Efficient Sovereign Debt Management in Emerging Economies[*]

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Abstract

This paper assesses the sovereign debt management of emerging economies in terms of constrained Pareto efficiency. I derive a market economy in which a sovereign borrower trades non-contingent bonds of different maturities with a foreign lender. The borrower is impatient and lacks commitment. I show that the market economy cannot implement the Planner's constrained efficient allocation through defaults but instead by changes in maturity and costly debt buybacks. Moreover, as the lender must enforce those buybacks, the implementation often requires history-dependent strategies. Nevertheless, interpreting the borrower's impatience as a form of bounded memory, small perturbations in the payoff of the market participants rule out any other strategies than Markov ones. In this case, the Planner's allocation can only be approximated by Markov debt management policies. I show that emerging economies such as Argentina and Brazil present evidence of such approximation albeit with different policies and outcome. In particular, conducting buybacks and avoiding defaults, the latter country comes closer to constrained Pareto efficiency.

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1 Introduction

The sovereign debt management of emerging economies has three main features. First and foremost, the better part of those economies defaults on their liabilities. Defaults are relatively frequent, last several years and are associated with markets exclusion as well as substantial debt reliefs.[1] Second, emerging economies rely more extensively on short-term debt during debt crises.[2] Third, they conduct debt buybacks to repurchase part of their debt on the secondary market. Even though such repurchases have become rare in the last few years, they are usually very costly and ineffective in reducing indebtedness.[3] The question that arises is how efficient is this debt management? I analyze the role of maturity, buyback and default in attaining or approximating the constrained (Pareto) efficient allocation in emerging economies.

In terms of maturity, the literature on fiscal policy with commitment suggests to trade non-contingent bonds of different maturities to replicate the return of Arrow securities.[4] The portfolio of bonds emanating from this approach is however empirically implausible.[5] To reconcile the model's prediction with the data, the literature has introduced different frictions such as limited commitment and trade constraints.[6] I provide an alternative explanation. I argue that the market participants often lack the strategical sophistication required to implement the aforementioned maturity management. Focusing on emerging economies, I show that one ought to consider Markov strategies. Under such strategies, market participants can usually only approximate the return of Arrow securities. Such approximation is consistent with the data, though.

In terms of default and buyback, the literature on sovereign debt argues that it might be optimal to conduct the former as this provides a source of state contingency, while the latter is suboptimal as it only benefits the lender.[7] I argue the opposite. A default generates deadweight losses which impact both the borrower and the lender. Hence, it is Pareto improving to avoid such event. Conversely, costly buybacks can generate state contingency without causing the aforementioned deadweight losses.[8] As the bond price incorporates the

[1] For default duration and haircuts, see Cruces and Trebesch (2013) and Asonuma and Trebesch (2016). For default frequency, see Tomz and Wright (2007, 2013) and Reinhart and Rogoff (2009).

[2] See Arellano and Ramanarayanan (2012), Broner et al. (2013), Perez (2017) and Bai et al. (2017).

[3] See Bulow and Rogoff (1988, 1991) and Cohen and Verdier (1995).

[4] See Kreps (1982), Angeletos (2002) and Barro (2003).

[5] Buera and Nicolini (2004) and Faraglia et al. (2010) show that the borrower ought to *sell* long-term bonds and *buy* short-term bonds in the magnitude of several multiples of GDP.

[6] Debortoli et al. (2017) introduce limited commitment in fiscal policy, Faraglia et al. (2019) limit the extent of debt repurchase and reissuance and Kiiashko (2022) adds limited commitment in repayment.

[7] See Grossman and Van Huyck (1988), Adam and Grill (2017), Roettger (2019) and Hatchondo et al. (2020a) for defaults as a source of risk sharing and footnote 3 for buybacks as suboptimal policy.

[8] I consider an exogenous cost of buyback and provide foundations for such cost in Appendix C.

cost of buybacks, it is possible to generate state-contingent capital losses and gains with the appropriate buyback policy. As a result, the optimal sovereign debt management consists of no default and occasional costly buybacks.

I consider an environment in which a foreign lender owns a production technology in a small open economy, provides the capital input and buys bonds issued by a sovereign government (i.e. the borrower). Conversely, the sovereign government takes decisions on behalf of the small open economy, runs the production technology and issues non-contingent defaultable bonds of different maturities. In addition, domestic production is subject to persistent productivity shocks and the government is impatient. In this set-up, I introduce one friction: the government cannot commit to repay the lender.

I first analyze the economy under Markov strategies and consider two Markov equilibria with opposite predictions. The first one is a version of Arellano and Ramanarayanan (2012). The borrower defaults on equilibrium path and never conducts costly buybacks. Defaults provide some form of state contingency and push the maturity towards the short end during debt crises. I therefore relate these predictions to the sovereign debt management of Argentina which repeatedly defaulted in the last few decades. In the second Markov equilibrium, the lender introduces a borrowing limit which becomes state contingent with the help of costly buyback programs. Defaults do not arise on equilibrium path, while costly buybacks do. I therefore relate these predictions to the debt management of Brazil which, unlike Argentina, has not defaulted since the end of the 1980s and conducted official buybacks since the 2000s.[9] I show that this second Markov equilibrium is Pareto superior to the first one demonstrating the inefficiency of default. Nonetheless, high costs of buyback foster the reliance on defaults instead of buybacks. I therefore conclude that the main difference between Argentina and Brazil ought to lie in the cost of buybacks and find supportive evidence in the data.

As I associate Argentina and Brazil with Markov equilibria, I provide a foundation for the use of Markov strategies in the context of emerging economies. Given that such economies suffer from important political instability, I interpret the borrower's impatience as a form of bounded memory. In addition, as emerging economies' fundamentals are difficult to assess for foreign creditors, I introduce small and independent perturbations in payoffs.[10] Under these two assumptions, I show that only Markov equilibria survive in my environment. This means that the study of emerging economies ought to be limited to Markov equilibria. Any non-Markov outcome can only be approximated by such economies.

[9] Argentina conducted buybacks at a discount (i.e. below par) which usually correspond to a default in the form of distressed debt exchange. My analysis does not consider such type of buyback.

[10] See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019) for the political instability and Tsyrennikov (2013) and Morelli and Moretti (2021) for the lack of financial transparency.

To assess the efficiency of the above Markov equilibria, I subsequently analyze the constrained efficient allocation. For this, I first derive an optimal contract between the borrower and the lender. The contract features state-contingent debt relief and production distortion. Particularly, the lender records capital losses in low productivity states and capital gains in high productivity states. In addition, when the borrower receives sufficiently high utility, the punishment of autarky is a real threat. The contract can therefore sustain the productivity-maximizing level of capital. Otherwise, the threat of autarky fades and the Planner reduces the level of capital to relax the participation constraint. The Planner never finds optimal to set capital to zero, though.

I then implement the optimal contract in the market economy. Given that the Planner never distorts capital to zero, defaults – which imply markets exclusion – cannot implement the constrained efficient allocation. Instead, the government adapts the maturity structure of its portfolio and conducts costly debt buybacks. Such buybacks implicitly introduce state contingency in the bond contract as the bond price incorporates the cost of buybacks. They occur in high productivity states implying that the price of long-term bonds increases after the realization of high productivity shocks. This in turn increases the value of outstanding long-term debt resulting in capital gains for the lender. The opposite happens after the realization of low productivity shocks. Thus, costly buybacks can generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract.

Nevertheless, the implementation of the constrained efficient allocation might not generally be Markov. The reason is that the borrower is not willing to conduct costly buybacks. Thus, the lender has to enforce them. I find that in a Markov equilibrium, such enforcement is possible only if the borrower does not issue any assets and buybacks are not too costly. In this situation, the lender can threaten the borrower with a sudden stop on debt if the buyback does not take place. However, to replicate the Planner's allocation, the borrower needs to hold short-term assets unless the buyback premium is sufficiently large. Particularly, I find that Markov strategies fail to implement the Planner's allocation under empirically plausible buyback premia. Thus, in light of my suggested equilibrium refinement, emerging economies can only approximate the constrained efficient allocation.

To gauge the goodness of the Markov approximation, I calibrate the Markov equilibrium with default to match moments of the Argentine economy over the period 1995-2019. The calibrated model fits well the data and features default episodes in which indebtedness increases with respect to output and maturity shortens. Conversely, during restructurings, the level of debt remains substantial and the maturity lengthens. In addition, using the calibration for Argentina, I find that the Markov equilibrium without default is quantitatively close to Brazil. I therefore interpret Brazil as the counterfactual of Argentina with buybacks and without defaults. Finally, in line with the literature on fiscal policy with commitment, I find that the implementation of the constrained efficient allocation generates unrealistic debt portfolios unlike Markov equilibria.

I then compare the Markov equilibria with the implementation of the optimal contract through various simulation exercises. I find important welfare gains for both the borrower and the lender to rely on costly buybacks instead of defaults. In light of this, I show that the Markov equilibrium without default is quantitatively the closest to the constrained efficient allocation. Nonetheless, the Markov equilibrium without default is far from the Pareto frontier indicating that the Markov approximation remains crude. I therefore conclude that the limitation to Markov strategies in emerging economies is very costly in terms of welfare.

The paper is organized as follows. I review the literature in Section 2. I describe the economic environment in Section 3 and introduce the market economy in Section 4. I present the Markov and the constrained efficient debt management in Sections 5 and 6, respectively. The calibration and quantitative analyses are in Section 7. Finally, I conclude in Section 8.

2 Literature Review

The paper combines elements of the literature on sovereign defaults and buybacks with elements of the literature about optimal contracts and their implementation.

The literature on sovereign defaults assumes that markets are incomplete and agents follow Markov strategies (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008).[11] There, the borrower has access to only non-contingent claims and can obtain limited state contingency through defaults. My study is the closest to Arellano and Ramanarayanan (2012) and Niepelt (2014) given that I adopt two bonds with different maturities and to Mendoza and Yue (2012) given that the default cost is endogenous.[12] I contribute to this literature in two ways. First, I show that the reliance on defaults to obtain state contingency is inefficient. Second, I provide a foundation for the use of Markov strategies interpreting the assumption of impatience as evidence of bounded memory and then implementing the refinement of Bhaskar et al. (2012) and Angeletos and Lian (2021). This second result relates to Krusell and Smith (1996) and Krusell et al. (2002) as it connects the equilibrium outcome with the sophistication of agents' strategies. In addition, it provides a robust equilibrium refinement and is therefore associated to the analysis of Passadore and Xandri (2021).

On a similar note, this paper relies on costly buybacks as a way to implement the constrained efficient allocation. It therefore relates to the seminal contribution of Bulow and

[11]See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

[12]See also Bohn (1990) and Barro (1995) for earlier work on optimal debt structure.

Rogoff (1988, 1991) who document that buybacks are suboptimal as they increase the recovery value per unit of bond and therefore fail to reduce indebtedness. In light of this, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Similarly, Aguiar et al. (2019) find that buybacks reduce welfare as they shift the maturity structure and therefore affect the default risk.[13] In opposition, Rotemberg (1991) shows that buybacks can be advantageous to all parties as they lower the bargaining costs. Moreover, Acharya and Diwan (1993) find that buybacks provide a positive signal about the borrower's willingness to repay. My analysis goes in this direction as it emphasizes the efficiency of buybacks as a source of risk sharing between the borrower and the lender.

The paper derives an optimal contract between a lender and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). My study accounts for limited commitment similar to Aguiar et al. (2009) and is close to Kehoe and Perri (2002) and Restrepo-Echavarria (2019) as it relies on the approach of Marcet and Marimon (2019) with the difference that I implement the contract in a market economy.[14]

The paper therefore addresses the literature on optimal contract's implementation. Note that I discuss the following studies in more details in Appendix A. Unlike Aguiar et al. (2019) and Müller et al. (2019), my implementation is not generally Markov. On the one hand, Aguiar et al. (2019) account for multiple maturities but consider a Planner's problem which does not take into consideration the legacy creditors in the objective function and has no participation constraint, unlike my Planner problem. On the other hand, Müller et al. (2019) use preemptive restructurings and GDP-linked bonds, whereas I rely on the maturity structure. An alternative to this approach is Dovis (2019) who develop an implementation through partial defaults and an active debt maturity management. He builds on Angeletos (2002) and Buera and Nicolini (2004) who show that one can replicate the state-contingency of Arrow securities using non contingent bonds of different maturities.[15] My implementation is the closest to Dovis (2019) with the difference that I rely on debt buybacks without defaults and haircuts. Moreover, similar to Hatchondo et al. (2020a), I connect my implementation to the Markov allocation. Especially, as buybacks need to be enforced, I explain why and when history dependence matters. I then relate this finding to the sovereign debt management of emerging economies and more broadly to the approach of Chari and Kehoe (1990, 1993).

[13]Furthermore, Aguiar et al. (2022) show that buybacks are rare as they only occur when there are no uncertainty surrounding debt auctions.

[14]The other difference is that I adopt a capital depreciation rate of 1 which simplifies the equilibrium computation and the proofs.

[15]See also the references in footnotes 4, 5 and 6.

3 Environment

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The small open economy is populated by a benevolent government and a large number of homogenous households which own domestic firms, while a foreign lender invests in the small open economy.[16]

The risk neutral lender discounts the future at rate $\frac{1}{1+r}$ and breaks even in expectations. It trades bonds with the government. In addition, it equips the small open economy with a production technology, $F(k_t, l_t)$, to produce goods and provides the capital input, k_t, at price p_t in every t. For tractability and without loss of generality, I assume that capital depreciates at rate 1. Domestic households provide the labor input, l_t.

The representative domestic household discounts the future at rate $\beta \leq \frac{1}{1+r}$. Preference over consumption is represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t corresponds to the consumption at time t. The instantaneous utility function $u(\cdot)$ is continuous, increasing and concave.

The government is benevolent and takes the decision on behalf of the small open economy. There is a tax on the import of capital made by domestic firms at rate $\tau_t = 1 - \frac{1}{p_t}$.[17] Thus, the household's after-tax income is given by

$$y(g_t, k_t) \equiv g_t F(k_t, l_t) - k_t.$$

Domestic households are endowed with one unit of labor in each period.[18] I therefore denote $f(k_t) \equiv F(k_t, 1)$. The production technology is continuous, increasing, concave, satisfies the Inada condition, $\lim_{k \to 0} f_k(k) = \infty$, and $f(0) > 0$. The fact that $f(\cdot)$ is concave implies that the production technology displays decreasing returns to scale. This means that there exists a level $k^*(g_t)$ which maximizes the net production such that $g f_k(k^*(g_t)) = p_t$.

Domestic production is subject to a shock g_t which takes value on the discrete set $G \equiv \{g_L, g_H\}$ with $0 < g_L < g_H$ and follows a Markov chain of order one with $\pi(g_{t+1}|g_t)$ corresponding to the probability of drawing g_{t+1} at date $t+1$ conditional on drawing g_t at

[16] The present environment is similar to the one of Quadrini (2004), Aguiar et al. (2009) and Dovis (2019).

[17] This technical assumption is necessary for the implementation of the constrained efficient allocation in the market economy. Having τ_t a choice variable would make taxation time inconsistent in Markov equilibria.

[18] As in Aguiar et al. (2009) and Dovis (2019), I combine the income of households and government together. Households provide labor inelastically and receive lump sum transfers from the government.

t. I further assume that shocks are persistent meaning that $\pi(g|g) > 0.5$ for all $g \in G$.[19]

The government has access to bonds with two different maturities. On the one hand, there is a one-period – i.e. short-term – bond, b_{st}, with unit price q_{st}. On the other hand, there is a perpetual – i.e. long-term – bond, $b_{lt} \leq 0$, with unit price q_{lt}, which pays a coupon of one every period. I denote debt as a negative asset meaning that $b_j < 0$ is a debt, while $b_j > 0$ is an asset for all $j \in \{st, lt\}$. The government can hold short-term assets but not long-term assets. The financial market is incomplete as bonds do not discriminate the returns across g.

The government can conduct *official* buybacks on the long-term bond at a specific price $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ with $\chi \in (0,1)$ being the *official* buyback premium.[20] Conversely, it can conduct *unofficial* buybacks in which it retires part of its debt at the market price, $q_{lt} \leq q_{lt}^{bb}$, without prior notice. This is a reduced form of what Cohen and Verdier (1995) call a "secret buyback". In both cases, the prospective value of long-term debt is such that $b_{lt,t+1} \geq b_{lt,t}$. Note that I provide foundations for $\chi > 0$ in Appendix C through a generalized Nash bargaining and alternatively through a signalling game.

I assume that the government lacks commitment. If the government defaults, it loses access to the capital and bond market. It subsequently consumes $c(g) = gf(0) > 0$ but can regain access to the markets with a fixed probability λ. The default cost is therefore endogenous as it entirely relates to markets access. Collecting all the assumptions,

Assumption 1 (General Settings)**.** *The risk neutral lender discounts at rate* $\frac{1}{1+r}$*. The risk averse government discounts at rate* $\beta \leq \frac{1}{1+r}$ *and has a utility function* $u(c)$ *which is continuous, increasing and concave. The productivity shock* $g \in G \equiv \{g_L, g_H\}$ *follows an Markovian process of order one with* $0 < g_L < g_H$ *and* $\pi(g|g) > 0.5$ *for all* $g \in G$*. The foreign production technology is continuous, increasing, concave, satisfies the Inada condition,* $\lim_{k\to 0} f_k(k) = \infty$*, and* $f(0) > 0$*. Capital depreciates at rate* 1 *and is taxed domestically at rate* $\tau = 1 - \frac{1}{p}$*.*

The timing of actions is the following. At the beginning of each period $t \geq 0$, the productivity shock, g_t, realizes and the lender provides k_t. Subsequently, domestic production takes place, capital depreciates, the government determines the debt repayment – including potential *official* buybacks – and collects the tax on capital. Conditional on repaying, a bond auction determines $b_{st,t+1}$ and $b_{lt,t+1}$.[21] Note that, if $b_{st,t+1} > 0$, the lender is in fact a short-term borrower and is the one auctioning to raise resources.

[19] The case in which $\pi(g|g) = 0.5$ corresponds to i.i.d shocks.

[20] The fact that *official* buybacks are settled above the risk-free price is consistent with the evidence that buybacks are costly for sovereign borrowers (Bulow and Rogoff, 1988, 1991).

[21] This timing rules out self-fulfilling debt crises (Ayres et al., 2018; Galli, 2021).

4 The Market Economy

In this section, I define the set of sustainable equilibrium outcomes in the market economy following the approach of Abreu (1988) and Chari and Kehoe (1990). Keeping track of the entire history of play, it is possible to sustain multiple equilibrium outcomes.

4.1 The Government's problem

Define $D_t \in \{0, 1\}$ as the default policy at time t. If $D_t = 0$, the government repays, while if $D_t = 1$, it defaults. Similarly, define $M_t \in \{0, 1\}$ as the *official* buyback policy at time t. If $M_t = 1$, the government officially buys its debt back, while if $M_t = 0$, it does not.

In addition, define the government's choice set as $\mathcal{C}_{b,t} = \{D_t, M_t, b_{st,t+1}, b_{lt,t+1}\}$ and the government's strategy as σ_b. Furthermore, let $h^t = (h^{t-1}, g_t, p_t, k_t, \mathcal{C}_{b,t})$ denote the history up to time t taking the initial debt $\{b_{j,0}\}_{j\in\{st,lt\}}$ as given. Due to the specific timing of actions, further define the history of the lender and the government as $h^t_l = (h^{t-1}, g_t)$ and $h^t_b = (h^{t-1}, g_t, p_t, k_t)$, respectively. I also define the history for the choice of capital as $h^t_k = (h^{t-1}, g_t, p_t)$. Finally, I denote the value of the lender and the government after any specific history by $W^l(h^t_l)$ and $W^b(h^t_b)$, respectively.

In the case in which the government decides to repay (i.e. $D_t = 0$), it determines its consumption and prospective borrowing given the realization of the history h^t_b. In the case of no *official* buyback (i.e. $M_t = 0$), the budget constraint reads

$$c_t + q_{st}(h^t_b, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h^t_b, \mathcal{C}_{b,t})(b_{lt,t+1} - b_{lt,t}) = y(g_t, k_t) + b_{st,t} + b_{lt,t}.$$

There is no restriction on the issue of long-term debt meaning that the government can potentially conduct *unofficial* buybacks. Conversely, in the case of an *official* buyback (i.e. $M_t = 1$), budget constraint is given by

$$c_t + q_{st}(h^t_b, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h^t_b, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t) + b_{st,t} + b_{lt,t}(1 + q^{bb}_{lt}) \ \wedge \ b_{lt,t+1} \geq b_{lt,t}.$$

The government retires the current long-term bond, $b_{lt,t}$, at $q^{bb}_{lt} = \frac{1}{(1-\chi)r}$ with $\chi \in (0, 1)$ and issues new long-term debt such that $b_{lt,t+1} \geq b_{lt,t}$. Conversely, if the government decides to default (i.e. $D_t = 1$), it gets excluded from the markets and consumes

$$c_t = g_t f(k_t).$$

Neither capital nor debt are repaid. Due to the specific timing of capital, the government enjoys $k_t \geq 0$ in the first period of autarky and then $k_t = 0$. The outstanding debt is restructured with probability λ. Upon restructuring, the government can regain access to the markets. In this case,

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t) + w_{st} + w_{lt}\frac{1+r}{r},$$

where w_{st} and $w_{lt}\frac{1+r}{r}$ are the recovery values of short-term and long-term debt, respectively. Thus, after any history h_b^t, the optimal strategy of the government, σ_b, is the solution of

$$W^b(h_b^t) = \max_{\{\mathcal{C}_{b,t}\}_{t=0}^{\infty}} \; u(c_t) + \beta \mathbb{E}\left[W^b(h_b^{t+1})\middle| h_b^t, \mathcal{C}_{b,t}\right], \tag{1}$$

subject to the budget constraint.

4.2 Sustainable equilibria

Having derived the government's problem, I can define and characterize the set of sustainable equilibria. The lender breaks even meaning that in expectations it makes zero profit. The price of one unit of bond can therefore be separated into two parts: the return when the government decides to repay and the recovery value when the government defaults. The price per unit of bond of maturity $j \in \{st, lt\}$ is given by,

$$q_j(h^t) = \mathbb{E}\left[(1 - D(h^{t+1}))q_j^P(h^{t+1}) + D(h^{t+1})q_j^D(h^{t+1})\middle| h^t\right]. \tag{2}$$

If the government decides to default, the recovery value for all $j \in \{st, lt\}$ is

$$q_j^D(h^{t+1}) = \frac{1}{1+r}\left[(1-\lambda)q_j^D(h^{t+1}) + \lambda \frac{w_j \frac{1+r}{1+r-\mathbb{1}_{j=lt}}}{b_{j,t}}\right],$$

where $\mathbb{1}_{j=lt}$ is an indicator function taking value one if $j = lt$ and zero otherwise. If the government restructures its debt, the lender receives $\frac{w_j \frac{1+r}{1+r-\mathbb{1}_{j=lt}}}{b_{j,t}}$ per unit of bond issued. Conversely, if it does not restructure, the government does not disburse anything now, but in present value it pays $q_j^D(h^{t+1})$.

