Workshop on Numerical and Theoretical Advances in Geophysical Fluid Dynamics and Nonlinear PDE Systems

February 7–8, 2026

Texas A&M University, Department of Mathematics

All talks will take place in BLOC 117. Each talk is 25 min + 5 min for questions/transition.

Time	Activities
Day 1 – Multiscale structure, instabilities and turbulence in geophysical flows	
8:00 – 8:50	Breakfast
8:50 – 9:00	Greeting
9:00 – 9:30	**Xin Liu** — *Fast–slow dynamics of the quasi-geostrophic approximation*
9:30 – 10:00	**Bingheng Yang** — *Ill-posed and well-posed free boundary shallow water flows*
10:00 – 10:30	**Mac Lee** — *SQG Point Vortex Dynamics with Order Rossby Corrections*
10:30 – 11:00	**Timo Sprekeler** — *Mixed multiscale finite element approximation of Darcy's problem*
11:00 – 11:30	Coffee break
11:30 – 12:00	**Prabir Daripa** — *A Numerical Study of Singularity Formation in an Euler Model of Shallow Water Wave Propagation: The Case of an Illposed Boussinesq Equation*
12:00 – 12:30	**Adrian Fraser** — *Spontaneous generation of helical flows by salt fingers in oceans and stars*
12:30 – 13:00	**Carolin Weber** — *Magneto-rotating finger convection in stable layers at the top of Earth's core*
13:00 – 14:00	Lunch break
14:00 – 14:30	**Chang Liu** — *Exact Coherent Structures of Sheared Double-Diffusive Convection*
14:30 – 15:00	**Philipp Vieweg** — *Emergent large-scale dynamics in stably and unstably stratified flows*
15:00 – 15:30	**Sam Lewin** — *Pathways to turbulence from internal gravity waves in geophysical flows*
15:30 – 16:00	**Sreenath Paleri** — *Modeling multi-scale exchanges in the hererogeneously heated atmospheric boundary layer*
16:00 – 18:00	Time for free discussions / leisure
18:00	Dinner at the Nam Café
Day 2 – Planetary dynamics, convection in spherical shells, large-scale circulation and magnetic fields	
8:30 – 9:00	Breakfast
9:00 – 9:30	**Krista Soderlund** — *Planetary Interior Fluid Dynamics: Models, Missions, and Open Questions*
9:30 – 10:00	**Chi Yan** — *Unraveling Ice Giant Magnetic Fields and Their Secular Variation with Dynamo Models*
10:00 – 10:30	**Arefe Nezami** — *Transport mechanisms in rotating convection under heterogeneous boundary forcing*
10:30 – 11:00	**Paula Wulff** — *Asymptotic Scaling Trends in Rotating Convection and Dynamo Experiments*
11:00 – 11:30	Coffee break
11:30 – 12:00	**Loren Matilsky** — *Superrotation & Jet Migration in Simulations of Jupiter's Convective Zone & Weather Layer*
12:00 – 12:30	**Florentin Daniel** — *New constraint on Europa's ice shell: Magnetic signature from the ocean*
12:30 – 12:40	Closing remarks
12:40 – 14:00	Lunch break
14:00 – ?	Open discussion: challenges ahead and potential collaborative research areas

Topics and Abstracts

Xin Liu (TAMU, Department of Mathematics)

Fast–slow dynamics of the quasi-geostrophic approximation

This work is devoted to investigating the rotating Boussinesq equations of inviscid, incompressible flows with both fast Rossby waves and fast internal gravity waves. The main objective is to establish a rigorous derivation and justification of a new generalized quasi-geostrophic approximation in a channel domain with no normal flow at the upper and lower solid boundaries, taking into account the resonance terms due to the fast and slow waves interactions. Under these circumstances, We are able to obtain uniform estimates and compactness without the requirement of either well-prepared initial data or a domain with no boundary. In particular, the nonlinear resonances and the new limit system, which takes into account the fast waves correction to the slow waves dynamics and boundary effect, are also identified without introducing Fourier series expansion. The key ingredient includes the introduction of (full) generalized potential vorticity.

