Discrete Math 3.1 Functions day 3

Review Exercise pg 131 #13-19 ALL - see back of book

Exercises: pg 133-134 # 39-47 odd, 51, 63, 65, 91, 93, 95

39. $f(n) = 2n+1$ $g(n) = 3n-1$

$f \circ f = 2(2n+1)+1 = 4n+2+1 = 4n+3$

$g \circ g = 3(3n-1)-1 = 9n-3-1 = 9n-4$

$f \circ g = 2(3n-1)+1 = 6n-2+1 = 6n-1$

$g \circ f = 3(2n+1)-1 = 6n+3-1 = 6n+2$

(41) $f(x) = \lfloor 2x \rfloor$ $g(x) = x^2$ * in back of book

$f \circ f = \lfloor 2\lfloor 2x \rfloor \rfloor = 2\lfloor 2x \rfloor$

$g \circ g = (x^2)^2 = x^4$

$f \circ g = \lfloor 2x^2 \rfloor$

$g \circ f = \lfloor 2x \rfloor^2$

(43) $f(x) = \frac{1}{x}$ $g(x) = 2x^2$

(45) $f(x) = 2x$ $g(x) = \sin x$

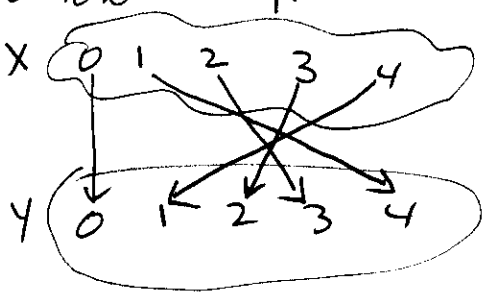
(47) $f(x) = x^3$ $g(x) = \frac{1}{\cos 6x}$

(51) $f(x) = 4x \bmod 5$ $f(0) = 0$ $f(1) = 4$ $f(2) = 3$ $f(3) = 2$ $f(4) = 1$

ordered pairs

$\{(0,0), (1,4), (2,3), (3,2), (4,1)\}$

arrow diagram



f is both one to one and onto

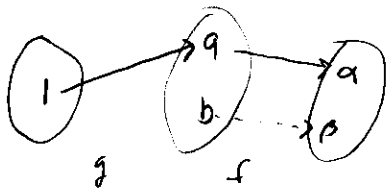
63 If f is onto, then $f \circ g$ is onto. False

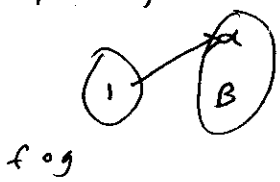
counter example

$X = \{1\}$ $Y = \{a, b\}$ $Z = \{\alpha, \beta\}$

$g = \{(1, a)\}$ $f = \{(a, \alpha), (b, \beta)\}$ ← which is onto

$f \circ g = \{(1, \alpha)\}$ - not onto, no arrow pointing to β





(65) If f and g are onto, then $f \circ g$ is onto. True

Let $z \in Z$. Since f is onto, $\exists$ a $y \in Y$ such that $f(y) = z$ $\forall z \in Z$

And g onto, $\exists$ $x \in X$ such that $g(x) = y$ $\forall y \in Y$.

$\therefore$ $f \circ g = f(g(x)) = f(y) = z$ and $f \circ g$ is onto

↑ $\exists$ $x \in X$ such that $f(g(x)) = z$ $\forall z \in Z$.

*65 also in back of book

(91) $f(x, y) = x - y$, $X = \{1, 2, \ldots 3$ not a binary operator

$f(1, 2) = 1 - 2 = -1 \notin X$.

(93) $f(x, y) = x/y$, $x = \{0, 1, 3, \ldots$ not a binary operator

$f(x, 0)$ - not defined

$f(1, 2) = 1/2$ - not in X

(95) Back of book says $g(x) = -x$

Does ① $g(x) = x - \langle$some Integer$\rangle$ also work?

② $g(x) = \lfloor x/2 \rfloor$ also work