Efficient Sovereign Debt Management: The Role of History, Maturity, Buyback and Default*

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Abstract

This paper identifies the role of past history, maturity, buyback and default in the context of constrained efficient sovereign borrowing. I derive a market economy in which a sovereign borrower trades non-contingent bonds of different maturities with a foreign lender. The borrower is impatient and lacks commitment. I show that the market economy cannot implement the Planner's constrained efficient allocation through defaults but instead by changes in maturity and costly debt buybacks. Moreover, as the lender must enforce those buybacks, the implementation often requires history-dependent strategies. Nevertheless, interpreting the borrower's impatience as a form of fading memory, small perturbations in the payoff of the market participants rule out any other strategies than Markov ones. In this case, the Planner's allocation can only be approximated by the market economy through Markov debt management policies. Emerging economies such as Argentina and Brazil present evidence of such approximation albeit with different policies and outcome.

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1 Introduction

The sovereign debt management of emerging economies has three main features. First and foremost, the better part of those economies defaults on their liabilities. Defaults are relatively frequent, last several years and are associated with output contractions as well as substantial debt reliefs.[1] Second, emerging economies rely more extensively on short-term debt during debt crises.[2] Third, they conduct debt buybacks to repurchase part of their debt on the market. Even though such events have become rare in the last few years, they are usually very costly and ineffective in reducing indebtedness.[3] The question that arises is whether such debt management policies are Pareto efficient? I analyze the role of the above policies in attaining or approximating the constrained efficient sovereign borrowing.

I consider an environment in which a foreign lender owns a production technology in a small open economy, provides the capital input and buys bonds issued by a sovereign government (i.e. the borrower). Conversely, the sovereign government takes the decision on behalf of the small open economy, runs the production technology and issues non-contingent defaultable bonds of different maturities. In addition, domestic production is subject to persistent productivity shocks and the government is impatient. In this set-up, I introduce one friction: the government cannot commit to repay the lender.

Relying on trigger strategies, I show that it is possible to sustain many different equilibria in this environment. I first focus on Markov equilibria to derive history-invariant debt management policies. I consider the case in which the lender inhibits the government's default incentives and the case in which it does not. I subsequently provide foundations for the use of Markov strategies. Particularly, I present a simple equilibrium refinement which rules out all non-Markov outcome. Secondly, I derive the debt management policies emanating from an optimal contract. Such policies correspond to the best achievable outcome in this environment but are usually highly history dependent.

To analyze the Markov debt management policies, I consider two types of Markov equilibria. The first one is a version of Arellano and Ramanarayanan (2012) with endogenous default cost. The lender does not inhibit default incentives and prices bonds accordingly. Default provides some form of state contingency and pushes the maturity towards the short end during debt crises. In the second Markov equilibrium, the lender introduces a borrowing limit which can be made state-contingent with the help of costly buyback programs. The borrowing limit prevents defaults while allowing for risk sharing as in Alvarez and Jermann

[1] For default duration and haircuts, see Cruces and Trebesch (2013) and Asonuma and Trebesch (2016). For default frequency, see Tomz and Wright (2007, 2013) and Reinhart and Rogoff (2009).

[2] See Arellano and Ramanarayanan (2012), Broner et al. (2013) and Bai et al. (2017).

[3] See Bulow and Rogoff (1988, 1991) and Cohen and Verdier (1995).

(2000). I show that this second equilibrium is Pareto superior demonstrating the inefficiency of default in generating risk sharing. I finally provide foundations for the use of Markov strategies. Interpreting the borrower's impatience as a form of fading memory, I show that only Markov equilibria are robust to small and independent perturbations in payoffs. The argument is that the lender conditions its actions on a past event only if the government does so and *vice versa*. Thus, conditioning on the past is only possible if both parties keep track of the entire history of play. As a result, there are two extremes: both parties either build on the entire history or do not at all.

To analyze the constrained efficient debt management policies, I first derive an optimal contract between the borrower and the lender. The Planner accounts for a borrower's participation constraint and keeps track of the binding constraint through a co-state variable – i.e. the relative Pareto weight – which is sufficient statistics for the history of play (Marcet and Marimon, 2019). The optimal contract determines the constrained efficient allocation. It features production distortion and state-contingent debt relief. Particularly, when the relative Pareto weight is sufficiently high, the punishment of autarky is a real threat. The contract can therefore sustain the productivity-maximizing level of capital. Otherwise, the threat of autarky fades. As a result, the Planner reduces the level of capital to relax the participation constraint. It never finds optimal to set capital to zero, though.

I then implement the constrained efficient allocation in the sovereign debt market economy. Given that the Planner never distorts capital to zero, defaults – which imply markets exclusion – cannot implement the constrained efficient allocation. Instead, the government adapts the maturity structure of its portfolio and conducts costly debt buybacks. Such buybacks implicitly introduce state contingency in the bond contract. It occurs at some very specific points in time implying that the price of long-term bonds evolves according to the likelihood of a buyback in the future. This in turns generates the term structure necessary to mimic the state contingency in liabilities of the optimal contract.

The implementation of the constrained efficient allocation might not generally rely on Markov strategies. The reason is that the borrower is not willing to conduct costly buybacks. Thus, the lender has to enforce them. I show that in a Markov equilibrium, such enforcement is possible only if the borrower does not issue any assets and buybacks are not too costly. In this situation, the lender can threaten the borrower with a sudden stop on debt if the buyback does not occur as planned. However, such threat ceases to be credible as soon as the borrower possesses some assets or the buyback becomes too onerous. I find that, to replicate the constrained efficient allocation, the borrower needs to hold short-term assets unless buybacks are uncommonly costly. As a result, Markov strategies fail to implement the Planner's allocation under empirically plausible buyback rates.

I calibrate the Markov equilibrium with default incentives to match moments of the Argentine economy over the period 1990-2019. The calibrated model fits well the data and features defaults episodes in which indebtedness increases with respect to output and maturity shortens. Conversely, during restructurings, the level of debt remains substantial and the maturity lengthens. In addition, I show that the Markov equilibrium without default incentives shares similarities with Brazil which has not defaulted since the end of the 1980s and conducted official buyback programs since the 2000s – unlike Argentina.

I then compare the Markov equilibria with the implementation of the optimal contract through various simulation exercises. In general, consumption is less volatile and corresponds to a lower share of output, while investment is more volatile in the constrained efficient allocation compared to the two Markov allocations. I find important welfare gains for the borrower to rely on costly buybacks instead of defaults. In light of this, I show that the Markov equilibrium without default incentives is quantitatively the closest to the constrained efficient allocation. This is consistent with the fact that Brazil's economic performance has been stronger than Argentina's in the last few decades.

The paper combines elements of the literature on sovereign defaults and buybacks with elements of the literature about optimal contracts and their implementation. The literature on sovereign defaults assumes that markets are incomplete and agents follow Markov strategies (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008).[4] There, the borrower has access to only non-contingent claims and can obtain limited state contingency through defaults. My study is the closest to Arellano and Ramanarayanan (2012) and Niepelt (2014) given that I adopt two bonds with different maturities and to Mendoza and Yue (2012) given that the default cost is endogenous.[5] I contribute to this literature in two ways. First, I show that the reliance on defaults to obtain state contingency is inefficient. Second, I provide foundations for the use of Markov strategies interpreting the assumption of impatience as evidence of fading memory and then implementing the refinement of Bhaskar et al. (2012) and Angeletos and Lian (2021). This second result relates to Krusell and Smith (1996) as it deals with the sophistication of agents' strategies.

On a similar note, this paper relies on costly buybacks as a way to implement the constrained efficient allocation. It therefore relates to the seminal contribution of Bulow and Rogoff (1988, 1991) who document that buybacks are suboptimal as they increase the recovery value per unit of bond and therefore fail to reduce indebtedness. In light of this, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Similarly, Aguiar et al. (2019) find that buybacks reduce welfare as they shift the maturity structure

[4]See also Aguiar and Amador (2014) and Aguiar et al. (2016).

[5]See also Bohn (1990) and Barro (1995) for earlier work on optimal debt structure.

and therefore affect the default risk.[6] In opposition, Rotemberg (1991) shows that buybacks can be advantageous to all parties as they lower the bargaining costs. Moreover, Acharya and Diwan (1993) find that buybacks provide a positive signal about the borrower's willingness to repay. My analysis goes in this direction as it emphasises the efficiency of buybacks as a source of risk sharing between the borrower and the lender.

The paper derives the optimal contract between a lender and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). My study accounts for limited enforcements similar to Aguiar et al. (2009) and is close to Kehoe and Perri (2002) and Restrepo-Echavarria (2019) as it relies on the approach of Marcet and Marimon (2019) with the difference that I implement the contract in a market economy.[7]

The paper therefore addresses the literature on optimal contract's implementation. Unlike Aguiar et al. (2019) and Müller et al. (2019), my implementation does not generally rely on Markov strategies. On the one hand, Aguiar et al. (2019) account for multiple maturities but consider a Planner's problem which does not take into consideration the legacy creditors in the objective function and has no participation constraint, unlike my Planner problem. On the other hand, Müller et al. (2019) use preemptive restructurings and GDP-linked bonds to mimic the return of Arrow securities, whereas I rely on the term structure. An alternative to this approach is Dovis (2019) who develop an implementation through partial defaults and an active debt maturity management. He builds on Angeletos (2002) and Buera and Nicolini (2004) who show that one can replicate the state-contingency of Arrow securities using non contingent bonds of different maturities.[8] My implementation is the closest to Dovis (2019) with the difference that I rely on debt buybacks without defaults and haircuts. Moreover, I connect my implementation with the Markov allocation. Especially, as buybacks need to be enforced, I explain why and when history dependence matters. See Appendix A for a more detailed discussion on this.

