Efficient Sovereign Debt Buybacks*

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Abstract

This paper challenges the conventional view that debt buybacks are detrimental to sovereign borrowers. Using a model of strategic lending, I show that buybacks can be rationalized as part of an optimal contract between a sovereign borrower and foreign lenders. In particular, buybacks allow bonds to function like Arrow securities. This is because they take place in the secondary market, where only legacy lenders operate, granting them market power, as opposed to the primary market, where new entrants ensure competitive returns. This mechanism can substitute for or complement standard implementations based on debt restructurings. The model is supported by recent empirical evidence from Brazil, where the buyback premium can be attributed to both limited secondary market competition and reduced default risk. These findings offer insights into sovereign debt management and the implementation of optimal contracts.

Keywords: sovereign debt, buyback, constrained Pareto efficiency, strategic lending
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1 Introduction

Countries regularly engage in the repurchase of previously issued bonds, a process known as a buyback.[1] Following the seminal contribution of Bulow and Rogoff (1988, 1991), the literature on sovereign debt generally considers buybacks to be suboptimal. The reason is that such operations actively enrich foreign lenders at the expense of sovereign borrowers. The purpose of this paper is to show that buybacks, although expensive for the borrower, can implement a constrained efficient allocation. I first expose this implementation in a model of strategic lending and then relate it to the recent empirical evidence in Brazil.

I show that the main aspects of sovereign debt buybacks can be rationalized as part of an optimal contract between a risk-averse sovereign borrower and risk-neutral foreign lenders. In particular, buybacks provide a clear interpretation for the binding participation constraint in optimal contracts under limited commitment. These contracts exhibit the characteristic that the borrower's participation constraint binds when the endowment increases. High endowment states are also associated with lower levels of indebtedness and capital outflows, aligning with the idea of a buyback.

More generally, buybacks allow bonds to function like Arrow securities. This is because issuances and buybacks occur in two distinct markets, creating state contingency through different payouts in these markets. More precisely, I distinguish between the primary market, where new bonds are issued, and the secondary market, where existing bonds can be repurchased before maturity. The crux of this distinction is that buybacks take place in the secondary market, where only legacy lenders operate, granting them market power. In contrast, new issuances occur in the primary market, where new entrants ensure competitive returns. Hence, legacy long-term debt yields higher payouts in the secondary market than in the primary market. I find supportive evidence of reduced competition in the secondary market during recent buybacks conducted by the Brazilian government.

The model assumes an endowment economy with one sovereign borrower and *two* foreign lenders. Endowment is stochastic with two states: high and low. The borrower is risk averse, impatient and lacks commitment. The lenders are risk neutral and trade non-contingent bonds of different maturities with the borrower. Only one of the two lenders holds the outstanding bonds every period. I call it the *incumbent* lender which represents the entire

[1] In 2024, Côte d'Ivoire repurchased bonds for 400 million USD with the support of the World Bank. In 2023, Argentina repurchased USD-denominated bonds maturing from 2025 for 1 billion USD. Between 2010 and 2022, the Mexican government repurchased bonds for a total of 44 billion USD according to World Bank International Debt Statistics. In 2009, Ecuador repurchased foreign-denominated bonds maturing in 2012 and 2030 for 3 billion USD. Between 2006 and 2019, Brazil conducted buybacks totalling 31 billion USD that I document in Section 5. In the early 2000s, there were also a series of buybacks of Brady bonds for Brazil, Mexico and Peru either directly on the secondary market or through call options.

group of legacy lenders. This supposes that this specific group of lenders can coordinate their actions. The other lender is the *outsider* which represents new entrants.

In the primary market, the two lenders engage in Bertrand competition, satisfying the borrower's demand for bonds at prices that ensure zero expected profit. In the secondary market, the incumbent lender is the sole holder of legacy long-term debt. It uses its market power to make take-it-or-leave-it buyback offers with a *markup* that makes the borrower indifferent between repurchasing debt and defaulting. The markup is zero only when the default threat is not credible. Similarly, during a default, the incumbent lender is the sole holder of the defaulted debt and can make take-it-or-leave-it restructuring offers with an *haircut*. As in Müller et al. (2019), for the market outcome to be efficient, the incumbent lender possesses all the market power ex post.

To disentangle the effect of the haircut and the buyback markup, I consider each case in isolation. This separation can be interpreted as reflecting the inability of legacy lenders to coordinate either during a restructuring or in the secondary market. The restructuring-based model corresponds to the re-contracting framework of Bulow and Rogoff (1989), in which notional payments arise in the high endowment state and lower payments arise in the low endowment state through renegotiation. Buybacks occur but without any markup.

The buyback-based model is the mirror image of the restructuring-based model. Notional payments arise in the low endowment state, while buybacks with a markup take place in the high-endowment state. Moreover, since the borrower is indifferent to default under a take-it-or-leave-it offer, such buybacks naturally emerge in complete markets. As noted by Arellano (2008), in incomplete markets, the surplus between the value of repayment and the value of autarky decreases when the endowment decreases and is i.i.d. In complete markets, the reverse holds as the threat of autarky fades in the low endowment state implying that buybacks at a markup arise in the high endowment state.

The market economy can implement the allocation of a Planner in three different ways: with buybacks entailing a markup, with restructurings entailing an haircut or with both. The Planner allocates resources between the borrower and the lenders, accounting for a participation constraint to ensure the borrower receives at least the value of autarky. This defines a constrained efficient allocation which features risk sharing in the form of state-contingent debt re-valuation. More precisely, debt is re-valuated upward in the high endowment state and downward in the low endowment state.

The decentralization in the restructuring-based model follows Müller et al. (2019) and Dovis (2019). Due to the debt relief in the form of an haircut, the restructuring reduces the value of outstanding short-term and long-term debt resulting in a valuation gain for the borrower in the low endowment state. In the high endowment state, the bond prices recover and the opposite argument holds. In steady state, buybacks are cheap as they occur without markup and in the low endowment state when the restructuring risk is elevated. Data from Brazil however contradict this dynamic as buybacks mostly occur in good times at a premium.

By reversing the payment schedule, the buyback-based model can also implement the constrained efficient allocation. Instead of receiving an haircut in the low endowment state, the borrower conducts buybacks at a markup in the high endowment state. The payment of the markup increases the value of outstanding long-term debt resulting in a valuation loss for the borrower. In the low endowment state, there is no markup paid and the opposite argument holds. Thus, the differentiated payout between the primary and the secondary market generates the valuation losses and gains necessary to replicate the state contingency in the Planner's allocation. In steady state, some buybacks are expensive because of the markup, while other occur at zero markup due to the non-credible default threat in the low endowment state. This better aligns with the evidence from Brazil.

The decentralization based on buybacks works without restructuring offer. The enforcement mechanism entirely relies on the incumbent lender's ability to make buyback offers with a markup. However, the resulting bond price dynamic is counterfactual. On the one hand, the long-term interest rate spread is negative owing to the absence of debt relief and the presence of the buyback markup. On the other hand, the long-term bond price is higher in the low endowment state since buybacks arise in the transition from a low to a high endowment. To remedy this, I combine markups with haircuts and assume a lenders' excess return. This corrects the price dynamic except in the immediate vicinity of the buyback.

I compare my theoretical findings with the experience of Brazil from 2006 to 2019, during which the country implemented a comprehensive buyback program. Focusing on the repurchase of USD-denominated bonds, I find that the dynamic of the observed buybacks aligns closely with the model's predictions. The Brazilian government primarily conducted buybacks during periods of economic growth and elevated bond prices. This contrasts sharply with the buybacks documented by Bulow and Rogoff (1988) in the 1980s, which were carried out amid debt crises.

I estimate the premium paid by the Brazilian government during buyback operations. In the model, this premium reflects the valuation gain realized by the incumbent lender. Building on the measure of creditor loss of Sturzenegger and Zettelmeyer (2008), I reconstruct the stream of cash flows associated with the bonds involved in each buyback. Specifically, I compare the present value of a bond with and without the buyback at the issue yield to maturity. The estimated premium averages 13.50%. It is due to the fact that most bonds were issued below par and repurchased above par. Buybacks were both frequent and substantial, with the government conducting operations nearly every quarter and repurchasing an average of 41.44% of issuance.

The model makes clear predictions about the source of this buyback premium. On the one hand, the premium may stem from reduced restructuring risk in high endowment states. On the other hand, it may reflect the market power of legacy lenders in the secondary market. Looking at the credit default swap spread, I find that the default risk reduces during buybacks which induces higher bond prices. For the assumption of non-competitive secondary market, I examine the bid-ask price spread which is commonly used to proxy market competitiveness and liquidity. This spread increases during buybacks both across and within bonds which I interpret as evidence of a reduced competition. Even if the Brazilian government targeted illiquid bonds to begin with, one would expect buybacks to improve, rather than impair, liquidity. The buyback premium thus appears to stem from both a reduction in default risk and diminished competition in the secondary market.

The paper is organized as follows. Section 1.1 reviews the related literature. Section 2 lays down the model environment. Sections 3 and 4 expose the market and the Planner economy, respectively. Section 5 presents the quantitative analysis. Section 6 concludes. The Online Appendix contains the proofs and the data sources.

1.1 Related literature

The paper contributes to the literature on sovereign defaults. It builds on the two-lender-one-borrower market structure of Kovrijnykh and Szentes (2007) with the standard timing of actions of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). Similar to Arellano and Ramanarayanan (2012) and Niepelt (2014), I adopt two bonds with different maturities. Moreover, I distinguish between the primary and the secondary bond markets as in Broner et al. (2010). I contribute to this literature in two ways. First, I depart from the standard assumption of competitive lending by modelling lenders with market power in the secondary market. Similar to Bi (2008), Yue (2010) and Benjamin and Wright (2013), I also assume market power during debt restructurings. Second, I analyze the consequences of long-term debt dilution in strategic lending. This complements the analysis of Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Chatterjee and Eyigungor (2015) focusing on purely competitive bond markets.

This paper focuses on sovereign debt buybacks and relates to the seminal contribution of Bulow and Rogoff (1988, 1991) who document that such operations are detrimental to borrowers since the reduction in future payments is less in expected value than the repurchase price. The rationale is that buybacks increase the recovery value per unit of bond given a fixed collateral value upon default. In that logic, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Without collateral value upon default, Aguiar et al. (2019) also find that buybacks reduce welfare as they shift the maturity structure and therefore affect the default risk. I challenge this view by establishing buybacks as a constrained efficient risk sharing agreement between the borrower and the lenders. Rotemberg (1991) and Acharya and Diwan (1993) also highlight potential benefits of buybacks through bargaining costs and signalling, respectively. However, like Bulow and Rogoff (1988, 1991), both studies primarily focus on a state of debt overhang, whereas I consider the full equilibrium dynamic in evaluating welfare. Additionally, I estimate buyback premia by adapting the framework of Sturzenegger and Zettelmeyer (2008) and inquire the source of such premia in the data.[2]

The paper addresses the literature on optimal contracts and their implementation. Deriving a contract between a borrower with limited commitment and foreign lenders, my study relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). Similar to Kehoe and Perri (2002) and Restrepo-Echavarria (2019), I use the Lagrangian approach of Marcet and Marimon (2019) with the difference that I implement the optimal contract in a market economy.

My implementation is most closely related to Müller et al. (2019) and Dovis (2019). In the former, the borrower faces both endowment and default cost shocks. Their decentralization uses a one-period defaultable bond to span the default costs via restructuring, and an endowment-contingent bond to span the endowment states. Like them, I assume the lender makes a take-it-or-leave-it offer. However, my formulation extends this offer beyond restructurings to include buybacks. Assuming only an endowment shock, I do not assume endowment-contingent bonds and rely on the maturity management developed by Angeletos (2002) and Buera and Nicolini (2004). Using a similar approach, Dovis (2019) proposes a decentralization in a production economy with privately observed productivity shocks using trigger strategies instead of Markov strategies. His decentralization generally fails to predict expensive buybacks in good times, though.

In other decentralizations, Alvarez and Jermann (2000) implement the allocation in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. In my case, short-term and long-term bonds together behave like an Arrow security due to the buyback markup or the haircut. There is also an implicit borrowing limit emanating from the incumbent lender's market power. Thus, the main difference with my analysis is that

[2] There is a small literature in empirical finance analysing buybacks of US treasury bonds. Similar to what I document for Brazil, Han et al. (2007) show that the US Treasury targeted illiquid bonds and these bonds became less liquid relative to other bonds during buybacks. While they find small impacts on bond prices, Connolly and Struby (2024) argue the opposite given spillover effects on nearby bonds.

the two authors assume a greater financial sophistication as securities are state contingent, while I rely on the maturity structure and the legacy lenders market power.

Finally, Aguiar et al. (2019) decentralize a constrained efficient allocation using a continuum of maturities.[3] They consider a Planner's problem with no participation constraint. This results in a constrained efficient allocation consistent with incomplete markets, while my allocation is consistent with complete markets. Moreover, the Planner does not take into consideration the legacy lenders in the surplus maximization, while such lenders represent the main actor in my environment. The reason is that the authors assume a competitive lending market meaning that the legacy lenders are unable to extract surplus from the borrower. Thus, the market economy cannot achieve overall efficiency for all contracting parties.

2 Environment

This section presents the different market participants, the organisation of the international bond market and the timing of actions.

2.1 Market participants

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The government of this economy (i.e. the borrower) is benevolent, receives an endowment every period and can trade bonds with two foreign lenders.

Endowment is stochastic. It takes value on the discrete set $Y \equiv \{y_L, y_H\}$ with $0 < y_L < y_H$ and is independent and identically distributed (i.i.d) with $\pi(y_{t+1})$ corresponding to the probability of drawing y_{t+1} at date $t+1$.

The borrower is impatient and discounts the future at rate $\beta < \frac{1}{1+r}$ with r being the exogenous risk-free rate. Preference over consumption is represented by $\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $c_t \geq 0$ denotes the consumption at time t and $\mathbb{E}(\cdot)$ denotes the expectation. The instantaneous utility function $u(\cdot)$ is continuous, strictly increasing, strictly concave and satisfies the Inada condition $\lim_{c \to 0} u_c(c) = \infty$ where $u_c(\cdot)$ denotes the first derivative of $u(\cdot)$.

The borrower cannot commit to repay the lenders. In case of default, the borrower suffers from permanent autarky. There is no endowment penalty upon default and the lenders are able to seize the borrower's assets, if any.

The two foreign lenders are risk neutral and discount the future at a rate $\frac{1}{1+r}$. As one will see, only one of the two lenders effectively holds bonds in a given period. This is the sole difference between the two lenders.

[3] See also the textbook treatment in Aguiar and Amador (2021).

2.2 Bond markets

The borrower can trade bond contracts with two different maturities: short-term and long-term.[4] The short-term bond b'_{st} has a unit price q_{st}, matures in the next period and pays a coupon of one. Following Hatchondo and Martinez (2009), the long-term bond b'_{lt} has a unit price q_{lt}, matures at a rate $(1-\delta) \in [0, 1)$ and pays a coupon of one every period. The risk-free return is given by $\overline{q} = \frac{1}{1+r-\delta}$. I denote debt as a negative asset meaning that $b'_j < 0$ is a debt, while $b'_j > 0$ is an asset for all $j \in \{st, lt\}$. The borrower is assumed to always be a net debtor, i.e. $b_{st} + b_{lt}(1 + \delta q_{lt}) \leq 0$.[5]

I distinguish between the primary and the secondary markets for bond contracts. In the former new bond contracts are issued, while in the latter (part of) perviously-issued bond contracts can be retired. Borrower's operations on the secondary market are called *buybacks* and are relevant only for long-term bond contracts since short-term bond contracts last one period. If the borrower enters the secondary market for long-term bond contracts, it can access the primary market for short-term bond contracts. Thus, the borrower decides to enter in either the primary market or both markets simultaneously.

If the borrower enters the primary market only, the two lenders simultaneously offer a pair of short-term and long-term bond contracts. The borrower can only accept one of the two offers. Similar to Kovrijnykh and Szentes (2007), I call a lender the *incumbent* at the beginning of a period if the borrower accepted its offer in the previous period. The other lender is called the *outsider*. I denote by $(b^{o\prime}_{st}, q^o_{st}; b^{o\prime}_{lt}, q^o_{lt})$ the offer made by the outsider and by $(b^{i\prime}_{st}, q^i_{st}; b^{i\prime}_{lt}, q^i_{lt})$ the offer made by the incumbent. If the borrower accepts the outsider's offer, the outsider takes over the long-term bond contract from the incumbent. For given outstanding bonds (b_{st}, b_{lt}), transfers are as follows. The borrower receives $-q^o_{st}b^{o\prime}_{st} - q^o_{lt}(b^{o\prime}_{lt} - \delta b_{lt})$ from the outsider and pays $b_{st} + b_{lt}$ to the incumbent. The outsider then pays $-q^o_{lt}\delta b_{lt}$ to the incumbent.

If the borrower enters both the primary and the secondary markets, the outsider and the incumbent simultaneously offer a short-term bond contract in the primary market. In the secondary market, the incumbent can make a take-it-or-leave-it offer $(b^{i\prime}_{lt}, q^i_{lt}, \chi)$ which consists of a debt buyback $b^{i\prime}_{lt} - \delta b_{lt} > 0$ with $b_{lt} < 0$ and a buyback markup $\chi \geq 0$. If the outsider's offer is accepted in the primary market, the outsider takes over the long-term bond contract from the incumbent as before. However, the outsider determines the price q^o_{lt} at which the new bond contract $b^{i\prime}_{lt}$ is taken over from the incumbent. That is when the incumbent offers $(b^{i\prime}_{st}, q^i_{st}; b^{i\prime}_{lt}, q^i_{lt}, \chi)$, one can say that the outsider effectively offers $(b^{o\prime}_{st}, q^o_{st}; b^{i\prime}_{lt}, q^o_{lt}, \chi)$. For given

[4]To decentralize the constrained efficient allocation, I need at least as many maturities as endowment states. See also footnote 11.