Bingheng Yang (TAMU, Department of Mathematics)

Ill-posed and well-posed free boundary shallow water flows

We consider the free boundary shallow water model derived by Li, Liu, and Peschka. In contrast to the well-posedness result shown in Li–Liu–Peschka, we show that the viscous shallow water model with the dynamic contact angle condition is classically ill-posed. Moreover, we also discuss recent progress made related to the inviscid model.

Mac Lee (UCSD, Physics)

SQG Point Vortex Dynamics with Order Rossby Corrections

Quasi-geostrophic flow is an asymptotic theory for flows in rotating systems that are in geostrophic balance to leading order. It is characterized by the conservation of (quasi-geostrophic) potential vorticity and weak vertical flows. Surface quasigeostrophy (SQG) is the special case when the flow is driven by temperature anomalies at a horizontal boundary. The next-order correction to QG, QG+, takes into account ageostrophic effects. We investigate point vortex dynamics in SQG+, building on the work of Weiss. The conservation laws for SQG point vortices that parallel the 2D Euler case no longer exist when ageostrophic effects are included. The trajectories of point vortices are obtained explicitly for the general two-vortex case in SQG and SQG+. For the three-vortex case, exact solutions are found for rigidly rotating and stationary equilibria consisting of regular polygons and collinear configurations. As in the 2D case, only certain collinear vortex configurations are rigid equilibria. Trajectories of passive tracers advected by point vortex systems are studied numerically, in particular their vertical excursions, which are non-zero because of ageostrophic effects. Surface trajectories can manifest local divergence even though the underlying fluid equations are incompressible.

Timo Sprekeler (TAMU, Department of Mathematics)

Mixed multiscale finite element approximation of Darcy's problem

In this talk, we discuss a computational multiscale method for the mixed formulation of a second-order linear elliptic equation subject to a homogeneous Neumann boundary condition, based on a stable localized orthogonal decomposition (LOD) in Raviart-Thomas finite element spaces. In the spirit of numerical homogenization, the construction provides low-dimensional coarse approximation spaces that incorporate fine-scale information from the heterogeneous coefficients by solving local patch problems on a fine mesh. The resulting numerical scheme is accompanied by a rigorous error analysis, and it is applicable beyond periodicity and scale-separation.

Prabir Daripa (TAMU, Department of Mathematics)

A Numerical Study of Singularity Formation in an Euler Model of Shallow Water Wave Propagation: The Case of an Illposed Boussinesq Equation

We numerically study the issue of finite-time existence of solutions associated with the initial value problem (IVP) of an ill-posed Boussinesq equation subject to small-amplitude sinusoidal initial disturbances over a 2π periodic domain. A spectral method in combination with the fourth-order Runge–Kutta method is used to perform numerical experiments using several time steps, several precision arithmetic calculations and filters with several cut-off levels. The calculations indicate that the solution blows up in a very short time t_c, the time of singularity formation. The value of t_c changes very little with the filter cut-off level. To validate our numerical results, we also analyse the evolution of Fourier coefficients through a perturbation analysis taking into account that the perturbation parameter ε (amplitude of the sinusoidal initial disturbance) is small. The location of the singularity where the solution blows up is found to be $3\pi/2$, independent of the parameter ε. It also appears that the profile of the solution at the location of the singularity is parabolic, of the form

$$\eta(x,t;\varepsilon) = \eta_0(t;\varepsilon) + c_1(t;\varepsilon)\big(x - 3\pi/2\big)^2$$

as the singularity forms, where $t_c \sim 1/\varepsilon^p$, $p \sim 0.28$. The functions $\eta_0(t;\varepsilon)$ and $c_1(t;\varepsilon)$ are determined numerically and discussed. From numerical calculations, we find that $c_1(t;\varepsilon) \sim (t_c - t)^{-\delta(\varepsilon)}$ as $t \to t_c$, where δ is independent of ε and is the rate of singularity formation. We obtain the following estimates for δ: $\delta = 2.46$ for $\varepsilon = 0.10$ and $\delta = 2.67$ for $\varepsilon = 0.025$.

Reference: A Numerical Study of Singularity Formation in an Euler model of shallow water wave propagation: The case of Illposed Boussinesq Equation, *R. Soc. Open Sci.* **13**: 251726, 2026; doi: `10.1098/rsos.251726`.

