pspipe notes : pol angle

Louis Thibaut

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1 Angle

When polarisation angles are rotated uniformly over the sky by an angle ϕ_α, spherical harmonics coefficients observed by the detector array α are related to the intrinsic ones by

$$a^{E,\alpha}_{\ell m} = a^{E}_{\ell m}\cos 2\phi_\alpha - a^{B}_{\ell m}\sin 2\phi_\alpha \tag{1}$$
$$a^{B,\alpha}_{\ell m} = a^{E}_{\ell m}\sin 2\phi_\alpha + a^{B}_{\ell m}\cos 2\phi_\alpha \tag{2}$$

The EB power spectrum between detector array α and detector array β is given by

$$\begin{aligned} C_\ell^{E_\alpha B_\beta} &= C_\ell^{EE}\cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB}\sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB}(\cos 2\phi_\alpha\cos 2\phi_\beta - \sin 2\phi_\alpha \sin 2\phi_\beta) \\ &= C_\ell^{EE}\cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB}\sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB}\cos 2(\phi_\alpha + \phi_\beta) \end{aligned} \tag{3}$$

Similarly

$$C_\ell^{B_\alpha E_\beta} = C_\ell^{EE}\sin 2\phi_\alpha \cos 2\phi_\beta - C_\ell^{BB}\cos 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{EB}\cos 2(\phi_\alpha + \phi_\beta) \tag{4}$$

Assuming the angle are small and the native C_ℓ^{EB} is small we get

$$C_\ell^{E_\alpha B_\beta} \sim 2\phi_\beta C_\ell^{EE} \tag{5}$$
$$C_\ell^{B_\alpha E_\beta} \sim 2\phi_\alpha C_\ell^{EE} \tag{6}$$

Similarly for TB we have

$$C_\ell^{T_\alpha B_\beta} = C_\ell^{TE}\sin 2\phi_\beta + C_\ell^{TB}\cos 2\phi_\beta \tag{7}$$
$$C_\ell^{B_\alpha T_\beta} = C_\ell^{TE}\sin 2\phi_\alpha + C_\ell^{TB}\cos 2\phi_\alpha \tag{8}$$

$$C_\ell^{T_\alpha B_\beta} \sim 2\phi_\beta C_\ell^{TE} \tag{9}$$
$$C_\ell^{B_\alpha T_\beta} \sim 2\phi_\alpha C_\ell^{TE} \tag{10}$$

For completness the effect of TE, EE and BB is given by :

$$C_\ell^{T_\alpha E_\beta} = C_\ell^{TE}\cos 2\phi_\beta - C_\ell^{TB}\sin 2\phi_\beta \tag{11}$$
$$C_\ell^{E_\alpha T_\beta} = C_\ell^{TE}\cos 2\phi_\alpha - C_\ell^{TB}\sin 2\phi_\alpha \tag{12}$$

$$\begin{aligned} C_\ell^{E_\alpha E_\beta} &= C_\ell^{EE}\cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{BB}\sin 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{EB}(\sin 2\phi_\beta\cos 2\phi_\alpha + \cos 2\phi_\beta \sin 2\phi_\alpha) \\ &= C_\ell^{EE}\cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{BB}\sin 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{EB}\sin 2(\phi_\beta + \phi_\alpha) \end{aligned} \tag{13}$$

$$\begin{aligned} C_\ell^{B_\alpha B_\beta} &= C_\ell^{EE}\sin 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{BB}\cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB}(\sin 2\phi_\alpha\cos 2\phi_\beta + \cos 2\phi_\alpha \sin 2\phi_\beta) \\ &= C_\ell^{EE}\sin 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{BB}\cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB}\sin 2(\phi_\beta + \phi_\alpha) \end{aligned} \tag{14}$$

2 Likelihood

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\begin{aligned}
C_b^{\alpha,EB} = \tilde{C}_b^{\alpha,EB} && (15)\\
\ln \mathcal{L}(C_b^{\alpha,EB}|\phi_\alpha) &\propto \left(C_b^{\alpha,EB} - C_b^{\mathrm{th,EB}}(\phi_\alpha)\right) \Sigma_{bb'}^{-1} \left(C_{b'}^{\alpha,EB} - C_{b'}^{\mathrm{th,EB}}(\phi_\alpha)\right) && (16)\\
\Sigma_{bb'} &= [\Sigma_{\mathrm{analytic}}]_{bb'}^{\mathrm{MC}} + \Sigma_{bb'}^{\mathrm{beam}} + \Sigma_{bb'}^{\mathrm{leakage\ beam}} && (17)
\end{aligned}
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