[5]Despite the fact that the borrower is a net debtor, it can be that $b_{st} > 0$ or $b_{lt} > 0$.

outstanding bonds (b_{st}, b_{lt}), the borrower receives $-q^o_{st}b^{o\prime}_{st} - q^o_{lt}b^{i\prime}_{lt}$ from the outsider and pays $b_{st} + b_{lt}(1+\delta\chi+\delta q^o_{lt})$ to the incumbent. The outsider then receives $q^o_{lt}b^{i\prime}_{lt}$ from the incumbent. This ensures that the incumbent's market power only pertains to the legacy debt δb_{lt} and not the new debt $b^{i\prime}_{lt}$.

If the borrower decides to default, the incumbent lender is the only lender affected and can make a take-it-or-leave-it restructuring offer. Formally, the restructuring offer consists of a face value haircut $(1-\phi) \leq 1$ of the outstanding short-term and long-term debt. With the restructuring, the borrower avoids autarky.[6]

Every period there is only one of the two lenders holding all the bond contracts. Hence, the incumbent can be interpreted as the entire group of legacy lenders. This supposes that this specific group of lenders can coordinate their actions. In opposition, the outsider corresponds to the entire group of lenders which hold no claim on the borrower. It reflects new entrants in the international bond market.

2.3 Timing of actions

I consider Markov equilibria. That is I restrict my attention to the payoff-relevant state vector $\Omega \equiv (y, b_{st}, b_{lt})$.[7] The timing of actions is the same as in Eaton and Gersovitz (1981). At the beginning of each period, y realizes and the borrower decides whether to default given (b_{st}, b_{lt}). Conditional on no default, the borrower decides to enter either the primary market only (i.e. no buyback) or both the primary and the secondary markets (i.e. buyback). Conditional on default, the incumbent lender can make a restructuring offer.

The timing of actions diverges from the one of Kovrijnykh and Szentes (2007) in which the lenders first offer bond contracts – stating a current payment and a value of the bond next period – and then the borrower decides whether to default. In my case, the default decision happens before offers are made and is taken as given by the two lenders. Furthermore, the bond contract specifies the amount lent and the price.

3 Market Economy

This section exposes the market economy. It first derives the borrower's and the lenders' problems and subsequently characterizes the underlying equilibrium.

[6] It is possible to model a period of market exclusion upon restructuring without affecting the argument about efficiency. See for instance Dovis (2019).

[7] To save up notation, I ignore the indicator function taking value one when the borrower is in default. Strictly speaking such variable should also be part of the state vector.

3.1 Borrower problem

The borrower's overall beginning of the period value is

$$V(y, b_{st}, b_{lt}) = \max\Big\{V^P(y, b_{st}, b_{lt}), V^{NP}(y, b_{st}, b_{lt})\Big\}, \tag{1}$$

where $V^P(\cdot)$ and $V^{NP}(\cdot)$ correspond to the value of repayment and non repayment, respectively. Under repayment, the value is $V^P(y, b_{st}, b_{lt}) = \max\{V^P_B(y, b_{st}, b_{lt}), V^P_{NB}(y, b_{st}, b_{lt})\}$ where $V^P_B(\cdot)$ and $V^P_{NB}(\cdot)$ are the value in case of buyback and no-buyback, respectively. Without buyback, the borrower enters the primary market only and chooses the offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ which maximizes its value subject to the budget constraint. Formally, one has that

$$\begin{aligned} V^P_{NB}(y, b_{st}, b_{lt}) = & \max_{(b'_{st}, q_{st}; b'_{lt}, q_{lt}) \in \Gamma} \Big\{u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right]\Big\} \\ & \text{s.t.} \quad c + q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}. \end{aligned} \tag{2}$$

The set Γ contains the offers of the two lenders. Given that there is no debt buyback, all offers in Γ are such that $b'_{lt} \leq \delta b_{lt}$ when $b_{lt} < 0$.

Under repayment with a buyback, the borrower enters both the primary market to issue new short-term bond contracts and the secondary market to repurchase existing long-term bond contracts. Formally,

$$\begin{aligned} V^P_B(y, b_{st}, b_{lt}) = & \max_{(b'_{st}, q_{st}; b'_{lt}, q_{lt}, \chi) \in \Psi} \Big\{u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right]\Big\} \\ & \text{s.t.} \quad c + q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1 + \delta\chi), \\ & \qquad b_{lt} < 0. \end{aligned} \tag{3}$$

The set Ψ contains the two offers for the short-term bond contract and the buyback offer. The incumbent's buyback offer is such that $\chi \geq 0$ and $b'_{lt} > \delta b_{lt}$ with $b_{lt} < 0$. If $b_{lt} \geq 0$, there is no possibility to buyback and $V^P_B(y, b_{st}, b_{lt}) = -\infty$.

Under non repayment, the value is $V^{NP}(y, b_{st}, b_{lt}) = \max\{V^R(y, b_{st}, b_{lt}), V^D(y)\}$ where $V^D(\cdot)$ and $V^R(\cdot)$ correspond to the value of outright default and restructuring, respectively. The former value is given by

$$V^D(y) = u(y) + \beta \mathbb{E}\left[V^D(y')\right].$$

The assumption that assets can be seized is to ensure that the value of default is independent of (b_{st}, b_{lt}). Finally, the value under restructuring is

$$V^R(y, b_{st}, b_{lt}) = \max_{(b'_{st}, q_{st}; b'_{lt}, q_{lt}) \in \Gamma} \left\{ u(c) + \beta \mathbb{E} \left[V(y', b'_{st}, b'_{lt}) \right] \right\} \tag{4}$$
$$\text{s.t.} \quad c + q_{st} b'_{st} + q_{lt} b'_{lt} = y + (\min\{0, b_{st}\} + \min\{0, b_{lt}\}[1 + \delta q_{lt}])\phi.$$

In a restructuring, the borrower obtains an haircut $(1 - \phi) \leq 1$ and can immediately issue new bonds on the primary market. The haircut only applies to debt and assets are seized.

Given $\Omega \equiv (y, b_{st}, b_{lt})$, I can define $D(\Omega)$ as the default policy which takes value 1 if $V^D(y) > \max\{V^P(\Omega), V^R(\Omega)\}$ and zero otherwise. Similarly, $R(\Omega)$ is the restructuring policy which takes value 1 if $V^R(\Omega) > V^P(\Omega)$ and $V^R(\Omega) \geq V^D(y)$ and zero otherwise. If the borrower is indifferent between a default and a restructuring, it chooses the latter. Also, $M(\Omega)$ is the buyback policy which takes value 1 if $V^P_B(\Omega) \geq \max\{V^P_{NB}(\Omega), V^{NP}(\Omega)\}$ and zero otherwise. Finally, $B_{st}(\Omega) = b'_{st}$ and $B_{lt}(\Omega) = b'_{lt}$ correspond to the short-term and long-term bond policy, respectively.

3.2 Lenders problem

The lenders' problem determines the offer sets Γ and Ψ as well as the optimal haircut $1 - \phi(\Omega)$. I first consider the offers on the primary market before turning to the secondary market and the restructuring.

In the primary market, the outsider is willing to offer bond contracts under which its expected payoff is large enough to cover the transfer to the incumbent. Define the value of the incumbent in state Ω by $W(\Omega)$. If the borrower enters the primary market only, the outsider's offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ has to be such that

$$q_{st} b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r} \mathbb{E} \left[W(y', b'_{st}, b'_{lt}) \right] \geq -q_{lt} \delta b_{lt}. \tag{5}$$

If the outsider's offer satisfies this constraint, the incumbent would offer the same contract. The reason is as follows. If the incumbent makes the same offer and it is accepted, the incumbent receives $-(b_{st} + b_{lt})$ plus the left-hand side of (5). Otherwise, the incumbent receives $-(b_{st} + b_{lt})$ plus the right-hand side of (5).

Similarly, if the borrower enters both the primary and the secondary markets, the outsider's offer $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ has to be such that $q_{st} b'_{st} + q_{lt} b'_{lt} + \frac{1}{1+r} \mathbb{E} \left[W(y', b'_{st}, b'_{lt}) \right] \geq 0$ which is the same as (5). One can therefore repeat the same argument as before. If the incumbent makes the same offer as the outsider and it is accepted, the incumbent receives $-(b_{st} + b_{lt}(1 + \delta \chi + \delta q_{lt}))$ plus the left-hand side of (5). Otherwise, the incumbent receives

$-(b_{st} + b_{lt}(1 + \delta\chi + \delta q_{lt}))$. Hence, if the outsider's offer satisfies (5), the incumbent would offer the same contract. Without loss of generality, one can assume that the incumbent's offer is always the one accepted by the borrower on the equilibrium path.

In the primary market, the two lenders compete with each other. This means that they both record zero expected profit. If one of the two lenders makes an offer leading to a strictly positive expected profit, the other lender can undercut this offer. As a result, prices are competitive in the primary market. For the short-term bond price, this means that

$$q_{st}(b'_{st}, b'_{lt}) = \begin{cases} \frac{1}{1+r}\mathbb{E}\big[P(\Omega') + (1 - P(\Omega'))K_{st}(\Omega')\big] & \text{if } b'_{st} < 0 \\ \frac{1}{1+r} & \text{else} \end{cases} \tag{6}$$

where $K_{st}(\Omega') \equiv \frac{\max\{0,(1+r)q_{lt}(b''_{st},b''_{lt})b'_{lt}\}}{-b'_{st}} + R(\Omega')\phi$ with $b''_{st} = B_{st}(\Omega')$ and $b''_{lt} = B_{lt}(\Omega')$ represents the recovery value upon non repayment and $P(\Omega') = 1 - D(\Omega') - R(\Omega')$ is the repayment policy. Since assets can be seized during defaults, the recovery value can be strictly positive even without restructuring. If $b'_{st} \geq 0$, the short-term bond price equates the risk-free return.[8] If $b'_{st} < 0$, the short-term bond price accounts for the default and restructuring risks. Conversely, the long-term bond price is given by

$$q_{lt}(b'_{st}, b'_{lt}) = \begin{cases} \frac{1}{1+r}\mathbb{E}\big[P(\Omega')\{Q_{lt}(\Omega') + M(\Omega')\delta\chi(\Omega')\} + (1 - P(\Omega'))K_{lt}(\Omega')\big] & \text{if } b'_{lt} < 0 \\ \frac{1}{1+r-\delta} & \text{else} \end{cases} \tag{7}$$

where $K_{lt}(\Omega') \equiv \frac{\max\{0,b'_{st}\}}{-b'_{lt}} + R(\Omega')(1+\delta q_{lt}(b''_{st}, b''_{lt}))\phi$ is the recovery value upon non repayment and $Q_{lt}(\Omega') \equiv 1 + \delta q_{lt}(b''_{st}, b''_{lt})$ is the notional return upon repayment. Notice that δ is restricted to be strictly larger than zero, otherwise (6) and (7) would be identical. Both bond prices are also independent of y given the i.i.d assumption.

In the secondary market, the incumbent is the sole holder of legacy long-term debt. It therefore acts as a monopolist for the repurchase of outstanding long-term bond contracts. I assume that the incumbent possesses all the market power ex post. The incumbent makes the following take-it-or-leave-it offer

$$\chi(y, b_{st}, b_{lt}) = \begin{cases} 0 & \text{if } V_B^P(y, b_{st}, b_{lt}) > V^D(y) \\ \varrho & \text{if } V_B^P(y, b_{st}, b_{lt}) = V^D(y) \\ \infty & \text{else} \end{cases} \tag{8}$$

The markup is zero when the default threat is not credible and strictly positive otherwise.

[8] Given that the borrower is a net debtor by assumption, the lenders are net creditors.

The incumbent imposes $\varrho > 0$ when the markup is *tight* in the sense of Alvarez and Jermann (2000). It corresponds to the highest possible markup the incumbent can charge consistent with no outright default. Alternatively, $\chi(y, b_{st}, b_{lt}) = \infty$ and the buyback offer is simply rejected. The parameter ϱ is fixed which enables the enforcement of the participation constraint as one will see.

Note that the market power of the incumbent solely pertains to the repurchase of the legacy debt δb_{lt}. In that logic the incumbent imposes a markup χ on δb_{lt} but still offers a new level of bond $b^{i\prime}_{lt}$ at a competitive price q^{i}_{lt}. The reason is the outsider values the new portfolio of bond whenever its offer is accepted for the short-term bond. Hence, the same logic applies as in the primary market.

The following proposition summarizes the main characteristics of the lender's offer in both the primary and the secondary markets.

Proposition 1 (Optimal Offers)**.** *In the primary and the secondary markets, the two lenders offer bond contracts with prices* (q_{st}, q_{lt}) *satisfying* (6)-(7) *implying that* (5) *hold with equality in all states* (y, b_{st}, b_{lt}).

Even though I assume two strategic lenders, the issuance of bonds is competitive meaning that (5) always holds with equality. The reason is that the two lenders are perfect Bertrand competitors for the issuance of new bond contracts. It is true that the incumbent can drive the outsider out of the primary market by making offers such that (5) cannot hold. Nevertheless, such offers lead to net losses. As a result, the incumbent never acts as a monopolist for the issuance of new bond contracts. This the main difference with respect to Kovrijnykh and Szentes (2007). The reason is that the borrower decides to default before the bond auction and the lenders take this decision as given.

Given this, the offer set in the primary market is $\Gamma = \{(b'_{st}, q_{st}; b'_{lt}, q_{lt}) : (b'_{lt} - \delta b_{lt}) \leq 0 \text{ if } b_{lt} < 0 \wedge (6)\text{-}(7)\}$ and in the secondary market is $\Psi = \{(b'_{st}, q_{st}; b'_{lt}, q_{lt}, \chi) : (b'_{lt} - \delta b_{lt}) > 0 \wedge (6)\text{-}(8)\}$.

Regarding restructuring, I apply the same idea as in the secondary market. The incumbent is the sole holder of debt and therefore acts as a monopolist for the restructuring of defaulted bond contracts. Again, the incumbent possesses all the market power ex post and makes a take-it-or-leave-it offer such that the borrower is indifferent between restructuring and default. The offer is such that

$$\phi(y, b_{st}, b_{lt}) = \begin{cases} \xi & \text{if } V^R(y, b_{st}, b_{lt}) = V^D(y) \text{ and } y = y_L \\ \infty & \text{else} \end{cases} \tag{9}$$

The incumbent offers $\xi \in [0,1)$ when the haircut is *tight* in y_L.[9] Otherwise, $\phi = \infty$ and the restructuring offer is simply rejected. Notice that for $(1-\phi) = 1$, $V^R(y, b_{st}, b_{lt}) \geq V^D(y)$ meaning that there is always an haircut capable of making the borrower indifferent to outright default. However, the parameter ξ is here fixed as for ϱ meaning that the incumbent does not tolerate all levels of indebtedness in restructurings.

The take-it-or-leave-it offer in Müller et al. (2019) is different from (9) as they model one-period bonds only. The authors assume that $\phi(y, b_{st}, b_{lt}) = \{\phi \in [0,1] : V^R(y, b_{st}, b_{lt}) = V^D(y)\}$ meaning that the haircut offer varies with the level of indebtedness. In particular, with more debt, the incumbent decreases ϕ to leave the borrower indifferent to default. This depresses the bond price such that the revenue raised with the new bond issuance, $q_{st}b'_{st} + q_{lt}b'_{lt}$, remains unchanged given (6)-(7). However, the reduction in the bond price reduces the value of legacy long-term debt, $q_{lt}\delta b_{lt}$. As a result, the borrower has an incentive to issue more debt to dilute legacy claims. Not allowing the haircut to adapt with the level of indebtedness makes the renegotiation protocol dilution proof.

In what follows, I consider cases in which the incumbent lender abstains from making haircut or markup offers. In the former case (9) does not hold and instead $\phi = \infty$. In the latter case (8) does not hold and $\chi = 0$. This enables me to disentangle the effect of each offer. More generally, as the incumbent represents the entire group of legacy lenders, such abstention can be interpreted as the inability of legacy lenders to coordinate either during a restructuring or in the secondary market.

3.3 Equilibrium properties

Combining the the previous two subsections together, I can reformulate the borrower's maximization problems in (2), (3) and (4) accounting for the definition of the offer sets Γ and Ψ. The problem is as if the borrower would offer bond contracts to the lenders subject to the constraint that bond prices are competitive.

Proposition 2 (Optimal Bond Contracts). *In equilibrium, if $M(y, b_{st}, b_{lt}) = R(y, b_{st}, b_{lt}) = D(y, b_{st}, b_{lt}) = 0$, the optimal bond contracts $B_{st}(y, b_{st}, b_{lt})$ and $B_{lt}(y, b_{st}, b_{lt})$ solve*

$$
\begin{aligned}
V^P_{NB}(y, b_{st}, b_{lt}) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right] \right\} \\
s.t. \quad & c + q_{st}(b'_{st}, b'_{lt})b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}, \\
& (6),\ (7),\ b'_{lt} \leq \delta b_{lt} \ \textit{if}\ b_{lt} < 0.
\end{aligned}
$$

[9] The constraint that restructuring cannot occur in y_H is also to ensure enforcement of the participation constraint. See the proof of Proposition 7 in the Online Appendix.

