

## Hub location - routing problem for LTL shipments of freight transport providers

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**Abstract :** The hub location-routing problem (HLRP) considers the location of hub facilities concentrating flows and through which flows are to be routed from origins to destinations, together with the design of both collection and distribution routes associated to each hub. The state of the art includes only very few works directly addressing the HLRP, contrary to the hub location problem (HLP) and the location-routing problem (LRP) which have been the subject of many works. Besides, the published research on the HLRP is focused on particular real cases and there is a need for generic models for this problem. Based on the literature review, the aims of this paper are to present the state of the art, to propose some generic mathematical models for the HLRP (the fixed cost HLRP and the p-HLRP with or without capacity single allocations) and to develop a memetic algorithm that combines a genetic algorithm (GA) with local search to solve it. In this memetic algorithm, the iterative local search methods are to diversify the solutions searched. The computational results based on (Australian post) instances validate the relevancy of this algorithm.

**Keywords:** *Hub location-routing problem, less than truckload shipments, memetic algorithm*

**Collaborations :** Institut de Recherche en Communication et Cybernétique de Nantes

## 1 Introduction

With the increasing pressure for performance of logistics systems in terms of reduction of costs and time for delivery while reducing the shipment loads, less than truckload (LTL) shipments get more and more attention, especially for the goods-transport industry (fresh food, drinks or piece goods) and postal service providers.

In logistic systems of LTL shipments, in order to achieve economies of scale, transportation of goods from one origin to its destination is performed through collection tours to a hub and delivery tours from the same or another hub, instead of undertaking direct shipment. The goods are then shipped between the two hubs using full truckload (FTL) shipments. In order to optimize this LTL shipments network, managers need to determine the location of the hubs, the allocation of non-hub nodes, and the optimal collection and delivery routes within the network. This problem, known as the hub location-routing problem (HLRP), is related to both the hub location problem (HLP) and the location-routing problem (LRP). For an extensive review on the HLP, ones can refer to Alumur and Kara [1] or Campbell and O'Kelly [2]. The goals of the LRP are to determine the locations of the depots, the allocation of customers to depots, and to design the distribution (or collection) routes in a system based on distribution of goods from warehouse to destinations, or collection the other way. Nagy and Salhi [10] presented a survey on issues, models and methods for the LRP. In their classification, they pointed out a particular case of the LRP where inter-facilities routes are considered and both pick-up and delivery routes must be determined: this refers to the many-to-many location-routing problem (MMLRP) that is similar to the HLRP. Different from the LRP which only considers one type of routes, the HLRP considers both collection and distribution routes. The differences among them can be seen in Table 1. From this table, it can be seen that the HLRP consider more decisions. So the objective of the HLRP is to minimize the total costs including hub costs, inter-hub transportation costs, and collection/distribution routing costs. Based on the literature review, the aims of this research are to analyze the state of the art, propose some generic mathematical models for the HLRP and develop a memetic algorithm to solve it effectively.

Table 1. Differences between HRP, LRP and HLRP

Decision Problem	Locate hub	Allocate non-hub nodes	Design vehicle routes	Consider flow between Supplier-Client
HLP	×	×		×
LRP	×	×	×	
HLRP	×	×	×	×

## 2 State of the art

In contrast to the HLP and the LRP which have been the subject of many works and comprise many variants

(see Table 2), only very few works directly address the HLRP. Liu et al. [9] introduced a heuristic algorithm to determine the delivery mode for each supplier-customer pair and to perform vehicle routing within a hub-and-spoke network with only one hub. Therefore, it can be treated as 1-HLRP with direct shipment. In addition, some researchers focused on the partitioning-hub location-routing problem (PHLRP). Gourdin et al. [8] firstly introduced and compared three integer programming (IP) models for this problem. Catanzaro et al. [3] explored possible valid inequalities to strengthen the IP model and introduced a branch-and-cut algorithm to solve the PHLRP that contains 20 vertices. Ta et al. [13] presented a binary integer linear programming model and a new method based on difference of convex functions algorithms to solve larger problems with up to 25 vertices. The hub location-routing problem (HLRP) has been also implemented for the application to postal service networks. Wasner et al. [14] developed a generalized and complex two-layer HLRP model of an Austrian parcel delivery service. This model divided the network into two interrelated design phases: line haul design and pickup/delivery design. Based on this two phases, it proposed a parallel heuristic solution which was proved effective for small and medium size instances. Çetiner et al. [5] developed a two-stage method that includes locating and routing for an uncapacitated/length limited HLRP for the Turkish postal delivery system. In summary, most of researches about the HLRP are modeled for postal systems and don't distinguish collection and delivery routes. Unlike the above particular cases of the HLRP, our research focus on developing and applying generic models for the hub location-routing problem .