The paper is organized as follows. I describe the economic environment in Section 2. Subsequently, I introduce the market economy and derive the set of sustainable equilibria in Section 3. I present the Markov debt management policies in Section 4. Thereafter, I derive the constrained efficient debt management policies in Section 5 and discuss alternative policies in Section 6. The calibration and quantitative analyses are in Section 7. Finally, I conclude in Section 8.

[6]Furthermore, Aguiar et al. (2022) show that buybacks are rare as they only occur when there are no uncertainty surrounding debt auctions.

[7]The other difference is that I adopt a capital depreciation rate of 1 which facilitates the equilibrium computation and the proofs.

[8]See also Faraglia et al. (2019) for an analysis of buyback and maturity with commitment.

2 Environment

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The small open economy is populated by a benevolent government and a large number of homogenous households which own domestic firms, while a foreign lender invests in the small open economy.[9]

The risk neutral lender discounts the future at rate $\frac{1}{1+r}$. It equips the small open economy with a production technology, $F(k, l)$, to produce goods and provides the capital input, k, at price p, which depreciates at rate $\delta = 1$. Domestic households provide the labor input, l. In addition, the lender trades bonds with the government.

The representative domestic household discounts the future at rate $\beta \leq \frac{1}{1+r}$. Preference over consumption is represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t corresponds to the consumption at time t. The instantaneous utility function takes the CRRA form, $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ is the coefficient of relative risk aversion.

The government is benevolent and takes the decision on behalf of the small open economy. It can tax the payment of capital made by the domestic firms at rate $\tau_t \in [0, 1]$. Thus, the household's after-tax income is given by

$$y(g_t, p_t, k_t, \tau_t) \equiv g_t F(k_t, l_t) - p_t(1 - \tau_t)k_t.$$

Domestic households are endowed with one unit of labor in each period.[10] I therefore denote $f(k) \equiv F(k, 1)$. The production technology has constant returns to scale, is continuous, increasing, $f_k(k) > 0$, concave, $f_{kk}(k) < 0$, satisfies the Inada condition, $\lim_{k \to 0} f_k(k) = \infty$, and $f(0) > 0$. The fact that $f(k)$ is concave implies that the production technology displays decreasing returns to scale with respect to k. This means that there exists a level $k^*(g)$ which maximizes the net production such that $g f_k(k^*(g)) = p$.

Domestic production is subject to a shock g_t which takes value on the discrete set $G \equiv \{g_L, g_H\}$ with $0 < g_L < g_H$ and follows a Markov chain of order one with $\pi(g_{t+1}|g_t)$ corresponding to the probability of drawing g_{t+1} at date $t + 1$ conditional on drawing g_t at t. I further assume that shocks are persistent meaning that $\pi(g|g) > 0.5$ for all $g \in G$.[11]

[9] The present environment is similar to the one of Quadrini (2004), Aguiar et al. (2009) and Dovis (2019).

[10] As in Aguiar et al. (2009) and Dovis (2019), I combine the income of households and government together. Households provide labor inelastically and receive lump sum transfers from the government.

[11] The case in which $\pi(g|g) = 0.5$ corresponds to i.i.d shocks.

The government has access to bonds with two different maturities. On the one hand, there is a one-period – i.e. short-term – bond, b_{st}, with unit price q_{st}. On the other hand, there is a perpetual – i.e. long-term – bond, $b_{lt} \leq 0$, with unit price q_{lt}, which pays a coupon of one every period (Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). I denote debt as a negative asset meaning that $b_j < 0$ is a debt, while $b_j > 0$ is an asset for all $j \in \{st, lt\}$. The government can hold short-term assets but not long-term assets. The financial market is incomplete as bonds do not discriminate the returns across g.

The government can conduct *official* buybacks on the long-term bond at a specific price $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ with $\chi \in (0, 1)$ being the *official* buyback premium.[12] Conversely, it can conduct *unofficial* buybacks in which it retires part of its debt at the market price, $q_{lt} < q_{lt}^{bb}$, without prior notice.[13] In both cases, the prospective value of long-term debt is such that $b'_{lt} \geq b_{lt}$.

I assume that the government lacks commitment. If the government defaults, it loses access to the capital and bond market. It subsequently consumes $c(g) = gf(0) > 0$ but can regain access to the markets with a fixed probability λ. The default cost is therefore endogenous as it entirely relates to markets access. Collecting all the assumptions,

Assumption 1 (General Settings)**.** *The risk neutral lender discounts at rate* $\frac{1}{1+r}$*. The risk averse government has a utility function* $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ *with* $\sigma > 0$ *and discounts at rate* $\beta \leq \frac{1}{1+r}$*. The productivity shock* $g \in G \equiv \{g_L, g_H\}$ *follows an Markovian process of order one with* $0 < g_L < g_H$ *and* $\pi(g|g) > 0.5$ *for all* $g \in G$*. The foreign production technology is continuous, increasing, concave, satisfies the Inada condition,* $\lim_{k\to 0} f_k(k) = \infty$*, and* $f(0) > 0$*. Capital depreciates at rate* $\delta = 1$*.*

The timing of actions is the following. At the beginning of each period $t \geq 0$, the productivity shock, g_t, realizes and the lender provides k_t. Subsequently, domestic production takes place, capital depreciates and the government determines the debt repayment – including potential *official* buybacks – and the tax τ_t. Conditional on repaying, a bond auction determines $b_{st,t+1}$ and $b_{lt,t+1}$.[14] Note that, if $b_{st,t+1} > 0$, the lender is in fact a short-term borrower and is the one auctioning to raise resources.

3 The Market Economy

In this section, I define the set of sustainable equilibrium outcome in the market economy following the approach of Abreu (1988) and Chari and Kehoe (1990). The state space

[12]The fact that *official* buybacks are settled above the risk-free price is consistent with the evidence that buybacks are costly for sovereign borrowers (Bulow and Rogoff, 1988, 1991).

[13]This is what Cohen and Verdier (1995) call a secret buyback.

[14]This timing rules out self-fulfilling debt crises (Ayres et al., 2018).

accounts for the entire history of play. Using the reversion to the worst equilibrium, it is possible to sustain many different equilibrium outcomes.

3.1 The Government's problem

Define $D_t \in \{0,1\}$ as the government's default policy at time t. If $D_t = 0$, the government repays, while if $D_t = 1$, it defaults. Similarly, define $M_t \in \{0,1\}$ as the government's *official* buyback policy at time t. If $M_t = 1$, the government officially buys its debt back, while if $M_t = 0$, it does not.

In addition, define the set of government's choices as $\mathcal{G}_t = \{D_t, M_t, b_{st,t+1}, b_{lt,t+1}, \tau_t\}$ and the government's strategy as σ_b. Furthermore, let $h^t = (h^{t-1}, g_t, p_t, k_t, \mathcal{G}_t)$ denote the history up to time t taking the initial debt $b_{st,0}$ and $b_{lt,0}$ as well as capital k_0 as given. Due to the specific timing of actions, further define the history of the lender and the government for debt as $h_l^t = (h^{t-1}, g_t, p_t, k_t, \mathcal{G}_t)$ and $h_b^t = (h^{t-1}, g_t, p_t, k_t)$, respectively. I also define the history of capital as $h_k^t = (h^{t-1}, g_t, p_t)$.

In the case in which the government decides to repay (i.e. $D_t = 0$), it determines its consumption and prospective borrowing given the realization of the history (h_b^t, g_t). In the case of no *official* buyback (i.e. $M_t = 0$), the budget constraint reads

$$c_t + q_{st}(h_b^t, \mathcal{G}_t) b_{st,t+1} + q_{lt}(h_b^t, \mathcal{G}_t)(b_{lt,t+1} - b_{lt,t}) = y(g_t, p_t, k_t, \tau_t) + b_{st,t} + b_{lt,t}.$$

There is no restriction on the issue of long-term debt meaning that the government can potentially conduct *unofficial* buybacks. Conversely, in the case of an *official* buyback (i.e. $M_t = 1$), budget constraint is given by

$$c_t + q_{st}(h_b^t, \mathcal{G}_t) b_{st,t+1} + q_{lt}(h_b^t, \mathcal{G}_t) b_{lt,t+1} = y(g_t, p_t, k_t, \tau_t) + b_{st,t} + b_{lt,t}(1 + q_{lt}^{bb}) \ \wedge \ b_{lt,t+1} \geq b_{lt,t}.$$

The government retires the current long-term bond, $b_{lt,t}$, at $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ with $\chi \in (0,1)$ and issues new long-term debt such that $b_{lt,t+1} \geq b_{lt,t}$. Conversely, if the government decides to default (i.e. $D_t = 1$), it gets excluded from the markets and consumes the after-tax income

$$c_t = y(g_t, p_t, k_t, \tau_t).$$

Due to the specific timing of capital, the government enjoys $k_t \geq 0$ in the first period of autarky and then $k_t = 0$. The outstanding debt is restructured with probability λ. Upon restructuring, the government can regain access to the markets. In this case,

$$c_t + q_{st}(h_b^t, \mathcal{G}_t) b_{st,t+1} + q_{lt}(h_b^t, \mathcal{G}_t) b_{lt,t+1} = y(g_t, p_t, k_t, \tau_t) + W_{st} + W_{lt} \frac{1+r}{r},$$

where W_{st} and $W_{lt}\frac{1+r}{r}$ correspond to the recovery value of short-term and long-term debt, respectively. Once the government has paid the recovery values, there is no remaining liabilities left.

