3.5 matrices of Relations day 2

- Exercises 16 – 18 <u>all</u>, 21 – 27 <u>all</u>

16 (a) $A_1 = \begin{array}{c|c} & x \;\; y \\ \hline \begin{matrix}1\\2\\3\end{matrix} & \begin{pmatrix}1 & 1\\1 & 0\\1 & 0\end{pmatrix}\end{array}$ (b) $A_2 = \begin{array}{c|c} & a \;\; b \;\; c \\ \hline \begin{matrix}x\\y\end{matrix} & \begin{pmatrix}0 & 1 & 0\\1 & 1 & 1\end{pmatrix}\end{array}$ (c) $A_1 \cdot A_2 = \begin{array}{c|c} & a \;\; b \;\; c \\ \hline \begin{matrix}1\\2\\3\end{matrix} & \begin{pmatrix}1 & 2 & 1\\0 & 1 & 0\\0 & 1 & 0\end{pmatrix}\end{array} \to$ (d) $\begin{array}{c|c} & a \;\; b \;\; c \\ \hline \begin{matrix}1\\2\\3\end{matrix} & \begin{pmatrix}1 & 1 & 1\\0 & 1 & 0\\0 & 1 & 0\end{pmatrix}\end{array}$

(e) $\{ (1,a), (1,b), (1,c), (2,b), (3,b)$

17 (a) $A_1 = \begin{array}{c|c} & 2 \;\; 3 \;\; 4 \;\; 5 \\ \hline \begin{matrix}2\\3\\4\\5\end{matrix} & \begin{pmatrix}1 & 0 & 1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{pmatrix}\end{array}$ (b) $A_2 = \begin{array}{c|c} & 1 \;\; 2 \;\; 3 \;\; 4 \\ \hline \begin{matrix}2\\3\\4\\5\end{matrix} & \begin{pmatrix}1 & 0 & 0 & 0\\1 & 1 & 0 & 0\\1 & 1 & 1 & 0\\1 & 1 & 1 & 1\end{pmatrix}\end{array}$ (c) $A_1 \cdot A_2 = \begin{array}{c|c} & 1 \;\; 2 \;\; 3 \;\; 4 \\ \hline \begin{matrix}2\\3\\4\\5\end{matrix} & \begin{pmatrix}4 & 3 & 1 & 1\\1 & 1 & 0 & 0\\1 & 1 & 0 & 0\\1 & 1 & 1 & 1\end{pmatrix}\end{array} \to$ (d) $\begin{pmatrix}1 & 1 & 1 & 1\\1 & 1 & 0 & 0\\1 & 1 & 0 & 0\\1 & 1 & 1 & 1\end{pmatrix}$

(e) $\{(2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (4,1), (4,2), (5,1), (5,2), (5,3), (5,4)\}$

18 (a) $A_1 \begin{array}{c|c} & 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \\ \hline \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{pmatrix}1 & 1 & 1 & 1 & 1\\1 & 1 & 1 & 1 & 0\\1 & 1 & 1 & 0 & 0\\1 & 1 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{pmatrix}\end{array}$ (b) $A_2 = \begin{array}{c|c} & 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \\ \hline \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{pmatrix}0 & 0 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 1 & 0\end{pmatrix}\end{array}$ (c) $A_1 \cdot A_2 = \begin{array}{c|c} \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{pmatrix}1 & 1 & 1 & 1 & 0\\1 & 1 & 1 & 0 & 0\\1 & 1 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0\end{pmatrix}\end{array}$ d $\to \begin{array}{c|c} & 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \\ \hline \begin{matrix}1\\2\\3\\4\\5\end{matrix} & \begin{pmatrix}1 & 1 & 1 & 1 & 0\\1 & 1 & 1 & 0 & 0\\1 & 1 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0\end{pmatrix}\end{array}$

(e) $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

21 see Back of Book.

22 Suppose ij entry of A is 1. Then the ij entry of A_1 <u>and</u> A_2 is 1, thus $(i,j) \in R_1$ <u>and</u> $(i,j) \in R_2$. Therefore $(i,j) \in R_1 \cap R_2$. Now suppose $(i,j) \in R_1 \cap R_2$. Then the ij^{th} entry of A_1 is 1 <u>and</u> ij^{th} entry of A_2 is 1, Therefore the ij^{th} entry of A is 1. It follows that A is the matrix of $R_1 \cap R_2$

23 $R_1 \cup R_2 = \begin{pmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 0 & 1\end{pmatrix}$

24 $R_1 \cap R_2 = \begin{pmatrix}0 & 0 & 0\\0 & 1 & 0\\1 & 0 & 1\end{pmatrix}$

25 each row of matrix has exactly one 1.

26 to be onto A is a function (see #25) & each column has <u>at least</u> one one in it.

27 To be 1-1 it is a function, each row and each column has exactly one 1 in it.