In case of repayment, the price depends on the maturity structure and the *official* buyback decision. Formally,

$$q_{st}^P(h^{t+1}) = \frac{1}{1+r} \quad \text{and} \quad q_{lt}^P(h^{t+1}) = \frac{1}{1+r}\left[1 + (1 - M(h^{t+1}))q_{lt}(h^{t+1}) + M(h^{t+1})q_{lt}^{bb}\right].$$

Having properly determined the price, I can define the a sustainable equilibrium.

Definition 1 (Sustainable Equilibrium). *Given $\{b_{j,0}\}_{j\in\{st,lt\}}$, a sustainable equilibrium in this environment consists of strategy for the government, σ_b, policy for the firm's capital, k as well as price schedule for capital, p, and for bonds, q_{st} and q_{lt} such that*

1. *Taking p, q_{st} and q_{lt} as given, σ_b is the solution to (1).*
2. *Taking p as given, the choice of capital by domestic firms is such that*

$$\mathbb{E}\Big[u_c(c(h_b^t))(gf_k(k(h^t)) - p(h^t))\Big|h_k^t\Big] = 0. \tag{3}$$

3. *The price of capital is consistent with*

$$\max_{k_t} \ \mathbb{E}\Big[p(1-\tau)k_t - k_t\Big|h_l^t\Big]. \tag{4}$$

4. *The price of each bond of maturity $j \in \{st, lt\}$ satisfies (2).*

Following the approach of Abreu (1988) and Chari and Kehoe (1990), I characterize the set of outcomes that can be sustained in equilibrium using reversion to the worst equilibrium. The following lemma shows that permanent autarky is the worst equilibrium outcome.

Lemma 1 (Worst Equilibrium Outcome). *In this environment, the worst possible outcome is permanent autarky which can be supported as an equilibrium.*

Proof. See Appendix J □

Keeping track of the entire history of play and relying on trigger strategies, I can sustain multiple equilibrium outcomes. The only two requirements are: the allocation and price satisfy the optimality conditions for all market participants and the borrower's value cannot be lower than the value of the worst equilibrium.

Lemma 2 (Sustainable Outcomes). *Given $\{b_{j,0}\}_{j\in st,lt}$, an allocation $\{\mathcal{C}_b(g^t), k(g^t)\}$ with prices $\{q_{st}(g^t), q_{lt}(g^t), p(g^t)\}$ is the outcome of a sustainable equilibrium if and only if it satisfies (1), (2), (3), (4) and for every t, h_b^t $W^b(h_b^t) \geq \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t)u(g_j f(0))$.*

Proof. See Appendix J □

I now focus on two specific types of equilibria: Markov equilibria which I find to be the relevant equilibrium concept for emerging economies and the constrained efficient equilibrium which is the best achievable outcome in this environment. I then highlight the similitudes and disparities between the two.

5 Markov Debt Management

In this section, I derive the Markov debt management policies. I consider the optimal debt management with and without default. I finally provide a foundation for the use of Markov strategies by the market participants in emerging economies.

5.1 Markov equilibrium with default

Markov equilibria rely on strategies conditioned on the state which only encodes payoff-relevant information (Maskin and Tirole, 2001). In this environment, the payoff-relevant state is $\Omega \equiv (g, b_{st}, b_{lt})$. Formally,

Definition 2 (Markov Equilibrium)**.** *A sustainable equilibrium is Markov if for any* $(h_b^t, h_l^t) \neq (\tilde{h}_b^t, \tilde{h}_l^t)$ *ending with the same* $\Omega_t \equiv (g_t, b_{st,t}, b_{lt,t})$*, strategies are the same such that*

$$W^b(h_b^t) = W^b(\tilde{h}_b^t) \;\wedge\; W^l(h_l^t) = W^l(\tilde{h}_l^t).$$

All Markov equilibria are sustainable equilibria as they restrict the information set to $\Omega_t \subset h^t$ for any $t > 0$.[22] However, the opposite is not true.

The Markov equilibrium with default is a version of Arellano and Ramanarayanan (2012) with endogenous default cost. The government's overall beginning of the period value is

$$W^b(\Omega) = \max_{D \in \{0,1\}} \Big\{ (1-D)V^P(\Omega) + DV^D(g,k) \Big\}, \tag{5}$$

where V^P and V^D correspond to the value of repayment and default, respectively. Under repayment, the government chooses whether to conduct *official* buybacks. Thus

$$V^P(\Omega) = \max_{M \in \{0,1\}} \Big\{ (1-M)V^{NB}(\Omega) + MV^B(\Omega) \Big\}, \tag{6}$$

where V^B and V^{NB} are the values under *official* buyback and no *official* buyback, respectively. If the government decides to officially repurchase its long-term debt,

$$\begin{aligned} V^B(\Omega) = &\max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{g'|g} \Big[W^b(\Omega') \Big] \\ &\text{s.t.} \quad c + q_{st}(g, b'_{st}, b'_{lt}) b'_{st} + q_{lt}(g, b'_{st}, b'_{lt}) b'_{lt} = y(g,k) + b_{st} + b_{lt}(1 + q_{lt}^{bb}), \\ &\qquad\; b'_{lt} \geq b_{lt}. \end{aligned}$$

[22]Definition 2 relates to a strong-Markov equilibrium as it requires that strategies – and not only payoffs – be the same (Chari and Kehoe, 1993). See also Definition J.6 in Appendix J.

Conversely, under no *official* buyback,

$$V^{NB}(\Omega) = \max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{g'|g}\Big[W^b(\Omega')\Big]$$
$$\text{s.t.} \quad c + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g, k) + b_{st} + b_{lt}.$$

Under default, the government is excluded from the capital and bond markets. It subsequently needs to restructure its debt. The value under default is given by

$$V^D(g, k) = u(gf(k)) + \beta \mathbb{E}_{g'|g}\Big[(1-\lambda)V^D(g', 0) + \lambda W^b(g', w_{st}, w_{lt}\frac{1+r}{r})\Big]. \tag{7}$$

I write $V^D(g, k)$ to make explicit the dependence on k. In Section 6, I use the value under default as the optimal contract's outside option.

To avoid redundancy with the previous section, the pricing equations and the equilibrium definition are presented in Appendix B. Note that the lender sets $p = \infty$ if $D(\Omega) = 1$ as the government does not repay the capital input in default. However, it sets $p = 1$ if $D(\Omega) = 0$ to not distort the production of the small open economy. Hence, in equilibrium, the level of capital is $k = k^*(g)$ if $D(\Omega) = 0$ and $k = 0$ otherwise.

In terms of sovereign debt management, this equilibrium is the closest to what is observed in Argentina as one will see in Section 7. On the one hand, defaults arise on equilibrium path and especially when productivity is low (Arellano, 2008). On the other hand, maturity shortens during debt crises. The repayment of long-term debt is laddered through multiple periods which implies a greater default risk than the short-term debt. As a result, close to default, the long-term debt price drastically drops which encourages shorter maturity (Arellano and Ramanarayanan, 2012).[23]

However, *official* buybacks do not arise on equilibrium path.[24] An *official* buyback is a reverse default as it corresponds to an overpayment, while a default is an underpayment of liabilities. While underpayments are sanctioned by markets exclusion, there is no direct reward after overpayments. As a result, the government is unwilling to conduct *official* buybacks. As one will see, the lender will have to enforce such buybacks.

Proposition 1 (No Official Buyback)**.** *Under Assumption 1, $M(\Omega) = 0$ for any Ω.*

Proof. See Appendix J □

[23] As shown by Niepelt (2014), this result is also a consequence of the fact that a default implicates the entire long-term and short-term debt. It would not arise if the government would only default on the maturing portion of the long-term debt. See also Perez (2017).

[24] Note that *unofficial* buybacks can arise on equilibrium path.

5.2 Markov equilibrium without default

I now consider the Markov equilibrium without default. The aim is to introduce an endogenous borrowing constraint that ensures no default on equilibrium path.

With the market structure at hand, it is possible to create some state contingency through *official* buybacks. However, since agents rely on Markov strategies, I cannot condition those buybacks on the history of play. Instead, I consider a simple rule stating that such debt repurchase occur only when $g = g_H$. Formally,

$$M(\Omega) = \begin{cases} 1 & \text{if} \quad g = g_H \\ 0 & \text{else} \end{cases} \tag{8}$$

Given this, the long-term debt becomes a pseudo Arrow security as it pays out more in the high productivity state than in the low productivity state. With this, I can define an endogenous borrowing limit in the form of

$$\begin{aligned} b'_{st} + b'_{lt}[1 + q_{lt}(g_L, b''_{st}, b''_{lt})] &\geq \mathcal{B}(g_L), \\ b'_{st} + b'_{lt}[1 + q^{bb}_{lt}] &\geq \mathcal{B}(g_H), \end{aligned} \tag{9}$$

where the borrowing limit is defined such that

$$\begin{aligned} V^P(g_L, b'_{st}, b'_{lt}) = V^D(g_L, k') \quad &\text{for all} \quad b'_{st} + b'_{lt}[1 + q_{lt}(g_L, b''_{st}, b''_{lt})] = \mathcal{B}(gL), \\ V^P(g_H, b'_{st}, b'_{lt}) = V^D(g_H, k') \quad &\text{for all} \quad b'_{st} + b'_{lt}[1 + q^{bb}_{lt}] = \mathcal{B}(g_H). \end{aligned}$$

This is what Zhang (1997) defines as a no-default borrowing constraint. The definition of $\mathcal{B}(g)$ is the one of Alvarez and Jermann (2000) as it allows for risk sharing between the two productivity states.[25] Hence, compared to the previous Markov equilibrium, *official* buybacks substitute defaults as a source of risk sharing.

The *official* buyback policy (8) imposes that the government repurchases its long-term debt every time g_H realizes. This can be unnecessarily costly. Instead, one can additionally condition *official* buybacks on a specific portfolio of debt. Formally,

$$M(\Omega) = \begin{cases} 1 & \text{if} \quad g = g_H,\ b_{st} = \overline{B}_{st} \text{ and } b_{lt} = \overline{B}_{lt} \\ 0 & \text{else} \end{cases} \tag{10}$$

[25]Note that, without *official* buyback, the endogenous borrowing limit is non-state contingent and corresponds to $(b'_{st}, b'_{lt}) \geq \min_{g' \in G} \Big\{ (b'_{st}, b'_{lt}) : V^P(\Omega') = V^D(g', k') \Big\}$.

where $\overline{B}_{st}$ and $\overline{B}_{lt}$ are some fixed level of short-term and long-term debt, respectively. Nonetheless, it is not clear *ex ante* whether such debt portfolio is attained with strictly positive probability on equilibrium path.

In addition, observe that lower k' reduce the value of autarky tomorrow. This relaxes the borrowing limit (9) today. However, lower k' also reduces the level of output tomorrow. There is therefore an optimal level of capital that balances those two forces and shapes the lender's choice of p. We come back to this in Section 6.

Provided that the lender has commitment, it will be the agent capable of implementing and enforcing the above borrowing constraints. On the one hand, the lender provides bonds as long as (9) holds. On the other hand, following Proposition 1, the lender needs to enforce *official* buybacks as the government is unwilling to conduct them.

In a Markov equilibrium, enforcement should be contingent on Ω only. To show how this can be achieved, define $B_{st}(\Omega) = b'_{st}$ and $B_{lt}(\Omega) = b'_{lt}$ as the short-term and long-term bond policy, respectively. The lender can enforce *official* buybacks through Markov strategies if there are no short-term assets and *official* buybacks are not too costly.

Lemma 3 (Official Buyback Enforcement)**.** *Under Assumption 1,*

I. If $B_{st}(g_H, b_{st}, b_{lt}) \geq 0$, an official buyback is not enforceable for any (b_{st}, b_{lt}).

II. If $B_{st}(g_H, b_{st}, b_{lt}) < 0$, an official buyback is enforceable when either $-b_{st}$ is sufficiently large or $-b_{lt}$ and χ are not too large.

Proof. See Appendix J $\square$

The rationale behind this result is that the lender is the second mover. Hence, it can threaten the government not to roll over debt (i.e. $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$) if the above buyback policy is violated. Obviously, this threat is credible if the government does not possess any assets and *official* buybacks are not too costly. This result can be interpreted as the standard no-saving argument of Bulow and Rogoff (1989) applied to *official* buybacks. I strengthen this result in Appendix C when I endogenize χ.

Lemma 3 imposes strong requirements for *official* buybacks to be enforceable with Markov strategies. As one will see in Section 6, the implementation of the constrained efficient allocation might violate such conditions, justifying the use of non-Markov strategies.

Besides this, I can show the Pareto superiority of the Markov equilibrium without default under the assumption that, in g_L, new issuance of long-term debt is large enough to compensate the cost of *official* buybacks in g_H.

Assumption 2 (Long-Term Borrowing). *In the Markov equilibrium with default, long-term borrowing is such that for any* (b_{st}, b_{lt})

$$B_{lt}(g_H, b_{st}, b_{lt}) \geq b_{lt} \quad \textit{and} \quad B_{lt}(g_L, b_{st}, b_{lt}) \leq \left[\frac{\pi(z_L|z_H)(\delta_2+\delta_3)}{\pi(z_L|z_L)\delta_1} + \pi(z_L|z_L)\delta_1\right] b_{lt} < 0,$$

where $\delta_3 = \frac{r[\pi(g_H|g_H)q_{lt}^{bb}(r+\pi(g_H|g_L))+\pi(g_L|g_H)\pi(g_H|g_L)q_{lt}^{bb}-\pi(g_H|g_H)]-\pi(g_H|g_L)}{r(1+r)(r+\pi(g_H|g_L))} > 0$, $\delta_2 = \frac{1}{r} - q_{lt}^{bb} - \delta_3 > 0$ *and* $\delta_1 = \frac{\pi(g_L|g_L)+\pi(g_H|g_L)q_{lt}^{bb}-1}{r-\pi(g_L|g_L)r+r^2} > 0$.

Given this, the allocation in the Markov equilibrium with default is a mean-preserving spread of the allocation in the Markov equilibrium without default.

Proposition 2 (Superior Markov Equilibrium). *Under Assumptions 1 and 2, the Markov equilibrium without default satisfying (8) is Pareto superior to the one with default. Plus, if official buybacks can only be conditioned on g, it is the best achievable Markov equilibrium.*

Proof. See Appendix J □

Hence, buybacks provide a better source of risk sharing than defaults. The analysis in Section 6 later confirms this point and explains why this is the case.

In terms of sovereign debt management, this equilibrium predicts the opposite of what the previous Markov equilibrium does. Defaults do not arise on equilibrium path, while *official* buybacks do. The predictions of the model are therefore the closest to what is observed in Brazil as one will see in Section 7. Unlike Argentina, Brazil has not defaulted since the late 1980s and conducted *official* buybacks since the 2000s.

Moreover whether maturity shortens in the low productivity state depends on the exact parameters of the model. There are two opposite effects. On the one hand, the long-term bond price varies with g owing to the *official* buyback, while the price of short-term debt does not owing to the absence of default. Particularly, when g_L realizes, the borrower relies more extensively on long-term debt as q_{lt} decreases. On the other hand, the borrower becomes relatively poorer in g_L and increases both its short and long-term indebtedness.

What differentiates the two Markov equilibria? In the equilibrium with default, the lender is *passive* as it lets defaults occur on equilibrium path and simply prices bonds accordingly. In the Markov equilibrium without default, the lender is *active* as it closely monitors the level of debt accumulated and enforce *official* buybacks whenever required. One could therefore argue that the difference between Argentina and Brazil is due to the fact that they face lenders with different behaviors or that the lender coordinates on sunspots. Those are not my interpretation. In light of Lemma 3, I would rather argue that the conditions to enforce *official* buybacks are not met in Argentina. For instance, the cost of an *official* buyback might be too high implying that the lender cannot implement the equilibrium without default – and therefore remains *passive*. I find supportive empirical evidence in Section 7.

5.3 Foundation of Markov equilibria

In light of section 4, the Markov equilibrium is a relatively unsophisticated equilibrium concept as it does not build on past history. However, I show that such equilibria particularly fit the study of emerging economies.

If $\beta(1+r) < 1$, the government is relatively more impatient than the lender. This assumption is standard in the literature on sovereign debt. Not only it is necessary to obtain empirically plausible debt ratios, it is also a property inherited from the general equilibrium analysis and the martingale convergence theorem. In addition, impatience has implications in terms of political economy. It is reduced form for the fact that governments are subject to re-elections and might lose office with positive probability (Alesina and Tabellini, 1990). In this situation, the government can be interpreted as a player whose recollection of past actions eventually fades. In other words, the government's memory goes back to a certain number of periods $\mathcal{T} = \frac{1-\psi}{\psi}$ with $\psi \in [0,1]$.[26]

Besides the bounded government's memory, I introduce a small perturbation in the existing environment. Following Bhaskar et al. (2012) and Angeletos and Lian (2021), in each period t, a utility shock $\epsilon\varrho_{b,t}$ and $\epsilon\varrho_{l,t}$ with $\epsilon \geq 0$ is drawn for the government and the lender, respectively. It has compact support $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ with absolutely continuous density $\varsigma_{P_i} > 0$ where $|\mathcal{C}_i|$ is the cardinality of the choice set of market participant $i \in \{b, l\}$. Moreover, it is independently distributed across market participants, histories and other shocks. If the market participant $i \in \{b, l\}$ chooses a particular action, say $D \in \mathcal{C}_i$, its utility is augmented by $\epsilon\varrho^D_{i,t}$.[27] Finally, the utility shock $\epsilon\varrho_{i,t}$ is privately observed by market participant $i \in \{b, l\}$.

Assumption 3 (Perturbation)**.** *The government's memory goes back to $\mathcal{T} = \frac{1-\psi}{\psi}$ periods in the past with $\psi \in [0,1]$. In addition, in each t, a utility shock $\epsilon\varrho_{i,t}$ with $\epsilon \geq 0$ is drawn from the compact support $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ with absolutely continuous and i.i.d. density $\varsigma_{P_i} > 0$ for each $i \in \{b, l\}$. The utility shock is additive and privately observed.*

Under Assumption 3, the benchmark case considered in Section 4 corresponds to $\psi = \epsilon = 0$. The equilibrium refinement boils down to what happens when $(\psi, \epsilon) > 0$ but arbitrarily small. It means that (a) the government eventually forgets the history of play in the very

[26] I consider here that the lender is a long-run player. However this is without loss of generality. The result holds as long as at most one market participant has unbounded memory.

[27] The instantaneous utility of the government taking action D is given by $u(c_t) + \epsilon\varrho^D_{b,t}$.

distant past and (b) market participants have imperfect knowledge of the other participant's fundamentals. These two assumptions particularly fit the case of emerging economies. On the one hand, such economies suffer from important political instability which can explain (a).[28] On the other hand, the fundamentals of these economies are difficult to assess from the perspective of foreign creditors. Particularly, governments of emerging economies often release data of poor quality or even distort some statistics which can explain (b).[29]

The presence of the privately observed shocks – albeit small and independent – coupled with the asymptotically bounded memory of the government prevent both participants to rely on past history. This causes all non-Markov equilibria to unravel.

Proposition 3 (Foundation of Markov equilibria)**.** *Under Assumptions 1 and 3, with* $(\psi, \epsilon) > 0$*, every sustainable equilibrium is a Markov equilibrium.*

Proof. See Appendix J □

The rationale behind that result follows Bhaskar (1998) and Bhaskar et al. (2012). Suppose the lender conditions its action at time t on a payoff-irrelevant past event, then the government must also condition on this past event given that moves are sequential. Nevertheless, as long as $\psi \neq 0$, the government eventually forgets everything that happened in an arbitrarily distant point in the past. This means that, asymptotically, the government – and consequently the lender – cannot condition on the entire past history. In addition, from an outside perspective, the utility shocks with $\epsilon \neq 0$ generate a non-degenerate probability distribution over actions. Market participants therefore behave is as if their counterpart were randomizing.

Both parts of Assumption 3 are necessary for Proposition 3 to hold. On the one hand, with $\psi = 0$ and $\epsilon \neq 0$, the markets participants could condition their actions on the entire history of play relying on the law of large numbers for the distribution of utility shocks. On the other hand, with $\psi \neq 0$ and $\epsilon = 0$, they could still condition their actions on a limited set of payoff-irrelevant states (e.g. yesterday's default policy).

Proposition 3 has two main consequences. First, I am left with two extremes: both parties either build on the entire history of play or do not at all. Second, it is without loss of generality to consider Markov equilibria in the study of emerging economies. Any non-Markov outcome can only be approximated – as opposed to implemented – by such economies. In what follows, I derive the constrained efficient allocation and show it is generally non-Markov. I then quantitatively assess the goodness of the Markov approximation.

[28]See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019).
[29]See notably Tsyrennikov (2013) and Morelli and Moretti (2021).

6 Constrained Efficient Debt Management

This section presents the constrained efficient debt management policies. I first derive the optimal contract and subsequently characterize the underlying constrained efficient allocation before implementing it the market economy.

6.1 The optimal contract

In what follows I derive the optimal contract which has to account for limited commitment in repayment. The participation constraint deals with the fact that the borrower can always break the contract and opt for autarky (Thomas and Worrall, 1994). Denoting g^t as the history of realized value of g at time t, it must hold that for all t and g^t

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(c(g^j)) \geq V^D(g_t, k_t). \tag{11}$$

If the borrower breaks the contract, it is sent to autarky for some time but can regain access to the market with probability λ and resumes the Markov equilibrium with default. $V^D(g_t, k_t)$ therefore corresponds to the value of default in the Markov equilibrium given by equation (7). As a result, the participation constraint ensures that the borrower's value of remaining in the contract is at least as large as the value of opting out.

Given the above constraint, the optimal contract between the borrower and the lender in sequential form is the result of

$$\max_{\{k(g^t),c(g^t)\}_{t=0}^{\infty}} \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) u(c(g^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) T(g^t) \tag{12}$$

$$\text{s.t.} \quad (11),\ T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t) \text{ for all } g^t, t \text{ with } (\mu_{b,0}, \mu_{l,0}) \geq 0 \text{ given.}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial non-negative Pareto weights assigned by the Planner to the borrower and the lender, respectively.

The Planner allocates capital and consumption to maximize the lender's and the borrower's weighted utility subject to the resource constraint and the participation constraint. The above maximization problem combines the utility function $u(\cdot)$ with the production function $f(\cdot)$ and therefore might not be convex.

Assumption 4 (Convexity)**.** *Define the optimal level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ and $h := gf(k) - k$ for $k \in [0, k^*(g)]$ with $h^*(g) = gf(k^*(g)) - k^*(g)$. Let $K(h)$ denote the inverse mapping from $[0, h^*(g)]$ to $[0, k^*(g)]$ such that $k = K(h)$. For all $g \in G$, $u(gf(k(h)))$*

is convex in h for $h \in [0, h^*(g)]$.

Following, Aguiar et al. (2009), Assumption 4 ensures that there is no need for randomization whenver the curvature of $u(\cdot)$ and $f(\cdot)$ is not too pronounced.

