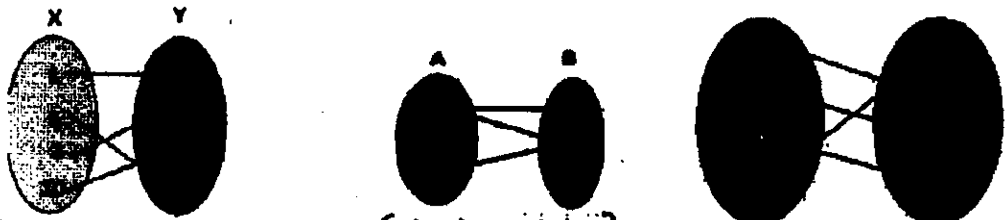
Name Allen Answers

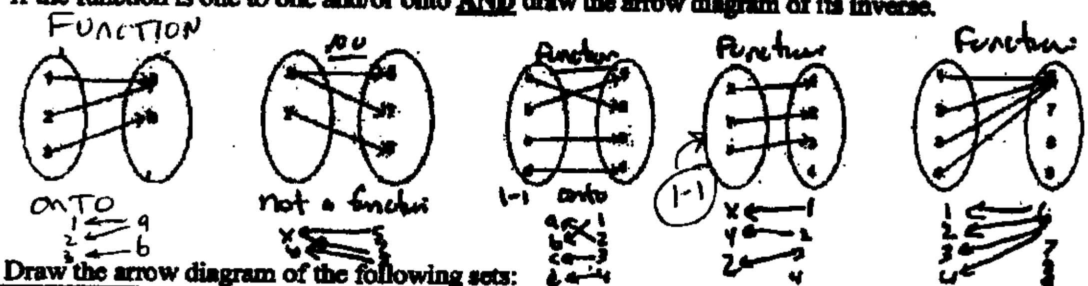
3.1 Functions

Rewrite each of the following arrow diagrams as a set of ordered pairs.

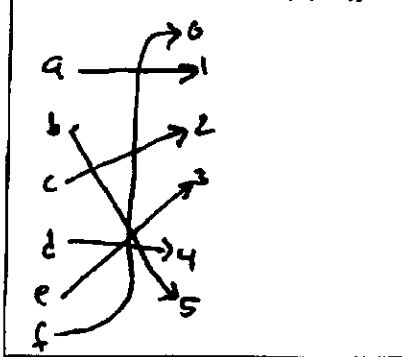
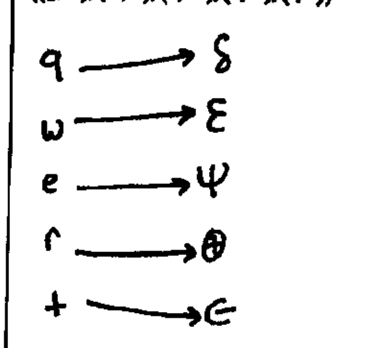
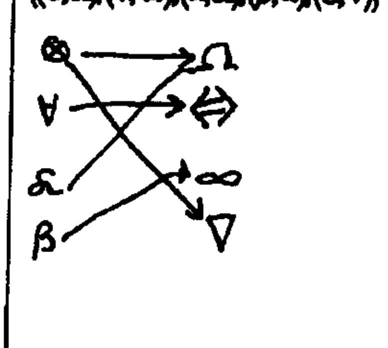


{(1,2),(5,7),(2,4),(10,8)} { (4,3),(4,1)(1,1)} {(3,2),(1,4),(10,8),(5,6)}

For each of the following arrow diagrams, determine if it is a function or not. If it is a function, determine if the function is one to one and/or onto **AND** draw the arrow diagram of its inverse.



Draw the arrow diagram of the following sets:

$\{(a,1),(b,5),(c,2),(d,4),(e,3),(f,0)\}$	$\{(q,\delta),(w,\varepsilon),(e,\Psi),(r,\oplus),(t,\in)\}$	$\{(\otimes,\Omega),(\forall,\Leftrightarrow),(\propto,\Omega),(\beta,\infty),(\otimes,\nabla)\}$
		

For each of the following sets, determine if it is a function from {1,2,3,4,5} to {a,b,c,d,e} or not. If it is a function, determine if the function is one to one and/or onto **AND** write set of order pairs of its inverse function.

{(1,c),(2,a),(3,b),(4,c),(2,d)} NOT A FUNCTION {(c,1),(a,2),(b,3),(c,4),(d,2)}	{(1,d),(2,d),(4,a)} Function inverse {(d,1),(d,2)(a,4)}
{(1,d),(2,a),(3,b),(4,e),(5,c)} Function 1-1 / onto (d,1), (a,2),(b,3)(e,4)(c,5)	{(1,a),(2,d),(3,b),(4,d),~~(5,e)~~(3,c)}, NOT A function inverse {(a,1),(d,2),(b,3),(d,4)(c,3)}

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Determine whether each function is one to one, onto or both. Prove your answer.

