Department ESR
Lecture: Fuzzy Control
Lab No. 2: Fuzzy Controller Design

Objectives

The goal of this lab is to design and implement a fuzzy controller for the swing-up and stabilization of an inverted pendulum. The control design is supported and implemented with the Matlab fuzzy toolbox and the Simulink environment of Matlab. For this lab the Simulink model of the first lab **Classical Controller Design** is used. The state space controller designed in the first lab constitutes the basis for the design of the fuzzy stabilizing controller.

Prerequisites

It is assumed that you are familiar with the basics of fuzzy control and with the design of a Mamdani and Takagi-Sugeno-Kang fuzzy controller. You have basic knowledge of the software environment Matlab, the simulation environment Simulink and the Matlab fuzzy toolbox. A detailed description of the inverted pendulum model is available in the first lab description **Classical Controller Design.**

If you need further information about Matlab, Simulink and the Fuzzy Toolbox, you should study the Matlab online help and tutorials (`Matlab Help -> Matlab -> Getting Started`, `Matlab Help -> Simulink -> Getting Started` and `Matlab Help -> Fuzzy Logic ToolBox -> Getting Started`) or the Matlab tutorials from the course web page.

In case you need to refresh your knowledge on fuzzy control, study the online-book „Fuzzy Control“, Kevin Passion, Prentice-Hall, 1998:

- chapter 3: „Case Studies in Design and Implementation“
  → case study: „Balancing a Rotational Inverted Pendulum“ in section 3.4
- section 2.4: „Simple Design Example: The Inverted Pendulum“.

Fuzzy controller for swing up of the inverted pendulum

The controller design is separated in two stages:

a) swing-up of the pendulum from the lower, stable equilibrium state to an upright position
b) stabilization of the pendulum and the cart in the upper, unstable equilibrium

The task of the swing-up controller is to bring the pendulum in the vicinity of the upper equilibrium point, where it can be stabilized by means of the stabilizing controller. If the cart moves back and forth at the right frequency and amplitude energy is pumped into the system and the magnitude of the pendulums oscillation increases. If the pendulum reaches the upper equilibrium with a small residual angular velocity, the control is switched to the stabilizing controller.
For the fuzzy controller, this type of gain scheduling is realized by separating the range of the input variable θ into two regions, the swing-up-area $|\theta|>0.5$ and the upper-equilibrium-area $|\theta|<0.5$. The swing-up controller causes an oscillation of the cart

position between a left and the right track point $+x_C$ and $-x_C$. The amplitude x_C depends on the parameters of the plant and is optimized manually by trial-and-error. The P-controller for position control calculates the force F which is applied to the cart:

$$F = K_p(\pm x_C - x) \quad , \qquad (1)$$

The sign of x_C depends on the current direction of the pendulums motion ($\mathrm{sgn}(\dot{\theta})$). If the cart moves toward the track point $+x_C$ the control action becomes $F = K_p(x_C - x)$ as long as $\dot{\theta} > 0$. When the pendulum has reached its maximum height $\dot{\theta} \approx 0$, the control action is reversed to a command force $F = K_p(-x_C - x)$ with the new reference position $-x_C$ and the cart is accelerated in the opposite direction. The proportional gain K_p is tuned manually. K_p and $\pm x_C$ are chosen such that the switching of the control action occurs before the cart reaches the extreme track points $\pm x_C$. As a starting point for the manual tuning of these parameters choose K_p=1 and $\pm x_C = \pm 1$.

The P-controller is realized by means of a Takagi-Sugeno-Kang-fuzzy controller. The state variables $\theta, \dot{\theta}, x, \dot{x}$ serve as input to the controller, the force F takes the role of the control variable. The gains k_i in the linear control law in the rule consequents

$$\textbf{if ... then } F = k_\theta \theta + k_{\dot{\theta}} \dot{\theta} + k_x x + k_{\dot{x}} \dot{x} + k_0$$

are derived from Equation (1), with $k_\theta = k_{\dot{\theta}} = k_{\dot{x}} = 0$ for the proportional position controller. The values for k_x, k_0 are determined from the parameters x_C and K_p. For the swing-up controller four rules are required for the following regions of the input space:

$$\begin{aligned}
&-\infty < \theta \le -0.5,\ -\infty < \dot{\theta} \le 0 \\
&0.5 \le \theta < \infty,\ -\infty < \dot{\theta} \le 0 \\
&-\infty < \theta \le -0.5,\ 0 < \dot{\theta} < \infty \\
&0.5 \le \theta < \infty,\ 0 < \dot{\theta} < \infty
\end{aligned}.$$