I now derive the recursive formulation of the above maximization problem. Following Marcet and Marimon (2019), I reformulate (12) as a saddle-point Lagrangian problem,

$$
\begin{aligned}
\mathcal{SP} \min_{\{\gamma(g^t)\}_{t=0}^{\infty}} \max_{\{k(g^t), c(g^t)\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) \mu_{b,t}(g^t) u(c(g^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) \mu_{l,t}(g^t) T(g^t) \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) \gamma(g^t) \Big[\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(c(g^j)) - V^D(g_t, k_t) \Big] \\
\text{s.t.} \quad & T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t), \\
& \mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t) \text{ and } \mu_{l,t+1}(g^t) = \mu_{l,t}(g^t) \text{ for all } g^t, t \\
& \text{with } \mu_{b,0}(g_0) \equiv \mu_{b,0} \text{ and } \mu_{l,0}(g_0) \equiv \mu_{l,0} \text{ given.}
\end{aligned}
$$

In this formulation, $\beta^t \pi(g^t|g_0)\gamma(g^t)$ is the Lagrange multiplier attached to the participation constraint of the borrower at time t. The above problem defines two new co-state variables, $\mu_{b,t}(g^t)$ and $\mu_{l,t}(g^t)$, which are the temporary non-negative Pareto weights the Planner attributes to the borrower and the lender, respectively. These variables are initialized at the original Pareto weights and subsequently become recursive.

Following Ábrahám et al. (2019), I define the relative Pareto weight of the borrower at time t as $x_t(g^t) = \frac{\mu_{b,t}(g^t)}{\mu_{l,t}(g^t)}$. Given the non-negativity and boundedness of the Lagrange multipliers, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} < \infty$.[30] Defining $\eta \equiv \beta(1+r) \leq 1$ and $\nu(g^t) \equiv \frac{\gamma(g^t)}{\mu_{b,t}(g^t)}$, the law of motion of the relative Pareto weight is given by

$$
x_{t+1}(g^t) = (1 + \nu(g^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}}. \tag{13}
$$

With this normalization, $\nu(g^t)$ represents the multiplier attached to the participation constraint. Following Marcet and Marimon (2019), the state vector for the problem reduces to (g, x) and the Saddle-Point Functional Equation is given by

$$
\begin{aligned}
FV(g, x) = \mathcal{SP} \min_{\nu(g)} \max_{k(g), c(g)} \; & x \Big[(1 + \nu(g)) u(c(g)) - \nu(g) V^D(g, k) \Big] \\
& + T(g) + \frac{1}{1+r} \sum_{g'} \pi(g'|g) FV(g', x')
\end{aligned} \tag{14}
$$

[30]The relative Pareto weight cannot be negative as $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t.

$$\text{s.t.} \quad x'(g) = (1+\nu(g))\eta x, \; T(g) = gf(k(g)) - c(g) - k \; \forall g.$$

The value function takes the form of $FV(g,x) = xV^b(g,x) + V^l(g,x)$ with $V^b(g,x) = u\left(c(g)\right) + \beta \mathbb{E}_{g'|g}\left[V^b\left(g',x'\right)\right]$ and $V^l(g,x) = T(g) + \frac{1}{1+r}\mathbb{E}_{g'|g}\left[V^l\left(g',x'\right)\right]$. I obtain the optimal consumption and capital policies by taking the first-order conditions in (14)

$$u_c(c(g)) = \frac{1}{x(1+\nu(g))} \quad \text{and} \quad gf_k(k(g)) - 1 = \nu(g)u_c(gf(k(g)))gf_k(k(g))x.$$

In terms of consumption, the binding participation constraint of the borrower (i.e. $\nu > 0$) induces an increase in consumption. Regarding capital, the economy does not reach the production-maximiging level of capital $k^*(g)$ as long as the participation constraint binds in g. In the next subsection, I formalize this argument in Proposition 5.

6.2 Equilibrium properties

I characterize the main properties of the contract in terms of Pareto frontier and risk sharing. Additional characterization can be found in Appendix D.

I start with the definition of two threshold values for the relative Pareto weight: the one for which the borrower's participation constraint binds with $k = 0$ and with $k = k^*(g)$.

Definition 3 (Threshold)**.** *Define $x_a(g)$ such that $V^b(g,x_a(g)) = V^D(g,0)$ and $x^*(g)$ such that $V^b(g,x^*(g)) = V^D(g,k^*(g))$.*

In words, $x^*(g)$ is the lowest relative Pareto weight that can sustain $k^*(g)$. Conversely, $x_a(g)$ is the weight associated with the autarkic allocation (i.e. $k = 0$) and is therefore the lowest possible weight in the contract. I show next that $x_a(g)$ is never attained.

Proposition 4 (Efficiency)**.** *Under Assumptions 1 and 4, the autarkic allocation is not optimal meaning that $x \in \tilde{X} \equiv [\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. Moreover, $V^l(g,x)$ is strictly decreasing, while $V^b(g,x)$ is strictly increasing in $x \in \tilde{X}$ for all $g \in G$.*

Proof. See Appendix J $\square$

The proposition is made of two parts. First, autarky (i.e. $k = 0$) is not optimal.[31] Due to the Inada condition on the production function, there are always strictly positive gains from trade between the borrower and the lender when k is close to zero. This means that

[31] If one would introduce domestic capital in the model, this result would continue to hold as long as foreign capital is complementary – as opposed to substitute – to domestic capital. Note further that the depreciation rate of capital is irrelevant here.

defaults – which imply markets exclusion – cannot implement the Planner's constrained efficient allocation. Second, the proposition states that the optimal contract is constrained efficient which makes it the best achievable outcome in this environment.

The following proposition highlights the main properties of the constrained efficient allocation. The contract features production distortions, risk sharing across states and state-contingent debt relief.

Proposition 5 (Constrained Efficient Allocation)**.** *Under Assumptions 1 and 4,*

I. (Production). $k(g, x) = k^*(g)$ *for* $x \geq x^*(g)$ *and* $x^*(g_H) > x^*(g_L)$. *Conversely, for all* $x, \ddot{x} \in \tilde{X}$ *with* $x^*(g) > x > \ddot{x}$, $0 < k(g, \ddot{x}) < k(g, x) < k^*(g)$.

II. (Risk-Sharing). $c(g_L, x) < c(g_H, x)$ *and* $x'(g_L, x) < x'(g_H, x)$ *for all* $x < x^*(g_H)$ *and* $c(g_L, x) = c(g_H, x)$ *and* $x'(g_L, x) = x'(g_H, x)$ *otherwise.*

III. (Liabilities). $V^l(g_L, x) < V^l(g_H, x)$ *for all* $x \in \tilde{X}$.

Proof. See Appendix J □

Part I of the above proposition states that the production-maximizing level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ is attained only if the relative Pareto weight, x, is above a certain threshold. Capital distortion is a consequence of the binding participation constraint (11). As the autarky value depends on the level of capital in the economy, the Planner finds optimal to reduce k to relax the constraint. It continues to decrease k as long as x decreases but never finds optimal to set $k = 0$. As already mentioned, this means that defaults cannot implement the Planner's allocation.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind in all productivity states. Otherwise, the Planner provides more consumption and a greater continuation value when the high productivity state realizes.

Part III relates to the liabilities of the borrower. In this environment, $T(g)$ corresponds to the borrower's current account balance. Hence, the value of the lender represents the net foreign asset position in the contract. A positive value of $V^l(g, x)$ therefore indicates that the extent to which the borrower is indebted. The proposition states that the liabilities increase when g is high. This implies that the Planner adopts a state-contingent policy as it provides debt relief in low productivity states. This state contingency will be replicated through *official* buybacks in the market economy.

Having determined the constrained efficient allocation, I now show that the long-term contract is characterized by an ergodic set of relative Pareto weights.

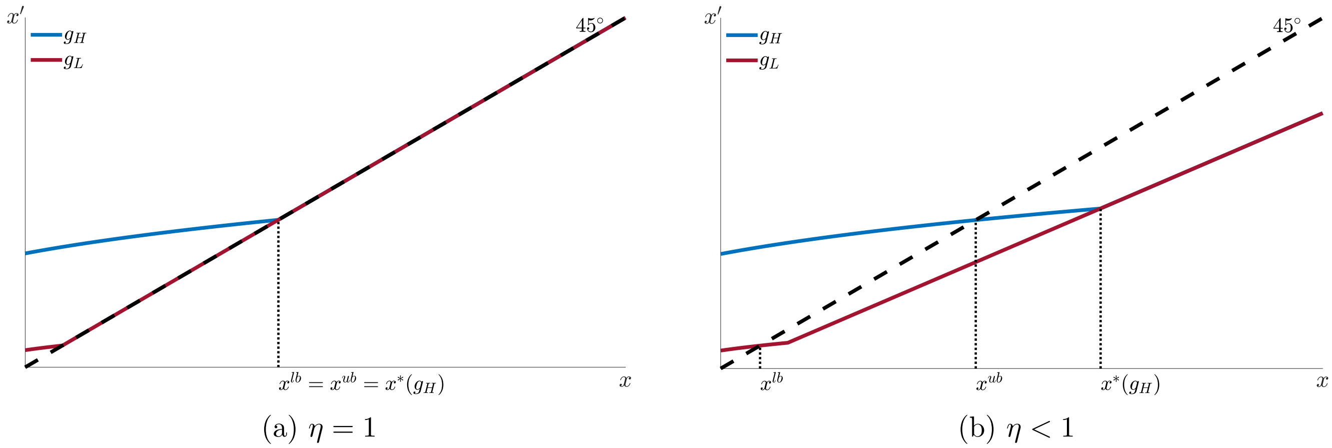


Figure 1: Steady State Dynamic

Proposition 6 (Steady State)**.** *A steady state is defined by an ergodic set of relative Pareto weights $x \in [x^{lb}, x^{ub}] \subset \tilde{X}$. Under Assumptions 1 and 4, it holds that $x'(g_H, x^{ub}) = x^{ub}$ and $x'(g_L, x^{lb}) = x^{lb}$ and*

I. If $\eta = 1$, then $x^{lb} = x^{ub} = x^*(g_H)$.

II. If $\eta < 1$, then $x^{lb} < x^{ub} < x^*(g_H)$.

Proof. See Appendix J □

The proposition states that whenever the borrower is sufficiently patient (i.e. $\eta = 1$), the steady state does not display any dynamic. Conversely, whenever the borrower is relatively impatient (i.e. $\eta < 1$), the steady state is dynamic. This dynamic is however bounded below by x^{lb} and above by x^{ub}. For instance, after a sufficiently long series of g_L (g_H), the contract hits x^{lb} (x^{ub}). It then stays there until g_H (g_L) realizes and that irrespective of the past realizations of the shock. The bounds of the ergodic set represent therefore regions of amnesia in the contract. Such regions will have specific interpretations in the implementation. Figure 1 illustrates each of the two steady states.

6.3 Equilibrium implementation

Having derived and characterized the constrained efficient allocation, I now construct a sustainable equilibrium in the market economy that implements the constrained efficient allocation. I also give the conditions for the implementation to be Markov as exposed in Section 5.

Proposition 7 (Implementation)**.** *Under Assumptions 1 and 4, given a constrained efficient allocation, a sustainable equilibrium exists that implements it. Moreover, if Part II of*

Lemma 3 applies at (g_H, x^{ub}), the sustainable equilibrium in question is the Markov equilibrium without default.

Proof. See Appendix J □

The implementation works as follows. The government conducts *official* buybacks when the economy hits the upper bound of the ergodic set (i.e. $x = x^{ub}$). As this bound is reached after a sufficiently long series of high productivity shocks, this buyback policy generates a specific term structure in which high productivity shocks are related to relatively larger long-term bond prices than low productivity shocks, while the short-term bond price remains unchanged. Given this, I can equalize the value of debt in the contract, $V^l(g, x)$, with the value of the debt in the market economy, $b_{st} + b_{lt}[1 + q_{lt}]$, for each (g, x). As I have two productivity states and two bonds, this gives a system of two equations with two unknowns which has a unique solution for each x given the specified term structure.

Lemma 4 (Official Buyback and No Default)**.** *Under Assumptions 1 and 4, the implementation features official buybacks on equilibrium path and no default. However, official buybacks cannot occur when g_L realizes.*

Proof. See Appendix J □

Lemma 4 is made of two parts. First, the implementation does not rely on defaults. As shown in Proposition 4, the Planner never finds optimal to distort capital to zero. This means that there is no proper markets exclusion in any points of the contract. It is therefore not possible to interpret the borrower's binding constraint as a default in my environment.[32]

Second, *official* buybacks need to occur to generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract. In particular, as they involve a premium χ, they arise in the high productivity state. Unlike defaults, they are an efficient source of risk sharing. Defaults entail costs for both the lender and the borrower, while *official* buybacks are solely costly for the latter. A default is therefore not renegotiation proof as both contracting parties would be strictly better off avoiding this event *ex post*.

A direct corollary of Lemma 4 is that the long-term bond spread is negative.[33] On the one hand, in the absence of default, there is no positive spread. On the other hand,

[32] Müller et al. (2019) and Restrepo-Echavarria (2019) interpret the borrower's binding constraint as a form of preemptive restructuring which does not trigger market exclusion. Nonetheless, Asonuma and Trebesch (2016) show that even preemptive restructurings are followed by markets exclusion in the data.

[33] This is a feature that one finds in other implementations such as the ones of Alvarez and Jermann (2000), Ábrahám et al. (2019) and Liu et al. (2020). The mechanism at work is different though. The negative spread restricts the trade of state-contingent securities in a two-sided limited commitment problem when the participation constraint of the risk-neutral lender binds.

official buybacks entail a premium χ implying that the long-term bond price exceeds the risk-free price. Nevertheless, as negative spreads have little empirical support in the context of emerging economies, I consider that the lender charges an excess return in the next section.

Another corollary is that whether maturity shortens in the low productivity state depends on the exact parameters of the model. As already explained in Section 5, there is a substitution effect which pushes the maturity towards the long end in g_L. In particular, every successive realization of g_L makes q_{lt} less sensitive to changes in g. This is because the lender anticipates that *official* buybacks are less likely to occur. More long-term debt is therefore required to replicate the state-contingent liabilities of the optimal contract. In opposition, there is an income effect which increases the total indebtedness in g_L. In steady state $x'(g_L) \leq x$ as shown in Proposition 6. This implies that the value of the lender increases as g_L realizes. Thus, more short-term and long-term debt are needed to replicate the liabilities of the optimal contract.

In Appendix E, I explore alternatives to *official* buybacks. Empirically, such alternatives do not exist or remain underdeveloped. Moreover, they raise similar enforcement issues as *official* buybacks. That is why I do not consider them in the main analysis.

Given the analysis of Section 5 and the importance of Markov strategies in the context of emerging economies, the following lemma determines whether the constrained efficient allocation can be implemented by means of Markov strategies.

Lemma 5 (Non-Markov Implementation)**.** *Under Assumptions 1 and 4, for a given implementation, Part II of Lemma 3 does not generally apply at the point of official buyback.*

Proof. See Appendix J ☐

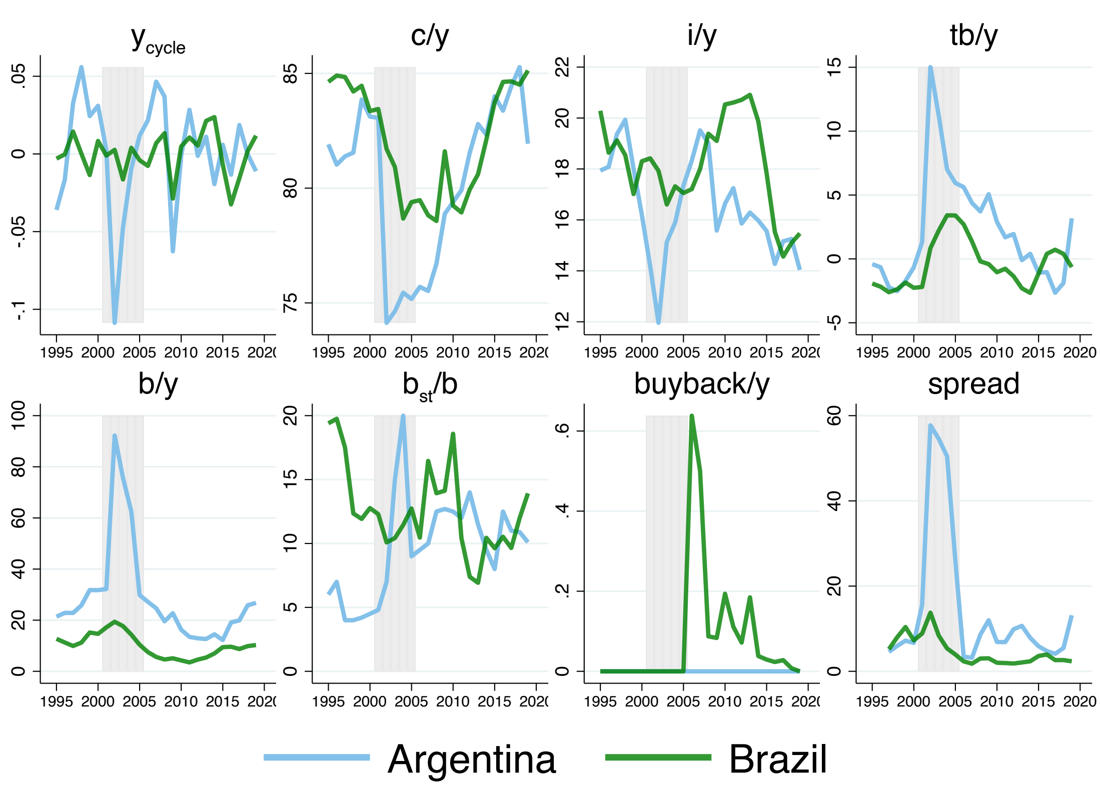
Lemma 3 states that *official* buybacks are enforceable in a Markov equilibrium when there is no short-term assets and *official* buybacks are not too costly. However, Lemma 5 shows that, at the point of *official* buyback, the borrower needs to hold short-term assets unless the *official* buyback premium is sufficiently large. Hence, *official* buybacks are not automatically enforceable through Markov strategies. Especially in the next section, I show quantitatively that that Markov strategies fail to implement the constrained efficient allocation under empirically plausible buyback premia. Trigger strategies are therefore more often than not necessary, which puts the implementation at the mercy of Proposition 3.

7 Quantitative Analysis

This section starts with a comparison of Argentina and Brazil since 1995. I then calibrate the Markov equilibrium with default to Argentina and assess the fit of the model to the data. I show that the Markov equilibrium without default is quantitatively close to Brazil and compare the two Markov equilibria with the constrained efficient allocation.

7.1 Argentina vs. Brazil

I compare the experience of Argentina and Brazil starting in 1995 as Brazil defaulted last in the 1980s and regained access to the international market after the implementation of the Brady Plan in 1994.[34] Additional results can be found in Appendix G.



Note: Output detrended using the Hodrick–Prescott filter with a smoothing parameter of 6.25. The spread corresponds to the EMBI spread, i to the investment, tb/y to the trade balance over output and b to the public sector external debt stock with $b = b_{st} + b_{lt}$.

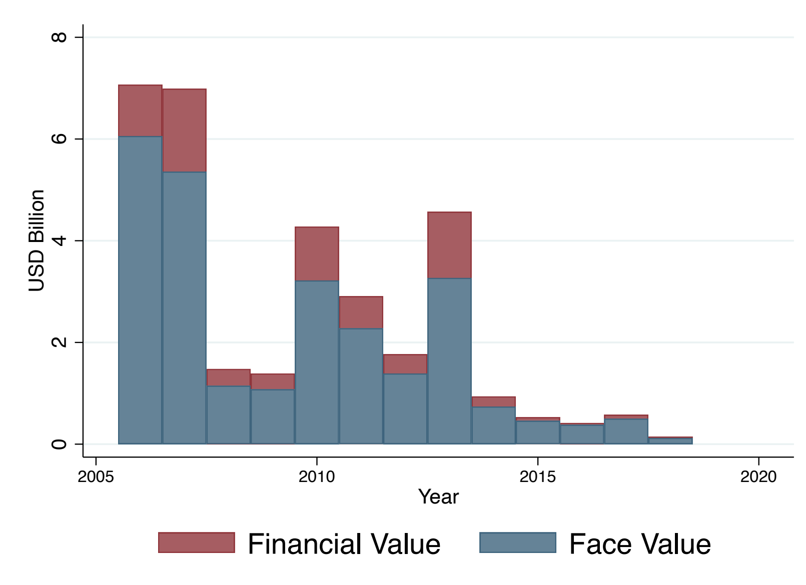
Source: Author's calculation, Buera and Nicolini (2021), Tesouro Nacional, Global Financial Data and World Bank.

Figure 2: Argentina vs. Brazil

Figure 2 presents the main statistics of interests for both countries. The blue (green) line represents Argentina (Brazil). As one can see, the two countries recorded a sudden drop in output at the end of the 1990s. Brazil experienced a major currency crisis following a

[34]In general, see Buera and Nicolini (2021) for for the economic history of Argentina and Ayres et al. (2021) for Brazil.

speculative attack on the real, while Argentina suffered from a banking crisis. Consumption and investment drastically reduced, while indebtedness and the spread largely increased. In addition, the average maturity shortened for both countries in the years preceding the crisis.[35] Most importantly, Argentina eventually defaulted in 2001 (represented by the grey area), whereas Brazil did not.



Source: Author's calculation and Tesouro Nacional.

Figure 3: Official Buyback in Brazil

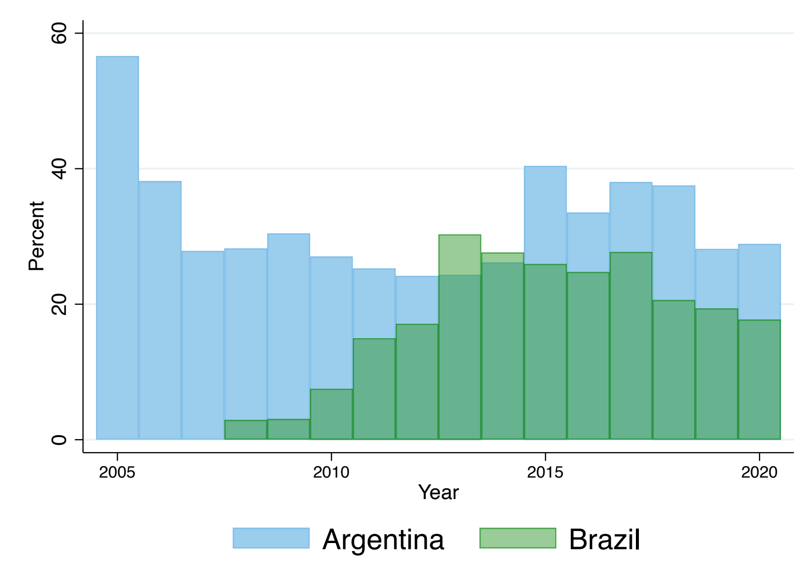
Moreover, Brazil conducted *official* buybacks while Argentina did not under the period considered.[36] Figure 3 depicts the *official* buybacks conducted by Brazil. In 2006, the country started the Early Redemption Program which aimed at correcting the average maturity of the debt and reducing the potential refinancing risk.[37] Repurchases were conducted by the Brazilian National Treasury either directly on the secondary market or indirectly through call options and special auctions. I identify two features in the Brazilian *official* buybacks. First, such buybacks are costly. The financial value (i.e. the red bar) is systematically above the face value (i.e. the blue bar) meaning that the Brazilian government always paid a premium to extract its debt out of the market. This premium is primarily explained by the fact that the repurchased Brazilian bonds entailed high coupon rates relative to the market interest rate. On average, the financial value is 24.5% above the face value. This figure

[35]See also the maturity regression analysis in Appendix G.

