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Numerical Solution of SIRS Model for Transmission of Dengue Fever using Homotopy Perturbation Method in Makassar

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Abstract. This study purposed to find a numerical solution for the Suspected–Infected-Recovered-Suspected (SIRS) model for transmission of dengue fever in the city of Makassar using the Homotopy Perturbation Method. The data used is secondary data on the number of dengue cases in Makassar City. The method used is the Homotopy Perturbation Method. The results obtained are numerical solutions of the SIRS model with the Perturbation Homotopy method. The results of the model analysis also show a graph of the trend in the trend of dengue spread in Makassar City with the SIRS model. These results predict the number of dengue cases in the city of Makassar, so that early prevention measures for the spread of dengue in the city of Makassar are of concern to the government.

Keywords: SIRS model, Perturbation Homotopy method, Dengue Fever Disease.

INTRODUCTION

Dengue Fever (DF) is a type of disease in the tropics area, especially indonesia, including in South Sulawesi [1]. Based on the data from Public Health Office the number, the number of cases of DF in 2016 was as many as 142, futhermore in 2017 it increased 248 cases [2] and until February 2018 it already recorded 62 cases DF in makassar city and include in 10 categories for the biggest disease in makassar regional public hospital [2].

Mathematical modeling can be used to detect and predict the number of dengue fever cases by the model of Suspected, Infected, Recovered and Suspected (SIRS) [3, 4, 5, 6, 7, 8, 9, 10]. In this research, the SIRS Model has been contruct and determineted by which numerical solution with homotopy pertubation method model SIRS. Determination of parameters, simulation and analysis of results is infered from secondary data on the number of dengue fever cases in makassar city. The result of the numerical solution on the SIRS model from the the two methods are then compared to see methods that are more accurate in control and handling transmission dengue fever in makassar city.

METHOD

This is an applied research. The model was built from the modification of the SIR model [8, 9]. The SIRS model for transmission of dengue fever was then determined solution by using homotopy pertubation method [11]. The data

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values of parameter and initial values used are secondary data the number of cases dengue faver in makassar. Model simulation was also conducted using MATLAB to predict the number of dengue faver cases in Makassar city.

RESULT AND DISCUSSION

Changes that occur in each human population in the transmission of DF for the SIRS model can be interpreted in the form of the following figure 1: Based on human and mosquito popualation diagram on figure 1, the rate of change

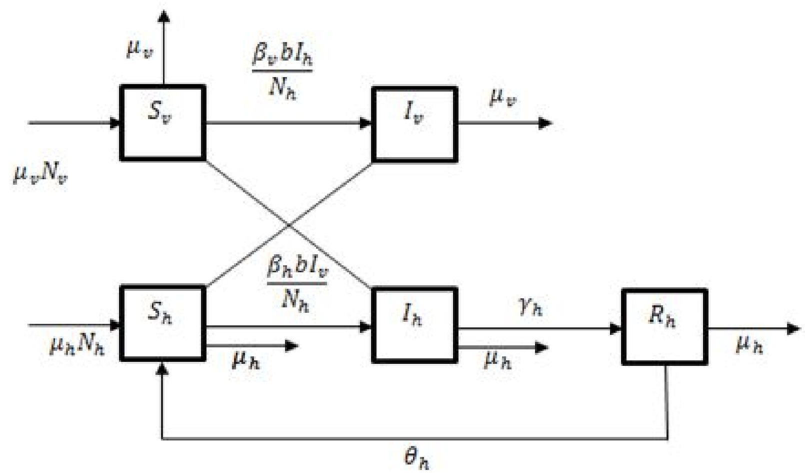

FIGURE 1. Human and mosquito population diagram for SIRS DF Transmission Model [8, 9]

humans which Suspected, Infected, Recovered and the rate of vector which Suspected and Infected dengue fever (DF) towards time can be interpreted as follows:

Human Population

$$\frac{dS_h}{dt} = \mu_h N_h - \frac{\beta_h b}{N_h} I_v S_h - \mu_h S_h + \theta_h R_h \tag{1}$$

$$\frac{dI_h}{dt} = \frac{\beta_h b}{N_h} I_v S_h - (\mu_h + \gamma_h) I_h \tag{2}$$

$$\frac{dR_h}{dt} = \gamma_h I_h - (\mu_h + \theta_h) R_h \tag{3}$$

Vector Population

$$\frac{dS_v}{dt} = \mu_v N_v - \frac{\beta_v b}{N_h} I_h S_v - \mu_v S_v \tag{4}$$

$$\frac{dI_h}{dt} = \frac{\beta_h b}{N_h} I_v S_h - \mu_v I_v \tag{5}$$

Procedures Homotopy Perturbation Method of Dengue Fever SIRS Model

Numerical solution of SIRS model of dengue fever transmission with homotopy perturbation method is shown by the following procedures:

