

## Online Identification of the Supercapacitor Ageing Based on Observer Theory

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**Abstract:** Supercapacitor (also called EDLC) is widely used in different domains such as energy and transportation domains, thanks to its high capacitance, high power density and long life cycle [1][2]. But the EDLC ageing is also a big issue that can lead to some catastrophic events [3] [4]. The supercapacitor ageing is visible by the evolution of some parameters such as the capacitances (C) or the equivalent series resistance ( $R_s$ ) [5]. To observe the state of health of the supercapacitors, offline techniques such as Electrochemical Impedance Spectroscopy (EIS) [6] are regularly used to characterize the supercapacitors. But they are not practical in real applications. This paper proposes and compares two online identification methods based on observer theory, to observe the evolution of the supercapacitors parameters. Two kinds of observers, an Extended Kalman Observer (EKO) and Interconnected Observers (IOs), are designed and compared. Identification results are presented for real experimental data obtained with 1F Nichicon supercapacitors.

**Keywords:** supercapacitor, ageing, online identification, extended Kalman observer, interconnected observers.

**Collaborations:** IMN

## 1 Introduction

This paper proposes to design online estimators of the EDLC parameters based on the observer theory, so as to diagnose the EDLC ageing. The supercapacitor parameters, which vary in the ageing process ( $R_s$  and C), will be estimated in real time and in situ during the charging period. It is then possible to inject an additional small current component (a pseudo random binary signal) which will make the information provided by the current and voltage signals rich enough to provide accurate parameter estimations (Fig. 1). By this way, the consumption of the energy stored in the supercapacitor by the user will not be disturbed by the supercapacitor diagnosis task. Once the evolution of the parameters is observed, it will be possible to determine the state of health (for the supercapacitor maintenance) and state of charge of the supercapacitor (to know the amount of energy stored). Some accelerated ageing experiments were carried out on 1F Nichicon Supercapacitors and experimental results are presented and analyzed.

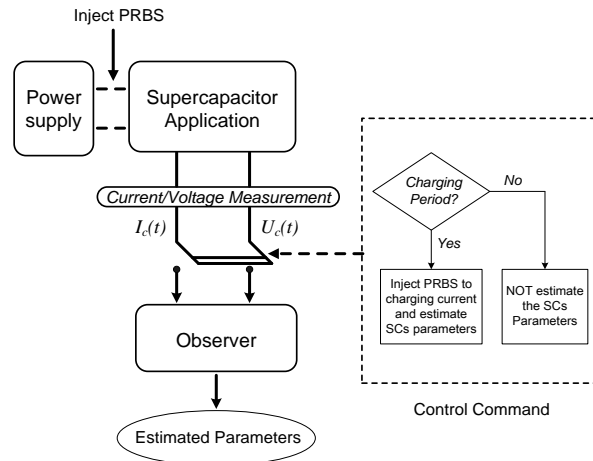


Fig. 1: The structure of the online identification method

## 2 Extended State Space Model of Supercapacitor

A first-order transmission line model (Fig. 2) [7] is used for the supercapacitor online characterization. It consists of three components: the series resistance  $R_s$ , the capacitance C and the resistance  $R_p$  paralleled to the capacitor.  $R_s$  represents the internal heating in the supercapacitor and is responsible for the immediate terminal voltage change phenomenon at the beginning of charging and discharging periods. C is the charge storage capability of a supercapacitor.  $R_p$ , which is often referred to as the leakage current resistance, corresponds to the energy loss due to the capacitor self-discharge.

