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Numerical resolution of the differential equations system of inverted pendulum dynamics relying the coordinates of the center of mass and the zero moment point

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Abstract: This paper presents a fast and efficient method to generate the trajectory of the robot center of mass for cyclic walking. We redefine the problem of the inverted pendulum model with tacking into account the vertical motion of the robot center of mass. This motion is specified depending on a bio-mechanical study that focuses on the properties of the vertical displacement of the robot center of mass as a function of the cycle time or the CoM advancement along the axis in the walking direction. As application of this method, we generate the motion of the humanoid robot Romeo for two forward steps. The simulation results prove the effectiveness of this method.

Keywords: *Humanoid robot, human-like walking, inverted pendulum model, CoM vertical motion, differential equations, numerical resolution.*

1 Introduction

The locomotion of humanoid robots is one of the most important topics in humanoid robots. These highly advanced robots are expected to coexist with humans. For this reason, we aim to obtain a human-like walking. Vukobratovic *et al.* [1] originally suggested the zero moment point (ZMP) as a criterion to study the dynamic stability of a biped robot. This criterion was widely used in the control of biped robots stability. The ZMP-depending methods represent the robot by an inverted pendulum. These methods specify the ZMP trajectory in advance and then calculate the center of mass (CoM) motion corresponding to the desired ZMP [2] and [3]. However, most of these methods do not take into account the vertical motion of the center of mass and this leads in most cases to a not very human-like walking gait. In recent years, some researches realized a method for walking generation with a vertical motion of CoM. In [4] Kim *et al.* realise a walking depending on the inverted pendulum mode. For them, the vertical motion of the CoM is captured from human and scaled so that, it could be applied to robot. The desired ZMP is calculated using a simplified Human model. The pendulum equation was solved numerically.
In this paper we also suggest to solve numerically the differential equation relying the horizontal components of the center of mass of the pendulum and the ZMP components. But the vertical motion of the CoM is defined as a function of time or as a function of the advancement x in the walking direction. These functions are based on a bio-mechanical study on human walking. In fact, the numerical solution allows us to generate a more human-like walking gait because the waist oscillates with a smaller magnitude and moves also vertically.

2 Inverted pendulum models

The dynamics balance of a mechanical system moving in contact with the ground leads to a relation linking the CoM position to the ZMP position, which can be expressed as

$$\begin{cases} x_p &= x - \dfrac{z - z_P}{\ddot{z} + g}\,\ddot{x} \\ y_p &= y - \dfrac{z - z_P}{\ddot{z} + g}\,\ddot{y} \end{cases} \tag{1}$$

where $P = [x_P \; y_P \; z_P]^t$ and $C = [x \; y \; z]^t$ denote the ZMP and CoM positions in the reference frame, respectively. The differential system (1) is non linear and difficult to solve analytically. A solution to use these equations for humanoid control [5] consists in linearizing the system in x and y. For example, Kajita *et al.* considered the motion of a linear inverted pendulum [6]. In such case, the vertical motion of the CoM is fixed with $z = z_c$, therefore $\ddot{z} = 0$. The differential system (1) becomes linear in x and y:

$$\begin{cases} x_p &= x - \dfrac{z_c - z_P}{g}\,\ddot{x} \\ y_p &= y - \dfrac{z_c - z_P}{g}\,\ddot{y} \end{cases} \tag{2}$$

The last differential equation system accepts a solution of the form:

$$\begin{cases} x &= x(0)\cosh(t/T_c) + T_c(\dot{x}_0)\sinh(t/T_c) \\ y &= y(0)\cosh(t/T_c) + T_c(\dot{y}_0)\sinh(t/T_c) \end{cases} \tag{3}$$

Where $T_c = \sqrt{\frac{z_c}{g}}$ and z_c is the constant height of the robot center of mass. This solution is fast and easy to compute, but leads to a not so human-like gait.

To our knowledge, no other analytical solution was proposed to solve the differential system 1.

3 Numerical resolution

The numerical solving considers two cases: case 1 is when $z = z(t)$ is a function of time and case 2 is when $z = z(x)$ which shows the cyclic relation between z and the advancement $x(t)$.