If $M(y,b_{st},b_{lt})=1$, $B_{st}(y,b_{st},b_{lt})$ *and* $B_{lt}(y,b_{st},b_{lt})$ *solve for a given* χ

$$\begin{aligned} V_B^P(y,b_{st},b_{lt}) = \max_{b'_{st},b'_{lt}} & \Big\{ u(c) + \beta \mathbb{E}\left[V(y',b'_{st},b'_{lt})\right] \Big\} \\ s.t. \quad & c + q_{st}(b'_{st},b'_{lt})b'_{st} + q_{lt}(b'_{st},b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1+\delta\chi), \\ & (6),\ (7),\ b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt}, \end{aligned}$$

and $\chi(y,b_{st},b_{lt})$ *satisfies* (8). *Finally, if* $R(y,b_{st},b_{lt})=1$, $B_{st}(y,b_{st},b_{lt})$ *and* $B_{lt}(y,b_{st},b_{lt})$ *solve for a given* ϕ

$$\begin{aligned} V^R(y,b_{st},b_{lt}) = \max_{b'_{st},b'_{lt}} & \Big\{ u(c) + \beta \mathbb{E}\left[V(y',b'_{st},b'_{lt})\right] \Big\} \\ s.t. \quad & c + q_{st}(b'_{st},b'_{lt})b'_{st} + q_{lt}(b'_{st},b'_{lt})b'_{lt} = y + (\min\{0,b_{st}\} + \min\{0,b_{lt}\}[1+\delta q_{lt}(b'_{st},b'_{lt})])\phi, \\ & (6),\ (7), \end{aligned}$$

and $\phi(y,b_{st},b_{lt})$ *satisfies* (9).

The next proposition specifies the conditions under which defaults and restructurings occur. It distinguishes the case in which the incumbent lender can make restructuring offers or not.

Proposition 3 (Default and Restructuring)**.** *If* $\phi(\Omega)=\infty$ *for all* Ω, $D(y_L,B_{st}(\Omega),B_{lt}(\Omega)) \geq D(y_H,B_{st}(\Omega),B_{lt}(\Omega)) = 0$. *If* $\phi(y_L,b_{st},b_{lt}) = \xi$ *for some* (b_{st},b_{lt}), $D(y_L,b_{st},b_{lt}) = 0$.

The proposition states that outright defaults occur in y_L when there is no restructuring offer. This is because of the strict concavity $u(\cdot)$. Since an outright default increases current consumption, repayment is more costly in y_L making it less likely. In addition, it holds that $D(y_L,b_{st},b_{lt}) \geq D(y_H,b_{st},b_{lt})$. Hence, a default is suboptimal in y_H since it would imply a default probability of one next period – i.e. no revenue raised from debt issuance. The possibility to offer restructuring substitutes to outright default as in Müller et al. (2019). The reason is that the incumbent possesses all the market power ex post and makes the borrower indifferent between a restructuring and a default.

Nevertheless buybacks can occur either in y_H or in y_L. The reason is that the borrower may issue more short-term debt during a buyback meaning that current consumption does not necessarily decrease during such operations. If the short-term debt issuance is not too pronounced such that the net indebtedness does not increase, one can use the argument in Chatterjee and Eyigungor (2012, Proposition 2) to show that a buyback cannot be optimal in y_L when it is not optimal in y_H.

Note that with neither markup nor haircut, the model reduces to the one of Arellano and Ramanarayanan (2012). In particular, there is no distinction between the primary and the secondary markets and the incumbent lender has no market power whatsoever.

4 Planner Economy

This section derives the constrained efficient allocation. I first characterize the Planner problem and subsequently decentralize the underlying allocation in the market economy.

4.1 Planner's problem

The constrained efficient allocation is the outcome of a problem in which a Planner allocates consumption to maximize the lenders' and the borrower's weighted utility subject to a participation constraint. The participation constraint accounts for the borrower's limited commitment in repayment (Thomas and Worrall, 1994). Denoting y^t as the history of realized endowment at time t, it must hold that for all t and y^t

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(c(y^j)) \geq V^D(y_t). \tag{10}$$

This condition ensures that the borrower's value is at least as large as the value of autarky. Given this, the Planner's maximization problem in sequential form reads

$$\max_{\{c(y^t)\}_{t=0}^{\infty}} \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) u(c(y^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{y^t} \pi(y^t) \left[y_t - c(y^t)\right] \tag{11}$$
$$\text{s.t.} \quad (10) \text{ for all } y^t, t \text{ with } (\mu_{b,0}, \mu_{l,0}) > 0 \text{ given.}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial Pareto weights assigned by the Planner to the borrower and the lenders, respectively. Following Marcet and Marimon (2019), I reformulate (11) as a saddle-point Lagrangian problem,

$$\mathcal{SP} \quad \min_{\{\gamma(y^t)\}_{t=0}^{\infty}} \max_{\{c(y^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) \mu_{b,t}(y^t) u(c(y^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{y^t} \pi(y^t) \mu_{l,t}(y^t) \left[y_t - c(y^t)\right]$$
$$+ \sum_{t=0}^{\infty} \beta^t \sum_{y^t} \pi(y^t) \gamma(y^t) \Big[\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(c(y^j)) - V^D(y_t) \Big]$$
$$\mu_{b,t+1}(y^t) = \mu_{b,t}(y^t) + \gamma(y^t) \text{ and } \mu_{l,t+1}(y^t) = \mu_{l,t}(y^t) \text{ for all } y^t, t$$

with $\mu_{b,0}(y_0) \equiv \mu_{b,0}$ and $\mu_{l,0}(y_0) \equiv \mu_{l,0}$ given.

In this formulation, $\gamma(y^t)$ denotes the Lagrange multiplier attached to the participation constraint at time t. As the value of the borrower appears in both the Planner's objective function and the participation constraint, there is a direct relationship between $\mu_{b,t}(y^t)$ and $\gamma(y^t)$. More precisely, the borrower's Pareto weight evolves according to $\mu_{b,t+1}(y^t) = \mu_{b,t}(y^t) + \gamma(y^t)$, while the lenders' Pareto weight, $\mu_{l,t+1}(y^t)$, remains constant.

Following Marcet and Marimon (2019), the saddle-point Lagrangian problem is homogenous of degree one in $(\mu_{b,t}(y^t), \mu_{l,t}(y^t))$. I can therefore redefine the maximization problem over $(x_t(y^t), 1)$ where $x_t(y^t) = \frac{\mu_{b,t}(y^t)}{\mu_{l,t}(y^t)}$ corresponds to the relative Pareto weight – i.e. the Pareto weight attributed to the borrower relative to the lenders. Given that $(\mu_{b,0}, \mu_{l,0}) > 0$ and $\gamma(y^t) \geq 0$ for all t, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} < \infty$. Moreover,

$$x_{t+1}(y^t) = (1 + \nu(y^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}}, \tag{12}$$

where $\eta \equiv \beta(1+r) < 1$ corresponds to the borrower's impatience relative to the lenders and $\nu(y^t) \equiv \frac{\gamma(y^t)}{\mu_{b,t}(y^t)}$ represents the normalized multiplier attached to the participation constraint. The state vector is then simply (y, x) and the Saddle-Point Functional Equation reads

$$\begin{aligned} PV(y, x) = \mathcal{SP} \min_{\nu(y)} \max_{c} \; & x\Big[(1 + \nu(y))u(c) - \nu(y)V^D(y)\Big] \\ & + y - c + \frac{1}{1+r}\sum_{y'} \pi(y')PV(y', x') \\ \text{s.t.} \quad & x'(y) = (1 + \nu(y))\eta x. \end{aligned} \tag{13}$$

The value function takes the form of $PV(y, x) = xV^b(y, x) + V^l(y, x)$ with $V^b(y, x) = u\left(c\right) + \beta\mathbb{E}\left[V^b\left(y', x'\right)\right]$ being the value of the borrower and $V^l(y, x) = y - c + \frac{1}{1+r}\mathbb{E}\left[V^l\left(y', x'\right)\right]$ being the value of the lenders. I obtain the optimal consumption by taking the first-order conditions in (13)

$$u_c(c) = \frac{1}{x(1 + \nu(y))}. \tag{14}$$

The binding participation constraint of the borrower (i.e. $\nu(y) > 0$) induces an increase in consumption. In what follows, I formalize this argument in Proposition 4.

4.2 Constrained efficient allocation

I characterize the main properties of the constrained efficient allocation. I start with the definition of a threshold value for the relative Pareto weight $x_D(y)$ which is such that $V^b(y, x_D(y)) = V^D(y)$. In words, $x_D(y)$ is the weight at which the participation constraint binds in y.

Proposition 4 (Constrained Efficient Allocation)**.**

I. (Efficiency). *$V^l(y, x)$ is strictly decreasing, while $V^b(y, x)$ is strictly increasing in $x \in \tilde{X} \equiv [x_D(y_L), \overline{x}]$ for all $y \in Y$ and $x_D(y_H) > x_D(y_L)$.*

II. (Risk-Sharing). *$c(y_L, x) < c(y_H, x)$ and $x'(y_L, x) < x'(y_H, x)$ for $x < x_D(y_H)$ and $c(y_L, x) = c(y_H, x)$ and $x'(y_L, x) = x'(y_H, x)$ otherwise. Also, $c(y_L, x_D(y_L)) = y_L$ and $c(y_H, x_D(y_H)) < y_H$.*

III. (Liabilities). *$V^l(y_L, x) < V^l(y_H, x)$ for all $x \in \tilde{X}$.*

Part I states that the allocation is constrained efficient. Accounting for the participation constraint, it is not possible to make one of the contracting parties better off without making the other worse off.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind in all endowment states. Otherwise, the Planner provides more consumption and a greater continuation value in the high endowment state. Moreover, when the participation constraint binds in y_L, the borrower consumes the value of its endowment. In opposition, when the participation constraint binds in y_H, it consumes less than its endowment.

Part III relates to the liabilities of the borrower. In this environment, the value of the lenders represents the net foreign asset position in the contract. A positive value of $V^l(y, x)$ indicates the extent towards which the borrower is indebted. The proposition states that the liabilities increase when y is high. This implies that the Planner adopts a state-contingent debt re-valuation. In the market economy, either the haircut $(1 - \xi)$ or the buyback markup ϱ will generate the necessary state contingency to replicate the Planner's allocation.

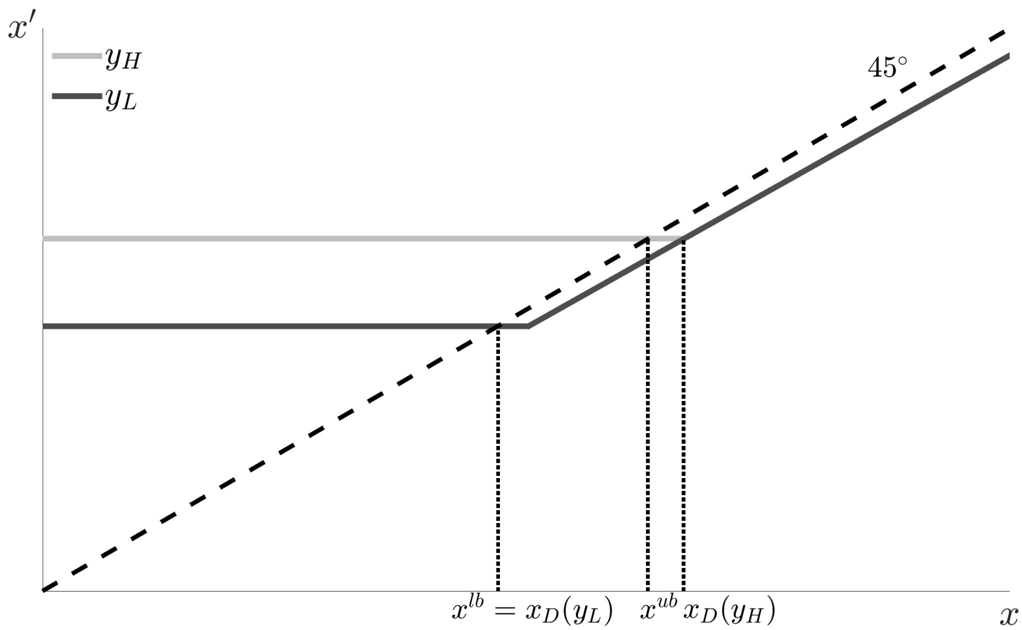
The Planner rules out the autarkic allocation whenever there are strictly positive rents to be shared among the contracting parties. That is why I assume the following.

Assumption 1 (Interiority)**.** *For all $y^t, t \geq 0$, there is a sequence $\{\tilde{c}(y^t)\}$ satisfying*

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{y^j} \pi(y^j) u(\tilde{c}(y^j)) - V^D(y_t) > 0.$$

Assumption 1 not only rules out autarky as a feasible allocation, it also ensures the uniform boundedness of the Lagrange multipliers. This guarantees existence and uniqueness of the Planner allocation.[10]

Proposition 5 (Existence and Uniqueness)**.** *Under Assumption 1, given initial conditions* (y_0, x_0)*, there exists a unique constrained efficient allocation.*



Note: The figure depicts the law of motion of the relative Pareto weight. The light grey line corresponds to the law of motion in y_H and the dark grey line to the law of motion in y_L. The black dotted line represents the 45° line. x^{lb} and x^{ub} correspond to the lower and upper bounds of the ergodic set, respectively. $x_D(y)$ corresponds to the relative Pareto weight at which the participation constraint binds in y.

Figure 1: Steady State Dynamic

I finally show that the long-term allocation is characterized by an ergodic set of relative Pareto weights. The term ergodic refers to the fact that the relative Pareto weights in this set are aperiodic and recurrent with non-zero probability.

Proposition 6 (Ergodic Set)**.** *The ergodic set of relative Pareto weights* $[x^{lb}, x^{ub}] \subset \tilde{X}$ *is such that* $x'(y_H, x^{ub}) = x^{ub}$*,* $x'(y_L, x^{lb}) = x^{lb}$ *with* $x^{lb} = x_D(y_L) < x^{ub} < x_D(y_H)$*.*

The proposition states that the steady state is dynamic and bounded in the interval $[x^{lb}, x^{ub}]$. For instance, after a sufficiently long series of y_L (y_H), the economy hits x^{lb} (x^{ub}). It then stays there until y_H (y_L) realizes and that irrespective of the past realizations of the shock. Figure 1 depicts the law of motion of the relative Pareto weight. The ergodic set corresponds to the interval $[x^{lb}, x^{ub}]$ and the basin of attraction to $[\underline{x}, x^{lb}) \cup (x^{ub}, \overline{x}]$.

[10]See also Proposition 4.10 in Alvarez and Jermann (2000).

4.3 Decentralization

Having derived and characterized the constrained efficient allocation, I construct two Markov equilibria that decentralize the Planner's allocation in the market economy. In particular, either buybacks based (8) or restructurings based on (9) can implement the constrained efficient allocation. I start with the restructuring-based decentralization.

Proposition 7 (Restructuring-Based Decentralization)**.** *Under Assumption 1 and $x_0 \leq x^{ub}$, a Markov equilibrium with $\chi(\Omega) = 0$ for all Ω decentralizes the constrained efficient allocation.*

The decentralization follows the ones of Müller et al. (2019) and Dovis (2019). The borrower restructures its debt when the economy hits x^{lb}. This is when the borrower's participation constraint binds enabling a restructuring offer with an haircut $1-\xi$ according to (9). The haircut reduces the value of outstanding debt. This generates the valuation losses and gains necessary to replicate the state contingency of the constrained efficient allocation. In particular, the borrower incurs a valuation gain in y_L through debt dilution and restructuring and a valuation loss in y_H when the bond prices recover.

As shown in the quantitative analysis next, the borrower conducts buybacks in y_L which is relatively cheap owing to the high probability of a restructuring depressing the bond prices. This goes against the evidence that buybacks conducted by the Brazilian government mostly occur in good times at a premium. That is why I consider an alternative decentralization in which there is no restructuring. Instead the borrower conduct buybacks at a markup which substitutes for the haircut. Markups and haircuts are the mirroring image of each other. The latter arise in the low endowment state and enable repayments below the notional payment, whereas the former arise in the high endowment state and enable repayments above the notional payment.

Proposition 8 (Buyback-Based Decentralization)**.** *Under Assumption 1 and $x_0 \leq x^{ub}$, a Markov equilibrium with $\phi(\Omega) = \infty$ for all Ω decentralizes the constrained efficient allocation.*

Proof. The proof of this proposition is by construction. I first determine the buyback policy necessary to implement the constrained efficient allocation and subsequently derive the underlying portfolio of bonds. Similar to Dovis (2019), I express the policy functions of the implemented Planner's allocation as a function of (y, x). Formally, define $\bar{D}, \bar{M}, \bar{R} : Y \times X \to \{0, 1\}$ and $\bar{q}_{st}, \bar{q}_{lt}, \bar{b}_{st}, \bar{b}_{lt}, \bar{\chi} : Y \times X \to \mathbb{R}$. Given the timing of actions, the bond policies and the price schedules can be rewritten as $\bar{b}_j(y, x) = \bar{b}_j(x'(y, x))$ and $\bar{q}_j(y, x) = \bar{q}_j(x'(y, x))$ for all $j \in \{st, lt\}$.

To generate the appropriate state contingency, buybacks must arise when the economy draws y_H and $x < x^{ub}$

$$\bar{M}(y, x) = 1 \quad \text{for} \quad y = y_H \text{ and } x < x^{ub}. \tag{15}$$

Condition (8) implies that $\bar{\chi}(y, x) = \varrho > 0$ when y_H and $x < x^{ub}$ as $V^b(y_H, x) = V^D(y_H)$. I show below that there are no other buybacks at a markup so the determination of $\bar{M}(y, x)$ in the rest of the state space is irrelevant for the bond price. Autarky is never optimal under Assumption 1 and there is no restructuring offer meaning that $\bar{D}(y, x) = \bar{R}(y, x) = 0$ for all (y, x). Because buybacks at a markup arise in the transition from y_L to y_H, the long-term bond price is higher in y_L than in y_H.

