1.3 Grade 9

1. Find the value of c that will make $16x^2 - 40x + c$ a perfect square.

2. Solve for x in $2(x^2 + 2) = 5x + 7$.

3. A number and its reciprocal have a sum of $\frac{58}{21}$. Find the larger of these two numbers.

4. Solve for x in $\left(\frac{x^4}{2} + 1\right)^2 - 2\left(\frac{x^4}{2} + 1\right) - 63 = 0$.

5. Solve for the inequality $3x^2 + 20x - 7 \geq 0$ for x.

6. Let r and s be the roots of $2x^2 + 3x - 6 = 0$. Find $r^3s^2 + r^2s^3$.

7. Find the range of values of the constant k so that $3x^2 + kx + 6$ is positive on any real x.

8. Find the values of the constant n so that $9x^2 - 3(2n + 3)x + 8n - 4 = 0$ has two equal roots.

9. One of the roots of $x^2 - bx + 36 = 0$ is one more that twice the other, where $b > 0$. Find b.

10. Find the vertex of the graph of $y = 3x^2 + 12x + 7$.

11. Find c if the x and y coordinates of the vertex of $y = cx^2 - 6cx + 21$ are equal.

12. The graph of $y = 2x^2 + 1$ is shifted 5 units to the left and 4 units up. Find the corresponding quadratic function (in the form $y = ax^2 + bx + c$) for the resulting graph.

13. An analyst determines that 250 students will buy a particular brand of calculator if these are sold at Php 400 each. Furthermore, for every Php 10 increase in the unit price, 5 less students would be willing to buy. What unit price will yield the largest possible revenue for the calculator seller?

14. Suppose that m varies directly as n. If $m = 4$ when $n = 6$, find m when $n = \frac{12}{7}$.

15. Suppose that q varies directly as r and inversely as the square of s. If $q = 12$ when $r = 1$ and $s = \frac{1}{2}$, find q when $r = 2$ and $s = \frac{1}{3}$.

16. Suppose that x ,y, and x are positive quantities such that x varies directly as the cube of y, and y varies inversely as twice z. If z is havled, what will be the corresponding change in x?

17. Rewrite the nonnegative exponents and simplify: $\frac{(x^2y^{-3}z^{1/2})^{-4}}{(x^{-1}yz^0)^2}$.

18. Simplify: $(5^{1/5}4^{2/5})^{5/3}5^{2/3}2^{8/3}$.

19. Rationalize the denominator of $\dfrac{2\sqrt{17}-\sqrt{3}}{\sqrt{17}+2\sqrt{3}}$ and simplify.

20. If $2 < x < 3$, simplify $\sqrt{4x^2-12x+9}-2\sqrt{x^2-7x+\dfrac{49}{4}}$.

21. Simplify $\sqrt[3]{250}+4\sqrt[3]{54}-5\sqrt[3]{16}$.

22. Solve for x in $8\sqrt{x+7}-8=5\sqrt{x+7}+7$.

23. Solve for x in $\sqrt{5x-10}=\sqrt{x-6}+6$.

24. If $r:s=2:5, s:t=3:5$, find $r:t$.

25. If $\dfrac{p+q}{2pq}=\dfrac{5}{3}$, find $\dfrac{p-q}{q}$.

26. Find all possible values of x in the proportion $(x-3):(x-1)=(x+6):(2x+2)$.

27. An angle of a quadrilateral has measure $10°$, while the others have degree measures in the ratio $3:4:7$. Find the measure of the largest angle.

28. In rhombus PQRS, $\angle QPR = 4\angle QSR$. Find $\angle PRQ$.

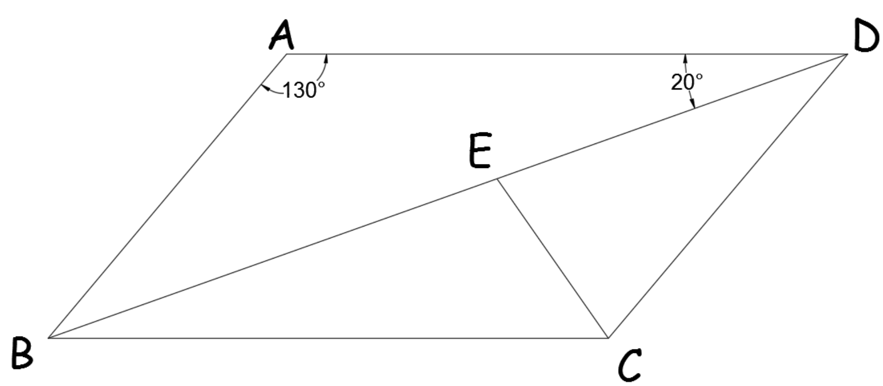
29. The diagonals of rhombus STAY intersect at X. If $AX = m+n, YX = 12, SX = 4m-n, TX = 4n$, find the length of the shorter diagonal.

