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Numerical Solution of SIRS model for Dengue Fever Transmission in Makassar Citywith Runge Kutta Method

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Abstract. The aim of this study was to obtain a SIRS model solution for the spread of dengue fever (DF) using the 4th order Runge-Kutta method. The data used was data on the number of dengue cases in Makassar city. The method used is the 4th order Runge Kutta Method. The results of this study are numerical solutions of the SIRS model on the spread of DF in Makassar city using the Runge Kutta method; Model simulation analysis shown that the estimated number of dengue cases in Makassar city, so that the government can take steps to prevent the spread of dengue in Makassar city.

Keywords: Numerical solutions, SIRS model, runge kutta Method

1. Introduction
Dengue fever (DF) is a type of disease in the tropics area, especially indonesia, including in south sulawesi [1]. Based on the data from Public Health Office, the number cases of DF in 2016 was142 cases, futhermore in 2017 it increased to 248 cases [2] and until February 2018 62 cases had been recorded DFwhich was included in 10 category the biggest disease in makassar regional public hospital [2].

Mathematical modeling can be used to detect and predict the number of dengue fever (DF) cases were done using the model of Suspected, Infected, Recovered and Suspected (SIRS) [3;4;5;6;7;8;9;10;11;12;13]. In this research, the SIRS model that has been constructed is determinated by which numerical solution with Runge-Kutta Method. Determination of parameters, simulation and analysis of result using secondary data on the number of dengue fever cases in makassar city. The result of numerical solution on the SIRS model with Rungge-Kutta method to show result of accurate in control and handling transmission dengue fever in makassar city.

2. Method
This research is applied research. The model was built from the modification of the SIR model [8,9]. The SIRS model for transmission of dengue fever was then determined solution by using the 4th order Runge Kutta Method [14]. The data values of the parameter and initial values used are secondary data the number of cases dengue fever in Makassar. The model simulation was also conducted using MAPLE 17 to predict the number of dengue fever cases in Makassar city.

3. Results and Discussion

3.1. SIRS Model for dengue fever

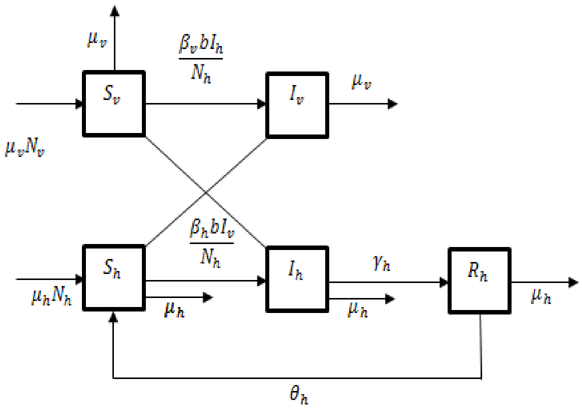

Figure 1. Human and mosquito population diagram for SIRS DF TransmissionModel [8,9]

Based on human and mosquito popualation diagram on Figure 1, the rate of change humans which Suspected, Infected, Recovered and the rate of vector which Suspected and Infected dengue fever (DHF) towards time can be interpreted as follows:

3.1.1. Human Population.

$$\frac{dS_h}{dt} = \mu_h N_h - \frac{\beta_h b}{N_h} I_v S_h - \mu_h S_h + \theta_h R_h \quad (1)$$

$$\frac{dI_h}{dt} = \frac{\beta_h b}{N_h} I_v S_h - (\mu_h + \gamma_h) I_h \quad (2)$$

$$\frac{dR_h}{dt} = \gamma_h I_h - (\mu_h + \theta_h) R_h \quad (3)$$

3.1.2. Vector Population

$$\frac{dS_v}{dt} = \mu_v N_v - \frac{\beta_v b}{N_h} I_h S_v - \mu_v S_v \quad (4)$$

$$\frac{dI_v}{dt} = \frac{\beta_v b}{N_h} I_v S_v - \mu_v I_v \quad (5)$$

3.2. Runge-Kutta Method

The thought of Runge-Kutta method is to maintain Taylor approximation. However in solving ordinary differential equation with the Taylor method not practical because the method requires derivative estimation f(x,y). Furthermore, not all a function are easy to calculate its derivatives, especially for difficult functions. The higher order of the Taylor method, its derivative must be calculated is also high [14].