Lemma 1 (Long-Term Bond Price With Buyback)**.** *Under* (15) *and* $\bar{D}(y, x) = \bar{R}(y, x) = 0$ *for all* (y, x)*, the long-term bond price is the unique fixed point of* $\bar{q}_{lt}$*, is decreasing and*

$$\frac{1 + \delta\varrho}{1 + r - \delta} > \bar{q}_{lt}(x'(y_L, x)) \geq \bar{q}_{lt}(x'(y_H, x)) \geq \bar{q},$$

with $\bar{q}_{lt}(x'(y_L, x)) > \bar{q}_{lt}(x'(y_H, x))$ *if* $\bar{b}_{lt}(x'(y_L, x)) < 0$ *and* $x \leq x^{ub}$*.*

When $x < x^{ub}$, the bond portfolio that replicates the constrained efficient allocation is

$$-V^l(y_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_H, x)) + \delta\varrho], \tag{16}$$
$$-V^l(y_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))]. \tag{17}$$

In opposition, for $x = x^{ub}$, the bond portfolio becomes

$$-V^l(y_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_H, x))],$$
$$-V^l(y_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma 1 ensures there exists a unique bond portfolio as long as $\bar{b}_{lt}(x'(y_L, x)) < 0$.[11] Combining (16) and (17), the short-term and long-term bond holdings when $x < x^{ub}$ are respectively

$$\bar{b}_{st}(x) = \frac{V^l(y_H, x)[1 + \delta\bar{q}_{lt}(x'(y_L, x))] - V^l(y_L, x)[1 + \delta\bar{q}_{lt}(x'(y_H, x)) + \delta\varrho]}{\delta[q_{lt}(x'(y_H, x)) + \varrho - \bar{q}_{lt}(x'(y_L, x))]},$$
$$\bar{b}_{lt}(x) = -\frac{V^l(y_H, x) - V^l(y_L, x)}{\delta[\bar{q}_{lt}(x'(y_H, x)) + \varrho - \bar{q}_{lt}(x'(y_L, x))]} < 0,$$

[11] Observe that there needs to be at least as many maturities as endowment states.

where $\bar{b}_{lt}(x) < 0$ comes from the fact that $V^l(y_H, x) > V^l(y_L, x)$ given Part III of Proposition 4 and from $\bar{q}_{lt}(x'(y_H, x)) + \varrho - \bar{q}_{lt}(x'(y_L, x)) = \frac{(1+r-\delta)(\bar{q}_{lt}(x'(y_H,x))+\varrho)-1}{1+r-\pi(y_L)\delta} > 0$ since $\bar{q}_{lt}(x'(y_H, x)) + \varrho > \bar{q}$.[12] In opposition, at $x \geq x^{ub}$,

$$\bar{b}_{lt}(x) = -\frac{V^l(y_H, x) - V^l(y_L, x)}{\delta[\bar{q}_{lt}(x'(y_H, x)) - \bar{q}_{lt}(x'(y_L, x))]} > 0,$$

From Proposition 6, $x'(y_H, x) = x^{ub}$ and $x'(y_L, x) < x^{ub}$ for all $x \leq x^{ub}$. The initial condition $x_0 \leq x^{ub}$ ensures that $x \leq x^{ub}$. This has two implications. First, $\bar{b}_{lt}(x'(y_L, x)) < 0$ guaranteeing that $\bar{q}_{lt}(x'(y_L, x)) > \bar{q}_{lt}(x'(y_H, x))$ for all $x \leq x^{ub}$ in Lemma 1. Second, there is a buyback with $\bar{b}_{lt}(x^{ub}) > 0$ and $\delta\bar{b}_{lt}(x) < 0$ whenever $y = y_H$ and $x < x^{ub}$ consistent with (15). Note further that at $x \geq x^{ub}$, the borrower possesses a long-term asset so there is no buyback and it is not possible to decentralize. Finally, to ensure that there is no buyback in x such that $x'(y_L, x) = x^{lb}$, it must be that $\delta \leq (V^l(y_H, x^{lb}) - V^l(y_L, x^{lb}))/(V^l(y_H, x) - V^l(y_L, x))$.[13]

Following Alvarez and Jermann (2000), to ensure that the participation constraint (10) holds, it should be that for all $y' \in Y$ and for all $x' \in \tilde{X}$

$$\bar{b}_{st}(x') + \bar{b}_{lt}(x')[1 + \delta\bar{q}_{lt}(x'(y', x')) + \bar{M}(y', x')\delta\bar{\chi}(y', x')] \geq \mathcal{G}(y'). \tag{18}$$

Equality holds when $x'(y', x') = x_D(y')$ which by definition gives for all $y' \in Y$ and for all $x' \in \tilde{X}$, $V^b(y', x') \geq V^D(y')$. The buyback policy (15) ensures the state contingency of $\mathcal{G}(y')$.

Let's verify the enforcement of (18). Fix y and assume that the borrower decides to issue more than the borrowing limit. First, consider that x is such that $x'(y_L, x) = x^{lb}$. In $y' = y_L$, since $V^b(y_L, x^{lb}) = V^D(y_L)$, the value under repayment falls below the value of default meaning that the borrower defaults. In $y' = y_H$, the key element is that $\mathcal{G}(y_H)$ is such that $\bar{\chi}(y_H, x^{ub}) = \varrho$. Given (8), any additional borrowing leads to $\bar{\chi}(y_H, x^{ub}) = \infty$. As a result, the borrower defaults as well in $y' = y_H$. Exceeding the borrowing limit is therefore unprofitable. Second, consider that x is such that $x'(y_L, x) > x^{lb}$. Using the same argument, the borrower will default in $y' = y_H$ which is suboptimal given Proposition 3.

This concludes the proof. I used the budget constraints in (2) and in (3) to determine the optimal bond holdings given the prices computed according to (6) and (7). Also, buybacks follow (8) and the participation constraint (10) holds. □

The decentralization works as follows. The borrower conducts buybacks when the economy transits from y_L to y_H. This is when the borrower's participation constraint binds

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[12]Under (15), $\bar{q}_{lt}(x'(y, x))$ only depends on y. Hence, $\bar{q}_{lt}(x'(y_L, x)) = \frac{1+\delta\pi(y_H)(\bar{q}_{lt}(x'(y_H,x))+\varrho)}{1+r-\delta\pi(y_L)}$ for $x \leq x^{ub}$.

[13]This condition is not necessarily binding. With the parameters used in Section 5, $\bar{b}_{lt}(x^{lb}) - \bar{b}_{lt}(x) < 0$ for x such that $x'(y_L, x) = x^{lb}$. Hence any $\delta \in (0, 1]$ is admissible there.

which enables a buyback with ϱ according to (8). The payment of the buyback markup increases the value of outstanding long-term debt, while the value of short-term bonds remain unchanged. This generates the valuation losses and gains necessary to replicate the state contingency of the constrained efficient allocation.[14]

The enforcement mechanism in the market economy is based on (8). In particular, when the borrower issues too much debt, the buyback deal vanishes (i.e. $\bar{\chi}(y,x) = \infty$) and the borrower receives the autarkic allocation. In the case of restructurings without buyback markups, enforcement is based on (9). When the borrower issues too much debt, the restructuring deal also vanishes (i.e. $\bar{\phi}(y,x) = \infty$). These two mechanisms imply that there is an implicit borrowing limit $\mathcal{G}(y)$ imposed by the incumbent lender's market power.[15]

Buybacks give a clear interpretation to the binding participation constraint in y_H. In this class of models, the borrower's participation constraint binds in the transition from y_L to y_H. This has often been wrongly interpreted as "defaults happen in good times". Under the above mechanism, I show that the borrower's binding constraint in y_H can be interpreted as a buyback. As shown in Proposition 4, the high endowment state is also associated with lower levels of indebtedness and capital outflows, aligning with the idea of a buyback. More generally, this implies that a buyback at a markup in y_H is inherent to complete markets.

Unlike the previous decentralization, buybacks can occur in both y_H and y_L. In bad times, there is zero markup owing to the non credible default threat. In opposition, in good times, buybacks are expensive given the markup. However, the presence of buybacks and the absence of defaults or restructurings make the long-term bond price dynamic counterfactual. First, the long-term bond interest rate spread is negative.[16] In the absence of defaults and restructurings, there is no positive spread. In addition, buybacks entail a markup implying that the long-term bond price exceeds $\overline{q}$. Second, the long-term bond price is higher in y_L than in y_H as shown in Lemma 1.

It is possible to decentralize the constrained efficient allocation with both markup and haircut offers. This approach enhances the predictions which are at odds with the data in the other two decentralizations. In particular, the markup enables expensive buybacks in good times, while the restructuring corrects part of the price dynamic. Given that χ and ϕ work against each other, the determination of the long-term bond price dynamic is delicate and depends on several parameters. For that reason, I do not provide a formal proof and present a quantitative illustration in the next section.

[14] In fact, the notional payment of the two bonds replicates the payout of a long position in Arrow securities, while combined with the buyback markup it replicates the payout of a short position.

[15] Notice that both decentralizations require that $x_0 \leq x^{ub}$. This ensures the bond price is strictly monotone in y implying the desired state contingency.

[16] This features in other decentralizations developed by Liu et al. (2023) and Ábrahám et al. (2025).

5 Quantitative Analysis

This section presents the numerical solution of the model and compares it with buybacks in Brazil. The details on the data can be found in the Online Appendix B.

5.1 Numerical solution

I provide a numerical example of the model. The utility function takes the CRRA form $u(c) = \frac{c^{1-\varpi}}{1-\varpi}$ and the lenders' return is $r = r^f + r^e$ where r^f is the risk-free rate and r^e is the excess return. The parametrization is as follows: $\varpi = 2$, $\beta = 0.935$, $r = r^f + r^e = 0.01 + 0.0246$, $\pi(y_H) = 0.7892$, $y_H = 0.3577$, $y_L = 0.3266$, $\delta = 0.975$, $\varrho = 0.01208$ and $1 - \xi = 0.293$. The value of ϖ and r^f are standard in the literature. The parameters related to the endowment process come from a 2-state Markov chain estimation from Brazil's real GDP between 1999q4 to 2019q4.[17] The maturity of long-term debt matches the highest maturity (40 years) issued by the Brazilian government since 1995. The discount factor comes from the calibration of Arellano and Ramanarayanan (2012) for Brazil. The excess return is the total real return on the US external position estimated by Gourinchas et al. (2017). The haircut comes from the estimation of Cruces and Trebesch (2013) for Brazil last restructuring in 1994. Finally, the buyback markup corresponds to the maximum cost of buyback estimated for US treasury bonds by Han et al. (2007).

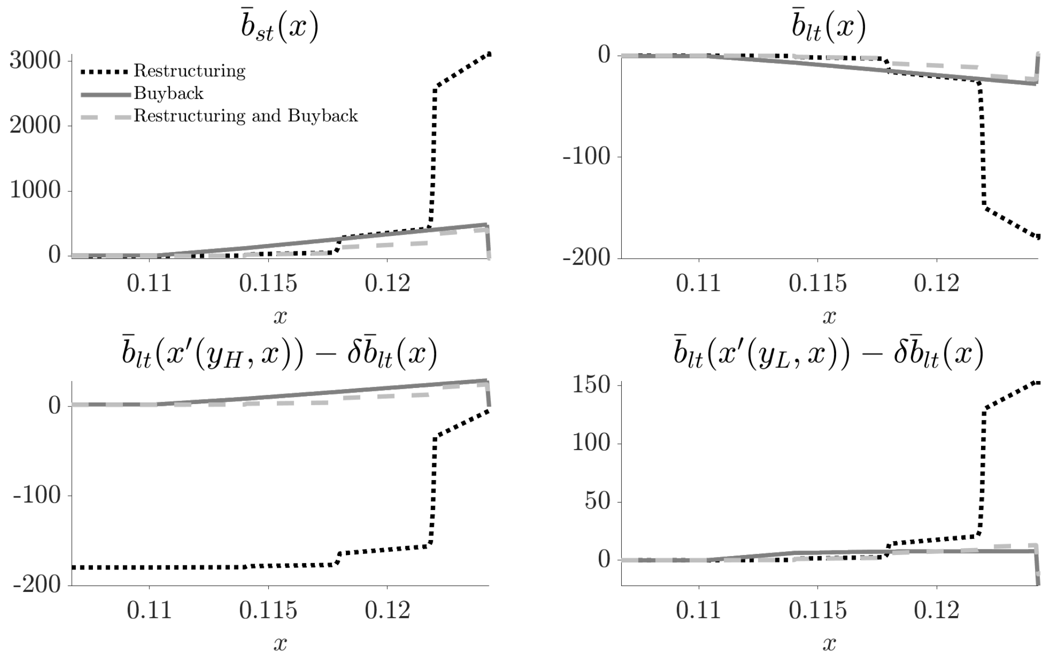
Since the interest rate spread is calculated with respect to the risk-free rate, modelling an excess return enables to correct the negative spread in the buyback-based decentralization.[18] In particular, the short-term bond interest rate spread is $\bar{r}_{st}(y,x) - r^f = \frac{1}{q_{st}(y,x)} - 1 - r^f$ and the long-term bond interest rate spread is $\bar{r}_{lt}(y,x) - r^f = \frac{1}{q_{lt}(y,x)} - (1-\delta) - r^f$.

Figure 2 depicts the bond policy functions that implement the constrained efficient allocation in the interval $[x^{lb}, x^{ub}]$. The dotted line corresponds to the restructuring-based decentralization, the solid line to the buyback-based decentralization and the dashed line combines both restructurings and buybacks. In terms of dynamics, the bond portfolios are similar across decentralizations.[19] The only difference is at x^{ub}. In the restructuring-based decentralization, the long-term bond position remains negative, while it becomes positive in the other two decentralizations. As a result there is no buyback in the transition for y_L

[17] The vector $(\pi(y_H), \pi(y_L))$ corresponds to the stationary distribution of the estimated transition matrix.

[18] This approach is simpler than assuming a pricing kernel similar to Arellano (2008) and Arellano and Ramanarayanan (2012). The reason is that I want to illustrate the state contingency emerging from the buyback and the restructuring, not from the lenders' marginal utility.

[19] Consistent with the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), the borrower holds long-term and short-term bonds in the magnitude of several multiples of GDP. Even though I consider an alternative environment without commitment, the bond portfolio implementing the constrained efficient allocation still implies large movements in bond holdings.



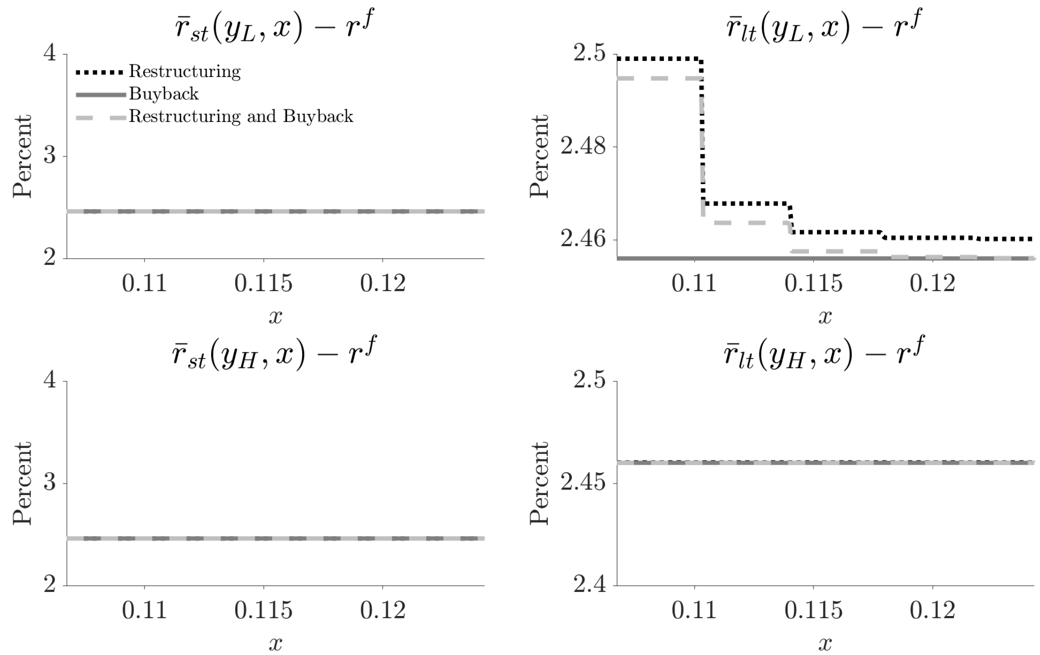
Note: The figure depicts the bond policy functions, which decentralize the constrained efficient allocation, as a function of the relative Pareto weight in the interval $[x^{lb}, x^{ub}]$.

Figure 2: Bond Policy Functions

to y_H in the restructuring-based decentralization. Buybacks occur in y_L when the default probability is high meaning that such operations are relatively cheap. Dovis (2019) also reports this dynamic. In the other two decentralizations, the borrower conducts buybacks at a markup whenever y_H realizes. There are also buybacks in y_L, when x is close enough to x^{ub}, but of smaller magnitude.

Figure 3 depicts the bond interest rate spreads that implement the constrained efficient allocation. The interest rate spread on the short-term bond is constant in all states. This is because buybacks only apply to long-term debt and the borrower holds short-term assets during restructurings. For the long-term bond, without restructuring, the excess return is enough to correct the negative spread. However, it remains true that $\bar{r}_{lt}(y_L, x) < \bar{r}_{lt}(y_H, x)$. This is counterfactual as noted earlier. Adding restructuring, we obtain that $\bar{r}_{lt}(y_L, x) \geq \bar{r}_{lt}(y_H, x)$ except in the vicinity of x^{ub} which ensures that $\bar{b}_{lt}(x^{ub}) > 0$ as shown in Proposition 8.[20] This implies that buybacks are expensive given the combination of the markup and the reduced restructuring risk.

[20]If $\bar{b}_{lt}(x^{ub}) < 0$ the bond portfolio dynamic is such that $\bar{b}_{lt}(x^{ub}) - \delta b_{lt}(x) \leq 0$ for all $x < x^{ub}$ invalidating the buyback implementation. Note further that without excess return, the decentralization combining markups and haircuts requires some negative spread to ensure that $\bar{b}_{lt}(x^{ub}) > 0$.



Note: The figure depicts the bond interest rate spreads, which decentralize the constrained efficient allocation, as a function of the relative Pareto weight in the interval $[x^{lb}, x^{ub}]$.