First, evaluate the performance of the swing-up controller over the entire state space including the region $-0.5 < \theta < 0.5$. Tune the parameters K_p and x_c such that the pendulum accumulates enough energy to overshoot the vertical position. Later, the region $-0.5 < \theta < 0.5$ is excluded from the input domain of the swing-up controller, as the stabilizing controller takes control. The parameters in the rule consequents is derived from the above described relationship between the motion of the pendulum and the reference position.

Implement the TSK fuzzy controller to swing up the inverted pendulum with the help of the fuzzy toolbox. Choose finite, reasonable universes of discourse for the input and output variables. For example, the input variable θ should cover at least the range $[-2\pi, 2\pi]$. Use trapezoidal membership functions for the input variables $\theta, \dot{\theta}$ with regard to the above mentioned partition of the input space. The left and right boundary membership functions assume a membership degree of μ=1 at the boundaries. For example, the left membership function for the mere swing-up controller for θ is defined by the characteristic points $[-2\pi,-2\pi,-0.5,0]$. Later, the transition from the swing-up controller to the stabilizing controller in the range $-0.5 \le \theta \le 0.5$ occurs in a small region with an overlap of $\Delta \approx 0.1$. The characteristic points of the corresponding, membership function are $[-2\pi,-2\pi,-0.55,-0.45]$ and [-0.55, -0.45, ...] for the triangular neighboring membership function.

Design and implement a Sugeno fuzzy controller with the help of the FIS-editor (Matlab command `fuzzy`) and save it as `fuzzy_cp_upswing.fis`. With the command `readfis` the saved controller is loaded to the workspace. Export the controller as

`fuzzy_cp_upswing` to the workspace. You can either use the FIS-editor to edit the rules and membership functions of the controller or manually edit the file `fuzzy_cp_upswing.fis` with a text editor.
To test the swing-up controller in the Simulink environment, add a fuzzy controller block to the model of the plant cart_pole.mdl (`View->Library browser->Fuzzy Logic Toolbox->Fuzzy Logic Controller`). As parameter of the fuzzy controller block either specify the FIS-file or the corresponding variable from the workspace `fuzzy_cp_upswing`. In the implementation of the fuzzy controller block boolean signals are processed incorrect. Therefore, turn the option Boolean Logic Signals to `off` (`Simulation->Simulation Parameters->Advanced->Boolean Logic Signals`).
How long does it take for the pendulum to reach the upper unstable equilibrium? Optimize the parameters K_p and x_C by trial-and-error such that the pendulum reaches the upper equilibrium within three oscillations (approx. 6 seconds). Concerning the force F, the saturation of the control variable $|F| \leq 10$ must be obeyed.

Fuzzy controller for stabilizing the pendulum and cart

The fuzzy controller for stabilization is designed as a state space controller based on the input variables $\theta, \dot{\theta}, x, \dot{x}$. This controller is also implemented as a TSK controller for reasons of compatibility with the swing-up controller, although the inherent structure of the controller suggests Mamdani type fuzzy rules. A Mamdani controller can be realized within the TSK-framework by using constant control actions $F=k_0$ in the rule consequents. This representation is equivalent to a Mamdani controller with singletons as output fuzzy sets, in which the constant values k_0 correspond to the Mamdani output singletons. At first, the fuzzy controller should approximate the linear function $F = k_\theta \theta + k_{\dot{\theta}} \dot{\theta} + k_x x + k_{\dot{x}} \dot{x} + k_0$ which is defined by the state space controller. The approximation takes place by fitting the gains of the fuzzy controller to those of the linear state space controller near the equilibrium point. Later the control surface becomes more non-linear with increasing deviation from the equilibrium.

The design of a fuzzy controller by copying a linear controller is described in „Fuzzy Control“, Kevin Passino, chapter 3: „Case Studies in Design and Implementation“, section 3.4 „Balancing a Rotational Inverted Pendulum“, pp. 147-149.