- $f(m,n)=m-n$ with domain Z x Z with codomain Z.

  onto $\forall\ x \in Z$ let $n = x$ & $m = n+2$
  not 1-1 $f(4,2) = f(5,3) = f(6,4) = \ldots$

- $f(x)=6x-9$ with domain and codomain the set of all real numbers.

  Both 1-1 and onto

Assuming that the codmain of each of the following functions has been properly restricted (Let range = codomain), find the inverse of the following:

- $f(x)=6x^5-19$

  $x = 6y^5 - 19$
  $6y^5 = x - 19$
  $5x-2$
  $y^5 = \frac{x+19}{6}$
  $\boxed{f^{-1}(x) = \sqrt[5]{\frac{x+19}{6}}}$

- $f(x)=6^{5x+2}$

  $x = 6^{5y+2}$
  $\ln x = (5y+2)\ln 6$
  $5y \ln 6 = \ln x - 2\ln 6$
  $y = \frac{\ln x - 2\ln 6}{5 \ln 6}$
  $f^{-1}(x) = \frac{\ln x - 2\ln 6}{5 \ln 6}$

Given the following functions:
$h=\{(a,c),(b,a),(c,b),(d,e),(e,d)\}$, $g=\{(a,e),(b,d),(c,a),(d,b),(e,c)\}$ and $k=\{(a,b),(b,c),(c,d),(d,a),(e,e)\}$
Find the following compositions (Write in set notation):

$h \circ g = h(g(x))$ $\{(a,d),(b,e),(c,c),(d,a),(e,b)\}$	$g \circ g$ $\{(a,c),(b,b),(c,e),(d,d),(e,a)\}$	$g \circ k = g(k(x))$ $\{(a,d),(b,a),(c,b),(d,e),(e,c)\}$
$h \circ k$ $\{(a,a),(b,b),(c,e),(d,c),(e,d)\}$	$g \circ k \circ h$ $g(k(h(x)))$ $\{(a,b),(b,d),(c,a),(d,c),(e,e)\}$	$k \circ g \circ k$ $k(g(k(x)))$ $\{(a,a),(b,b),(c,c),(d,e),(e,d)\}$

Decompose each function $f(x)$ into simpler functions $g(x)$ and $h(x)$ such that $f(x)=g \circ h$. $= g(h(x))$

$\log(\sqrt{x})$ $g(x) = \log x$ $h(x) = \sqrt{x}$	$\sqrt{\sin(x)}$ $g(x) = \sqrt{x}$ $h(x) = \sin x$	$\frac{\sin(x^2+1)}{1+\tan(x^2+1)}$ $g(x) = \sin(x)/1+\tan x$ $h(x) = x^2+1$

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3.2 Sequences and Strings

For the sequence: $s_n = -2(-1)^n + 2(3)^n$, $n > 1$ find (without a calculator).

$s_4 = (-2)(-1)^4 + 2 \cdot 3^4$ $= -2 + 2 \cdot 81 = 160$	$s_8 = -2(-1)^8 + 2 \cdot 3^8$ $-2 + 13122 = \boxed{13120}$	$\sum_{i=2}^{5} s_i = 16 + 56 + 160 + 488 = 720$
$\prod_{i=2}^{4} s_i = 16 \cdot 56 \cdot 160 = 143360$	Find a formula for s_{n-1} = $S_{n-1} = -2(-1)^{n-1} + 2(3^{n-1})$	Find a formula for s_{n-2} = $S_{n-2} = -2(-1)^{n-2} + 2 \cdot 3^{n-2}$

Show $s_n = 2s_{n-1} + 3s_{n-2}$

$= 2(-2(-1)^{n-1} + 2(3^{n-1})) + 3(-2(-1)^{n-2} + 2 \cdot 3^{n-2})$

$= -4(-1)^{n-1} + 4 \cdot 3^{n-1} + (-6)(-1)^{n-2} + 2 \cdot 3^{n-1}$

$= 4(-1)^n - 6(-1)^n + (4+2) \cdot 3^{n-1}$

$\to (-2)(-1)^n + 2 \cdot 3^n$

For the sequence: $g_n = (n+1)2^n$, $n \geq 0$ find (without a calculator).

