

Part I. Write your answer on the space provided before each item. Each correct answer earns 2 points.

- 1 A and B are positive integers, where A is between 3 and 18 (inclusive), and B is between 5 and 20 (inclusive). Find the positive difference between the largest and smallest possible values of $\frac{A}{B}$.
- 2 A rectangle has area $(2x^2 - x - 21) \text{ cm}^2$. Find its perimeter if it has a base of $(x + 3) \text{ cm}$.
- 3 Find the coordinates of a point N on the segment connecting $M(-3, 4)$ and $P(7, -11)$ such that $\frac{MN}{NP} = \frac{2}{3}$.
- 4 Find the area of a rectangle with width 18 cm if it is inscribed in a circle with radius 15 cm.
- 5 Insert two arithmetic means between 24 and 57
- 6 The midpoints of the three sides of a triangle are connected to form a smaller triangle. If the smaller triangle has perimeter $\frac{5}{6} + \frac{\sqrt{5}}{8} \text{ cm}$, find the perimeter of the larger triangle.
- 7 The sum of the interior angles of a polygon is between 2013° and 2020° . How many sides does it have?
- 8 The difference between the 51st and 16th term terms of an arithmetic sequence is 135. Find the positive common difference.
- 9 Find the radius of a cylindrical can whose volume is $54\pi^4 \text{ cm}^3$ if its height is twice its radius.
- 10 Find the value of x so that $x + 6$, $2x + 4$, and $4x - 4$ will form a geometric sequence.
- 11 Simplify $(1 + \sqrt{3})(1 + \sqrt[4]{3})(1 + \sqrt[8]{3})(1 + \sqrt[16]{3})(1 - \sqrt[16]{3})$.
- 12 Find the largest among a set of three numbers, which, when added in pairs, give the sums 26, 27, and 31.
- 13 A square and an equilateral triangle have the same perimeter. If the square has area 144 cm^2 , what is the area of the triangle.
- 14 The legs of a right triangle are 3 cm and $\sqrt{3} \text{ cm}$. Find the altitude to the hypotenuse.
- 15 $\triangle PQR$ is inscribed in a circle. The measures of the nonoverlapping arcs \widehat{PQ} , \widehat{QR} , and \widehat{RP} are $(2x + 36)^\circ$, $(3x - 6)^\circ$, and $(x + 30)^\circ$, respectively. Find $\angle PQR$.

Part II. Write a complete and neat solution to each problem. Each correct solution earns 3 points.

- 1 Find the sum of the series $\frac{3}{2} - \frac{2}{3} + \frac{3}{2^2} - \frac{2}{3^2} + \frac{3}{2^3} - \frac{2}{3^3} + \dots$.
- 2 In regular hexagon $ABCDEF$, square $BCMN$ is drawn, sharing side BC with the hexagon. Find $\angle FAN$.
- 3 Quadrilateral $MNPQ$ is inscribed in a circle. Its diagonal $NQ = 2$ is a diameter of the circle. If $\angle MNQ = 30^\circ$ and $\angle PNQ = 45^\circ$, find the ratio of the area of $MNPQ$ to the area of the circle.
- 4 The sum of the first 12 terms of an arithmetic sequence is 6 times the sum of the first 5 terms. Find the ratio of the common difference to the first term.
- 5 Find the radius of the circle inscribed in a right triangle with legs 5 cm and 12 cm.

Part III. Write a complete and neat solution to each problem. Each correct solution earns 5 points.

- 1 The first, fourth, and twelfth terms of an arithmetic sequence form a geometric sequence. Find the ratio of the second term of the first term of the arithmetic sequence.
- 2 Square $ABCD$ has area 2 units². E and F are points on AB and BC , respectively, such that $AE = CF$. If the square is folded along DE and DF , sides AD and CD coincide along diagonal BD . Find the common length of AE and CF .
- 3 Triangle ABC is an isosceles right triangle with right angle at B . Through a point D on BC , AD is extended to a point P so that $\angle APC = 90^\circ$. If $\triangle APC \sim \triangle CPD$ and $BD = 3\sqrt{2}$, find CD .