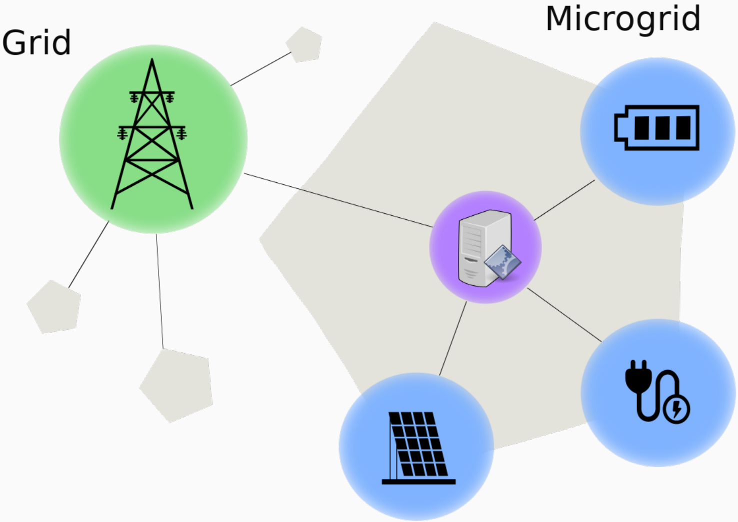
Stochastic Optimization for Energy Management Systems

Methods, results and perspectives

Adrien Le Franc, Michel De Lara

CERMICS ENPC x Efficacity

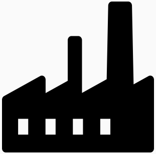

Grid
Microgrid

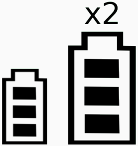
Schneider provides a set of realistic microgrid configurations

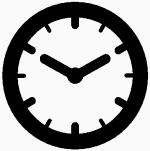
11 sites
to manage
separately

2 settings
of battery
per site

$\sim$ 10 periods
of 10 days
per site







Our achievement

We write stochastic optimization problems
for managing microgrids at least expected cost
and test our solutions on Schneider's data

Outline of the presentation

1. A stochastic optimization problem for microgrid management
2. Resolution with Stochastic Dynamic Programming (SDP)
3. Modeling stagewise dependency of the noise process
4. Numerical results
5. Conclusion and perspectives

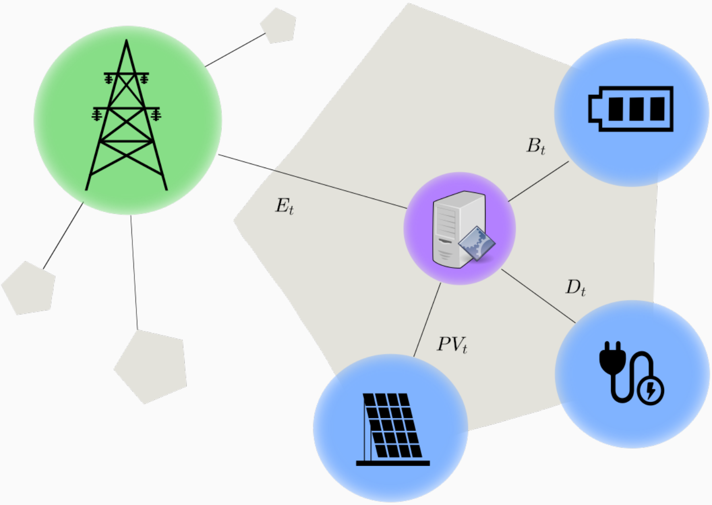
Outline of the presentation

1. **A stochastic optimization problem for microgrid management**
2. Resolution with Stochastic Dynamic Programming (SDP)
3. Modeling stagewise dependency of the noise process
4. Numerical results
5. Conclusion and perspectives