After any history (h_b^t, g_t), the optimal strategy of the government, σ_b, is the solution of

$$W^b(h_b^t, g_t) = \max_{\mathcal{G}_t = \{D_t, M_t, b_{st,t+1}, b_{lt,t+1}, \tau_t\}} u(c_t) + \beta \mathbb{E}\left[W^b(h_b^{t+1}, g_{t+1}) \middle| h_b^t, \mathcal{G}_t \right], \tag{1}$$

subject to the budget constraint.

3.2 Sustainable equilibria

This subsection aims at defining and characterizing the set of sustainable equilibria. The lender is competitive meaning that in expectations it makes zero profit. The price of one unit of bond can therefore be separated into two parts: the return when the government decides to repay and the recovery value when the government defaults. The price per unit of bond of maturity $j \in \{st, lt\}$ is given by,

$$q_j(h^t) = \mathbb{E}\left[(1 - D(h^{t+1})) q_j^P(h^{t+1}) + D(h^{t+1}) q_j^D(h^{t+1}) \middle| h_b^t, \mathcal{G}_t \right]. \tag{2}$$

If the government decides to default, the recovery value for all $j \in \{st, lt\}$ is

$$q_j^D(h^{t+1}) = \frac{1}{1+r} \left[(1-\lambda) q_j^D(h^{t+1}) + \lambda \frac{W_j \frac{1+r}{1+r-\mathbb{I}_{j=lt}}}{b_{j,t}} \right],$$

where $\mathbb{I}_{j=lt}$ is an indicator function taking value one if $j = lt$ and zero otherwise. If the government restructures its debt, the lender receives $\frac{W_j \frac{1+r}{1+r-\mathbb{I}_{j=lt}}}{b_{j,t}}$ per unit of bond issued. Conversely, if it does not restructure, the government does not disburse anything now, but in present value it pays $q_j^D(h^{t+1})$.

In case of repayment, the price depends on the maturity structure and the *official* buyback decision. For the one-period bond,

$$q_{st}^P(h^{t+1}) = \frac{1}{1+r},$$

while for the long-term bond,

$$q_{lt}^{P}(h^{t+1}) = \frac{1}{1+r}\Big[1 + (1 - M(h^{t+1}))q_{lt}(h^{t+1}) + M(h^{t+1})q_{lt}^{bb}\Big],$$

where $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ with $\chi \in (0,1)$ being the *official* buyback premium. Having properly determined the price, I can define the a sustainable equilibrium in the market economy.

Definition 1 (Sustainable Equilibrium). *Given $\{b_{j,0}\}_{j\in\{st,lt\}}$ and k_0, a sustainable equilibrium in this environment consists of*

- *Strategy for the government, σ_b.*
- *Policy for the firm's capital, k.*
- *Price schedule for capital, p, and for bonds, q_{st} and q_{lt}.*

such that

1. *Taking p, q_{st} and q_{lt} as given, σ_b is the solution to (1).*
2. *Taking p as given, the choice of capital by domestic firms is such that*

$$\mathbb{E}\Big[u_c(c(h_b^t, g_t))(gf_k(k(h^t)) - p(h^t))\Big|h_k^t\Big] = 0. \tag{3}$$

3. *Taking σ_b as given, the price of capital is consistent with*

$$\max_{k_t}\ \mathbb{E}\Big[p(1-\tau(h_b^t, g_t))k_t - k_t\Big|h^t\Big]. \tag{4}$$

4. *The prices of bond satisfy (2).*

Following the approach of Abreu (1988) and Chari and Kehoe (1990), I characterize the set of outcomes that can be sustained in equilibrium using reversion to the worst equilibrium. The following lemma shows that permanent autarky is the worst equilibrium outcome.

Lemma 1 (Worst Equilibrium Outcome). *In this environment, the worst possible outcome is permanent autarky which can be supported as an equilibrium.*

Proof. See Appendix G ☐

Keeping track of the entire history of play, I can sustain many different equilibrium outcome relying on trigger strategies. I now focus on two specific types of equilibria: Markov equilibria which do not build on past history and the constrained efficient equilibrium which generally relies on past history and is the best achievable outcome in this environment.

4 Markov Debt Management

In this section, I derive the Markov debt management policies. I consider the optimal debt management with and without default incentives. I finally provide foundation for the use of Markov strategies by the market participants.

4.1 Markov equilibrium with default incentives

Markov equilibria rely on memoryless strategies conditioned on the state which only encodes payoff-relevant information (Maskin and Tirole, 2001). In this environment, the payoff-relevant state is $\Omega \equiv (g, b_{st}, b_{lt})$. All Markov equilibria are sustainable equilibria as they restrict the information set to $\Omega_t \subset h^t$ for any $t > 0$.[15] However, the opposite is not true.

The Markov equilibrium with default incentives is a version of Arellano and Ramanarayanan (2012) with endogenous default cost. The government's overall beginning of the period value is given by

$$V(\Omega) = \max_{D \in \{0,1\}} \Big\{ (1-D)V^P(\Omega) + DV^D(g) \Big\}, \tag{5}$$

where V^P and V^D correspond to the value of repayment and default, respectively. Under repayment, the government chooses whether to conduct *official* buybacks. Thus

$$V^P(\Omega) = \max_{M \in \{0,1\}} \Big\{ (1-M)V^{NB}(\Omega) + MV^B(\Omega) \Big\}, \tag{6}$$

where V^B and V^{NB} are the values under *official* buyback and no *official* buyback, respectively. If the government decides to officially repurchase its long-term debt,

$$\begin{aligned} V^B(\Omega) = \max_{\tau, b'_{st}, b'_{lt}} \; & u(c) + \beta \mathbb{E}_{g'|g}\Big[V(\Omega')\Big] \\ \text{s.t.} \quad & c + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})b'_{lt} = y(g, p, k, \tau) + b_{st} + b_{lt}(1 + q^{bb}_{lt}), \\ & b'_{lt} \geq b_{lt}. \end{aligned}$$

In Appendix C, I endogenize the *official* buyback premium χ through a generalized Nash bargaining. Conversely, under no *official* buyback,

$$\begin{aligned} V^{NB}(\Omega) = \max_{\tau, b'_{st}, b'_{lt}} \; & u(c) + \beta \mathbb{E}_{g'|g}\Big[V(\Omega')\Big] \\ \text{s.t.} \quad & c + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g, p, k, \tau) + b_{st} + b_{lt}. \end{aligned}$$

[15] See Definition G.4 in Appendix G for a formal definition of a Markov equilibrium.

Under default, the government is excluded from the capital and bond markets. It subsequently needs to restructure its debt. The value under default is given by

$$V^D(g) = \max_{\tau}\ u(y(g,p,k,\tau)) + \beta \mathbb{E}_{g'|g}\Big[(1-\lambda)V^D(g') + \lambda V(g', W_{st}, W_{lt}\frac{1+r}{r})\Big]. \qquad (7)$$

To avoid redundancy with the previous section, the pricing equations and the equilibrium definition are presented in Appendix B.

The government is tempted to tax capital only in the case of default. It therefore sets $\tau = 1$ if $D(\Omega) = 1$ as it loses access to the capital market in the next period. However, it sets $\tau = 0$ if $D(\Omega) = 0$ as any tax on capital would directly reduce the small open economy's output. It does not internalize the fact that more k raises the value of autarky. The lender therefore provides k such that $gf_k(k) = 1$ if $D(\Omega) = 0$ and $k = 0$ otherwise.

In terms of debt management, this equilibrium concept is the closest to what is observed in most emerging economies. On the one hand, defaults arise on equilibrium path and especially when productivity is low (Arellano, 2008). On the other hand, maturity shortens during debt crises. The repayment of long-term debt is laddered through multiple periods which implies a greater default risk than the short-term debt. As a result, close to default, the long-term debt price drastically drops which encourages shorter maturity (Arellano and Ramanarayanan, 2012).[16]

However, unlike defaults, *official* buybacks do not arise on equilibrium path.[17] An *official* buyback is a reverse default as it corresponds to an overpayment, while a default is an underpayment of liabilities. Nonetheless, there is no instantaneous reward after such overpayment, while underpayment are sanctioned by markets exclusion. As a result, the government is unwilling to conduct *official* buybacks. As one will see next, the lender will need to enforce such buybacks.

Proposition 1 (No Official Buyback)**.** *Under Assumption 1, in any Ω, the borrower is unwilling to conduct official buybacks.*

Proof. See Appendix G □

In Section 5, I use the value under default (7) as the optimal contract's outside option. Thus, the Markov equilibrium with default incentives represents the reference point for comparing the different equilibria analyzed in this paper.

[16] As shown by Niepelt (2014), this result is also a consequence of the fact that a default implicates both the long-term and the short-term debt.

[17] Note that *unofficial* buybacks can arise on equilibrium path.