Runge-Kutta Method is another alternative to the Taylor series method that doesn't require derivative. This method is trying to obtain a high degree of accuracy, and also to avoid looking for higher derivatives by evaluating functions f(x,y) with a fixed point on each interval. In general, the Runge-Kutta method has three characteristics [14]:

- This method is a one-step method, that its to obtain y_{n+1} only requires information available on the previous point (x_0, y_0).
- This method is corresponding with the taylor series until h^p term, with p is different for a different method.
- This method doesn't require evaluation of each derivative f(x,y), but the function itself.

3.2.1. The 4th Order Runge-Kutta Method.

The 4th order Runge-Kutta is the most accurate method compared to the previous order Runge-Kutta Method. Therefore, The 4th Runge-Kutta is often used to solve a differential equation. This method is obtained from the 2nd Runger-Kutta method second order with n=4, which has a form of an equation [13].

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{6}$$

with

$$\begin{aligned} k_1 &= hf(x_r, y_r) \\ k_2 &= hf(x_r + \tfrac{1}{2}h, y_r + \tfrac{1}{2}k_1) \\ k_3 &= hf(x_r + \tfrac{1}{2}h, y_r + \tfrac{1}{2}k_2) \\ k_4 &= hf(x_r + h, y_r + k_3) \end{aligned} \tag{7}$$

The 4th order Runge-Kutta method is having a high accuracy level of the solution reatherthan the previous order Runge-Kutta method, this method is also easy, stable and its have a small cutting and rounding error.

An equations (6) and (7) are resolved using the 4th order Runger-Kutta method have obtained degression from the general form the 4th orderRunge-Kutta method with $n = 4$. The general form of the 4th order Runge-Kutta as in equation (8).

$$y_{r+1} = y_r + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4)h \tag{8}$$

with

$$k_1 = f(x_r, y_r) \tag{9}$$

$$k_2 = f(x_r + p_1h, y_r + q_{11}k_1h) \tag{10}$$

$$k_3 = f(x_r + p_2h, y_r + q_{21}k_1h + q_{22}k_2h) \tag{11}$$

$$k_4 = f(x_r + p_3h, y_r + q_{31}k_1h + q_{32}k_2h + q_{33}k_3h) \tag{12}$$

To obtain solution with using Runge-Kutta method fourth order, required Taylor series fourth order, such that by following general form of Taylor series equation is obtained equation (13).

$$y_{r+1} = y_r + f(x_r, y_r)h + f'(x_r, y_r)\frac{h^2}{2!} + f''(x_r, y_r)\frac{h^3}{3!} + f'''(x_r, y_r)\frac{h^4}{4!} + 0(h^5) \tag{13}$$

With

$$f'(x_r, y_r) = \frac{\partial f}{\partial x} + f(x_r, y_r)\frac{\partial f}{\partial y} \tag{14}$$

$$f''(x_r,y_r)=\left[\frac{\partial^2 f}{\partial x^2}+2f(x_r,y_r)\frac{\partial^2 f}{\partial x\partial y}+\left(f(x_r,y_r)\right)^2\frac{\partial^2 f}{\partial y^2}\right]+\left[\left(\frac{\partial f}{\partial x}+f(x_r,y_r)\frac{\partial f}{\partial y}\right)\frac{\partial f}{\partial y}\right]$$