30. Find the perimeter of rhoumbus STAY from the previous problem.

31. In parallelogram ABCD, AB = 9, $BC = 3x-5y, CD = 4x-3y, DA = x+y$. Find x

32. In parallelogram HIJK, $\angle J = (5x+30)°, \angle K = (3x-10)°$. Find $\angle H$.

33. In parallelogram ABCD, $\angle BAD = 130°, \angle ADB = 20°$, and E is chosen on diagonal BD so that $DE = DC$. Find $\angle CEB$.



34. In an isosceles trapezoid, the lengths of the diagonals are $3x+2$ and $5x-8$. Find x.

35. In trapezoid MNPQ, $MN \parallel PQ$ and R and S are the midpoints of MQ and NP, repsectively. If $MN = x + 1, RS - 6, PQ = 5x - 1$, find the length of PQ.

36. The diagonals of quadrilateral POST are perpendicular, and intersect at E. Suppose that $TE = OE = 3$, SE is twice as long as PE, and the area of the quadrilateral is 18 square units. Find the length of SE.

For problems 37 to 39: in $\triangle PQR$, S and T are points on PQ and PR, respectively, so that $ST \parallel QR$.

37. Suppose ST = 2, QR =6, PT = 5. Find PR.

38. Suppose PT = 5, TR = 4, and the perimeter of $\triangle PST$ is 12. Find the perimeter of $\triangle PQR$.

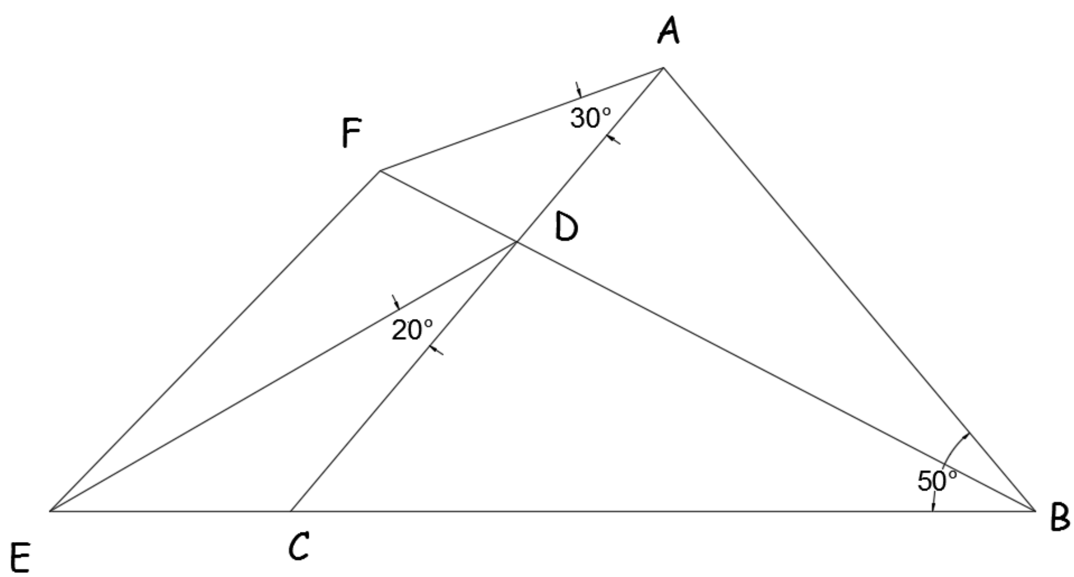
39. Suppose $PS = x - 3$, SQ = 6, and $TR = x + 2$. Find x.

40. The sides of $\triangle ABC$ are 9 cm, 10 cm, and 12 cm. If $\triangle ABC \sim \triangle DEF$, find the length of the longest side of $\triangle DEF$ is its shortest side is 6 cm.

41. Two similar triangles have their perimeters in the ratio 4 : 5. Find the ratio of their areas.

42. Given the points A(0,0), B(10,0),C(15,0),D(4,6) in the plane, the point E is chosen such that $AD \parallel BE$ and $BD \parallel CE$. Find the length of BE.

43. In the figure, $AB = AC$ and $DB = DE$. IF $\angle EDC = 20°, \angle FAD = 30°, \angle ABC = 50°$, which two triangles (whose sides are already shown) are similar to each other?



For problems 44 to 46: In $\triangle ABC, \angle B = 90°$. Let E be the point on AC so that $BE \perp AC$.

44. Suppose AE = 9 and CE = 2. Find BE.

45. Suppose AE = 30 and CE = 6, Find BC.

46. Suppose AB = 12 and $\dfrac{AE}{CE} = 8$. Find AE.

47. Two sides of a rectangle are 9 cm and 12 cm long. Find the length of a diagonal.

48. A ladder is leaning against a vertical wall with the top 5 m above the ground. The top of the ladder slides all the way down the wall so that the bottom of the ladder slides 1 m away from the wall. How long is the ladder?