4.2 Markov equilibrium without default incentives

I now consider the Markov equilibrium without default incentives. The aim is to introduce an endogenous borrowing constraint that ensures no default on equilibrium path. I consider two types of constraints: a non-contingent and a contingent one. The former is defined as

$$b'_{st} + b'_{lt} \geq \min_{g' \in G} \Big\{ (b'_{st} + b'_{lt}) : V^P(\Omega') = V^D(g') \Big\}. \tag{8}$$

This limit ensures that the government never accumulates a level of debt for which a default is optimal. This is what Zhang (1997) defines as a no-default borrowing constraint.

The above constraint is relatively unsophisticated, though. Especially, it does not allow for much risk sharing. With the market structure at hand, it is possible to create some state contingency through *official* buybacks. However, since agents rely on Markov strategies, I cannot condition those buybacks on the history of play. Instead, I consider a simple rule stating that such debt repurchase occur only when $g = g_H$. Formally,

$$M(\Omega) = \begin{cases} 1 & \text{if} \quad g = g_H \\ 0 & \text{else} \end{cases} \tag{9}$$

Given this, the long-term debt becomes a pseudo Arrow security as it pays out more in the high productivity state than in the low productivity state. With this, I can define an endogenous borrowing limit in the form of

$$\begin{aligned} b'_{st} + b'_{lt}[1 + q_{lt}(g_L, b''_{st}, b''_{lt})] &\geq \mathcal{B}(g_L), \\ b'_{st} + b'_{lt}[1 + q^{bb}_{lt}] &\geq \mathcal{B}(g_H), \end{aligned} \tag{10}$$

where the borrowing limit is defined such that

$$\begin{aligned} V^P(g_L, b'_{st}, b'_{lt}) &= V^D(g_L) \quad \text{for all} \quad b'_{st} + b'_{lt}[1 + q_{lt}(g_L, b''_{st}, b''_{lt})] = \mathcal{B}(gL). \\ V^P(g_H, b'_{st}, b'_{lt}) &= V^D(g_H) \quad \text{for all} \quad b'_{st} + b'_{lt}[1 + q^{bb}_{lt}] = \mathcal{B}(g_H). \end{aligned}$$

The definition of the endogenous borrowing limit is the one of Alvarez and Jermann (2000). It plays the same role as (8) with the difference that it allows for some risk sharing between the two productivity states.[18]

The *official* buyback policy (9) imposes that the government repurchases its long-term

[18]With the above *official* buyback policy and the fact that $q^{bb}_{lt} > \frac{1}{r}$, in equilibrium, $q_{lt}(g_H, b_{st}, b_{lt}) = q_{lt}(g_H) > q_{lt}(g_L) = q_{lt}(g_L, b_{st}, b_{lt})$ for all (b_{st}, b_{lt}).

debt every time g_H realizes. This can be unnecessarily costly. Instead, one can additionally condition *official* buybacks on a specific portfolio of debt. Formally

$$M(\Omega) = \begin{cases} 1 & \text{if} \quad g = g_H,\ b_{st} = \overline{B}_{st} \text{ and } b_{lt} = \overline{B}_{lt} \\ 0 & \text{else} \end{cases} \tag{11}$$

where $\overline{B}_{st}$ and $\overline{B}_{lt}$ are some fixed level of short-term and long-term debt, respectively. Nonetheless, it is not clear *ex ante* whether such debt portfolio is attained with positive probability on equilibrium path.

Provided that the lender has commitment, it will be the agent capable of implementing and enforcing the above borrowing constraints. In the case of non-contingent borrowing limit, the lender simply provides bonds as long as (8) holds. The same holds true for the state-contingent borrowing limit with the additional requirement that the government implements the appropriate buyback policy. Nevertheless, following Proposition 1, the lender needs to enforce *official* buybacks.

In a Markov equilibrium, enforcement should be contingent on Ω only. To show how this can be achieved, define $B_{st}(\Omega) = b'_{st}$ and $B_{lt}(\Omega) = b'_{lt}$ as the short-term and long-term bond policy, respectively. The lender can enforce *official* buybacks through Markov strategies if there are no short-term assets and *official* buybacks are not too costly.

Lemma 2 (Official Buyback Enforcement). *Under Assumption 1,*

I. If $B_{st}(\Omega) \geq 0$, an official buyback is not enforceable.

II. If $B_{st}(\Omega) < 0$, an official buyback is enforceable when either $-b_{st}$ is sufficiently large or $-b_{lt}$ and χ are not too large.

Proof. See Appendix G □

The rationale behind this result is that the lender is the second mover. Hence, it can threaten the government not to roll over debt if the above buyback policy is violated. Obviously, this threat is credible if the borrower does not possess any assets and *official* buybacks are not too costly relative to the no-roll-over punishment. This result can be interpreted as the standard no-saving argument of Bulow and Rogoff (1989) applied to *official* buybacks. I further strengthen this result in Appendix C when I endogenize χ with a generalized Nash bargaining.

Lemma 2 imposes strong requirements for *official* buybacks to be enforceable with Markov strategies. As one will see in Section 5, the implementation of the constrained efficient allocation might violate such conditions, justifying the use of history-dependent strategies.

Besides this, I can show that the Pareto superiority of the Markov equilibrium without default incentives under the assumption that the borrower decreases its long-term debt in g_H.[19] The rationale behind this result is the following. First, the government cannot do worse than the Markov equilibrium with default incentives given that (10) holds. Second, as the buyback price is above the risk-free price and there is no default on equilibrium path, the lender's value is strictly greater here than in the previous Markov equilibrium.

Proposition 2 (Superior Markov Equilibrium)**.** *Under Assumption 1 and $B_{lt}(g_H, b_{st}, b_{lt}) \geq b_{lt}(1-\chi)$ for all (b_{st}, b_{lt}), if official buybacks are enforceable, the Markov equilibrium without default incentives satisfying (9) is Pareto superior to the one with default incentives. Plus, if official buybacks can only be conditioned on g, it is the best achievable Markov equilibrium.*

Proof. See Appendix G □

Hence, buybacks provide a better source of risk sharing than defaults. The analysis in Section 5 later confirms this point and explains why this is the case.

4.3 Foundation of Markov equilibria

In light of section 3, the Markov equilibrium is a relatively unsophisticated equilibrium concept as it does not build on past history. However, I show that, under asymptotically fading government's memory, only Markov equilibria survive small and independent perturbations in payoffs.[20]

Under $\beta(1+r) < 1$, the government is relatively more impatient than the lender. This assumption is standard in the literature on sovereign debt. Not only it is necessary to obtain empirically plausible debt ratios, it is also a property inherited from the general equilibrium analysis and the martingale convergence theorem. In addition, impatience has implications in terms of political economy. It is reduced form for the fact that governments are subject to re-elections and might lose office with positive probability (Alesina and Tabellini, 1990).[21] In this situation, the government can be interpreted as a short-term player whose recollection of past actions eventually fades. In other words, the government's memory goes back to a certain number of periods $\mathcal{T} \sim \text{Geo}(\psi)$ with $\psi \in [0,1]$ (Angeletos and Lian, 2021).[22]

Besides the fading government's memory, I introduce a small perturbation in the existing environment. Following Bhaskar et al. (2012), in each period t, a utility shock $\epsilon\varrho_{b,t}$ and $\epsilon\varrho_{l,t}$

[19]The standard behavior in equilibrium for this class of models is to reduce indebtedness when the productivity state is relatively high. See Niepelt (2014).

[20]I consider here that the lender is a long-run player. However this is without loss of generality. The result holds as long as at most one market participant has unbounded memory.

[21]See also Levy and Razin (2021) who explain political polarization through short-term memory.

[22]If $X \sim \text{Geo}(\psi)$ then the probability mass function is $\Pr(X = x) = (1-\psi)^x\psi$.

with $\epsilon \geq 0$ is drawn for the government and the lender, respectively. It has compact support $P_i \subset \mathbb{R}^{|A_i|}$ with absolutely continuous density $\mu_{P_i} > 0$ where $|A_i|$ is the cardinality of the choice set of market participant $i \in \{b, l\}$. Moreover, it is independently distributed across market participants, histories and other shocks. If the market participant $i \in \{b, l\}$ chooses a particular action, say $d \in A_i$, its utility is augmented by $\epsilon \varrho^d_{l,t}$.[23] Finally, the utility shock $\epsilon \varrho_{i,t}$ is privately observed by market participant $i \in \{b, l\}$.

Assumption 2 (Perturbation)**.** *The government's memory goes back to $\mathcal{T} \sim Geo(\psi)$ periods in the past with $\psi \in [0, 1]$. Plus, in each t, a utility shock $\epsilon \varrho_{i,t}$ with $\epsilon \geq 0$ is drawn from the compact support $P_i \subset \mathbb{R}^{|A_i|}$ with absolutely continuous and i.i.d. density $\mu_{P_i} > 0$ for each $i \in \{b, l\}$. The utility shock is additive and privately observed.*

Under Assumption 2, the benchmark case considered in Section 3 corresponds to $\psi = \epsilon = 0$. The case of perturbed memory boils down to what happens when $\psi, \epsilon > 0$ but arbitrarily small. It means that (a) the government eventually forgets the history of play in the very distant past and (b) market participants have imperfect knowledge of the other participant's fundamentals. The presence of the privately observed shock – albeit small and independent – coupled with the fading memory of the government prevent both participants to rely on past history. This causes all non-Markov equilibria to unravel.

