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DISTRIBUTED RESOURCE ALLOCATION OVER SWITCHED NETWORKS[1]

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Abstract: Decision making is required in many engineering problems, especially in those that seek to optimize certain criteria, such as increasing economic benefits, improving the quality of service, or reducing energy consumption. Since the complexity and scale of systems have increased in the last years, traditional centralized approaches for decision making are no longer appropriate, and distributed schemes are becoming more prevalent. Our work focuses on an important problem that involves distributed decision making, the optimal resource allocation over networks of agents. Concretely, we propose to use a continuous–time algorithm that does not require a centralized coordinator. Moreover, we analyze the case where the communication network used by the agents to coordinate their decisions is not completely reliable. Some results on the robustness and optimality of the considered method are provided. Finally, we apply the technique to the management of a smart energy system.

Keywords: *Distributed optimization, resource allocation, switched systems, smart energy systems.*
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1 Introduction

The growing interest on distributed decision making problems has lead the development of a large number of distributed optimization methods (e.g., see [1], [2] and the references therein) as those based on multi–agent systems. An important problem within this area is the optimal resource allocation over networks of agents, that is widely used in several fields (a comprehensive survey on the applications of resource allocation can be found in [3]). There is a long tradition in the literature on distributed methods for resource allocation that use multi-agent schemes. Among all the methods, the appropriate coordination of agents is a crucial issue to avoid suboptimal solutions.

In order to carry out the aforementioned coordination, several techniques use a centralized agent. However, in some real situations (especially when we have a large number of agents and the amount of communications allowed between them is strongly limited), the implementation of this centralized coordinator is impractical, too expensive, or even infeasible [4]. In these cases, it is necessary to apply fully decentralized approaches. In our work, we have focused on distributed resource allocation methods over networks of agents that do not require a central coordinator. More precisely, we propose to use a continuous time version of the *center–free* algorithm [5], [6]. The considered technique is basically a gradient–like method that belongs to a class of algorithms studied in [7], where each node of the network is associated with an agent, that is responsible for managing the resource allocated in the node by taking into account a given cost function. Starting with a feasible allocation, each agent updates the corresponding resource in proportion to the difference between the marginal costs perceived by itself and its neighbors. The simple idea behind the method makes it appropriate for applications where the agents have elementary computation capabilities. Moreover, the use of a continuous time approach leads a natural interaction of the considered methodology with dynamical systems, e.g., when the communication network that allows the agents to share information changes over the time due to failures (i.e., a network with switching topology).

In this document, we present the continuous time version of the *center–free* algorithm and some preliminary results regarding its optimality. Finally, in order to illustrate the flexibility and robustness of the method, we show an application to the management of smart energy systems. More specifically, we present a distributed solution to the optimal chiller loading problem in multiple chiller systems [8], which are widely used in large air–conditioning systems.

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2 Problem Statement

The problem of optimal resource allocation over networks of agents arises when we have a limited amount of a certain resource, and it is necessary to establish a distribution policy between a set of agents, which are connected by a communication network that is modeled as a directed graph $\mathcal{G}$. The objective is to minimize an overall cost function subject to some technical constraints. For simplicity of the formulation, we assume that each agent in the network handles a decision variable of the problem. Mathematically, the resource allocation problem considered in our work is given by [3],

$$\begin{array}{ll} \min & J(x) \\ \text{s. t.} & \mathbf{1}^\top x = d \end{array} \tag{1}$$

where $J : \mathbb{R}^n \to \mathbb{R}$ is a strictly convex cost function, and $d \in \mathbb{R}$ is a given constant related to the total amount of available resource. Hence, in the above formulation, x_i can be viewed as the amount of resource allocated to the i^{th} agent. Moreover, the local information structure imposed by the digraph $\mathcal{G}$ must be considered as part of the problem statement. In this case, an algebraic digraph $\mathcal{G}_x = (\mathcal{G}, x)$ emerges, where the node i is associated with the i^{th} agent, x_i is the i^{th} decision variable, and the edges of $\mathcal{G}$ are the available communication channels.

In order to deal with non ideal communications, we also analyze the case when some edges of the digraph $\mathcal{G}$ are removed or added (which models the failure/creation of communication links). To this end, we define the set $\Gamma = \{\mathcal{G}_1, \ldots, \mathcal{G}_m\}$ comprised by all the digraphs of order n resulting from adding/removing edges of $\mathcal{G}$, such that $\mathcal{G}_k$ is weight–balanced and strongly connected, for $k = 1, \ldots, m$ [9].

