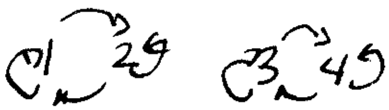
3.4b Equivalence Relations day 2 

Exercises pg 164–165 #15–31 odd,

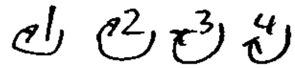
(15) $\{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$

$[1] = [2] = \{1,2\}$ $[3] = [4] = \{3,4\}$



(17) $\{(1), (2), (3), (4)\}$

$\Rightarrow \{(1,1), (2,2), (3,3), (4,4)\}$



(19) $\{\{1,2,3,4\}\}$

$\{(1,1), (1,2) (1,3), (1,4),$
$(2,1), (2,2), (2,3) (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1) (4,2) (4,3) (4,4)\}$

(21) $ARA$ since $A \cup Y = A \cup Y$ (Reflexive)

$ARB \Rightarrow BRA$ since $A \cup Y = B \cup Y$ (Symmetric)
$\Rightarrow B \cup Y = A \cup Y$

$ARB$ and $BRC \Rightarrow ARC$
since $A \cup Y = B \cup Y$ and $B \cup Y = C \cup Y$
then $A \cup Y = C \cup Y$

(23) 8 An equivalence is determined by presence or absences of 1, 2 and 5

→ see last page for additional help

(25) $R = X \times X$ – all order pairs

(27) $(1,1)$ $(2,2)$ $(3,3)$ $(4,4)$ $(5,5)$ $(6,6)$
$(1,2)$ $(2,1)$ $(3,4), (4,3)$

1b



29) Given $xRy$, $yRz$ then $zRx$ prove eq. Relation with $R$ reflexive

Suppose $xRy$, since $R$ is reflexive $yRy$. Taking $z=y$

$xRy$ & $yRz \Rightarrow zRx$ but $z=y \Rightarrow yRx \Rightarrow R$ is symmetric

now $xRy$ and $yRz \Rightarrow zRx$. but $R$ is symmetric $\{zRx \Rightarrow xRz\}$

So $xRy$ & $yRz \Rightarrow zRx \Rightarrow xRz$ ∴ transitive

∴ $R$ is an equivalence relation.

(31) Let $X = \{1, 2, \ldots 10\}$. Define a relation $R$ on $X \times X$ by
$(a,b)\,R\,(c,d)$ if $a+d=b+c$

a) $R$ is an equivalence relation

reflexive $(a,b)\,R\,(a,b)$ if $a+b=b+a$ ✓

symmetric
$(a,b)\,R\,(b,a)$ if $a+b=b+a$ ✓

transitive
$(a,b)\,R\,(c,d)$ and $(c,d)\,R\,(e,f)$ then $(a,b)\,R\,(e,f)$

$\underbrace{(a,b)\,R\,(c,d)}_{a+d=b+c}$ $\quad c+f=d+e$ $\quad$ does $a+f=b+e$ ?

$a+d+c+f = b+c+d+e$

$a+f=b+e$ ✓

b) (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (1,7), (1,8) (1,9) (1,10)
(2,1) (3,1) (4,1) (5,1) (6,1) (7,1) (8,1) (9,1) (10,1)

not assigned

$Y = \{3,4\}$

$\{1,2,3,4,5\}$ $A \cup Y = B \cup Y$

all equwalence classes for problem 23

$[1] = \{ \{1\} \{1,3\} \{1,4\}, \{1,3,4\} \}$

$[2] = \{ \{2\} \{2,3\}, \{2,4\} \{2,3,4\} \}$

$[3] = \{ \{3\} \{4\} \{3,4\} \} = [3]$

$[4] = \{ \{3\} \{4\} \{3,4\} \} = [4]$

$[5] = \{ \{5\}, \{5,3\}, \{5,4\}, \{3,4,5\} \}$

$[1,2] = \{ \{1,2\}, \{1,2,3\} \{1,2,4\}, \{1,2,3,4\} \}$

$[1,3] = \{ \{1\}, \{1,3\}, \{1,4\}, \{1,3,4\} \} = [1]$

$[1,4] = \{ \{1\}, \{1,3\}, \{1,4\} \{1,3,4\} \} = [1]$

$[1,5] = \{ \{1,5\}, \{1,3,5\}, \{1,4,5\} \{1,3,4,5\} \}$

$[1,2,3] = \{ \{1,2\} \{1,2,3\} \{1,2,4\} \{1,2,3,4\} \} = [1,2]$

$[1,2,4] = \{ \{1,2\} \{1,2,3\} \{1,2,4\} \{1,2,3,4\} \} = [1,2]$

$[1,2,5] = \{ \{1,2,5\} \{1,2,3,5\} \{1,2,4,5\}, \{1,2,3,4,5\} \}$

$[2,3,4] = \{ \{2\}, \{2,3\}, \{2,4\}, \{2,3,4\} \} = [2]$

$[2,3,5] = \{ \{2,5\}, \{2,3,5\}, \{2,4,5\}, \{2,3,4,5\} \} = [2,5]$

$[3,4,5] = \{ \{5\} \{3,5\} \{4,5\} \{3,4,5\} = [5]$

$[2,3] = [2]$

$[2,4] = [2]$

$[2,5] = \{ \{2,5\}, \{2,3,5\}, \{2,4,5\}$ and $\{2,3,4,5\} \}$

$[3,4] = [3]$

$[3,5] = [5]$

$[4,5] = [5]$

$[1,2,3,4] = [1,2]$

$[1,2,3,5] = [1,2,5]$

$[2,3,4,5] = [2,5]$

$[1,2,3,4,5] = [1,2,5]$

8 equivalence classes

$[1], [2], [3], [5], [1,2], [1,5], [2,5], [1,2,5]$