## 2019 Metrobank-MTAP-DepEd Math Challenge
Elimination Round • Grade 9 • Category A

Name: _____ School: _____ Score: _____

Instruction: Write your answer on the space provided before each item. Give all fractions and ratios in lowest terms. All denominators should be rationalized. Figures are not drawn to scale.

_____ 1. Simplify $\dfrac{3^0 - 3^{-2}}{3^{-1} + 3^{-3}}$.

_____ 2. Rationalize the denominator of $\dfrac{\sqrt{15} - 3}{\sqrt{5} - \sqrt{3}}$ and simplify.

_____ 3. Simplify $\sqrt{72} \pm \sqrt{32} - \sqrt{2}$.

_____ 4. Simplify $\left(\dfrac{b^{4x-3y}}{b^{3x+2y}}\right)^x \left(\dfrac{b^{3x+6y}}{b^{-2x+4y}}\right)^y$.

_____ 5. What constant should be added to $x^2 - 10x$ to make it a perfect square?

_____ 6. Find the *larger* root of $10x^2 - x = 3$.

_____ 7. Solve the inequality $2x^2 - 7x - 4 \leq 0$. Express your answer in interval notation.

_____ 8. Find the *sum* of the *distinct* roots of $\left(x + \dfrac{2}{x}\right)^2 - 6\left(x + \dfrac{2}{x}\right) + 9 = 0$.

_____ 9. List down all possible values of the constant k so that $3x^2 - 2kx + 2 = 0$ has two equal roots.

_____ 10. Find the range of values of b if $2x^2 + bx + 8 = 0$ has no real roots. Express your answer as an interval.

_____ 11. What is the sum of the roots of $x^2 + 5x - 7 = 0$?

_____ 12. If r and s are the roots of $x^2 - 6x + 4 = 0$, find $r + s - rs$.

_____ 13. The roots of $2x^2 - 3kx - 2 = 0$, where $k \neq 0$ is a constant, are $x = \dfrac{3k \pm \sqrt{36k + 16}}{4}$. Find k.

_____ 14. The roots of $x^2 - 6x + 7 = 0$ are r and s, while the roots of $x^2 + bx + c = 0$ are $-r$ and $-s$. Find b.

_____ 15. List down all the roots of $\dfrac{2x - 3}{3x - 8} = \dfrac{5x - 11}{4x - 6}$.

_____ 16. List down all the roots of $x + 3 = \sqrt{4x + 17}$.

_____ 17. A rectangular garden is 9 m by 11 m. A pathway of uniform width will be constructed around the garden, forming a larger rectangle with area 120 m². How wide (in m) should the pathway be?

_____ 18. What is the vertex of $y = 2x^2 - 8x - 3$?

_____ 19. Find c if the range of $y = -3x^2 + 30x + c$ is $(-\infty, 40]$.

_____ 20. The vertex of the graph of $y = 3x^2 + 12 + c$ is $(-2, -5)$. Find the constant c.

_____ 21. A quadratic function $f(x) = ax^2 + bx + c$ has zeros -2 and 4, and its graph has vertex $(1, -18)$. Find the constant a.

_____ 22. The graph of $y = x^2 - 3$ is shifted 3 units to the right and 7 units down. Find the corresponding quadratic function (in the form $y = ax^2 + bx + c$) for the resulting graph.

_____ 23. Find the largest possible product of two numbers if the sum of the first and twice the second is 12.

_____ 24. Suppose y varies directly as x. If $y = 7$ when $x = 4$, find x when $y = 28$.

_____ 25. Suppose r varies directly as the square of s, and inversely as t. If $r = 4$ when $s = 3$ and $t = 6$, find r when $s = 6$ and $t = 4$.

_____ 26. Let a, b, and c be positive quantities such that a varies directly as b and inversely as c. If c increases and b decreases, will a increase or decrease?

_____ 27. It takes 10 hours for 6 painters to paint a house. How many hours would it take 15 painters?

_____ 28. The sides of a triangle are in the ratio $3 : 4 : 6$. Find the longest side (in cm) if its perimeter is 52 cm.

_____ 29. The angles of a quadrilateral are in the ratio $2 : 3 : 3 : 4$. Find its smallest angle.

_____ 30. If $(x + y) : y = 17 : 6$, find $(x - y) : y$.

_____ 31. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4, and 5, if the digits are unique?

_____ 32. A class has 20 girls and 30 boys. Half of the girls and a third of the boys wear eyeglasses. If a student is picked at random from this class, what is the probability of this student wearing eyeglasses?

_____ 33. The three sides of a triangle have integer lengths. If two sides have lengths 4 and 9, what is the least possible length of the third side?

