3.2 Discrete Math day 2

Review Exercise pg 144 # 10-16 all - see Back of book page 1

Exercise pg 145-146 # 59-91 odd, # 95-110 all

(59) $\Omega_n = 3 \quad \sum_{i=1}^{3} \Omega_i = 3+3+3 = 9$

(63) is Ω increasing - NO

(67) $x_1 = 2,\ x_n = 3 + x_{n-1},\ n \geq 2 \quad \sum_{i=1}^{3} x_i = 2 + \overset{5}{(3+2)} + 3 + \overset{5}{(3+2)} = 15$

(71) x is increasing

(75) $w_n = \frac{1}{n} - \frac{1}{n+1} = \frac{(n+1) - n}{n(n+1)} = \frac{1}{n(n+1)}$

$\sum_{i=1}^{10} w_i = \frac{1}{2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \frac{1}{6\cdot 7} + \frac{1}{7\cdot 8} + \frac{1}{8\cdot 9} + \frac{1}{9\cdot 10} + \frac{1}{10\cdot 11}$

$\frac{1}{2} + \frac{1}{6} = \frac{4}{6} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4} + \frac{1}{20} = \frac{16}{20} = \frac{4}{5} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} = \boxed{\frac{10}{11}}$

(79) w is decreasing

(83) $s_n = 2n - 1 \quad n \geq 1$

First seven terms: $s_1 = 1 \quad s_2 = 3, \quad s_3 = 5 \quad s_4 = 7 \quad s_5 = 9 \quad s_6 = 11, \ s_7 = 13$

(87) $t_n = 2^n,\ n \geq 1$

first seven terms: 2, 4, 8, 16, 32, 64, 128

(91) $y_n = 2^n - 1 \quad z_n = n(n-1)$

$\left(\sum_{i=1}^{3} y_i\right)\left(\sum_{i=1}^{3} z_i\right) = (1+3+7)(0+2+6) = 11 \cdot 8 = 88$

(95) $r_n = 3 \cdot 2^n - 4 \cdot 5^n,\ n \geq 0$

$r_0 = 3 - 4 = -1$

(96) $r_1 = 3 \cdot 2 - 4 \cdot 5 = 6 - 20 = -14$

(97) $r_2 = 3 \cdot 4 - 4 \cdot 25 = 12 - 100 = -88$

(98) $r_3 = 3 \cdot 8 - 4 \cdot 125 = -476$

(99) $r_p = 3 \cdot 2^p - 4 \cdot 5^p$

(100) $r_{n-1} = 3 \cdot 2^{n-1} - 4 \cdot 5^{n-1}$

(101) $r_{n-2} = 3 \cdot 2^{n-2} - 4 \cdot 5^{n-2}$

(102) see Next page

102 show $r_n = 7r_{n-1} - 10r_{n-2}$, $n \ge 2$

$$= 7(3\cdot 2^{n-1} - 4\cdot 5^{n-1}) - 10(3\cdot 2^{n-2} - 4\cdot 5^{n-2})$$

$$= 21\cdot 2^{n-1} - 28\cdot 5^{n-1} - 30\cdot 2^{n-2} + 40\cdot 5^{n-2}$$

$$= \underbrace{(2\cdot 21 - 30)}_{42-30} 2^{n-2} + (-28\cdot 5 + 40)\cdot 5^{n-2} = \underbrace{12}_{4\cdot 3}\cdot 2^{n-2} - \underbrace{100}_{25\cdot 4}\cdot 5^{n-2}$$

$$= 3\cdot 2^n - 4\cdot 5^n \quad \checkmark$$

$z_n = (2+n)3^n$ $n \ge 0$

103 $z_0 = 2$ 104 $z_1 = 3\cdot 3 = 9$ 105 $z_2 = 4\cdot 3^2 = 36$

106 $z_3 = 5\cdot 3^3 = 135$ 107 $z_i = (2+i)3^i$ 108 $z_{n-1} = (2+n-1)\cdot 3^{n-1} = (1+n)\cdot 3^{n-1}$

109 $z_{n-2} = (2+n-2)\cdot 3^{n-2} = n\cdot 3^{n-2}$

110 $$z_n = 6z_{n-1} - 9z_{n-2} = \underbrace{6}_{2\cdot 3}(1+n)\cdot 3^{n-1} - \underbrace{9}_{3\cdot 3}(n\cdot 3^{n-2})$$

$$= 2(1+n)\cdot 3^n - n\cdot 3^n = \{\underbrace{2(1+n) - n}_{2+2n}\}\cdot 3^n$$

$$= (2+n)\cdot 3^n$$