What we will cover in this section

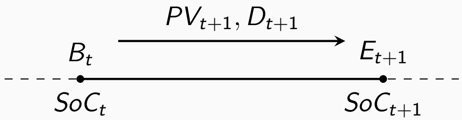
Outline of the section

Energy flows in the microgrid



Decision chronology

- SoC_t is the battery state at time t
- decisions B_t are taken at the beginning of $[t, t + \Delta_t]$
- PV_{t+1}, D_{t+1} are observed at the end of $[t, t + \Delta_t]$
- eventually energy E_{t+1} is imported from the grid



EMS deterministic (anticipative) optimization problem

$$
\begin{aligned}
\min_{B_0 \dots B_{T-1}} \quad & \sum_{t=0}^{T-1} p_{buy,t} E_{t+1}^{+} - p_{sell,t} E_{t+1}^{-} \\
& E_{t+1} = {\color{red} D_{t+1}} - {\color{red} PV_{t+1}} + B_t \\
& SoC_0 = soc_0 \\
& SoC_{t+1} = SoC_t + \frac{\rho_c}{c} B_t^{+} - \frac{1}{c \rho_d} B_t^{-} \\
& 0 \leq SoC_t \leq 1 \\
& - p\Delta_t \leq B_t \leq p\Delta_t
\end{aligned}
$$

In practice the outcome of (D_t, PV_t) is **uncertain**

Outline of the section

We model uncertainties as random processes

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space

Uncertainties are modeled as random processes

$$\mathbf{D} : \omega \mapsto (\mathbf{D}_1(\omega), ..., \mathbf{D}_T(\omega))$$
$$\mathbf{PV} : \omega \mapsto (\mathbf{PV}_1(\omega), ..., \mathbf{PV}_T(\omega))$$

At intermediate stages $1 \leq t < T$ the decision maker observes the **history process**

$$\mathbf{H}_t = (\mathbf{D}_1, \mathbf{PV}_1, ..., \mathbf{D}_t, \mathbf{PV}_t)$$

Non-anticipativity constraint

Decisions must be **non-anticipative**

i.e. for identical history observations h_t we must take the same decision

$$\mathbf{B}_t \text{ is measurable w.r.t. } \sigma(\mathbf{H}_t)$$

This can be formulated in functional form with **policies**

$$\mathbf{B}_t = \gamma_t(\mathbf{H}_t)$$

We can restrict policies γ_t to special cases e.g. **state feedbacks**

$$\mathbf{B}_t = \pi_t(\mathbf{SoC}_t)$$

Outline of the section

EMS stochastic optimization problem

$$
\begin{aligned}
\min_{\pi_0 \dots \pi_{T-1}} \quad & \mathbb{E}\Big[\sum_{t=0}^{T-1} p_{buy,t} \mathbf{E}^{+}_{t+1} - p_{sell,t} \mathbf{E}^{-}_{t+1} \Big] \\
& \mathbf{E}_{t+1} = \mathbf{D}_{t+1} - \mathbf{PV}_{t+1} + \mathbf{B}_t \\
& \mathbf{SoC}_0 = soc_0 \\
& \mathbf{SoC}_{t+1} = \mathbf{SoC}_t + \frac{\rho_c}{c} \mathbf{B}^{+}_t - \frac{1}{\rho_d c} \mathbf{B}^{-}_t \\
& 0 \leq \mathbf{SoC}_t \leq 1 \\
& -p\Delta_t \leq \mathbf{B}_t \leq p\Delta_t \\
& \mathbf{B}_t = \pi_t(\mathbf{SoC}_t)
\end{aligned}
$$

Compact formulation

- **State**

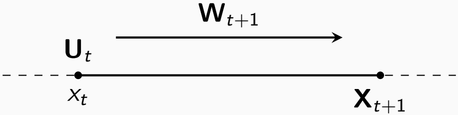
$$\mathbf{X}_t = \mathbf{SoC}_t$$

- **Noise**

$$\mathbf{W}_t = \mathbf{D}_t - \mathbf{PV}_t$$

- **Control**

$$\mathbf{U}_t = \mathbf{B}_t$$



Stochastic optimization problem in generic form

$$
\begin{aligned}
\min_{\pi_0 \dots \pi_{T-1}} \quad & \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) \Big] \\
& \mathbf{X}_0 = x_0 \\
& \mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) \\
& \mathbf{U}_t \in \mathcal{U}_t^{\mathrm{ad}}(\mathbf{X}_t) \\
& \mathbf{U}_t = \pi_t(\mathbf{X}_t)
\end{aligned}
$$