Proposition 3 (Foundation of Markov equilibria)**.** *Under Assumption 2, with $\psi, \epsilon > 0$, all non-Markov equilibria unravel.*

Proof. See Appendix G ☐

The rationale behind that result follows Bhaskar (1998) and Bhaskar et al. (2012). Suppose the lender conditions its action at time t on a payoff-irrelevant past event, then the government must also condition on this past event. Nevertheless, as long as $\psi \neq 0$, the borrower eventually forgets everything that happened in an arbitrarily distant point in the past. This means that, asymptotically, the borrower has no memory of past actions and therefore cannot rely on past history. The utility shock then ensures that each optimal choice is a singleton along the path of play.

This result has two main consequences. First, I am left with two extremes: both parties either build on the entire (infinite) history of play or do not at all. Second, the constrained efficient allocation can often only be approximated by means of Markov strategies. This relates to Maskin and Tirole (2001) who show that there always exists a Markov equilibrium in the vicinity of the unperturbed initial equilibrium. In view of this, in Section 7, I quantitatively assess how close Markov equilibria are from the constrained efficient allocation.

[23] The instantaneous utility of the government taking action d (e.g. default) is given by $u(c_t) + \epsilon \varrho^d_{b,t}$.

5 Constrained Efficient Debt Management

This section presents the constrained efficient debt management policies. I first derive the optimal contract. I subsequently characterize the underlying constrained efficient allocation before implementing it the market economy.

5.1 The optimal contract

In what follows I derive the optimal contract which has to account for limited enforcement in repayment. The participation constraint deals with the fact that the borrower can always break the contract and opt for autarky (Thomas and Worrall, 1994). Denoting g^t as the history of realized value of g at time t, it must hold that for all t and g^t

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(c(g^j)) \geq V^D(g_t, k_t). \tag{12}$$

If the borrower breaks the contract, the government is sent to autarky for some time but can regain access to the market with probability λ and resume the Markov equilibrium with default incentives. $V^D(g_t, k_t)$ therefore corresponds to the value of default in the Markov equilibrium given by equation (7). Note that I write $V^D(g_t, k_t)$ instead of $V^D(g_t)$ to make explicit the dependence on k_t. As a result, the participation constraint ensures that the borrower's value of remaining in the contract is at least as large as the value of opting out.

Given the above constraint, the optimal contract between the borrower and the lender in sequential form is the result of

$$\begin{aligned}
\max_{\{k(g^t), c(g^t)\}_{t=0}^{\infty}} \; & \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) u(c(g^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) T(g^t) \\
\text{s.t.} \quad & (12),\ T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t),\ \forall g^t, t \\
& \text{with } \mu_{b,0} \text{ and } \mu_{l,0} \text{ given.}
\end{aligned}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial non-negative Pareto weights assigned by the Planner to the borrower and the lender, respectively.

The Planner allocates capital and consumption to maximize the lender's and the borrower's weighted utility subject to the resource constraint and the participation constraint. The above maximization problem combines the utility function $u(\cdot)$ with the production function $f(\cdot)$ and therefore might not be convex.

Assumption 3 (Convexity). *Define the optimal level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ and $h := gf(k) - k$ for $k \in [0, k^*(g)]$ with $h^*(g) = gf(k^*(g)) - k^*(g)$. Let $K(h)$ denote the inverse mapping from $[0, h^*(g)]$ to $[0, k^*(g)]$ such that $k = K(h)$. For all $g \in G$, $u(gf(k(h)))$ is convex in h for $h \in [0, h^*(g)]$.*

Following, Aguiar et al. (2009), Assumption 3 ensures that there is no need for randomization whenver the curvature of $u(\cdot)$ and $f(\cdot)$ is not too pronounced.

I now derive the recursive formulation of the above maximization problem. Following Marcet and Marimon (2019), I rewrite the sequential problem as a saddle-point Lagrangian problem,

$$
\begin{aligned}
\mathcal{SP} \quad \min_{\{\gamma(g^t)\}_{t=0}^{\infty}} \max_{\{k(g^t),c(g^t)\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0)\mu_{b,t}(g^t)u(c(g^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0)\mu_{l,t}(g^t)T(g^t) \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0)\gamma(g^t)\Big[u(c(g^t)) - V^D(g_t, k_t)\Big] \\
\text{s.t.} \quad & T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t), \\
& \mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t) \text{ and } \mu_{l,t+1}(g^t) = \mu_{l,t}(g^t), \ \forall g^t, t \\
& \text{with } \mu_{b,0}(g_0) \equiv \mu_{b,0} \text{ and } \mu_{l,0}(g_0) \equiv \mu_{l,0} \text{ given.}
\end{aligned}
$$

In this formulation, $\beta^t\pi(g^t|g_0)\gamma(g^t)$ is the Lagrange multiplier attached to the participation constraint of the borrower at time t. The above formulation of the problem defines two new co-state variables, $\mu_{b,t}(g^t)$ and $\mu_{l,t}(g^t)$, which are the temporary non-negative Pareto weights the Planner attributes to the borrower and the lender, respectively. These variables are initialized at the original Pareto weights and subsequently become recursive.

Following Ábrahám et al. (2019), I further simplify the above problem. I define the relative Pareto weight of the borrower as

$$x_t(g^t) = \frac{\mu_{b,t}(g^t)}{\mu_{l,t}(g^t)},$$

Given the non-negativity and boundedness of the Lagrange multipliers, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} < \infty$.[24] Defining $\eta \equiv \beta(1+r) \leq 1$ and $\nu(g^t) \equiv \frac{\gamma(g^t)}{\mu_{b,t}(g^t)}$, the law of motion of the relative Pareto weight is given by

$$x_{t+1}(g^t) = (1 + \nu(g^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}} \tag{13}$$

[24] I later show that setting $\underline{x} \geq 0$ this is without loss of generality as the continuation of an efficient allocation is itself efficient.

With this normalization, $\nu(g^t)$ represents the multiplier attached to the participation constraint. Following Marcet and Marimon (2019), the state vector for the problem reduces to (g, x) and the Saddle-Point Functional Equation is given by

$$\begin{aligned} FV(g,x) = \mathcal{SP} \min_{\nu(g)} \max_{k(g),c(g)} \; & x\Big[(1+\nu(g))u(c(g)) - \nu(g)V^D(g,k)\Big] \\ & + T(g) + \frac{1}{1+r}\sum_{g'} \pi(g'|g) FV(g',x') \\ \text{s.t.} \quad & T(g) = gf(k(g)) - c(g) - k, \\ & x'(g) = (1+\nu(g))\eta x \; \forall g. \end{aligned} \tag{14}$$

The value function takes the form of $FV(g,x) = xV^b(g,x) + V^l(g,x)$ with $V^b(g,x) = u\left(c(g)\right) + \beta \mathbb{E}_{g'|g}\left[V^b\left(g',x'\right)\right]$ and $V^l(g,x) = T(g) + \frac{1}{1+r}\mathbb{E}_{g'|g}\left[V^l\left(g',x'\right)\right]$. I obtain the optimal consumption and capital policies by taking the first-order conditions in (14)

$$u_c(c(g)) = \frac{1}{x(1+\nu(g))} \quad \text{and} \quad gf_k(k(g)) - 1 = \nu(g)u_c(gf(k(g)))gf_k(k(g))x.$$

In terms of consumption, the binding participation constraint of the borrower (i.e. $\nu > 0$) induces an increase in consumption.

Regarding capital, the economy does not reach the production-maximiging level of capital $k^*(g)$ as long as the participation constraint binds in g. Furthermore, the more this constraint binds, the more distorted is capital.

5.2 Equilibrium properties

I characterize the main properties of the contract. I first determine the Pareto frontier in this environment. I subsequently present how the contract provides risk sharing across states. Finally, I show how the contract's allocation can be implemented as a sustainable equilibrium in the market economy. Additional characterization can be found in Appendix D.

Proposition 4 (Efficiency)**.** *Under Assumptions 1 and 3, for all $x \in X$, the utility possibility frontier is strictly increasing and the autarkic allocation is not optimal.*

Proof. See Appendix G ☐

The proposition is made of two parts. First, autarky (i.e. $k = 0$) is not optimal. Due to the Inada conditions on the production function, there are strictly positive gains from trade between the borrower and the lender. This means that defaults – which imply markets exclusion – cannot implement the Planner's constrained efficient allocation.

Second, the proposition states that the optimal contract is constrained efficient which makes it the best achievable outcome in this environment. Hence, the debt management policies that I later derive from the optimal contract are the (constrained) efficient ones.

The following proposition defines the main properties of the constrained efficient allocation. The contract features production distortions and risk sharing.

Proposition 5 (Constrained Efficient Allocation)**.** *Under Assumptions 1 and 3,*

I. (Production). *There exists a level of relative Pareto weight $x^*(g)$ such that $k(g,x) = k^*(g)$ for $x \geq x^*(g)$ and $x^*(g_H) > x^*(g_L)$. Conversely, for all $x, \tilde{x} \in X$ with $x^*(g) > x > \tilde{x}$, $0 < k(g,\tilde{x}) < k(g,x) < k^*(g)$.*

II. (Risk-Sharing). *$c(g_L,x) \leq c(g_H,x)$ and $x'(g_L,x) \leq x'(g_H,x)$ for all x with equality when $x \geq x^*(g_H)$.*

III. (Liabilities). *$V^l(g_L,x) < V^l(g_H,x)$ for all x.*

Proof. See Appendix G □

Part I of the above proposition states that the production-maximizing level of of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ is attained only if the relative Pareto weight, x, is above a certain threshold. Capital distortion is a consequence of binding participation constraint (12). As the autarky value depends on the level of capital in the economy, the Planner finds optimal to reduce k to relax the constraint. It continues to decrease k as long as x decreases but never finds optimal to set $k = 0$. As already mentioned, this means that defaults cannot implement the Planner's allocation.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind. Otherwise, the Planner provides more consumption and a greater continuation value when the high productivity state realizes.