49. The two legs of a right triangle are in the ratio $\dfrac{\sqrt{5}}{2}$. If the area is $9\sqrt{5}$ square units, find the length of the hypotenuse.

50. In $\triangle ABC, \angle C = 90°$ and $\tan A = \dfrac{2\sqrt{6}}{5}$. Find $\cos B$

2.3 Grade 9

1. Find the value of c that will make $16x^2 - 40x + c$ a perfect square.

   For it to be a perfect square, the discriminant should be zero.

$$\begin{aligned}(-40)^2 - 4(16)(c) &= 0\\ 16(10)^2 - 4(16)c &= 0\\ 10^2 - 4c &= 0\\ 100 - 4c &= 0\\ c &= \boxed{25}\end{aligned}$$

2. Solve for x in $2(x^2 + 2) = 5x + 7$

$$\begin{aligned}2x^2 - 5x - 3 &= 0\\ (2x+1)(x-3) &= 0\\ x &= \boxed{-\frac{1}{2}, 3}\end{aligned}$$

3. A number and its reciprocal have a sum of $\frac{58}{21}$. Find the larger of these two numbers.

$$\begin{aligned}n + \frac{1}{n} &= \frac{58}{21}\\ 21n^2 - 58n + 21 &= 0\\ (3n-7)(7n-3) &= 0\\ n &= \frac{3}{7}, \frac{7}{3}\end{aligned}$$

   Thus the larger of the two numbers is $\boxed{\frac{7}{3}}$

4. Solve for x in $\left(\frac{x^4}{2} + 1\right)^2 - 2.\left(\frac{x^4}{2} + 1\right) - 63 = 0$

   Let $u = \frac{x^4}{2} + 1$

$$\begin{aligned}u^2 - 2u - 63 &= 0\\ (u-9)(u+7) &= 0\\ u &= 9\\ \frac{x^4}{2} + 1 &= 9\\ x &= \pm 2\\ \frac{x^4}{2} + 1 &= -7\\ x &\text{ is } \text{imaginary}\end{aligned}$$

   Thus the real values of x is $\boxed{x = \pm 2}$

5. Solve for the inequality $3x^2 + 20x - 7 \geq 0$ for x.

$$\begin{aligned} 3x^2 + 20x - 7 &\geq 0 \\ (3x-1)(x+7) &\geq 0 \end{aligned}$$

Creating the table of signs for the appropriate intervals,

x	$x \leq -7$	$-7 \leq x \leq \frac{1}{3}$	$x \geq \frac{1}{3}$
$3x-1$	-	-	+
$x+7$	-	+	+
$(3x-1)(x+7)$	+	-	+

Therefore, the solution set is $\boxed{x \leq -7, x \geq \frac{1}{3}}$

6. Let r and s be the roots of $2x^2 + 3x - 6 = 0$. Find $r^3s^2 + r^2s^3$.

$$\begin{aligned} r+s &= -\frac{3}{2} \\ rs &= \frac{-6}{2} = -3 \\ r^3s^2 + r^2s^3 &= r^2s^2(r+s) \\ &= (-3)^2\left(-\frac{3}{2}\right) \\ &= \boxed{-\frac{27}{2}} \end{aligned}$$

7. Find the range of values of the constant k so that $3x^2 + kx + 6$ is positive on any real x.

The expression will be positive for any real number x if and only if the quadratic equation

$$3x^2 + kx + 6 = 0$$

has no real roots. Which means, the discriminant is negative. That is

$$b^2 - 4ac < 0$$

$$\begin{aligned} b^2 - 4ac &< 0 \\ k^2 - 4(3)(6) &< 0 \\ k^2 - 72 &< 0 \\ (k+6\sqrt{2})(k-6\sqrt{2}) &< 0 \end{aligned}$$

Thus, the values of k are $\boxed{-6\sqrt{2} < k < 6\sqrt{2}}$

8. Find the values of the constant n so that $9x^2 - 3(2n+3)x + 8n - 4 = 0$ has two equal roots.

   There will be two roots if the discriminant will be exactly equal to zero.

$$\begin{aligned} b^2 - 4ac &= 0 \\ (3(2n+3))^2 - 4(9)(8n-4) &= 0 \\ (2n+3)^2 - 4(8n-4) &= 0 \\ 4n^2 + 12n + 9 - 32n + 16 &= 0 \\ 4n^2 - 20n + 25 &= 0 \\ (2n-5)^2 &= 0 \\ n &= \boxed{\frac{5}{2}} \end{aligned}$$

9. One of the roots of $x^2 - bx + 36 = 0$ is one more that twice the other, where $b > 0$. Find b.

   Let the roots be a and $2a+1$. Since the product of the roots is 36,

$$\begin{aligned} a(2a+1) &= 36 \\ 2a^2 + a - 36 &= 0 \\ (2a+9)(a-4) &= 0 \\ a &= 4 \end{aligned}$$

   And since b is the sum of the roots, then $a + (2a+1) = 3a + 1 = 3(4) + 1 = \boxed{13}$

