Metrobank-MTAP-DepEd Math Challenge 2015
Elimination Stage Grade 9

Name: _____ **School:** _____ **Score:** _____

Instruction: Write your answer on the space provided before each item. Give all fractions in lowest terms, and all equations of lines in the form $ax + by = c$ where a, b and c are relatively prime integers, with $a > 0$.

_____ **1.** Simplify $\sqrt{\frac{3}{2}} - \sqrt{\frac{2}{3}}$.

_____ **2.** Evaluate $\frac{2^0 + 2^{-1}}{2^{-2} + 2^{-3}}$.

_____ **3.** Simplify $\sqrt{\frac{1}{9} + \frac{1}{16}}$.

_____ **4.** Simplify $\frac{x^{-1} - y^{-1}}{x^{\frac{1}{3}} - y^{\frac{1}{3}}}$.

_____ **5.** If a, b and c are real numbers such that $\frac{b}{a} = 5$ and $\frac{b}{c} = 2$, what is the value of $\frac{a+b}{b+c}$?

_____ **6.** If $x \neq 0$, $\frac{x}{9} = y^2$ and $\frac{x}{3} = 3y$, what is x?

_____ **7.** If $f(x) = x^2 + 6x + 9$, what is $f(x - 3)$?

_____ **8.** If $f(x) = x^2 + 1$ and $g(x) = x - 1$, for all real numbers x, for what real number a does $f(g(-a)) = g(f(-a))$?

_____ **9.** Solve for x in $\sqrt{3 + \sqrt{x}} = 3$.

_____ **10.** Solve for x in the equation $\frac{2x}{x+2} + \frac{x+2}{2x} = 2$.

_____ **11.** If $x \neq 1$, solve for x in $2\sqrt{x} + \frac{3}{\sqrt{x}} = 5$.

_____ **12.** Evaluate $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$.

_____ **13.** Find two consecutive positive integers whose product is 506.

_____ **14.** If $c > a > 0$ and if $a - b + c = 0$, find the larger root of $ax^2 + bx + c = 0$.

_____ **15.** Solve for x in $2x^2 + x < 6$.

_____ **16.** Solve for real numbers x satisfying the inequality $x - 2\sqrt{x} \leq 3$.

_____ **17.** Find the minimum of value of $x^2 - 8x + 3$.

_____ **18.** Find the smallest value of $x + \frac{5}{x}$, for all real numbers x.

_____ **19.** Solve for b in the equation $(x + 1)(x + a) = x^2 + bx + 3$.

_____ **20.** Write the quadratic equation with integer coefficents whose roots are the reciprocal of the roots of $2x^2 - 3x + 1 = 0$.

_____ **21.** Compute the sum of all the roots of $(x - 2)(x + 1) + (x - 1)(x + 4) = 0$.

_____ **22.** If r and s are the roots of $x^2 + x - 1 = 0$, evaluate $(r + s)^2$.

_____ **23.** For what value(s) of m are the roots of $(m - 1)x^2 - mx + 1 = 0$ equal?.

_____ **24.** It is known that y varies as the square of x and that $y = 8$ when $x = 1$. What is y when $x = 8$?

_____ **25.** Suppose that x and y are inversely proportional and are positive quantities. By what percent does y decrease if x is increased by 25%?

_____ **26.** If 4 men can paint a house in 5 days, in how many days can 10 men paint the same house?

_____ **27.** If y is proportional to the cube of x and x is proportional to the fourth power of z, then y is proportional to which power of z?

28. Running at uniform speed in a race, Allan can beat Ben by 20 m, Ben can beat Carlo by 10 m and Allan can beat Carlo by 28 m. How long is the race?

29. Find the measure of the vertex angle of an isosceles triangle whose base angles measure 65°.

30. Find x if the angles of a quadrilateral measure $x°$, $(2x + 10)°$, $(3x + 20)°$ and $(4x - 30)°$.

31. An equilateral triangle and a square have the same perimeter. What is the ratio of the length of a sides of the triangle to the length of a side of the square?