$$\begin{aligned}f'''(x_r,y_r)=&\left[\frac{\partial^3 f}{\partial x^3}+3f(x_r,y_r)\frac{\partial^3 f}{\partial x^2\partial y}+3\left(f(x_r,y_r)\right)^2\frac{\partial^3 f}{\partial x\partial y^2}+\frac{\partial^3 f}{\partial y^3}\left(\frac{dy}{dx}\right)^3\right]+\\&\left[3\left(\frac{\partial f}{\partial x}+f(x_r,y_r)\frac{\partial f}{\partial y}\right)\left(\frac{\partial^2 f}{\partial x\partial y}+f(x_r,y_r)\frac{\partial^2 f}{\partial y^2}\right)\right]+\\&\left[\frac{\partial^2 f}{\partial x^2}+2f(x_r,y_r)\frac{\partial^2 f}{\partial x\partial y}+\left(f(x_r,y_r)\right)^2\frac{\partial^2 f}{\partial y^2}\right]+\left[\left(\frac{\partial f}{\partial x}+f(x_r,y_r)\frac{\partial f}{\partial y}\right)\left(\frac{\partial f}{\partial y}\right)^2\right]\end{aligned} \quad (15)$$

Then we obtained Taylor series fourth order in equation (16).

$$\begin{aligned}y_{r+1}=&\,y_r+f(x_r,y_r)h+\left[\frac{\partial f}{\partial x}+f(x_r,y_r)\frac{\partial f}{\partial y}\right]\frac{h^2}{2}+\\&\left[\frac{\partial^2 f}{\partial x^2}+2f(x_r,y_r)\frac{\partial^2 f}{\partial x\partial y}+\left(f(x_r,y_r)\right)^2\frac{\partial^2 f}{\partial y^2}\right]\frac{h^3}{6}+\left[\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+f(x_r,y_r)\left(\frac{\partial f}{\partial y}\right)^2\right]\frac{h^3}{6}+\\&\left[\frac{\partial^3 f}{\partial x^3}+3f(x_r,y_r)\frac{\partial^3 f}{\partial x^2\partial y}+3\left(f(x_r,y_r)\right)^2\frac{\partial^3 f}{\partial x\partial y^2}+\frac{\partial^3 f}{\partial y^3}\left(\frac{dy}{dx}\right)^3\right]\frac{h^4}{24}+\\&\left[3\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+f(x_r,y_r)\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+f(x_r,y_r)\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\left(f(x_r,y_r)\right)^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right)\right]\frac{h^4}{24}+\\&\left[\frac{\partial^2 f}{\partial x^2}+2f(x_r,y_r)\frac{\partial^2 f}{\partial x\partial y}+\left(f(x_r,y_r)\right)^2\frac{\partial^2 f}{\partial y^2}\right]\frac{h^4}{24}+\\&\left[\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+f(x_r,y_r)\left(\frac{\partial f}{\partial y}\right)^3\right]\frac{h^4}{24}\end{aligned} \quad (16)$$

After obtaining Taylor series fourth order in equation (16), its equation identified with equations (10)-(12) so we have a values $a_1,a_2,a_3,a_4,p_1,p_2,p_3,q_{11},q_{21},q_{31},q_{22},q_{32}$, and q_{33}, where:

$$k_1=f(x_r,y_r) \quad (17)$$

$$k_2=f(x_r+p_1h,y_r+q_{11}k_1h)=f(x_r,y_r)+p_1h\frac{\partial f}{\partial x}+q_{11}hf(x_r,y_r)\frac{\partial f}{\partial y} \quad (18)$$

$$k_3=f(x_r+p_2h,y_r+q_{21}k_1h+q_{22}k_2h)$$

$$k_3=f(x_r,y_r)+p_2h\frac{\partial f}{\partial x}+q_{21}hf(x_r,y_r)\frac{\partial f}{\partial y}+q_{22}hf(x_r,y_r)\frac{\partial f}{\partial y}+p_1q_{22}h^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+ \quad (19)$$

