

# Markov Chain Monte Carlo for Inverse Problems

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## 2 Theory

### 2.1 Papers

#### 2.1.1 Diaconis 1988

Bayesian Numerical Analysis (?)

#### 2.1.2 Stuart et al: Inverse Problems: A Bayesian Perspective

Theoretical Background

#### 2.1.3 Cotter et al: MCMC for functions

Implementation, MCMC in infinite dimensions

#### 2.1.4 Schneider et al: Earth System Modeling 2.0

Example for MCMC on ODE

## 2.2 Small results

### 2.2.1 Bayes' Theorem & Radon-Nikodym Derivative

### 2.2.2 Acceptance Probabilities for different MCMC Proposers

### 2.2.3 Different formulations of multivariate Gaussians

## 3 Implementation

### 3.1 Framework/Package Structure

#### 3.1.1 Densities

#### 3.1.2 Accepters

#### 3.1.3 Proposers

#### 3.1.4 Sampler

### 3.2 Results

#### 3.2.1 Analytic sampling from a bimodal Gaussian

#### 3.2.2 Bayesian inverse problem for $G(u) = \langle g, u \rangle$

#### 3.2.3 Bayesian inverse problem for $G(u) = g(u + \beta u^3)$

#### 3.2.4 Geophysics example