3.1 z as a function of time t

We represent the vertical motion of the center of mass during normal walking by a sinusoidal function oscillating around z_c. z_c is the average height of the center of mass of the robot. Therefore, the vertical displacement of CoM (shown in figure 1) can be expressed as

$$z(t) = z_c + A\cos(\omega t + \phi) \tag{4}$$

Where 2A is the difference between the maximum and the minimum height of the CoM; $\omega = 2\pi/T_{sup}$; T_{sup} is the period of one step because z completes 2 cycles within one walking cycle and ϕ is some angle depending on the first phase of the motion (Single Support SS or Double Support DS).

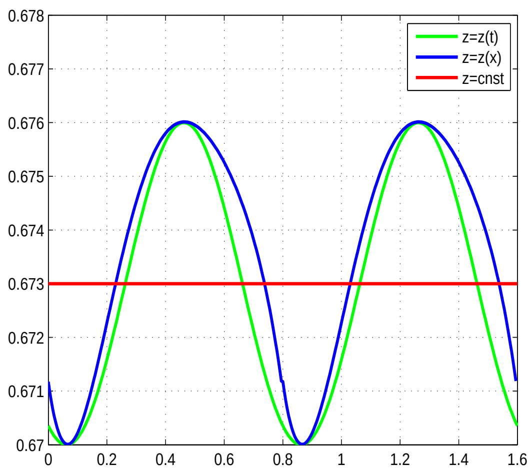

Figure 1: Vertical motion of the robot center of mass during one walking cycle.

Hence,

$$\begin{aligned} \dot{z}(t) &= \tfrac{dz}{dt} = -\omega A\sin(\omega t + \phi) \\ \ddot{z}(t) &= -\omega^2 A\cos(\omega t + \phi) \end{aligned} \tag{5}$$

Substituting Eq 4 and Eq 5 into Eq 1 (same treatment for y), we obtain:

$$\ddot{x} = \frac{g - \omega^2 A\cos(\omega t + \phi)}{z_c + A\cos(\omega t + \phi)}(x - x_p) \tag{6}$$

3.2 z as a function of the advancement $x(t)$

In the cyclic walking, the vertical motion of the center of mass z can be expressed as a function of the advancement $x(t)$. As an example, let us consider z as a polynomial function in x (as shown in figure 1).

$$z = z(x) = \sum_{i=0}^{i=n} a_i x^i \tag{7}$$

$$\dot{z} = \frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} = \frac{dz}{dx}\dot{x} = \sum_{i=1}^{i=n} i a_i x^{(i-1)} \dot{x} \tag{8}$$

$$\ddot{z} = \frac{d^2 z}{dx^2} + \frac{dz}{dx}\ddot{x} = \sum_{i=2}^{i=n} i(i-1) a_i x^{(i-2)} \dot{x}^2 + \sum_{i=1}^{i=n} i a_i x^{(i-1)} \ddot{x} \tag{9}$$

Where a_i are the polynomial coefficients, they should be calculated depending on the bio-mechanical study of Hayot *et al.* [7] and taking into account the continuity conditions in position and in velocity. Substituting z and $\ddot{z}$ in the differential equation system 1, we obtain

$$\ddot{x} = \frac{\sum_{i=2}^{i=n} i(i-1) a_i x^{(i-2)} (x - x_p)}{\sum_{i=0}^{i=n} a_i x^i - (\sum_{i=1}^{i=n} i a_i x^{(i-1)})(x - x_p)} \dot{x}^2 + \frac{g(x - xp)}{\sum_{i=0}^{i=n} a_i x^i - (\sum_{i=1}^{i=n} i a_i x^{(i-1)})(x - x_p)} \tag{10}$$

Equations (10) and (6) are 2^{nd} order non-linear and non-homogeneous differential equations of high complexity and can not be solved analytically to our knowledge because they do not belong to any famous form of differential equations. But they could be solved numerically. The numerical solution of the differential equation system 1 allow as to take into account the effect of the vertical motion of the center of mass, which was neglected by many authors in order to linearize the system and make it possible to solve analytically. This method does not constraint the pelvis to stay at a fixed height while walking.