1. The model is considered as a system of ordinary differential equations first order in general can be written as an equation:

$$\frac{du_1}{dt} + g_1(t, u_1, u_2, \cdots, u_m) = f_1(t)$$

$$\frac{du_2}{dt} + g_2(t, u_1, u_2, \cdots, u_m) = f_2(t)$$

$$\ddots$$

$$\frac{du_m}{dt} + g_m(t, u_1, u_2, \cdots, u_m) = f_m(t)$$

and satisfy initial condition $u_1(t_0) = c_1$, $u_2(t_0) = c_2$, $\cdots$, $u_m(t_0) = c_m$.

2. System of equation (1) following the operator form:

$$L(u_1) + g_1(t, u_1, u_2, \cdots, u_m) = f_1(t)$$

$$L(u_2) + (t, u_1, u_2, \cdots, u_m) = f_2(t)$$

$$\ddots$$

$$L(u_m) + (t, u_1, u_2, \cdots, u_m) = f_m(t)$$

with $L = d/dt$ is an operator linear and $N_1, N_2, \cdots, N_m$ are nonlinear operator.

3. The form of differential equation system that satisfies of the following equation:

$$Du_i(t) = L_i(t, u_1, u_2, \cdots, u_n) + N_i(t, u_1, u_2, \cdots, u_n) + g_i(t)$$

With L_i is a linear operator, N_i is a nonlinear operator and g_i is an unknow analytic function. Futhermore, in homotopy pertubation technique, a homotopy form is contructed:

$$Du_i(t) = p\left[L_i(t, u_1, u_2, \cdots, u_n) + N_i(t, u_1, u_2, \cdots, u_n) + g_i(t)\right],$$

$1 \leq i \leq n$ where p is immersions parameter is changed from zero to unit.

4. Suppose the initial approach as the following equations:

$$u_{1,0}(t) = v_1(t) = u_1(t_0) = c_1$$

$$u_{2,0}(t) = v_2(t) = u_2(t_0) = c_2$$

$$\ddots$$

$$u_{m,0}(t) = v_m(t) = u_m(t_0) = c_m$$

and

$$u_1(t) = u_{1,0}(t) + pu_{1,1}(t) + p^2u_{1,2}(t) + \cdots$$
$$u_2(t) = u_{2,0}(t) + pu_{2,1}(t) + p^2u_{2,2}(t) + \cdots$$

$$\ddots$$

$$u_m(t) = u_{m,0}(t) + pu_{m,1}(t) + p^2u_{m,2}(t) + \cdots$$

5. Coefficient arragement to the order of p, we have:

$$L(u_{1,1}) + L(v_1) + N_1\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_1 = 0, \quad u_{1,1}(t_0) = 0$$

$$L(u_{2,1}) + L(v_2) + N_2\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_2 = 0, \quad u_{2,1}(t_0) = 0$$

$$L(u_{3,1}) + L(v_3) + N_3\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_3 = 0, \quad u_{3,1}(t_0) = 0$$

$$\ddots$$

$$L(u_{m,1}) + L(v_m) + N_m\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_m = 0, \quad u_{m,1}(t_0) = 0$$

then

$$L(u_{1,2}) + N_1\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_1 = 0, \quad u_{1,2}(t_0) = 0$$

$$L(u_{2,2}) + N_2\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_2 = 0, \quad u_{2,2}(t_0) = 0$$

$$L(u_{3,2}) + N_3\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_3 = 0, \quad u_{3,2}(t_0) = 0$$

$$\ddots$$

$$L(u_{m,2}) + N_m\left(u_{1,0}, u_{2,0}, \cdots, u_{m,0}\right) - f_m = 0, \quad u_{m,2}(t_0) = 0$$

and soon.

