szemenyei.adrian.laszlo

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1 Introduction

The social force model was introduced in 1995 by Helbing and Molnár to model the dynamics/evolution of crowds - partly in the case for evacuations[1]. The model is very well-known and popular in practice mostly because it can reproduce several self-organization pedestrian behavior (e.g. lane formation, faster-is-slower effect, stop-and-go waves etc.). It is a microscopic pedestrian model, meaning that the individuals are the "units" - unlike in the case of macroscopic models, where we only can talk about densities. The model is based on Newton's second law ($F = ma$) - mathematically described as a system of ordinary differential equations (ODE) - where the pedestrians move in continuous space and the forces - which accelerate the pedestrians similar to molecular dynamics - are not only physical but can be social. Adding another behavioral aspect to these models means adding another force and/or manipulating the existing ones.

2 The model formulation

Suppose that there are N pedestrians. The spatial position, velocity, and acceleration of the j-th pedestrian at time $t \geq 0$ are denoted by $x_j(t) = (x_j^1(t), x_j^2(t))$, $v_j(t) = (v_j^1(t), v_j^2(t))$ and $a_j(t) = (a_j^1(t), a_j^2(t))$, respectively. From now on, we won't always write out the time variable t of the above functions. Based on Newtons's second law, the ODE system reads as:

$$a_j(t) = \dot{v}_j(t) = F_j^{all}(t) \tag{2.1}$$ {eq: m1}

$$\dot{x}_j(t) = w_j(v_j(t)), \tag{2.2}$$ {eq: m2}

where

$$w_j(v_j) = \begin{cases} v_j & \text{if } \|v_j\| < v_{max} \\ \frac{v_j}{\|v_j\|} v_{max} & \text{otherwise} \end{cases} \tag{2.3}$$ {eq: m3}

where v_{max} is a known constant, denoting the maximal speed, for which pedestrian j can travel (i.e. it can depend on j). The function w is called the speed velocity cut-off auxiliary function. Note that imposing a limit on the pedestrian speed introduces a first deviation from Newtonian mechanics and it makes the system non-differentiable. Helbing and Molnár formulated the model with the addition of random fluctuations to the equation (2.1), but this is usually omitted in analysis and simulations.

Now, we will talk about the different forces, which in the simplest case are

$$F_j^{all} = F_j^{dest} + F_j^{soc} + F_j^{wall} \tag{2.4}$$

2.1 F_j^{dest} force

The F_j^{dest} force "moves" the pedestrian in the direction of his/her goal r_j (e.g. door), which in the simplest case (when the "room" has "no geometry" and no obstacles) has the form:

$$F_j^{dest} = \frac{1}{\tau}\left(\frac{r_j - x_j(t)}{\|r_j - x_j(t)\|} v_{0,j} - v_j(t)\right) \tag{2.5}$$ {eq: dist_simple}

where $\tau\,[1/sec]$ is the relaxation time, while $v_0, j \in \mathbb{R}^2$ constant is the desired speed of pedestrian j i.e. the speed with which he/she wants to walk, if there is no other force "motivating" him/her to do otherwise.

In the more complex case - when the room has some geometry, one can attain the shortest path from all points by solving the auxiliary Eikonal equation:

$$\|\boldsymbol{\nabla} T(x)\|_2 = 1 \tag{2.6}$$ {eq: eikonal}

where $T(x)$ is called the arrival time for the point in the space x, while the "goals" are incorporated to the motel with 0 boundary value condition. Thus in this case (2.5) reads as:

$$F_j^{dist} = \frac{1}{\tau}\left(\frac{\boldsymbol{\nabla} T(x)}{\|\boldsymbol{\nabla} T(x)\|} v_{0,j} - v_j(t)\right) \tag{2.7}$$ {eq: dist_shortest}

It is also possible to incorporate the fact that the shortest distance is not always the quickest path in a non-empty space (i.e. travel time depends on the distributions and speed of others). Pedestrians tend to incorporate the above when moving in some situations. In this case the auxiliary Eikonal equation reads as:

$$\|\boldsymbol{\nabla} T(x)\| = \frac{1}{F(x)} \tag{2.8}$$ {eq: eikonal}

where F is [2]. Makmul modeled the evacuation of pedestrians affected by smoke spreading. The spreading of the smoke was incorporated with advection-diffusion equation and coupled to the social force model by the eikonal eqation, by the dependence of F by the smoke density and pedestrian density[3].

