# MTH1002

# UNIVERSITY OF EXETER

# COLLEGE OF ENGINEERING, MATHEMATICS AND PHYSICAL SCIENCES

# **MATHEMATICS**

May 2019

Mathematical Methods

Module Leader: Dr P. Philipp and Professor K. Zhang
Duration: 2 HOURS.

Answer Section A (50%) and any TWO of the three questions in Section B (25% for each).

Marks shown in questions are merely a guideline. Candidates are permitted to use approved portable electronic calculators in this examination.

This is a **CLOSED NOTE** examination.

## SECTION A

1. (a) Find the critical point of

$$f(x,y) = xy - x,$$

and determine whether it is a local minimum, a local maximum, or a saddle point. (8)

(b) Express the definite integral

$$\int_0^9 (2x+1) \, \mathrm{d}x$$

as a limit of Riemann sums and evaluate that limit.

(c) Find the following integrals.

(i) 
$$\int x \cos x \, dx$$
, (ii)  $\int_0^3 x^2 \sqrt{9 - x^2} \, dx$ . (10)

(d) Solve the following ordinary differential equations.

(i) 
$$\frac{dy}{dx} + \frac{3}{x}y = 4 + x^2$$
 with  $y(1) = \frac{7}{6}, x > 0;$ 

(ii) 
$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = \mathrm{e}^{-x}$$
 with  $y(0) = 1, y'(0) = 0.$  (12)

(e) Suppose that  $f(x) = x^2 - 2x - 3$  and consider

$$A = \begin{bmatrix} -1 & 0 \\ 4 & 3 \end{bmatrix}.$$

Find the matrix f(A). (6)

(f) Evaluate the following integral.

$$\int_{-1}^{1} \int_{-1}^{1} 3(x^2 + y^2) \, \mathrm{d}y \, \mathrm{d}x.$$

(6)

[50]

(8)

### SECTION B

2. (a) Let

$$f(x) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} \, dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} \, dt, \quad 0 < x < \frac{\pi}{2}.$$
Show that  $f(x)$  is constant. (10)

(b) Evaluate the expression

$$I = \lim_{n \to \infty} \frac{1}{n} \left[ (n+1)(n+2)(n+3)...(n+n) \right]^{1/n}$$

by associating it with a definite integral.

(15) [**25**]

3. (a) Find the values of a and b in

$$\xi = x + ay$$
,  $\eta = x + by$ ,

such that switching from (x,y) to  $(\xi,\eta)$  transforms the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial x^2} = 0$$

to

$$\frac{\partial^2 u}{\partial \xi \partial \eta} = 0.$$

(10)

(b) Find the general solution x(t) of the third-order differential equation

$$t\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + 3\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - t\frac{\mathrm{d}x}{\mathrm{d}t} - x = 0, \qquad t > 0.$$

[Hint: Consider y(t) = t x(t).] (15)

[25]

4. (a) Find all the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{13}$$

(b) Find the eigenvectors corresponding to those eigenvalues. (12)

[25]

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