Table 2 the recent variants of HLP and LRP

Problems	Research Variants
HLP	The p-hub median problem
	The hub location problem with fixed costs
	The p-hub center problem
	Hub covering problems
LRP	Classical LRP
	Planar LRP
	Set partitioning LRP
	Dynamic LRP

### 3 Mathematical models

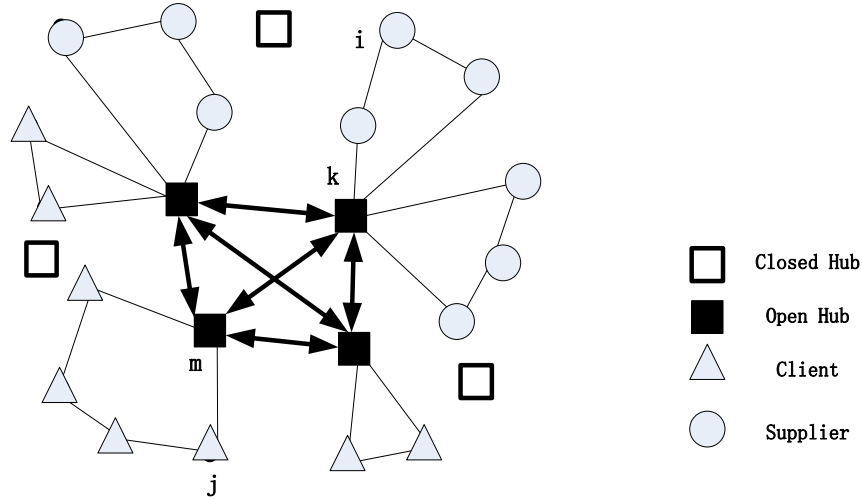


Figure 1 the network of the hub location-routing problem

Based on the above description, the aim of this research is to design an optimal network for LTL shipments through hubs. In our study, this network is defined on a complete graph  $G = (V, E)$  (see Figure 1). The vertex set is  $V = \{H\} \cup \{I\} \cup \{J\}$ , where  $H$  is the set of possible hubs,  $I$  is the set of suppliers (origins) and  $J$  is the set of customers (destinations). Associated with the network, a distance matrix is defined, where its elements  $d_{ij}$  are the distance between two vertices  $(i, j) \in E$ . This distance matrix is assumed to be symmetric and satisfying the triangle inequality. We consider also a demand matrix with elements  $q_{ij}$  being the flow quantity for each supplier-client pair, where  $i, j \in J$ . So the HLRP is to select the hubs and design the optimal routing from supplier  $i$  to client  $j$  with collection and delivery tours beginning and ending the hubs. The problem considers

both the strategic and operational level of the whole network design. At the strategic level, it consists of the determination of the number, location of the hubs, and the collection and delivery area covered by each hub. At the operational level, the vehicle routing problems of the collection and delivery process are handled.

In order to address and solve the HLRP with a generic approach, this research proposes mathematical models based on the following hypothesis: each supplier and client have to be allocated to a single hub and to one collection or delivery route respectively; there is no direct connection between suppliers and clients; each flow can be transported through two hubs at most; all vehicles used for the collection and the delivery are identical and capacitated; each vehicle tour starts and ends at the same open hub and collection/delivery routes are distinguished. Based on different assumptions for the hubs, we have developed several mixed integer programming models for the fixed cost HLRP and the p-HLRP with or without hub capacity and with single supplier/client allocations. In these models, there are three main variables to determine: flow variables, hub location variables and routing variables. And the objective is to minimize the total cost including the fixed hub and vehicle costs, the transportation costs between the hubs and the costs of collection/delivery tours.

