A Genetic Algorithms Approach for Inverse Shortest Path Length Problems

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ABSTRACT

Inverse Combinatorial Optimization has become a relevant research subject over the past decades. In graph theory, the Inverse Shortest Path Length problem becomes relevant when people don't have access to the real cost of the arcs and want to infer their value so that the system has a specific outcome, such as one or more shortest paths between nodes. Several approaches have been proposed to tackle this problem, relying on different methods, and several applications have been suggested. This study explores an innovative evolutionary approach relying on a genetic algorithm. Two scenarios and corresponding representations are presented and experiments are conducted to test how they react to different graph characteristics and parameters. Their behaviour and differences are thoroughly discussed. The outcome supports that evolutionary algorithms may be a viable venue to tackle Inverse Shortest Path problems.

Keywords: *Computer Science, Genetic Algorithms, Graph Theory, Inverse Combinatorial Optimization, Inverse Shortest Path Length*

INTRODUCTION

Graph theory is highly appraised in Computer Science as well as several other fields, with applications on a large number of well documented problems (Gross & Yellen, 2005). A formal definition of a graph has been given for instance by Gross and Yellen (2005), but as succinctly as possible, a graph is a collection of nodes connected (or partially connected) by arcs with an associated cost such that successive arcs form paths between nodes. There are

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many widely known problems in the literature related to graph theory, such as that of finding the shortest path between two nodes (source and sink). This problem has been widely studied in the past and several efficient solutions have been developed by the scientific community (Bellman, 1956; Bertsekas, 1991; Dijkstra, 1959; Floyd, 1962; Johnson, 1977; Moore, 1959).

However, when solving such optimization problems, we often assume that we have access to all the information required and that this data is accurate. Often in real applications this is not the case. In shortest path problems, for instance, we may not have access to the real cost of the arcs (distances, travel times, etc.), we may have estimates or only partial knowledge of their values. A possible solution to extend our knowledge on the arc costs is to consider the inverse problem. This requires only that we have access to the topology of the graph and a desired outcome for the system, such as one or more shortest paths between pairs of $\left(source, sink\right)$ nodes. Inverse optimization allows a vector of arc costs to be calculated, or at least estimated, so that our desired outcome is optimal and the path costs as close as possible to previous estimations that we may have access to.

Inverse optimization problems are highly complex and widely relevant, but so far have failed to attract the attention of the Evolutionary Computation community. The need to test the viability of evolutionary approaches to tackle inverse optimization problems or in this case Inverse Shortest Path Length (ISPL) problems specifically is the main motivation for this study. We propose an unprecedented Genetic Algorithm approach which is tested on two different scenarios and hope to provide a baseline for future work on the subject. We experiment with several design choices, compare the obtained results, and discuss the resulting behaviours. The following section covers Inverse Combinatorial Optimization, previous work on both Inverse Solution Optimization Problems and Inverse Objective Value Optimization Problems is presented and discussed. Afterwards, the ISPL problem is introduced and our Genetic Algorithms approach described. Our experimentation setup, including the applied methodology, is described and the obtained results are presented and analysed. Finally, conclusions are drawn and future work proposed.

INVERSE COMBINATORIAL OPTIMIZATION

On a broad analysis, inverse optimization problems can be branched into two classes: inverse solution optimization problems and inverse objective value optimization problems (Hung & Director-Sokol, 2003). Briefly, inverse solution optimization problems require finding an arc costs vector that makes a desired solution optimal. Variations include the minimum cut problem (Zhang & Cai, 1998), inverse maximum flow problem (Yang, Zhang, & Ma, 1997), inverse center location problem (Cai, Yang, & Zhang, 1999), the inverse shortest path problem (ISPP) (Cai & Yang, 1994), among others. Inverse objective value optimization problems require that a solution is found that matches a desired value for the objective function rather than assuming there is a desired optimal solution. This type of problem has been much less explored than inverse solution optimization problems but the reverse center location (Zhang, Yang, & Cai, 1999) and the Inverse Shortest Path Length (ISPL) (Fekete, Hochstättler, Kromberg, & Moll, 1999) are examples, with ISPL probably being the most widely known. Heuberger (2004) presented various methods that had been successfully applied in the past to a set of inverse combinatorial problems on an extended survey. The following subsections further explore the two classes of inverse combinatorial optimization and previous work regarding each one.

Inverse Solution Optimization Problems

Burton and Toint (1992) were likely the first to address the ISPP. They present a specialization of the dual Quadratic Programming method by Goldfarb and Idnani (1983), which computes a sequence of optimal solutions based on con-