[36]Note that Argentina repurchased external debt at a discount (i.e. below par) on the secondary market in various occasions. However, this corresponds to a default in the form of distressed debt exchange.

[37]See https://www.gov.br/tesouronacional/en/federal-public-debt/external-market/buyback-program.

provides the basis of calibration of $\chi > 0$ in the next subsection. Second, those buybacks were the largest when the output of the Brazilian economy was on or above trend consistent with the model's prediction. I provide more details on that in Appendix G.



Source: Author's calculation and Onen et al. (2023).

Figure 4: Share of Domestic Holdings of Foreign-Currency Sovereign Bonds

What can explain the absence of *official* buybacks in Argentina? I provide one potential explanation. Argentina suffers from a larger home bias than Brazil in the holdings of external sovereign debt. As one can see on Figure 4, in the early 2000s roughly 35% of the Argentinian foreign-currency sovereign bonds were held domestically. In opposition, for that same period, this figure amounts to 10% for Brazil. Linking Figures 3 and 4, one clearly sees that the amount bought back reduces as the share of domestic holdings increases.[38] In other words, there is a negative association between the share of domestic holdings and *official* buybacks in Brazil. Especially, *official* buybacks become negligible in Brazil around the years in which the share of domestic holdings align with the one in Argentina. This supports the claim made in Section 5 that the difference between Argentina and Brazil can be linked to the cost of *official* buybacks.

As a result, the experience of Argentina and Brazil qualitatively relate to the predictions of the Markov equilibrium with and without default, respectively. In what follows, I gauge the quantitative fit.

[38]See also the regression analysis in Appendix G.

7.2 Calibration

I calibrate the Markov equilibrium with default as it corresponds to the workhorse model in the literature on sovereign defaults. The calibration aims at matching some specific moments of the Argentine economy over the period 1995-2019. Table 1 summarizes each parameter.

Table 1: Calibration

Parameter	Value	Description	Targeted Moment
A. Based on Literature			
ϑ	2.00	Risk aversion	
r^f	0.01	Risk-free rate	
W_{st} and W_{lt}	0.00	Recovery value	
B. Direct Measure from the Data			
$\pi(g_H\|g_H)$	0.93	Probability staying high state	
$\pi(g_L\|g_L)$	0.68	Probability staying low state	Real total factor productivity
g_L	0.44	Productivity in low state	
$1-\alpha$	0.70	Labor share	Labor income share
χ	0.197	*Official* buyback premium	Financial over face value of debt
r^e	0.04	Excess return	US excess return on debt
C. Based on Model solution			
β	0.80	Discount factor	Debt-to-GDP ratio
g_H	1.12	Productivity in high state	Correlation consumption and output
ϕ	1.50	CES production	Investment-to-GDP ratio
λ	0.281	Probability re-accessing market	Average spread

In accordance with Assumption 1, the instantaneous utility function takes the CRRA form with a coefficient of relative risk aversion of ϑ, i.e. $u(c) = \frac{c^{1-\vartheta}}{1-\vartheta}$. I adopt $\vartheta = 2$ as it is standard in the real business cycle literature and set the discount factor to $\beta = 0.8$ to match the average public sector external debt-to-GDP ratio of 28.71%. This corresponds to a quarterly discounting of 0.945 which is standard in studies on emerging economies. In addition, the production function has the CES form

$$F(k,l) = \left[\alpha k^{\frac{\phi-1}{\phi}} + (1-\alpha) l^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where α represents the capital share and ϕ the CES parameter. The value of $1-\alpha$ is set to the standard labor share in GDP adopted in the literature on emerging economies (Mendoza and Yue, 2012). The CES parameter is $\phi = 1.5$ to match the share of investment in GDP and is within the range of admissible values in the business cycle literature. In addition, I estimate the Markov transition matrix by means of a Markov-switching AR(1) process with two states. For this, I use data on the real total factor productivity of Argentina from 1990 to 2019 from the Penn World Table 10.0 (Feenstra et al., 2015). I then select g_H to match the correlation between consumption and output and accordingly set g_L to obtain the average real TFP of Argentina given the estimated transition matrix.

Regarding the exogenous rate r, I relax the assumption of break-even lending and set $r = r^f + r^e$ where r^f represents the risk-free rate and r^e corresponds to the lender's excess return. This means that the lender borrows at r^f and lend at $r > r^f$. This has two purposes. First, it better captures the potential risk premium US investors demand on emerging market bonds. I therefore set $r^e = 0.0434$ consistent with the US excess return on debt instruments estimated by Gourinchas et al. (2017) and $r^f = 0.01$ as it is standard in the literature. Second, modelling an excess return enables to correct the negative spread which has little empirical support for the countries under study.[39]

As it is standard in the literature on sovereign defaults, I set $w_{st} = w_{lt} = 0$ meaning that the recovery value of debt is nil. This also avoids large increases in indebtedness and consumption boom prior to default.[40] Besides this, I choose $\lambda = 0.281$ to match the average (EMBI) spread of 14.17%. The value selected implies an expected default length of roughly 3.5 years. This is below the value of 5.1 years Cruces and Trebesch (2013) find in the data.

Finally, there is one parameter left to calibrate: the *official* buyback premium. This parameter only matters for equilibria without default.[41] I set $\chi = 0.197$ to match the wedge between the financial and the face value recorded on the Brazilian Early Redemption program highlighted in the previous subsection.

In what follows I use the calibration to compute the two Markov equilibria presented in Section 5 as well as the constrained efficient allocation derived in Section 6. Note that I compute the Markov equilibrium without default according to the *official* buyback policy (8) as (10) requires to define $\overline{B}_{st}$ and $\overline{B}_{lt}$ which are difficult to characterize *ex ante*.[42]

7.3 Numerical results

This subsection presents the result of the calibration. It gauges the fit of the model with respect to the data for both targeted and non targeted moments. It also compares the outcome of the Markov allocation with default (MA), without default (MAND) and the constrained efficient allocation (CEA) together.

The upper part of Table 2 presents the fit of the MA with respect to the Argentine economy in terms of targeted moments. It also reports the result of the CEA and the MAND. As one can see, the MA replicates relatively well the main features of the Argentine economy in terms of consumption, investment, spreads and indebtedness.

[39] The spread is calculated with respect to the risk-free rate.

[40] See Hatchondo et al. (2016), Dvorkin et al. (2021) and Fourakis (2021).

[41] Following the discussion in Section 5, one could say that $\chi \to 1$ in the Markov equilibrium with default.

[42] Note further that I consider the same capital policy (i.e. $k = 0$ in default and $k = k^*(g)$ otherwise) for both Markov equilibria as this enables a better comparison between the two.

Table 2: Targeted and Non-Targeted Moments

A. Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
i/y	14.26	14.22	17.98	16.22	14.61
$-b/y$	28.71	28.15	10.12	7.18	-353.20
Spread	14.17	12.88	4.97	3.85	3.95
$\text{corr}(c, y)$	0.96	0.94	0.88	0.95	0.68
B. Non-Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
c/y	78.90	85.80	81.43	82.90	82.52
$\sigma(c)/\sigma(y)$	1.04	0.84	0.83	0.71	0.21
$\sigma(i)/\sigma(y)$	3.31	0.08	3.21	0.08	1.25
$\sigma(tb/y)/\sigma(y)$	1.22	0.40	1.38	0.40	0.96
$\sigma(spread)/\sigma(y)$	4.53	0.77	2.47	0.00	0.01
$\text{corr}(i, y)$	0.97	0.91	0.93	1.00	0.99
$\text{corr}(tb/y, y)$	-0.60	0.53	-0.34	0.79	0.97
$\text{corr}(spread, y)$	-0.63	-0.24	-0.20	-0.71	-0.67

Note: The variable $\sigma(\cdot)$ denotes the volatility, tb/y denotes the trade balance over output and i the investment which corresponds to k in the model given the depreciation of 1. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25.

The lower part of Table 2 presents the fit of the MA in terms of non-targeted business-cycle moments. In general, the fit is poor. This is because I only consider 2 productivity states meaning that I rule out tail events. The MA generates too low volatilities for most variables. Moreover, the trade balance is pro-cyclical unlike the data. The model however produces empirically plausible correlations for the spread and investment relative to output.

Having said that, the MA generates a realistic debt dynamic. Table 3 depicts the underlying debt structure of the Markov equilibria and the CEA. Two points deserve to be noted. First, the MA replicates well the data as maturity shortens during debt crises, while indebtedness relative to GDP increases. Second, during a restructuring, the maturity lengthens and the level of debt remains substantial.[43]

Turning to the MAND, Table 2 presents the similarities with Brazil. As discussed at the beginning of the section, Brazil has not defaulted since the end of the 1980s, whereas Argentina roughly defaulted 3 times since 1995 with the most recent episode being in 2023. Second, Brazil conducted an official buyback program from 2006 to 2018. Third, maturity shortens in the low productivity state. Fourth, in terms of economic fundamentals, Brazil records a lower average debt ratio, a greater average investment ratio and a lower average spread than Argentina for the period 1995 to 2019. The MAND is capable of matching most

[43] See Benjamin and Wright (2013), Mihalache (2020) and Dvorkin et al. (2021) for related results.

Table 3: Debt Structure

	Mean $-b/y$ (percent)	Mean $-b/y$ in g_H (percent)	Mean $-b/y$ in default (percent)	Mean $-b/y$ in restructuring (percent)
Argentina	28.7	22.0	65.7	29.9
MA	28.2	24.1	216.4	17.5
Brazil	10.1	9.0	-	-
MAND	7.2	2.3	-	-
CEA	-353.2	-353.3	-	-
	Mean b_{st}/b (percent)	Mean b_{st}/b in g_H (percent)	Mean b_{st}/b in default (percent)	Mean b_{st}/b in restructuring (percent)
Argentina	9.7	8.3	11.7	9.0
MA	44.0	43.6	84.1	64.5
Brazil	12.6	12.4	-	-
MAND	21.7	11.3	-	-
CEA	112.5	112.0	-	-

Note: In the CEA, $b_{st}/b > 1$ as $b_{st} > 0$ in some states while $b_{lt} \leq 0$.

of the main moments of the Brazilian economy despite the fact that none of them where directly targeted.[44] This suggests that Brazil can be interpreted as the counterfactual of Argentina with buybacks and without default in the period 1995-2019.

Looking at the CEA in the last column of Table 2, one directly observes that it predicts an empirically implausible average indebtedness. In fact, the borrower holds a net asset position. Such prediction is well known in the literature on fiscal policy under commitment as highlighted by notably Buera and Nicolini (2004) and Faraglia et al. (2010). Even though I consider an alternative environment without commitment, the bond portfolio implementing the CEA remains at odds with the data.

The MAND and the CEA achieve better risk sharing than the MA. In the latter, consumption corresponds to a lower share of output, correlates less with output and is less volatile. Investment corresponds to a larger share of output, correlates more with output and is more volatile. Finally, the bond spread is lower than in the MA given that defaults do not arise on equilibrium path and *official* buybacks exceed the risk-free price. The same holds true for the MAND with the exception of a slightly larger consumption correlation than in the MA.

7.4 Implementation and buyback cost

In this subsection, I discuss the implementation of the CEA in the market economy. I show that Part II of Lemma 3 is not satisfied under empirically plausible *official* buyback premia.

Figure 5 depicts the main policy functions related to the optimal contract. The law

[44] Note that the excess return corrects the negative spread in the model.

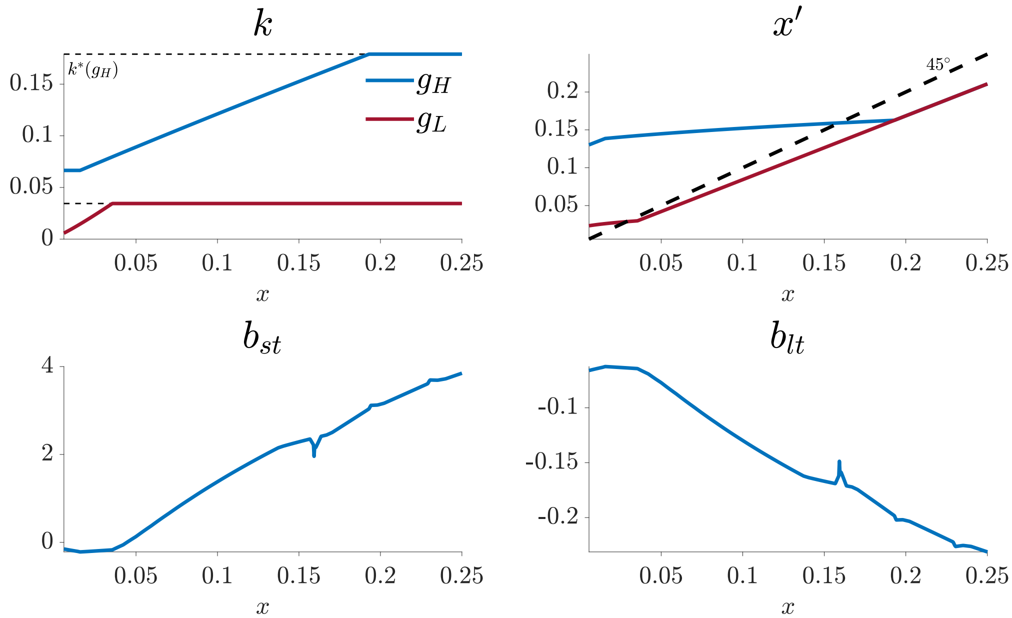
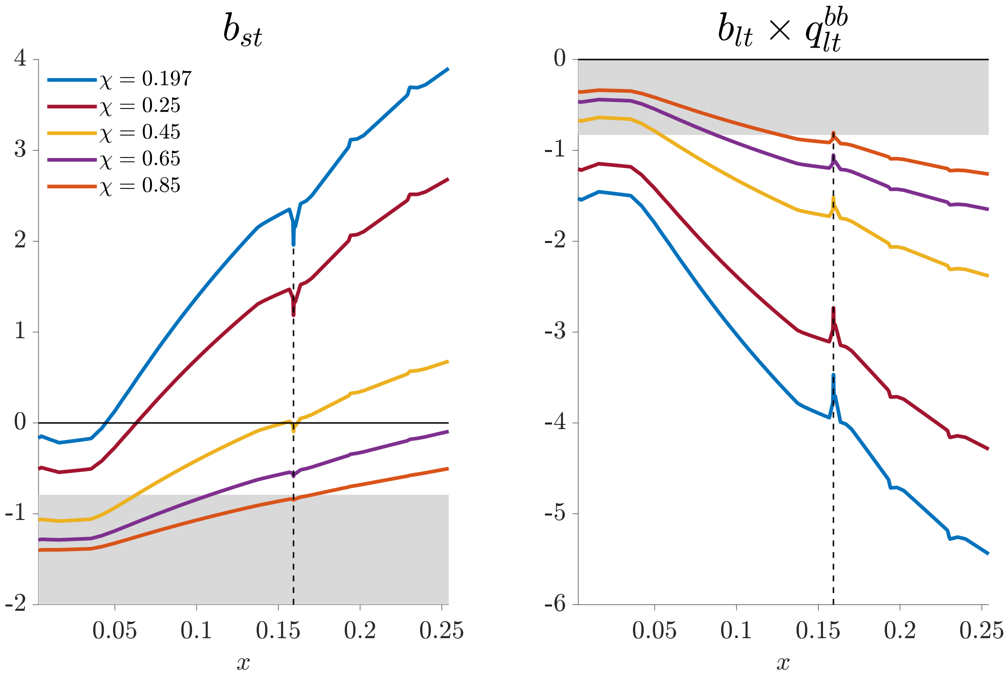


Figure 5: Main Policy Functions of the CEA

of motion of the relative Pareto weight is consistent with the fact that $\eta < 1$. Capital is distorted for low values of x. However, as soon as x is sufficiently high the productivity-maximizing level, $k^*(g)$, is reached. Nevertheless, such values of x are outside the steady state and therefore capital remains distorted in the long-run when $g = g_H$. Regarding borrowing, when x is low, the government accumulates more short-term debt and less long-term debt. In opposition, when x gets larger, the opposite is true. Furthermore, the borrower holds short-term assets – especially when *official* buybacks occur. This violates Part II of Lemma 3 and explains the reliance on trigger strategies.

To obtain short-term debt holdings when *official* buybacks occur, the premium χ should be larger than the calibrated one. The rationale behind this is that with a larger χ, the long-term bond price is more sensitive to the realization of g. As a result, more short-term debt and less long-term debt are required to replicate the state-contingent liabilities of the optimal contract. Figure 6 depicts the portfolio of bonds necessary to implement the CEA for different values of χ. The black dashed line represents the relative Pareto weight at which the *official* buyback occurs – i.e. $x = x^{ub}$. As one can see, it is possible that the borrower holds short-term debt – and not asset – by more than doubling χ relative to the calibration benchmark. This means that Markov strategies fail to implement the Planner's allocation under empirically plausible *official* buyback premia in emerging economies.

Furthermore, for *official* buybacks to be enforceable with Markov strategies, the premium χ should be close to 1. The grey area in Figure 6 represents the region in which Part II of



Note: The grey area depicts the region in which the implementation of the optimal contract is Markov – i.e. Part II of Lemma 3 applies when $g = g_H$.

Figure 6: Implementation and χ

Lemma 3 applies. In this region, the holding of short-term debt is substantial, while there is little long-term debt and χ gets closer to 1. This ensures that the threat of a sudden stop on debt is credible. Thus, for the implementation to be Markov, the borrower ought to accumulate primarily short-term debt and needs to conduct small *official* buybacks at very large premia – i.e. of the order of 650% of the face value. Again, such high premia are at odds with what is observed in the data.

Thus, with respect to the literature on fiscal policy under commitment and the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), I reconcile the model's prediction with the data by arguing that the borrower lacks the strategical sophistication to implement the CEA. Under an empirically plausible cost of *official* buyback, the borrower can only approximate – as opposed to replicate – the returns of Arrow securities with non-contingent bonds of multiple maturities. Nevertheless, such approximation is consistent with the sovereign debt management of emerging economies as shown previously.

7.5 Equilibria comparison

In this subsection, I explore in more details the differences between the Markov equilibria and the CEA. For this purpose, I conduct two main exercises. First, I compute welfare gains with respect to the MA. Second, I measure the distance of each equilibria from the Pareto frontier. Additional results can be found in Appendix H.

Table 4 depicts the welfare gains of the CEA and the MAND in consumption equivalent terms with respect to the MA for both the borrower and the lender. Welfare gains are computed through the simulation of 5,000 independent shock histories starting with initial debt holdings and relative Pareto weights drawn from the ergodic set. The details of the welfare computations are presented in Appendix I.

Table 4: Welfare Analysis

State	Borrower welfare gains (percent)		Lender welfare gains (percent)		$\mathscr{F}(g)$ (percent)		
	MAND	CEA	MAND	CEA	MA	MAND	CEA
g_H	0.04	0.36	10.2	28.3	23.6	26.3	100.0
g_L	4.14	4.16	32.2	49.2	18.7	21.2	100.0
average	0.82	1.08	14.3	32.2	22.6	25.3	100.0

As one can see, the CEA and the MAND imply substantial welfare gains compared to the MA, on average for both the lender and the borrower. The CEA leads to the largest welfare gains in all states for all market participants. Those are more pronounced when g_L realizes. Hatchondo et al. (2020a) find similar results when comparing the MA with a Ramsey plan. Note that in the MAND, the borrower's welfare gains become negligible compared to the MA when g_H realizes. This is due to the fact that *official* buybacks occur whenever g_H realizes, unlike the implementation of the CEA in which such buybacks occur conditional on a certain portfolio holding in addition to the realization of g_H. It therefore seems that the borrower in the MAND conducts *official* buybacks too frequently.

Table 5: Borrower Welfare Decomposition

State	MAND		CEA	
	State contingency (percent)	Cost of default (percent)	State contingency (percent)	Cost of default (percent)
g_H	1.88	98.12	15.51	84.49
g_L	99.13	0.87	98.73	1.27
average	20.36	79.64	31.32	68.68

In Table 5, I decompose the borrower's welfare gains of the MAND and the CEA by calculating the percentage of gains that can be attributed to the following two factors: cost of default and state-contingency. I isolate those two factors in the following way. To compute the gain related to state contingency, I compute the MAND with the non-contingent borrowing constraint discussed in footnote 25. The residual welfare gains can then be attributed to the cost of default. Doing so I find that, in the MAND, 20% of the welfare gains come from the state contingency on average and the remaining part can be attributed to the cost of default. In the CEA, we find that 31% of the gains come from state contingency and the rest comes from the cost of default. In both cases, the share of gains related to state contingency is the highest in g_L.

Besides the welfare gains, I can compute the distance with respect to the Pareto frontier. For this purpose, I derive a metric measuring the distance between the constrained efficient allocation and any alternative allocation. From Proposition 7, I derive a direct correspondence between x and (b_{st}, b_{lt}). I can therefore express the value of the lender in any Markov equilibrium as a function of x instead of (b_{st}, b_{lt}), i.e. $\ddot{V}^l : G \times X \to \mathbb{R}$. I then define

$$\mathcal{F}(g) = \frac{\int_{\underline{x}}^{\overline{x}} \ddot{V}^l(g,x)dx}{\int_{\underline{x}}^{\overline{x}} V^l(g,x)dx}.$$

The metric $\mathcal{F}(g)$ measures the distance between the Markov allocation and the CEA. Given Proposition 4, it is bounded between 0 and 1. A value of $\mathcal{F}$ near 1 indicates that an allocation is close to the constrained efficient benchmark, whereas a value close to 0 indicates the opposite.[45] I compute $\mathcal{F}(g)$ for $b_{st} \leq 0$ and $b_{lt} \leq 0$ in the Markov equilibrium without default risk to stay consistent with Lemma 3.

Figure 7 depicts the different frontiers: in red the Pareto frontier and in blue and green the utility possibility frontier related to the MA and the MAND, respectively. Defaults in the MA produce an upward sloping part of the frontier in which both the borrower and the lender can be made better off. Neither the CEA nor the MAND display such upward slope. This shows the inefficiency of default (see Fudenberg et al. 1990).

Looking at the metric $\mathcal{F}(g)$ in the last column of Table 4, the MAND is superior to the MA but not to the CEA. More precisely, the MA is relatively far from the CEA and the MAND can get the economy closer to it. The MAND therefore provides a better approximation of the CEA than the MA. Nevertheless, the MAND remains far from the CEA meaning that the Pareto improvement is small relative to what can be achieved with trigger startegies. The metric $\mathcal{F}(g)$ is important as it relates to the entire value of the debt contract (i.e. the combined value for the borrower and the lender) and not only on the steady state unlike the welfare gains computed above.

[45] The metric $\mathcal{F}(g)$ is based on the same concept as the Gini coefficient which measures the distance between the Lorenz curve and the equity line.