3 Distributed Solution

In order to solve the problem stated in Equation (1), we use a continuous time version of the *center–free* algorithm proposed in [5], as follows,

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} a_{i,j} \left(\frac{\partial J}{\partial x_j} - \frac{\partial J}{\partial x_i} \right), \text{ for all } i = 1, \ldots, n, \tag{2}$$

where $\mathcal{N}_i$ is the neighborhood of i, i.e., the set of nodes from which the i^{th} node can receive information; and $a_{i,j}$ is the element of the i^{th} row and j^{th} column of the adjacency matrix associated with the communication digraph $\mathcal{G}_k$. This algorithm is distributed if we assume that the expression in brackets can be calculated by the i^{th} agent using local information.

3.1 Convergence Analysis

If we apply the algorithm described in Equation (2) to the problem given in Equation (1), a dynamic multi–agent network emerges, and it can be modeled as the following switched system [9],

$$\dot{x}(t) = -L(\mathcal{G}_k)\nabla J(x), \quad k = \sigma(t), \mathcal{G}_k \in \Gamma, \tag{3}$$

where $\sigma(t) : \mathbb{R}_{\geq 0} \to \{1, \ldots, m\}$ is a piecewise continuous switching signal related to the events that modify the communication topology. The following proposition states that the described switched system asymptotically reaches the optimal solution of the problem given in Equation (1).

Proposition 3.1. *Let $x(0)$ be a feasible solution of the problem given in Equation (1). For any arbitrary switching signal $\sigma(\cdot)$, the solution of the switched system in Equation (3) asymptotically converges to x^*, where $x^* = \arg\min_{\mathbf{1}^\top x = d} (J(x))$.*

A consequence of the above proposition is that if the directed graph representing the communication network topology remains weight-balanced and strongly connected for all time, then the algorithm described in Equation (2) is robust under failures in the communication channels.

4 Application to Energy Efficiency in Chilled–water Plants

In this section we present a distributed solution to the optimal chiller loading problem in multiple chiller plants [8], which are widely used in large air–conditioning systems. The goal is to distribute the cooling load among the chillers that comprise the plant for minimizing the total amount of power used by them. For a better understanding of the problem, first we present a brief description of the system.

The purpose of a decoupled chilled–water plant comprised by n chillers is to provide a water flow f_T at a certain temperature T_s to the rest of the air–conditioning system. To do that, the plant needs to meet a cooling load C_L that is given by the following expression

$$C_L = mf_T(T_r - T_s) \tag{4}$$

where m is the specific heat of the water and T_r is the temperature of the water returning to the chillers. Since there are multiple chillers, the total cooling load C_L is split among them, i.e., $C_L = \sum_{i=1}^{n} Q_i$, where Q_i is the cooling power provided by the i^{th} chiller, which, in turn, is given by $Q_i = mf_i(T_r - T_i)$, where f_i and T_i are, respectively, the flow rate of chilled water and the water supply temperature of the i^{th} chiller. Furthermore, in these kinds of plants, $f_T = \sum_{i=1}^{n} f_i$. In order to meet the corresponding cooling load, the i^{th} chiller consumes a power P_i that can be calculated using the following expression [10]

$$\begin{aligned} P_i = \; & \left(k_{0,i} + k_{1,i} m f_i T_r + k_{2,i}(m f_i T_r)^2 +\right) + \left(k_{3,i} - k_{1,i} m f_i - k_{4,i} m f_i T_r - 2k_{2,i}(m f_i)^2 T_r\right) T_i + \\ & \left(k_{5,i} + k_{6,i} m f_i + k_{2,i}(m f_i)^2\right) T_i^2 \end{aligned} \tag{5}$$

where $k_{j,i}$, for $j = 0, \ldots, 6$, are constants related to the i^{th} chiller. If we assume that the flow rate f_i of each chiller is constant, then P_i is a quadratic function of the temperature T_i. The optimal chiller loading problem involves the calculation of the chillers' water supply temperatures that meet the total cooling load given in Equation (4), and minimize the total amount of power consumed by the chillers, i.e., $\sum_{i=1}^{n} P_i$. Moreover, given the fact that the cooling output of a chiller needs to be higher than 30% of its capacity (to avoid operating problems [10]), we have the following additional constraints

$$0.3\overline{Q}_i \leq mf_i(T_r - T_i) \leq \overline{Q}_i \quad \text{for } i = 1, \ldots, n \tag{6}$$

where $\overline{Q}_i$ is the capacity (rated value) of the i^{th} chiller. Notice that these constraints can be addressed by means of barrier functions.