The fuzzy controller for stabilizing the pendulum employs four input variables. The universe of discourse for the input variable θ is partitioned into five membership functions, the universes of discourse of the other input variables $\dot{\theta}, x, \dot{x}$ are partitioned into three membership functions each. Considering, the possible combinations of input terms, the universe of discourse of the control variable is partitioned into eleven membership functions $Y^{-5}, ..., Y^{0}, ... Y^{5}$. The following centers of gravity are assigned to these membership functions within a range scaled to the interval [-1,1]:

$$\begin{gathered}\{Y_{cog}^{-5} = -1, Y_{cog}^{-4} = -0.8, Y_{cog}^{-3} = -0.6, Y_{cog}^{-2} = -0.4, Y_{cog}^{-1} = -0.2, \\ Y_{cog}^{0} = Y_{cog}^{1} = 0.2, Y_{cog}^{2} = 0.4, Y_{cog}^{3} = 0.6, Y_{cog}^{4} = 0.8, Y_{cog}^{5} = 1\}\end{gathered}.$$

This fuzzy controller operates with normalized output range, which means it generates outputs in the range [-1,1]. The output corresponds to the sum of the signed deviations from the null position, represented by the indices of the input variables. The remaining task is to determine proper scaling factors for the input and output variables such that the desired linear behavior matches the gains $k_\theta,\ k_{\dot\theta},\ k_x,\ k_{\dot x}$.

We start the design task considering the feedback gains suggested by the state space controller from the first lab $k_\theta = 20,\ k_{\dot\theta} = 5,\ k_x = 0.5,\ k_{\dot x} = 1$. These parameters were previously obtained by means of pole placement.

The universes of discourse for the input and output fuzzy variables are computed in such a way that for small deviations from the equilibrium the gradient of the fuzzy control surface corresponds to the feedback gains k_i. The rule base (rule table) given in „Fuzzy Control“, Kevin Passion, chapter 3: „Case Studies in Design and Implementation“, section 3.4 „Balancing a Rotational Inverted Pendulum“, pp. 147-149, originates from input output ranges normalized on the interval [-1,1]. The actual range of the input θ however is given by the interval [-0.5,0.5], i.e., the original range needs to be scaled with a factor $g_\theta = 2$. As shown in figure 2a, the scaled ranges are covered by five equidistant, triangular membership functions with the centers of gravity :

$$X_\theta{}^{-2}{}_{\mathrm{cog}} = -0.5,\ X_\theta{}^{-1}{}_{\mathrm{cog}} = -0.25,\ X_\theta{}^{0}{}_{\mathrm{cog}} = 0,\ X_\theta{}^{1}{}_{\mathrm{cog}} = 0.25,\ X_\theta{}^{2}{}_{\mathrm{cog}} = 0.5\,.$$

For the controller a change of the input θ by 0.25 (change in the index by +1) corresponds to a change of the output by 0.2 (change in the index of the output fuzzy set by +1). In other words, the actual gain factor for F=kθ amounts to $\mathrm{k}=\frac{1}{0.25}0.2=0.8$. In order to realize the desired gain of $k_\theta = 20$ suggested by the state space controller, g_F must be increased by a factor 25 to g_F=5. Therefore, the corresponding singleton output fuzzy sets shown in figure 3 are located at center points

$$\begin{gathered} Y^{-5} = -25,\ Y^{-4} = -20,\ Y^{-3} = -15,\ Y^{-2} = -10,\ Y^{-1} = -5, \\ Y^{0} = 0,\ Y^{1} = 5,\ \ Y^{2} = 10,\ \ Y^{3} = 15,\ Y^{4} = 20,\ Y^{5} = 25 \end{gathered}\ .$$

The specification of the output singletons fixes the output gain to g_F=5. In order to mimic the feedback gains associated with the other inputs, their input ranges must be scaled according to the gains of the state space controller.

Repeating the same calculation as above, the input scaling factors become

$$g_{\dot\theta} = \frac{k_{\dot\theta}}{g_F} = 1$$

$$g_x = \frac{k_x}{g_F} = 0.1$$

$$g_{\dot x} = \frac{k_{\dot x}}{g_F} = 0.2$$

.Figures 2b,c,d depict the membership functions and universe of discourse for $-1 \le \dot\theta \le 1,\ -10 \le x \le 10,\ -5 \le \dot x \le 5$.