Figure 3: Bond Interest Rate Spreads

5.2 Buyback, default risk and market power

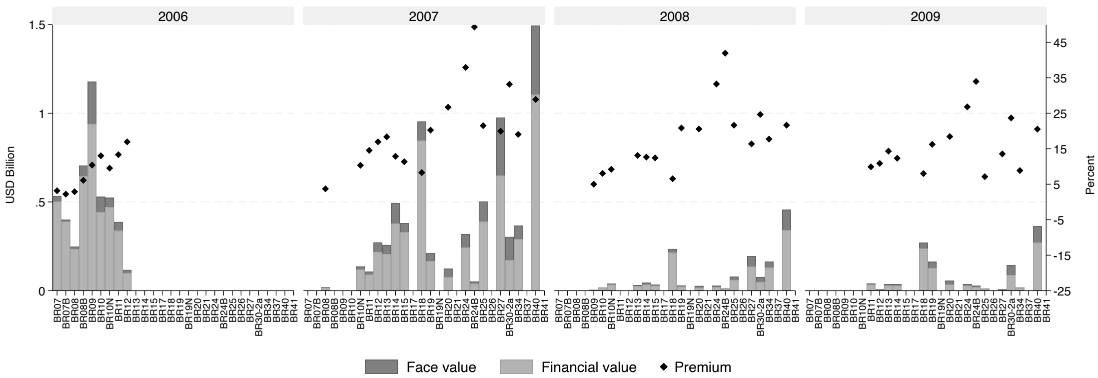
I investigate buybacks in Brazil and how they relate to the model's predictions. Brazil defaulted last in the 1980s and regained access to the international market after the implementation of the Brady Plan in 1994.[21] In 2006, the Brazilian government started a buyback program called Early Redemption Program. Repurchases were conducted by the Brazilian National Treasury and reported at the bimonthly frequency.

Figure 4 depicts the amount bought back by the Brazilian government between 2006 and 2017 for bonds denominated in USD.[22] The dark grey bar represents the face value, while the light grey bar corresponds to the financial value. In the period considered, buybacks amounted to a total of 24 USD billion in face value and 31 USD billion in financial value. Financial (face) value buybacks were the largest in 2007 with 7 (5) USD billion, in 2006 with 5 (4) USD billion and in 2013 with 5 (3) USD billion. In addition, buybacks were the largest for bonds with a relatively long residual maturity. For instance, buybacks for bonds due in

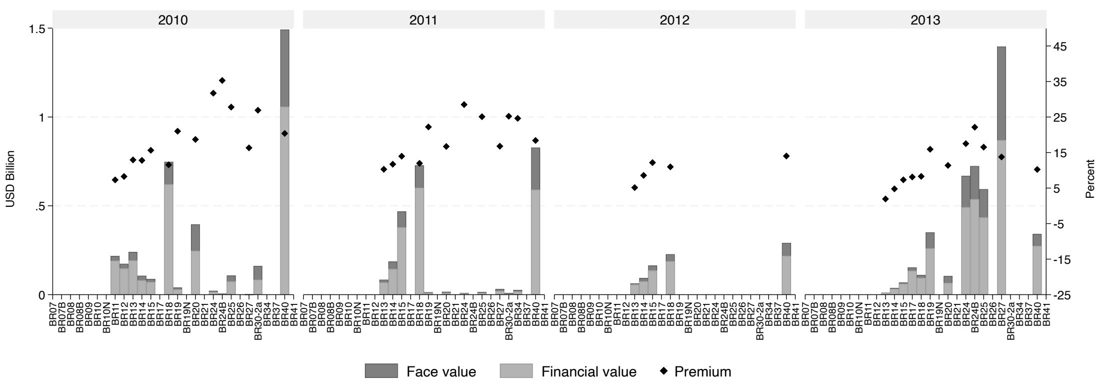
[21] The Brady Plan is an extensive debt restructuring program aimed at resolving the numerous of sovereign debt defaults in 1980s especially in Latin America. See Ayres et al. (2021) for a detailed economic history of Brazil.

[22] I omit a bond with floating coupon (BR09F) due to the lack of information on the coupon structure. Note further that there were only two buybacks in 2018 summing to 0.13 (0.11) USD billion in financial (face) value and no buyback in 2019.

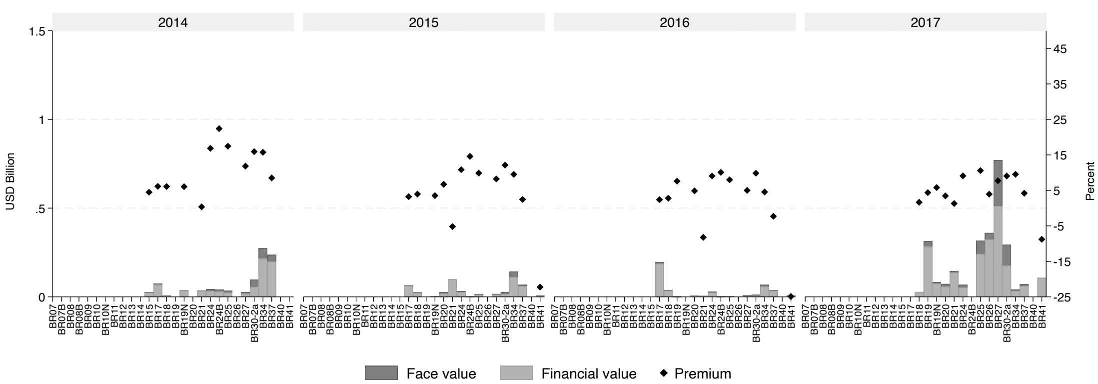
10 years or more amounted to 16 (11) USD billions in financial (face) value. On average, the Brazilian government bought back 41.44% of the total issuance of bonds in the program.



(a) 2006-2009



(b) 2010-2013



(c) 2014-2017

Note: The figure depicts the buyback amount and premium by year and by bonds. All bonds are denominated in USD. The Financial value corresponds to the amount required for payment of the securities redeemed, while the face value corresponds to the value of debt in the national statistics. The buyback premium is computed according to (19) and expressed in percent. There were only two buybacks in 2018 summing to 0.13 (0.11) USD billion in financial (face) value and no buyback in 2019.

Figure 4: Buyback Premium by Bond and Year

Most of the Brazilian buybacks occured when GDP was growing and the bond price was high. Table 1 depicts the outcome of fixed effects regressions of the face value of buybacks in USD million on the nominal GDP growth, the bond price and the EMBI+ spread for the period 1997m2 to 2019m12.[23] There is a positive association between buybacks and GDP growth both seasonally (when looking at bimonthly growth) and over the longer time horizon (when looking at yearly growth). Similarly, buybacks correlates positively with the individual bond price and negatively with the EMBI+ spread. As a result, there is evidence that the Brazilian government repurchased its debt in good times at high prices.

Table 1: Face Value Buyback Regressions

	(1) Face Value Buyback	(2) Face Value Buyback	(3) Face Value Buyback	(4) Face Value Buyback
GDP Growth, Bimonthly	4.59***	3.62***		
	[1.19]	[1.33]		
GDP Growth, Yearly			1.82***	1.61***
			[0.43]	[0.48]
Bond Price	0.29***		0.28***	
	[0.06]		[0.06]	
EMBI+ Spread		-2.25***		-2.13***
		[0.47]		[0.47]
Bond FE	Yes	Yes	Yes	Yes
Observations	2348	2348	2348	2348
R^2 adjusted	0.03	0.03	0.03	0.04

Note: The table depicts the outcome of OLS regressions of buybacks in USD billion on the nominal GDP growth and the EMBI+ spread for the period 1997m2 to 2019m12. *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

The decentralization with buybacks and restructurings in Section 4 makes clear predictions about the timing and the cost of buybacks. As in the data, the model predicts costly buybacks in good times. The source of such buybacks is twofold: the reduced restructuring risk in good times and the market power of legacy lenders in the secondary market.

To assess the assumption of reduced competitiveness in the secondary market, I use the secondary-market bid and ask prices and the primary-market issuance price. I first compare the secondary-market price immediately after issuance to the primary-market price to measures the efficiency of primary-market pricing. Specifically, if the bond trades at a higher price in the secondary market shortly after issuance, it suggests that the bond was underpriced initially implying a relatively less competitive primary market. The daily mean difference in bond price between the secondary and primary markets at issuance is -0.019 percent with a standard error of 0.20. It is not significantly different from zero and

[23] Results are similar if one use the financial value of buybacks.

Table 2: Bid-Ask and Credit Default Swap Spreads Regressions

	(1) BAP Spread	(2) CDS Spread	(3) BAP Spread	(4) CDS Spread
Buyback	1.03**	-1.37***	1.14***	-1.02***
	[0.43]	[0.13]	[0.44]	[0.13]
Volume at Issuance	-0.12	0.29***		
	[0.08]	[0.07]		
Price at Issuance	0.01***	-0.04***		
	[0.00]	[0.01]		
Original Maturity	0.05***	-0.07***		
	[0.01]	[0.01]		
Coupon	0.01	23.40***		
	[1.68]	[3.14]		
Constant	-1.16**	5.80***	0.55***	3.07***
	[0.47]	[0.92]	[0.03]	[0.09]
Bond FE	No	No	Yes	Yes
Observations	2348	2348	2348	2348
R^2 adjusted	0.02	0.07	0.05	0.19

Note: The table depicts the outcome of OLS and fixed effects regressions of respectively the bid-ask price (BAP) spread and the 5-year credit default swap (CDS) spread on a buyback dummy taking value 1 in case of a buyback and other bond-specific control variables. *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

the magnitudes is economically small – i.e. less than 2 basis points. I therefore reject the hypothesis that the primary market is less competitive than the secondary market.

Second, I examine the bid-ask price (BAP) spread in the secondary market, a commonly used proxy for market competition and liquidity. A wider spread implies weaker market competition and a larger mismatch between supply and demand. Table 2 reports the results of OLS and fixed effects regressions using the BAP spread as the dependent variable and a buyback dummy variable taking value one during buybacks as the main explanatory variable. Two comments are in order. First, the spread increases during buybacks in the pooled OLS regression. This indicates that Brazil targeted less liquid bonds in the buyback. Second, in the fixed effects regression, the BAP spread also increases during buybacks. I interpret this positive relationship as evidence of reduced competition in the secondary market. Even if the repurchased bonds were illiquid to begin with, one would expect buybacks to improve, rather than impair, liquidity.

To assess the assumption of reduced default risk, I use the 5-year credit default swap (CDS) spread on Brazilian external bonds. This spread refers to the annual premium paid to insure against the default risk. In that logic, a higher CDS spread indicates a higher perceived default risk. Results are again reported in Table 2. In both OLS and fixed effects regression, a buyback is associated with a reduced CDS spread meaning a reduced default risk. The effect is both statically and economically significant. I interpret this as evidence that the default risk decreases during buyback operations. This additionally says that the widened BAP spread during buybacks cannot be explained by debt distress. This contrasts sharply with the buybacks documented by Bulow and Rogoff (1988) in the 1980s, which were carried out amid debt crises.

Table 3: Bond Price Regressions

	(1) Price	(2) Price
CDS Spread	-2.59***	-2.54***
	[0.10]	[0.10]
BAP Spread	-0.43***	-0.78***
	[0.07]	[0.15]
Δ Volume Outstanding	-1.66	-1.21
	[1.07]	[0.99]
Buyback	2.48***	7.57***
	[0.59]	[0.96]
Buyback $\times$ CDS Spread		-2.82***
		[0.40]
Buyback $\times$ BAP Spread		0.34**
		[0.17]
Constant	123.57***	123.58***
	[0.40]	[0.41]
Bond FE	Yes	Yes
Observations	2348	2348
R^2 adjusted	0.72	0.72

Note: The table depicts the outcome fixed effects regressions of the bond price on the bid-ask price (BAP) spread, the 5-year credit default swap (CDS) spread, a buyback dummy taking value 1 in case of a buyback and its interaction with the spread variables. *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

Finally, I study the impact of the CDS and the BAP spreads on the bond price. Table 3 depicts the outcome of fixed effects regressions using the bond price as the dependent variable. There is a negative association between the CDS spread and the bond price within bonds. This association becomes even more negative when interacted with the buyback dummy variable. This is consistent with the model's prediction: the bond price increases with lower default risk, a situation in which expensive buybacks arise. For the BAP spread the result is different. In general, the BAP spread is also negatively associated with the bond price. This is a standard result: a large BAP spread reflects low liquidity and thus a low bond price. However, during buybacks, the dampened negative effect of the BAP spread suggests a distortion to competitive pricing. Although the total effect remains negative, this is again evidence that buybacks are associated with less competitive conditions in the secondary market.

It is difficult to gauge the relative importance of the default risk and the secondary market competition on the bond price. While the CDS spread gives an accurate estimation of default risk, the BAP spread is an imperfect proxy for competitiveness. Alternative measures such as coverage ratios and trading volumes are unfortunately not readily available.

5.3 Buyback premium

In the model, the buyback premium reflects the valuation gain of the incumbent lender. I estimate such premium by comparing the cashflow stream of a bond with and without a buyback. A bond contract specifies a sequence of coupon payments and a principal repayment at maturity. A borrower issues bonds on the primary market at a given price. In the absence of a buyback, the borrower receives the value of the bond at the primary-market price and transfers to the lenders the coupons and the principal when due up to maturity. In the presence of a buyback, the borrower transfers the coupons due up to the buyback and the secondary-market price of the bond during the buyback.

Formally, consider a bond with a face value b, a coupon rate κ, a yield to maturity i and a maturity T. The present value of the coupons is given by $FVC(T,\kappa,b;i) = \sum_{t=1}^{T} \frac{b\kappa}{(1+i)^t} = \frac{1-(1+i)^{-T}}{i} b\kappa$. The present value of the face value corresponds to $FVP(T,b;i) = \frac{b}{(1+i)^T}$. Given this, in the absence of a buyback, the financial value of a bond for a given i is

$$FV^{NB}(T,\kappa,b;i) = FVC(T,\kappa,b;i) + FVP(T,b;i).$$

Evaluated at the issue yield to maturity, this equation gives the value the lenders get if they hold the bond from the issuance to the maturity.

In the case of a buyback, the borrower repurchases the bond prior to maturity, say $t_B < T$. The bond is repurchased on the secondary market at a financial value $B = FVC(T - t_B, \kappa, b; i_B) + FVP(T - t_B, b; i_B)$ where i_B is the yield to maturity at t_B. Thus, in the presence of buybacks, the financial value of a bond for a given i is

$$FV^{B}(t_B,\kappa,b,B;i) = FVC(t_B,\kappa,b;i) + FVP(t_B,B;i).$$

The yield to maturity represents the internal rate of return of the bond. Hence, whenever $FV^{B}(t_B,\kappa,b,B;i) > FV^{NB}(T,\kappa,b;i)$ for a given i, the borrower pays a premium in the buyback operation. In the opposite case, there is a discount.

As the borrower issues bonds on the primary market and potentially buys them back on the secondary market, there are two yields to maturity to consider. The first one is the issue yield to maturity, i_I, which gives the bond's expected total return at issuance. The second one is i_B necessary to compute B. The buyback premium at issuance is therefore

$$\frac{FV^B(t_B, \kappa, b, B, i_I)}{FV^{NB}(T, \kappa, b, i_I)} - 1. \tag{19}$$

The computation of the buyback premium follows a similar logic as the computation of haircuts in Sturzenegger and Zettelmeyer (2008). Equation (19) compares the present value of the "old" debt (denominator) with the present value of the "new" debt (numerator). In that logic the premium corresponds to a negative haircut. I do not compute a "market" premium which would consist of replacing $FV^{NB}(T, \kappa, b, i_I)$ by its face value in (19). The reason is that this measure would exaggerate the premium paid by the borrower.

Figure 4 depicts the average buyback premium in percent by year and by bond. The premium averages 13.50% and has a median at 12.03% overall. The 5-th and the 95-th percentiles amount to 1.94% and 31.97%, respectively. The buyback premium is positive in the majority of cases. Only three bonds (BR21, BR37 and BR41) were repurchased at a discount. Hence, the lenders made substantial valuation gains according to this metric. This comes from the fact that bonds were usually issued below the par value and bought back above the par value. As one can see in Table B.2 in the Online Appendix B, the issue yield to maturity is almost always larger than the coupon rate. However, the opposite is true for the buyback yield to maturity.

My estimation strategy differs from the one of Bulow and Rogoff (1988, 1991). The two authors take the difference in the value of debt in the secondary market before and after the buyback. They then compare this difference with the face value of the buyback. This comparison reflects a more marked-to-market approach. However, it does not properly capture the capital transfer between the borrower and the lenders in the model as it computes the premium as if the debt was issued in the secondary market prior to the buyback.

Han et al. (2007) also develop a notion of buyback markup they call *buyback cost*. They estimate such cost as the difference between the weighted-average price paid during the buyback and the ask price in the secondary market at the time the auction results were announced. This requires exact data about the timing of buybacks which I do not have.

6 Conclusion

This paper shows that buybacks can be rationalized as part of an optimal contract between a sovereign borrower and foreign lenders. The bottom line is that buybacks allow bonds to function like Arrow securities. This is because issuances and buybacks occur in two distinct markets. On the one hand, buybacks take place in the secondary market, where the borrower can only deal with legacy lenders. On the other hand, new issuances occur in the primary market, where the borrower deals with legacy lenders and new entrants. Assuming that legacy lenders coordinate, the borrower faces a monopolist in the secondary market and competitors in the primary market leading to a higher payout of legacy long-term debt during buybacks than during issuances. As legacy lenders are the sole holder of defaulted debt, it also faces a monopolist during restructurings which can complement the above implementation.

Furthermore, buybacks provide a clear interpretation for the binding participation constraint in optimal contracts. The property that the borrower's participation constraint binds when endowment increases has often been wrongly interpreted as "defaults happen in good times". Instead, buybacks align with the fact that indebtedness decreases and capital flows out in the high endowment state.

There is evidence of reduced competition in the secondary market during buybacks when looking at Brazil's Early Redemption Program between 2006 and 2019. However, this is not the only factor explaining the presence of a buyback premium. The reduced default risk also plays an important role.