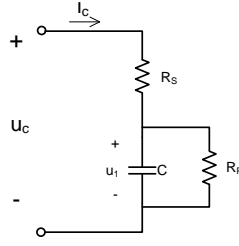


Fig. 2: First-order transmission line model of a supercapacitor

The supercapacitor dynamic behavior is described as:

$$\begin{cases} \frac{du_1}{dt} = -\frac{1}{R_p C} u_1 + \frac{1}{C} i_c \\ u_c = u_1 + R_s i_c \end{cases} \quad (1)$$

where  $u_1$  is the voltage across the capacitor  $C$ ,  $i_c$  and  $u_c$  are respectively the current and voltage of the supercapacitor. The parameters  $R_s$ ,  $R_p$  and  $C$  are slowly varying in the ageing process of a supercapacitor [5] [6]. Thus, they are considered as piecewise constants. Then, the dynamic behavior of the three parameters can be expressed as:

$$\frac{d}{dt}[R_s(t)] = 0, \quad \frac{d}{dt}\left[\frac{1}{C(t)}\right] = 0, \quad \frac{d}{dt}\left[-\frac{1}{R_p(t)C(t)}\right] = 0 \quad (2)$$

Adjoining these three terms into the linear system (1), an extended nonlinear supercapacitor state space model is obtained:

$$\dot{x}_1 = x_1 x_4 + x_3 i_c, \quad \dot{x}_2 = 0, \quad \dot{x}_3 = 0, \quad \dot{x}_4 = 0 \quad \text{and} \quad u_c = x_1 + x_2 i_c \quad (3)$$

where  $x_1 = u_1$ ,  $x_2 = R_s$ ,  $x_3 = \frac{1}{C}$ ,  $x_4 = -\frac{1}{R_p C}$  are the state variables,  $i_c$  is the process input and  $u_c$  is the process output of the extended system. The observability [8] of this nonlinear system is studied. It is complex because it depends on the input and the states values. In some cases, the extended system is unobservable because the rank of the Jacobian matrix is not full. For example, the unobservable case happens when the current signal is constant, a sinusoidal signal or an exponential signal.

### 3 Online Characterization of Supercapacitor Ageing

#### 3.1 Extended Kalman observer design

The extended supercapacitor model (3) is a nonlinear system  $\begin{cases} \dot{x}_a = f(x_a, i_c) \\ u_c = h(x_a, i_c) \end{cases}$ , where  $x_a = [x_1 \ x_2 \ x_3 \ x_4]^T$  is the augmented state vector. The continuous-time extended Kalman observer is based on first-order Taylor approximations of the functions  $f(x_a, i_c)$  and  $h(x_a, i_c)$ :

$$F(\hat{x}_a, i_c) = \frac{\partial f(x_a, i_c)}{\partial x_a} \Big|_{x_a = \hat{x}_a} = \begin{bmatrix} \hat{x}_4 & 0 & i_c & \hat{x}_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad H(i_c) = \frac{\partial h(x_a, i_c)}{\partial x_a} \Big|_{x_a = \hat{x}_a} = [1 \quad i_c \quad 0 \quad 0] \quad (4)$$

Then, the extended Kalman observer (EKO) [9] is usually written as:

$$\begin{cases} \dot{\hat{x}}_a = f(\hat{x}_a, i_c) + K(u_c - \hat{u}_c) & (5.a) \\ \hat{u}_c = h(\hat{x}_a, i_c) & (5.b) \\ K = PH^T R^{-1} & (5.c) \\ P = F(\hat{x}_a, i_c)P + PF^T(\hat{x}_a, i_c) + Q - PH^T(i_c)R^{-1}H(i_c)P & (5.d) \end{cases}$$

where  $\hat{x}_a = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4]^T$  is an estimation of  $x_a$ .  $R \in \mathcal{R}^{1 \times 1}$  is the measurement noise covariance, while  $Q \in \mathcal{R}^{4 \times 4}$  is the process noise covariance. The estimation error covariance  $P \in \mathcal{R}^{4 \times 4}$  is the solution to the Riccati differential equation and  $K$  is the correction gain of the observer.

