

# pspipe notes : pol angle

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## 1 Angle

When polarisation angles are rotated uniformly over the sky by an angle  $\phi_\alpha$ , spherical harmonics coefficients observed by the detector array  $\alpha$  are related to the intrinsic ones by

$$a_{\ell m}^{E,\alpha} = a_{\ell m}^E \cos 2\phi_\alpha - a_{\ell m}^B \sin 2\phi_\alpha \quad (1)$$

$$a_{\ell m}^{B,\alpha} = a_{\ell m}^E \sin 2\phi_\alpha + a_{\ell m}^B \cos 2\phi_\alpha \quad (2)$$

The EB power spectrum between detector array  $\alpha$  and detector array  $\beta$  is given by

$$\begin{aligned} C_\ell^{E_\alpha B_\beta} &= C_\ell^{EE} \cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB} \sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB} (\cos 2\phi_\alpha \cos 2\phi_\beta - \sin 2\phi_\alpha \sin 2\phi_\beta) \\ &= C_\ell^{EE} \cos 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{BB} \sin 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB} \cos 2(\phi_\alpha + \phi_\beta) \end{aligned} \quad (3)$$

Similarly

$$C_\ell^{B_\alpha E_\beta} = C_\ell^{EE} \sin 2\phi_\alpha \cos 2\phi_\beta - C_\ell^{BB} \cos 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{EB} \cos 2(\phi_\alpha + \phi_\beta) \quad (4)$$

Assuming the angle are small and the native  $C_\ell^{EB}$  is small we get

$$C_\ell^{E_\alpha B_\beta} \sim 2\phi_\beta C_\ell^{EE} \quad (5)$$

$$C_\ell^{B_\alpha E_\beta} \sim 2\phi_\alpha C_\ell^{EE} \quad (6)$$

Similarly for TB we have

$$C_\ell^{T_\alpha B_\beta} = C_\ell^{TE} \sin 2\phi_\beta + C_\ell^{TB} \cos 2\phi_\beta \quad (7)$$

$$C_\ell^{B_\alpha T_\beta} = C_\ell^{TE} \sin 2\phi_\alpha + C_\ell^{TB} \cos 2\phi_\alpha \quad (8)$$

$$C_\ell^{T_\alpha B_\beta} \sim 2\phi_\beta C_\ell^{TE} \quad (9)$$

$$C_\ell^{B_\alpha T_\beta} \sim 2\phi_\alpha C_\ell^{TE} \quad (10)$$

For completeness the effect of TE, EE and BB is given by :

$$C_\ell^{T_\alpha E_\beta} = C_\ell^{TE} \cos 2\phi_\beta - C_\ell^{TB} \sin 2\phi_\beta \quad (11)$$

$$C_\ell^{E_\alpha T_\beta} = C_\ell^{TE} \cos 2\phi_\alpha - C_\ell^{TB} \sin 2\phi_\alpha \quad (12)$$

$$\begin{aligned} C_\ell^{E_\alpha E_\beta} &= C_\ell^{EE} \cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{BB} \sin 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{EB} (\sin 2\phi_\beta \cos 2\phi_\alpha + \cos 2\phi_\beta \sin 2\phi_\alpha) \\ &= C_\ell^{EE} \cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{BB} \sin 2\phi_\alpha \sin 2\phi_\beta - C_\ell^{EB} \sin 2(\phi_\beta + \phi_\alpha) \end{aligned} \quad (13)$$

$$\begin{aligned} C_\ell^{B_\alpha B_\beta} &= C_\ell^{EE} \sin 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{BB} \cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB} (\sin 2\phi_\alpha \cos 2\phi_\beta + \cos 2\phi_\alpha \sin 2\phi_\beta) \\ &= C_\ell^{EE} \sin 2\phi_\alpha \sin 2\phi_\beta + C_\ell^{BB} \cos 2\phi_\alpha \cos 2\phi_\beta + C_\ell^{EB} \sin 2(\phi_\beta + \phi_\alpha) \end{aligned} \quad (14)$$