02610 Optimization and Data Fitting

Homework assignment 1

Hand in on DTU Learn before 1 October 10pm.

1 One-page report on the exercises for week 5 (60%)

2 Convexity (by hand, 10%)

Consider the function

$$f(x_1, x_2) = x_1 x_2$$

on $\mathbb{R}^2_{++}$, i.e. $x_1, x_2 > 0$. Determine whether it is convex or concave.

3 Stationary points and steepest descent method (20%)

Consider the test function

$$f(\boldsymbol{x}) = f_1(\boldsymbol{x})^2 + f_2(\boldsymbol{x})^2, \qquad \boldsymbol{x} \in \mathbb{R}^2$$

where

$$f_1(\boldsymbol{x}) = x_1 + x_2^2 - 1,$$
$$f_2(\boldsymbol{x}) = x_1 + 3.$$

(4%) Compute the gradient $\nabla f(\boldsymbol{x})$ and Hessian $\nabla^2 f(\boldsymbol{x})$ by hand.

(7%) Find the three stationary points by calculating the first-order optimality condition, and prove that two are global minimizers and one is a saddle point.

(9%) Use Matlab to apply the steepest descent method with backtracking line search on the problem of minimizing f. Set the stopping criteria as $\|\nabla f(\boldsymbol{x}_k)\|_\infty \leq 10^{-5}$, and the parameters in the backtracking are $\bar{\alpha} = 1$, $\rho = 0.5$ and $c = 0.5$. Test with three different starting points: $[-2, -1]^T$, $[-2, 0]^T$, and $[-2, 1]^T$. For each of the three starting points, show the number of iterations and the point to which the method converged.

4 Newton method (by hand, 10%)

Consider a univariate function $f(x) = x^s$ with $s \geq 2$ and $x \in \mathbb{R}_{++}$, i.e. $x > 0$. Prove that, for any starting point $x_0 \in \mathbb{R}_{++}$, Newton's method is well-defined and converges linearly to zero. Compute the convergence factor.

5 Inverse of an increasing convex function (Bonus, 10%)

Suppose $f : \mathbb{R} \to \mathbb{R}$ is increasing and convex on its domain. Let g denote its inverse, i.e., $g(f(x)) = x$. Suppose both f and g are twice differentiable. What can we say about convexity or concavity of g?