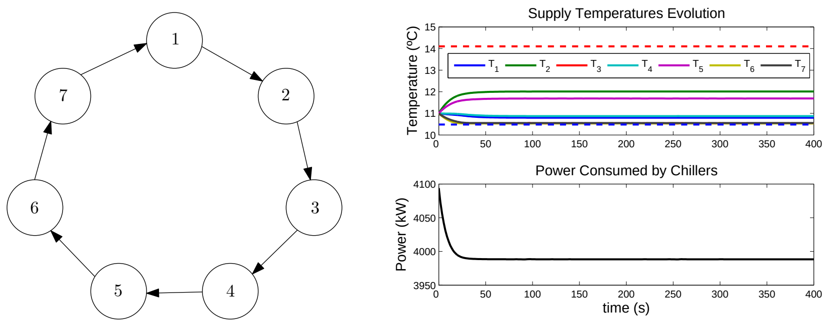
Now, let us consider that we want to solve the aforementioned problem in a distributed way by using a multi–agent system, in which each chiller is managed by an agent that decides the value of the water supply temperature. We assume that the i^{th} agent knows (e.g., by measurements) the temperature of the water returning to the chillers, i.e., T_r, and the flow rate of chilled water, i.e., f_i. The coordination of the agents' decisions is done through a communication network with a topology given by the graph $\mathcal{G}$. If each $P_i(T_i)$ is a convex function, then the problem is in the form stated in Equation (1), and it can be solved by using the algorithm proposed in Equation (2). The main advantage of this approach is to increase the resilience of the whole system against possible failures, due to the fact that the plant operation does not rely on a single control center but on multiple individual controllers without the need for a centralized coordinator.

4.1 Illustrative Example

We simulate a chilled–water plant comprised by 7 chillers, the cooling capacity and the water flow rate of each chiller are, respectively, $\overline{Q}_i = 1406.8$ kW and $f_i = 65$ kg.s^{-1} for $i = 1, \ldots, 7$; the specific heat of the water is $m = 4.19$ kW.s.kg^{-1}.^{o}C^{-1}; the supply temperature of the system is $T_s = 11$ ^{o}C; and the coefficients $k_{j,i}$ of Equation (5) are based on [10]. We operate the system at 90% of its total capacity, i.e., $C_L = 0.9 \sum_{i=1}^{n} \overline{Q}_i$. In order to apply the distributed solution presented in Equation (2), we use an agent per chiller (i.e., the i^{th} agent controls the supply temperature T_i of the i^{th} chiller) and the communication network with ring topology shown in Figure 1a. The initial conditions of the chillers' supply temperatures are $T_i(0) = T_s$, for $i = 1, \ldots, 7$. The results are depicted in Figure 1b, where it is shown that the cooling load is properly allocated among the chillers by adjusting the supply temperatures. More efficient chillers (i.e., chiller 3, chiller 6, and chiller 7) are more loaded than the less efficient ones (i.e., chiller 2 and chiller 5), this can be noticed from the fact that their supply temperatures, in steady state, reach the minimum value. Furthermore, the energy consumption is minimized and power saving reaches to 2.6%.

5 Conclusion

In this document, we have summarized our current research on distributed resource allocation over networks using a multi–agent system approach. Convergence and optimality of the considered method



a) *b)*

Figure 1: *a)* Communication network topology, *b)* Evolution of supply temperatures and total power consumed by the chillers.

is guaranteed under some conditions on the topology of the communication network that enables the agents to share information, even in the face of possible failures on this network. The main advantage of the studied technique is that it does not require a centralized coordinator, which makes the method appropriate to be applied in large scale distributed systems. As future directions we plan to characterize the performance of the algorithm by using, for instance, the ideas exposed in [11]. Moreover, we propose to apply the method to dynamic problems, and to analyze a more general class of switching that relaxes the assumptions in Proposition 3.1, as in [12].

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