$$q_{11}q_{22}h^2f(x_r,y_r)\left(\frac{\partial f}{\partial y}\right)^2$$

$$k_4 = f(x_r + p_3 h, y_r + q_{31}k_1 h + q_{32}k_2 h + q_{33}k_3 h)$$

$$= f(x_r, y_r) + p_3 h\frac{\partial f}{\partial x} + q_{31}hf(x_r, y_r)\frac{\partial f}{\partial y} + q_{32}hf(x_r, y_r)\frac{\partial f}{\partial y} + p_1 q_{32}h^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} +$$
$$q_{11}q_{32}f(x_r, y_r)h^2\left(\frac{\partial f}{\partial y}\right)^2 + q_{33}f(x_r, y_r)h\frac{\partial f}{\partial y} + p_2 q_{33}h^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} +$$
$$q_{21}q_{33}f(x_r, y_r)h^2\left(\frac{\partial f}{\partial y}\right)^2 + q_{22}q_{33}f(x_r, y_r)h^2\left(\frac{\partial f}{\partial y}\right)^2 + p_1 q_{22}q_{33}h^3\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2 + \qquad (20)$$
$$q_{11}q_{22}q_{33}f(x_r, y_r)h^3\left(\frac{\partial f}{\partial y}\right)^3$$

Subtituting the values $k_1, k_2, k_3,$ and k_4 to equation (8).

$$y_{r+1} = y_r + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4)h = y_r + a_1k_1h + a_2k_2h + a_3k_3h + a_4k_4h$$

$$y_{r+1} = y_r + \left[a_1 f(x_r, y_r) + a_2 f(x_r, y_r) + a_3 f(x_r, y_r) + a_4 f(x_r, y_r)\right]h +$$
$$\begin{bmatrix} a_2 p_1\frac{\partial f}{\partial x} + a_3 p_2\frac{\partial f}{\partial x} + a_4 p_3\frac{\partial f}{\partial x} + a_2 q_{11}f(x_r, y_r)\frac{\partial f}{\partial y} + \\ a_3 q_{21}f(x_r, y_r)\frac{\partial f}{\partial y} + a_3 q_{22}f(x_r, y_r)\frac{\partial f}{\partial y} + a_4 q_{31}f(x_r, y_r)\frac{\partial f}{\partial y} + \\ a_4 q_{32}f(x_r, y_r)\frac{\partial f}{\partial y} + a_4 q_{33}f(x_r, y_r)\frac{\partial f}{\partial y} \end{bmatrix} h^2 +$$
$$\begin{bmatrix} a_3 p_1 q_{22}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + a_4 p_1 q_{32}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + a_4 p_2 q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + \\ a_3 q_{11}q_{22}f(x_r, y_r)\left(\frac{\partial f}{\partial y}\right)^2 + a_4 q_{11}q_{32}f(x_r, y_r)\left(\frac{\partial f}{\partial y}\right)^2 + \\ a_4 q_{21}q_{33}f(x_r, y_r)\left(\frac{\partial f}{\partial y}\right)^2 + a_4 q_{22}q_{33}f(x_r, y_r)\left(\frac{\partial f}{\partial y}\right)^2 \end{bmatrix} h^3 +$$
$$\left[a_4 p_1 q_{22}q_{33}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2 + a_4 q_{11}q_{22}q_{33}f(x_r, y_r)\left(\frac{\partial f}{\partial y}\right)^3\right]h^4 \qquad (21)$$

According to the comparison with two equations, we obtaining equation (22)

$$
\begin{aligned}
&p_1 = q_{11},\\
&p_2 = q_{21} + q_{22},\\
&p_3 = q_{31} + q_{32} + q_{33},\\
&a_1 + a_2 + a_3 + a_4 = 1,\\
&a_2 p_1 + a_3 p_2 + a_4 p_3 = \frac{1}{2},\\
&a_3 p_1 q_{22} + a_4 \left(p_1 q_{32} + p_2 q_{33}\right) = \frac{1}{6},\\
&a_1 p_1 q_{22} q_{33} = \frac{1}{24}.
\end{aligned} \tag{22}
$$

So that from equation (23) we suppose that:

$$q_{11} = \frac{1}{2},\ q_{21} = 0,\ q_{22} = \frac{1}{2},\ q_{31} = 0,\ q_{32} = 0, \text{ dan } q_{33} = 1,$$