$$\dim \mathbb{X} = 1 \quad \dim \mathbb{U} = 1 \quad \dim \mathbb{W} = 1$$

Outline of the presentation

1. A stochastic optimization problem for microgrid management
2. Resolution with Stochastic Dynamic Programming (SDP)
3. Modeling stagewise dependency of the noise process
4. Numerical results
5. Conclusion and perspectives

What we will cover in this section

Outline of the section

Value functions

Value functions are defined as the cost-to-go at time step $t \in \{1, ..., T\}$ and state $x \in \mathbb{X}_t$

$$
\begin{aligned}
V_t(x) = \min_{\pi_t \dots \pi_{T-1}} \quad & \mathbb{E}\Big[\sum_{s=t}^{T-1} L_s(\mathbf{X}_s, \mathbf{U}_s, \mathbf{W}_{s+1})\Big] \\
& \mathbf{X}_t = x \\
& \mathbf{X}_{s+1} = f_s(\mathbf{X}_s, \mathbf{U}_s, \mathbf{W}_{s+1}) \\
& \mathbf{U}_s \in \mathcal{U}_s^{\mathrm{ad}}(\mathbf{X}_s) \\
& \mathbf{U}_s = \pi_s(\mathbf{X}_s)
\end{aligned}
$$

Bellman equations

Assuming uncertainties $(\mathbf{W}_1, ..., \mathbf{W}_T)$ are **stagewise independent**
V_t and V_{t+1} are connected by the **Bellman equations**

$$V_t(x) = \min_u \ \mathbb{E}_{\mathbf{W}_{t+1}}\Big[L_t(x, u, \mathbf{W}_{t+1}) + V_{t+1}\big(f_t(x, u, \mathbf{W}_{\mathbf{t+1}})\big)\Big]$$

We set $V_T(x) = K(x_T)$ and compute backward $\big(V_t\big)_{0 \le t \le T-1}$

Outline of the section

Discrete spaces for computing numerical solutions

We consider discrete stationary spaces $\mathbb{X}$, $\mathbb{U}$ and discrete fixed cardinal noise spaces $\mathbb{W}_1, \dots, \mathbb{W}_T$

- complexity to compute value functions $\big(V_t(x)\big)_{\substack{0 \le t \le T-1 \\ x \in \mathbb{X}}}$

$$\mathcal{O}\big(T \times |\mathbb{X}| \times |\mathbb{W}| \times |\mathbb{U}|\big)$$

