Investigating the behaviour of lenses using an optical ray tracer

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Abstract

The refraction at interfaces with different refractive indices in a spherical refracting surface and a planoconvex lens with focal lengths ~ 100 mm were modelled using optical ray tracing, in which light waves were approximated as rays and their motion was calculated using the principles of geometrical optics. It was found that for both the surface and the lens, rays propagating parallel to the optical axis nearly converged to a focal point on the axis, as can be theoretically calculated. However, there were some deviation from the ideal behaviour due to spherical aberration. This produced a spread at the focus, which was found to be increasing exponentially for all surfaces used. It was found that a single spherical surface had the smallest spread, and when the convex side of a planoconvex lens was facing the light source, the spread was only around 1/4 of that of the opposite case, so would produce sharper images.

Introduction

Optical ray tracing is a method for simulating the path of light waves through various dielectric media with different refractive indices. It is an idealised approximation [1] because light waves have a finite beam width while rays are infinitely thin. The approximation works when the wavelength of the light is much smaller than the dimensions of the objects the light is propagated through [2], which means it works well for visible light considered. This method can simulate refraction, reflection, but not diffraction as it is not described by ray optics. However, it is still useful in designing a lens with optimised properties.

Theory

A refracting surface is defined by five quantities: its position along the optical *z*-axis z_0, its curvature, the refractive indices on either side of the surface n_1 and n_2, and its aperture radius. From the curvature of a surface, the radius of curvature can be worked out as 1/curvature.

Snell's law is used in determining the refracted angle from the incident angle. It states that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{1}$$

where θ_1 and θ_2 are the angles of incidence and refraction respectively for a ray propagating from medium 1 to 2. If $n_2 < n_1$, by setting θ_2 to the marginal case $\pi/2$, then a critical angle θ_c meeting the condition that $\sin\theta_c = n_2/n_1$ can be found, and any incident angle greater than θ_c will be subject to total internal reflection, i.e. there is no refraction.

In order to deduce the focal length from the above, paraxial approximation has to be taken into account. This assumes that rays lie close enough to the optical axis so a small angle approximation can be made when the rays strike the axis [3][4]. Then, the theoretical focal length f of a spherical surface can be found as [5]

$$f \approx \frac{Rn_2}{n_2 - n_1}, \tag{2}$$

where R is the radius of curvature. For a planoconvex lens this relation becomes [6]

$$f \approx \frac{R}{n_2 - n_1}, \tag{3}$$

where R is the radius of curvature of the convex surface.

The rays for which the above relations hold true are called paraxial rays. Rays further away from the optical axis are refracted more than the paraxial ones. This creates a spread of points at the focus instead of a single focal point, which is called spherical aberration. The spread can be characterised y its root-mean-square (RMS) radius r_{RMS}, which is given by