Part III relates to the liabilities of the borrower. In this environment, $T(g)$ corresponds to the borrower's current account balance. Hence, the net present value of the lender corresponds to the net foreign asset position in the contract.[25] A positive value $V^l(g,x)$ therefore indicates that the borrower owes money to the lender and in that logic the greater is this value the greater is the liabilities towards the lender. The proposition states that the liabilities increase when g is high. This implies that the Planner adopts a state-contingent policy

[25] In the balance of payments statistics, this object is the net international investment position.

as it provides debt relief in low productivity states. This state contingency will be replicated through *official* buybacks in the market economy.

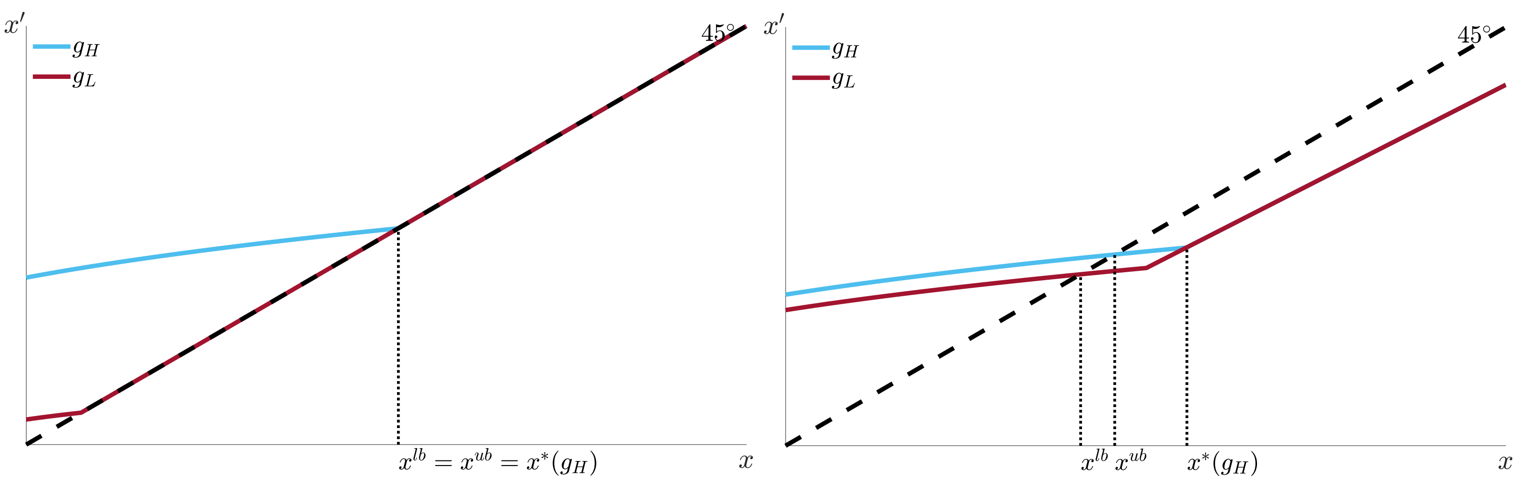
Having determined the constrained efficient allocation, I now characterize the steady state of the optimal contract. The long-term contract is characterized by an ergodic set of relative Pareto weights.

Proposition 6 (Steady State)**.** *A steady state is defined by an ergodic set of relative Pareto weights* $x \in [x^{lb}, x^{ub}] \subset X$*. Under Assumptions 1 and 3, it holds that* $x'(g_H, x^{ub}) = x^{ub}$ *and* $x'(g_L, x^{lb}) = x^{lb}$ *and*

I. If $\eta = 1$*, then* $x^{lb} = x^{ub} = x^*(g_H)$*.*

II. If $\eta < 1$*, then* $x^{lb} < x^{ub} < x^*(g_H)$*.*

Proof. See Appendix G □



(a) $\eta = 1$ (b) $\eta < 1$

Figure 1: Steady State Dynamic

The proposition states that whenever the borrower is sufficiently patient (i.e. $\eta = 1$), the steady state does not display any dynamic. Conversely, whenever the borrower is relatively impatient (i.e. $\eta < 1$), the steady state is dynamic. Note however that when the contract hits one of the bounds, history becomes irrelevant. For instance, after a sufficiently long series of g_L (g_H), the contract hits x^{lb} (x^{ub}). It then stays there until g_H (g_L) realizes and that irrespective of the past history. The bounds of the ergodic set represent therefore regions of amnesia in the contract. Such regions will have specific interpretations in the implementation. Figure 1 illustrates each of the two steady states.

5.3 Equilibrium implementation

Having derived and characterized the constrained efficient allocation, I am now interested in implementing the optimal contract as a sustainable equilibrium in the market economy presented in Section 3. Obviously, to replicate the constrained efficient allocation, the following conditions have to be met.

Lemma 3 (Conditions for Implementation)**.** *Given* $\{b_{j,0}\}_{j\in st,lt}$ *and* k_0*, an equilibrium allocation* $\{D(g^t), M(g^t), c(g^t), k(g^t), \tau(g^t),\ b_{st}(g^t), b_{lt}(g^t)\}$ *with prices* $\{q_{st}(g^t), q_{lt}(g^t), p(g^t)\}$ *implements a constrained efficient outcome if and only if it satisfies the resource constraint, the participation constraint, (12), together with the budget constraint, the bond pricing equation, (2), the capital policy, (3), and the capital pricing equation, (4).*

Proof. See Appendix G □

In light of this lemma, I construct a sustainable equilibrium in the market economy that implements the constrained efficient allocation. I also give the conditions for the implementation to be Markov as exposed in Section 4.

Proposition 7 (Implementation)**.** *Under Assumptions 1 and 3, given a constrained efficient allocation, a sustainable equilibrium exists that implements it. Moreover, if Part II of Lemma 2 applies, the sustainable equilibrium in question is the Markov equilibrium without default incentives.*

Proof. See Appendix G □

The implementation works as follows. The government conducts *official* buybacks when the economy hits the upper bound of the ergodic set. As this bound is reached after a sufficiently long series of high productivity shocks, this buyback policy generates a specific term structure in which high productivity shocks are related to relatively larger long-term bond prices than low productivity shocks, while the short-term bond price remains unchanged. Given this, I can equalize the value of debt in the contract, $V^l(g,x)$, with the value of the debt in the market economy, $b_{st} + b_{lt}[1 + q_{lt}]$, for each (g, x). As I have two productivity states and two bonds, this gives a system of two equations with two unknowns which has a unique solution for each x given the specified term structure.

The implementation following Proposition 7 has many features in terms of past history, maturity, default, debt buyback, borrowing and spread:

– *History dependence.*

Are *official* buybacks enforceable through Markov strategies? From (G.1) and (G.2) in Appendix G, the short-term and long-term holdings at the *official* buyback are respectively

$$\bar{b}_{st}(x) = \frac{V^l(g_H, x)[1 + \bar{q}_{lt}(x'(g_L, x))] - V^l(g_L, x)[1 + q_{lt}^{bb}]}{q_{lt}^{bb} - \bar{q}_{lt}(x'(g_L, x))},$$
$$\bar{b}_{lt}(x) = -\frac{V^l(g_H, x) - V^l(g_L, x)}{q_{lt}^{bb} - \bar{q}_{lt}(x'(g_L, x))}.$$

From Part III of Proposition 5, it holds that $V^l(g_H, x) > V^l(g_L, x)$ meaning that $b_{lt} < 0$. However, it is not guaranteed that $b_{st} < 0$. Particularly, b_{st} can be negative only if q_{lt}^{bb} is very large with respect to q_{lt}. I later show quantitatively that this requires χ to be closer to one than to zero.[26] Moreover, recall that the *official* buyback takes place in the point of amnesia meaning that $b'_{st} = b_{st}$ and $b'_{lt} = b_{lt}$.

Thus, Part II of Lemma 2 does not generally apply. In other words, *official* buybacks might not automatically be enforced by Markov strategies. Trigger strategies are usually necessary, which puts the implementation at the mercy of Proposition 3.

– *No default.*

The implementation does not rely on defaults. As previously shown, the Planner never finds optimal to distort capital to zero. This means that there is no proper markets exclusion, even when the contract hits the borrower's participation constraint. It is therefore not possible to interpret the borrower's binding constraint as a default *stricto sensu* in my environment. Other studies however do. For instance, Müller et al. (2019) and Restrepo-Echavarria (2019) view the borrower's binding constraint as a form of preemptive restructuring which does not trigger market exclusion. Nonetheless, Asonuma and Trebesch (2016) show that even preemptive restructurings are followed by some periods of market exclusion.

– *Costly buybacks.*

I consider that the government conducts *official* buybacks when it hits the upper bound of the ergodic set – i.e. $x = x^{ub}$. Moreover, buybacks are costly. In light of this, could *official* buybacks occur in the lower bound of the ergodic set – i.e. $x = x^{lb}$ – at a discount? The answer is negative. To reach the lower point of amnesia, the relative Pareto weight needs to decrease which means the government's indebtedness increases.