6. We have description of solution of SIRS model with homotopy pertubation method can be expressed by the following equation:

$$\phi_{1,n}(t) = u_1(t) = \lim_{p\to 1} u_1(t) = \sum_{k=0}^{n-1} u_{1,k}(t)$$

$$\phi_{2,n}(t) = u_2(t) = \lim_{p\to 1} u_2(t) = \sum_{k=0}^{n-1} u_{2,k}(t)$$

$$\phi_{3,n}(t) = u_3(t) = \lim_{p\to 1} u_3(t) = \sum_{k=0}^{n-1} u_{3,k}(t)$$

$$\ddots$$

$$\phi_{m,n}(t) = u_m(t) = \lim_{p\to 1} u_m(t) = \sum_{k=0}^{n-1} u_{m,k}(t)$$

Homotopy Pertubation Method

Homotopy Pertubation Method is a semi analytic numeric method, in implement of iteration on this method, there are several steps as following: First step, write SIRS model dengue fever in form of:

$$\frac{dx}{dt} = \mu_h(1 - x(t)) - \alpha x(t)z(t) + \theta_h w(t) \tag{6}$$

$$\frac{dy}{dt} = \alpha x(t)z(t) + \beta y(t) \tag{7}$$

$$\frac{dz}{dt} = \gamma(1 - z(t))y(t) - \delta z(t) \tag{8}$$

$$\frac{dw}{dt} = \gamma_h y(t) - \varphi w(t) \tag{9}$$

With initial conditions:
$x_0(t) = c_1$, $y_0(t) = c_2$, $z_0(t) = c_3$, $w_0(t) = c_4$. According to homotopy pertubation method, built a homotopy for SIRS model from dengue fever as following condition:

$$v_1' - x_0' + p\left(x_0' - \mu_h(1 - v_1) + \alpha v_1 v_3 - \theta_h v_4\right) = 0 \tag{10}$$

$$v_2' - y_0' + p\left(y_0' - \alpha v_1 v_3 + \beta v_2\right) = 0 \tag{11}$$

$$v_3' - z_0' + p\left(z_0' - \gamma(1 - v_3) v_2 + \delta_1 v_3\right) = 0 \tag{12}$$

$$v_4' - w_0' + p\left(w_0' - \gamma_h v_2 + \varphi v_4\right) = 0 \tag{13}$$

We choose initial Approximation:

$$v_{1,0}(t) = x_0(t) = v_1(0) = c_1, \tag{14}$$

$$v_{2,0}(t) = y_0(t) = v_2(0) = c_2, \tag{15}$$

$$v_{3,0}(t) = z_0(t) = v_3(0) = c_3, \tag{16}$$

$$v_{4,0}(t) = w_0(t) = v_4(0) = c_4, \tag{17}$$

and

$$v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2 v_{1,2}(t) + p^3 v_{1,3}(t) + \cdots \tag{18}$$

$$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2 v_{2,2}(t) + p^3 v_{2,3}(t) + \cdots \tag{19}$$

$$v_3(t) = v_{3,0}(t) + pv_{3,1}(t) + p^2 v_{3,2}(t) + p^3 v_{3,3}(t) + \cdots \tag{20}$$

where $v_{i,j}$ $(i = 1, 2; j = 1, 2, 3, \cdots)$ are function but has been determined. Substituting equations (14)-(20) into equation (10)-(13), we obtained:

$$v_{1,1}' + \alpha v_{1,0} v_{3,0} + \mu_h v_{1,0} - \mu_h - \theta_h v_{4,0} = 0, \quad v_{1,1}(0) = 0, \tag{21}$$

$$v_{2,1}' - \alpha v_{1,0} v_{3,0} + \beta v_{2,0} = 0, \quad v_{2,1}(0) = 0, \tag{22}$$

$$v_{3,1}' - \gamma v_{2,0} + \delta_1 v_{3,0} + \gamma v_{2,0} v_{3,0} = 0, \quad v_{3,1}(0) = 0, \tag{23}$$

$$v_{4,1}' - \gamma_h v_{2,0} + \varphi v_{4,0} = 0, \quad v_{4,1}(0) = 0, \tag{24}$$

$$v_{1,2}' + \alpha v_{1,1} v_{3,0} + \alpha v_{1,0} v_{3,1} + \mu_h v_{1,1} - \mu_h - \theta_h v_{4,1} = 0, \quad v_{1,2}(0) = 0, \tag{25}$$

$$v_{2,2}' - \alpha v_{1,1} v_{3,0} - \alpha v_{1,0} v_{3,1} + \beta v_{2,1} = 0, \quad v_{2,2}(0) = 0, \tag{26}$$

$$v_{3,2}' - \gamma v_{2,1} + \delta_1 v_{3,1} + \gamma v_{2,1} v_{3,0} + \gamma v_{2,0} v_{3,1} = 0, \quad v_{3,2}(0) = 0, \tag{27}$$

$$v_{4,2}' - \gamma_h v_{2,1} + \varphi v_{4,1} = 0, \quad v_{4,2}(0) = 0, \tag{28}$$