This study stresses the fact that the strategic interaction of the lenders is key. The literature on sovereign debt and default has focused on the borrower's side. However, it is possible to explain a variety of alternative dynamics in equilibrium by looking at the lenders and the way they interact.

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Online Appendix

(Not for Publication)

A Proofs

A.1 Proof of Proposition 1

Proposition 1 (Optimal Offers)**.** *In the primary and the secondary markets, the two lenders offer bond contracts with prices (q_{st}, q_{lt}) satisfying (6)-(7) implying that (5) hold with equality in all states (y, b_{st}, b_{lt}).*

Proof. I first prove that the two lenders always offer competitive prices. For this, consider that the two lenders offer the same quantity $(b^{o\prime}_{st}, b^{o\prime}_{lt}) = (b^{i\prime}_{st}, b^{i\prime}_{lt}) = (b'_{st}, b'_{lt})$ but at potentially different prices. Denote by (q_{st}, q_{lt}) the competitive price vector satisfying (6)-(7). I start with the case without buyback meaning that $b'_{lt} < \delta b_{lt}$ if $b_{lt} < 0$ and distinguish 3 cases:

1. Suppose the incumbent offers $(q^i_{st}, q^i_{st}) = (q_{st}, q_{lt})$. If the incumbent's offer is accepted, its payoff is given by

   $$-b_{st} - b_{lt} + q_{st}b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

   Observe that if $b'_{lt} - \delta b_{lt} < 0$, the borrower would be strictly better off with a price higher than q_{lt}. In opposition, if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$, the borrower would be strictly better off with a price lower than q_{lt}. Similarly if $b_{st} < 0$ ($b_{st} > 0$) the borrower would be strictly better off with a price higher (lower) than q_{st}. Hence, depending on the sign of $b'_{lt} - \delta b_{lt}$ and b_{st}, the outsider can undercut the incumbent offer by appropriately offering a higher or a lower price than the competitive price.

   However, if the outsider offers such prices, it incurs a loss. When $b'_{lt} - \delta b_{lt} < 0$ or $b_{st} < 0$, the outsider is selling debt at a price which is lower than the present value of expected returns. Similarly, when $b'_{lt} - \delta b_{lt} > 0$ or $b_{st} > 0$, it is taking debt at a price which is higher than the present value of expected payouts. If the outsider's offer is accepted, its payoff is

   $$q^o_{st}b'_{st} + q^o_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

   In opposition, if the outsider offers (q_{st}, q_{lt}) and its offer is accepted, its payoff is

   $$q_{st}b'_{st} + q_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})].$$

Note that there is no buyback meaning that $b'_{lt} - \delta b_{lt} < 0$ if $b_{lt} < 0$. As $(q_{st} - q^o_{st})b'_{st} + (q_{lt} - q^o_{lt})(b'_{lt} - \delta b_{lt}) > 0$, the outsider is strictly worse off than offering (q_{st}, q_{lt}).

2. Suppose the incumbent offers $q^i_{st} < q_{st}$ if $b'_{st} < 0$, $q^i_{st} > q_{st}$ if $b'_{st} > 0$, $q^i_{lt} < q_{lt}$ if $b'_{lt} - \delta b_{lt} < 0$ and $q^i_{lt} > q_{lt}$ if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$. The outsider can undercut the incumbent's offer by offering $(q^o_{st}, q^o_{lt}) = (q_{st}, q_{lt})$. The borrower strictly prefers the outsider's offer, meaning that the incumbent's payoff is

$$-b_{st} - b_{lt}(1 + \delta q^o_{lt}),$$

which by (5) is weakly lower than the payoff when the incumbent offers (q_{st}, q_{lt}).

3. Suppose the incumbent offers $q^i_{st} > q_{st}$ if $b'_{st} < 0$, $q^i_{st} < q_{st}$ if $b'_{st} > 0$, $q^i_{lt} > q_{lt}$ if $b'_{lt} - \delta b_{lt} < 0$ and $q^i_{lt} < q_{lt}$ if $b'_{lt} - \delta b_{lt} > 0$ with $b_{lt} > 0$. If the outsider offers $(q^o_{st}, q^o_{lt}) = (q_{st}, q_{lt})$, the borrower strictly prefers the incumbent's offer. Repeating the argument of the first case, the incumbent is strictly worse than offering (q_{st}, q_{lt}).

Consequently, the two lenders offer competitive prices. For $b_{st} < 0$ and $b'_{lt} - \delta b_{lt} < 0$ ($b_{st} > 0$ and $b'_{lt} - \delta b_{lt} > 0$), they have no incentive to offer a price higher (lower) than (q_{st}, q_{lt}) as this would result to losses. There is also no reason to offer a lower (higher) price than (q_{st}, q_{lt}) as one lender can undercut the other.

Observe however that the incumbent and the outsider have slightly different objective functions. For a given quantity of bonds (b'_{st}, b'_{lt}), the incumbent solves

$$\max_{(q^i_{st}, q^i_{lt})} \Big\{ - b_{st} - b_{lt} + q^i_{st} b'_{st} + q^i_{lt}(b'_{lt} - \delta b_{lt}) + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})] \Big\},$$

while the outsider solves

$$\max_{(q^o_{st}, q^o_{lt})} \Big\{ q^o_{st} b'_{st} + q^o_{lt} b'_{lt} + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})] \Big\}.$$

The difference is the value of outstanding debt $-b_{st} - b_{lt}(1 + \delta q^i_{lt})$ which depends on the offered long-term bond price. When $b_{lt} < 0$, the incumbent has therefore an incentive to offer a higher long-term bond price to maximize the value of outstanding long-term debt. At the same time, it can offer a lower short-term bond price such that the net expected profit of new issuances is zero. Such an offer would violate (6)-(7). If the outsider offers prices satisfying (6)-(7), the borrower will prefer that offer and the outsider becomes the incumbent. The reason is that maximizing the value of outstanding long-term debt corresponds to maximizing the borrower's repayment which reduces the borrower's value. Thus, bond prices always satisfy (6)-(7).

In the case with buyback, the two lenders compete on the short-term bond price. It is straightforward to repeat the previous argument in that case. On the secondary market, suppose the incumbent offer a non-competitive price $\ddot{q}_{lt} > q_{lt}$ to maximize the value of outstanding debt. If the incumbent offers $(b'_{st}, q_{st}; b'_{lt}, \ddot{q}_{lt}, \chi)$, the outsider can undercut the incumbent by offering $(b'_{st}, q_{st}; b'_{lt}, q_{lt}, \chi)$. The borrower will prefer the outsider's offer as $\ddot{q}_{lt} > q_{lt}$ implies a more costly buyback. As a result, irrespective on whether the borrower enters the secondary market, it holds that

$$q_{st}b'_{st} + q_{lt}b'_{lt} + \frac{1}{1+r}\mathbb{E}[W(y', b'_{st}, b'_{lt})] = 0.$$

The equality comes from the fact that competitive prices imply zero expected returns on new bond contracts. Hence, (5) holds with equality. □

A.2 Proof of Proposition 2

Proposition 2 (Optimal Bond Contracts)**.** *In equilibrium, if $M(y, b_{st}, b_{lt}) = R(y, b_{st}, b_{lt}) = D(y, b_{st}, b_{lt}) = 0$, the optimal bond contracts $B_{st}(y, b_{st}, b_{lt})$ and $B_{lt}(y, b_{st}, b_{lt})$ solve*

$$\begin{aligned} V^P_{NB}(y, b_{st}, b_{lt}) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right] \right\} \\ s.t. \quad & c + q_{st}(b'_{st}, b'_{lt})b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}, \\ & (6),\ (7),\ b'_{lt} \leq \delta b_{lt} \text{ if } b_{lt} < 0. \end{aligned}$$

If $M(y, b_{st}, b_{lt}) = 1$, $B_{st}(y, b_{st}, b_{lt})$ and $B_{lt}(y, b_{st}, b_{lt})$ solve for a given χ

$$\begin{aligned} V^P_{B}(y, b_{st}, b_{lt}) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right] \right\} \\ s.t. \quad & c + q_{st}(b'_{st}, b'_{lt})b'_{st} + q_{lt}(b'_{st}, b'_{lt})(b'_{lt} - \delta b_{lt}) = y + b_{st} + b_{lt}(1 + \delta\chi), \\ & (6),\ (7),\ b_{lt} < 0 \text{ and } b'_{lt} > \delta b_{lt}, \end{aligned}$$

and $\chi(y, b_{st}, b_{lt})$ satisfies (8). Finally, if $R(y, b_{st}, b_{lt}) = 1$, $B_{st}(y, b_{st}, b_{lt})$ and $B_{lt}(y, b_{st}, b_{lt})$ solve for a given ϕ

$$\begin{aligned} V^R(y, b_{st}, b_{lt}) = \max_{b'_{st}, b'_{lt}} & \left\{ u(c) + \beta \mathbb{E}\left[V(y', b'_{st}, b'_{lt})\right] \right\} \\ s.t. \quad & c + q_{st}(b'_{st}, b'_{lt})b'_{st} + q_{lt}(b'_{st}, b'_{lt})b'_{lt} = y + (\min\{0, b_{st}\} + \min\{0, b_{lt}\}[1 + \delta q_{lt}(b'_{st}, b'_{lt})])\phi, \\ & (6),\ (7), \end{aligned}$$

and $\phi(y, b_{st}, b_{lt})$ satisfies (9).

Proof. Proposition 1 shows that lenders always offer bond contracts satisfying (6) and (7). I additionally need to show that the lenders' offer needs to maximize the borrower's utility. First consider the case without buyback. Assume by contradiction that the incumbent offers $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that (6) and (7) hold but does not satisfy (2). Then the outsider can make a counter offer such that (6), (7) and (2) hold. The borrower would prefer this offer and the outsider becomes the incumbent.

In the case of a restructuring, the same argument holds true as the borrower immediatly re-access the primary market. The optimal haircut satisfies (9).

In the case of a buyback, the outsider can only compete with the incumbent on the short-term bond contract. Hence, for the short-term bond contract, the same argument as before holds. For the long-term bond contract, the buyback happens only if the borrower decides to enter the secondary market for a given χ. As a result, the incumbent's offer satisfies (3) and the optimal buyback markup satisfies (8). □

A.3 Proof of Proposition 3

Before proving Proposition 3, I present some intermediate results. First, I show the monotonicity of the value under buyback, no buyback and non repayment.

Proposition A.1. *$V^P_{NB}(y, b_{st}, b_{lt})$ and $V^P_B(y, b_{st}, b_{lt})$ are strictly increasing in (y, b_{st}, b_{lt}) and $V^D(y)$ is strictly increasing in y.*

Proof. Consider that the optimal offer under y_H is $(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})$ and under y_L is $(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})$. One the has that

$$\begin{aligned}
V^P_{NB}(y_H, b_{st}, b_{lt}) &= u(y_H + b_{st} + b_{lt} - q^H_{st} b^{H\prime}_{st} - q^H_{lt}(b^{H\prime}_{lt} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b^{H\prime}_{lt}, b^{H\prime}_{st})] \\
&\geq u(y_H + b_{st} + b_{lt} - q^L_{st} b^{L\prime}_{st} - q^L_{lt}(b^{L\prime}_{lt} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b^{L\prime}_{lt}, b^{L\prime}_{st})] \\
&> u(y_L + b_{st} + b_{lt} - q^L_{st} b^{L\prime}_{st} - q^L_{lt}(b^{L\prime}_{lt} - \delta b_{lt})) + \beta \mathbb{E}[V(y', b^{L\prime}_{lt}, b^{L\prime}_{st})] \\
&= V^P_{NB}(y_L, b_{st}, b_{lt}),
\end{aligned}$$

where the first inequality comes from optimality and the second from $y_H > y_L$. The exact same argument can be repeated for $V^P_B(y, b_{st}, b_{lt})$. Moreover,

$$V^D(y_H) = u(y_H) + \beta \mathbb{E}[V^D(y')] > u(y_L) + \beta \mathbb{E}[V^D(y')] = V^D(y_L),$$

where the inequality comes from $y_H > y_L$.

Consider $(b^1_{st}, b^1_{lt}) < (b^2_{st}, b^2_{lt})$. Denote the optimal offer under (b^2_{st}, b^2_{lt}) by $(b^{2\prime}_{st}, q^2_{st}; b^{2\prime}_{lt}, q^2_{lt})$ and under (b^1_{st}, b^1_{lt}) by $(b^{1\prime}_{st}, q^1_{st}; b^{1\prime}_{lt}, q^1_{lt})$. One the has that

$$\begin{aligned} V^P_{NB}(y, b^2_{st}, b^2_{lt}) &= u(y + b^2_{st} + b^2_{lt} - q^2_{st} b^{2\prime}_{st} - q^2_{lt}(b^{2\prime}_{lt} - \delta b^2_{lt})) + \beta \mathbb{E}[V(y', b^{2\prime}_{lt}, b^{2\prime}_{st})] \\ &\geq u(y + b^2_{st} + b^2_{lt} - q^1_{st} b^{1\prime}_{st} - q^1_{lt}(b^{1\prime}_{lt} - \delta b^2_{lt})) + \beta \mathbb{E}[V(y', b^{1\prime}_{lt}, b^{1\prime}_{st})] \\ &> u(y + b^1_{st} + b^1_{lt} - q^1_{st} b^{1\prime}_{st} - q^1_{lt}(b^{1\prime}_{lt} - \delta b^1_{lt})) + \beta \mathbb{E}[V(y', b^{1\prime}_{lt}, b^{1\prime}_{st})] \\ &= V^P_{NB}(y, b^1_{st}, b^1_{lt}), \end{aligned} \tag{A.1}$$

where the first inequality comes from optimality and the second from $(b^1_{st}, b^1_{lt}) < (b^2_{st}, b^2_{lt})$. The exact same argument can be repeated for $V^P_B(y, b_{st}, b_{lt})$. □

Second, I show that there is no contract with capital inflow under default risk. This is a repetition of Proposition 2 in Arellano (2008). Define $\mathfrak{D}(b_{st}, b_{lt}) = \{y : D(y, b_{st}, b_{lt}) = 1\}$ as the default set.

Proposition A.2. *If $\mathfrak{D}(b_{st}, b_{lt}) \neq \emptyset$ for some (b_{st}, b_{lt}), then there are no bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $b_{st} + b_{lt} - q_{st} b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) > 0$.*

Proof. The proof follows Arellano (2008, Proposition 2). Suppose by contradiction that there is a contract $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) = b_{st} + b_{lt} - q_{st} b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) > 0$ but the borrower prefers an alternative contract $(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})$ such that $\Delta(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt}) = b_{st} + b_{lt} - \tilde{q}_{st} \tilde{b}'_{st} - \tilde{q}_{lt}(\tilde{b}'_{lt} - \delta b_{lt}) \leq 0$ and finds default to be optimal because $u(y + \Delta(\tilde{b}'_{st}, \tilde{q}_{st}; \tilde{b}'_{lt}, \tilde{q}_{lt})) + \beta \mathbb{E}[V(y', \tilde{b}'_{lt}, \tilde{b}'_{lt})] < u(y) + \beta \mathbb{E}[V^D(y')]$.

Observe that under all contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) > 0$, the borrower prefers to repay than to default. This is because $u(y + \Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt})) > u(y)$ and $\mathbb{E}[V(y', b'_{lt}, b'_{lt})] \geq \mathbb{E}[V^D(y')]$ for $b'_{st} + b'_{lt}(1 - \delta q'_{lt}) \leq 0$. This contradicts the fact that both $(\tilde{b}'_{st}, \tilde{q}'_{st}; \tilde{b}'_{lt}, \tilde{q}'_{lt})$ and default are optimal. □

Given Proposition A.2, I can show that the default risk diminishes when y increases if there is no bond contract with capital inflow.

Proposition A.3. *If all bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ are such that $b_{st} + b_{lt} - q_{st} b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) \leq 0$, then $V^P_{NB}(y_H, b_{st}, b_{lt}) - V^D(y_H) \geq V^P_{NB}(y_L, b_{st}, b_{lt}) - V^D(y_L)$.*

Proof. Suppose that all bond contracts $(b'_{st}, q_{st}; b'_{lt}, q_{lt})$ are such that $\Delta(b'_{st}, q_{st}; b'_{lt}, q_{lt}) = b_{st} + b_{lt} - q_{st} b'_{st} - q_{lt}(b'_{lt} - \delta b_{lt}) \leq 0$. Now consider that the optimal offer under y_H is $(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})$ and under y_L is $(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})$. From optimality, one has that

$$u(y_H + \Delta(b^{H\prime}_{st}, q^H_{st}; b^{H\prime}_{lt}, q^H_{lt})) + \beta \mathbb{E}\left[V(y', b^{H\prime}_{st}, b^{H\prime}_{lt})\right] \geq u(y_H + \Delta(b^{L\prime}_{st}, q^L_{st}; b^{L\prime}_{lt}, q^L_{lt})) + \beta \mathbb{E}\left[V(y', b^{L\prime}_{st}, b^{L\prime}_{lt})\right].$$

So if

$$u(y_H)+\beta\mathbb{E}\left[V^D(y')\right]-\Big[u(y_L)+\beta\mathbb{E}\left[V^D(y')\right]\Big]\leq \tag{A.2}$$
$$u(y_H+\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L))+\beta\mathbb{E}\left[V(y',b_{st}^{L\prime},b_{lt}^{L\prime})\right]-\Big[u(y_L+\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L))+\beta\mathbb{E}\left[V(y',b_{st}^{L\prime},b_{lt}^{L\prime})\right]\Big].$$

then one gets that

$$u(y_H)+\beta\mathbb{E}\left[V^D(y')\right]-\Big[u(y_L)+\beta\mathbb{E}\left[V^D(y')\right]\Big]\leq$$
$$u(y_H+\Delta(b_{st}^{H\prime},q_{st}^H;b_{lt}^{H\prime},q_{lt}^H))+\beta\mathbb{E}\left[V(y',b_{st}^{H\prime},b_{lt}^{H\prime})\right]-\Big[u(y_L+\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L))+\beta\mathbb{E}\left[V(y',b_{st}^{L\prime},b_{lt}^{L\prime})\right]\Big].$$

Simplifying (A.2), one obtains that

$$u(y_H)-u(y_L)\leq u(y_H+\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L))-u(y_L+\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L)),$$

which holds true given the strict concavity of $u(\cdot)$ and the fact that all bond contracts are such that $\Delta(b_{st}^{L\prime},q_{st}^L;b_{lt}^{L\prime},q_{lt}^L)\leq 0$. □

Given Propositions 1, A.1, A.2 and A.3, I can prove Proposition 3 showing the occurrence of defaults and restructurings.