Adrian Fraser (CU Boulder, Applied Mathematics)

Spontaneous generation of helical flows by salt fingers in oceans and stars

We present the dynamics of salt fingers in the regime of slow salinity diffusion (small inverse Lewis number) and moderate-to-strong stratification (large density ratio), relevant to Earth's oceans and stellar interiors. Using three-dimensional direct numerical simulations in periodic domains, we show that salt fingers exhibit rich, multiscale dynamics in this regime, with vertically elongated fingers that are twisted into helical shapes at large scales by mean flows and disrupted at small scales by isotropic eddies. We use a multiscale asymptotic analysis to motivate a reduced set of partial differential equations that filters internal gravity waves and removes inertia from all parts of the momentum equation except for the Reynolds stress that drives the helical mean flow. When simulated numerically, the reduced equations capture the same dynamics and fluxes as the full equations in the appropriate regime. The reduced equations enforce zero helicity in all fluctuations about the mean flow, implying that the symmetry-breaking helical flow is spontaneously generated by strictly non-helical fluctuations. We also show that these dynamics are found in both oceanographic and astrophysical regimes.

Carolin Weber (Institute of Geophysics, University of Münster)

Magneto-rotating finger convection in stable layers at the top of Earth's core

While most of the Earth's fluid outer core is vigorously convecting, there is evidence of a stably stratified layer at its very top. This stable layer is often assumed to be a result of a sub-adiabatic thermal stratification, which overcompensates a destabilizing compositional gradient. In such a scenario, small-scale double-diffusive modes known as finger instabilities are likely to be excited. Fingering convection has been studied extensively in oceanography and more recently in astrophysics, but little work has been done so far in the context of planetary cores, where both rotational and magnetic effects complicate the dynamics.

In this talk, I will argue that the pre-existing knowledge on fingering convection is not directly applicable in the outer core of the Earth. Instead, linear stability analysis reveals that both the morphology and the growth rates of the fingers are significantly affected by the combined effects of rotation and magnetic field. For a large range of latitudes, we observe a shift away from purely vertically moving, horizontally isotropic fingers to tilted, sheet-like structures. Direct numerical simulations confirm the existence of these novel convection patterns. They also reveal that the resulting turbulent fluxes differ significantly from cases considered previously in the literature, in terms of both magnitude and direction.

The analysis of these numerical results is currently ongoing, with the goal of deriving flux laws and determining whether large-scale structures like layers and internal gravity waves could form in the stable layer at the top of the Earth's core.

Co-authors: Stephan Stellmach (U. Münster)

Chang Liu (U. Connecticut, Department of Mechanical Engineering)

Exact Coherent Structures of Sheared Double-Diffusive Convection

The interaction between shear and double-diffusive convection (DDC) in the diffusive regime (cold fresh water on top of hot salty water) plays an important role in the heat and mass transport of polar region oceans. This study computes exact coherent structures (ECS) of diffusive-regime DDC with a uniform background shear in a vertically wall-bounded flow layer. We focus on the shear-influenced regime and present two-dimensional (2D) ECS consisting of steady-state solutions and periodic orbits. The steady-state solutions include tilted convective rolls with various horizontal wavenumbers, and they are invariant under horizontal translation. All tilted convective roll states undergo saddle-node bifurcation, leading to a stable upper branch and an unstable lower branch, suggesting that they originate from the subcritical bifurcation of conduction base states. Hopf bifurcations appear on the stable upper branch of tilted convective rolls, leading to periodic orbits. Bifurcation diagrams for dimensionless parameters, including the Rayleigh number, the Prandtl number, and the diffusivity ratio, are established, suggesting subcritical behavior. Increasing shear strength stabilizes the 2D tilted convective roll, while these tilted convective rolls continue to exist in the limit of zero density ratio corresponding to sheared Rayleigh-Benard convection. Extension to the three-dimensional (3D) domain leads to 3D streamwise-elongated roll solutions; one of them originates from a subcritical bifurcation of the corresponding 2D roll solution. Chaotic solutions from direct numerical simulations generally visit neighborhoods of these steady or periodic solutions, and these visits leave an imprint on the flow statistics.