If an information matrix  $S = RP^{-1}$  is used, then since:  $\dot{S}P + \dot{P} = 0$ , Eq. (5.c) and Eq. (5.d) become:

$$K = S^{-1}H^T \quad (6)$$

$$\dot{S} = -SF(\hat{x}_a, i_c) - F^T(\hat{x}_a, i_c)S - SQR^{-1}S + H^T(i_c)H(i_c) \quad (7)$$

### 3.2 Interconnected Observers Design

The extended supercapacitor model (3) can also be seen as the interconnection of two subsystems  $\Sigma_1$  and  $\Sigma_2$ .

$$\Sigma_1 : \begin{cases} \dot{X}_1 = F_1(X_2, i_c)X_1 \\ u_c = H_1(i_c)X_1 \end{cases}, \quad \Sigma_2 : \begin{cases} \dot{X}_2 = F_2(X_1, i_c)X_2 + g_1(X_1, i_c) \\ u_c = H_2(i_c)X_2 + g_2(X_1, i_c) \end{cases} \quad (8)$$

With

$$X_1 = [x_1 \quad x_2 \quad x_3]^T, \quad X_2 = [x_1 \quad x_4]^T, \quad F_1(X_2, i_c) = \begin{bmatrix} x_4 & 0 & i_c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad F_2(X_1) = \begin{bmatrix} 0 & x_1 \\ 0 & 0 \end{bmatrix},$$

$$H_1(i_c) = [1 \quad i_c \quad 0], \quad H_2 = [1 \quad 0], \quad g_1(X_1, i_c) = [x_3 i_c \quad 0]^T, \quad g_2(X_1, i_c) = x_2 i_c$$

Then, for the interconnected subsystems, interconnected observers (IOs) can be designed. To estimate  $x_2=R_s$  and  $x_3=I/C$ , a Kalman-like observer [10] is designed for the first subsystem  $\Sigma_1$ :

$$O_1 : \begin{cases} \dot{\hat{X}}_1 = F_1(\hat{X}_2, i_c)\hat{X}_1 + S_1^{-1}H_1^T(i_c)(u_c - \hat{u}_c) \\ \dot{S}_1 = -\rho_1 S_1 - F_1^T(\hat{X}_2, i_c)S_1 - S_1 F_1(\hat{X}_2, i_c) + H_1^T(i_c)H_1(i_c) \\ \hat{u}_c = H_1(i_c)\hat{X}_1 \end{cases} \quad (9)$$

with  $\hat{X}_1 = [\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3]^T$  is an estimation of  $X_1$ . The positive constant  $\rho_1$  is the only tuning parameter of the Kalman-like observer.  $S_1$  is a symmetric positive definite matrix.

**Remark:** Kalman-like observer used here is based on the Kalman observer. Kalman-like observer neglects the process noise  $Q$  and adds another term  $-\rho_1 S_1$  in the differential equation of  $S$ .

The second subsystem is a second order system with two variables  $x_1$  and  $x_4$ . It is noticed that one state variable  $x_1$  can be estimated from the measurement:  $x_1 = u_c - x_2 i_c$ . Therefore, the order of the system  $\Sigma_2$  is reduced from two to one. Then, the reduced order system is expressed as:

$$\Sigma_2^r : \begin{cases} \dot{x}_4 = 0 \\ y = \dot{x}_1 = x_4(u_c - x_2 i_c) + x_3 i_c \end{cases} \quad (10)$$

Where,  $x_4 = -\frac{1}{(R_p C)}$  is the only remaining state variable to estimate by the observer and  $y$  is the new output of the reduced order system. Then, a reduced order Luenberger observer for the reduced order system is designed:

$$O_2 : \begin{cases} \dot{\hat{x}}_4 = K(y - \hat{y}) \\ \hat{y} = \hat{x}_4(u_c - \hat{x}_2 i_c) + \hat{x}_3 i_c \end{cases} \quad (11.a)$$

$$(11.b)$$

Where,  $K$  is the observer gain. To eliminate the differential part in Eq. (11.a), a new estimation variable  $\hat{x}_5$  is defined as:  $\hat{x}_5 = \hat{x}_4 - Kx_1$ . Eq. (11.a) can then be rewritten as:

$$\dot{\hat{x}}_5 = -K^2(u_c - \hat{x}_2 i_c)^2 - K(u_c - \hat{x}_2 i_c)\hat{x}_5 - K\hat{x}_3 i_c \quad (12)$$