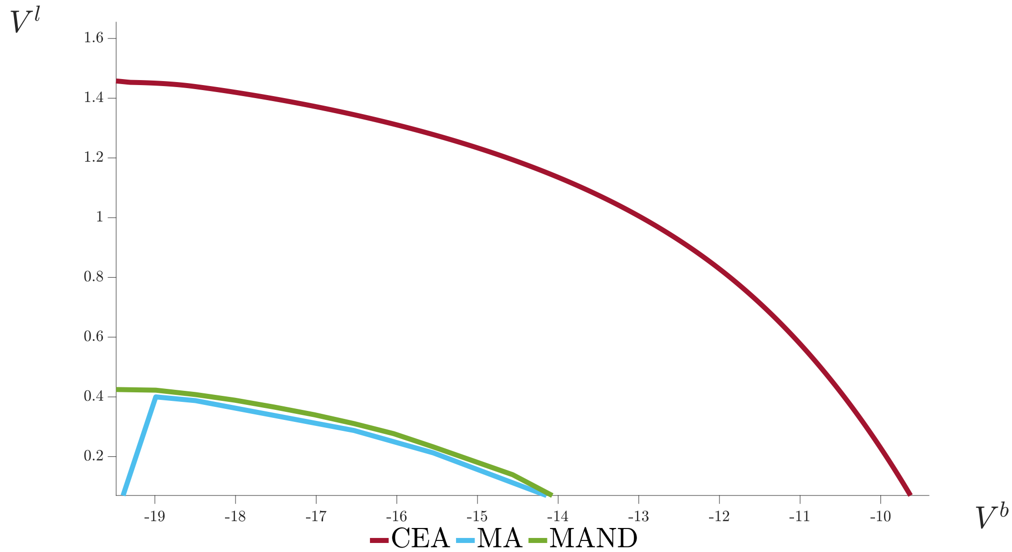


Figure 7: Distance to Pareto Frontier in g_L

8 Conclusion

This paper derives the constrained efficient allocation emanating from an optimal contract to deduce the optimal sovereign debt management policy. The bottom line is that the reliance on defaults on equilibrium path is inefficient. Instead, changes in maturity and costly debt buybacks can implement the constrained efficient allocation. Nevertheless, the implementation often requires highly sophisticated agents capable of building on past history. I show that less sophisticated agents – in the spirit of emerging economies – would in fact rely on Markov strategies. Given this, I derive history-invariant debt management policies inspired by the optimal contract and assess their efficiency. I show that a Markov equilibrium with a no-default constraint and an *official* buyback program provides a better approximation of the constrained efficient allocation than a Markov equilibrium with default. The comparison of Argentina and Brazil since 1995 supports this evidence.

This paper stresses the fact that incomplete markets might not be the reason why a market economy fails to attain constrained efficiency. Rather it can be linked to the incapacity of market participants to build on past history. I show that such restriction in the strategies followed by the market participants makes sense in the context of emerging economies. In that logic, Markov equilibria as (time-invariant) approximation of the constrained efficient allocation are not only the empirically-relevant but also the policy-relevant equilibrium concept for such economies.

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Appendix

A Discussion on Alternative Implementations

This section discusses the relationship between the implementation presented in Section 6 and the main alternatives that exist in the literature.

Dovis (2019) considers an environment similar to the one presented in Section 3 with the only difference that g is privately observed by the borrower. He derives an optimal contract subject to a participation and an incentive compatibility constraint to account for limited commitment and adverse selection, respectively. He subsequently decentralizes the aforementioned contract through partial defaults and an active debt maturity management. The main difference with my study is that he explicitly uses defaults – instead of costly buy-backs – to implement the constrained efficient allocation. This is because the combination of limited commitment and adverse selection generates a region of *ex post* inefficiencies in which the Planner sets $k = 0$.[46] As I only consider limited commitment, this region does not exist in my analysis – as shown in Proposition 4. Nevertheless, my implementation works in the environment of Dovis (2019), while the opposite is not true. In general, his implementation does not apply to renegotiation-proof contracts, while mine applies to contracts with or without *ex post* inefficiencies.

Besides this, Alvarez and Jermann (2000) propose a way to implement the allocation derived in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. I apply their approach in my environment in Appendix F. The main difference with my analysis is that the two authors assume a greater financial sophistication as securities are state contingent, while I generally need higher sophistication in the strategy of the market participants – unless the implementation works under Markov strategies.

The study of Müller et al. (2019) considers a small open economy with a stochastic default cost and two productivity states: recession and normal time. The authors assume a financial market formed by two securities: a one-period non-contingent defaultable bond and a state-contingent bond which pays out only in normal time (i.e. GDP-linked bond). The authors additionally assume that the borrower lacks commitment only in recession and renegotiation upon default is endogenous. This coupled with the aforementioned market structure, enables the two bonds to act as Arrow securities. In other words, the defaultable bond is recession contingent and spans the different stochastic default costs through renegotiation, while the contingent bond spans the good state which is free from default risk. Hence, as the bonds act as proper Arrow securities, there is no need to rely on past history.

[46] Using different environments, Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007) and Yared (2010) also characterize a region of *ex-post* inefficiencies in optimal contracts.

The last study that I would like to discuss is the one of Aguiar et al. (2019) who consider a small open economy with a stochastic default cost and two productivity states as in Müller et al. (2019). The authors assume a continuum of maturities. They show the equivalence between the Markov equilibrium and the constrained efficient equilibrium. The Planner's problem is nonetheless peculiar as it does not take into consideration the legacy creditors in the objective function. In other words, the Planner problem is sequential and only accounts for the current creditors, taking as given the inherited debt level. Furthermore, there is no participation constraint of the borrower. That is, the Planner cannot prevent the occurrence of defaults on equilibrium path. Hence, in the absence of a participation constraint – i.e. a forward-looking constraint – the Planner needs not build on past history. This combined with the disregard of legacy creditors directly leads to the Markov equilibrium in the spirit of Eaton and Gersovitz (1981).

B Price in Markov Equilibrium

The definition of price and equilibrium directly follow from Definition 2 stating that Markov equilibria are sustainable equilibria restricted to the payoff-relevant space $\Omega = (g, b_{st}, b_{lt})$. Thus, the price of one unit of bond of maturity $j \in \{st, lt\}$ is given by

$$q_j(g, b'_{st}, b'_{lt}) = \mathbb{E}_{g'|g}\left[(1 - D(\Omega'))q^P_j(g', b'_{st}, b'_{lt}) + D(\Omega')q^D_j(g', b'_{st}, b'_{lt})\right], \tag{B.1}$$

where recovery value given by

$$q^D_j(g', b'_{st}, b'_{lt}) = \frac{1}{1+r}\left[(1-\lambda)q^D_j(g', b'_{st}, b'_{lt}) + \lambda \frac{w_j \frac{1+r}{1+r-\mathbb{I}_{j=lt}}}{b'_j}\right],$$

where $\mathbb{I}_{j=lt}$ is an indicator function taking value one if $j = lt$ and zero otherwise. In case of repayment, the price depends on the maturity, the repayment productivity and the buyback decision.

$$q^P_{st}(g', b'_{st}, b'_{lt}) = \frac{1}{1+r} \quad \text{and} \quad q^P_{lt}(g', b'_{st}, b'_{lt}) = \frac{1}{1+r}\left[1 + (1 - M(\Omega'))q_{lt}(g', b''_{st}, b''_{lt}) + M(\Omega')q^{bb}_{lt}\right],$$

where $b''_j = B_j(g', b'_{st}, b'_{lt})$ for $j \in \{st, lt\}$. Given this as well as Definitions 1 and 2, a Markov equilibrium can be defined as

Definition B.4 (Markov Equilibrium). *In this environment, a Markov equilibrium consists of a set of prices, $\{p(\Omega), q_{st}(g, b'_{st}, b'_{lt}), q_{lt}(g, b'_{st}, b'_{lt})\}$, a set of policy functions $\mathcal{G}(\Omega) = \{D(\Omega), M(\Omega), B_{st}(\Omega), B_{lt}(\Omega)\}$ such that, at every possible state $\Omega = (g, b_{st}, b_{lt})$,*

1. *Taking p, q_{st} and q_{st} as given, $\mathcal{G}(\Omega)$ solves the government's problem* (5)-(7).

2. *Taking p as given, the choice of capital by domestic firms is such that $gf_k(k) = p$.*

3. *The price of capital is consistent with* $\max_k\{p(\Omega)(1-\tau(\Omega))k - k\}$.

4. *The price of each bond of maturity $j \in \{st, lt\}$ satisfies* (B.1)

C Foundations for Costly Debt Buybacks

In what follows, I endogenize the cost of *official* buybacks in two ways. First, I develop a standard Nash bargaining in the Markov equilibrium. Second, I present a signalling game in which costly *official* buybacks enable the borrower to signal its productivity.

Before that, I present the mechanism of Bulow and Rogoff (1988, 1991) and highlight why this does not suit my framework. The two authors show that a buyback increases the value of debt as the recovery value is divided among fewer creditors.

To see this, consider a Markov equilibrium in which $\mathbb{E}_{g'|g}D(g', b'_{st}, b'_{lt}) > 0$ for all (g, b'_{st}, b'_{lt}). The buyback premium naturally emerges from the bond price as the recovery value is

$$q^D_{lt}(g', b'_{st}, b'_{lt}) = \frac{1}{1+r}\left[(1-\lambda)q^D_{lt}(g', b'_{st}, b'_{lt}) + \lambda \frac{w_{lt}\frac{1+r}{r}}{b'_{lt}}\right].$$

Provided that w_{lt} is constant, when $-b'_{lt}$ increases, q^D_{lt} decreases which implies that q_{lt} decreases given the strictly positive default probability. This is what the literature calls dilution. With a buyback the opposite happens as $-b'_{lt}$ decreases. There is a *reverse* dilution which increases q^D_{lt} and therefore q_{lt}.

This mechanism however works as long as there is a strictly positive default probability. If defaults never arise on equilibrium path, the long-term bond price remains constant. Moreover, this mechanism can only rationalize buybacks at a discount (i.e. below par) on the secondary market. For instance, it cannot explain the case of Brazil which bought back its debt at a premium (i.e. when the financial value is above the face value) as shown in Section 7.

C.1 Nash Bargaining in Markov Equilibrium

In this subsection, I introduce a Nash bargaining game in the Markov equilibrium. This first shows how to endogenize the *official* buyback premium. It also reinforces the argument made in Lemma 3 about the enforcement of *official* buybacks in Markov equilibria.

The threat point of the game is that the borrower is not able to roll over its debt in the current period if the *official* buyback does not take place. In such circumstance, the borrower's value is given by

$$
\begin{aligned}
\bar{V}^{NB}(\Omega) = & \max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{g'|g}\Big[W^b(\Omega')\Big] \\
& \text{s.t.} \quad c + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g,k) + b_{st} + b_{lt}, \\
& \qquad\quad b'_{lt} \geq b_{lt}, \\
& \qquad\quad b'_{st} \geq 0.
\end{aligned}
$$

Notice that the borrower can issue short-term assets. For any *official* buyback premium χ, I define the surplus of the borrower as

$$
\Delta^b(\Omega;\chi) = V^B(\Omega;\chi) - \max\Big\{\bar{V}^{NB}(\Omega), V^D(g,k)\Big\}.
$$

The borrower's surplus corresponds to the difference between the value of conducting the *official* buyback and the value of rejecting it and suffering the underlying sudden stop.

To define the surplus of the lender, I first need to derive the lender's value under *official* buyback, under no *official* buyback and under default. The former reads

$$
\begin{aligned}
V_l^B(\Omega) = & \max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[W^l(\Omega')\Big] \\
& \text{s.t.} \quad c_l + b_{st} + b_{lt}(1 + q_{lt}^{bb}) = q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},
\end{aligned}
$$

while under no *official* buyback

$$
\begin{aligned}
\bar{V}_l^{NB}(\Omega) = & \max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[W^l(\Omega')\Big] \\
& \text{s.t.} \quad c_l + b_{st} + b_{lt} = q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}), \\
& \qquad\quad b'_{lt} \geq b_{lt}, \\
& \qquad\quad b'_{st} \geq 0.
\end{aligned}
$$

and finally, under default

$$
V_l^D(g,k) = -k + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[(1-\lambda)V_l^D(g',0) + \lambda W^l(g', w_{st}, w_{lt}\frac{1+r}{r})\Big]
$$

The continuation value under repayment is then $W^l(\Omega) = (1 - D(\Omega) - M(\Omega))\bar{V}_l^{NB}(\Omega) + M(\Omega)V_l^B(\Omega) + D(\Omega)V_l^D(g,k)$ if $g = g_H$ and $W^l(\Omega) = (1 - D(\Omega))V_l^{NB}(\Omega) + D(\Omega)V_l^D(g,k)$

otherwise. The surplus of the lender corresponds to the difference in the value under *official* buyback and no *official* buyback

$$\Delta^l(\Omega;\chi) = V_l^B(\Omega;\chi) - \Big[(1-D(\Omega))\bar{V}_l^{NB}(\Omega) + D(\Omega)V_l^D(g,k)\Big].$$

If the lender has all the bargaining power, then it could extract a large *official* buyback premium (i.e. $\chi \to 1$). In opposition, if the borrower has all the bargaining power, it can conduct *official* buybacks at low cost (i.e. $\chi \to 0$). To consider the case in between those two extremes, I assume that the lender has a bargaining power of $\zeta \in [0,1]$ and the borrower of $1-\zeta$. In Ω, the *official* buyback premium $\chi(\Omega)$ is the solution to

$$\begin{aligned}
\chi(\Omega) = \arg\max_{\tilde{\chi}\in(0,1)} & \Big[\Delta^l(\Omega;\tilde{\chi})^{\zeta} + \Delta^b(\Omega;\tilde{\chi})^{1-\zeta}\Big] \\
\text{s.t.} \quad & \Delta^l(\Omega;\tilde{\chi}) \geq 0, \\
& \Delta^b(\Omega;\tilde{\chi}) \geq 0.
\end{aligned}$$

In light of Lemma 3, the above bargaining problem has a solution only if the threat of the sudden stop is credible. If the threat is not credible in a given state Ω, $\Delta^b(\Omega;\chi) < 0$ for all $\chi \in (0,1)$ meaning that there is no $\chi > 0$ for which the borrower is willing to conduct *official* buybacks instead of being punished. In other words, there is no solution to the Nash bargaining program meaning that *official* buybacks are not enforceable.

C.2 Signalling in Markov Equilibrium

Besides Nash Bargaining, I can rationalize costly *official* buybacks with a signalling game. For this purpose, consider that g is privately observed by the borrower. The lender must therefore form beliefs on g – i.e. the borrower's type.

To be an equilibrium, beliefs have to be consistent with the market participants' strategies and, given the beliefs, each market participant's strategy must be optimal. A belief system for the lender, $\Gamma(b_{st}, b_{lt})$, specifies the a probability distribution over G,

$$\Gamma(b_{st}, b_{lt}) = Pr(g = g_H | b_{st}, b_{lt}).$$

The lender relies on the debt repayment, say S, as signal for the borrower's type. I therefore construct a separating equilibrium in which the borrower signals its type through debt repayment as in Cole et al. (1995) and Phan (2017a,b). The timing of actions is the following. First, g realizes and is privately observed by the borrower which then decides how much debt to repay, S. Conditional on the repayment, the lender offers capital $k(S)$ and a bond price schedule $q_{st}(S, b'_{st}, b'_{lt,H})$ and $q_{lt}(S, b'_{st}, b'_{lt})$ for the short-term bond, b'_{st}, and the long-term bond, b'_{lt}, respectively.[47]

The repayment signal works as follows. If the repayment is sufficiently large, then the lender believes that g_H realized. In opposition, a low repayment signals that g_L realized. The signal therefore fully reveals the shock. However, to be an equilibrium, the low type should not be willing to choose a high repayment and *vice versa*.

I assume the following. If the borrower draws g_H, it chooses to conduct an *official* buyback. In opposition, if it draws g_L, there is neither *official* buyback nor default. Hence, the repayment of the high type for a given (b_{st}, b_{lt}) is

$$S_H(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q_{lt}^{bb}),$$

and for the low type,

$$S_L(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q_{lt}).$$

With $\chi > 0$, it directly follows that $S_L(b_{st}, b_{lt}) > S_H(b_{st}, b_{lt})$ for all (b_{st}, b_{lt}). Thus, costly *official* buybacks are necessary to signal types in the absence of defaults. Whenever the lender receives a repayment lower than $-S_H(b_{st}, b_{lt})$, it believes that g_L realized. Obviously, those beliefs are consistent only if the high type has no incentive to repay according to the low type and *vice versa*. Thus, it must hold that for any (b_{st}, b_{lt}),

$$\begin{aligned} &u(y(g_H, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) = \\ &u(y(g_H, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}), \end{aligned} \tag{C.1}$$

where $b'_{j,i}$ denotes the bond choice of maturity $j \in \{st, lt\}$ of reported borrower's type $i \in \{L, H\}$. Equation (C.1) makes the high type indifferent between paying S_H or S_L. Moreover, given that $y(g_H, k(S_H)) > y(g_H, k(S_L))$ as $g_H > g_L$, (C.1) implies by concavity of the utility function that

$$\begin{aligned} &u(y(g_L, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) \leq \\ &u(y(g_L, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}). \end{aligned}$$

As a result, if the the high type indifferent between paying S_H or S_L, the low type has no

[47] Note that similar to the timing in Eaton and Gersovitz (1981), it is implicitly assumed that the borrower can commit to repayment decision made before the debt auction. See Cole and Kehoe (2000) and Ayres et al. (2018) for more details.

incentive to pay S_H instead of S_L. Thus, if (C.1) holds, the beliefs are updated according to

$$\Gamma(b_{st}, b_{lt}) = \begin{cases} 1 & \text{if} \quad S \leq S_H(b_{st}, b_{lt}) \\ 0 & \text{else} \end{cases}$$

By Proposition 3 in Phan (2017b), the set of strategies and beliefs presented in this subsection constitutes a separating Markov equilibrium.

We see that from the definition of S_H and S_L, the *official* buyback premium χ ought to be strictly larger than zero for the signal to be informative.[48] On the other hand, as $\chi \to 1$, $S_H(b_{st}, b_{lt}) \to \infty$. By (C.1) this would imply that the low type has to accumulate an infinite amount of assets. There is therefore a cap on how large χ can be and – similar to Lemma 3 – the lower is the *official* buyback premium, the easier is (C.1) satisfied in a given state Ω.

D Further Characterization of the Optimal Contract

I start this section with the existence and uniqueness of the optimal contract. For this, following Marcet and Marimon (2019), I need the following interiority assumption.

Assumption D.1 (Interiority). *There is an $\epsilon > 0$, such that, for all $g^t, t \geq 0$, there is a sequence $\{\tilde{c}(g^t), \tilde{k}(g^t)\}$ satisfying,*

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j | g_t) u(\tilde{c}(g^j)) \geq V^D(g_t, \tilde{k}_t) + \epsilon.$$

Assumption D.1 ensures the uniform boundedness of the Lagrange multipliers. It states that there are strictly positive rents to be shared among the contracting parties. In my environment, this assumption is satisfied given the Inada condition on the production function.[49]

Proposition D.1 (Existence and Uniqueness). *Under Assumptions 1, 4 and D.1, there exists a unique contract allocation with initial condition (g_0, x_0).*

Proof. See Appendix J $\square$

Having shown existence and uniqueness of the contract allocation, the following lemma derives the inverse Euler Equation which gives the consumption dynamic in the contract.

[48] Otherwise, instead of *official* buybacks, the borrower would need to rely on defaults to signal its type as in Cole et al. (1995) and Phan (2017a,b).

[49] See Proposition 4.

Lemma D.1 (Inverse Euler Equation)**.** *Under Assumptions 1 and 4, the inverse Euler equation for a given* $g \in G$ *reads*

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

Proof. See Appendix J ☐

If the participation constraint of the borrower never binds, I obtain that for all (g, x),

$$\frac{1}{u_c(c(g))} \geq \mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))}\right]$$

with strict inequality when $\eta < 1$. In this case, the inverse Euler Equation is a positive super-martingale. Immiseration is a consequence of Doob's theorem stating that such super-martingales converge almost surely. With $\eta < 1$, the inverse of the marginal utility of consumption converges to 0. Alternatively, when $\eta = 1$ consumption remains constant. Under limited commitment of the borrower (i.e. $\nu(g) \geq 0$), one obtains a left bounded positive submartingale. The borrower's participation constraints therefore sets an upper bound on the supermartingale and prevents immiseration.

E Alternatives to Official Buybacks

In this section, I provide alternatives to *official* buybacks: "excusable" defaults, variable-coupon bonds and variable-maturity bonds.

First, Grossman and Van Huyck (1988) develop the concept of "excusable" defaults. The idea is that defaults which are on the path of play agreed by all market participants are not punished. In other words, the debt contract specifies *ex ante* the circumstances in which the borrower is allowed to repudiate its debt without suffering from markets exclusion. Given this, if defaults were "excusable", then the borrower's binding constraint – i.e. $x = x^{lb}$ – could be interpreted as a default. The issue is that the borrower might be willing to repudiate debt more often than what the debt contract specifies. To deal with this, one can either use trigger strategies or introduce an endogenous borrowing limit similar to (9). Nevertheless, the concept of "excusable" defaults has little empirical relevance. The closest policy that has been implemented to this date is a sovereign debt standstill analyzed by Hatchondo et al. (2020b) with the only difference that there is no arrears accumulation in "excusable" defaults. In addition, Mateos-Planas et al. (2023) show that if the borrower were to choose the conditions for "excusable" defaults, such events would be extremely rare if not inexistent.

Second, the long-term debt can have variable coupon as in Faraglia et al. (2019) and Aguiar et al. (2021). Particularly, assume that the coupon payment is a choice variable, say $\kappa \in [0,1]$, for the borrower. Obviously, the variability of the coupon is a covenant in the debt contract. In other words, changes in coupon are agreed by the contracting parties *ex ante* and do not pertain to a contract renegotiation – e.g. an outright default in case of reduced coupon payment. With such debt contract, it is possible to implement the constrained efficient allocation in two ways: the borrower sets a standard coupon payment $\tilde{\kappa}$ and either increases it to $\bar{\kappa} > \tilde{\kappa}$ when $x = x^{ub}$ or decreases it to $\underline{\kappa} < \tilde{\kappa}$ when $x = x^{lb}$. In the former case, a variant of Proposition 1 applies as the borrower is not willing to pay a larger coupon payment. Hence, the same enforcement issue arises as with *official* buybacks and trigger strategies remain necessary in general. In opposition, in the case of reduced coupon payment, the borrower might be tempted to reduce the coupon payment more frequently than the Planner would. Thus, the lender would also need to supervise the coupon policy.

Lastly, bonds can have variable maturities. That is, the maturity of outstanding short-term (long-term) debt can be lengthened (shortened). Similar to variable-coupon bonds, this is a feature which should be explicitly mentioned in the debt contract. To implement the constrained efficient allocation, the borrower ought to either lengthen the maturity of short-term debt when $x = x^{ub}$ or shorten the maturity of long-term debt when $x = x^{lb}$. Implicitly, by shortening the maturity, the borrower pays less coupons than it initially promised. In other words, the claim of legacy creditors is reduced. The opposite happens in the case of maturity lengthening. Thus, similar to variable-coupon bonds, maturity lengthening would need to be enforced, while maturity shortening should be closely supervised to avoid lowering legacy creditors' claim too frequently.

