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2.5 page 114 #9-12

#9 $c_1 = 0$ $c_n = c_{\lfloor n/2 \rfloor} + n^2$

$c_2 = 0 + 2^2 = 4$

$c_3 = c_{\lfloor 3/2 \rfloor} + 3^2 = c_1 + 9 = 0 + 9 = 9$

$c_4 = c_{\lfloor 4/2 \rfloor} + 4^2 = c_2 + 16 = 4 + 16 + 20$

$c_5 = c_{\lfloor 5/2 \rfloor} + 5^2 = c_2 + 25 = 4 + 25 = 29$

ALSO BACK of Book

#10) prove $c_n = c_{\lfloor n/2 \rfloor} + n^2 < 4n^2$ for all $n > 1$

for inductive step we must have $1 \leq k < n$

$k = \lfloor n/2 \rfloor$, $1 \leq n/2 < n$ $n=1$ - false $n=2$ true

BASIS step for $n=1$. see problem #9 above

INDUCTIVE step: Assume $c_n < 4n^2$ fo $1 \leq k < n$

prove for $k = n$: $c_n = c_{\lfloor n/2 \rfloor} + n^2 < 4(\lfloor n/2 \rfloor^2) + n^2$

$\leq 4\left(\frac{n}{2}\right)^2 + n^2 = 2n^2 < 4n^2$

* My Question to you. Why didn't we prove $c_n \leq 2n^2$
what changes in the proof to prove $c_n \leq 2n^2$
hint: think BASIS step!

#11 $c_1 = 0$ $c_n = 4c_{\lfloor n/2 \rfloor} + n$ for all $n > 1$

$c_2 = 4 \cdot 0 + 2 = 2$

$c_3 = 4 \cdot c_{\lfloor 3/2 \rfloor} + 3 = 4 \cdot c_1 + 3 = 3.$

$c_4 = 4 \cdot c_{\lfloor 4/2 \rfloor} + 4 = 4 \cdot c_2 + 4 = 4 \cdot 2 + 4 = 12$

$c_5 = 5 \cdot c_{\lfloor 5/2 \rfloor} + 5 = 5 \cdot c_2 + 5 = 5 \cdot 2 + 5 = 15$

#12 on back of pape

$$C_n = 4C_{\lfloor n/2 \rfloor} + n$$

#2 Prove $C_n \leq 4(n-1)^2$ for all $n \geq 1$

BASIS step: $1 \leq k < n$ $\quad 1 \leq \lfloor \frac{n}{2} \rfloor < n$

$n = 1$, $\cancel{n \geq 2}$ $\quad C_1 = 0 \leq 4(1-1)^2 = 0$ ✓

INDUCTIVE step: $C_n = 4C_{\lfloor n/2 \rfloor} + n \leq 4(n-1)^2$ for $1 \leq k < n$

prove true for n

$$C_n = 4C_{\lfloor n/2 \rfloor} + n \leq 4\left[4(\lfloor n/2 \rfloor - 1)^2\right] + n$$
$$\leq 4\left[4\left(\left(\tfrac{n}{2}\right) - 1\right)^2\right] + n$$
$$= 4\left[4\left(\tfrac{n^2}{4} - n + 1\right)\right] + n$$
$$= 4n^2 - 16n + 16 + n$$
$$= 4n^2 - 15n + 16$$
$$\leq 4(n-1)^2$$

$$\begin{array}{r} 4n^2 - 8n + 4 \\ -(4n^2 - 15n + 16) \\ \hline 7n - 12 > 0 \text{ for } n = 2, 3, \ldots \end{array}$$