Philipp Vieweg (University of Cambridge, DAMTP)

Emergent large-scale dynamics in stably and unstably stratified flows

Gravity, the weakest of the four fundamental interactions in physics, pervades the whole universe while being inescapable by acting as a body force. Depending on the mass density stratification of a configuration, gravity may thus either stabilise or destabilise a fluid and affect geophysical and astrophysical flows. This talk will cover simplified yet meaningful configurations which correspond to either one of these cases and are accessible to direct numerical simulations.

In the first part of this talk, we will focus on unstably stratified Rayleigh–Bénard convection as the paradigm of thermal convection. We will cover some recent results from studying the impact of thermal (and mechanical) boundary conditions on large-scale flow structures. It will be shown that thermal boundary conditions are crucial to the formation of long-living large-scale (turbulent) flow structures. In particular, a slow transient aggregation process — that only stops once the horizontal extent of the domain is reached — can be found once the fluid layer is subjected to Neumann-type constant heat flux boundary conditions. We trace this mechanism of self-organisation of flow structures back to secondary instabilities as well as an inverse cascade in spectral space.

In the second part of this talk, we will focus on stably stratified shear-driven turbulence that exhibits a Kelvin–Helmholtz instability. In this set-up, two layers of fluid interact only due to an imposed shear profile which inevitably renders the flow anisotropic even in the horizontal directions. Traditionally, this configuration has been studied by means of initial value problems where the ensuing turbulence decays eventually due to dissipation. Here, in contrast, we consider the presence of a forcing that enables entering a statistically stationary state. It will be shown that an analysis of characteristic length and time scales reveals highly anisotropic dynamics associated with the self-organising flow.

Selected references

1. P. P. Vieweg, J. D. Scheel and J. Schumacher, "Supergranule aggregation for constant heat flux-driven turbulent convection", *Phys. Rev. Research* **3**, 013231 (2021). DOI: `10.1103/PhysRevResearch.3.013231`
2. P. P. Vieweg, "Large-scale flow structures in turbulent Rayleigh–Bénard convection: Dynamical origin, formation, and role in material transport", PhD thesis, TU Ilmenau, Germany (2023). DOI: `10.22032/dbt.58334`
3. P. P. Vieweg and C. P. Caulfield, "Anisotropy of emergent large-scale dynamics in forced stratified shear flows", *J. Fluid Mech.*, accepted. arXiv:`2507.14991`

Sam Lewin (UC Berkeley, Department of Mechanical Engineering)

Pathways to turbulence from internal gravity waves in geophysical flows

Internal gravity waves, which arise in continuously stratified fluids with buoyancy as the restoring force, are a fundamental mechanism for transporting energy and momentum through Earth's atmosphere and oceans. These waves also provide a critical link in the transfer of energy from large to small scales, eventually generating turbulence and mixing in regions where they become unstable and break. In this talk I will briefly review our understanding of wave-driven mixing in the ocean interior and its relationship with the stability of linear internal waves. In particular, I will discuss how wave instability may be greatly expedited through interactions with background currents. I will focus on the unusual case of a horizontally sheared background current, where the total amount of energy dissipated by turbulence may in fact be much larger than the initial wave energy, suggesting a significant release of energy from the background flow. The results demonstrate how internal waves may act as catalysts as well as energy sources for turbulence production in geophysical flows.

Sreenath Paleri (TAMU, Atmospheric Sciences)

Modeling multi-scale exchanges in the hererogeneously heated atmospheric boundary layer

We investigate how effective surface length scales (Leff) and atmospheric boundary layer stability modulate surface-induced secondary circulations over a realistic heterogeneous surface. The evolution of the circulations and their impact on surface-atmosphere fluxes are studied using coupled large eddy simulations of the CHEESEHEAD19 field campaign. The heterogeneity-induced circulations were diagnosed using time and ensemble averaging of the atmospheric fields. Simulations were performed for summer (August) and autumn (September) Intensive Observation Periods of the field campaign, characterised differently in terms of normalised surface length scales and ABL stability. Quasi-stationary and persistent circulations were diagnosed in the daytime ABL that span the entire mixed layer height (zi). Their variation in time and space are presented. Homogeneous control runs were also performed to compare and contrast spatial organisation and validate the time-ensemble averaging operation. In the convective boundary layers simulated during the summer time simulations, wavelengths that scale as the effective surface heterogeneity length scales contribute the most to the heterogeneity-induced transport. Contributions from surface-induced circulations were lower in the simulated near-neutral BL for the autumn simulations. We find that both Leff/zi and ABL static stability control the relative contribution of surface-induced circulations to the area averaged vertical transport. This scale analysis supports prior work over the study domain on scaling tower measured fluxes by including low frequency contributions. We believe that the conceptual framework presented here can be extended to include the effects of sub-grid land surface heterogeneity in numerical weather prediction and climate models and also to further explore scale-aware scaling methodologies for near surface-atmosphere exchanges.

