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Efficient Power Allocation Strategy in Multiuser MIMO Broadcast Interference Channel

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Abstract: In this paper, sum rate optimization of multiuser multiple-input multiple-output broadcast (MU-MIMO) communication systems is investigated with perfect channel state information (CSI) at the transmitter. Since power allocation is a signomial optimization problem in the presence of the multiuser interference (MUI), it is not convex in general. We propose an iterative water-filling algorithm that takes advantage of the classical water-filling principle. The proposed algorithm reduces the computational complexity significantly compared with the methods in the literature only with a negligible performance degradation. In addition, the generalized eigenvalue technique for beamforming design is utilized in this paper. Simulations show that the sum rate of the proposed method also achieves the sum capacity of the MU-MIMO broadcast channel, especially in low signal-to-noise ratio (SNR) region.

Keywords: *Multiuser multiple-input multiple-output (MU-MIMO), beamforming, power allocation, interference suppression.*
Collaborations : Organismes de recherche académique, partenaires industriels, programmes de travail.

1 Introduction

In this paper, motivated by [1], a new power allocation method in the MU-MIMO broadcast interference channel is proposed. We relax the zero-forcing constraint, and adopt the generalized eigenvalue beamforming technique in [1], which is a very effective strategy for such optimization problem [1], [2], and [3]. In addition, unlike the GP power allocation in [1], the proposed power allocation method allocates the total transmit power iteratively through the very simple water-filling principle, by which more power is allocated to the subchannels with larger channel gains. This strategy substantially reduces the computational complexity compared with the GP method. Also, since the proposed method allocates the total transmit power iteratively taking the interference from the previous allocated power into account, which also results in a very near-capacity sum rate. Therefore, the proposed algorithm is attractive for practical implementation. Additionally, notice that the beamforming vectors are determined separately for each data stream in the proposed method, the transmit antennas number and the receive antennas number can be arbitrary.

2 System model

2.1 Downlink transmission

Consider the downlink MU-MIMO broadcast channel with K users, where a base station is equipped with N_t transmit antennas, each user has N_r receive antennas and receives L_k data streams. L_k is determined by the proposed iterative water-filling algorithm described in section 3. In the downlink transmission, the received signal $\boldsymbol{y}_k^{\mathrm{DL}}$ at the kth user after receive beamforming filter can be written as [1], [2]

$$\boldsymbol{y}_k^{\mathrm{DL}} = \boldsymbol{U}_k^H \boldsymbol{H}_k \boldsymbol{V}_k \sqrt{\boldsymbol{P}_k}\boldsymbol{x}_k^{\mathrm{DL}} + \boldsymbol{U}_k^H \boldsymbol{H}_k \Big(\sum_{i=1, i\neq k}^{K} \boldsymbol{V}_i \sqrt{\boldsymbol{P}_i}\boldsymbol{x}_i^{\mathrm{DL}} \Big) + \boldsymbol{U}_k^H \boldsymbol{n}_k^{\mathrm{DL}} \tag{1}$$

where the $N_r \times N_t$ matrix $\boldsymbol{H}_k$ denotes the channel between the transmitter and the kth user, $\boldsymbol{x}_k^{\mathrm{DL}} = [x_{k1}^{\mathrm{DL}}, \ldots, x_{kL_k}^{\mathrm{DL}}]^T$ is the transmit vector for the kth user and satisfies $\mathbb{E}|x_{kl}^{\mathrm{DL}}|^2 = 1$ $(1 \leq l \leq L_k)$. $\boldsymbol{V}_k = [\boldsymbol{v}_{k1}, \ldots, \boldsymbol{v}_{kL_k}]$ with denotes the transmit beamforming vectors with $\|\boldsymbol{v}_{kl}\| = 1$ $(1 \leq l \leq L_k)$. $\boldsymbol{U}_k = [\boldsymbol{u}_{k1}, \ldots, \boldsymbol{u}_{kL_k}]$ is the $N_r \times L_k$ receive beamforming matrix with $\|\boldsymbol{u}_{kl}\| = 1$ $(1 \leq l \leq L_k)$. $\boldsymbol{P}_k$ is a diagonal matrix $\mathrm{diag}\{p_{k1}, \ldots, p_{kL_k}\}$ and represents the assigned power for the kth user's data streams. $\boldsymbol{n}_k^{\mathrm{DL}}$ is the zero-mean complex white Gaussian noise (AWGN) vector with covariance matrix $\sigma^2 \boldsymbol{I}_{N_r}$ observed at each user.