10. Find the vertex of the graph of $y = 3x^2 + 12x + 7$

$$\begin{aligned} y &= 3x^2 + 12x + 7 \\ &= 3(x^2 + 4x) + 7 \\ &= 3(x^2 + 4x + 4 - 4) + 7 \\ &= 3(x^2 + 4x + 4) - 3(4) + 7 \\ &= 3(x+2)^2 - 5 \end{aligned}$$

   Thus the vertex is at $\boxed{(-2, -5)}$

11. Find c if the x and y coordinates of the vertex of $y = cx^2 - 6cx + 21$ are equal.

$$\begin{aligned} y &= cx^2 - 6cx + 21 \\ &= c(x^2 - 6x) + 21 \\ &= c(x^2 - 6x + 9) - 9c + 21 \\ &= c(x-3)^2 - 9c + 21 \\ 3 &= -9c + 21 \\ c &= \boxed{2} \end{aligned}$$

12. the graph of $y = 2x^2 + 1$ is shifted 5 units to the left and 4 units up. Find the corresponding quadratic function (in the form $y = ax^2 + bx + c$) for the resulting graph.

$$\begin{aligned} y - 4 &= 2(x+5)^2 + 1 \\ y &= 2(x+5)^2 + 5 \\ y &= 2(x^2 + 10x + 25) + 5 \end{aligned}$$

Thus the corresponding quadriatic function is $\boxed{y = 2x^2 + 20x + 55}$

13. An analyst determines that 250 students will buy a particular brand of calculator if these are sold at Php 400 each. Furthermore, for every Php 10 increase in the unit price, 5 less students would be willing to buy. What unit price will yield the largest possible revenue for the calculator seller?

Since Revenue = Number of students buying $\times$ Unit price, and let x be the number of times the unit price is increased by Php 10, that is also the number of times 5 students will refuse to buy.

$$\begin{aligned} R &= (400 + 10x)(250 - 5x) \\ &= -50(x + 40)(x - 50) \end{aligned}$$

Note that the vertex of a quadratic function is located at the midpoint of the roots. Also, since the coefficient of x^2 is negative, it the quadratic function will have its maximum value.

$$x = \frac{-40 + 50}{2} = 5$$

Thus the unit price that yields to the highest possible revenue is

$$400 + 10(5) = \boxed{\text{Php}\,450}$$

14. Suppose that m varies directly as n. If $m = 4$ when $n = 6$, find m when $n = \frac{12}{7}$.

$$\begin{aligned} m &= kn \\ k &= \frac{m}{n} \\ k &= \frac{m_1}{n_1} = \frac{m_2}{n_2} \\ \frac{4}{6} &= \frac{m}{\frac{12}{7}} \\ \frac{7m}{12} &= \frac{2}{3} \\ m &= \boxed{\frac{8}{7}} \end{aligned}$$

15. Suppose that q varies directly as r and inversely as the square of s. If $q = 12$ when $r = 1$ and $s = \frac{1}{2}$, find q when $r = 2$ and $s = \frac{1}{3}$.

$$\begin{aligned}
q &= k\frac{r}{s^2} \\
k &= \frac{qs^2}{r} \\
\frac{12\left(\frac{1}{2}\right)^2}{1} &= \frac{q\left(\frac{1}{3}\right)^2}{2} \\
q &= \boxed{54}
\end{aligned}$$

16. Suppose that x ,y, and z are positive quantities such that x varies directly as the cube of y, and y varies inversely as twice z. If z is havled, what will be the corresponding change in x?

$$\begin{aligned}
x &= k_1 y^3 \\
y &= \frac{k_2}{2z} \\
z &\to \frac{1}{2}z \\
y_1 &= \frac{k_2}{2 \times \frac{1}{2}z} \\
&= \frac{k_2}{z} \\
y_1 &= 2y \\
x_1 &= k_1 y_1^3 \\
&= k_1 (2y)^3 \\
&= 8k_1 y^3 \\
x_1 &= 8x
\end{aligned}$$

Thus, x is $\boxed{\text{increased to 8 times}}$

17. Rewrite the nonnegative exponents and simplify: $\dfrac{(x^2y^{-3}z^{1/2})^{-4}}{(x^{-1}yz^0)^2}$

$$\begin{aligned}
\frac{(x^2y^{-3}z^{1/2})^{-4}}{(x^{-1}yz^0)^2} &= \frac{x^8y^{12}z^{-2}}{x^{-2}y^2} \\
&= \boxed{\frac{x^{10}y^{10}}{z^2}}
\end{aligned}$$

18. Simplify: $(5^{1/5}4^{2/5})^{5/3}5^{2/3}2^{8/3}$

$$\begin{aligned}
(5^{1/5}4^{2/5})^{5/3}5^{2/3}2^{8/3} &= (5^{1/5}2^{4/5})^{5/3}5^{2/3}2^{8/3} \\
&= 5^{1/3}2^{4/3}5^{2/3}2^{8/3} \\
&= 5 \times 2^4 \\
&= \boxed{80}
\end{aligned}$$