2.2 F_j^{soc} force

Considering the forces what other pedestrians exerts onto pedestrian j, to model that he/she wants to keep a certain distance from other pedestrians. We suppose that the force is the superposition of the forces that pedestrian i puts onto pedestrian j ($i = 1, \ldots, N$), mathematically:

$$F_j^{soc} = \sum_{i \neq j} F_j^{soc,i}$$

In the literature there are numerous ways to implement this force. The above ambiguity occurs by the following factors:

1. It makes sense to have the force *angle-dependent* where the angle can be either between the speed vectors of the people or the speed of the jth pedestrian and the location of the ith. In the later case one suppose that pedestrian j does not know where pedestrian i is walking toward, he/she only know him/her position.

2. In a good pedestrian model, the pedestrians are not "point-like" - they at least "circle-like"[4].

3. e.t.c.

One way to implement the above force is the following[5]:

$$F_j^{soc,i} = A_j exp\left(\frac{r_{ji} - d_{ji}}{B_j}\right) n_{ji} \left(\lambda_j + (1 - \lambda_j) \frac{1 + cos\,\theta_{ji}}{2}\right), \qquad (2.9) \quad \texttt{\{eq: fsoc\}}$$

where

- $A_j, B_j > 0$ are the parameters.
- $r_{ji} := r_j + r_i$ the sum of radii's of pedestrian i and j.
- $d_{ji} = \|x_j - x_i\|$ the distance between the centers of mass of the pedestrians i and j.
- n_{ji} is the normed vector pointing from pedestrian i to pedestrian j:

$$n_{ji} = \frac{x_j - x_i}{d_{ji}}$$

- The parameter $\lambda_j \in [0, 1]$ that makes the force larger when pedestrian j is walking towards pedestrian i, because
- θ_{ji} ="The angle between the direction of motion of pedestrian j and the position of pedestrian i", thus

$$cos\,\theta_{ji} = -n_{ji} \cdot \frac{v_j}{\|v_j\|}$$

One problem with the above formulation that pedestian j knows more than only the position of pedestrian i, namely his direction of movement and possibly his speed. One moves differently whether he sees that he will collide with the other or only follows him.

In the case of panicking crowds, one can consider extra physical forces, which only occur, when pedestrians touch. In [5] these physical forces between two pedestrians were formulated as$d_{ij} < r_{ij}$:

1. The body force, which counters the body compression:

$$k\, g(r_{ji} - d_{ji})n_{ji} \tag{2.10}$$ {eq: body_force}

, where k is a constant and $g(x) = max(0, x)$, thus only has effect when $d_{ij} < r_{ij}$.

2. The sliding friction force impeding relative tangential motion

$$\kappa\, g(r_{ji} - d_{ji})t_{ji}\Delta v^t_{ij} \tag{2.11}$$ {eq: sliding_friction_forc

where $t_j i = (-n^2_{ji}, n^1_{ji})$ the tangential direction and $\Delta v^t_i j = (v_i - v_j) \cdot t_{ji}$ is the tangential velocity difference and $g(x) = max(0, x)$, thus only has effect when $d_{ij} < r_{ij}$.

2.3 F^{wall}_j force

The F^{wall}_j force makes the pedestrians to keep a distance from the wall. It can be similarly formulated as (2.9) and (2.10) and (2.11) can be incorporated similarly:

$$f^{wall}_j = A_j exp\left(\frac{r_j - d_{jw}}{B_j}\right)n_{jw}\left(\lambda_j + (1 - \lambda_j)\frac{1 + cos\,\theta_{jw}}{2}\right), \tag{2.12}$$ {eq: wall_force}

where d_{jw} is the distance from the wall, n_{jw} is the direction perpendicular to the wall and

$$cos\,\theta_{jw} = -n_{jw} \cdot \frac{v_j}{\|v_j\|}$$

Note that here the dependence on the above angle makes more sense than for two pedestrians (since the wall has no speed), but the fact that the force only depends on the closest point from the wall is somewhat conflicting with reality, but its not that simple since the below case won't happen in the model in general, since there is also the force which guides the pedestrian to the goal and the below is not the shortest distance route.

2.4 Generalizations of the above model

In general, most papers in the literature are about the generalizations of the above model, in the sense that they research what extra social forces can be added/modified for different specific situations.A review of the above can is [6]. Some examples are the following:

1. Makmul modeled the evacuation of pedestrians affected by smoke spreading. The spreading of the smoke was incorporated with advection-diffusion equation and coupled to the social force model by the eikonal eqation, by the dependence of F by the smoke density and pedestrian density[3].

2. Mussaid et al. considered how groups and interactions in them change the movement of pedestrians[7]. They were able to model the pattern that in low-densities small groups tend to walk side-by-side, while in larger densities in 'V' shape.

In general the analysis consists of the evaluation of simulations. Parameter optimisation is also a well-researched question[8][9](and references therein). In the literature there are only handful of articles which investigate the mathematical aspects of the model and its different possible discretizations - which used to solve the ODE system.

Koster et al. [10].

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