[26] Based on the calibration in section 7, I would need χ to be at least 8 times larger than the calibrated one to get $b_{st} < 0$ at the point of buyback.

This goes against the idea of a debt repurchase which aims at reducing indebtedness. In addition, buybacks at discount rates are a form of default. As discussed previously, the point in which $x = x^{lb}$ cannot be interpreted as such.

Unlike defaults, buybacks are an efficient source of risk sharing. Defaults entail cost for both the lender and the borrower, while *official* buybacks are solely costly for the latter. A default is therefore not renegotiation proof as both contracting parties would be better off avoiding this event *ex post*.

My argument goes against Bulow and Rogoff (1988, 1991) in the sense that costly buybacks are necessary to implement the constrained efficient allocation. Nevertheless, in my analysis, it is true that *official* buybacks are completely ineffective in reducing the government's indebtedness. In fact, when $x = x^{ub}$, $b'_{lt} = b_{lt}$ which means that indebtedness remains unchanged.

– *Endogenous borrowing limits.*

The bounds of the ergodic set implicitly define endogenous borrowing and lending limits. On the one hand, when the borrower hits x^{lb}, borrowing remains the same every period as long as g_L realizes. Similarly, when the borrower hits x^{ub}, no additional lending takes place as long as g_H realizes. The borrowing limit attached to x^{lb} ensures that the borrower has no incentive to default. On the other hand, the lending limit attached to x^{ub} naturally arises due to discounting.

– *Negative spread.*

Given the buyback rate and the absence of defaults, the long-term bond spread is negative. This is a feature that one finds in other implementations such as the one of Alvarez and Jermann (2000). The mechanism at work is different, though. In my case, the negative spread enables to mimic the state-contingency in the contract provided that defaults do not arise on equilibrium path (i.e. no positive spread). In the case of Alvarez and Jermann (2000), the negative spread restricts the trade of state-contingent securities in a two-sided limited enforcement problem when the participation constraint of the lender binds (Krueger et al., 2008).

6 Alternatives to Official Buybacks

In this section, I explore alternatives to *official* buybacks. Empirically, such alternatives do not exist or remain underdeveloped. That is why I do not consider them in the main analysis.

First, Grossman and Van Huyck (1988) develop the concept of “excusable” defaults. The idea is that defaults which are on the path of play agreed by all market participants are not punished. In other words, the debt contract specifies *ex ante* the circumstances in which the borrower is allowed to repudiate its debt without suffering from markets exclusion. Given this, if defaults were “excusable”, then the borrower’s binding constraint – i.e. $x = x^{lb}$ – could be interpreted as a default. The issue is that the borrower might be willing to repudiate debt more often than what the debt contract specifies. To deal with this, one can either use trigger strategies or introduce an endogenous borrowing limit similar to (10). Nevertheless, the concept of “excusable” defaults has little empirical relevance.

Second, the long-term debt can have variable coupon as in Faraglia et al. (2019). In the main analysis, I normalized the coupon payment to one following Hatchondo and Martinez (2009). However, assume that the coupon payment is a choice variable, say $\kappa \in [0, 1]$, for the borrower. Obviously, the variability of the coupon is a covenant in the debt contract. In other words, changes in coupon are agreed by the contracting parties *ex ante* and do not pertain to a contract renegotiation – e.g. an outright default in case of reduced coupon payment. With such debt contract, it is possible to implement the constrained efficient allocation in two ways: the borrower sets a standard coupon payment $\tilde{\kappa}$ and either increases it to $\bar{\kappa} > \tilde{\kappa}$ when $x = x^{ub}$ or decreases it to $\underline{\kappa} < \tilde{\kappa}$ when $x = x^{lb}$. In the former case, a variant of Proposition 1 applies as the borrower is not willing to pay a larger coupon payment. Hence, the same enforcement issue arises as with *official* buybacks and trigger strategies remain necessary. Similarly, in the case of reduced coupon payment, the borrower might be tempted to reduce the coupon payment too frequently.

Lastly, bonds can have variable maturities. That is, the maturity of outstanding short-term (long-term) debt can be lengthened (shortened). Similar to variable-coupon bonds, this is a feature which should be explicitly mentioned in the debt contract. To implement the constrained efficient allocation, the borrower ought to either lengthen the maturity of short-term debt when $x = x^{ub}$ or shorten the maturity of long-term debt when $x = x^{lb}$. Implicitly, by shortening the maturity, the borrower pays less coupons than it initially promised. In other words, the claim of legacy creditors is reduced. The opposite happens in the case of maturity lengthening. Thus, similar to variable-coupon bonds, maturity lengthening would need to be enforced, while maturity shortening should be closely supervised to avoid lowering legacy creditors’ claim more frequently than the Planner would.

In sum, there exist alternatives to *official* buybacks. However, such alternatives are not common in debt contracts and raise similar enforcement issues as *official* buybacks.

7 Quantitative Analysis

This section starts with the calibration of the Markov equilibrium with default incentives to Argentina. It then assesses the fit of the model to the data and shows that the Markov equilibrium without default incentives shares similarities with Brazil. Subsequently, it compares the two Markov equilibria with the constrained efficient allocation. Particularly, it gauges the goodness of Markov equilibria in approximating the constrained efficient debt management policies. Finally, it compares the implementation of Section 5 with an alternative one.

7.1 Calibration

I calibrate the Markov equilibrium with default incentives as it corresponds to the workhorse model in the literature on sovereign debt and defaults. The calibration aims at matching some specific moments of the Argentine economy over the period 1990-2019. Table 1 summarizes each parameter.

Table 1: Calibration

Parameter	Value	Description	Targeted Moment
A. Based on Literature			
σ	1.00	Risk aversion	
r^f	0.01	Risk-free rate	
W_{st} and W_{lt}	0.00	Recovery value	
B. Direct Measure from the Data			
$\pi(g_H\|g_H)$	0.93	Probability staying high state	Real total factor productivity
$\pi(g_L\|g_L)$	0.68	Probability staying low state	
g_H	1.02	Productivity in high state	
g_L	0.91	Productivity in low state	
$1-\alpha$	0.70	Labor share	Labor income share
r^e	0.02	Excess return	US excess return on debt
C. Based on Model solution			
β	0.70	Discount factor	Debt-to-GDP ratio
ϕ	1.80	CES production	Investment-to-GDP ratio
λ	0.19	Probability re-accessing market	Average spread

As exposed in Section 2, the utility function takes the CRRA form with a coefficient of relative risk aversion of σ. I adopt $\sigma = 2$ as it is standard in the real business cycle literature. In addition, in accordance with Assumption 1, the production function is CES

$$F(k,l) = \left[\alpha k^{\frac{\phi-1}{\phi}} + (1-\alpha) l^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where α represents the capital share and ϕ the CES parameter. The value of $1-\alpha$ is set to standard labor share in GDP adopted in the literature on emerging economies (Mendoza and Yue, 2012). The CES parameter is $\phi = 1.8$ to match the share of investment in GDP and is within the range of admissible values in the business cycle literature. In addition, I estimate the Markov chain by means of a Markov-switching AR(1) process with two states. For this, I use data on the real total factor productivity of Argentina from 1990 to 2019 from the Penn World Table 10.0 (Feenstra et al., 2015). Finally, I set the discount factor to $\beta = 0.7$ to match the average external debt-to-GDP ratio of 48.9%.

Regarding the exogenous rate r, I distort the assumption of competitive lending and set $r = r^f + r^e$ where r^f represents the risk-free rate and r^e corresponds to the lender's excess return. This means that the lender borrows at r^f and lend at r. This has two proposes. First, it better captures the potential risk premium US investors demand on emerging market bonds. I therefore set $r^e = 0.0434$ consistent with the US excess return on debt instruments estimated by Gourinchas et al. (2017) and $r^f = 0.01$ as it is standard in the literature. Second, modelling an excess returns enables to correct the negative spread which has little empirical support for the countries under study.

As it is standard in the literature on sovereign defaults, I set $W_{st} = W_{lt} = 0$ meaning that the recovery value of debt is nil. This also avoids large increases in indebtedness and consumption boom prior to default.[27] Besides this, I choose $\lambda = 0.19$ to match the average (Embi) spread of 13.5%. The value selected implies an expected default length of roughly 5 years, similar to what Cruces and Trebesch (2013) find in the data.

Finally, there is one parameter left to calibrate: the buyback premium χ. However, the government in the Markov equilibrium with default does not conduct *official* buybacks. Furthermore, there exist few estimates on buyback costs. I therefore set $\chi = 0.1$ as the maximum value of χ such that Part II of Lemma 2 applies.

7.2 Numerical results

This subsection presents the result of the calibration. It gauges the fit of the model with respect to the data for both targeted and non targeted moments. It also compares the outcome of the Markov allocation with default incentives (MA), without default incentives (MAND) and the constrained efficient allocation (CEA) together.

The upper part of Table 2 present the fit of the MA with respect to the Argentine economy in terms of targeted moments. It also reports the result of the CEA and the MAND. As one can see, the MA replicates relatively well the main features of the Argentine economy in terms of investment, spreads and indebtedness.

The lower part of Table 2 present the fit of the MA in terms of non-targeted business-

[27] See Hatchondo et al. (2016), Dvorkin et al. (2021) and Fourakis (2021).