- complexity to compute the online decision $\pi_t(x_t)$

$$\mathcal{O}\big(|\mathbb{W}| \times |\mathbb{U}|\big)$$

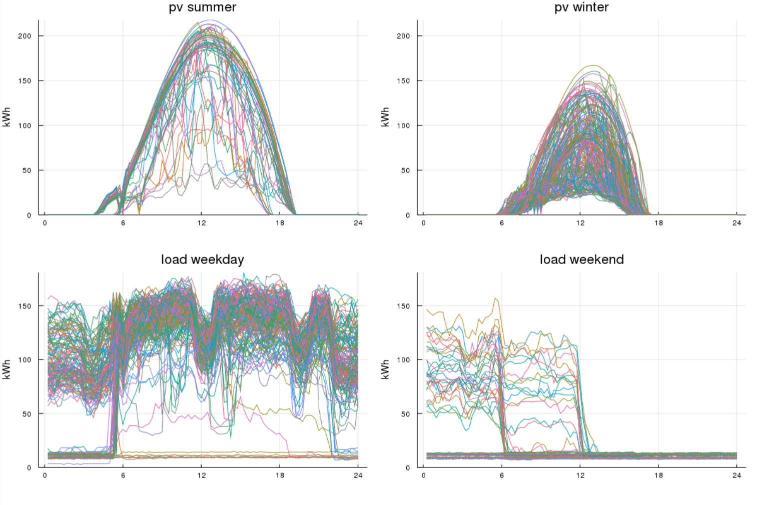
Computing value functions offline

$$V_t(x) = \min_u \int_{\mathbb{W}_{t+1}} \Big[L_t(x, u, w_{t+1}) + V_{t+1}\Big(f_t(x, u, w_{t+1}) \Big) \Big] \mu^{off}_{\mathbf{W}_{t+1}}(dw_{t+1})$$

- We have a collection of observations $\left(w^1_{t+1}, \ldots, w^{N_{t+1}}_{t+1}\right)$ from calibration data
- We use Kmeans for optimal quantization of the noise in 10 values

$$\hat{\mu}_{\mathbf{W}^{off}_{t+1}} = \sum_{k=1}^{10} P^k_t \delta_{c^k_t}$$

Partitioning the calibration data for computing $\hat{\mu}^{off}_{\mathbf{W}_t}$



Computing policies online

$$\pi_t(x_t) \in \arg\min_u \int_{\mathbb{W}_{t+1}} \Big[L_t(x_t, u, w_{t+1}) + V_{t+1}\Big(f_t(x_t, u, w_{t+1}) \Big) \Big] \mu^{on}_{\mathbf{W}_{t+1}}(dw_{t+1})$$

- We have forecasts $\widetilde{w}_t$ about $\mathbf{W}_{t+1}$ available online

$$\hat{\mu}_{\mathbf{W}^{on}_{t+1}} = \delta_{\widetilde{w}_t}$$

- We use B-spline interpolation for computing $V_{t+1}\Big(f_t(x_t, u, w_{t+1}) \Big)$ from $\big(V_{t+1}(x) \big)_{x \in \mathbb{X}}$

Outline of the presentation

What we will cover in this section

Outline of the section

We introduce stagewise dependency with an AR-1 process

We model the noise as an AR-1 process $\mathbf{W}_{t+1} = \alpha_t \mathbf{W}_t + \beta_t + \boldsymbol{\varepsilon}_{t+1}$
We assume **stagewise independence** of the error $(\varepsilon_1, ..., \varepsilon_T)$

$$\min_{\hat{\pi}_0 \dots \hat{\pi}_{T-1}} \quad \mathbb{E}_{\boldsymbol{\varepsilon}} \Big[\sum_{t=0}^{T-1} p_{buy,t} \big[\mathbf{U}_t + \mathbf{W}_{t+1}\big]^+ - p_{sell,t} \big[\mathbf{U}_t + \mathbf{W}_{t+1}\big]^- \Big]$$

$$\mathbf{X}_0 = x_0$$

$$\mathbf{X}_{t+1} = \mathbf{X}_t + \frac{\rho_c}{c} \mathbf{U}_t^+ - \frac{1}{c \rho_d} \mathbf{U}_t^-$$

$$\mathbf{W}_0 = w_0$$

$$\mathbf{W}_{t+1} = \alpha_t \mathbf{W}_t + \beta_t + \boldsymbol{\varepsilon}_{t+1}$$

$$\mathbf{U}_t \in \mathcal{U}_t^{\mathrm{ad}}(\mathbf{X}_t)$$

$$\mathbf{U}_t = \hat{\pi}_t(\mathbf{X}_t, \mathbf{W}_t)$$

Compact formulation

- **State**

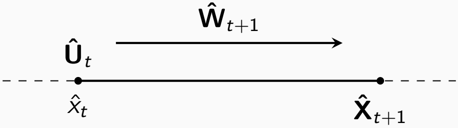
$$\hat{\mathbf{X}}_t = (\mathbf{SoC}_t, \mathbf{D}_t - \mathbf{PV}_t)$$

