pspipe notes : pol angle

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1   Angle

When polarisation angles are rotated uniformly over the sky by an angle ϕ_α, spherical harmonics coefficients observed by the detector array α are related to the intrinsic ones by

$$
\begin{aligned}
a_{\ell m}^{E,\alpha} &= a_{\ell m}^E \cos 2\phi_\alpha - a_{\ell m}^B \sin 2\phi_\alpha & (1) \\
a_{\ell m}^{B,\alpha} &= a_{\ell m}^E \sin 2\phi_\alpha + a_{\ell m}^B \cos 2\phi_\alpha & (2)
\end{aligned}
$$

The EB power spectrum between detector array α and detector array β is given by

$$
\begin{aligned}
C_\ell^{E_\alpha B_\beta} &= C_\ell^{EE} \cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB} \sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB}(\cos 2\phi_\alpha \cos 2\phi_\beta - \sin 2\phi_\alpha \sin 2\phi_\beta) \\
&= C_\ell^{EE} \cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB} \sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB} \cos 2(\phi_\alpha + \phi_\beta) & (3)
\end{aligned}
$$

Similarly

$$
C_\ell^{B_\alpha E_\beta} = C_\ell^{EE} \sin 2\phi_\alpha \cos 2\phi_\beta - C_\ell^{BB} \cos 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{EB} \cos 2(\phi_\alpha + \phi_\beta) \tag{4}
$$

Assuming the angle are small and the native C_ℓ^{EB} is small we get

$$
\begin{aligned}
C_\ell^{E_\alpha B_\beta} &\sim 2\phi_\beta C_\ell^{EE} & (5) \\
C_\ell^{B_\alpha E_\beta} &\sim 2\phi_\alpha C_\ell^{EE} & (6)
\end{aligned}
$$

Similarly for TB we have

$$
\begin{aligned}
C_\ell^{T_\alpha B_\beta} &= C_\ell^{TE} \sin 2\phi_\beta + C_\ell^{TB} \cos 2\phi_\beta & (7) \\
C_\ell^{B_\alpha T_\beta} &= C_\ell^{TE} \sin 2\phi_\alpha + C_\ell^{TB} \cos 2\phi_\alpha & (8)
\end{aligned}
$$

$$
\begin{aligned}
C_\ell^{T_\alpha B_\beta} &\sim 2\phi_\beta C_\ell^{TE} & (9) \\
C_\ell^{B_\alpha T_\beta} &\sim 2\phi_\alpha C_\ell^{TE} & (10)
\end{aligned}
$$