Then the solution with remaining parameters are:

$$p_1 = \frac{1}{2},\ p_2 = \frac{1}{2},\ p_3 = 1, \tag{23}$$

$$a_1 = \frac{1}{6},\ a_2 = \frac{1}{3},\ a_3 = \frac{1}{3},\ a_4 = \frac{1}{6}. \tag{24}$$

The values of parameters are subtituted to equation (13), so we have standard form Runge-Kutta method:

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{25}$$

with

$$k_1 = hf(x_r, y_r) \tag{26}$$

$$k_2 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1) \tag{27}$$

$$k_3 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2) \tag{28}$$

$$k_4 = hf(x_r + h, y_r + k_3) \tag{29}$$

Futhermore, by substituting mathematical model for dengue fever transmission in equation (13) then, we have an equations:

$$x_{t+1} = x_t + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{30}$$

$$y_{t+1} = y_t + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)h \tag{31}$$

$$z_{t+1} = z_t + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h \tag{32}$$

$$w_{t+1} = w_t + \frac{1}{6}(n_1 + 2n_2 + 2n_3 + n_4)h \tag{33}$$

with

$$k_1 = \mu_h(1 - x_t) - \alpha x_t z_t + \theta_h w_t \tag{34}$$

$$l_1 = \alpha x_t z_t - \beta y_t \tag{35}$$

$$m_1 = \gamma(1 - z_t)y_t - \delta z_t \tag{36}$$

$$n_1 = \gamma_h y_t - \varphi w_t \tag{37}$$

$$k_2 = \mu_h\left(1 - (x_t + k_1\frac{h}{2})\right) - \alpha(x_t + k_1\frac{h}{2})(z_t + m_1\frac{h}{2}) + \theta_h(w_t + n_1\frac{h}{2}) \quad (38)$$

$$l_2 = \alpha(x_t + k_1\frac{h}{2})(z_t + m_1\frac{h}{2}) - \beta(y_t + l_1\frac{h}{2}) \quad (39)$$

$$m_2 = \gamma\left(1 - (z_t + m_1\frac{h}{2})\right)(y_t + l_1\frac{h}{2}) - \delta(z_t + m_1\frac{h}{2}) \quad (40)$$

$$n_2 = \gamma_h(y_t + l_1\frac{h}{2}) - \varphi(w_t + n_1\frac{h}{2}) \quad (41)$$

$$k_3 = \mu_h\left(1 - (x_t + k_2\frac{h}{2})\right) - \alpha(x_t + k_2\frac{h}{2})(z_t + m_2\frac{h}{2}) + \theta_h(w_t + n_2\frac{h}{2}) \quad (42)$$

$$l_3 = \alpha(x_t + k_2\frac{h}{2})(z_t + m_2\frac{h}{2}) - \beta(y_t + l_2\frac{h}{2}) \quad (43)$$

$$m_3 = \gamma\left(1 - (z_t + m_2\frac{h}{2})\right)(y_t + l_2\frac{h}{2}) - \delta(z_t + m_2\frac{h}{2}) \quad (44)$$

$$n_3 = \gamma_h(y_t + l_2\frac{h}{2}) - \varphi(w_t + n_2\frac{h}{2}) \quad (45)$$

$$k_4 = \mu_h(1 - (x_t + k_3h)) - \alpha(x_t + k_3h)(z_t + m_3h) + \theta_h(w_t + n_3h) \quad (46)$$

$$l_4 = \alpha(x_t + k_3h)(z_t + m_3h) - \beta(y_t + l_3h) \quad (47)$$

$$m_4 = \gamma(1 - (z_t + m_3h))(y_t + l_3h) - \delta(z_t + m_3h) \quad (48)$$

$$n_4 = \gamma_h(y_t + l_3h) - \varphi(w_t + n_3h) \quad (49)$$

The initial value and parameters used in SIRS model for dengue fever transmission shown in table 1:

Table 1.Initial condition used in SIRS model for transmission dengue fever

Initial Value SIRS Model		
Variabeland Parameter	**Value**	**Source**
$x(0)$	$\frac{7675406}{7675893}$	[15]
$y(0)$	$\frac{487}{7675893}$	[15]
w(0)	0.000001	[10]
$z(0)$	0.056	[10]
N	7675893	[15]
μ_h	0,000046	[10]
α	0,232198	[10]
β	0,328879	[10]
γ	0,375	[10]
γ_h	0.328833	[10]
θ_h	0.001	[10]
φ	0.001046	[10]
δ	0,0323	[10]

By using a time interval$0 \leq t \leq 100$ month, with h = 0,01, where initial condition and initial value in the form of parameter value in table 1, so that we obtained:

$$x_{0+1} = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \quad (50)$$

$$y_{0+1} = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)h \tag{51}$$

$$z_{0+1} = z_0 + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h \tag{52}$$

$$w_{0+1} = w_0 + \frac{1}{6}(n_1 + 2n_2 + 2n_3 + n_4)h \tag{53}$$

with

$$k_1 = 0{,}000046\left(1 - \frac{7675406}{7675893}\right) - 0{,}232198\left(\frac{7675406}{7675893}\right)0{,}056 + (0{,}001)(0.000001) \tag{54}$$

$$l_1 = 0{,}232198(\frac{7675406}{7675893})0{,}056 - 0{,}328879(\frac{487}{7675893}) \tag{55}$$

$$m_1 = 0{,}375(1 - 0{,}056)(\frac{487}{7675893}) - 0{,}0323(0{,}056) \tag{56}$$

$$n_1 = (0.328833)(\frac{487}{7675893}) - (0.001046)(0.000001) \tag{57}$$

$$k_2 = (0{,}000046)\left(1 - \left(\left(\frac{7675406}{7675893}\right) + k_1\frac{0{,}01}{2}\right)\right) - (0{,}232198)((\frac{7675406}{7675893}) + k_1\frac{0{,}01}{2})(0{,}056 + m_1\frac{0{,}01}{2}) + (0.001)(0.000001 + n_1\frac{0{,}01}{2}) \tag{58}$$

$$l_2 = (0{,}232198)((\frac{7675406}{7675893}) + k_1\frac{0{,}01}{2})(0{,}056 + m_1\frac{0{,}01}{2}) - (0{,}328879)((\frac{487}{7675893}) + l_1\frac{0{,}01}{2}) \tag{59}$$

$$m_2 = (0{,}375)\left(1 - \left(0{,}056 + m_1\frac{0{,}01}{2}\right)\right)((\frac{487}{7675893}) + l_1\frac{0{,}01}{2}) - (0{,}0323)(0{,}056 + m_1\frac{0{,}01}{2}) \tag{60}$$

$$n_2 = (0.328833)((\frac{487}{7675893}) + l_1\frac{0{,}01}{2}) - (0.001046)(0.000001 + n_1\frac{0{,}01}{2}) \tag{61}$$

$$k_3 = (0{,}000046)\left(1 - \left(\left(\frac{7675406}{7675893}\right) + k_2\frac{0{,}01}{2}\right)\right) - (0{,}232198)((\frac{7675406}{7675893}) + k_2\frac{0{,}01}{2})(0{,}056 + m_2\frac{0{,}01}{2}) + (0.001)(0.000001 + n_2\frac{0{,}01}{2}) \tag{62}$$

$$l_3 = (0{,}232198)((\frac{7675406}{7675893}) + k_2\frac{0{,}01}{2})(0{,}056 + m_2\frac{0{,}01}{2}) - (0{,}328879)((\frac{487}{7675893}) + l_2\frac{0{,}01}{2}) \tag{63}$$

$$m_3 = (0{,}375)\left(1 - \left(0{,}056 + m_2\frac{0{,}01}{2}\right)\right)((\frac{487}{7675893}) + l_2\frac{0{,}01}{2}) - (0{,}0323)(0{,}056 + m_2\frac{0{,}01}{2}) \tag{64}$$