$$r_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2}, \tag{4}$$

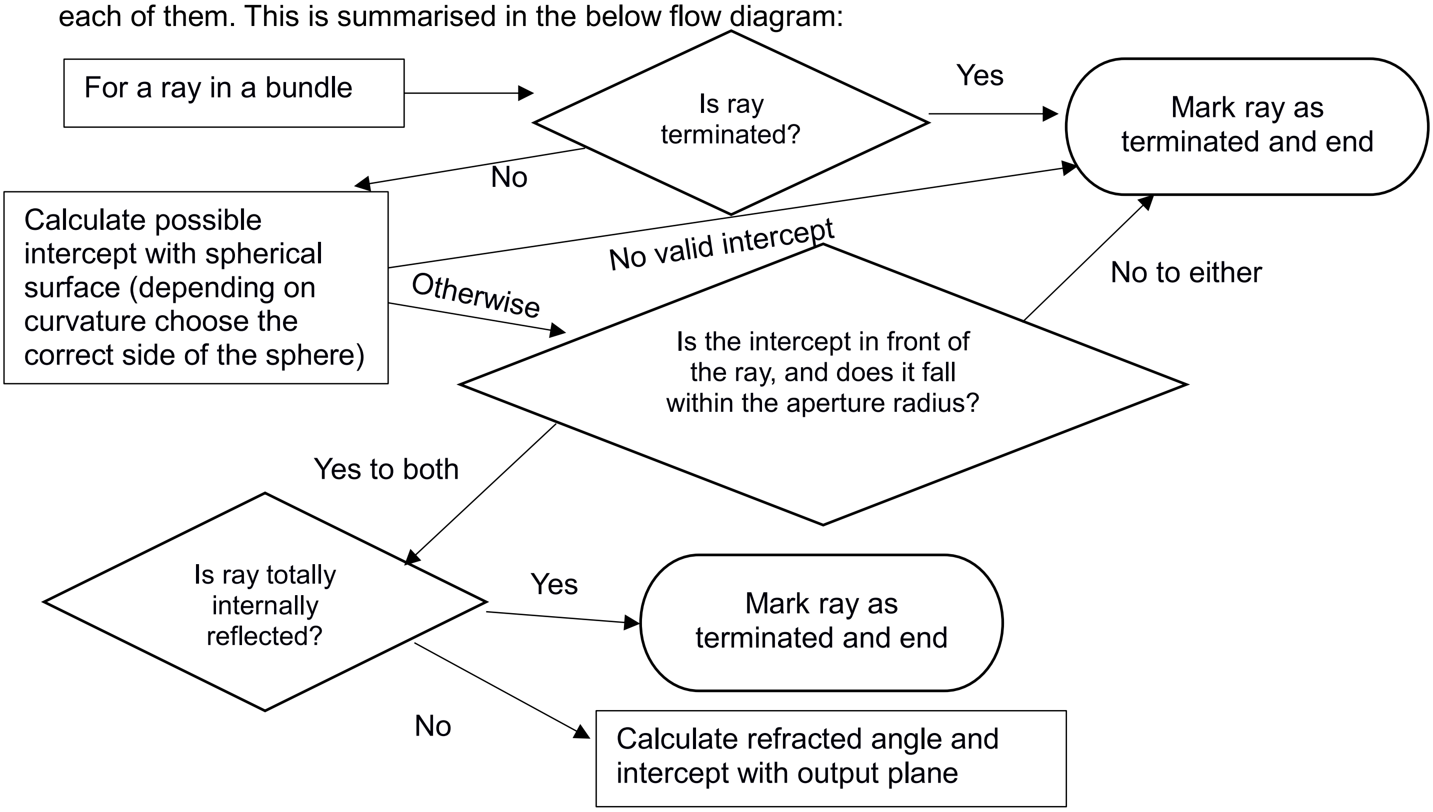
where r_i is the distance of a point from the optical axis at the focus, and N is the number of points at the focus.

Method

A ray was modelled as an object under the class `ray`, and contained lists of its positions and its current direction as attributes. It also had an attribute which stated whether the ray was terminated or not. A collimated beam was represented by a bundle of rays under the class `raybun`, which stored all `ray` objects in a list.

There were two optical elements in this model, a spherical refracting surface under the class `SphericalRefraction`, and an output plane under the class `OutputPlane`. A refracting surface was defined by the five parameters as above, while an output plane was defined by its position. Both optical elements had a method `propagate_ray`, which calculated the first possible intercept with the surface using the current position and direction of the ray by calling the method `intercept`. Then for a refracting surface, if a valid intercept was found, it would attempt to refract the ray using the refractive indices given by calling the method `refract`. If a valid refracted angle is found, the intercept would be stored as the new position and the new direction would be calculated from this refracted angle. If there was either no intercept or no refracted angle, then the ray would be marked as terminated.

The method `propagate_raybun` took all the rays in the bundle and repeated the above process for each of them. This is summarised in the below flow diagram:



In the model, a paraxial ray with distance 0.1 mm away from the optical axis was sent in initially to find the paraxial focus. The output plane was then set there. Three cases were considered, as shown below:

Surface	z_0 / mm	Curvature / mm^{-1}	n_1	n_2	Radius of aperture / mm	Focal length calculated using equations (2) and (3) / mm
Spherical	20	0.03	1.0	1.5	10	100
Two orientations of a planoconvex lens, with surfaces defined below:						
Convex	20 / 25	± 0.02	1.0 ($z < 20$ and $z > 25$) 1.5168 ($20 < z < 25$)		5.5	100
Plane	25 / 20	0			5.5	100

The collimated beam used for each case had the following properties:

Surface / lens	Number of layers excluding centre n	Radius of beam $r_{\max}$ / mm	Number of points in innermost ring m
Spherical	5	3	6
Planoconvex	5	5	6

In addition to a beam propagating parallel to the *z*-axis, for the spherical case, another parallel beam with the centre at $x = -2$ in the beginning was also used.