3 Power allocation

In this section, the transmit and receive beamforming vectors which can be obtained by solving the generalized eigenvalue problems are supposed to be fixed. We investigate the power allocation optimization problem that is described as

$$\begin{aligned} &\max_{\{p_{kl}\}\ \substack{k=1,\cdots,K\\ l=1,\cdots,L_k}} \sum_{k=1}^{K}\sum_{l=1}^{L_k} \log_2(1+\frac{p_{kl}g_{kl}(k,l)}{\mathrm{In}_{kl}+\sigma^2}) \\ &\text{subject to } \sum_{k=1}^{K}\sum_{l=1}^{L_k} p_{kl} = P_T,\ p_{kl}\geq 0,\ \forall k, \text{and } \forall l. \end{aligned} \tag{2}$$

where $\mathrm{In}_{kl} = \sum_{i=1}^{K}\sum_{\substack{j=1\\(i,j)\neq(k,l)}}^{L_i} p_{ij}g_{kl}(i,j)$, and $g_{kl}(i,j) = |\boldsymbol{u}_{kl}^{H}\boldsymbol{H}_k\boldsymbol{v}_{ij}|^2$.

Note that this optimization problem is non-concave and the optimal solution needs solving a GP iteratively [1], [4], this pretty high computational complexity makes it hard to implement for practice. while the well-known uncomplicated water-filling power allocation algorithm assumes that the subchannels are parallel and interference free. Motivated by these, in this paper we propose an effective iterative power allocation method (Algorithm 1) that not only reduces the computational complexity substantially but retains the high global throughput as well. The proposed method approximates the original non-concave problem as a solvable concave one at each iteration, and the optimal solution is obtained through classical water-filling algorithm.

In the proposed method, the total transmit power constraint P_T is divided into N equal portions, that is $\Delta P = \frac{P_T}{N}$. At iteration step n $(1\leq n\leq N)$, for the lth data stream of the kth user, we consider the interference and Gaussian noise together as $\beta_{kl}^{(n-1)}$, that is

$$\beta_{kl}^{(n-1)} = \mathrm{In}_{kl}^{(n-1)} + \sigma^2 \tag{3}$$

where $\mathrm{In}_{kl}^{(n-1)} = \sum_{i=1}^{K}\sum_{\substack{j=1\\(i,j)\neq(k,l)}}^{L_i} p_{ij}^{(n-1)}g_{kl}(i,j)$, and $p_{ij}^{(n-1)}$ is the already allocated power to the jth data stream of the ith user in the previous $n-1$ steps. Then we solve the power allocation problem

$$\begin{aligned} &\max_{\{\Delta p_{kl}\}\ \substack{k=1,\cdots,K\\ l=1,\cdots,L_k}} \sum_{k=1}^{K}\sum_{l=1}^{L_k} \log_2(1+\frac{\Delta p_{kl}g_{kl}(k,l)}{\beta_{kl}^{(n-1)}}) \\ &\text{subject to } \sum_{k=1}^{K}\sum_{l=1}^{L_k} \Delta p_{kl} = \Delta P,\ \Delta p_{kl}\geq 0,\ \forall k, \text{and } \forall l. \end{aligned} \tag{4}$$

The objective function in (4) can be maximized by the simple water-filling algorithm. We iteratively use water-filling algorithm until all the total transmit power is allocated. Since the maximum value of the current sum rate is obtained at each iteration. And it is known that water-filling algorithm allocates more power to the subchannels who have larger channel gains. This proposed technique attempts to optimize the global throughput and suppress the interference simultaneously at each iteration. Therefore, the more portions the total transmit power is divided into, the larger global throughput we can get. This observation is validated by the simulation results in the next section.

An amount of ΔP power is allocated during one iteration. Finally, The available number of data streams $L_k(\leq \min\{N_t,N_r\})$ of kth user is determined by the number of subchannels that the allocated $p_{kl}>0$ $(1\leq l\leq L_k)$. Algorithm 1 summarizes the above proposed iterative water-filling power allocation algorithm. For the virtual uplink, the transmit power can also be allocated for each data stream similarly to (4).

Thus, the algorithm for joint beamforming design and power allocation is summarized in Algorithm 2.

4 Simulation results

This section presents the simulation results of the proposed iterative power allocation method. We consider a MU-MIMO broadcast channel system with $N_t = 4$, $N_r = 2$, and $K = 4$. The total transmit power P_T is divided into 5 equal portions (i.e. $N = 5$). In table 1 we compare the processing time of the

Algorithm 1 The proposed iterative power allocation algorithm

$\Delta P = \frac{P_T}{N}$, $p_{kl}^{(0)} = 0$, $\forall k$ and $\forall l$.
for iteration $n = 1 : N$
  for the kth user's lth data stream
    Calculate the term $\beta_{kl}^{(n-1)}$ (3).
  end
  Allocate power ΔP by the water-filling algorithm (4).
  Calculate $p_{kl}^{(n)} = p_{kl}^{(n-1)} + \Delta p_{kl}$, $\forall k$, and $\forall l$.
end

Algorithm 2 The proposed iterative algorithm for beamforming design and power allocation