Therefore, the estimation for state  $x_4$  can be obtained from the following equation:

$$\hat{x}_4 = \hat{x}_5 + K(u_c - \hat{x}_2 i_c) \quad (13)$$

The state  $x_4$  estimation error of the reduced order Luenberger observer and their differential equations are:

$$e(t) = x_4(t) - \hat{x}_4(t), \quad \dot{e}(t) = \dot{x}_4(t) - \dot{\hat{x}}_4(t) = -K(u_c - \hat{x}_2 i_c) e(t) \quad (14)$$

Thus, by tuning the observer gain  $K$ , the observer could obtain a good dynamical behavior.

For EKO, the convergence is not guaranteed while for the IOs, the convergence can be proved by showing that the derivative of the Lyapunov function of the IOs is exponentially convergent. Furthermore, the IOs with reduced order Luenberger sub-observer has a lower computational cost compared to the EKO.

## 4 Experimental results

100 Nichicon 1F/2.7V supercapacitors have been placed in a stove at 60°C to accelerate the ageing process during 12 weeks. Then the current and voltage signals of different aged supercapacitors have been recorded during several charging and discharging cycles, with the help of a multipotentiostat (Biologic VMP3) to estimate the parameters with the observer. In the charging period, the supercapacitors are charged with a constant current (50 mA) with a small additional Pseudo Random Binary Signal (PRBS) added to help the parameter

identification.

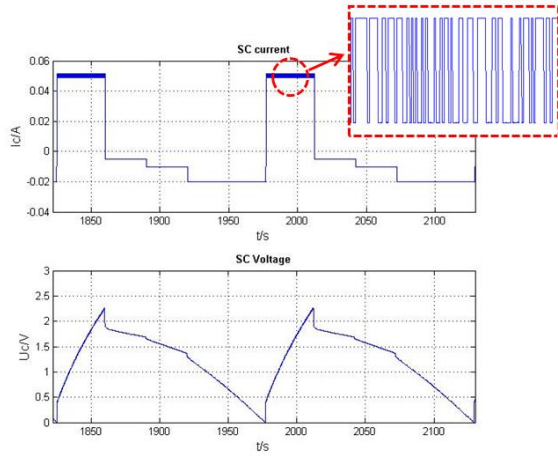


Fig. 3: Current and voltage signals recorded on an EDLC (before accelerated ageing) during two charging/discharging cycles

Table. 1: Parameter estimations provided by IOs at different ageing periods

	Before ageing	3 weeks ageing	6 weeks ageing	9 weeks ageing	12 weeks ageing
$R_s/\Omega$	2.30	3.58	5.16	6.34	6.8
$C/F$	0.835	0.812	0.824	0.814	0.808
$R_p/\Omega$	149.63	129.68	121.30	120.49	117.80

The parameter estimation provided by the interconnected observers at different ageing periods is shown in Table. 1. It is clear that  $R_s$  has a significant increase in the ageing process and  $R_p$  is decreasing with time. The capacitance  $C$  is only slightly decreasing.

## 5 Conclusion

To observe ageing of the supercapacitors online, an extended state space model is built. To perform an online observation of the ageing of a supercapacitor, the nonlinear observability of such a system is analyzed. Two kinds of observers (EKO and IOs) are designed. Compared to EKO, IOs have a lower computational cost and their convergence is proved. 12 weeks long accelerated ageing experiments are carried out on 1F Nichicon supercapacitors. The estimation results of the observers based on the experimental data showed that both observers succeed to estimate the parameters evolution in real time. It is revealed from the result that the series resistance of supercapacitor varies a lot with ageing time while the evolution of the other two parameters is much slower.

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