For collimated beam with diameters varying between 5 × 10^{-3} to 3 mm, the RMS radius spread of all the points were calculated this was compared to the diffraction limit given by $\lambda f/D$, where $\lambda = 588\ \mathrm{nm}$ is the wavelength of the light used (although its value did not affect the propagation in any way), f the focal length as found from a paraxial ray, and D the diameter of the beam. The diffraction limit is an upper performance limit of the lens [7]. It limits the angular resolution and light beams with a diameter above it cannot be resolved, in which case it is known as 'diffraction limited'.

Results & Discussion

Referring to the graph on p. 44 in the lab book, the focal point for the single spherical surface was found to be at around 120 mm, giving a focal length of about 100 mm, which was equal to the theoretical value using equation (2), whereas looking at p. 47 and 50, this was found to be at 121.75 mm and 118.45 mm for the planoconvex lens, with the convex side facing light source having a smaller value. These gave focal lengths of 99.25 mm and 95.95 mm respectively, which was close to the theoretical value. The difference was probably due to the separation between the two surfaces.

The upper part of each spot diagram in p. 41, 42, 48 and 51 plotted the collimated beam used at the beginning, while the lower plotted the beam at the focus. Comparing the two diagrams obtained for a single spherical surface (p. 41, 42), it can be seen that if a beam propagated parallel to the optical axis, the points at focus were symmetrical, and the spread was also smaller. In this example, it was about a half of the non-parallel case. If the two diagrams obtained using either orientation of the planoconvex lens (p. 48, 51) were compared, it can be seen that aligning the convex side of the lens towards the light source result only in about 1/4 of the spread of the other orientation.

This was echoed in the RMS plots (p. 48-49, 51-52). As seen above, the diffraction limit falls with the reciprocal of diameter. From the plots, the RMS radius increased quite exponentially, and the intersection between the two can be taken as the point where the maximum width of the beam for which the lens can resolve. This maximum diameter was found to be 1.10 and 1.76 mm respectively, with the convex side facing source having the larger value. This showed that larger objects could be resolved by the same lens if the convex lens was facing the object. Also, the RMS radius increased at a much slower rate than the other case, which was also 1/4 for the same range of diameters considered. For the single spherical surface, the RMS radius was much smaller than the planoconvex case. This is probably due to the fact that it was only refracted once instead of twice. The maximum diameter was found to be 1.39 mm. Interestingly, this lies between the two values in the planoconvex case. This showed that using a planoconvex lens in a correct orientation (where the convex side faced the lens) can help reduce the spread and result in a sharper image.

This was also the same as result obtained from experiment [2], which showed that ray optics worked correctly for this simple modelling. However, this method relied on a number of simplifications, which resulted in some limitations. First of all, the width of a ray was zero, while a light wave had a finite width. Secondly, as stated above, the wavelength did not affect the propagation of the ray in ray optics, as the assumption that the light had an extremely small wavelength was made. However, the wavelength had an effect in wave propagation if the full solution of Maxwell's equations was considered. Thirdly, this worked for ideal situations with short ranges only, in order to assume that the refractive index is constant within a medium considered. More complex algorithms would be needed for longer wavelengths or longer ranges. However, this simple modelling had proved to be useful in visualising optics concept such as refraction, focus, and aberration.

Conclusion

The performances of a spherical refracting surface and a planoconvex lens were analysed using optical ray tracing. It was found that for all spherical surfaces, spherical aberration occurred which resulted in a spread of points at the focus and less sharp images. Also, using a planoconvex lens with the convex side facing the object would result in a more focussed image while the opposite would result in about 4 times the spread. More advanced modelling would be required to result in a design of an optimal lens.

References

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