Initialization: $[\boldsymbol{U}_k, \boldsymbol{\Sigma}_k, \boldsymbol{V}_k] = \text{SVD}(\boldsymbol{H}_k)$, $\boldsymbol{P}_k = 0$, $\forall k$.
Repeat $i \leftarrow i + 1$
1, Downlink power allocation.
  Suppose $\boldsymbol{V}_k$ and $\boldsymbol{U}_k$ are fixed, $\forall k$.
2, Downlink receive beamforming design.
  Suppose power allocation and $\boldsymbol{V}_k$ are fixed, $\forall k$.
3, Uplink power allocation.
  Suppose $\boldsymbol{V}_k$ and $\boldsymbol{U}_k$ are fixed, $\forall k$.
4, Uplink receive beamforming design.
  Suppose power allocation and $\boldsymbol{U}_k$ are fixed, $\forall k$.
Until the sum rate convergence, i.e. $|C_i^{DL} - C_{i-1}^{DL}| \leq \epsilon$, or $i > i_{\max}$.

proposed method and GP method. The processing time is recorded for 8 iterations of each method, which is a moderate number of iterations in [1]. Note that the proposed method uses water-filling principle for N times, and the computational complexity of the classical water-filling algorithm is $\mathbb{O}(KN_r\log_2 KN_r)$ [5]. It can be calculated that the computational complexity of the proposed method is $\mathbb{O}(NKN_r\log_2 KN_r)$. In table 1 we can observe that the proposed method performs more than 2×10^3 times faster than GP method.

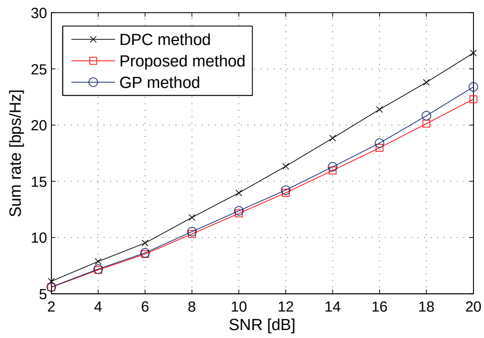
Fig. 1 illustrates the sum capacity by the DPC algorithm, the achievable sum rates of GP algorithm and the proposed algorithm, respectively. We can observe that GP algorithm performs only slightly better than the proposed method. Therefore, the proposed method is practical due to the greatly reduced computational complexity.

To demonstrate that the sum rate of the proposed method is close to the sum capacity, in Fig. 2, we show the simulation results via different power portion N, where we consider a MU-MIMO broadcast channel system with $N_t = 4$, $N_r = 4$, and $K = 4$. Note that $N = 1$ is the classical water-filling algorithm, it assumes that the subchannels are parallel and interference free, and ignores the interference that comes from other subchannels, so it does not work here. The proposed method ($N \geq 2$) assigns transmit power iteratively by considering the interference contributed in the previous iterations, and then follows the water-filling principle. Fig. 2 illustrates that the gap between the sum rate of the proposed method and sum capacity is reduced when the power portions number is increased. Note that most of the gain is achieved even the number of equal power portions N is quite small, Therefore, the proposed technique is promising for practical implementation.

Considering the convergence behavior, it is shown in [1] that the sum rate is monotonically increased at each iteration, i.e., $C_i^{UL} \geq C_i^{DL} \geq C_{i-1}^{UL} \geq C_{i-1}^{DL}$, we suppose that C_i^{DL} and C_i^{UL} denote the temporary

Table 1: Processing time comparison (seconds)

Algorithm	Proposed method	GP method
Processing Time	0.0261	64.7424



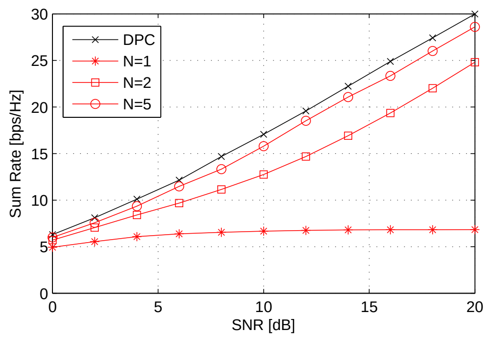


Figure 1: Sum rate comparison of DPC method [6], GP method [1], and the proposed method. $N_t = 4$, $N_r = 2$, and $K = 4$.

Figure 2: Illustration of the sum rate approaching the sum capacity with power portions $N = 1$, 2 and 5, respectively.

sum rate of the downlink channel and the corresponding virtual uplink after i iterations, respectively. Since the sum rate under the sum power constraint is bounded, the algorithm's convergence is guaranteed by the monotonic convergence theorem.

5 Conclusion

In this paper, we propose a practical power allocation method taking advantage of water-filling principle. The proposed method first divides the total transmit power into smaller equal portions, then each portion is allocated by approximating the original problem as a concave solvable one and the very simple water-filling algorithm is adopted to obtain the optimal solution. The proposed method substantially reduces the computational complexity compared with GP method in the literature, only with a slight performance degradation. Additionally, since beamforming vectors are determined separately for each data stream in the proposed method, the number of transmit and receive antennas can be arbitrary. It is shown that this proposed method is attractive in practice. Some numerical results validate the proposed technique.

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