19. Rationalize the denominator of $\dfrac{2\sqrt{17} - \sqrt{3}}{\sqrt{17} + 2\sqrt{3}}$ and simplify.

$$\begin{aligned}
\frac{2\sqrt{17} - \sqrt{3}}{\sqrt{17} + 2\sqrt{3}} &= \frac{2\sqrt{17} - \sqrt{3}}{\sqrt{17} + 2\sqrt{3}} \times \frac{\sqrt{17} - 2\sqrt{3}}{\sqrt{17} - 2\sqrt{3}} \\
&= \frac{(2\sqrt{17} - \sqrt{3})(\sqrt{17} - 2\sqrt{3})}{17 - 12} \\
&= \frac{2(17) - 5\sqrt{51} + 2(3)}{5} \\
&= \frac{40 - 5\sqrt{51}}{5} \\
&= \boxed{8 - \sqrt{51}}
\end{aligned}$$

20. If $2 < x < 3$, simplify $\sqrt{4x^2 - 12x + 9} - 2\sqrt{x^2 - 7x + \dfrac{49}{4}}$

$$\begin{aligned}
\sqrt{4x^2 - 12x + 9} - 2\sqrt{x^2 - 7x + \frac{49}{4}} &= \sqrt{(2x-3)^2} - \sqrt{4x^2 - 28x + 49} \\
&= \sqrt{(2x-3)^2} - \sqrt{(2x-7)^2}
\end{aligned}$$

Since $2 < x < 3$

$$\begin{aligned}
\sqrt{(2x-3)^2} &= 2x - 3 \\
\sqrt{(2x-7)^2} &= 2x - 7 \\
(2x-3) - (2x-7) &= \boxed{4}
\end{aligned}$$

21. Simplify $\sqrt[3]{250} + 4\sqrt[3]{54} - 5\sqrt[3]{16}$

$$\begin{aligned}
\sqrt[3]{250} + 4\sqrt[3]{54} - 5\sqrt[3]{16} &= \sqrt[3]{2 \times 5^3} + 4\sqrt[3]{2 \times 3^3} - 5\sqrt[3]{2^4} \\
&= 5\sqrt[3]{2} + 12\sqrt[3]{2} - 10\sqrt[3]{2} \\
&= \boxed{7\sqrt[3]{2}}
\end{aligned}$$

22. Solve for x in $8\sqrt{x+7} - 8 = 5\sqrt{x+7} + 7$

$$\begin{aligned}
8\sqrt{x+7} - 8 &= 75\sqrt{x+7} + 7 \\
3\sqrt{x+7} &= 15 \\
\sqrt{x+7} &= 5 \\
x + 7 &= 25 \\
x &= \boxed{18}
\end{aligned}$$

23. Solve for x in $\sqrt{5x-10} = \sqrt{x-6} + 6$

$$\begin{aligned}
\sqrt{5x-10} &= \sqrt{x-6} + 6 \\
5x-10 &= x-6+12\sqrt{x-6}+36 \\
5x-10 &= x+30+12\sqrt{x-6} \\
4x-40 &= 12\sqrt{x-6} \\
x-10 &= 3\sqrt{x-6} \\
x^2-20x+100 &= 9x-54 \\
x^2-29x+154 &= 0 \\
(x-22)(x-7) &= 0 \\
x &= 22, 7
\end{aligned}$$

Substitute $x = 22$ gives $10 = 10$.
Substitute $x = 7$ gives $5 = 7$. EXTRANEOUS
Thus the only solution is $\boxed{x = 22}$

24. If $r : s = 2 : 5, s : t = 3 : 5$, find $r : t$

$$\begin{aligned}
\frac{r}{s} &= \frac{2}{5} \\
\frac{s}{t} &= \frac{3}{5} \\
\frac{r}{s} \times \frac{s}{t} &= \frac{2}{5} \times \frac{3}{5} \\
\frac{r}{t} &= \boxed{6 : 25}
\end{aligned}$$

25. If $\dfrac{p+q}{2p} = \dfrac{5}{3}$, find $\dfrac{p-q}{q}$

$$\begin{aligned}
\frac{p+q}{2p} &= \frac{5}{3} \\
3p+3q &= 10p \\
7p &= 3q \\
p &= \frac{3}{7}q \\
\frac{p-q}{q} &= \frac{\frac{3}{7}q - q}{q} \\
&= \boxed{-\frac{4}{7}}
\end{aligned}$$

26. Find all possible values of x in the proportion $(x-3):(x-1)=(x+6):(2x+2)$

$$\begin{aligned}(x-3)(2x+2) &= (x-1)(x+6)\\ 2x^2-4x-6 &= x^2-5x-6\\ x^2+x &= 0\\ x &= \boxed{0,-1}\end{aligned}$$