$g_2 = 3 \cdot 2^2 = 12$	$g_{14} = 15 \cdot 2^{14}$ $\boxed{245760}$	$\sum_{i=1}^{4} g_i = 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + 5 \cdot 2^4$ $= 4 + 12 + 32 + 80 = 128$
List the first five terms of the for the subsequence obtained by taking the second, fourth, sixth, eight, … terms. $3 \cdot 2^2;\ 5 \cdot 2^4,\ 7 \cdot 2^6,\ 9 \cdot 2^8,\ 11 \cdot 2^{10}$	Find a formula for the expression n_k. $n_k = 2K$	Find a formula for the k^{th} term of the subsequence. $g_{n_k} = g_{2k} = (2k+1) \cdot 2^{2k}$ 12, 80, 448, 2304

Show $g_n = 4g_{n-1} - 4g_{n-2}$, $g_0 = 1, g_1 = 4$

$4 \cdot (n) 2^{n-1} - 4[(n-2+1) 2^{n-2}]$

$2n \cdot 2^n - 4[(n-1)2^{n-2}]$

$2n \cdot 2^n - (n-1) \cdot 2^n$

$\to 2n \cdot 2^n - n2^n + 1 \cdot 2^n$

$= (2n - n + 1) \cdot 2^n$

$= (n+1) \cdot 2^n = g_n$

Compute the given quantity using the strings

$\alpha = babab$	$\beta = abba$	$\gamma = bba$
a) $\beta\gamma$ = abba bba	b) $\alpha\beta$ = babab abba	c) $\gamma\alpha$ = bba babab
d) $\gamma\beta\alpha$ = bba abba babab	e) $\alpha\beta\gamma$ = babab abba bba	f) $\beta\beta\gamma\alpha$ = abba abba bba babab
g) $\lvert\alpha\beta\gamma\beta\rvert = 5+4+3+4 = 16$	h) $\lvert\gamma\alpha\alpha\gamma\beta\rvert = 3+5+5+3+4 = 20$	i) $\lvert\gamma\alpha^3\gamma^3\beta\alpha^5\rvert = 3+3\cdot5+5\cdot3+4+5\cdot5$

$= 3 + 15 + 15 + 4 + 25$

$= 33 + 29$

$= 62$

List all substring of the string: *aaccb*

empty string (λ) a b c

aa ac cc cb

aac acc ccb

aacc accb

aaccb

(14)

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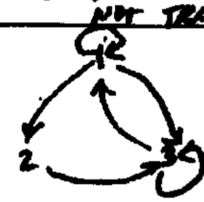
3.3 Relations / 3.4 Equivalence Relations

Determine whether each relation on the set of integers is reflexive, symmetric, antisymmetric or transitive. In addition, denote if the relation is a partial order or equivalence relation.

$(x,y)\in R$ if $x=y^2$, (0,0) (1,1) Anti symmetric	$(x,y)\in R$ if $x-y=4$ (5,1), (6,2) (10,6), Anti symmetric	$(x,y)\in R$ if 5 divides $2x+y$ (1,1) -no (2,1) (1,3) not (2,3) (2,1) (1,3) (10,5) ! (5,10) none

$\{(1,2),(2,3),(1,1),(1,3),(2,2),(3,3),(3,1),(2,1)\}$  ✓ Reflexive (3,2) → NOT symmetric (3,1) (1,2) ≠ (3,2) NOT TRAN	$\{(x,y),(y,x),(x,z),(y,x),(z,x),(z,y),(x,x)\}$  NOT REFLEXIVE symmetric (y,x)(x,y) ≠ (y,y) NOT TRANSITIVE

Consider the following relation: $\{(1,2),(2,3),(1,1),(1,3),(3,3),(3,1)\}$



Which ordered pairs need to be added to the relation for the relation to be?

A) reflexive (2,2)
B) symmetric (3,2), (2,1)
C) transitive (2,1), (3,2), (2,2)
D) an equivalence relation (2,1), (3,2), (2,2)
E) which ordered pairs need to be deleted for the relation to be antisymmetric

3.5 Matrices of Relations

Find the matrix of the relation from X to Y relative to the ordering given.