- **Noise**

$$\hat{\mathbf{W}}_t = \varepsilon_t$$

- **Control**

$$\hat{\mathbf{U}}_t = \mathbf{B}_t$$



We augmented the state dimension

$$
\begin{aligned}
\min_{\hat{\pi}_0 \ldots \hat{\pi}_{T-1}} \quad & \mathbb{E}\Big[\sum_{t=0}^{T-1} \hat{L}_t(\hat{\mathbf{X}}_t, \hat{\mathbf{U}}_t, \hat{\mathbf{W}}_{t+1})\Big] \\
& \hat{\mathbf{X}}_0 = \hat{x}_0 \\
& \hat{\mathbf{X}}_{t+1} = \hat{f}_t(\hat{\mathbf{X}}_t, \hat{\mathbf{U}}_t, \hat{\mathbf{W}}_{t+1}) \\
& \hat{\mathbf{U}}_t \in \mathcal{U}_t^{\mathrm{ad}}(\hat{\mathbf{X}}_t) \\
& \hat{\mathbf{U}}_t = \hat{\pi}_t(\hat{\mathbf{X}}_t)
\end{aligned}
$$

$\dim \hat{\mathbb{X}} = 2 \quad \dim \hat{\mathbb{U}} = 1 \quad \dim \hat{\mathbb{W}} = 1$

Outline of the section

3. Modeling stagewise dependency of the noise process

3.1 Modeling noise with an AR-1 process

3.2 Numerical implementation for the EMS problem

Linear regression for AR-1 calibration

$$\mathbf{W}_{t+1} = \alpha_t \mathbf{W}_t + \beta_t + \varepsilon_{t+1}$$

- We have collections of observations $\left(w_t^1, ..., w_t^N\right)_{1 \leq t \leq T}$ from calibration data
- We use linear regression to fit optimal weights

$$(\alpha_t, \beta_t) \in \underset{(\alpha,\beta)}{\arg\min} \sum_{i=1}^{N} ||w_{t+1}^i - \alpha w_t^i - \beta||^2$$

Computing offline and online laws

$$\mathbf{W}_{t+1} = \alpha_t \mathbf{W}_t + \beta_t + \boldsymbol{\varepsilon}_{t+1}$$

- We compute offline laws using Kmeans from calibration data

$$\hat{\mu}_{\boldsymbol{\varepsilon}_{t+1}^{off}} = \sum_{k=1}^{10} P_t^k \delta_{\mathbf{c}_t^k}$$

- We have forecasts $\widetilde{w}_t$ about $\mathbf{W}_{t+1}$ available online and deduce a forcast $\widetilde{\varepsilon}_t$ about $\boldsymbol{\varepsilon}_{t+1}$

$$\hat{\mu}_{\boldsymbol{\varepsilon}_{t+1}^{on}} = \delta_{\widetilde{\varepsilon}_t}$$

Outline of the presentation

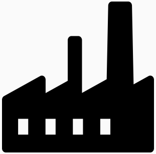
1. A stochastic optimization problem for microgrid management
2. Resolution with Stochastic Dynamic Programming (SDP)
3. Modeling stagewise dependency of the noise process
4. **Numerical results**
5. Conclusion and perspectives

What we will cover in this section

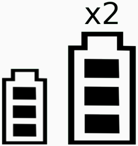
Outline of the section

Testing data is organized in periods of 10 consecutive days

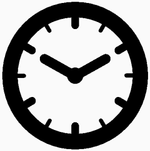
11 sites
to manage
separately



2 settings
of battery
per site



$\sim$ 10 periods
of 10 days
per site



Schneider's score metric for evaluating a policy

Score computed over **one period** against the naive policy
where energy must be consumed or sold (no storage)

$$naive_t = p_{buy,t}\left[d_{t+1} - pv_{t+1}\right]^{+} - p_{sell,t}\left[d_{t+1} - pv_{t+1}\right]^{-}$$
$$cost_t^{\pi} = p_{buy,t}\left[d_{t+1} - pv_{t+1} + \pi_t(soc_t)\right]^{+} - p_{sell,t}\left[d_{t+1} - pv_{t+1} + \pi_t(soc_t)\right]^{-}$$

