pspipe notes : pol angle

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9 octobre 2023

1 Angle

When polarisation angles are rotated uniformly over the sky by an angle ϕ_α, spherical harmonics coefficients observed by the detector array α are related to the intrinsic ones by

$$\begin{aligned} a^{E,\alpha}_{\ell m} &= a^{E}_{\ell m}\cos 2\phi_\alpha - a^{B}_{\ell m}\sin 2\phi_\alpha && (1)\\ a^{B,\alpha}_{\ell m} &= a^{E}_{\ell m}\sin 2\phi_\alpha + a^{B}_{\ell m}\cos 2\phi_\alpha && (2)\end{aligned}$$

The EB power spectrum between detector array α and detector array β is given by

$$\begin{aligned} C^{E_\alpha B_\beta}_\ell &= C^{EE}_\ell \cos 2\phi_\alpha \sin 2\phi_\beta - C^{BB}_\ell \sin 2\phi_\alpha \cos 2\phi_\beta + C^{EB}_\ell (\cos 2\phi_\alpha \cos 2\phi_\beta - \sin 2\phi_\alpha \sin 2\phi_\beta)\\ &= C^{EE}_\ell \cos 2\phi_\alpha \sin 2\phi_\beta - C^{BB}_\ell \sin 2\phi_\alpha \cos 2\phi_\beta + C^{EB}_\ell \cos 2(\phi_\alpha + \phi_\beta) && (3)\end{aligned}$$

Similarly

$$C^{B_\alpha E_\beta}_\ell = C^{EE}_\ell \sin 2\phi_\alpha \cos 2\phi_\beta - C^{BB}_\ell \cos 2\phi_\alpha \sin 2\phi_\beta + C^{EB}_\ell \cos 2(\phi_\alpha + \phi_\beta) \quad (4)$$

Assuming the angle are small and the native C^{EB}_ℓ is small we get

$$\begin{aligned} C^{E_\alpha B_\beta}_\ell &\sim 2\phi_\beta C^{EE}_\ell && (5)\\ C^{B_\alpha E_\beta}_\ell &\sim 2\phi_\alpha C^{EE}_\ell && (6)\end{aligned}$$

Similarly for TB we have

$$\begin{aligned} C^{T_\alpha B_\beta}_\ell &= C^{TE}_\ell \sin 2\phi_\beta + C^{TB}_\ell \cos 2\phi_\beta && (7)\\ C^{B_\alpha T_\beta}_\ell &= C^{TE}_\ell \sin 2\phi_\alpha + C^{TB}_\ell \cos 2\phi_\alpha && (8)\end{aligned}$$

$$\begin{aligned} C^{T_\alpha B_\beta}_\ell &\sim 2\phi_\beta C^{TE}_\ell && (9)\\ C^{B_\alpha T_\beta}_\ell &\sim 2\phi_\alpha C^{TE}_\ell && (10)\end{aligned}$$

For completness the effect of TE, EE and BB is given by :

$$\begin{aligned} C^{T_\alpha E_\beta}_\ell &= C^{TE}_\ell \cos 2\phi_\beta - C^{TB}_\ell \sin 2\phi_\beta && (11)\\ C^{E_\alpha T_\beta}_\ell &= C^{TE}_\ell \cos 2\phi_\alpha - C^{TB}_\ell \sin 2\phi_\alpha && (12)\end{aligned}$$

$$\begin{aligned} C^{E_\alpha E_\beta}_\ell &= C^{EE}_\ell \cos 2\phi_\alpha \cos 2\phi_\beta + C^{BB}_\ell \sin 2\phi_\alpha \sin 2\phi_\beta - C^{EB}_\ell (\sin 2\phi_\beta \cos 2\phi_\alpha + \cos 2\phi_\beta \sin 2\phi_\alpha)\\ &= C^{EE}_\ell \cos 2\phi_\alpha \cos 2\phi_\beta + C^{BB}_\ell \sin 2\phi_\alpha \sin 2\phi_\beta - C^{EB}_\ell \sin 2(\phi_\beta + \phi_\alpha) && (13)\end{aligned}$$

$$\begin{aligned} C^{B_\alpha B_\beta}_\ell &= C^{EE}_\ell \sin 2\phi_\alpha \sin 2\phi_\beta + C^{BB}_\ell \cos 2\phi_\alpha \cos 2\phi_\beta + C^{EB}_\ell (\sin 2\phi_\alpha \cos 2\phi_\beta + \cos 2\phi_\alpha \sin 2\phi_\beta)\neg\dagger\\ &= C^{EE}_\ell \sin 2\phi_\alpha \sin 2\phi_\beta + C^{BB}_\ell \cos 2\phi_\alpha \cos 2\phi_\beta + C^{EB}_\ell \sin 2(\phi_\beta + \phi_\alpha) && (14)\end{aligned}$$