Easy

E1 In a parallelogram, the midpoints of three sides are connected to form a triangle having area 7 cm^2. Find the area of the parallelogram. [28 cm^2]

E2 Factor $2x^3 + 7x^2 + 2x + 7$ completely into polynomials with integer coefficients. $[(2x + 7)(x^2 + 1)]$

E3 Two numbers have sum 3 and product 7. What is the average of their reciprocals? $\left[\frac{3}{14}\right]$

E4 A geometric sequence has first term $\frac{1}{10}$ and common ratio -2. Find the 1st term bigger than 1. [1.6]

E5 An equilateral triangle has perimeter 30 cm. Find its area. $[25\sqrt{3}\,\text{cm}^2]$

E6 An arithmetic sequence has 1st term 3 and 5th term 27. Find the 4th term. [21]

E7 Find the sum of the arithmetic mean $2 + 4 + 6 + \cdots + 38$. [380]

E8 Suppose the (positive) integer n is a perfect square. What is the next perfect square? $[n + 2\sqrt{n} + 1]$

E9 What is the largest possible value of the function $f(x) = -\sqrt{x^2 - 2x + 5}$? [2]

E10 Two similar polygons have areas 90 cm^2 and 160 cm^2. If the smaller polygon has perimeter 36 cm, find the perimeter of the larger polygon. [48 cm]

EX A cube has diagonal $4\sqrt{3}$ cm. Find its volume. [64 cm^3]

Average

A1 Find the length of the tangent line segment from $(9, 13)$ to the circle $x^2 + y^2 = 25$. [15]

A2 For what real values of k will $x^2 + k^2x + 3k^2 - k^3 = 0$ have all its roots imaginary? $[(-6, 0) \cup (0, 2)]$

A3 In the list $2, 6, 9, x, 15, 18, 20$, no three terms form an arithmetic sequence. Find all possible values of x if it is an integer strictly between 9 and 15. [11 and 14]

A4 In $\triangle ABC$, $BC = 9$. Point D is chosen on BC so that $CD = 5$. If $\angle CAB = \angle ADB$, find AB, [6]

A5 The line ℓ is parallel to and above the graph $y = \frac{3}{4}x + \frac{3}{4}$. If the shortest distance between any two points from these lines is 5 units, determine the equation of ℓ. $\left[y = \frac{3}{4}x + 7\right]$

AX A two-digit number is 22 more than the product of its digits. If its units digit is prime, find this number. [57]

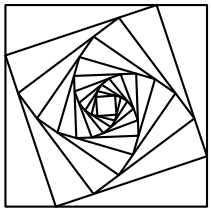
Difficult

D1 Three distinct numbers, in arithmetic sequence, have a sum of 12. If their squares form a geometric sequence, find the largest of the three originally given numbers. $[4 + 4\sqrt{2}]$

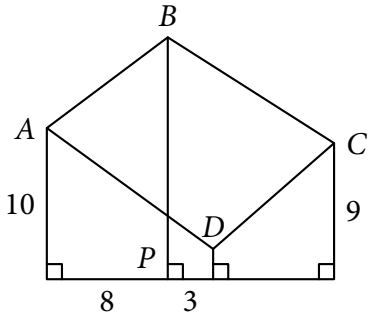
D2 A right triangle has area 7.5 and hypotenuse $5\sqrt{2}$. How long are its two legs? $[\sqrt{5}, 3\sqrt{5}]$

D3 On horizontal segment AB which has length 20, point C is chosen so that $AC = 15$. Points D and E are chosen above A and B, respectively, so that $AD = 20$ and $BE = 15$. Find the area of the intersection of $\triangle CAD$ and $\triangle ABE$. [54]

D4 An infinite sequence of inscribed squares is constructed, as follows: each side of a square is divided by a point in the ratio 1 : 3, and the four points connected to form the inscribed square. If the largest square in the figure has area 64 cm^2, find the total area of the squares formed. $\left[\frac{512}{3}\text{ cm}^2\right]$



D5 In the figure, $BP = 16$, and $ABCD$ is a parallelogram. Find the area of $ABCD$. [122]



DX Solve the following system for x and y: $x + y + \frac{x}{y} = 13$ and $\frac{x(x+y)}{y} = 42$. $\left[(6,1), \left(\frac{21}{4}, \frac{3}{4}\right)\right]$

Tiebreaker

TB1 Three different faces of a rectangular box has areas 4 cm^2, 25 cm^2, and 36 cm^2. Find its volume. [60 cm^2]

TB2 The sum of four numbers is 48. If the first is increased by 3, the second is decreased by 3, the third is multiplied by 3, and the fourth is divided by 3, the results are all equal. What are the four original numbers, in order? [6, 12, 3, 27]

TB3 Triangle ABC has a right angle at C, with $AC = 3$ and $BC = 4$. The altitude from C is drawn, with its foot at point D on AB. The angle bisector of the smallest angle of $\triangle ABC$ intersects CD at E. Find DE. $\left[\frac{16}{15}\right]$

Do-or-Die

DoD An isosceles triangle has sides $4x + 4$, $2x + 5$, and $2x^2 + x + 2$. Find all possible values of its perimeter. [15, 26, 33]