27. An angle of a quadrilateral has measure 10°, while the others have degree measures in the ratio $3:4:7$. Find the measure of the largest angle.

$$\begin{aligned}10+3k+4k+7k &= 360\\ 14k &= 350\\ k &= 25\\ 7k &= \boxed{175^\circ}\end{aligned}$$

28. In rhombus PQRS, $\angle QPR = 4\angle QSR$. Find $\angle PRQ$.

Using properties of a rhombus,

$$\begin{aligned}\angle QPR &= \angle QRP\\ \angle QSR &= \angle SQR\end{aligned}$$

If we let T be the intersection of the diagonals, $\triangle QTR$ is a right triangle at T. Thus,

$$\angle QPR + \angle QSR = 90$$

Meaning, $\angle QSR = 18^\circ$. Thus, $\angle PRQ = \angle QPR = \boxed{72^\circ}$

29. The diagonals of rhombus STAY intersect at X. If $AX = m+n, YX = 12, SX = 4m-n, TX = 4n$, find the length of the shorter diagonal.

$$\begin{aligned}4n &= 12\\ n &= 3\\ 4m-n &= m+n\\ 3m &= 2n\\ m &= 2\\ SA &= (4m-n)+(m+n) = 5m = 10\\ TY &= 2(12) = 24\end{aligned}$$

Thus the length of the shorter diagonal is $\boxed{10}$

30. Find the perimeter of rhoumbus STAY from the previous problem.

$$\begin{aligned}4s^2 &= 10^2+24^2\\ s^2 &= 5^2+12^2\\ s^2 &= 169\\ s &= 13\\ 4s &= \boxed{52}\end{aligned}$$

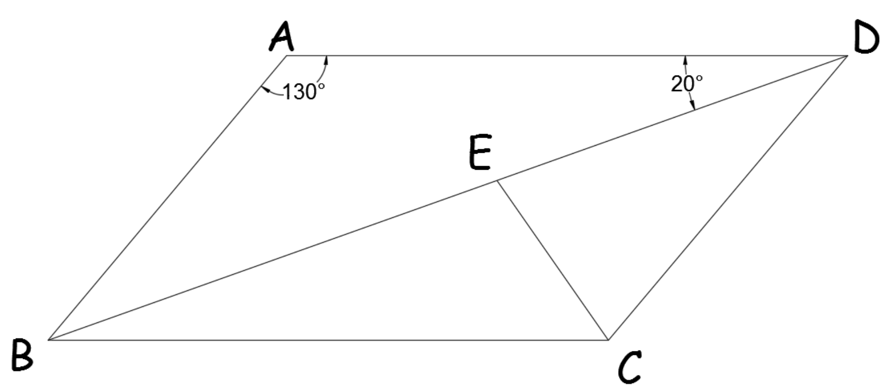
31. In parallelogram ABCD, AB = 9, $BC = 3x - 5y, CD = 4x - 3y, DA = x + y$. Find x

$$\begin{aligned} AB &= CD \\ 4x - 3y &= 9 \\ BC &= AD \\ 3x - 5y &= x + y \\ 2x &= 6y \\ x &= 3y \\ x &= \boxed{3} \\ y &= 1 \end{aligned}$$

32. In parallelogram HIJK, $\angle J = (5x + 30)^\circ, \angle K = (3x - 10)^\circ$. Find $\angle H$.

$$\begin{aligned} \angle K + \angle J &= 180 \\ (5x + 30) + (3x - 10) &= 180 \\ 8x - 20 &= 180 \\ x &= 25 \\ \angle H &= \angle J \\ &= 5(25) + 30 \\ &= \boxed{155^\circ} \end{aligned}$$

33. In parallelogram ABCD, $\angle BAD = 130^\circ, \angle ADB = 20^\circ$, and E is chosen on diagonal BD so that $DE = DC$. Find $\angle CEB$.



$$\begin{aligned} \angle ABD &= 180 - 130 - 20 = 30 \\ \angle BDC &= \angle ABD = 30 \\ DE &= DC \\ \angle DEC &= \angle DCE \\ \angle DEC &= \angle DCE = 75 \\ \angle CEB &= 180 - 75 = \boxed{105^\circ} \end{aligned}$$

34. In an isosceles trapezoid, the lengths of the diagonals are $3x+2$ and $5x-8$. Find x.

$$\begin{aligned} 3x+2 &= 5x-8 \\ 2x &= 10 \\ x &= \boxed{5} \end{aligned}$$

35. In trapezoid MNPQ, $MN \parallel PQ$ and R and S are the midpoints of MQ and NP, repsectively. If $MN = x+1, RS = 6, PQ = 5x-1$, find the length of PQ.

$$\begin{aligned} \frac{(x+1)+(5x-1)}{2} &= 6 \\ 3x &= 6 \\ x &= 2 \\ PQ &= \boxed{9} \end{aligned}$$

36. The diagonals of quadrilateral POST are perpendicular, and intersect at E. Suppose that $TE = OE = 3$, SE is twice as ling as PE, and the area of the quadrilateral is 18 square units. Find the length of SE.

$$\begin{aligned} \frac{PS \times OT}{2} &= 18 \\ \frac{(3+3)(2x+x)}{2} &= 18 \\ 9x &= 18 \\ x &= PE = 2 \\ SE &= 2x = \boxed{4} \end{aligned}$$

For problems 37 to 39: in $\triangle PQR$, S and T are points on PQ and PR, respectively, so that $ST \parallel QR$.