$$n_3 = (0.328833)((\frac{487}{7675893}) + l_2\frac{0{,}01}{2}) - (0.001046)(0.000001 + n_2\frac{0{,}01}{2}) \tag{65}$$

$$k_4 = (0{,}000046)\left(1-\left(\left(\frac{7675406}{7675893}\right)+k_3(0{,}01)\right)\right)-(0{,}232198)((\frac{7675406}{7675893}) + k_3(0{,}01))(0{,}056+m_3(0{,}01))+(0.001)(0.000001+n_3(0{,}01)) \quad (66)$$

$$l_4 = (0{,}232198)((\frac{7675406}{7675893})+k_3(0{,}01))(0{,}056+m_3(0{,}01)) - (0{,}328879)((\frac{487}{7675893})+l_3(0{,}01)) \quad (67)$$

$$m_4 = (0{,}375)\left(1-\left(0{,}056+m_3(0{,}01)\right)\right)((\frac{487}{7675893})+l_3(0{,}01)) - (0{,}0323)(0{,}056+m_3(0{,}01)) \quad (68)$$

$$n_4 = (0.328833)((\frac{487}{7675893})+l_3(0{,}01))-(0.001046)(0.000001+n_3(0{,}01)) \quad (69)$$

3.3. SIRS model simulation for dengue fever transmission

Based on exiting data and parameters,simulation model was completed with using MAPLE 17. The initial condition with used to model of this simulation are based on reported by Ministry of Health Republic Indonesia Makassar city 2017, as shown in table 1 and the result of simulation model are given by Figure 2 to Figure 5 is below