_____ 34. In $\triangle ABC$, $\angle A = 71°$ and $\angle C = 67°$. Which of the three sides is the shortest?

_____ 35. In the figure on the right, $\angle ADB = 120°$, $\angle CDB = 140°$, and $AD = CD$. If $AB = 3x - 6$ and $CB = 24$, what are the possible values of x?
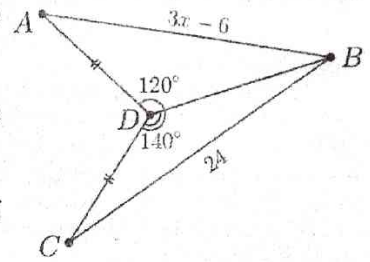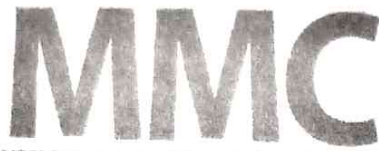
For Problems 36 to 41, use the figure below on the right. The four segments marked with arrow heads are parallel. The problems below related to it do not depend on each other.

_____ 36. If $\angle AGF = (3x)°$, $\angle GFH = 48°$ and $\angle FEC = (2x + 4y)°$, find the value of y.

_____ 37. If $AB = 5$, $BC = 6$, $CD = 4$, and $GJ = 21$, find HJ.

_____ 38. If $GH = 3$, $HI = 5$, and $HF = 2.25$, find IE.

_____ 39. If $GH = 15$, $HI = 25$, $IJ = y$, $AB = x$, $BC = 20$, and $CD = 8$, find $x + y$.
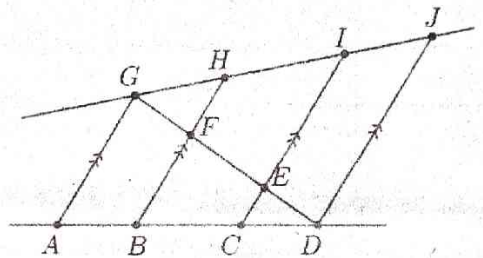
_____ 40. If $AB = 2x - 3$, $BC = 2x + 7$, $GF = 3x - 3$, and $FE = 3x + 13$, find x.

_____ 41. Suppose I is the midpoint of HJ. If $HF = 3x$, $IE = 5x$, and $JD = 5x + 12$, find x.

_____ 42. The diagonals of parallelogram $ABCD$ meet at P. If $PA = x^2$, $PC = x + 6$, and $PD = 2x - 1$, find PB.

_____ 43. The diagonals of a rhombus have lengths 10 and 14. What is the perimeter?

_____ 44. Two similar triangles have perimeters 16 and 18. Find the ratio (smaller to larger) of their areas.

_____ 45. The sides of a right triangle have lengths $x - 2$, $2x$, and $2x + 2$. Find its area.

_____ 46. In a right triangle, the altitude to the hypotenuse divides the hypotenuse into segments with lengths 5 and 20. Find the length of this altitude.

_____ 47. The sides of a triangle have lengths 7, 11, and 16. Find the perimeter of a similar triangle whose longest side is 24.

_____ 48. In $\triangle ABC$, D is chosen on AB and E is chosen on AC such that $\angle AED = \angle B$. If $AE = 9$, $EC = 3$, $DE = 7$, and $AD = 8$, find BD.

_____ 49. An equilateral triangle and a square have the same perimeter. Find the area of the triangle if the square has area 36.

_____ 50. In *isosceles* trapezoid $STAY$ with parallel sides ST and AY, $ST = 3m - 7n$, $SA = 3m - 2n$, $TY = 2m + 3n$, and $AY = 3m + 7n$. Find $\dfrac{m}{n}$.

1. $\frac{12}{5} = 2\frac{2}{5} = 2.4$

2. $\sqrt{3}$

3. $9\sqrt{2}$

4. $b^{x^2+2y^2}$

5. 25

6. $\frac{3}{5}$

7. $[-1/2, 4]$

8. 3

9. $\pm\sqrt{6}$

10. $(-8, 8)$

11. -5

12. 2

13. 4

14. 6

15. 2, 5

16. 2

17. $\frac{1}{2} = 0.5$

18. $(2, -11)$

19. -35

20. 7

21. 2

22. $y = x^2 - 6x - 1$

23. 18

24. 16

25. 24

26. decrease

27. 4

28. 24

29. $60°$

30. $5 : 6$ or $\frac{5}{6}$

31. 60

32. $\frac{2}{5} = 0.4 = 40\%$

33. 6

34. AC or b

35. $2 < x < 10$ or $(2, 10)$

36. 25

37. 14

38. 6

39. 22

40. 9

41. 6

42. 5

43. $4\sqrt{74}$

44. $64 : 81$ or $\frac{64}{81}$

45. 120

46. 10

47. 51

48. $11/2 = 5.5$

49. $16\sqrt{3}$

50. 5