4 Results

The vertical motion of the CoM is specified according to a study of human walking carried out by Hayot *et al.* [7]. Depending on this study, the height of the CoM reaches its maximum value in the middle of the single support phase and reaches its minimum value in the middle of the double support phase. Also, the magnitude of the vertical oscillation of CoM is about 0.006 m. We use the model of the humanoid robot Romeo. Its total weight is 40.53 kg and the height is 1.43 m. Romeo has 33 degrees of freedom (DOF): each leg has 7 DOF (3 DOF in the hip joint, 1 DOF in the knee, 2 DOF in the ankle and 1 in the toe pitch). Each arm has 7 DOF (3DOF in the shoulder, 2 DOF in the elbow and 3 DOF in the wrist). The trunk has 1 DOF, the neck has 2 and the head has 2 DOF.
We generate the $3D$ CoM displacement for the robot Romeo in the two cases : $z = z(t)$ and $z = z(x)$ and we built the two models corresponding to equations 6 and 10. The desired trajectory of ZMP is defined as a polynomial function, it move forward in the x direction in the single support phase as illustrated in the left graph of figure 4. The step length is 34.8 cm and the cycle time is 1.6 s. 0.1 s for the double support phase and 0.7 s for the single support phase. The resulting horizontal trajectories of CoM in the two cases $z = z(t)$ and $z = z(x)$ are shown in the right graph of figure 4. In the same figure, we show the CoM trajectory calculated using the Linear inverted pendulum LIP and for the same gait parameters (cycle time and step length). When comparing these three CoM trajectories to the one of human walking given in [7], we notice that the magnitude of the horizontal oscillation of the robot center of mass is smaller in the two cases when taking into account the vertical component z of the center of mass than the case when $z = cnst$. In addition, the case $z = z(x)$ give the smallest magnitude of CoM horizontal oscillation.

5 conclusion

We proposed a new form of the inverted pendulum model. The vertical motion of the center of mass was specified according to some characteristics extracted of a bio-mechanical study of the human walking.

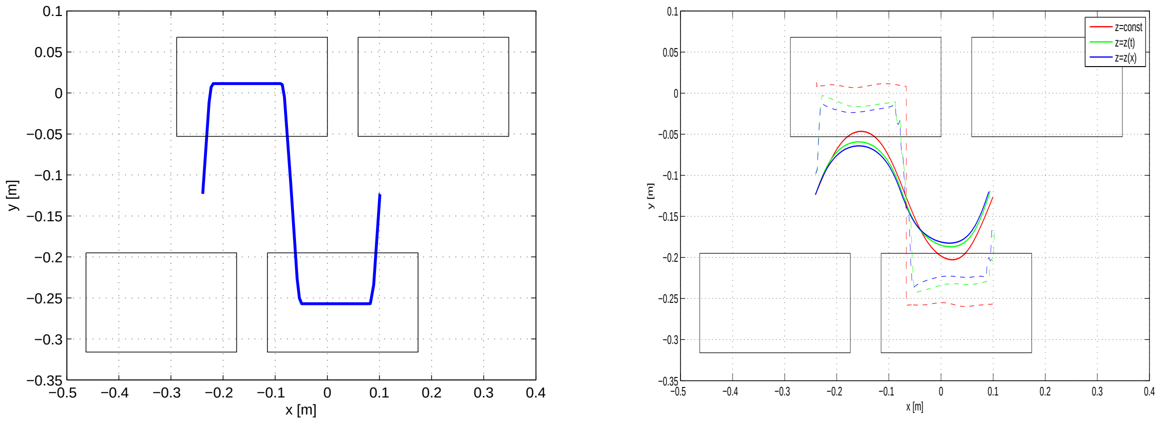

Figure 2: In the left graph, desired trajectory of ZMP for one walking cycle with feet placements. In the right graph, resulting trajectories of CoM during one walking cycle for a step length $L = 34.8$ cm: the red curve corresponds to the linear inverted pendulum, the green one corresponds to the case $z = z(t)$ and the blue one corresponds to the case $z = z(x)$

The numerical resolution of the differential equations system was adopted in order to overcome the mathematical complexity. The validity of this method was demonstrated by generating the motion of the humanoid robot Romeo in computer simulation.
Our future work will be to generate robot motion using the numerical method for different step lengths and to find the optimal step length which minimizes the energy consumed by the system.

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