37. Suppose ST = 2, QR =6, PT = 5. Find PR.

$$\begin{aligned} \frac{PT}{PR} &= \frac{ST}{QR} \\ \frac{5}{PR} &= \frac{2}{6} \\ PR &= \boxed{15} \end{aligned}$$

38. Suppose PT = 5, TR = 4, and the perimeter of $\triangle PST$ is 12. Find the perimeter of $\triangle PQR$.

$$\begin{aligned} \frac{PT}{TR} &= \frac{P_{PST}}{P_{PQR}} \\ \frac{5}{9} &= \frac{12}{P_{PQR}} \\ P_{PQR} &= \boxed{\frac{108}{5}} \end{aligned}$$

39. Suppose $PS = x - 3$, SQ = PT= 6, and $TR = x + 2$. Find x.

$$\begin{aligned}
\frac{PS}{SQ} &= \frac{PT}{TR} \\
\frac{x-3}{6} &= \frac{6}{x+2} \\
x^2 - x - 6 &= 36 \\
x^2 - x - 42 &= 0 \\
(x-7)(x+6) &= 0 \\
x &= \boxed{7}
\end{aligned}$$

40. The sides of $\triangle ABC$ are 9 cm, 10 cm, and 12 cm. If $\triangle ABC \sim \triangle DEF$, find the length of the longest side of $\triangle DEF$ is its shortest side is 6 cm.

$$\begin{aligned}
\frac{6}{9} &= \frac{x}{12} \\
x &= \boxed{8}
\end{aligned}$$

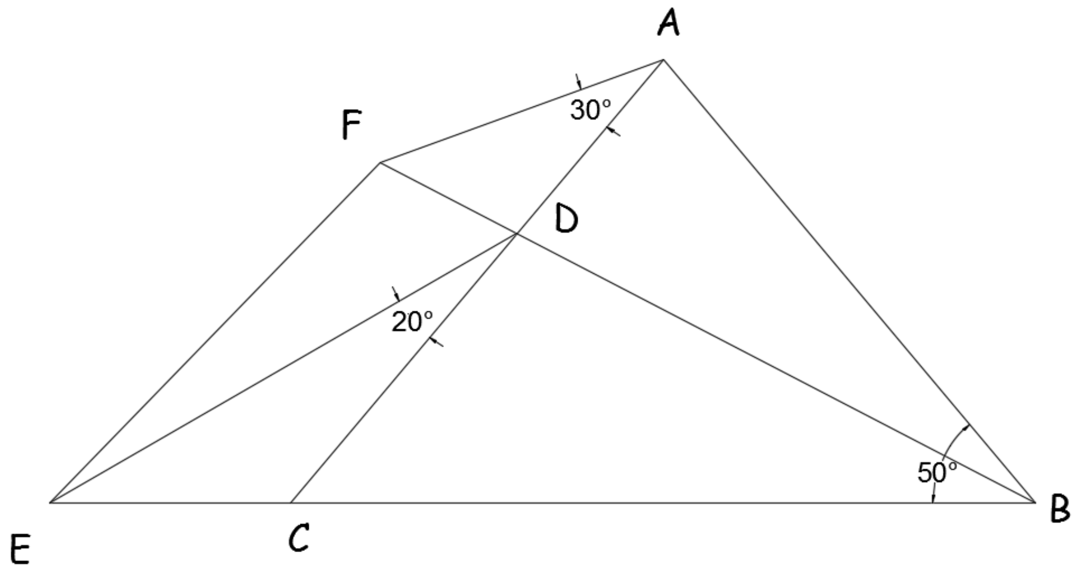
41. Two similar triangles have their perimeters in the ratio 4 : 5. Find the ratio of their areas.

