parameters and data in the formula.

Part 3

- Binomial negative log likelihood

$$\log\left(\mathcal{L}\left(n_s \mid \sigma_I, c_)\right)\right) = \sum_{i=1}^{N_s} \log(i) - \sum_{i=1}^{n_s} \log(i) - \sum_{i=1}^{N_s - n_s} \log(i) + n_s \log(P_s) + (N_s - n_s)\log(1 - P_s) \quad (1.26)$$

- Where P_s is the response probability for a given stimulus
  - Sum over stimuli

- The equations for P_s $_d$epends on the model

Part 3 – Early MLE

The free parameters are c_a, c_v, σ_a and σ_v.

From these, we can calculate $\tilde{\mu}_{a,i} = I_i - c_a$ where $i = 1, ..., 5$ and $\tilde{\mu}_{v,j} = I_j - c_v$ where $j = 1, ..., 5$.

Then we calculate

$$\tilde{\mu}_{av,i,j} = w_a \tilde{\mu}_{a,i} + (1 - w_a)\tilde{\mu}_{v,j}$$

where the weight, w_a, is given by

$$w_a = \frac{\sigma_v^2}{\sigma_a^2 + \sigma_v^2}$$

We also calculate

$$\sigma_{av}^2 = w_a^2 \sigma_a^2 + (1 - w_a)^2 \sigma_v^2 = \frac{\sigma_a^2 \sigma_v^2}{\sigma_a^2 + \sigma_v^2}$$

Finally we calculate the response probabilities as

$$P_{a,i} = \Phi\left(\frac{\tilde{\mu}_{a,i}}{\sigma_a}\right)$$

$$P_{v,j} = \Phi\left(\frac{\tilde{\mu}_{v,j}}{\sigma_v}\right)$$

$$P_{av,i,j} = \Phi\left(\frac{\tilde{\mu}_{av,i,j}}{\sigma_{av}}\right)$$

Part 3 – Probability matching (FLMP)

The free parameters are $z_{a,i}$ where $i = 1, ..., 5$ and $z_{v,j} = I_j - c_v$ where $j = 1, ..., 5$.

From the free parameters we calculate the response probabilities

$$P(\hat{S}_i \mid x_a) =$$

Finally we calculate the response probabilities as

$$P_{a,i} = \frac{\exp z_{a,i}}{\exp z_{a,i} + 1}$$

$$P_{a,i} = \frac{\exp z_{v,j}}{\exp z_{v,j} + 1}$$

$$P_{av,i,j} = \frac{P_{a,i} P_{v,j}}{P_{a,i} P_{v,j} + (1 - P_{a,i})(1 - P_{v,j})}$$

Part 3 – Late MLE

The free parameters are c_a, c_v, σ_a and σ_v.

From these, we can calculate $\tilde{\mu}_{a,i} = I_i - c_a$ where $i = 1, ..., 5$ and $\tilde{\mu}_{v,j} = I_j - c_v$ where $j = 1, ..., 5$.

From this we calculate the response probabilities as

$$P_{a,i} = \Phi\left(\frac{\tilde{\mu}_{a,i}}{\sigma_a}\right)$$

$$P_{v,j} = \Phi\left(\frac{\tilde{\mu}_{v,j}}{\sigma_v}\right)$$

$$P_{av,i,j} = \frac{P_{a,i}P_{v,j}}{P_{a,i}P_{v,j} + (1 - P_{a,i})(1 - P_{v,j})}$$

Part 3 – cross validation

- As in part 1
  - Fit the model to all the data except the responses to one stimulus
  - Calculate the neg log likelihood for the stimulus left out
  - Repeat for all stimuli