$$v_{1,3}' + \alpha v_{1,2} v_{3,0} + \alpha v_{1,1} v_{3,1} + \alpha v_{1,0} v_{3,2} + \mu_h v_{1,2} - \mu_h - \theta_h v_{4,2} = 0, \quad v_{1,3}(0) = 0, \tag{29}$$

$$v_{2,3}' - \alpha v_{1,2} v_{3,0} - \alpha v_{1,1} v_{3,1} - \alpha v_{1,0} v_{3,2} + \beta v_{2,2} = 0, \quad v_{2,3}(0) = 0, \tag{30}$$

$$v_{3,3}^{'} - \gamma v_{2,2} + \delta_1 v_{3,2} + \gamma v_{2,2} v_{3,0} + \gamma v_{2,1} v_{3,1} + \gamma v_{2,0} v_{3,2} = 0, \quad v_{3,3}(0) = 0, \tag{31}$$

$$v_{4,3}^{'} - \gamma_h v_{2,2} + \varphi v_{4,2} = 0, \quad v_{4,3}(0) = 0, \tag{32}$$

$$v_{1,4}^{'} + \alpha v_{1,3} v_{3,0} + \alpha v_{1,2} v_{3,1} + \alpha v_{1,1} v_{3,2} + \alpha v_{1,0} v_{3,3} + \mu_h v_{1,3} - \mu_h - \theta_h v_{4,3} = 0, \quad v_{1,4}(0) = 0, \tag{33}$$

$$v_{2,4}^{'} - \alpha v_{1,3} v_{3,0} - \alpha v_{1,2} v_{3,1} - \alpha v_{1,1} v_{3,2} - \alpha v_{1,0} v_{3,3} + \beta v_{2,3} = 0, \quad v_{2,4}(0) = 0, \tag{34}$$

$$v_{3,4}^{'} - \gamma v_{2,3} + \delta_1 v_{3,3} + \gamma v_{2,3} v_{3,0} + \gamma v_{2,2} v_{3,1} + \gamma v_{2,1} v_{3,2} + \gamma v_{2,0} v_{3,3} = 0, \quad v_{3,4}(0) = 0, \tag{35}$$

$$v_{4,4}^{'} - \gamma_h v_{2,3} + \varphi v_{4,3} = 0, \quad v_{4,4}(0) = 0 \tag{36}$$

So obtained solution of differential equation as in equation (37)-(45), then,

$$v_{1,1} = \int_0^t [-\alpha v_{1,0} v_{3,0} - \mu_h v_{1,0} + \mu_h + \theta_h v_{4,0}]\, ds, \tag{37}$$

$$v_{2,1} = \int_0^t [\alpha v_{1,0} v_{3,0} - \beta v_{2,0}]\, ds \tag{38}$$

$$v_{3,1} = \int_0^t [\gamma v_{1,0} v_{2,0} - \delta_1 v_{3,0} - \gamma v_{2,0} v_{3,0}]\, ds \tag{39}$$

$$v_{4,1} = \int_0^t [\gamma_h v_{2,0} - \varphi v_{4,0}]\, ds \tag{40}$$

$$v_{1,2} = \int_0^t [-\mu_h v_{1,1} - \alpha v_{1,1} v_{3,0} - \alpha v_{1,0} v_{3,1} + \theta_h v_{4,1}]\, ds, \tag{41}$$

$$v_{2,2} = \int_0^t [-\beta v_{2,1} + \alpha v_{1,1} v_{3,0} + \alpha v_{1,0} v_{3,1}]\, ds \tag{42}$$

$$v_{3,2} = \int_0^t [\gamma v_{2,1} - \gamma v_{2,1} v_{3,0} - \delta_1 v_{3,1} - \gamma v_{2,0} v_{3,1}]\, ds \tag{43}$$

$$v_{4,2} = \int_0^t [\gamma_h v_{2,1} - \varphi v_{4,1}]\, ds \tag{44}$$

$$v_{1,3} = \int_0^t [\alpha v_{1,0} v_{3,2} - \alpha v_{1,1} v_{3,1} - \alpha v_{1,2} v_{3,0} - \mu_h v_{1,2} + \theta_h v_{4,2}]\, ds, \tag{45}$$

$$v_{2,3} = \int_0^t [-\beta v_{2,2} + \alpha v_{1,2} v_{3,0} + +\alpha v_{1,1} + \alpha v_{1,0} v_{3,2}]\, ds \tag{46}$$

$$v_{3,3} = \int_0^t [\gamma v_{2,2} - \gamma v_{2,2} v_{3,0} - \gamma v_{2,1} v_{3,1} - \delta_1 v_{3,2} - \gamma v_{2,0} v_{3,2}]\, ds \tag{47}$$