Notice, that with the implementation of the fuzzy controller the true input ranges need to be chosen according to actual behavior of the plant. The gain factors only determine the actual position of the membership functions. In order to cover the actual entire input range, the left and right boundary membership functions for $x, \dot x, \dot\theta$ are defined as trapezoidal fuzzy sets with a wider left and right support that extends to the universe of discourse boundaries. For the input θ the boundary membership functions are specified in a way that guarantees a smooth transition between stabilizing and swing-up controller.

The above mentioned intervals define the point on the X-axis, at which the boundary membership functions acquire their maximum degree of membership (see figures 2).

The interval borders correspond to the centers of gravity of the left and right triangular or trapezoidal membership functions $X_{\dot{\theta},x,\dot{x}}^{\pm 1}$. One selects the remaining overlapping membership functions such that the center of the triangles correspond to the rightmost point of the right neighbor and the leftmost point of the left neighbor's membership function.

At the boundaries the trapezoidal membership functions assume a constant degree of membership (μ=1), as shown in figures 2.

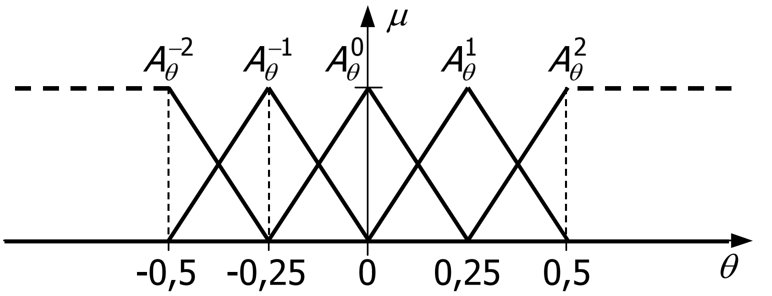

Figure 2a: Membership functions of the input variable θ

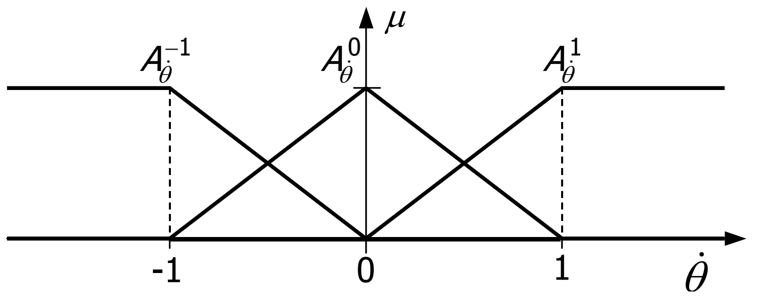

Figure 2b: Membership functions of the input variable $\dot{\theta}$

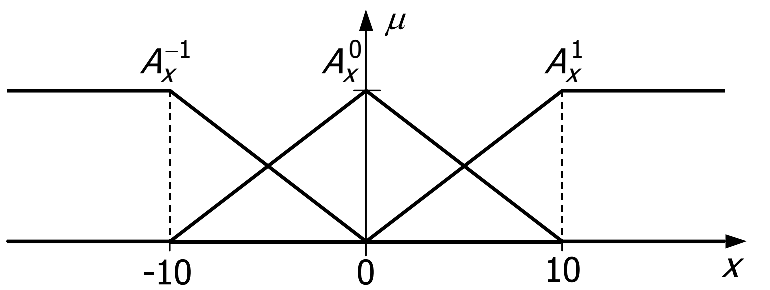

Figure 2c: Membership functions of the input variable x

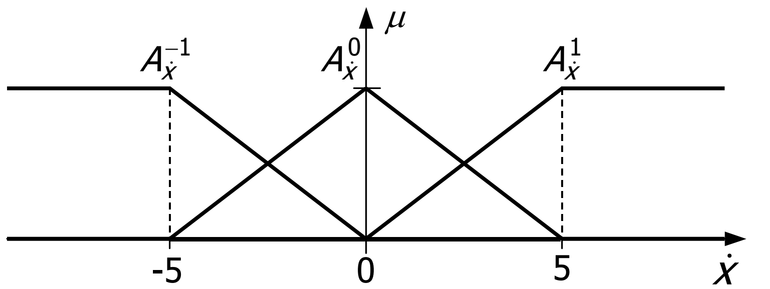

Figure 2d: Membership functions of the input variable $\dot{x}$

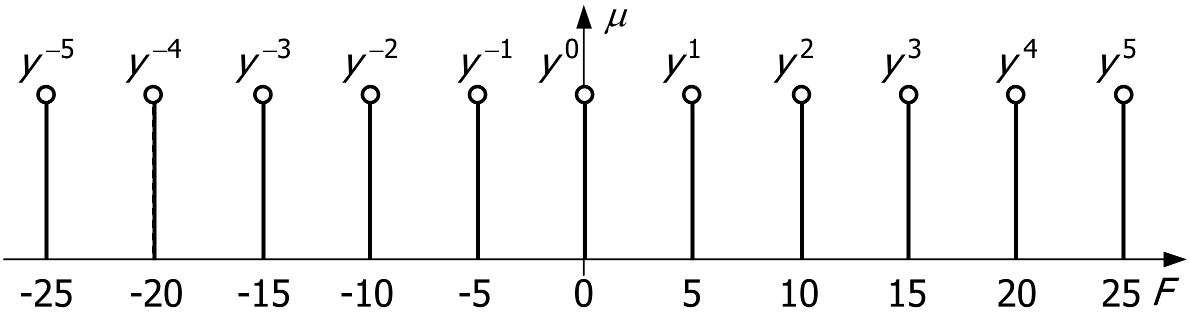

Figure 3: Singleton membership functions of the control variable F

Augment the Tagaki-Sugeno-Kang controller `fuzzy_cp_upswing.fis`, designed in the first part of the assignment. Add the Mamdani rules necessary for the stabilization of the pendulum. The gain factors in the consequents of the TSK rules are set to zero, in order to mimic Mamdani type rules. The constant offsets k in the TSK-consequent correspond to the singletons shown in figure 3.
Test the entire fuzzy controller in the Simulink environment. Evaluate its stability behavior for small deviations $-0.3 \le \theta_0 \le 0.3$ from the equilibrium. Afterwards, evaluate the control behavior for swinging up from the lower equilibrium of the inverted pendulum.

Optimization of the fuzzy controller for the stabilization of the pendulum and the cart