Proposition 3 (Default and Restructuring)**.** *If $\phi(\Omega)=\infty$ for all Ω, $D(y_L,B_{st}(\Omega),B_{lt}(\Omega))\geq D(y_H,B_{st}(\Omega),B_{lt}(\Omega))=0$. If $\phi(y_L,b_{st},b_{lt})=\xi$ for some (b_{st},b_{lt}), $D(y_L,b_{st},b_{lt})=0$.*

Proof. Consider first the case that $\phi(\Omega)=\infty$. This implies that $R(\Omega)=0$ as $V^R(\Omega)=-\infty$. Denote the bond policy vector $\boldsymbol{B}(\Omega)=(B_{st}(\Omega),B_{lt}(\Omega))$ and fix Ω. From Propositions A.2 and A.3, if there is a risk of default $V_{NB}^P(y_H,\boldsymbol{B}(\Omega))-V^D(y_H)\geq V_{NB}^P(y_L,\boldsymbol{B}(\Omega))-V^D(y_L)$. This means that if $D(y_H,\boldsymbol{B}(\Omega))=1$ then $D(y_L,\boldsymbol{B}(\Omega))=1$. The opposite is however not true. Hence, defaults arise in y_H only if they arise in y_L too. This implies that when $D(y_H,\boldsymbol{B}(\Omega))=1$, $\mathbb{E}[D(y,\boldsymbol{B}(\Omega))]=1$.

At these odds, bond prices are as follows. First, if $b_{st}'<0$ and $b_{lt}'<0$, then $q_{st}(b_{st}',b_{lt}')=q_{lt}(b_{st}',b_{lt}')=0$ given (6)-(7). Second, if $b_{st}'\geq 0$ and $b_{lt}'<0$, then $q_{st}(b_{st}',b_{lt}')=\frac{1}{1+r}$ and $q_{lt}(b_{st}',b_{lt}')=\frac{1}{1+r}\frac{b_{st}'}{-b_{lt}'}$. This implies that $q_{st}(b_{st}',b_{lt}')b_{st}'=-q_{lt}(b_{st}',b_{lt}')b_{lt}'$. Third, if $b_{st}'<0$ and $b_{lt}'\geq 0$, then $q_{st}(b_{st}',b_{lt}')=\frac{1}{1+r}\frac{(1+r)\overline{q}b_{lt}'}{-b_{st}}$ and $q_{lt}(b_{st}',b_{lt}')=\overline{q}$ implying that $q_{st}(b_{st}',b_{lt}')b_{st}'=-\overline{q}b_{lt}'$ as well.

Under these bond prices, it is optimal to set $\boldsymbol{B}(\Omega)=(B_{st}(\Omega),B_{lt}(\Omega))=0$. By assumption the borrower is never a net saver. Hence, the borrower either chooses $\boldsymbol{B}(\Omega)=0$ or defaults today. In the former case $D(y_H,\boldsymbol{B}(\Omega))=D(y_L,\boldsymbol{B}(\Omega))=0$, while in the latter case $D(\Omega)=$

1. If $y = y_L$, the proof is done. Otherwise, one can repeat the same argument backward as $D(y_H, b_{st}, b_{lt}) = 1$ implies $D(y_L, b_{st}, b_{lt}) = 1$.

Consider now that $\phi(y_L, b_{st}, b_{lt}) = \xi < \infty$. Given (9), by definition, restructurings substitute for defaults in y_L.

$\square$

A.4 Proof of Proposition 4

Proposition 4 (Constrained Efficient Allocation)**.**

I. (Efficiency). *$V^l(y,x)$ is strictly decreasing, while $V^b(y,x)$ is strictly increasing in $x \in \tilde{X} \equiv [x_D(y_L), \overline{x}]$ for all $y \in Y$ and $x_D(y_H) > x_D(y_L)$.*

II. (Risk-Sharing). *$c(y_L, x) < c(y_H, x)$ and $x'(y_L, x) < x'(y_H, x)$ for $x < x_D(y_H)$ and $c(y_L, x) = c(y_H, x)$ and $x'(y_L, x) = x'(y_H, x)$ otherwise. Also, $c(y_L, x_D(y_L)) = y_L$ and $c(y_H, x_D(y_H)) < y_H$.*

III. (Liabilities). *$V^l(y_L, x) < V^l(y_H, x)$ for all $x \in \tilde{X}$.*

Proof. I prove each part of the proposition one by one:

– Part I

The law of motion of the relative Pareto weight is given by $x'(y,x) = (1 + \nu(y,x))\eta x$, while the first-order condition on consumption reads $u_c(c(y,x)) = \frac{1}{(1+\nu(y,x))x} = \frac{\eta}{x'(y,x)}$.

Consider the interval $[x_D(y_L), \overline{x}]$. From the law of motion of the relative Pareto weight, x' is strictly increasing in x. From the first-order conditions on consumption, c is strictly increasing in x. Hence, so does the value of the borrower. Moreover, observe that

$$\partial_x PV(y,x) = V^b(y,x) + x\partial_x V^b(y,x) + \partial_x V^l(y,x) = V^b(y,x).$$

This comes from the envelope condition stating that $\partial_x PV = V^b(y,x)$. At the same time, the decomposition in PV leads to $\partial_x PV(y,x) = Vb(y,x) + x\partial_x V^b(y,x) + \partial_x V^l(y,x)$. Combining these two equations delivers the above equation. This implies the efficient risk-sharing property: $x\partial_x V^b(y,x) = -\partial_x V^l(y,x)$ implying that $V^l(x,s)$ is strictly decreasing in x.

Moreover, as $V^D(y_H) > V^D(y_L)$ and the value of the borrower is strictly increasing in x, it must be that $x_D(y_H) > x_D(y_L)$.

– Part II

Observe that, given the first-order condition, $c(y_L, x) \leq c(y_H, x)$ only when $\nu(y_L) \leq \nu(y_H)$. Assume by contradiction that $\nu(y_L) > \nu(y_H)$. This implies that $c(y_L, x) > c(y_H, x)$ and $x'(y_L, x) > x'(y_H, x)$. Given this, from Part I, $V^b(y', x'(y_L, x)) > V^b(y', x'(y_H, x))$. As a result.

$$u(c(y_L, x)) + \beta \mathbb{E} V^b(y', x'(y_L, x)) > u(c(y_H, x)) + \beta \mathbb{E} V^b(y', x'(y_H, x)).$$

Moreover as $\nu(y_L) > \nu(y_H) \geq 0$

$$\begin{aligned} u(c(y_H, x)) + \beta \mathbb{E} V^b(y', x'(y_H, x)) &\geq V^D(y_H), \\ u(c(y_L, x)) + \beta \mathbb{E} V^b(y', x'(y_L, x)) &= V^D(y_L). \end{aligned}$$

This implies that $V^D(y_L) > V^D(y_H)$, a contradiction. Hence, $\nu(y_L) \leq \nu(y_H)$ which gives $c(y_L, x) \leq c(y_H, x)$ and $x(y_L, x) \leq x(y_H, x)$ as desired.

Furthermore, by definition, when $x \geq x_D(y_H)$, then $\nu(y) = 0$ for all y implying that $c(y_L, x) = c(y_H, x)$ and $x(y_L, x) = x(y_H, x)$. Otherwise, $c(y_L, x) < c(y_H, x)$ and $x(y_L, x) < x(y_H, x)$.

Finally, observe that if $x = x_D(y_L)$, then $V^b(y_L, x_D(y_L)) = V^D(y_L)$ given the definition of $x_D(y_L)$ and $V^b(y_H, x_D(y_L)) = V^D(y_H)$ given that $x_D(y_L) < x_D(y_H)$. As a result, $\mathbb{E}[V^b(y', x_D(y_L))] = \mathbb{E}[V^D(y')]$ implying that $c(y_L, x_D(y_L)) = y_L$. In addition, if $x = x_D(y_H)$, then $V^b(y_H, x_D(y_H)) = V^D(y_H)$ given the definition of $x_D(y_H)$ and $V^b(y_L, x_D(y_H)) > V^D(y_L)$ since $x_D(y_H) > x_D(y_L)$. As a result, $\mathbb{E}[V^b(y', x')] > \mathbb{E}[V^D(y')]$ implying that $c(y_H, x_D(y_H)) < y_H$.

– Part III

This proof is a modified version of Thomas and Worrall (1990, Lemma 4). Assume by contradiction that for a given x it holds that $V^l(y_H, x) \leq V^l(y_L, x)$. Consider the pooling allocation in which $u(\ddot{c}(y_H, x)) = u(\ddot{c}(y_L, x)) = u(c(y_H, x))$ and $\ddot{V}^b(y_H, x) = \ddot{V}^b(y_L, x) = V^b(y_H, x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to $\ddot{V}^l(y_H, x) > \ddot{V}^l(y_L, x)$ which is a direct contradiction. Hence, $V^l(y_H, x) > V^l(y_L, x)$.

$\square$

A.5 Proof of Proposition 5

Proposition 5 (Existence and Uniqueness)**.** *Under Assumption 1, given initial conditions* (y_0, x_0)*, there exists a unique constrained efficient allocation.*

Proof. Existence and uniqueness follow from Theorem 3 in Marcet and Marimon (2019). The two authors make the following assumptions: A1 a well defined Markov chain process for y, A2 continuity in $\{c\}$ and measurability in y, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lenders and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given my environment. Since x is bounded, consumption and payoffs functions are bounded as well. This combined with the fact that the borrower's outside option is also bounded ensures that A4 is met. Whether A3 is satisfied depends on the initial condition (y_0, x_0). Assumption 1 ensures feasibility and that the strict interiority condition is satisfied.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding to the original contract problem (11). To obtain the recursive formulation of the contract, I have normalized the co-state variable. I relied on the the homogeneity of degree one in (μ_b, μ_l) to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if I define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}^2_+\}$ and the set of feasible consumption by $A = \{c \in \mathbb{R}_+\}$, the correspondence $SP : A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Marcet and Marimon (2019) additionally show that the the saddle point functional equation (13) is a contraction mapping. Thus, given the strict concavity assumptions of $u(\cdot)$, the allocation is unique. □

A.6 Proof of Proposition 6

Before proving Proposition 6, I derive the inverse Euler Equation which characterizes the consumption dynamic in the Planner's allocation.

Proposition A.4 (Inverse Euler Equation)**.** *The inverse Euler equation is given by*

$$\mathbb{E}\left[\frac{1}{u_c(c(y', x'))(1 + \nu(y', x'))}\right] = \eta \frac{1}{u_c(c(y, x))},$$

Proof. The law of motion of the relative Pareto weight is given by $x'(y, x) = (1 + \nu(y, x))\eta x$

and the level of consumption by $u_c(c(y,x)) = \frac{1}{x(1+\nu(y,x))}$. Isolating x leads to

$$x = \frac{1}{u_c(c(y,x))(1+\nu(y,x))}. \tag{A.3}$$

Plugging this back into the law of motion gives $x'(y,x) = (1+\nu(y,x))\eta\frac{1}{u_c(c(y,x))(1+\nu(y,x))}$. Replacing $x'(y,x)$ by with the forward equivalent of (A.3) gives $\frac{1}{u_c(c(y',x'))(1+\nu(y',x'))} = \eta\frac{1}{u_c(c(y,x))}$. Taking expectations on both sides gives the inverse Euler equation.

If the participation constraint never binds, I obtain that $\frac{1}{u_c(c(y,x))} > \mathbb{E}[\frac{1}{u_c(c(y',x'))}]$. In this case, the inverse marginal utility of consumption is a positive super-martingale which converges almost surely to 0 by Doob's theorem. Hence, there is a form of immiseration coming from the borrower's relative impatience.[24] With the borrower's limited commitment (i.e. $\nu(y,x) \geq 0$) and $\eta = 1$, one obtains a left bounded positive sub-martingale. The borrower's participation constraints therefore sets an upper bound on the super-martingale and limits immiseration. □

Proposition 6 (Ergodic Set)**.** *The ergodic set of relative Pareto weights $[x^{lb}, x^{ub}] \subset \tilde{X}$ is such that $x'(y_H, x^{ub}) = x^{ub}$, $x'(y_L, x^{lb}) = x^{lb}$ with $x^{lb} = x_D(y_L) < x^{ub} < x_D(y_H)$.*

Proof. The law of motion of the relative Pareto weight $x'(y,x) = (1+\nu(y,x))\eta x$ is dictated by the relative impatience, η, and the binding participation constraint, $\nu(y,x)$. Given that $\eta < 1$, the relative Pareto weight increases only if $\nu(y,x) > 0$ is sufficiently large to overcome impatience. By definition, when $x \geq x_D(y)$, $\nu(y,x) = 0$ meaning that impatience eventually dominates the limited commitment. Hence, impatience prevents the contract to reach $x_D(y_H)$ as $\nu(y_H, x_D(y_H)) = 0$. This implies that $x_D(y_H) > x^{ub}$. Moreover, $x'(y_L, x) < x'(y_H, x)$ when $x < x_D(y_H)$ implying that $x^{ub} > x^{lb}$. Finally, as $x'(y,x)$ is bounded below by $x_D(y_L)$, $x^{lb} = x_D(y_L)$. Said differently, $x^{lb} < x_D(y_L)$ would violate (10).

To show the existence of a unique ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as any point in the interval $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \to \mathbb{R}$ as

$$Q(x, G) = \sum_{y'} \pi(y')\mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of y_L, the relative Pareto weight transit to x^{lb} through impatience. Hence

[24]The term immiseration is usually used in the context of moral hazard (Thomas and Worrall, 1990; Phelan, 1995, 1998).

for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(y_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M = 1$ y_H, the relative Pareto weight transit to x^{ub} through the binding participation constraint (i.e. $x^{lb} < x^{ub} < x_D(y_H)$) meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}]) = \pi(y_H) > 0$. Setting $\epsilon = \min\{\pi(y_L)^M, \pi(y_H)\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. □

A.7 Proof of Proposition 7

Before proving Proposition 7, the following lemma shows that the long-term bond price is state contingent and increases in the realisation of y_H when there are restructuring offers and $\chi(\Omega) = 0$ for all Ω. For this, define the policies $\bar{D}, \bar{M}, \bar{R} : Y \times X \to \{0, 1\}$ and $\bar{\chi}, \bar{\phi}, \bar{q}_{st}, \bar{q}_{lt}, \bar{b}_{st}, \bar{b}_{lt} : Y \times X \to \mathbb{R}$. Given the timing of actions, the bond policies and the price schedules can be rewritten as $\bar{b}_j(y, x) = \bar{b}_j(x'(y, x))$ and $\bar{q}_j(y, x) = \bar{q}_j(x'(y, x))$ for all $j \in \{st, lt\}$.

Lemma A.1 (Long-Term Bond Price With Restructuring). *Under (A.4), $\bar{D}(y, x) = \bar{\chi}(y, x) = 0$ for all (y, x), the long-term bond price is the unique fixed point of $\bar{q}_{lt}$, is inceasing and*

$$\bar{q} \geq \bar{q}_{lt}(x'(y_H, x)) \geq \bar{q}_{lt}(x'(y_L, x)) \geq 0,$$

with $\bar{q} > \bar{q}_{lt}(x'(y_H, x)) > \bar{q}_{lt}(x'(y_L, x))$ if $\bar{b}_{lt}(x'(y_H, x)) < 0$, $\bar{b}_{lt}(x'(y_L, x)) < 0$ and $x \leq x^{ub}$.

Proof. Recall that with $\bar{D}(y, x) = \bar{\chi}(y, x) = 0$ for all (y, x), the long-term bond price reads

$$\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}\mathbb{E}\Big[\bar{P}(y', x')\{1 + \delta\bar{q}_{lt}(y', x')\} + (1 - \bar{P}(y', x'))\left(\bar{K}_{lt} + \big\{1 + \delta\bar{q}_{lt}(y', x')\big\}\bar{\phi}(y', x')\right)\Big] & \text{if } \bar{b}_{lt}(x) < 0 \\ \frac{1}{1+r-\delta}, & \text{else} \end{cases}$$

where $\bar{K}_{lt} = \frac{\max\{0, \bar{b}_{st}(x)\}}{-\bar{b}_{lt}(x)}$ and $\bar{P}(y', x') = 1 - \bar{D}(y', x') - \bar{R}(y', x')$. By assumption, $\bar{D}(y', x') = 0$ for all (y', x') and (A.4) hold. From Proposition 6, y_H and $x = x^{lb}$ arises with strictly positive probability for any (y, x). Given (9), $\bar{\phi}(y_L, x^{lb}) = \xi < 1$ which implies for $\bar{b}_{lt}(x'(y, x)) < 0$

$$\bar{q} > \bar{q}_{lt}(x'(y, x)) \geq 0.$$

Define $\mathcal{Q}$ as the space of bounded functions $\bar{q}_{lt} : [\underline{x}, \bar{x}] \to [0, \frac{1}{1+r-\delta}]$ and $\mathbb{T} : \mathcal{Q} \to \mathcal{Q}$ as

$$\mathbb{T}\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}\Big[\pi(y_H)[1 + \delta\bar{q}_{lt}(x'(y_H, x))] + \pi(y_L)\left(\bar{K}_{lt} + [1 + \delta\bar{q}_{lt}(x'(y_L, x))]\xi\right)\Big] & \text{if } x'(y_L, x) = x^{lb} \\ \frac{1}{1+r}\sum_{y'}\pi(y')[1 + \delta\bar{q}_{lt}(x'(y', x))] & \text{else} \end{cases}$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$. Moreover, a closer inspection $\bar{q}_{lt}$ indicates that it is increasing.