$$\left(\frac{4}{5}\right)^2 = \boxed{16 : 25}$$

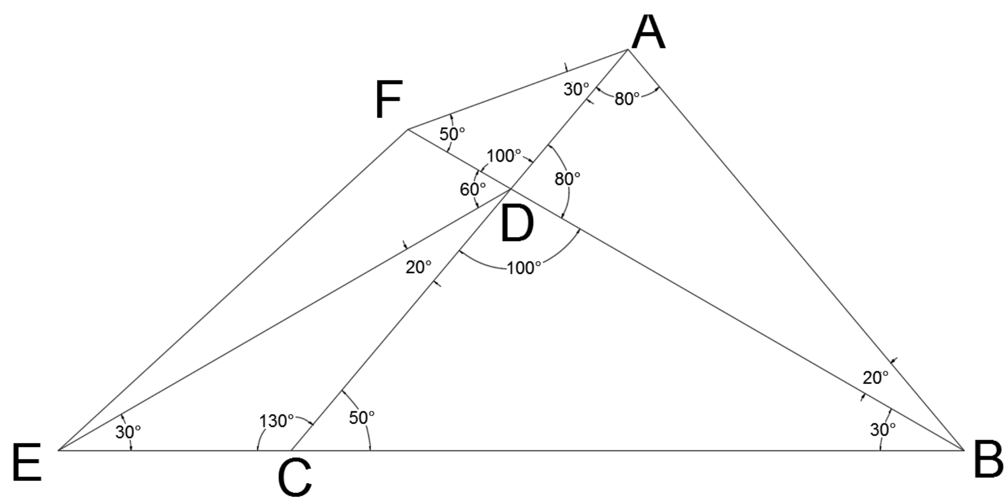
42. Given the points A(0,0), B(10,0),C(15,0),D(4,6) in the plane, the point E is chose such that $AD \parallel BE$ and $BD \parallel CE$. Find the length of BE.

$$\begin{aligned}
AD &\parallel DE \\
\frac{0-0}{10-0} &= \frac{6-y}{x-4} \\
y &= 6 \\
BD &\parallel CE \\
\frac{6-0}{4-10} &= \frac{y-0}{x-15} \\
-1 &= \frac{6}{x-15} \\
x &= 9 \\
BE &= \sqrt{(10-9)^2 + (0-6)^2} \\
&= \boxed{\sqrt{37}}
\end{aligned}$$

43. In the figure, $AB = AC$ and $DB = DE$. IF $\angle EDC = 20°, \angle FAD = 30°, \angle ABC = 50°$, which two triangles (whose sides are already shown) are similar to each other?



Using properties of triangles and using the facts that $AB = AC, DB = DE$ that is when $\triangle ABC$ and $\triangle BED$ are both isosceles.



Thus the similar triangles are $\boxed{\triangle AFD \sim \triangle BCD}$

For problems 44 to 46: In $\triangle ABC, \angle B = 90°$. Let E be the point on AC so that $BE \perp AC$.

44. Suppose AE = 9 and CE = 2. Find BE.

$$BE = \sqrt{9 \times 2} = \boxed{3\sqrt{2}}$$

45. Suppose AE = 30 and CE = 6, Find BC.

$$BC = \sqrt{AC \times CE} = \boxed{6\sqrt{6}}$$

46. Suppose AB = 12 and $\dfrac{AE}{CE} = 8$. Find AE.

$$\sqrt{8x \times 9x} = 12 \to x = 2$$

That is when $x = CE$. Thus $AE = 8x = \boxed{16}$

47. Two sides of a rectangle are 9 cm and 12 cm long. Find the length of a diagonal.

$$\sqrt{9^2 + 12^2} = \boxed{15}$$

48. A ladder is leaning against a vertical wall with the top 5 m above the ground. The top of the ladder slides all the way down the wall so that the bottom of the ladder slides 1 m away from the wall. How long is the ladder?

If we let L be the length of the ladder and b be the distance between the foot of the ladder and the base of the wall,

$$\begin{aligned} L^2 &= b^2 + 5^2 \\ L^2 &= (b+1)^2 + 4^2 \end{aligned}$$

Subtracting the two equations,

$$\begin{aligned} 2b + 1 - 9 &= 0 \\ b &= 4 \\ L &= \sqrt{5^2 + 4^2} \\ L &= \boxed{\sqrt{61}} \end{aligned}$$

49. The two legs of a right triangle are in the ratio $\dfrac{\sqrt{5}}{2}$. If the area is $9\sqrt{5}$ square units, find the length of the hypotenuse.

$$\begin{aligned} \frac{a}{b} &= \frac{\sqrt{5}}{2} \\ a &= b\frac{\sqrt{5}}{2} \\ \frac{ab}{2} &= 9\sqrt{5} \\ ab &= 18\sqrt{5} \\ b^2\frac{\sqrt{5}}{2} &= 18\sqrt{5} \\ b &= 6 \\ a &= 3\sqrt{5} \\ c &= \sqrt{a^2 + b^2} \\ &= \boxed{3\sqrt{10}\text{ units}} \end{aligned}$$

50. In $\triangle ABC$, $\angle C = 90^\circ$ and $\tan A = \dfrac{2\sqrt{6}}{5}$. Find $\cos B$

$$\begin{aligned}
\tan A &= \frac{2\sqrt{6}}{5} \\
\tan B &= \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12} \\
\cos B &= \frac{12}{\sqrt{12^2 + (5\sqrt{6})^2}} \\
&= \frac{12}{\sqrt{294}} \\
&= \frac{12}{7\sqrt{6}} \\
&= \boxed{\frac{2\sqrt{6}}{7}}
\end{aligned}$$