F Alternative Implementation

In what follows, I propose an alternative implementation as the one derived in Section 6. More precisely, I rely on the approach of Alvarez and Jermann (2000) using trade in state-contingent securities and an endogenous borrowing limit.

The structure of the financial market is the following. At the start of a period, the government holds a perpetual security a.[50] The government can trade $|G|$ state contingent securities $a'(g')$ with a unit price of $q(g', a'(g')|g)$. The portfolio $a'(g')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q(g, a') \equiv \sum_{g'|g} q(g', a'(g')|g)$, and an insurance portfolio of $|G|$ Arrow securities

[50]The maturity is unimportant in this implementation. The security a could also be a one-period security.

$\hat{a}'(g')$. Thus we have that $a'(g') = \bar{a}' + \hat{a}(g')$ with

$$\bar{a}' = \frac{\sum_{g'|g} q(g', a'(g')|g) a'(g')}{q(g, a')} \quad \text{and} \quad \sum_{g'|g} q(g', a'(g')|g) \hat{a}'(g') = 0.$$

The last equation represents the market clearing condition of the Arrow securities.

The capital market is the same as in the main text: the lender provides k at price p which is taxed at rate $\tau = 1 - \frac{1}{p}$. The government's problem therefore reads

$$W^b(g, a) = \max_{c, \{a'(g')\}_{g' \in g}} u(c) + \beta \mathbb{E}_{g'|g} \left[W^b(g', a'(g')) \right] \tag{F.1}$$

$$\text{s.t. } c + \sum_{g'|g} q(g', a'(g')|g)(a'(g') - a) \leq y(g, k) + a$$

$$\bar{a}' + \hat{a}(g') \geq \mathcal{A}(g', k'), \tag{F.2}$$

where $\mathcal{A}(g', k')$ represents the endogenous borrowing limit and is defined such that

$$W^b(g', \mathcal{A}(g', k')) = V^D(g', k'). \tag{F.3}$$

One can see here the similarity with the borrowing limit defined in Section 5. The lender's problem is static. I nonetheless express it in recursive form.

$$W^l(g, a_l) = \max_{c_l, k_l, \{a_l'(g')\}_{g' \in g}} c_l + \frac{1}{1+r} \mathbb{E}_{g'|g} \left[W^l(g', a_l'(g')) \right] \tag{F.4}$$

$$\text{s.t. } c_l + \sum_{g'|g} q(g', a_l'(g')|g)(a_l'(g') - a_l) \leq p(1-\tau)k_l - k_l + a_l.$$

Given this environment, I can determine a recursive competitive equilibrium in the following terms.

Definition F.5 (Recursive Competitive Equilibrium (RCE))**.** *A* recursive competitive equilibrium *is a sequence of prices $q(g', a'(g')|g)$ and $p(g, a)$, value functions, $W^b(g, a)$ and $W^l(g, a)$, an endogenous borrowing limit, $\mathcal{A}(g', k')$, as well as policy functions for (i) consumption, $c(g, a)$ and $c_l(g, a)$, (ii) capital, $k = k(g, a)$ and $k_l = k_l(g, a)$, (iii) asset holdings $a'(g') = A(g', g, a)$ and $a_l'(g') = A_l(g', g, a)$ such that,*

1. *Given the value function for the outside option of the government, $V^D(g', k')$ as well as the prices $q(g', a'(g')|g)$ and $p(g, a)$,*
   (a) the policy functions $c(g, a)$ and $A(g', g, a)$, together with the value function $W^b(g, a)$, solve the government problem (F.1) with the endogenous limit, $\mathcal{A}(g', k')$.

(b) the policy functions $c_l(g, a_l)$, $k_l(g, a)$, and $A_l(g', g, a_l)$, together with the value function $W^l(g, a_l)$, solve the lender's problem (F.4) and

2. *Taking p as given, $k(g, a)$ is such that $u_c(c)(gf_k(k) - p) = 0$.*

3. *The price of capital is consistent with $\max_k\{p(1-\tau)k - k\}$.*

4. *The asset market clears, $a'(g') + a_l'(g') = 0$ for all $g' \in G$.*

5. *The product and capital markets clear, $c(g, a) + c_l(g, a_l) = gf(k(g, a))$ and $k(g, a) = k_l(g, a)$.*

For the government's problem, taking the first-order conditions with respect to consumption and assets, one obtains

$$
\begin{aligned}
u_c(c) &= \mu_{BC}(g, a), \\
q(g', a(g')|g) &= \beta\pi(g'|g)\frac{u_c(c')}{u_c(c)}[1 + \sum_{g''|g} q(g'', a''(g'')|g')] + \frac{\mu_{EBL}(g', a'(g'))}{u_c(c)},
\end{aligned}
$$

where μ_{BC} and μ_{EBL} are the Lagrange multipliers attached to the budget constraint and the endogenous borrowing limit, respectively. Especially, $\mu_{EBL}(g', a'(g')) \geq 0$ with $\mu_{EBL}(g', a'(g')) = 0$ if $a'(g') > \mathcal{A}(g', k')$.

Conversely, taking the first-order conditions with respect to consumption, capital and assets of the lender's problem

$$
\begin{aligned}
1 &= \mu^l_{BC}(g, a_l), \\
1 &= p(1-\tau), \\
q(g', a(g')|g) &= \frac{1}{1+r}\pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')).
\end{aligned}
$$

Following Krueger et al. (2008), the price is determined by the agent whose constraint is not binding. Therefore the price is determined by

$$
q(g', a(g')|g) = \pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')) \max\left\{\beta\frac{u_c(c(g', a'(g'))}{u_c(c(g, a))}, \frac{1}{1+r}\right\}. \tag{F.5}
$$

The following lemma states that the constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.

Proposition F.2 (Alternative Implementation)**.** *Given initial conditions* (g_0, x_0)*, a constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.*

Proof. See Appendix J ∎

Table F.1: Alternative Implementation

	Benchmark	Alternative
$-b/y$	-353.20	15.87
Spread	3.95	4.34
$\sigma(b/y)/\sigma(y)$	8.34	0.31
$\sigma(spread)/\sigma(y)$	0.00	0.00
$\mathrm{corr}(b/y, y)$	-0.72	0.67
$\mathrm{corr}(spread, y)$	-0.67	0.00

Note: The variable $\sigma(\cdot)$ denotes the volatility. In the alternative implementation, $\bar{a} = b$. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25.

The benchmark implementation presented in Section 6 relies on changes in the term premium to mimic the state-contingency in the optimal contract, while this alternative implementation relies on changes in security holdings provided that securities are state-contingent. More importantly, given that securities are state contingent, the assumption that the borrower and the lender keep track of the entire history of play is not anymore necessary. The implementation of the constrained efficient allocation now lies on the assumption of a greater financial sophistication.

Having properly defined the alternative implementation, I now compare it quantitatively to the one presented in Section 6 using the calibration in Section 7.

Table F.1 presents the main difference between the two implementations. The benchmark case is related to a net asset position and a larger volatility of the debt ratio. This comes from the fact that bonds are non-contingent and the borrower alternates between short-term assets and long-term debt. Thus, large movements in debt holdings are necessary to replicate the state contingency in the contract as shown by Buera and Nicolini (2004) and Faraglia et al. (2010). Particularly, we see that the volatility of debt-to-GDP ratio is 26 times larger in the benchmark than in the alternative implementation. Besides this, the benchmark implementation displays a lower spread owing to *official* buybacks. As explained before, the reason behind this is that the alternative implementation does not rely on changes in prices

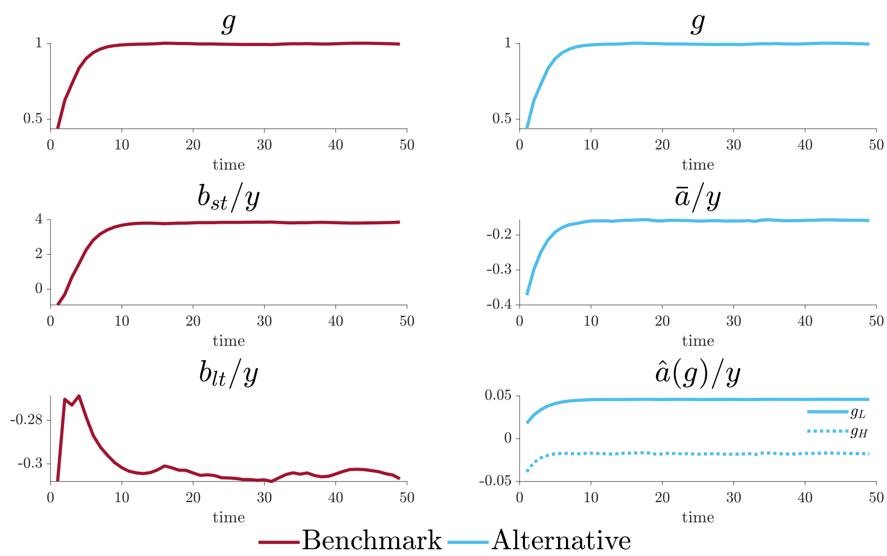

Figure F.1: Impulse Response Functions to a Negative g Shock

to mimic the state-contingency of the contract given that securities are state-contingent by definition.

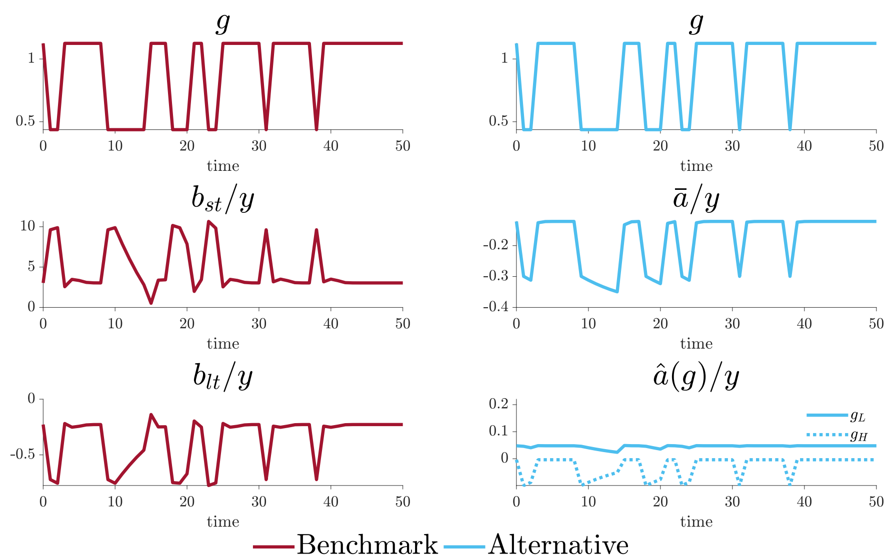

Figure F.2: Simulation of a Steady State Path

Similar to the previous section, I construct impulse responses to see how the two implementations work. Figure F.1 depicts the responses in red for the benchmark implementation and in blue for the alternative one. The Arrow securities complement the accumulation of bonds in the alternative case, while the benchmark implementation needs to adapt both the long-term and the short-term bonds in opposite directions.

Turning to the simulation in Figure F.2, one can see that the level of long-term bond in the benchmark case closely follows the pattern of bonds in the alternative case. The magnitude of change in the former is nonetheless larger than in the latter. In terms of Arrow securities, $\hat{a}(g_H)$ closely follows the evolution of $\bar{a}$, while $\hat{a}(g_L)$ has the opposite sign. The evolution of $\hat{a}(g_H)$ is therefore closely mimicking the evolution of b_{lt}. Given that $\hat{a}(g)$ is state contingent, the alternative implementation needs to change the debt portfolio with lower magnitude.

G Empirical Analysis on Maturity and Buyback in Brazil

In this section, I derive and assess four main facts related to the prediction of the model for Brazil. The first fact relates to the sovereign debt maturity management. As already noted by Arellano and Ramanarayanan (2012), Broner et al. (2013), Perez (2017) and Bai et al. (2017), maturity shortens during debt crises and lengthens otherwise.

Fact I. *Average maturity shortens during debt crises and lengthens otherwise.*

Table G.2 presents the result of a typical maturity regression analysis. The dependent variable corresponds to the average maturity (in years) on new external debt retrieved from the World Bank's International Debt Statistics from 1995 to 2019. The explanatory variable is the EMBI spread which I obtain from the Global Financial Data. As one can see, when the spread increases, the average maturity of new issuance shortens.

The second fact relates to *official* buybacks. As already highlighted in Section 7, the Brazilian government always paid a premium to extract its debt out of the market. More precisely, on average, the financial value is 24.5% above the face value.

Fact II. *Official buybacks are costly.*

While the second fact states that *official* buybacks are costly, the third fact explains when *official* buybacks occur. Consistent with the predictions of the model, *official* buybacks tend to arise in good times. Particularly, there is a positive association between the amount repurchased and the economic situation of the country.

Fact III. *Substantial official buybacks are more likely to arise in good economic times.*

Table G.2: Regression Analysis

	(1) Maturity	(2) Buyback	(3) Buyback
EMBI Spread	-0.52**	-0.33***	-1.95***
	[0.20]	[0.10]	[0.59]
Foreign Share of External Debt			9.05*
			[4.55]
Observations	25	25	15
R^2 adjusted	0.17	0.24	0.45

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

Source: Author's calculation, Global Financial Data, Tesouro Nacional and World Bank.

Table G.2 presents the result of the regression analysis on the Brazilian buyback. The dependent variable corresponds to the amount of external debt bought back in USD billion. I use the same explanatory variables as before. As one can see, when the EMBI spread increases, the amount bought back diminishes.

Using the database of Onen et al. (2023), I include as an additional explanatory variable the share of foreign holdings of long-term external government bonds. This variable only starts in 2005, though. Given this, I come up with the last fact

Fact IV. *Substantial official buybacks are more likely with a low share of domestic holdings of external debt.*

As shown in Table G.2, a larger share of foreign holdings is associated to higher buybacks. Given the larger home bias in Argentinian external debt holdings shown in Figure 4, this can explain why Argentina did not conduct *official* buybacks in the period considered. Obviously, there can be alternative explanations. However, the one presented here is consistent with Lemma 3.

H Additional Quantitative Results

This subsection provides additional quantitative results. First, I construct impulse response functions following a stark negative shock in both the Markov equilibria and the CEA. Second, I look at the dynamic of a specific shock path in steady state.

Figure H.3 depicts the impulse response functions resulting from a stark negative shock on selected key variables. The responses are computed as the mean of 2,000 independent shock histories starting with the lowest shock as well as initial debt holdings and relative Pareto weight drawn from the ergodic set. The blue line represents the Markov equilibrium with default (MA), the green line the Markov equilibrium without default (MAND) and the red line the constrained efficient allocation (CEA).

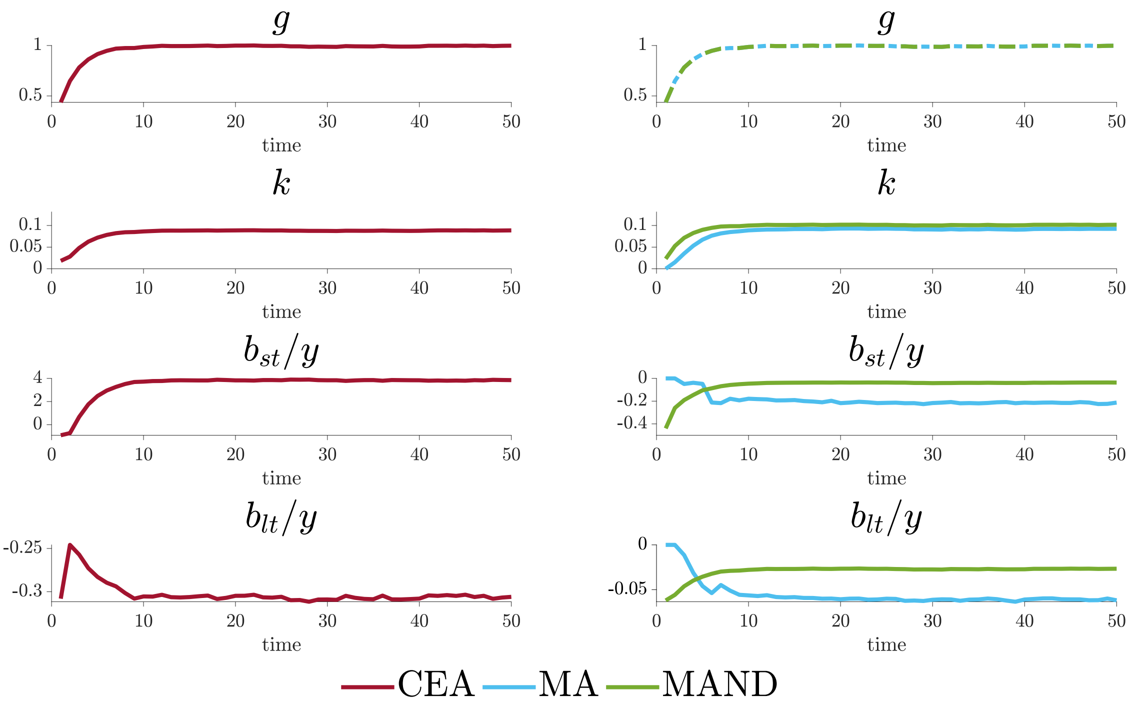

Figure H.3: Impulse Response Functions to a Negative g Shock

We see that at the outbreak of the shock's realization, capital decreases in the MA, the MAND and the CEA – albeit to a lesser extent in the latter two cases. Capital and debt go to zero as economies in the MA fall into default. In opposition, defaults do not arise in the CEA and the MAND which can both increase the indebtedness on impact. Maturity shortens at the outbreak of the bad shock's realization in both the MAND and the CEA. The Markov equilibria rely mostly on short-term debt, while the CEA uses both in opposite directions. Note further that the CEA switches from short-term debt to asset holdings.

The impulse response functions give an idea of the long-run dynamic of the economy. However, it does not tell how the economy reacts in the short run especially when there is a transition between two values of g. Thus, I simulate the economy and generate one history of shocks for 200 periods. To avoid that the initial conditions blur the results, the first 150 periods are discarded. Again, the blue line represents the MA, the green line the MAND and the red line the CEA. In addition, the grey area represents the period in which the MA is in default.

Figure H.4 depicts the simulation results. One observes that, in the MA, the economy defaults in the transition from g_H to g_L. This causes market exclusion and therefore $b_{st} =$

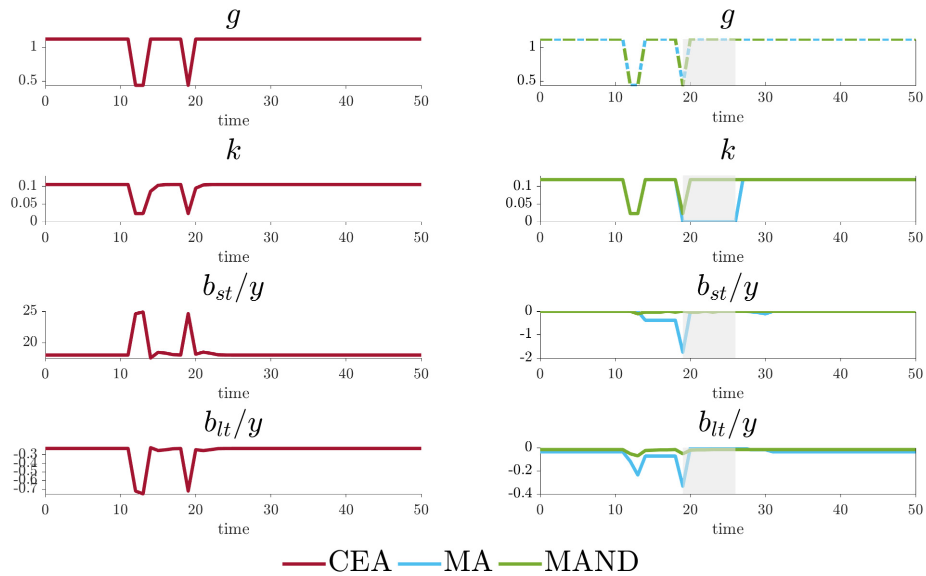

Figure H.4: Simulation of a Typical Path

$b_{lt} = k = 0$. In opposition, there are no defaults in the CEA and the MAND. In the transition from g_H to g_L, the government adapts the maturity of the debt and increases its indebtedness. Especially, one sees that the level of short-term bonds have opposite movements in the MAND and the CEA. The magnitude of the changes is also very different. Consistent with the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), the movements in debt holdings are the most pronounced for the CEA. Particularly, holdings of short-term debt oscillate between 2000% and 2500% of output.

I Welfare Analysis

To compute the borrower's welfare, first define the borrower's value for a sequence of consumption $\{c(g^t)\}$ starting from an initial state at $t = 0$ as

$$W^b(\{c(g^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c(g^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c(g^t)^{1-\vartheta}}{1-\vartheta},$$

I denote the borrower's consumption allocation in the benchmark model by $\{c^b(g^t)\}$ and the consumption allocation in the alternative model by $\{c^a(g^t)\}$. In addition, I define the consumption-equivalent welfare gain of the alternative model with respect to the benchmark model by ι such that

$$W^b(\{(1+\iota)c^b(g^t)\}) = W^b(\{c^a(g^t)\}).$$

Given the functional form of the instantaneous utility one obtains

$$(1+\iota)^{1-\vartheta}\Big[W^b(c^b(g^t)\})\Big] = W^b(\{c^a(g^t)\}).$$

The borrower's welfare gain therefore boils down to

$$\iota = \left[\frac{W^b(\{c^a(g^t)\})}{W^b(\{c^b(g^t)\})}\right]^{\frac{1}{1-\vartheta}} - 1.$$

The lender's welfare gains can be computed in the same way by setting $\vartheta = 0$ owing to the risk neutrality.

J Proofs

Proof of Lemma 1

The value of permanent autarky is given by

$$v_a(g_t) = \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0)), \tag{J.1}$$

as the lender sets $k = 0$ in case of default. Permanent autarky is the worst equilibrium outcome as the government could always be better off with $k = \epsilon$ for small $\epsilon > 0$ given the Inada conditions on the production function. I show this in Proposition 4.

Permanent autarky is an equilibrium of the market economy. Suppose that the lender believes that $D_t = 1$ for all t. Then, it sets $p_t = \infty$ and $q_{j,t} = 0$ for all $j \in \{st, lt\}$. Given this, the government finds optimal to choose $D_t = 1$ for all t confirming the lender's beliefs. □

Proof of Lemma 2

Necessity:

> Conditions (1), (2), (3) and (4) follow directly from the equilibrium's definition. The budget constraints in the repayment (including *official* buybacks) and default states is required by feasibility. Finally, the fact that the value of the borrower is larger than the value under the worst equilibrium – i.e. (J.1) – ensures that the allocation can be sustained by trigger strategies.

