

Lesson 10: Radicals and Rational Exponents

Objectives

- To be able to identify the parts of a radical expression.
- To be able to translate expressions between radical and exponential form.
- To simplify square roots and higher order radical expressions.

Terms

- Index
- Root
- Radical
- Radicand
- Equivalent Expressions
- Rational Exponent
- Simplify

Think about this: What does Area have to do with Radical Expressions?

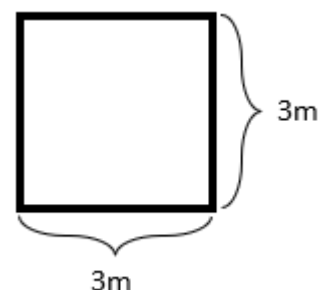
- When finding the area of a square, we square the length of the side:

○ $Area = \underline{\hspace{2cm}}$

- **Radical expressions** allow us to reverse this process.

- How can you find the side length of a square given the area?

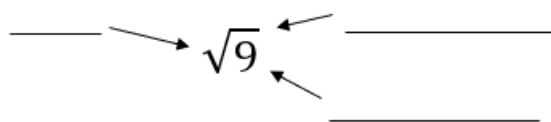
▪ $\sqrt{Area} = \underline{\hspace{2cm}}$



Definitions

- **Index:** The number you are trying to find.
 - If there is no index or root indicated, you are finding the second or square root.
- **Radical:** The symbol used to indicate finding the root of a number
- **Radicand:** The value inside the radical symbol.

Label the Diagram



This tells us that we are finding:

the square root of 9

How does this relate to Area and Side Length?

- What does the radicand of a square root represent?
 - Area
- What does the simplified square root expression represent?
 - Side Length
- When simplifying this expression, we can ask ourselves:
 - What number, when multiplied by itself gives us 9?
 - The answer to this question is the simplified radical expression.

$$\sqrt{9} = \sqrt{3^2} = 3$$

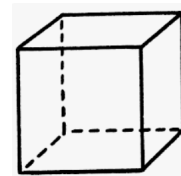
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How does this relate to Volume and Side Length?

- When finding the volume of a cube, we “cube” the length of the side:

○ $Volume = \underline{\hspace{2cm}}$

○ $\sqrt[3]{Volume} = \underline{\hspace{2cm}}$



- What does the radicand of a cube root represent?

○ $\underline{\hspace{2cm}}$

$\sqrt[3]{8} = \underline{\hspace{1cm}}$

- What does the simplified square root expression represent?

○ $\underline{\hspace{2cm}}$

- When simplifying this expression, we can ask ourselves:

- What number, when multiplied by itself $\underline{\hspace{2cm}}$ gives us 8?

- The answer to this question is the simplified radical expression.

Evaluate the following expressions:

1. $\sqrt[3]{\hspace{1cm}}$

2. $\sqrt{\hspace{1cm}}$

3. $\sqrt[5]{\hspace{1cm}}$

4. $\sqrt[4]{\hspace{1cm}}$

****You cannot take the $\underline{\hspace{2cm}}$ root of a $\underline{\hspace{2cm}}$ number.

Think about this: How can we determine if radical expressions are equivalent?

Remember, to determine if two expressions are equivalent:

- Collect evidence by testing values in both expressions.
 - If each expression returns a different value, then they are not equivalent.
- Justify why the expressions are equivalent.

Which expressions are equivalent?
Check with the value $x = \underline{\hspace{1cm}}$.

$x + 2x$

$4x - 1$

$3x$

What can we do to maintain equivalent radical expressions?

We can show some properties of radicals by using the properties of equivalency.

- Are these expressions equivalent?

a. Are they equivalent? Yes or No

b. Write a statement showing your conclusion:

Use $a = \underline{\hspace{1cm}}$ and $b = \underline{\hspace{1cm}}$ to determine if the expressions are equivalent.

$$\sqrt{a + b} = \sqrt{a} + \sqrt{b}$$

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2. Are these expressions equivalent?

a. Are they equivalent? Yes or No

b. Write a statement showing your conclusion:

Use $a = \underline{\hspace{1cm}}$ and $b = \underline{\hspace{1cm}}$ to determine if the expressions are equivalent.

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

3. Are these expressions equivalent?

a. Are they equivalent? Yes or No

b. Write a statement showing your conclusion:

Use $a = \underline{\hspace{1cm}}$ and $b = \underline{\hspace{1cm}}$ to determine if the expressions are equivalent.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

***The properties from #2 and #3 also apply to higher order radical expressions (root > 2).*

Extra Practice: Use $x = \underline{\hspace{2cm}}$ to determine if the following expressions are equivalent.

$\sqrt{x+2}$	$\sqrt{x} + 2$	$\sqrt{x} + \sqrt{2}$	$\sqrt{\frac{x}{4}}$	$\frac{\sqrt{x}}{\sqrt{4}}$
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Try this: Use your calculator to determine if the following expressions are equivalent.

$\sqrt[3]{8}$ and	and $27^{2/3}$	$\sqrt{4^2}$ and
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Definition:

- **Rational Exponent:** exponents that are $\underline{\hspace{2cm}}$.
 - **Numerator:** $\underline{\hspace{2cm}}$.
 - **Denominator:** $\underline{\hspace{2cm}}$.
 - **Base:** $\underline{\hspace{2cm}}$
- **Translate:** You can translate between radical expressions and expressions with rational exponents.
 - The expressions are $\underline{\hspace{2cm}}$ to each other.
 - Index or root is the $\underline{\hspace{2cm}}$.

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Practice: Translate each expression into either a radical expression or a rational exponent.

Given	$50^{1/2}$	$\sqrt[3]{392x^5}$	$256^{1/4}$	$\sqrt[5]{32y^6}$
Equivalent Expression				

Think about this: What does it mean to simplify and evaluate radical expressions? How does factoring help us simplify radical expressions?

Example

- Simplify the expression: $\sqrt{50}$
 - Identify the root: _____
 - What are the factors of 50?
 - What factors are perfect squares?
 - Rewrite the radical as a product of radicals.
 - ex. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$
 - What is the simplified form of $\sqrt{50}$?
- Simplify the expression: $(392)^{1/3}$
 - Rewrite as a radical expression.
 - Identify the root: _____
 - Factor 392 using a factor tree and rewrite the radical in factored form.
 - Simplify the expression.

$$\sqrt{50}$$

$$392^{1/3}$$

Practice: Simplify the following radical expressions. Leave your answer in radical form.

$\sqrt[3]{270}$	$\sqrt{48}$	$96^{1/5}$
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Where will you see this in upcoming material?

What are the calculator skills you needed?