We compute the total cost over the period for both policies
and deduce a score (the higher the better)

$$score^{\pi} = \frac{naive - cost^{\pi}}{naive} \times 100\ \%$$

Outline of the section

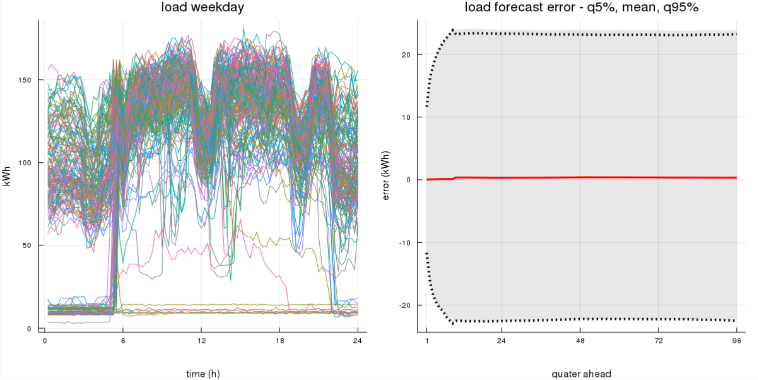
Our results

Our first SDP-based attempts score lower than MPC
Modeling noise stagewise dependence helps bridging the gap

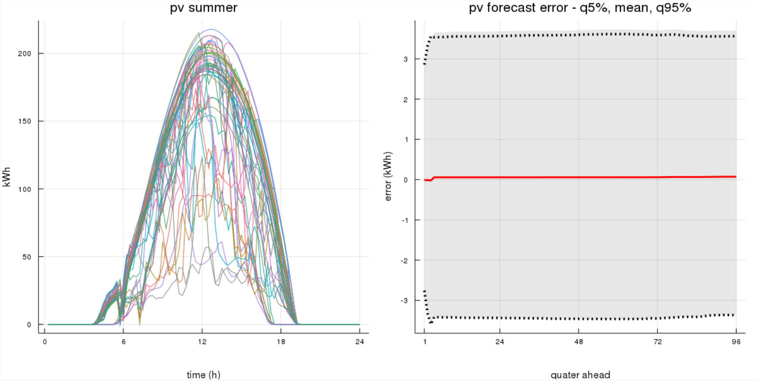
Method	**SDP**	**SDP AR-1**	**MPC**
Score (%)	16.1	17.1	18.2

Table 1: Average score on all testing periods. Higher is the better.

Load forecast quality for MPC



PV forecast quality for MPC



Outline of the presentation

Conclusion - what we did

- We wrote a stochastic optimization problem to manage microgrids at least cost
- We introduced Stochastic Dynamic Programming to solve the EMS problem
- We modeled stagewise dependency of the noise with an AR-1
- Under modeling assumptions we computed numerical solutions of our problem
- We gain on average **17.1%** of energy expenses vs a naive policy whereas MPC reaches **18.2%**

Perspectives

- We will compare with MPC on sites where the forecasts are less accurate
- We could improve the lags in the AR process to strengthen stagewise dependency (improve dim $\mathbb{X}$, will be using SDDP)
- We could introduce coherent risk measures instead of expectation
- Other constraints arise from grid operators (penalty on high energy importation, demand charge...)

P. Carpentier, J.-P. Chancelier, G. Cohen, and M. De Lara.
Stochastic multi-stage optimization.
In *Probability Theory and Stochastic Modelling*, volume 75.
Springer, 2015.

N. Löhndorf and A. Shapiro.
Modeling time-dependent randomness in stochastic dual dynamic programming.
European Journal of Operational Research, 273(2):650–661, 2019.

T. Rigaut, P. Carpentier, J. P. Chancelier, M. De Lara, and J. Waeytens.
Stochastic optimization of braking energy storage and ventilation in a subway station.
IEEE Transactions on Power Systems, 34(2):1256–1263, 2019.