Sufficiency:

Let's rely on trigger strategy (Abreu, 1988). That is, each player is punished by the worst outcome of the game (i.e. permanent autarky which is an equilibrium as shown in Lemma 1) if it decides to deviate. Since the outcome satisfies (1), (2), (3) and (4), it is optimal. Also as it satisfies the different budget constraints it is feasible. Finally, no deviations from play is profitable given that the value of the borrower is larger than the value under the worst equilibrium.

□

Proof of Proposition 1

Assume by contradiction that in a given state Ω, the borrower wants to conduct an *official* buyback. That is, the borrower picks a pair (b'_{st}, b'_{lt}) such that

$$V^{NB}(\Omega) < V^{B}(\Omega).$$

The consumption under *official* buyback is given by

$$c^{B}(\Omega) = y(g,k) + b_{st} + b_{lt}(1 + q^{bb}_{lt}) - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},$$

and the expected continuation value by

$$\mathbb{E}_{g'|g}\Big[W^{b}(g', b'_{st}, b'_{lt})\Big].$$

Now consider the alternative strategy of picking the same pair (b'_{st}, b'_{lt}) but conducting an *unofficial* buyback. In such circumstance, consumption is given by

$$c^{NB}(\Omega) = y(g,k) + b_{st} + b_{lt} - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}).$$

It is clear from the *official* buyback premium that $c^{NB}(\Omega) > c^{B}(\Omega)$. Moreover, as the borrower chooses the same (b'_{st}, b'_{lt}), the continuation value is the same as before. Hence,

$$V^{NB}(\Omega) = u(c^{NB}(\Omega)) + \beta\mathbb{E}_{g'|g}\Big[W^{b}(g', b'_{st}, b'_{lt})\Big] > u(c^{B}(\Omega)) + \beta\mathbb{E}_{g'|g}\Big[W^{b}(g', b'_{st}, b'_{lt})\Big] = V^{B}(\Omega),$$

which contradicts the fact that an *official* buyback is ever optimal. □

Proof of Lemma 3

The main element that has to be understood is that an *official* buyback represents a reverse-default as it corresponds to an overpayment – in opposition to an underpayment – of liabilities. Not conducting *official* buybacks does not lead to market exclusion as the borrower repaid its debt. The only punishment the lender can impose in case of no *official* buyback is to not roll over the debt, meaning that $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$. This punishment is clearly Markov as it only pertains to the current period, unlike trigger strategies that rely on all future paths of play.

In addition, when the borrower issues short-term assets, there is no threat available to the lender as the borrower is in fact the lender of short-term debt. In other words, when $b'_{st} > 0$, it is the lender who is auctioning and it cannot exclude the borrower from the auction – as the borrower repaid the debt and has market access. In opposition, when $b'_{st} < 0$, the borrower is auctioning – as it seeks to raise resources – and the lender can decide not to participate to this auction.

– Part I

The proof follows the same logic as the one of Proposition 1. Suppose by contradiction that the lender can enforce *official* buybacks in a state Ω such that $B_{st}(\Omega) \geq 0$. Formally, in the case of an *official* buyback, the borrower chooses $b'_{st} = B_{st}(\Omega) \geq 0$ and $b'_{lt} = B_{lt}(\Omega) \geq b_{lt}$ to maximize its utility.

Now consider the case in which the borrower does not conduct the *official* buyback but mimics the debt choice in the case of *official* buyback. As explained before, this is possible as the lender cannot prevent the borrower to issue assets.

The contradiction is immediate as the continuation value is the same in the two cases and $\bar{c}^{NB} < c^B$. Thus, *official* buybacks are not enforceable in case of short-term asset issuance.

– Part II

I consider a state Ω in which, $B_{st}(\Omega) < 0$ and $B_{lt}(\Omega) \geq b_{lt}$. Moreover, I assume without loss of generality that the choice of private debt is the same in the case with and without *official* buyback. Given this, we have that for all $g' \in G$, $W^b(g', B_{st}(\Omega), B_{lt}(\Omega)) \leq W^b(g', 0, B_{lt}(\Omega))$. In words, the continuation value under the no-roll-over punishment is weakly larger than the continuation value under an *official* buyback.

I consider two cases. First, for a given g and $b_{lt} < 0$, if $b_{st} \leq \{b_{st} < 0 : \bar{V}^{NB}(g, b_{st}, b_{lt}) = V^D(g) | g, b_{lt} < 0\}$, the *official* buyback is enforceable given the definition of the endogenous borrowing limit (9).

Second, given that the continuation value under punishment is weakly larger, to obtain that $c^B > \bar{c}^{NB}$, it must be that

$$q_{st}(g, b'_{st}, b'_{lt})b'_{st} < b_{lt}(q^{bb}_{lt} - q_{lt}(g, b'_{st}, b'_{lt})).$$

Hence, provided that $q^{bb}_{lt} = \frac{1}{(1-\chi)r}$, if $\chi \to 0$ and $b_{lt} \to 0$, it is possible to have $V^B(\Omega) > \bar{V}^{NB}(\Omega)$ ensuring the enforcement of *official* buybacks.

□

Proof of Proposition 2

I prove the proposition by construction. Denote the objects related to the Markov equilibrium with default with "MA" and without default with "MAND". Given equation (9), for all Ω,

$$V^P_{MAND}(\Omega) \geq V^D(g, k).$$

Hence, there is no default on equilibrium path. This combined with the state contingent buyback program in (8) implies that

$$q^{MAND}_{lt}(g_L, b'_{st}, b'_{lt}) = q^{MAND}_{lt}(g_L) = \frac{\pi(g_H|g_L)q^{bb}_{lt} + 1}{r + \pi(g_H|g_L)} > \frac{1}{r}, \tag{J.2}$$

$$q^{MAND}_{lt}(g_H, b'_{st}, b'_{lt}) = q^{MAND}_{lt}(g_H) = \frac{1}{1+r}\left[1 + \pi(g_L|g_H)q^{MAND}_{lt}(g_L) + \pi(g_H|g_H)q^{bb}_{lt}\right] > \frac{1}{r}, \tag{J.3}$$

$$q^{MAND}_{st}(g, b'_{st}, b'_{lt}) = q^{MAND}_{st} = \frac{1}{1+r}.$$

As a result the long-term (short-term) bond price in the Markov equilibrium without default is always strictly (weakly) greater the in the Markov equilibrium with default. Formally for all (g, b'_{st}, b'_{lt}),

$$\begin{aligned} q^{MAND}_{lt}(g) &> q^{MA}_{lt}(g, b'_{st}, b'_{lt}), \\ q^{MAND}_{st} &\geq q^{MA}_{st}(g, b'_{st}, b'_{lt}). \end{aligned}$$

First, consider that there is no default in the MA for any Ω meaning that

$$\begin{aligned} q^{MAND}_{lt}(g) > q^{MA}_{lt} &\equiv q^{rf}_{lt} = \frac{1}{r}, \\ q^{rf}_{st} \equiv q^{MAND}_{st} = q^{MA}_{st} &= \frac{1}{1+r}. \end{aligned}$$

Define the consumption in the MA as

$$c_{MA}(\Omega) = y(g,k) + b_{st} + b_{lt} - q_{st}^{rf} B_{st}^{MAND}(\Omega) - q_{lt}^{rf}(B_{lt}^{MAND}(\Omega) - b_{lt}).$$

Given the absence of default, the maturity structure is indeterminate in the MA. Conversely, the consumption in the MAND under (8) when $g = g_L$ is

$$\begin{aligned} c_{MAND}(g_L, b_{st}, b_{lt}) = & y(g_L, k) + b_{st} + b_{lt} - q_{st}^{rf} B_{st}^{MAND}(g_L, b_{st}, b_{lt}) - \\ & q_{lt}^{MAND}(g_L)(B_{lt}^{MAND}(g_L, b_{st}, b_{lt}) - b_{lt}), \end{aligned}$$

and when $g = g_H$

$$\begin{aligned} c_{MAND}(g_H, b_{st}, b_{lt}) = {} & y(g_H, k) + b_{st} + b_{lt}(1 + q_{lt}^{bb}) - q_{st}^{rf} B_{st}^{MAND}(g_H, b_{st}, b_{lt}) - \\ & q_{lt}^{MAND}(g_H) B_{lt}^{MAND}(g_H, b_{st}, b_{lt}), \\ B_{lt}^{MAND}(g_H, b_{st}, b_{lt}) \geq b_{lt}. & \end{aligned}$$

Under Assumption 2 and the fact that there is no default, it is always possible for the borrower in the MAND to mimic the choice of debt given in the MA. Hence, consider that in the MAND, the borrower adopts the following policy: $\tilde{B}_{lt}^{MAND}(\Omega) = B_{lt}^{MA}(\Omega) \equiv B_{lt}(\Omega)$ and $\tilde{B}_{st}^{MAND}(\Omega) = B_{st}^{MA}(\Omega) \equiv B_{st}(\Omega)$ for all Ω such that

$$\begin{aligned} \tilde{c}_{MAND}(g_L, b_{st}, b_{lt}) &= c_{MA}(g_L, b_{st}, b_{lt}) - [q_{lt}^{MAND}(g_L) - q_{lt}^{rf}](B_{lt}(g_L, b_{st}, b_{lt}) - b_{lt}) \\ &> c_{MA}(g_L, b_{st}, b_{lt}), \\ \tilde{c}_{MAND}(g_H, b_{st}, b_{lt}) &= c_{MA}(g_H, b_{st}, b_{lt}) - [q_{lt}^{MAND}(g_H) - q_{lt}^{rf}](B_{lt}(g_H, b_{st}, b_{lt}) - b_{lt}) + [q_{lt}^{bb} - q_{lt}^{MAND}(g_H)]b_{lt} \\ &< c_{MA}(g_H, b_{st}, b_{lt}). \end{aligned}$$

Now define

$$\begin{aligned} \delta_1 &= q_{lt}^{MAND}(g_L) - q_{lt}^{rf} = q_{lt}^{MAND}(g_L) - \frac{1}{r}, \\ \delta_2 &= q_{lt}^{MAND}(g_H) - q_{lt}^{rf} = q_{lt}^{MAND}(g_H) - \frac{1}{r}, \\ \delta_3 &= q_{lt}^{bb} - q_{lt}^{MAND}(g_H). \end{aligned}$$

Replacing the expression of $q_{lt}^{MAND}(g_L)$ and $q_{lt}^{MAND}(g_H)$ with (J.2) and (J.3), respectively, we obtain after some rearranging the expressions in Assumption 2. For the MAND to be superior to the MA, we need for any $g_- \in G$

$$\mathbb{E}_{g|g_-} c_{MA}(\Omega) \leq \mathbb{E}_{g|g_-} \tilde{c}_{MAND}(\Omega).$$

Given that $\tilde{c}_{MAND}(g_H, b_{st}, b_{lt}) < c_{MA}(g_H, b_{st}, b_{lt})$, for the above expression to hold, it should be that, at least

$$\mathbb{E}_{g|g_H} c_{MA}(\Omega) = \mathbb{E}_{g|g_H} \tilde{c}_{MAND}(\Omega),$$

which implies that

$$-\pi(g_L|g_H)\delta_1(B_{lt}(g_L, b_{st}, b_{lt}) - b_{lt}) - \pi(g_H|g_H)\Big[\delta_2(B_{lt}(g_H, b_{st}, b_{lt}) - b_{lt}) - \delta_3 b_{lt}\Big] = 0.$$

Given that, $B_{lt}(g_L, b_{st}, b_{lt}) \geq b_{lt}$ by Assumption 2 and $B_{lt}(g_L, b_{st}, b_{lt}) \leq 0$ by Assumption 1, one gets that

$$B_{lt}(g_L, b_{st}, b_{lt}) \leq \Big[\frac{\pi(z_L|z_H)(\delta_2 + \delta_3)}{\pi(z_L|z_L)\delta_1} + \pi(z_L|z_L)\delta_1\Big] b_{lt}.$$

Given the properties of the borrower's utility, Assumption 2 implies that for all Ω and $g_- \in G$

$$\mathbb{E}_{g|g_-} \tilde{V}^P_{MAND}(g, b_{st}, b_{lt}) > \mathbb{E}_{g|g_-} V^P_{MA}(g, b_{st}, b_{lt}),$$

where $\tilde{V}^P_{MAND}$ is the value in the MAND when the borrower follows $\tilde{B}^{MAND}_{lt}(\Omega) = B^{MA}_{lt}(\Omega)$ and $\tilde{B}^{MAND}_{st}(\Omega) = B^{MA}_{st}(\Omega)$. Such behavior might not be optimal and therefore

$$\mathbb{E}_{g|g_-} V^P_{MAND}(g, b_{st}, b_{lt}) \geq \mathbb{E}_{g|g_-} \tilde{V}^P_{MAND}(g, b_{st}, b_{lt}) > \mathbb{E}_{g|g_-} V^P_{MA}(g, b_{st}, b_{lt}).$$

Under the strategy, $\tilde{B}^{MAND}_{lt}(\Omega) = B^{MA}_{lt}(\Omega) \equiv B_{lt}(\Omega)$ and $\tilde{B}^{MAND}_{st}(\Omega) = B^{MA}_{st}(\Omega) \equiv B_{st}(\Omega)$, the lender extracts a larger rent than in the MA given that $\chi > 0$. Hence, for the same Ω, the value of the lender is strictly larger in the MAND than in the MA.

Considering no default in the MA gives an upper bound for the price. Hence, the argument derived above generally applies when there is a strictly positive probability of default in the MA. Note that one cannot compare the MAND and the MA for debt portfolio in which (9) would be violated. Hence, the MA and the MAND do not necessarily span each other.

The last part of the proposition follows directly from the fact that *official* buybacks can only be conditioned on g. Otherwise, one could condition *official* buybacks on a specific (g, b_{st}, b_{lt}) which would probably be less recurrent than (8) and therefore less costly for the borrower. □

Proof of Proposition 3

I prove the proposition following Bhaskar et al. (2012). I first show that every equilibrium under Assumption 3 with $(\psi, \epsilon) > 0$ is essentially sequentially strict. I then prove that every essentially sequentially strict equilibrium is a Markov (perfect) equilibrium.

I start the proof with some definitions. Given the information structure, I split the histories into two categories: public and private. Public histories are the ones defined in Section 4 – that is h_b^t and h_l^t. Private histories of the borrower and the lender at time t are the ones tracking the utility shocks – that is $p_b^t = (p_b^{t-1}, \varrho_{b,t})$ and $p_l^t = (p_l^{t-1}, \varrho_{l,t})$, respectively. Finally, I denote he entire history of the play including the privately observed utility shocks by $\hat{h}^t$.

In addition, I denote σ_b and σ_l as the strategy profile of the borrower and the lender, respectively. Besides this, $\mathcal{C}_i$ corresponds to the countable set of actions with typical element a_i for market participant $i \in \{b, l\}$. For instance, actions taken by the borrower relate to borrowing, defaults and buybacks, while the lender chooses capital and its price. Moreover, $W^b(\sigma_b, \sigma_l | h_b^t, p_b^t)$ and $W^l(\sigma_b, \sigma_l | h_l^t, p_l^t)$ represent respectively the value of the borrower and the lender from the strategy profile (σ_b, σ_l) at the relevant histories.

Given that each market participant has some private information regarding their payoff, they need to form beliefs about the unobserved utility shock of the other participants. Denote the belief of agent $i \in \{b, l\}$ over the entire history $\hat{h}^t$ as $\omega_i^{(h_i^t, p_i^t)}$. I follow Bhaskar et al. (2012) and put the least structure possible on such beliefs. They simply need to be independent of the private payoff shocks and put zero weight to events that history $\hat{h}^t$ is inconsistent with h_i^t. With this, I define a Markov equilibrium as

Definition J.6 (Markov Equilibrium)**.** *A strategy σ_i for $i \in \{b, l\}$ is Markov if for any two histories $(h_i^t, p_i^t) \neq (\tilde{h}_i^t, \tilde{p}_i^t)$ ending with the same state Ω_t,*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(\tilde{h}_i^t, \tilde{p}_i^t).$$

A strategy profile (σ_b, σ_l) is a Markov equilibrium if (σ_b, σ_l) is Markov and for any alternative strategy $(\tilde{\sigma}_b, \tilde{\sigma}_l)$,

$$W^b(\sigma_b, \sigma_l) \geq W^b(\tilde{\sigma}_b, \tilde{\sigma}_l) \ \wedge \ W^l(\sigma_b, \sigma_l) \geq W^l(\tilde{\sigma}_b, \tilde{\sigma}_l).$$

Note that this definition is equivalent to Definition 2 in the main text accounting for Assumption 3. Furthermore, I define a sequential best response as

Definition J.7 (Sequential Best Response). *A strategy σ_i is a sequential best response to (σ_{-i}, ω_i), if for each history (h_i^t, p_i^t) and each alternative strategy $\tilde{\sigma}_i$*

$$\int W^i(\sigma_i, \sigma_{-i}|\hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t) \geq \int W^i(\tilde{\sigma}_i, \sigma_{-i}|\hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Strategy σ_i is a sequential best response to σ_{-i} if strategy σ_i is a sequential best response (σ_{-i}, ω_i) for some ω_i.

Given the information structure, there is no general solution concept which can be used here. That is why, Bhaskar et al. (2012) appeal to the very weak concept of sequential optimality. Nonetheless, a profile of mutual sequential best response for the borrower and the lender represents a perfect Bayesian equilibrium.

The other concept defined by the aforementioned authors is the current shock strategy which relies at most on the current value of the private shock. Formally

Definition J.8 (Current Shock Strategy). *A strategy σ_i is a current shock strategy, if for any public history (h_i^t, p_i^t) and for any two histories, p_i^t and $\tilde{p}_i^t$, both finishing with the same ϱ_i, then for almost all ϱ_i*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(h_i^t, \tilde{p}_i^t).$$

The next lemma links Definitions J.7 and J.8. In words, any sequential response relies at most on the current value of the private shock. As a result the history of past private shocks becomes irrelevant.

Lemma J.2 (Sequential Strictness and Current Shock Strategy). *If σ_i is a sequential best response to σ_{-i}, then σ_i is a current shock strategy.*

Proof. Consider a market participant i with history (h_i^t, p_i^t). The expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t) = \mathbb{E}_{g'|g} \int \int \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', \hat{h}^t) d\varsigma_{P_i}(\varrho_i') d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Since σ_{-i} and $\omega_i^{(h_i^t, p_i^t)}$ do not depend on the private history, the value $W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t)$ is also independent of private history. Furthermore, since the density of ϱ_i is absolutely continuous, the market participant i can only be indifferent between two actions on a zero measure of the support. For different values of ϱ_i, the action is unique and independent of the past values of the shock. $\square$

Given that beliefs on the history of past private shock do not matter, I can suppress the dependence on the beliefs and the private shock realization in the value function. Thus, the expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i}|h_i^t) = \int \mathbb{E}_{g'|g} \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i}|a_i, g', \varrho_i', h_i^t) d\varsigma_{P_i}(\varrho_i').$$

I then arrive to the first step of the proof. Given that beliefs over private histories are irrelevant for optimality, every perfect Bayesian equilibrium (i.e. a profile of mutual sequential best responses) satisfying Assumption 3 with $(\psi, \epsilon) > 0$ are essentially sequentially strict.

Lemma J.3 (Sequential Best Response and Perfect Bayesian Equilibrium)**.** *Every perfect Bayesian equilibrium satisfying Assumption 3 with $(\psi, \epsilon) > 0$ is essentially sequentially strict.*

Proof. I need to show that for any period, history and for almost all values of the private shock, the optimal action is unique. I consider the case of the borrower first. The borrower's value from action a_b after the realization of ϱ_b is given by

$$W^b(a_b, \varrho_b, \sigma_b|h_b^t) = u(a_b, g) + \epsilon\varrho_b^a + \beta\mathbb{E}_{g'|g} W^i(\sigma_b|a_b, g', h_b^t).$$

Suppose two actions a_b and $\tilde{a}_b$, the equality $W^b(a_b, \varrho_b, \sigma_b|h_b^t) = W^b(\tilde{a}_b, \varrho_b, \sigma_b|h_b^t)$ implies that

$$\epsilon(\varrho_b^{a_b} - \varrho_b^{\tilde{a}-b}) = u(a_b, g) - u(\tilde{a}_b, g) + \beta\mathbb{E}_{g'|g}\left[W^b(\sigma_b|a_b, g', h_b^t) - W^b(\sigma_b|\tilde{a}_b, g', h_b^t)\right].$$

The set of actions is countable, whereas the set of values of private shocks for which a market participant can be indifferent has measure zero. Hence, for almost all values of ϱ_i, the set of maximizing actions must be a singleton, and the profile is essentially sequentially strict. The proof naturally extends to the case of the lender and is therefore omitted. □

Now that we have that all equilibria satisfying Assumption 3 with $(\psi, \epsilon) > 0$ are essentially sequentially strict, I simply need to show that sequentially strict equilibria are Markov equilibria.

Lemma J.4 (Sequential Strictness and Markov Equilibrium)**.** *Every essentially sequentially strict perfect Bayesian equilibrium is a Markov perfect equilibrium.*

Proof. Consider a t period history h^t. As shown previously, the private history matters, so the focus is on public history. Under Assumption 3 with $(\psi, \epsilon) > 0$, the borrower's behavior will not depend on h^t anymore from $t + \mathcal{T} + 1$ periods onward given that its memory is bounded to $\mathcal{T}$ periods back. This means that the lender's value will not depend on h^t from $t + \mathcal{T} + 1$ periods for sure. As a result, if the lender strategy is sequentially strict, then h^t becomes irrelevant from $t + \mathcal{T} + 1$ periods.

What happens in period $t + \mathcal{T}$? This represents the last period in which strategies could be conditioned on h^t. However, at that time, the borrower's maximization problem is independent of h^t as no conditioning is possible next period. In addition, sequential strictness implies that the maximizing action is a singleton. Applying this argument recursively completes the proof. □

I have therefore shown that, under the assumption of bounded memory of the borrower, small perturbations in the payoff of the market participants suppress all equilibria except Markov ones. □

Proof of Proposition 4

I first show that the autarkic allocation is not optimal. The proof follows Aguiar et al. (2009). Consider a version of the optimal contract in which the outside option corresponds to the value of permanent autarky is given by (J.1). In autarky, $k = 0$ and from Definition 3 there is an $x_a(g)$ such that $u(c(g, x_a(g))) = u(gf(0))$ at all histories. Using the definition of h in Assumption 4, consider that one increases h by Δh and $u(c(g, x_a(g)))$ by $\theta u_c(gf(0))\Delta h$ where

$$\theta = \frac{u_c(g_L f(0))}{u_c(g_L f(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g_L} u_c(g' f(0))} < 1.$$

I defined θ such that the borrower's participation constraint holds. To see this, note that the increase of h increases the borrower's outside option by $u_c(gf(0))\Delta h$ as it can benefit from the additional level of capital before going to autarky forever. However, if the borrower does not choose autarky, its value increases by $\theta(u_c(gf(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g} u_c(g' f(0)))\Delta h \geq u_c(gf(0))\Delta h$ by definition of θ. Hence the borrower's participation constraint is satisfied. Furthermore, the value of the lender changes by

$$\Delta h \frac{1}{1+r}\left(1 - \mathbb{E}_{g'|g_L}\left[\frac{u_c(g' f(0))}{u_c(c(g', x_a(g)))}\right]\theta\right) = \Delta h \frac{1}{1+r}(1-\theta) > 0,$$

where the equality comes from the fact that we consider the autarkic allocation (i.e. $c(g, x_a(g)) = gf(0)$) and the inequality from the fact that $\theta < 1$. As a result, the autarkic allocation is not optimal. The contract is therefore restricted to relative Pareto weights $x > \max_{g \in G}\{x_a(g)\} = x_a(g_H)$.