32. John cuts an equilateral triangular paper whose sides measure 2 cm. into pieces. He then rearranges the pieces to form a square without overlapping. How long is the side of the square formed?

33. The sides of a triangle are of lengths 5, 12 and 13 cm. What is the length of the shortest altitude?

34. Each side of triangle ABC measures 8 cm. If D is foot of the altitude drawn from A to the side BC and E is the midpoint of AD, how long is the segment BE?

35. A point E is chosen inside a square of side 8 cm. such that it is equidistant from to adjacent vertices of the square and the side opposite these vertices. Find the common distance.

36. A point E is drawn inside rectangle $ABCD$ and its distances from A, B and C are 2 cm., 3 cm. and 4 cm. How far is E from D?

37. Find the area of a rectangle with diagonal of length 10 cm is twice as long as it is wide.

38. Triangle ABC has a right angle at C. If $\sin A = \frac{1}{3}$, what is $\cos A$?

39. A side of a triangle measures 3 cm. A line segment is drawn parallel to this side, forming a trapezoid whose area is $\frac{2}{3}$ of the area of the triangle. How long is the line segment?

40. A cone has volume 64 m^3. If the cone is cut parallel to the base at a distance from the vertex equal to $\frac{1}{4}$ of the height of the cone, what is the volume of the resulting cone?

41. In figure 1 below, $ABCD$ is a square with diagonal 2 cm. long and AEC is an equilateral triangle. Find the area of the quadrilateral $AECB$.

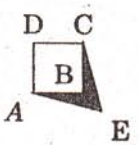

Figure 1

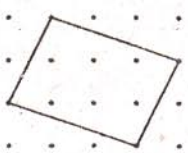
Figure 2

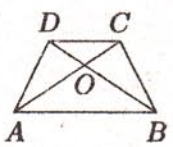

Figure 3

42. Pegs are put on a board 1 cm apart both horizontally and vertically. What is the area of the quadrilateral formed by stretching a rubber band over four pegs as shown in figure 2 above?

43. In figure 3 above, $ABCD$ is an isosceles trapezoid with AB as its longest side and O divides the diagonals AC and BD in the ratio 1 : 2. What is the area of $ABCD$ if the area of BOC is 2 m^2?

44. On triangle ABC, D and E are the midpoints of BC and AB, respectively. The median AD meets CE at F. If the area of the triangle EFA is 1 cm^2, what is the area of triangle ABC?

45. Equilateral triangle ABC has side of length 2 cm, M is the midpoint of of AC and B is the midpoint of CD. What is the area of $\triangle MCD$?

46. Points D, E and F are drawn one on each side of triangle ABC so that $ADEF$ is a rhombus. If the lengths of AC, AB and BC are 6, 12 and 8 respectively, how long is a side of $ADEF$?

47. A rectangular piece of paper 4 cm. in length and 3 cm. in width is folded so that a pair of diagonally opposite vertices coincide. How long is the crease(fold-mark) formed?

48. In trapezoid $ABCD$, $AD \parallel BC$ and $AB = BC = \frac{AD}{2}$. Find $\angle ACD$.

49. Two angles of a triangle measures 30° an 105° and the side between these two angles measures 2 cm. What is the perimeter of the triangle?

50. How many noncongruent triangles with perimeter 9 have sides of integer length?

2015 Metrobank-MTAP-DepED Math Challenge
Division Orals - Grade 9

15-second Questions. [2 points each]