$$v_{4,3} = \int_0^t [\gamma_h v_{2,2} - \varphi v_{4,2}]\, ds \tag{48}$$

$$v_{1,4} = \int_0^t [-\alpha v_{1,3} v_{3,0} - \alpha v_{1,2} v_{3,1} - \alpha v_{1,1} v_{3,2} - \alpha v_{1,0} v_{3,3} - \mu_h v_{1,3} + \theta_h v_{4,3}]\, ds, \tag{49}$$

$$v_{2,4} = \int_0^t [\alpha v_{1,3} v_{3,0} + \alpha v_{1,2} v_{3,1} + \alpha v_{1,1} v_{3,2} + \alpha v_{1,0} v_{3,3} - \beta v_{2,3}]\, ds \tag{50}$$

TABLE 1. Initial condition used in SIRS model for transmission dengue fever

Initial Value SIRS Model		
Variabel and Parameter	**Value**	**Source**
$x(0)$	$\frac{7675406}{7675893}$	KKRI Makassar City [12]
$y(0)$	$\frac{487}{7675893}$	KKRI Makassar City [12]
$w(0)$	0.000001	Side, 2013 [8]
$z(0)$	0.056	Side, 2013 [8]
N	7675893	KKRI Makassar City [12]
μ_h	0.000046	Side, 2013 [8]
α	0.232198	Side, 2013 [8]
β	0.328879	Side, 2013 [8]
γ	0.375	Side, 2013 [8]
γ_h	0.328833	Side, 2013 [8]
θ_h	0.001	Side, 2013 [8]
φ	0.001046	Side, 2013 [8]
δ	0.0323	Side, 2013 [8]

$$v_{3,4} = \int_0^t \left[\gamma v_{2,3} - \delta_1 v_{3,3} - \gamma v_{2,3} v_{3,0} - \gamma v_{2,2} v_{3,1} - \gamma v_{2,1} v_{3,2} - \gamma v_{2,0} v_{3,3}\right] ds \tag{51}$$

$$v_{4,4} = \int_0^t \left[\gamma_h v_{2,3} - \varphi v_{4,3}\right] ds \tag{52}$$

The initial value and parameters used in SIRS model for transmission dengue fever as shown in Table 1.
From initial value in Table 1, we have $c_1 = \frac{7675406}{7675893}$, $c_2 = \frac{487}{7675893}$, $c_3 = 0.056$ and parameters value $\alpha = 0.232198$, $\beta = 0.328879$, $\gamma = 0,375$, $\delta = 0.0323$, $\gamma_h = 0.328833$, $\theta_h = 0.001$ and $\varphi = 0.001046$ obtain

$$v_{1,1} = 0.999936555 - 0.01304826t \tag{53}$$

$$v_{2,1} = (6.34454E - 0.5) + 0.013023129t \tag{54}$$

$$v_{3,1} = 0.056 - 0.00178634t \tag{55}$$

$$v_{4,1} = 0.000001 + (2.08629E - 05)t \tag{56}$$

$$\ddots$$

Generally, obtained SIRS model solution for transmission of dengue fever are:

$$x(t) = \sum_{j=0}^{4} v_{1,j} = 0.999936555 - 0.01304826t + 0.000585026t^2 - 0.001096818t^3 - 0.000242282t^2 \tag{57}$$

$$y(t) = \sum_{j=0}^{4} v_{2,j} = (6.34454E - 05) + 0.013023129t + 0.003698608t^2 + 0.002313229t^3 + 0.001003117t^4 \tag{58}$$

$$z(t) = \sum_{j=0}^{4} v_{3,j} = 0.056 - 0.00178634t + 0.004667929t^2 + 0.001167146t^3 + 0.000760837t^4 \tag{59}$$

$$w(t) = \sum_{j=0}^{4} v_{4,j} = 0.000001 + (2.08629E - 05)t + 0.004282433t^2 + 0.001215984t^3 + 0.000760598t^4 \tag{60}$$

An Equation (57),(58),(59) and (60) are solutions of of SIRS model for transmission of dengue fever using homotopy perturbation method.

CONCLUSION

Mathematical model of SIRS is a model for transmission of dengue fever with assumption that dengue fever sufferers who have recovered infected again. Solutions of SIRS model for transmission of dengue fever using homotopy perturbation method gives description about the number of individual are Suspected Exposed, Infected and Recovered dengue fever case on month until next year, the number of hinghest and lowest cases of individual infected dengue fever so that the goverment of makassar city can take precautionary measuares so that transmission of dengue fever could be solved.

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