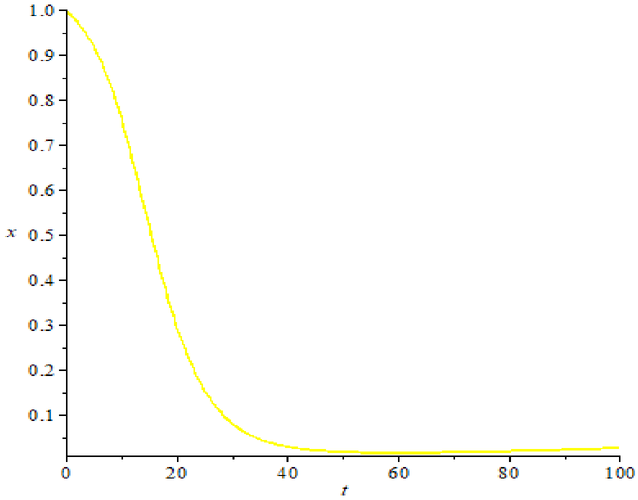

Figure 2. Prediction of the number of suspected dengue fever transmission

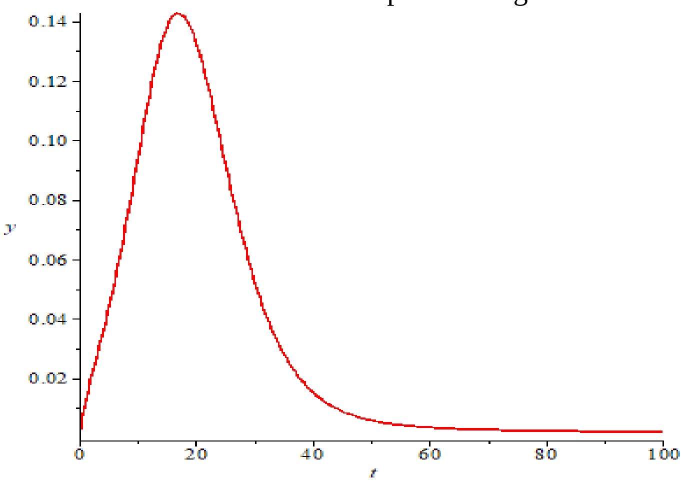

Figure 3. Prediction of the number of infected dengue fever transmission

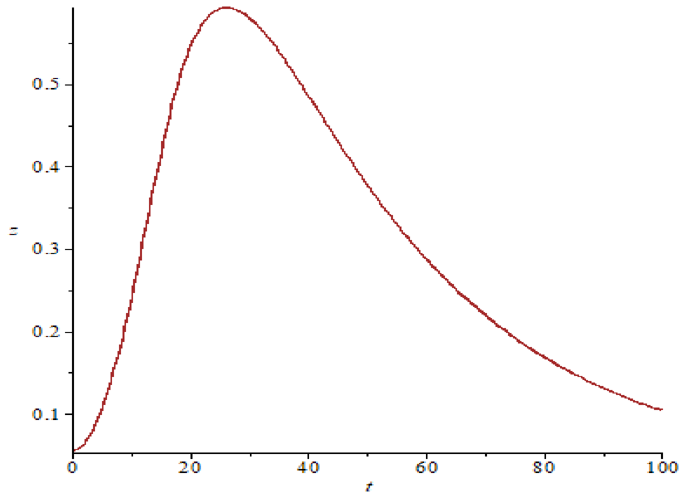

Figure 4. Prediction of the number of infected vectors dengue fever transmission

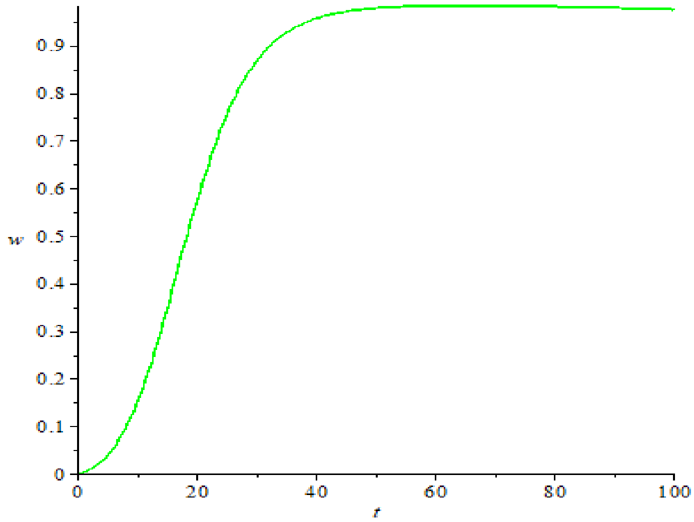

Figure 5. Prediction of the number of recovered vectors dengue fever transmission

Figure 2 shown that the number of initial value suspect is 0,9999365 and capable to make a description about the next suspect number, and which suspects reaches the minimum on the month 60-th (5 years), this cause of the number of host population that switch status become infected and also at the same time the number of host population recovers, but must be remembered that those population still have the chance to get infected again.

The result of the SIRS model in Figure 3 shown that the number of dengue fever (DF) cases, in other words, the number of infected people is 14 to 18 months to reach the highest stage (12% from the human population) and would decrease to zero in the 45-th months. As for, the number of mosquitoes infected suitable in Figure 4, only takes time less than 30 months, more precisely the 28-th month to reach the highest number, i.e approach 60% from the mosquito population for Makassar city takes time 160 months (13 years) to approach zero. Figure 5 shown of a most significant increase in the population recovering from dengue fever, the positive trend would attain top in the 4 years (50-th month) and afterward, but this graph would slowly decline because in the SIRS, model the recovered population can be contracting again. Which could also be seen in the suspected graph, where its graph would slowly increasing in the 50-th month.

4. Conclusion

By using the 4th order Runge-Kutta would be obtained an illustration the number of suspected, exposed, infected and recovered in next month and year, the number of increase and decrease of dengue fever cases in Makassar city. So, the government of makassar city can take precautionary measures so that the transmission of dengue fever could be solved. The 4th order Runge-Kutta method also shown the rate of development of the number of infected vectors.

5. Acknowledgements

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