This implies that $\bar{q}_{lt}(x'(y_H, x)) \geq \bar{q}_{lt}(x'(y_L, x))$ as $x'(y_H, x) \geq x'(y_L, x)$ for all x. The inequality becomes strict when $\bar{b}_{lt}(x'(y_L, x)) < 0$ and $x \leq x^{ub}$. Assume by contradiction that there exists a x such that $\bar{q}_{lt}(x'(y_H, x)) = \bar{q}_{lt}(x'(y_L, x))$ and $b_{lt}(x'(y_L, x)) < 0$. This requires that $x'(y_H, x)$ and $x'(y_L, x)$ belongs to a subset $[x_t, x_{t+1}]$ where $\bar{q}_{lt}$ stays constant. Hence, for any $\ddot{x} \in [x_t, x_{t+1}]$, it must be that $x'(y_H, \ddot{x}), x'(y_L, \ddot{x}) \in [x_t, x_{t+1}]$. Since $x \leq x^{ub}$, the contradiction is immediate as $x'(y_H, x) = x_D(y_H)$, whereas $x'(y_L, x) < x_D(y_H)$ for any $x \in [x^{lb}, x^{ub}]$. □

The long-term bond price is higher in y_H than in y_L because a restructuring arises after a sufficiently long series of y_L. Given this, I can prove Proposition 7 by construction.

Proposition 7 (Restructuring-Based Decentralization)**.** *Under Assumption 1 and $x_0 \leq x^{ub}$, a Markov equilibrium with $\chi(\Omega) = 0$ for all Ω decentralizes the constrained efficient allocation.*

Proof. Given Assumption 1, autarky is never optimal meaning that $\bar{D}(y, x) = 0$ for all (y, x). To generate the appropriate state contingency, I assume that a restructuring arises when the economy draws y_L and $x = x^{lb}$ where,

$$\bar{R}(y, x) = \begin{cases} 1 & \text{if} \quad y = y_L \text{ and } x = x^{lb} \\ 0 & \text{else} \end{cases} \tag{A.4}$$

This policy ensures that $\bar{\phi}(y_L, x^{lb}) = \xi$ given (9). Buybacks occur but without any markup – i.e. $\bar{\chi}(y, x) = 0$ for all (y, x) – so $\bar{M}(y, x)$ is irrelevant for the determination of the bond price. For the bond portfolio, it should hold when $x > x^{lb}$ that

$$\begin{aligned} -V^l(y_H, x) &= \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta \bar{q}_{lt}(x'(y_H, x))], \\ -V^l(y_L, x) &= \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta \bar{q}_{lt}(x'(y_L, x))]. \end{aligned}$$

This is a system of 2 equations with 2 unknowns for which Lemma A.1 ensures that there exists a unique solution. In particular, the long-term bond position is given by

$$\bar{b}_{lt}(x) = -\frac{V^l(y_H, x) - V^l(y_L, x)}{\delta[\bar{q}_{lt}(x'(y_H, x)) - \bar{q}_{lt}(x'(y_L, x))]} < 0,$$

where the inequality comes from the fact that $V^l(y_H, x) > V^l(y_L, x)$ given Part III of Proposition 4 and from Lemma A.1. In addition $x_0 \leq x^{ub}$ ensures $x \leq x^{ub}$ given Proposition 6. In opposition, for $x = x^{lb}$, the relationship is given by

$$-V^l(y_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \delta \bar{q}_{lt}(x'(y_H, x))], \tag{A.5}$$

$$-V^l(y_L,x) = \xi \min\{0, \bar{b}_{st}(x)\} + \xi \min\{0, \bar{b}_{lt}(x)\}[1 + \delta \bar{q}_{lt}(x'(y_L,x))]. \tag{A.6}$$

Note that $\bar{b}_{st}(x^{lb})$ can be positive or negative. However, Lemma A.1 requires $\bar{b}_{lt}(x^{lb}) < 0$. For this, it must be that $\xi > V^l(y_L, x^{lb})/V^l(y_H, x^{lb}) < 1$. Since ξ is a free parameter, it can be chosen to satisfy this inequality.

Following Alvarez and Jermann (2000), to ensure that the participation constraint (10) holds, there should be an endogenous borrowing limit $\mathcal{G}(y')$ where $\mathcal{G}(y_H) = \bar{b}_{st}(x_D(y_H)) + \bar{b}_{lt}(x_D(y_H))[1 + \delta \bar{q}_{lt}(x_D(y_H))]$ and $\mathcal{G}(y_L) = \xi \min\{0, \bar{b}_{st}(x_D(y_L))\} + \xi \min\{0, \bar{b}_{lt}(x_D(y_L))\}[1 + \delta \bar{q}_{lt}(x_D(y_L))]$. The restructuring policy (A.4) ensures the state contingency of $\mathcal{G}(y')$.

Let's verify the enforcement of $\mathcal{G}(y')$. Fix y and assume that the borrower decides to issue more than the borrowing limit. First, consider that x is such that $x'(y_L, x) = x^{lb}$. In $y' = y_H$, since $V^b(y_H, x^{lb}) = V^D(y_H)$ and there is no restructuring in y_H, the value under repayment falls below the value of default meaning that the borrower defaults.[25] In $y' = y_L$, the key element is that $\mathcal{G}(y_L)$ is such that $\bar{\phi}(y_L, x^{lb}) = \xi$. Given (9), any additional borrowing leads to $\bar{\phi}(y_L, x^{lb}) = \infty$. As a result, the borrower defaults in $y' = y_L$. Hence, exceeding the borrowing limit in that case is unprofitable. Second, consider that x is such that $x'(y_L, x) > x^{lb}$. Using the same argument as before, the borrower will default in $y' = y_H$ which is suboptimal given Proposition 3.

This concludes the proof. I used the budget constraints in (2) and in (4) to determine the optimal bond holdings given the prices computed according to (6) and (7). Also, restructurings follow (9) and the participation constraint (10) holds. □

A.8 Proof of Lemma 1

Lemma 1 (Long-Term Bond Price With Buyback)**.** *Under (15) and $\bar{D}(y,x) = \bar{R}(y,x) = 0$ for all (y,x), the long-term bond price is the unique fixed point of $\bar{q}_{lt}$, is decreasing and*

$$\frac{1+\delta\varrho}{1+r-\delta} > \bar{q}_{lt}(x'(y_L,x)) \geq \bar{q}_{lt}(x'(y_H,x)) \geq \bar{q},$$

with $\bar{q}_{lt}(x'(y_L,x)) > \bar{q}_{lt}(x'(y_H,x))$ if $\bar{b}_{lt}(x'(y_L,x)) < 0$ and $x \leq x^{ub}$.

Proof. Given that $\bar{D}(y',x') = \bar{R}(y',x') = 0$ for all (y',x'), the long-term bond price is

$$\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}\mathbb{E}\left[1 + \bar{M}(y',x')\delta\bar{\chi}(y',x') + \delta\bar{q}_{lt}(y',x')\right], & \text{if } \bar{b}_{lt}(x) < 0 \\ \frac{1}{1+r-\delta}, & \text{else} \end{cases}$$

[25] There is no restructuring in y_H by assumption as I cannot rule out that there is an alternative level of debt $(\tilde{b}'_{st}, \tilde{b}'_{lt})$ such that $V^R(y_H, \tilde{b}'_{st}, \tilde{b}'_{lt}) = V^D(y_H)$ which could make the deviation profitable.

From Proposition 6, y_H and $x \leq x^{ub}$ arises with strictly positive probability for any (y, x). Thus, given (15), it holds that

$$\frac{1+\delta\varrho}{1+r-\delta} > \bar{q}_{lt}(x'(y,x)) \geq \bar{q},$$

with strict inequality for $\bar{b}_{lt}(x'(y,x)) < 0$. Define $\mathcal{Q}$ as the space of bounded functions $\bar{q}_{lt} : [\underline{x}, \bar{x}] \to [0, \frac{1+\delta\varrho}{1+r-\delta}]$ and $\mathbb{T} : \mathcal{Q} \to \mathcal{Q}$ as

$$\mathbb{T}\bar{q}_{lt}(x) = \begin{cases} \frac{1}{1+r}(\pi(y_L)[1+\delta\bar{q}_{lt}(x'(y_L,x))] + \pi(y_H)[1+\delta\varrho+\delta\bar{q}_{lt}(x'(y_H,x))]) & \text{if } x < x^{ub} \\ \frac{1}{1+r}\sum_{y'}\pi(y')[1+\delta\bar{q}_{lt}(x'(y',x))] & \text{else} \end{cases}$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$. Moreover, a closer inspection $\bar{q}_{lt}$ indicates that it is decreasing. This implies that $\bar{q}_{lt}(x'(y_H,x)) \leq \bar{q}_{lt}(x'(y_L,x))$ as $x'(y_H,x) > x'(y_L,x)$ for all x in the above specified domain. The inequality is strict whenever $b_{lt}(x'(y_L,x)) < 0$ and $x \leq x^{ub}$. Assume by contradiction that there exists a x such that $\bar{q}_{lt}(x'(y_H,x)) = \bar{q}_{lt}(x'(y_L,x))$ and $b_{lt}(x'(y_L,x)) < 0$. This requires that there exists a subset of $[x^{lb}, x^{ub}]$ where $\bar{q}_{lt}$ stays constant. The contradiction is immediate as $x'(y_H,x) = x_D(y_H)$, whereas $x'(y_L,x) < x_D(y_H)$ for any $x \in [x^{lb}, x^{ub}]$. $\square$

B Data

Table B.1 specifies the source of the data used in the analysis. For GDP data, I rely on OECD National Accounts for monthly nominal GDP series and quarterly real GDP series. I detrend the logarithm of the real GDP series using the HP filter with a smoothing parameter of 1600. EMBI+ spreads data come from the Global Financial Database.

Regarding the bonds issued and repurchased by the Brazilian government there are three data sources. First, I use the Monthly Debt Report published by the Brazilian National Treasury to retrieve the amount bought back for each bond. Reports of buyback are bimonthly and specify the face and the financial value of each bond repurchased.[26] In 2006, the report only specifies the aggregated financial value. In this year, I estimate the financial value using the face value and the average ask price on the secondary market in the bimonth of the buyback. Note that I do not account for bonds denominated in foreign currency other than USD.[27]

[26]Starting 2018, buybacks reports are monthly.

[27]There were some bonds in EUR and JPY involved in the buyback program. However, they correspond to a negligible amount compared to the ones denominated in USD.

Table B.1: Data Sources and Definitions

Series	Sources	Unit
Output	OECD National Accounts[a]	volumes and normalized
Global bonds	Bloomberg[b]	current USD
Global bonds	Brazilian National Treasury[c]	current USD
Buybacks	Brazilian National Treasury[d]	current USD
EMBI+	Global Financial Database[e]	basis point

[a] Real GDP, nominal GDP. Measure: VPVOBARSA, RS.
[b,c] USD-denominated bonds issued by the government of Brazil. See Table B.2.
[d] Financial and face value buybacks by bimonth (month starting 2018) and by bond. See Monthly Debt Report.
[e] Government bond spread. Series code: EMBPBRAD.

Second, the Brazilian National Treasury publishes the list of all foreign denominated bonds issued by the Brazilian government since 1995. The list contains the coupon rate, the issuance yield and price, the issuance and maturity dates, the ISIN code, the re-openings and the volumes in USD.

Third, I use Bloomberg to find all the bonds issued by the Brazilian National Treasury. This enables me to obtain the complete history of bond prices and the yields to maturity as well as more information on the bond structure. Given this and the second dataset, I can re-construct the cashflow stream of each bond. Table B.2 indicates all the USD-denominated bonds issued by the Brazilian government. Recall that I exclude BR09F from the analysis given the lack of information about the coupon structure.

For the regression analysis, I compute the difference between the primary-market price and the secondary-market price immediately after issuance at the daily frequency.[28] Denoting the secondary-market price by q_{SM} and the primary-market price by q_{PM}, I compute the difference as

$$\frac{q_{SM} - q_{PM}}{q_{PM}} \times 100.$$

This difference is averaged across re-openings for each bond. Regarding the bid-ask price spread, I follow the standards in the financial literature. Denoting the bid price by q_{SMB} and the ask price by q_{SMA}, I compute the bid-ask price spread as

$$\frac{q_{SMA} - q_{SMB}}{q_{SMB}} \times 100.$$

[28]I consider the price on the secondary market up to 30 days after the issuance. Further restricting this time window does not significantly affect the estimation.

Table B.2: Bonds

Bond	ISIN	Maturity (year)	Coupon (percent)	Issuance (USD billion)	Mean i_I (percent)	Buyback (USD billion)	Mean i_B (percent)	Mean Premium (percent)
BR01	US105756AD24	5	8.88	0.75	0.00	0.00	-	-
BR05	US105756AS92	4	9.63	1.00	11.25	0.00	-	-
BR06	US105756AQ37	5	10.25	1.50	10.54	0.00	-	-
BR07	US105756AM23	7	11.25	1.50	11.25	0.50	5.35	3.16
BR07B	US105756AW05	4	10.00	1.00	10.70	0.39	5.32	2.22
BR08	US105756AG54	10	9.38	1.25	9.40	0.25	3.28	3.31
BR08B	US105756AU49	6	11.50	1.25	11.74	0.65	5.75	6.10
BR09	US105756AJ93	10	14.50	2.00	14.61	0.95	5.18	7.71
BR09F	US105756BC32	5	-	0.75	-	0.31	-	-
BR10	US105756AV22	8	12.00	1.00	12.38	0.46	3.53	10.53
BR10N	US105756BA75	7	9.25	1.50	9.45	0.63	4.14	9.92
BR11	US105756AY60	8	10.00	1.25	10.69	0.65	3.58	12.00
BR12	US105756AT75	10	11.00	1.25	12.60	0.47	3.28	13.23
BR13	US105756AX87	10	10.25	1.25	10.58	0.59	1.89	10.78
BR14	US105756BD15	10	10.50	1.25	8.33	0.78	2.49	11.00
BR15	US105756BG46	10	7.88	2.10	7.77	1.03	2.58	11.41
BR16	US105756BJ84	10	12.50	1.48	12.75	0.00	-	-
BR17	US105756BM14	10	6.00	2.55	5.40	0.45	1.51	4.11
BR18	US105756BH29	13	8.00	4.51	7.58	2.90	4.79	7.83
BR19	US105756BE97	15	8.88	1.50	8.84	0.90	4.62	16.19
BR19N	US105756BQ28	10	5.88	2.30	4.85	0.12	1.95	5.15
BR20	US105756AK66	20	12.75	1.00	13.27	0.53	4.31	15.71
BR21	US105756BS83	11	4.88	2.99	3.63	0.27	4.29	-2.98
BR22	US105756BL31	15	12.50	1.39	11.67	0.00	-	-
BR23	US105756BU30	10	2.63	2.15	2.75	0.00	-	-
BR24	US105756AR10	23	8.88	2.15	12.91	0.94	5.14	22.79
BR24-BRL	US105756BT66	12	8.50	1.69	8.60	0.00	-	-
BR24B	US105756AZ36	21	8.88	0.82	12.59	0.65	5.41	32.49
BR25	US105756BF62	20	8.75	2.25	8.53	1.26	5.20	17.47
BR25K	US105756CD06	5	2.88	1.75	2.28	0.00	-	-
BR25L	US105756BV13	11	4.25	4.30	3.92	0.00	-	-
BR26	US105756BX78	10	6.00	2.50	5.12	0.32	4.37	3.94
BR27	US105756AE07	29	10.13	3.50	10.29	2.25	5.58	13.84
BR28	US105756BZ27	10	4.63	3.00	4.68	0.00	-	-
BR28-BRL	US105756BN96	21	10.25	2.51	8.84	0.00	-	-
BR29	US105756CA66	10	4.50	2.00	3.91	0.00	-	-
BR30-1a	US105756CC23	10	3.88	3.50	3.51	0.00	-	-
BR30-2a	US105756AL40	30	12.25	1.60	12.47	0.65	5.65	21.61
BR31-1a	US105756CG37	7	6.25	2.00	6.50	0.00	-	-
BR31-2a	US105756CE88	10	3.75	1.50	3.88	0.00	-	-
BR32	US105756CK49	30	6.13	2.00	6.38	0.00	-	-
BR33	US105756CF53	11	6.00	2.25	6.15	0.00	-	-
BR34	US105756BB58	30	8.25	2.70	8.28	0.93	6.07	13.13
BR34A	US105756CH10	10	6.13	2.25	6.35	0.00	-	-
BR37	US105756BK57	31	7.13	3.03	6.51	0.37	5.93	3.77
BR40	US105756AP53	40	11.00	5.16	13.73	3.86	8.10	18.79
BR41	US105756BR01	31	5.63	2.93	4.80	0.14	6.33	-13.13
BR45	US105756BW95	31	5.00	3.55	5.13	0.00	-	-
BR47	US105756BY51	31	5.63	3.00	5.65	0.00	-	-
BR50	US105756CB40	30	4.75	4.00	4.88	0.00	-	-
BR54	US105756CJ75	30	7.13	2.25	7.15	0.00	-	-

Note: The table depicts all the USD-denominated bonds issued by Brazil. For each bond the table gives the ISIN code, the maturity at issuance, the coupon rate, the total amount issued (including re-openings), the mean issue yield to maturity (including re-openings), the total amount bought back, the mean buyback yield to maturity and the mean buyback premium computed according to (19). For the bond name $-1a$ and $-2a$ indicate the bond tranche, $-BRL$ indicates that either the principal or the coupon payment is in BRL. The other letters were attributed by the Brazilian National Treasury.