Krista Soderlund (UT Austin, School of Geophysics)

Planetary Interior Fluid Dynamics: Models, Missions, and Open Questions

Planetary fluid dynamics governs the generation of magnetic fields, the transport of heat and momentum, and the coupling between deep interiors and observable surface and magnetospheric phenomena. This talk will highlight recent modeling efforts from our group that span terrestrial planets, giant planets, and ocean worlds. For terrestrial bodies, we investigate coupled core-basal magma ocean dynamos and their implications for the early magnetic histories of Earth, Venus, and the Moon, as well as testing whether Mercury's present-day dynamo could be sustained by iron snow processes in its core. For the ice giants, we investigate multipolar dynamo regimes relevant to Uranus and Neptune, with particular emphasis on time variability and observational signatures in the context of the Uranus Orbiter and Probe mission concept prioritized by the Origins, Worlds, and Life Decadal Survey. Finally, we discuss fluid dynamics in ocean worlds, focusing on ocean circulation and ice–ocean–seafloor coupling, with direct relevance to observations by the Europa Clipper and Juice missions that are enroute to the Jovian system. Together, these studies illustrate how advances in numerical and theoretical fluid dynamics inform mission science across a wide range of planetary environments.

Chi Yan (UT Austin, School of Geophysics)

Unraveling Ice Giant Magnetic Fields and Their Secular Variation with Dynamo Models

The magnetic fields of the two ice giant planets, Uranus and Neptune, are unique due to their strongly non-axisymmetric and non-dipole-dominated features, in contrast to other planetary magnetic fields in the solar system. Multiple hypotheses have been proposed to explain these features, including e.g. stably stratified interior, vigorous turbulent convection, and density stratification. Furthermore, insights from ab initio simulations and high-pressure experiments indicate that water transitions smoothly from its molecular to an ionic phase under ice giant interior conditions, leading to radially variable electrical conductivity profiles.

Here, we use 3D numerical dynamo simulations to investigate the effects of different interior structures tailored for Uranus and Neptune on dynamo behavior, with particular attention to the magnetic field morphology and their secular variation. We find that including the radially variable electrical conductivity profile in our dynamo models helps reproduce key features of ice giant-like magnetic fields, including the dominant field morphology and magnetic power spectra. Furthermore, we highlight the potential for detecting secular variation in Uranus' and Neptune's magnetic fields, and point to current space telescopes and the future Uranus Orbiter and Probe mission as key opportunities to observe secular variation beyond Earth.

Co-authors: K. M. Soderlund (UT Austin), H. Cao (UCLA), J. M. Aurnou (UCLA), A. Masters (Imperial College London), H. Melin (Northumbria University).

Arefe Nezami (UT Austin, School of Geophysics)

Transport mechanisms in rotating convection under heterogeneous boundary forcing

Subsurface oceans in icy moons such as Europa and Enceladus likely experience spatially varying heat flux from their rocky mantles, giving rise to rotating convection under heterogeneous boundary forcing—a regime in which momentum and heat transport remain poorly understood. We analyze buoyancy-driven flows governed by the rotating Boussinesq equations in a spherical shell subject to spatially varying thermal forcing at the inner boundary. Using three-dimensional numerical simulations that vary the Rayleigh and Ekman numbers, we systematically explore how flow organization and transport depend on the degree of rotational constraint, the amplitude of imposed heat-flux heterogeneity, and the choice of kinematic boundary conditions. We find that no-slip boundaries enable efficient coupling between thermal forcing at the inner boundary and transport at the outer boundary, whereas free-slip boundaries favor zonally dominated states that homogenize the thermal field. As the Rayleigh number increases, this organization breaks down: transport reorganizes toward radially dominated fluxes, and the sign structure of the Reynolds stresses reverses, indicating a transition in the angular-momentum balance. Reynolds stresses thus provide a sensitive diagnostic of dynamical regime transitions. These results establish how boundary conditions and rotation compete to control transport in planetary ocean worlds and provide a framework for interpreting observational constraints on ice-shell dynamics and internal heating patterns.