$\{(1,\alpha),(2,\alpha),(4,\alpha),(1,\beta),(2,\beta),(3,\gamma),(4,\gamma),(5,\gamma)\}$

Ordering of X: 1,2,3,4,5

Ordering of Y: α,β,γ

$$\begin{array}{c} \\ 1\\2\\3\\4\\5 \end{array}\begin{array}{c} \begin{array}{ccc} \alpha & \beta & \gamma \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

not 1st

$\{(a,1),(a,2),(b,2),(b,3),(b,4),(c,3),(d,1),(d,2),(d,4)\}$

Ordering of X: a,b,c,d

Ordering of Y: 1,2,3,4

$$\begin{array}{c} \\ a\\b\\c\\d \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Using a calculator: AND matrices

- Find the relation $R_1 \circ R_2$ if $R_1 = \{(x,y) \mid x+y \le 5\}$; R_1 is from X to Y; $R_2 = \{(y,z) \mid y = z+1\}$; R_2 is from Y to Z; ordering of X, Y and Z: 1, 2, 3 ,4, 5

$R_1 \circ R_2 = A_2 A_1$

$$R_1 \to A_1 = \begin{array}{c} \\ 1\\2\\3\\4\\5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R_2 \to A_2 \quad Y \begin{array}{c} \\ 1\\2\\3\\4\\5 \end{array}\begin{array}{c} \begin{array}{ccccc} & & z & & \\ 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_2 \cdot A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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For each of the following Matrices, determine if the relation represented by the Matrix is reflexive, symmetric or transitive.

A^2

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

diagonals = 1 ⇒ $\boxed{\text{reflexive}}$

not symmetric $(a,b) \neq (b,a)$

anti symmetric – NO $(5,3)=(3,5)$

transitive $A^2 = 1$ then $A = 1$

NOT TRANSITIVE

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

NOT reflexive

$\boxed{\text{Symmetric}}$

NOT ANTI SYMMETRIC

NOT TRANSITIVE

A^2

$$\begin{matrix} x & 0 & x \\ 0 & x & x \\ 0 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$$

reflexive – NOT

Symmetric – NO

ANTI symmetric – NO

$\boxed{\text{TRANSITIVE}}$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

NOT Reflexive

NOT symmetric $(2,1) \neq (1,2)$

NOT AntiSymmetric $(5,1)=(1,5)$

NOT TRANSITIVE

The following Matrix is not **reflexive**. Which ordered pairs need to be added to the Matrix/relation making the relation/matrix **reflexive**?

$(1,1)$, $(4,4)$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The following Matrix is not **symmetric**. Which ordered pairs need to be added to the Matrix/relation making the relation/matrix **symmetric**?

$(3,1)$, $(4,1)$, $(4,2)$, $(2,5)$, $(5,3)$, $(4,5)$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The following Matrix is not **transitive**. Which ordered pairs need to be added to the Matrix/relation making the relation/matrix **transitive**?

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

There is still more

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Consider the following relations given by each table.

Rooms		
max capacity	room	dept
100	A0	Mathematics
40	A1	Mathematics
40	A2	Mathematics
40	A3	Comp. Sci.
100	B1	Foreign Lang
40	B2	English
100	B3	English

Courses		
dept	class	sections
Mathematics	Trig/Precal	3
Mathematics	Algebra II	3
Mathematics	Calculus I	2
Comp. Sci.	CS 1	1
Foreign Lang	French 1	2
English	AP Eng Lit	1
English	Bible as Lit	2

Instructors	
dept	instructor
Mathematics	Gauss
Mathematics	Euler
Mathematics	Fermat
Comp. Sci.	Gates
Foreign Lang	Le Peregrine
English	C.S. Lewis
English	Tolkien

Prerequisites	
prerequisite	class
Algebra II	Trig/Precal
none	Algebra II
Trig/Precal	Calculus I
Algebra II	CS 1
none	French 1
none	AP Eng Lit
AP Eng Lit	Bible as Lit

Write a sequence of operations to answer the query. Also provide an answer to the query.

- Find all English instructors. Instructors [Dept = English]
  → CS Lewis, Tolkien
- Find all instructors who can teach in room A0.
  TEMP = Rooms [room = A0]
  TEMP 1 = Temp [~~dept~~ = instructor.dept] Instructor
  Temp1 [instructor] → Gauss, Euler, Fermat
- Find all classrooms in which Bible as Lit can be taught.
  Temp = Courses [class = Bible as Lit]
  Temp 1 = Temp [dept = Rooms.dept] Rooms
  Temp1 [room] → B2, B3
- Find all Departments with classrooms that can hold 100 students.
  Temp = Rooms [max capacity = 100]
  Temp [dept] → Mathematics, Foreign Language, English
- Find all classes that C.S. Lewis can teach.
  Temp = Instructors [instructor = C.S. Lewis]
  Temp 1 = Temp [dept = Courses.dept] Courses
  Temp1 [class] → AP English Lit, Bible as Lit
- Find all class and the number of sections for any class with no prerequisites.
  Temp = Prerequisites [prerequisite = none]
  Temp1 = Temp [class = Courses.class] Courses
  Temp1 [class, sections] → Algebra II, 3
  French 1, 2
  AP Eng Lit, 1