

2015 Grade 9 MTAP Math Challenge – Division Orals

15-second question (2 points each)

1.) Determine all positive number x that satisfy $5x^2 = 10x$.

Answer: $x = 2$

2.) What is the fourth power of $\sqrt{2 + \sqrt{2}}$?

Answer: $6 + 4\sqrt{2}$

3.) Simplify $4^{\frac{-k}{2}} + 8^{-\frac{k-1}{3}}$

Answer: $\frac{3^k}{2}$

4.) If $a \spadesuit b = \sqrt{a^2 + b^2}$, what is the value of $(3 \spadesuit 4) \spadesuit 12$?

Answer: 13

5.) Suppose that x, y and z are positive integers such that $xy = 6$, $xz = 10$ and $yz = 15$. What is the value of xyz ?

Answer: 30

6.) The yearly changes in population of a certain town for two consecutive years are 20% increase on the first year and 20% decrease on the second year. What is the net change in percent over the two year period?

Answer: 4% decrease

7.) What is the slope of the line parallel to $2x + 5y + 2 = 0$.

Answer: $-2/5$

8.) The area of a triangle is 100 sq. cm. What will be its area if its altitude is increased by 10% and its base is decreased by 10%?

Answer: 99 sq. cm.

9.) The sum of two numbers is 2015. If 9 is added to each of the numbers and then each of the resulting numbers is doubled, what is the sum of the final two numbers?

Answer: 4066

10.) A square and a triangle have the same perimeter. If the square has area 144 sq. cm., what is the area of the triangle?

Answer: $64\sqrt{3}$ sq cm.

11.) Let r and s be the solutions of $x^2 - 3x + 1 = 0$. What is the value of $(r + 1)(s + 1)$?

Answer: 5

30-second question (3 points each)

1.) If $f(x) = x^2 = x + 1$, find the sum of all numbers y that satisfies $f(2y) = 2$.

Answer: $1/2$

2.) A man walks 1 km east then 1 km northwest. How far is he from his starting point?

Answer: $2 - \sqrt{2}$ km

3.) Four men working for four days can paint 4 cars. How many cars can 6 men working for 6 days paint?

Answer: 9 cars

4.) The longer base of a trapezoid measures 10 cm and the line segment joining the midpoint of the diagonals measures 3 cm. What is the length of the shorter base?

Answer: 4 cm

5.) What is the least possible value of $x^2 + 3x + 2$ if $x^2 - 3x - 2 \leq 0$?

Answer: 6, -1/4

6.) The point D is the midpoint of the side BC of equilateral triangle ABC and E is the midpoint of AD. How long is BE if a side of $\triangle ABC$ measures 8 cm?

Answer: $2\sqrt{7}$ cm

1-minute question (5 points each)

1.) If the roots of $x^2 + nx + m = 0$ are twice those of $x^2 + mx + 1 = 0$, what is the value of n?

Answer: 8

2.) The lengths of the sides of a triangle are 10, 17 and 21 cm. How long is the altitude of the triangle to longest side?

Answer: $\frac{4\sqrt{70}}{5}$

3.) Triangle ABC is isosceles. If $\angle A = 50^\circ$, what are the possible measures of $\angle B$?

Answer: $50^\circ, 80^\circ$

4.) The medians AD and BE of $\triangle ABC$ are perpendicular. Find the length of AB if $BC = 3$ cm and $AC = 4$ cm.

Answer: $\sqrt{5}$ cm

5.) The product of three consecutive positive integers is 16 times their sum. What is the sum of the three numbers?

Answer: 21

6.) Point E is on the side AC of $\triangle ABC$ and points D and F are chosen on the side AB such that $DE \parallel BC$ and $EF \parallel CD$. Find the length of BD if $AF = FD = 3$ cm.

Answer: 6 cm

Clincher Question

1.) In $\triangle ABC$, $\angle C = 30^\circ$. If D is the foot of the altitude from A to BC and E is the midpoint of AC, find the measure of $\angle EDC$.

Answer: 30°

2.) One candle will burn completely at a uniform rate in 4 hours while another in 3 hours. At what time should the two candles be simultaneously lighted so that one will be half the length of the other at 6:00 PM?

Answer: 3:36 PM

3.) Points P and Q are drawn on the sides BC and AC of triangle ABC such that $\angle AQB$ and $\angle APB$ measures 110° and 80° respectively. If point R is chosen inside $\triangle ABC$ such that AR and BR bisect $\angle CAP$ and $\angle CAQ$ respectively, what is the measure of $\angle ARB$?

Answer: 95°

Do or Die Question

Point E is the midpoint of the side BC of $\triangle ABC$ and F is the midpoint of AE. The line thru BF intersects AC at D. Find the area of $\triangle AFD$ if the area of the triangle is 48 cm^2 .
Answer: 4 cm^2