1. Determine all positive number x that satisfy $5x^2 = 10x$. [x = 2]
2. What is the fourth power of $\sqrt{2+\sqrt{2}}$? [$6+4\sqrt{2}$]
3. Simplify $4^{-\frac{k}{2}} + 8^{-\frac{k-1}{3}}$. [$\frac{3}{2^k}$]
4. If $a \spadesuit b = \sqrt{a^2+b^2}$, what is the value of $(3 \spadesuit 4) \spadesuit 12$? [13]
5. Suppose that x, y and z are positive integers such that xy = 6, xz = 10 and $y^{yz}x = 15$. What is the value of xyz? [30]
6. The yearly changes in the population of a certain town for two consecutive years are 20% increase on the first year and 20% decrease on the second year. What is the net change in percent over the two year period? [4% decrease]
7. What is the slope of the line parallel to $2x + 5y + 2 = 0$. [- 2/5]
8. The area of a triangle is 100 cm^2. What will be its area if its altitude is increased by 10% and its base is decreased by 10%? [99 cm^2]
9. The sum of two numbers is 2015. If 9 is added to each of the numbers and then each of the resulting numbers is doubled, what is the sum of the final two numbers? [4066]
10. A square and a triangle have the same perimeter. If the square has area 144 cm^2, what is the area of the triangle? [$64\sqrt{3}$ cm^2]
11. Let r and s be the solutions of $x^2 - 3x + 1 = 0$. What is the value of $(r + 1)(s + 1)$? [5]

30-second Questions [3 points each]

1. If $f(x) = x^2 = x + 1$, find the sum of all numbers y that satisfies $f(2y) = 2$. [$\frac{1}{2}$]
2. A man walks 1 km east then 1 km northwest. How far is he from his starting point? [$\sqrt{2-\sqrt{2}}$ km]
3. Four men working for four days can paint 4 cars. How many cars can 6 men working for 6 days paint? [9 cars]
4. The longer base of a trapezoid measures 10 cm and the line segment joining the midpoint of the diagonals measures 3 cm. What is the length of the shorter base? [4 cm]
5. What is the least possible value for $x^2 + 3x + 2$ if $x^2 - 3x - 2 \leq 0$? [6] -1/4
6. The point D is the midpoint of the side BC of equilateral triangle ABC and E is the midpoint of AD. How long is BE if a side of ΔABC measures 8 cm? [$2\sqrt{7}$cm]

1-minute Questions [5 points each]

1. If the roots of $x^2 + nx + m = 0$ are twice those of $x^2 + mx + 1 = 0$, what is the value of n? [8]
2. The lengths of the sides of a triangle are 10, 17 and 21 cm. How long is the altitude of the triangle to longest side? [$\frac{4\sqrt{70}}{5}$]
3. Triangle ABC is isosceles. If $\angle A = 50°$, what are the possible measures of $\angle B$? [50°, 80°]
4. The medians AD and BE of ΔABC are perpendicular. Find the length of AB if BC = 3 cm and AC = 4 cm. [$\sqrt{5}$ cm]
5. The product of three consecutive positive integers is 16 times their sum. What is the sum of the three numbers? [21]
6. Point E is on the side AC of ΔABC and points D and F are chosen on the side AB such that DE ∥ BC and EF ∥ CD. Find the length of BD if AF = FD = 3 cm. [6 cm]

Clincher Questions

C.1. In ΔABC, $\angle C = 30°$. If D is the foot of the altitude from A to BC and E is the midpoint of AC, find the measure of $\angle EDC$. [30°]

C.2. One candle will burn completely at a uniform rate in 4 hours while another in 3 hours. At what time should the two candles be simultaneously lighted so that one will be half the lenght of the other at 6:00 P.M.? [3:36 P.M.]

C.3. Points P and Q are drawn on the sides BC and AC of triangle ABC such that $\angle AQB$ and $\angle APB$ measures 110° and 80° respectively. If point R is chosen inside ΔABC such that AR and BR bisects $\angle CAP$ and $\angle CAQ$ respectively, what is the measure of $\angle ARB$? [95°]

Do-or-Die Questions

Point E is the midpoint of the side BC of ΔABC and F is the midpoint of AE. The line thru BF intersects AC at D. Find the area of ΔAFD if the area of the triangle is 48 cm^2. [4 cm^2]