Co-authors: Daphne Lemasquerier (U. St. Andrews), Doughlas Hemingway (UT Austin), Krista Soderlund (UT Austin).

Paula Wulff (UCLA, EPSS)

Asymptotic Scaling Trends in Rotating Convection and Dynamo Experiments

Asymptotic analysis of rotating convective turbulence, as exists in planets and stars, yields scaling predictions that all hinge on the value of the convective Rossby number, Ro_c. These asymptotic scalings are tested here using a broad compilation of laboratory-numerical rotating convection and dynamo experiments in planar, cylindrical, and spherical systems. We find good agreement between the different rapidly rotating and slowly rotating Ro_c-dependent trends for convective heat transfer, convective velocities, length scales, and local Rossby numbers in Boussinesq fluids. Further, we show that the predicted rapidly rotating velocity trend also holds in anelastic systems, albeit with an additional dependence on the density stratification. Our tests demonstrate that, given reasonable estimates of Ro_c in a geophysical fluid system, accurate first-order predictions of the convective flows and dynamics can be made.

Loren Matilsky (UC Santa Cruz)

Superrotation & Jet Migration in Simulations of Jupiter's Convective Zone & Weather Layer

Abstract: Observations of the mean zonal flow in Jupiter's weather layer (WL) reveal an intricate pattern of latitudinally stacked "jets", or concentrated regions of zonal flow moving prograde or retrograde compared to the bulk rotation of the planet. The strongest flow is a strongly "superrotating" (i.e., prograde) jet straddling the equator, which is flanked by 6–7 retrograde/prograde pairs of weaker jets in each hemisphere. The two primary drivers of the zonal flows on Jupiter are thought to be "shallow" baroclinically-driven 2D turbulence in the WL and "deep" rotationally constrained buoyantly driven Taylor columns in the convective zone (CZ) just underneath the WL. To make progress in resolving this "deep versus shallow driving" debate, here we study the generation of mean zonal flows in two rotating, 3D, spherical-shell, anelastic convection simulations of a Jovian-like planet. In one case, the CZ is isolated, but in the other case, the upflows are allowed to overshoot into a stably-stratified near-surface WL. We find that in both cases, homogenization of an appropriately defined potential vorticity over a Rhines scale creates multiple jets early in the simulation at high latitudes inside the "tangent cylinder" (the axially-oriented cylinder circumscribing the base of the CZ). At low latitudes (outside the tangent cylinder), there is equatorial superrotation due to the angular momentum transport by Taylor columns. The presence of a WL significantly alters the thermal wind balance and meridional circulation profiles, which result in stronger deviations of the jets from the Taylor–Proudman state of alignment with the rotation axis. Finally, although the superrotation remains stable, the weaker high-latitude jets slowly migrate poleward on a thermal diffusion time-scale.

Florentin Daniel (Northwestern, CIERA)

New constraint on Europa's ice shell: Magnetic signature from the ocean

Jupiter's icy moons are believed to host subsurface liquid oceans, and among them, Europa stands out as one of the most promising candidates for extraterrestrial life. Yet, the processes driving oceanic flows beneath its ice shell, as well as the factors controlling the thickness of this ice, remain incompletely understood. One especially distinctive feature of Europa is that its salty ocean is electrically conducting and thus influenced by Jupiter's time-varying magnetic field, which is believed to drive a large-scale zonal flow. In this presentation, I will examine how this magnetically induced jet affects both the heat flux and the dynamics of the convective flow within Europa's ocean. I'll first discuss how the magnetically-driven jet efficiently transports heat in stably stratified regions near the top of the ocean, and may alter the expected convective scaling laws in deeper layers. Second, by analysing the latitudinal distribution of heat flux and relating it to ice-thickness variations, I'll present some predictions that can be compared with current observations. In anticipation of the upcoming JUICE and Europa Clipper missions, improved measurement precision could help further constrain the ocean's properties and refine our model-based forecasts.