

Markov Chain Monte Carlo for Inverse Problems

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1 Theory

1.1 Papers

1.1.1 Stuart et al: Inverse Problems: A Bayesian Perspective [3]

Theoretical Background

1.1.2 Cotter et al: MCMC for functions [1]

Implementation, MCMC in infinite dimensions

1.1.3 Schneider et al: Earth System Modeling 2.0 [2]

Example for MCMC on ODE

1.2 Small results

1.2.1 Bayes' Theorem & Radon-Nikodym Derivative

1.2.2 Acceptance Probabilities for different MCMC Proposers

1.2.3 Different formulations of multivariate Gaussians

2 Implementation

2.1 Framework/Package Structure

2.1.1 Densities

2.1.2 Accepters

2.1.3 Proposers

2.1.4 Sampler

2.2 Results

2.2.1 Analytic sampling from a bimodal Gaussian

2.2.2 Bayesian inverse problem for $\mathcal{G}(u) = \langle g, u \rangle$

2.2.3 Bayesian inverse problem for $\mathcal{G}(u) = g(u + \beta u^3)$

2.2.4 Geophysics example

References

- [1] S. L. Cotter, G. O. Roberts, A. M. Stuart, and D. White. MCMC Methods for Functions: Modifying Old Algorithms to Make Them Faster. *Statistical Science*, 28(3):424–446, August 2013. Publisher: Institute of Mathematical Statistics.
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