Consider the interval $[\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. From the law of motion of the relative Pareto weight, x' is strictly increasing in x. From the first-order conditions on consumption, c is strictly increasing in x. Hence, so does the value of the borrower. In opposition, with a greater c or x', the value of the lender decreases. That is the lender's value is strictly decreasing in x. Moreover, note that the relative Pareto weight, $x_{t+1} = \frac{\mu_{b,t}+\gamma(g_t)}{\mu_{l,t}}$, cannot be negative as $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t. Hence, any continuation of an efficient allocation is itself efficient. $\square$

Proof of Proposition 5

– Part I

The optimal level of capital is given by

$$g f_k(k(g)) - 1 = \nu(g) u_c(g f(k(g))) g f_k(k(g)) x.$$

As one can see, as soon as the participation constraint does not bind (i.e. $\nu(g) = 0$), the contract can attain the production-maximizing level of capital $k^*(g)$ such that $g f_k(k^*(g)) = 1$. When this condition is not met, $k < k^*(g)$. Thus, define $x^*(g)$ such that

$$V^b(g, x^*(g)) = V^D(g, k^*(g)).$$

By the above definition, if $x < x^*(g)$, capital is distorted in state g, while if $x \geq x^*(g)$, capital is at the production-maximizing level. Moreover as $V^D(g_L, k^*(g_L)) < V^D(g_H, k^*(g_H))$, $x^*(g_H) > x^*(g_L)$.

Assume by contradiction that for $x_1 < x_2 < x^*(g)$ one has that $k(g, x_1) \geq k(g, x_2)$ for all $g \in G$. From Proposition 4, we have that $V^b(g, x_1) < V^b(g, x_2)$. Moreover, as $k^*(g) > k(g, x_1) \geq k(g, x_2)$, the borrower's participation constraint binds which gives

$$\begin{aligned} V^b(g, x_1) &= u(g f(k(g, x_1))) + \beta \mathbb{E}_{g'|g} V^D(g', k') \\ &< V^b(g, x_2) \\ &= u(g f(k(g, x_2))) + \beta \mathbb{E}_{g'|g} V^D(g', k'), \end{aligned}$$

where the two equalities come from the binding borrower's participation constraint and the inequality for the fact that $x_1 < x_2$ for a given g. This is contradiction. Hence, it must be that $k(g, x_1) < k(g, x_2)$ for any $x_1 < x_2$.

The fact that that $k(g, x) > 0$ for all (g, x) follows directly from Proposition 4 which shows that the autarkic allocation is not optimal.$\square$

– Part II

The law of motion of the relative Pareto weight is given by $x'(g) = (1+\nu(g))\eta x$, while the first-order condition on consumption reads $u_c(c(g)) = \frac{1}{1+\nu(g)}$.

Given the first-order condition, $c(g_L, x) \leq c(g_H, x)$ only when $\nu(g_L) \leq \nu(g_H)$. Assume by contradiction that $\nu(g_L) > \nu(g_H)$. This implies that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Especially, consider that $\nu(g_L) > \nu(g_H) \geq 0$. In this case,

$$u(c(g_H, x)) + \beta \mathbb{E}_{g'|g_H} V^b(g', x'(g_H, x)) \geq u(g_H f(k(g_H))) + \beta \mathbb{E}_{g'|g_H} V^D(g', k'),$$
$$u(c(g_L, x)) + \beta \mathbb{E}_{g'|g_L} V^b(g', x'(g_L, x)) = u(g_L f(k(g_L))) + \beta \mathbb{E}_{g'|g_L} V^D(g', k').$$

Given that $g_H > g_L$ and $\pi(g|g) > 0.5$ for all $g \in G$, $u(g_H f(k(g_H))) > u(g_L f(k(g_L)))$ and $V^D(g_H, k) > V^D(g_L, k)$. This implies that

$$u(c(g_H, x)) + \beta V^b(g', x'(g_H, x)) > u(c(g_L)) + \beta V^b(g', g_L, x'(g_L, x)).$$

which contradicts the fact that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Hence, $\nu(g_L) \leq \nu(g_H)$ which gives $c(g_L, x) \leq c(g_H, x)$ and $x(g_L, x) \leq x(g_H, x)$ as desired.

Especially, by definition, when $x \geq x^*(g_H)$, then $\nu(g) = 0$ for all g implying that $c(g_L, x) = c(g_H, x)$ and $x(g_L, x) = x(g_H, x)$. Otherwise, $c(g_L, x) < c(g_H, x)$ and $x(g_L, x) < x(g_H, x)$. $\square$

– Part III

This proofs is a modified version of Thomas and Worrall (1990, Lemma 4). The value of liabilities in the optimal contract is given by

$$V^l(g, x) \equiv g f(k(g)) - k(g) - c(g, x) + \frac{1}{1+r} \mathbb{E}_{g'|g} V^l(g', x'(g, x)).$$

Assume by contradiction that for a given x it holds that $V^l(g_H, x) \leq V^l(g_L, x)$. For $x \geq x^*(g_H)$, one directly reaches a contradiction as $c(g_L, x) = c(g_H, x)$ and $x(g_L, x) = x(g_H, x)$ which implies that $V^l(g_H, x) > V^l(g_L, x)$.

For $x < x^*(g_H)$, consider the pooling allocation in which $u(\ddot{c}(g_H, x)) = u(\ddot{c}(g_L, x)) = u(c(g_H, x))$ and $\ddot{V}^b(g_H, x) = \ddot{V}^b(g_L, x) = V^b(g_H, x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to

$$\ddot{V}^l(g_H, x) > \ddot{V}^l(g_L, x)$$

which is a direct contradiction. Hence, $V^l(g_H, x) \geq V^l(g_L, x)$. However, $V^l(g_H, x) = V^l(g_L, x)$ is ruled out by fact that there is no perfect risk sharing when $x < x^*(g_H)$. $\square$

Proof of Proposition 6

Recall the law of motion of the relative Pareto weight

$$x'(g) = (1 + \nu(g))\eta x.$$

The motion of the relative Pareto weight is dictated by the relative impatience, η, and the binding participation constraint, ν. I consider two cases. On the one hand, if $\eta < 1$, the relative Pareto weight increases only if $\nu(g) > 0$ is sufficiently large to overcome impatience. As we know, when $x \geq x^*(g)$, $\nu(g, x) = 0$ meaning that impatience eventually dominates the limited commitment issue. On the other hand, if $\eta = 1$ immiseration due to impatience does not exist and the relative Pareto weight remains constant.

When $\eta = 1$, the upper bound of the ergodic set coincides with the lower bound. As shown in Proposition 5, $x'(g_L, x) \leq x'(g_H, x)$. Moreover, by definition of $x^*(g_H)$ in Proposition 5, $x^{ub} = x^{lb} = x^*(g_H)$. Conversely, when $\eta < 1$, impatience prevents the contract to reach $x^*(g_H)$ as $\nu(g_H, x^*(g_H)) = 0$. Hence, $x^*(g_H) > x^{ub}$. Moreover, $x'(g_L, x) < x'(g_H, x)$ when $x < x^*(g_H)$ implying that $x^{ub} > x^{lb}$. In other words, impatience immiserates the relative Pareto in the low productivity state implying that $x^*(g_H) > x^{ub} > x^{lb}$.

To show the existence of the ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as the midpoint of $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \to \mathbb{R}$ as

$$Q(x, G) = \sum_{g'|g} \pi(g'|g)\mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of g_L, the relative Pareto weight transit to x^{lb} (either through impatience or because $x^{lb} = x^{ub}$). Hence for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(g_L|g_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M < \infty$ g_H, the relative Pareto weight transit to x^{ub} (either through the binding constraint or because $x^{lb} = x^{ub}$) meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}])^M \geq \pi(g_H|g_H)^M > 0$. Setting $\epsilon = \min\{\pi(g_L)^M, \pi(g_H)^M\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. $\square$

Proof of Proposition 7

The proof of this proposition is by construction. Similar to Dovis (2019), I express the policy functions of the implemented contract as a function of the relative Pareto weight, x, and the productivity state, g. Formally, define

$$\begin{aligned}
&\bar{D}, \bar{M} : G \times X \to \{0, 1\}, \\
&\bar{k}, \bar{p}, \bar{q}_{st}, \bar{q}_{tl}, \bar{b}_{st}, \bar{b}_{lt} : G \times X \to \mathbb{R}.
\end{aligned}$$

Given the timing of actions, the price schedules and bond policies depend on the prospective relative Pareto weights after the productivity shock realizes. Those objects can therefore be rewritten as

$$\begin{aligned}
&\bar{b}_j(g, x) = \bar{b}_j(x'(g, x)), \\
&\bar{q}_j(g, x) = \bar{q}_j(x'(g, x)) \quad \text{for all } j \in \{st, lt\}.
\end{aligned}$$

I first determine the default and *official* buyback policies. Subsequently, I compute the underlying prices. I then define the portfolio of bonds to match the total value of debt $V^l(g, x)$ implied by the constrained efficient allocation. Finally, I determine the optimal capital pricing from the optimality conditions of the lender and the domestic firms.

Given Proposition 4, autarky is never optimal in the contract. Hence, the government never enters into default. That is $\bar{D}(g, x) = 0$ for all (g, x). The government will therefore rely on changes in the maturity structure and *official* buybacks as in the Markov equilibrium without default. I assume that *official* buybacks arise only if the economy hits the upper bound of the ergodic set,

$$\bar{M}(g, x) = \begin{cases} 1 & \text{if} \quad g = g_H \text{ and } x = x^{ub} \\ 0 & \text{else} \end{cases}$$

Given the above policies, the short-term bond price equates the risk-free price,

$$\bar{q}_{st}(x) = \frac{1}{1 + r},$$

while the long-term bond price,

$$\bar{q}_{lt}(x) = \frac{1}{1 + r} \mathbb{E}_{g'|g}[1 + \bar{q}_{lt}(x')].$$

Note further that, the long-term bond price has the following properties.

Lemma J.5 (Bond Price)**.** *Under Assumption 1, with* $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ *and* $\chi \in (0,1)$*, the long-term bond price is the unique fixed point of* $\bar{q}_{lt}$*, is decreasing and is such that*

$$\frac{1}{(1-\chi)r} > \bar{q}_{lt}(x'(g_H,x)) \geq \bar{q}_{lt}(x'(g_L,x)) > \frac{1}{r},$$

with strict inequality when $\eta < 1$*.*

Proof. Recall that the long-term bond price is given by

$$\bar{q}_{lt}(g,x) = \frac{1}{1+r}\mathbb{E}_{g'|g}\left[(1-\bar{D}(g',x'))\left\{1+(1-\bar{M}(g',x'))\bar{q}_{lt}(g',x')+\bar{M}(g',x')q_{lt}^{bb}\right\}+\bar{D}(g',x')\bar{q}_{lt}^{D}(g',x')\right],$$

I consider that $\bar{D}(g',x') = 0$ for all (g',x') and $\bar{M}(g',x') = 1$ if $g' = g_H$ as well as $x' = x^{ub}$ and $\bar{M}(g',x') = 0$ otherwise. From Proposition 6, g_H and $x = x^{ub}$ arises with strictly positive probability for any (g,x),

$$\frac{1}{(1-\chi)r} > \bar{q}_{lt}(g,x) > \frac{1}{r}.$$

Define Q_{lt} as the space of bounded functions $\bar{q}_{lt} : [\underline{x},\bar{x}] \to [0, \frac{1}{(1-\chi)r}]$ and $\mathbb{T} : Q_{lt} \to Q_{lt}$ as

$$\mathbb{T}\bar{q}_{lt}(g,x) = \frac{1}{1+r}\sum_{g'}\pi(g'|g)[1+\bar{q}_{lt}(g',x')].$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$ which is increasing as $\mathbb{T}$ maps increasing functions into increasing functions. This implies that $\bar{q}_{lt}(x'(g_H,x)) \geq \bar{q}_{lt}(x'(g_L,x))$ as $x'(g_H,x) > x'(g_L,x)$ for all x in the above specified domain. Assume now that there exists a x such that $\bar{q}_{lt}(x'(g_H,x)) = \bar{q}_{lt}(x'(g_L,x))$. This requires that $x'(g_H,x)$ and $x'(g_L,x)$ belongs to a subset $[x_t, x_{t+1}]$ where $\bar{q}_{lt}$ stays constant. Hence, for any $\ddot{x} \in [x_t,x_{t+1}]$, it must be that $x'(g_H,\ddot{x}), x'(g_L,\ddot{x}) \in [x_t,x_{t+1}]$ which is a contradiction as $x'(g_H,x_{t+1}) > x_{t+1}$ when $\eta < 1$. Therefore it must be that $\bar{q}_{lt}(x'(g_H,x)) > \bar{q}_{lt}(x'(g_L,x))$ when $\eta < 1$ and $\bar{q}_{lt}(x'(g_H,x)) \geq \bar{q}_{lt}(x'(g_L,x))$ otherwise. ☐

Having properly determined the different price schedules, I can now determine the bond holdings and the maturity in order to match the total value of the debt implied by the constrained efficient allocation. Particularly, it must hold that when $x = x^{ub}$,

$$-V^{l}(g_H,x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1+q_{lt}^{bb}], \tag{J.4}$$

$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))]. \tag{J.5}$$

Otherwise, the relationship is given by

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_H, x))],$$
$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma J.5 ensures that there exists a unique solution. The maturity structure of the bond portfolio is therefore properly determined.

To complete the proof, I determine the optimal capital price. Given that $\tau = 1 - \frac{1}{\bar{p}(g,x)}$, from the optimality conditions of the domestic firms and the lender I get

$$g f_k(k) = \bar{p}(g, x). \tag{J.6}$$

Hence, the constrained efficient allocation can be replicated with the above policies for default, *official* buyback, and bond holdings. The optimality conditions of the lender, the government and the domestic firms are satisfied as well as the price schedules.

This concludes the proof as the market allocation satisfies the necessary and sufficient conditions provided in Lemma 2. Especially, I used the budget constraints to determine the optimal bond holdings given the prices computed according to (2). The capital pricing is set to match the conditions (3) and (4). Finally, the resource constraint and the condition for reversion to the worst equilibrium outcome are satisfied as the constrained efficient allocation meet those requirements. Especially, note that $V^D(g_t, 0) \geq v_a(g_t)$ as defined in (J.1) given that $\lambda \geq 0$. Thus, the participation constraint (11) ensures that the outcome can be sustained by trigger strategies.

For the second part of the proposition, assume that Part II of Lemma 3 applies. For $\eta < 1$, (10) implements the constrained efficient allocation with Markov strategies under $\overline{B}_{st} = B_{st}(g_H, \overline{B}_{st}, \overline{B}_{lt})$ and $\overline{B}_{st} = B_{lt}(g_H, \overline{B}_{st}, \overline{B}_{lt})$. For $\eta = 1$, (8) implements the constrained efficient allocation with Markov strategies. □

Proof of Lemma 4

The fact that default never occurs is a direct corollary of Proposition 4. Regarding buybacks, I consider that the government conducts *official* buybacks when it hits the upper bound of the ergodic set – i.e. $x = x^{ub}$ and $g = g_H$. I therefore need to consider two alternatives.

First, could *official* buybacks occur in the lower bound of the ergodic set – i.e. $x = x^{lb}$?

The answer is negative. To reach the lower point of amnesia, the relative Pareto weight needs to decrease. More precisely, in steady state $x'(g_L) \leq x$ as shown in Proposition 6. This implies that the value of the lender increases as g_L realizes. This goes against the idea of a debt repurchase which (weakly) reduces indebtedness. Furthermore, from Part III of Proposition 5, it holds that $V^l(g_H, x) > V^l(g_L, x)$ which implies debt relief in the low productivity state. However, the price of long-term debt would increases as g_L realizes (i.e. the reverse of Lemma J.5) which goes against the idea of a debt relief. Hence, it is not possible to have an *official* buyback at any point related to the realization of g_L.

Second, could *official* buybacks occur before the contract hits the upper bound of the ergodic set – i.e. $x < x^{ub}$ and $g = g_H$? The answer is positive as the realization of g_H is associated with a debt decrease. However, one has to be careful that each *official* buyback should be such that $b'_{lt} \geq b_{lt}$. Moreover, note that if *official* buybacks happen at say $\tilde{x} < x^{ub}$ and $g = g_H$, then for all $x \in (\tilde{x}, x^{ub}]$ $\bar{q}_{lt}(x'(g_H, x)) \leq \bar{q}_{lt}(x'(g_L, x))$. $\square$

Proof of Lemma 5

From (J.4) and (J.5), the short and long-term holdings at the *official* buyback are respectively

$$\bar{b}_{st}(x) = \frac{V^l(g_H, x)[1 + \bar{q}_{lt}(x'(g_L, x))] - V^l(g_L, x)[1 + q^{bb}_{lt}]}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} < 0,$$

$$\bar{b}_{lt}(x) = -\frac{V^l(g_H, x) - V^l(g_L, x)}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} \lesseqqgtr 0.$$

From Part III of Proposition 5, it holds that $V^l(g_H, x) > V^l(g_L, x)$ meaning that $b_{lt} < 0$. However, it is not guaranteed that $b_{st} < 0$. Particularly, b_{st} can be negative only if q^{bb}_{lt} is very large with respect to q_{lt}. Moreover, recall that the *official* buyback takes place in the point of amnesia meaning that $b'_{st} = b_{st}$ and $b'_{lt} = b_{lt}$. As a result, Part II of Lemma 3 does not generally apply as b'_{st} is negative when χ is sufficiently large. $\square$

Proof of Lemma D.1

The law of motion of the relative Pareto weight is given by

$$x'(g) = (1 + \nu(g))\eta x.$$

and the level of consumption by

$$u_c(c(g)) = \frac{1}{x(1 + \nu(g))}.$$

Isolating x leads to

$$x = \frac{1}{u_c(c(g))(1+\nu(g))}. \tag{J.7}$$

Plugging this back into the law of motion gives

$$x'(g) = (1+\nu(g))\eta \frac{1}{u_c(c(g))(1+\nu(g))}.$$

Replacing $x'(g)$ by with the forward equivalent of (J.7) gives

$$\frac{1}{u_c(c(g'))(1+\nu(g'))} = \eta \frac{1}{u_c(c(g))}.$$

Taking expectations on both sides,

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

which gives the inverse Euler equation. □

Proof of Proposition D.1

Existence and uniqueness follow from Theorem 3 in Marcet and Marimon (2019). The two authors make the following assumptions: A1 a well defined Markov chain process for g, A2 continuity in $\{c, k\}$ and measurability in g, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lender and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given Assumptions 1 and 4. Since feasible c and k are bounded, payoffs functions are bounded as well. This combined with the fact that the outside options are also bounded ensure that A4 is met. Whether A3 is satisfied depends on the initial condition (g_0, x_0). Assumption D.1 ensures feasibility and that the strict interiority condition is satisfied.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding to the original contract problem (12). To obtain the recursive formulation of the contract, I have normalized the co-state variable. I relied on the the homogeneity of degree one in (μ_b, μ_l) to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if I define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}^2_+\}$ and the set of feasible consumption and capital by $A = \{(c, k) \in \mathbb{R}^2_+\}$, the correspondence $SP : A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Marcet and Marimon (2019) additionally show that the the saddle point functional equation (14) is a contraction mapping. Thus, given the concavity assumptions of $u(\cdot)$ and $f(\cdot)$, the allocation is unique. □

Proof of Proposition F.2

Following Alvarez and Jermann (2000) we prove the proposition by construction. First, define the asset price as

$$q(g',x'|g) = \frac{\pi(g'|g)}{1+r}\left[1 + \sum_{g''|g'} q(g'',x''|g')\right] \max\left\{\frac{u'(c(g',x'))}{u'(c(g,x))}\eta, 1\right\}.$$

Second, iterating over the budget constraint of the government and applying the transversality condition gives

$$a(g^t) = \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[c(g^{t+j}, x(g^{t+j})) - Y(g^{t+j}, x(g^{t+j}))], \tag{J.8}$$

where, $Y(g^t, x(g^t)) = g_t f(k(g^t, x(g^t))) - k(g^t, x(g^t))$ for all t and g^t. Similarly, iterating over the budget constraint of the lender leads to

$$\begin{aligned} a_l(g^t) &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t) c_l(g^{t+j}, x(g^{t+j})) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[Y(g^{t+j}, x(g^{t+j})) - c(g^{t+j}, x(g^{t+j}))] \\ &= -a(g^t). \end{aligned} \tag{J.9}$$

The market clearing condition implies that $a_l(g^t) + a(g^t) = 0$ for all t and g^t.

To ensure that the capital level is the same as in the constrained efficient allocation, I set the capital price according to

$$g f_k(k) = p(g, a).$$

I now need to establish the correspondence between the initial conditions, x_0, in the contract and the initial conditions in the RCE, a_0. Given (J.8) and (J.9) evaluated at $t = 0$, one can determine $\bar{a}'$ using the budget constraint

$$c(g_0, x_0) + q(g_0, a_1)(\bar{a}' - a_0) + \sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) \leq y(g_0, k) + a_0.$$

and the fact that $\sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) = 0$. Once, $\bar{a}'$ is determined, one can find the holdings of Arrow securities $\hat{a}'(g', g_0, a_0)$ for all $g' \in G$. We can then retrieve the entire portfolio recursively for $t > 0$.

Third, define the endogenous borrowing limits such that

$$\mathcal{A}(g, k) = a(g, \tilde{x}(g, k)),$$

where $\tilde{x}(g, k)$ is the relative Pareto weight when the participation constraint binds at (g, k). This definition implies that $a'(g', g, a) \geq \mathcal{A}(g', k')$. Hence, the constructed asset holdings satisfy the competitive equilibrium constraints.

Fourth, to ensure optimality of the policy functions by setting

$$\mu_{BC}(g, a) = \frac{1}{x(1 + \nu(g))}$$

Hence, since $c(g, x)$ satisfies the optimality conditions in the Planner's problem, it is also optimally determined in the RCE. For the lenders, $c_l(g, x)$ is optimal if the asset portfolio is optimally determined. For this observe that

$$\begin{aligned} q(g', a'(g')|g) &= \frac{1}{1+r}\pi(g'|g)\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &> \frac{1}{1+r}\pi(g'|g)\frac{u'(c(g', a'(g')))}{u'(c(g, a))}\eta\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &\quad \text{if } a'(g', g, a) = \mathcal{A}(g', k'). \end{aligned}$$

Hence the portfolio is optimally determined. We therefore obtain a one-to-one map between x and a for a given g. More precisely, $c(g, a) = c(g, x)$, $c_l(g, a) = T(g, x)$ and $k(g, a) = k(g, x)$. Thus, $W^b(g, a) = W^b(g, x)$ and $W^l(g, a) = W^l(g, x)$. Furthermore, the endogenous limits binds uniquely and exclusively when the participation constraints of the government binds. $\square$