Fuzzy controllers possess the advantage of a nonlinear control surface, which allows it, to exploit the range of the control variable better than in the case of a conventional, linear controller. A linear controller employs a constant gain across the entire input space. Therefore the maximum gain is limited by the range of the inputs in conjunction with the saturation of the control variable. In the case of small deviations, however, a larger gain is possible, without violating the saturation limits of the control variable. The gain of a fuzzy controller due to its nonlinear characteristic can be varied across the input space. This advantage applies in particular in case of small deviations from the equilibrium, which a fuzzy controller can compensate with a large response of the control variable. In extreme cases this non-linearity leads to a two-point controller, which switches between the upper and lower limits of the control variable depending upon the input state. In our case, the gain between the input θ and the control variable F is to be increased for small deviations $-0.25 \le \theta \le 0.25$. The influence of the scaling factors and the location of the membership functions on the control surface of a fuzzy controller are described in section 2.4 "Simple Design Example: The Inverted Pendulum" in the book "Fuzzy Control". From the above aspects for the design of the controller it is intuitively clear that the gain increases locally, if the **input membership functions** move closer to the origin. Conversely, it applies that the gain is increased locally by shifting the **output membership functions** towards the boundary regions.

Improve the control behavior regarding the transient time by properly adapting the central membership functions $X_\theta^{-1}, X_\theta^{1}$ of the input variables θ and the central singletons $Y^{-3}, Y^{-2}, Y^{-1}, Y^{1}, Y^{2}, Y^{3}$ of the output variable F. How does the transient time improve, for an initial deviation from the equilibrium of $\theta_0 = 0.3$? Sketch the control surface for small deviations from the equilibrium state as function of the input variables

$\theta, \dot{\theta}$. Compare the control surfaces of the original linear fuzzy controller and the modified controller. Compare in each case the control behavior of the fuzzy controller with that of the state space controller from the first lab for modified system parameters, for a start state of $\theta_0 = 0.3$. Change the system parameters, by switching to the detail view of the plant and modify the pendulum length to l=0.5 and the mass of the pendulum to m=0.2. Which influence on the swing up behavior of the pendulum do the altered pendulum length and mass have?