## 4 Memetic algorithm and computational experiments

To prove the validity of models, some preliminary experiments have been conducted with the fixed-cost capacitated version of the HLRP models using CPLEX. The obtained results based on small instances prove the effectiveness of our approach. In order to solve larger problems, a memetic method based on Genetic Algorithm (GA) is proposed. GA has been successfully used to solve the HLPs and the LRP in the past years [4, 11, and 12]. Our algorithm (see Algorithm 1) is inspired from Derbel et al. [6]. It is coded with natural number for each chromosome which includes two parts representing location/allocation decisions and collection/delivery routing decisions respectively. The location section  $L(x)$  represents an assignment configuration that gives the set of open depots and the nodes affected to each of them. The second routing section  $R(x)$  is a permutation vector fixing the rank of node on a given route. Let  $L(x) = \{l_1, l_2, l_3, \dots, l_n\}$  and  $R(x) = \{r_1, r_2, r_3, \dots, r_n\}$ , where  $n$  is the number of non-hub nodes,  $l_i$  is the index of open hub and  $r_i$  is the index of suppliers and clients. Then for each  $i \in \{1, 2, \dots, n\}$ , the allocation solution is  $r_i \in l_i$ , that it means that non-hub node  $r_i$  is assigned to hub  $l_i$ . And the routes can be deduced from  $R(x)$  and  $L(x)$  according to the order of non-hub nodes in  $R(x)$ . To illustrate the representation of a solution, an example is considered in Figure 2. In this example, 3 possible hubs, 6 suppliers and 6 clients are considered. To distinguish the three kinds of set, we use a series of integers in ascending order to express them (see Figure 2). So  $H = \{1, 2, 3\}$  is the set of potential hubs;  $I = \{4, 5, 6, 7, 8, 9\}$  is the set of suppliers and  $J = \{10, 11, 12, 13, 14, 15\}$  is the set of clients. Because of collection and delivery distinguished processes, the routing section will be divided into two parts to produce routes. For example, since suppliers 4, 7 and 6 are allocated to hub 1, the route serving these suppliers is obtained based on the permutation order in  $R(x)$ . More precisely, if we don't consider the capacity of the vehicle and the hub, supplier 4 is followed by supplier 7 then supplier 6 in  $R(x)$  meaning that this collection route serving those suppliers from hub 1 is  $\{1 - 4 - 7 - 6 - 1\}$ . Similarly, the delivery route from hub 1 is  $\{1 - 14 - 13 - 12 - 1\}$ .

$P(x)$  :

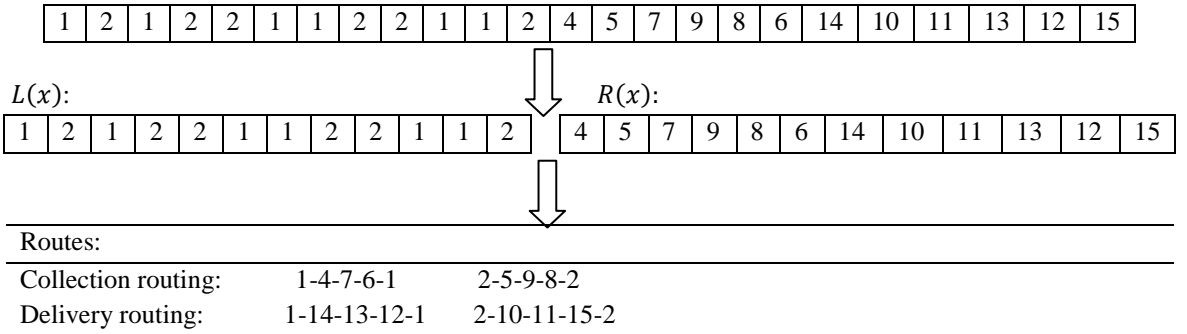


Figure 2 An example of HLRP solution representation

Firstly, our GA starts with a population of initial random solutions, and then it selects the best individuals from each generation based on fitness including a penalty component, and applies crossover and mutation operators to produce next generation. Most importantly, to overcome the weaknesses of the GA heuristics, some local search modules are applied to each new generated individual to improve the assignment and routing solutions. Computational experiments have been conducted based on some randomly generated data sets and Australian Post data sets [7]. The results are compared with ones obtained by CPLEX and show that memetic algorithm can solve larger problems while obtaining good solutions compared to the CPLEX bounds.

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**Algorithm 1: Memetic Algorithm for Hub Location-Routing Problem**

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1 set the global parameters;
2 let evaluation function  $F_{eval} \leftarrow \infty$ ;
3 Initialize first generation of individuals randomly, define it as  $Pop$ ;
4 Initialize the best fitness  $F_{evalbest} = \min F_{eval}(x), x \in (1 \dots Pop)$ ;
5 While termination criterion is not satisfied Do
6     Select individuals  $x_1$  and  $x_2$  from  $Pop$  following probability;
7     Apply Crossover to  $x_1$  and  $x_2$  to obtain a new child  $x_{new}$ ;
8     Apply Mutation to  $x_{new}$ ;
9     if  $F_{eval}(x_{new}) < F_{evalbest} (1 + \delta)$  then
10         Apply local search with  $x_{new}$  to obtain improve solution;
11         Update the best fitness  $F_{evalbest}$ ;
12 Apply Replacement to  $Pop$  to produce new generation.
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## 5 Conclusion and prospects

In this paper, we presented the hub location-routing problem (HLRP) for LTL shipments in a transportation network. We analyzed the state of the art and observed that there is very little published research on this practical network design problem. Then in order to solve it, we have proposed some general mathematic models based on different variants of the problem. We have then developed a memetic method that combines a GA with local search to solve the fixed-cost capacitated single allocation HLR. Computational results based on randomly generated and Australian Post instances prove the efficiency of our memetic algorithm. This algorithm can be adapted to solve other variants of the problem. In the future, we plan to use his algorithm to solve larger and real-world problems. Some more factors will also be considered, for example time factor and environmental factors.

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