Table 2: Targeted and Non-Targeted Moments

A. Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
i/y	13.90	13.40	17.90	17.64	13.37
$-b/y$	48.90	46.33	27.70	11.64	-614.40
Spread	13.50	12.17	5.10	4.09	4.14
B. Non-Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
c/y	78.60	86.60	81.30	78.33	84.04
tb/y	1.80	13.40	-0.10	21.67	2.58
$\sigma(c)/\sigma(y)$	1.11	0.93	0.95	0.95	0.38
$\sigma(i)/\sigma(y)$	3.38	0.22	3.16	0.21	1.27
$\sigma(tb/y)/\sigma(y)$	1.17	0.74	1.26	0.04	0.84
$\sigma(spread)/\sigma(y)$	4.29	0.32	2.34	0.00	0.00
$\rho(c, y)$	0.95	0.84	0.89	1.00	0.73
$\rho(i, y)$	0.97	0.84	0.95	1.00	0.93
$\rho(tb/y, y)$	-0.63	0.33	-0.41	1.00	0.92
$\rho(spread, y)$	-0.64	-0.02	-0.09	-1.00	-0.99

Note: For the volatilities and correlation statistics, I filter the simulated data through the HP filter with a smoothness parameter of 1600.

cycle moments. In general, the fit is poor. This is because I only consider 2 productivity states meaning that I rule out tail events. The MA generates too low volatilities for most variables. Moreover, the trade balance is pro-cyclical unlike the data. The model however produces empirically plausible correlations for consumption and investment.

Having said that, the MA generates a realistic debt dynamic. Table 3 depicts the underlying debt structure of the Markov equilibria. Two points deserve to be noted. First, the MA replicates well the data as maturity shortens during debt crises, while indebtedness relative to GDP increases. Second, during a restructuring, the maturity lengthens and the level of debt does not substantially decrease.

Turning to the MAND, Table 2 presents the similarities with Brazil. First and foremost, Brazil has not defaulted since the end of the 1980s, whereas Argentina roughly defaulted 3 times since the 1980s with the most recent episode being in 2019. Second, Brazil conducted an official buyback program from 2006 to 2015. Plus, debt repurchases where the largest when GDP productivity was high.[28] Third, in terms of economic fundamentals, Brazil records a lower average debt ratio (27.7%), a greater average investment ratio (17.9%) and a lower average spread (5.1%) than Argentina for the period 1990 to 2019.

More importantly, the MAND is capable of matching most of the main moments of the

[28]See https://www.gov.br/tesouronacional/en/federal-public-debt/external-market/buyback-program.

Table 3: Debt Structure

	Mean $-b$ (percent of y)	Mean $-b$ in g_H (percent of y)	Mean $-b$ in default (percent of y)	Mean $-b$ in restructuring (percent of y)
Argentina	48.9	38.3	56.8	46.6
MA	46.5	38.1	46.6	35.0
Brazil	27.7	27.4	-	-
MAND	11.6	8.8	-	-
	Mean b_{st}/b (percent)	Mean b_{st}/b in g_H (percent)	Mean b_{st}/b in default (percent)	Mean b_{st}/b in restructuring (percent)
Argentina	18.9	18.2	20.2	14.8
MA	74.9	72.9	80.8	68.7
Brazil	13.9	14.5	-	-
MAND	71.4	71.4	-	-
	$\rho(-b_{st}, y)$	$\rho(-b_{st}, spread)$	$\rho(-b_{lt}, y)$	$\rho(-b_{lt}, spread)$
Argentina	-0.05	0.19	0.18	-0.12
MA	-0.25	0.15	0.78	-0.14
Brazil	-0.05	0.52	0.03	0.59
MAND	0.00	0.00	0.00	0.00

Brazilian economy despite the fact that none of them where directly targeted.[29] This suggests that Brazil can be interpreted as the counterfactual of Argentina with buybacks and without default.

The MAND and the CEA achieve better risk sharing than the MA. In the latter, consumption corresponds to a lower share of output and is less volatile. Investment corresponds to a lower share of output, correlates more with output and is more volatile. Moreover, the borrower holds a net assets position.[30] Finally, the bond spread is lower than in the MA given that defaults do not arise on equilibrium path and *official* buybacks exceed the risk-free price. The same holds true for the MAND with the exception of a lower investment volatility than in the MA and a net debt position as in the MA.

7.3 Policy functions

In this section, I present the different policy functions relevant for my analysis. I start with the CEA and then shift to the MA and MAND. The main focus is to see how debt evolves between the different equilibria.

Figure 2 depicts the main policy functions related to the optimal contract. The law of motion of the relative Pareto weight is consistent with the fact that $\eta < 1$. In that logic, the steady state of the contract is not degenerate and is located around $x = 0.19$ and $x = 0.65$.

[29]Note that the excess return corrects the negative spread in the model.
[30]This empirically unrealistic prediction as already be noted by Buera and Nicolini (2004).

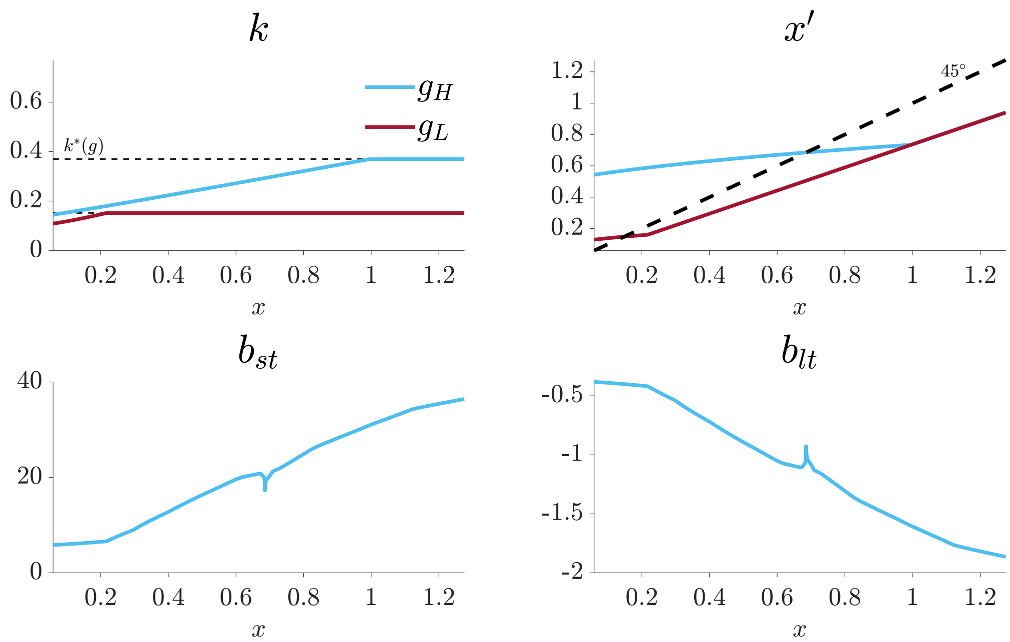
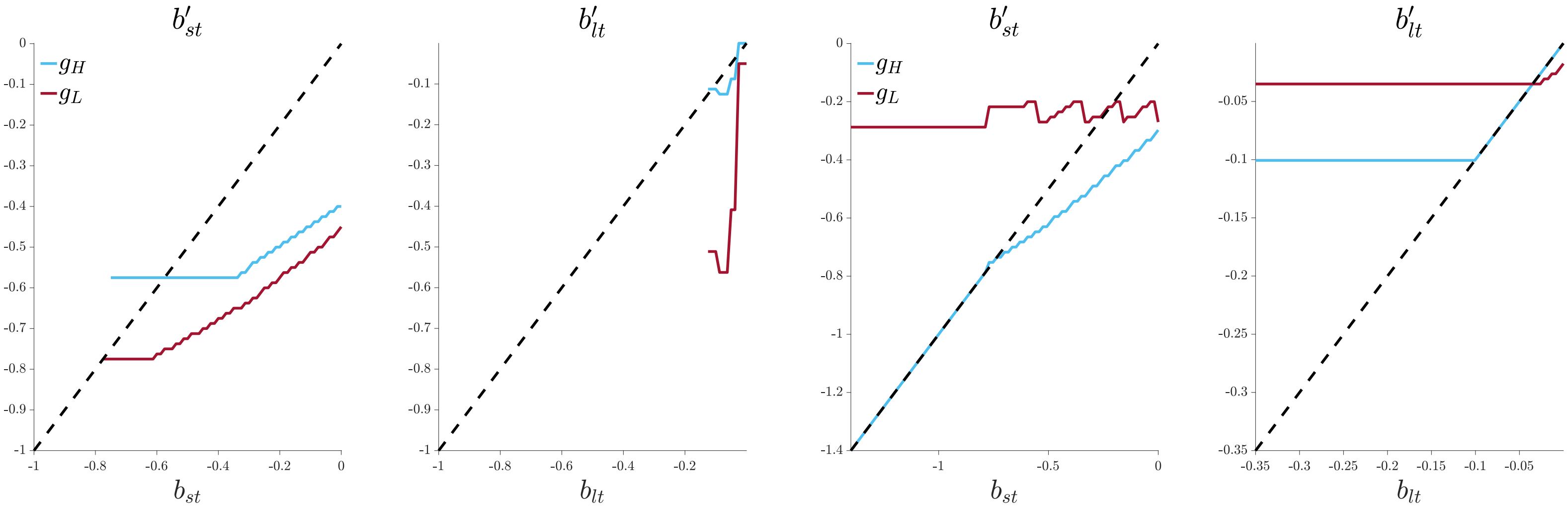


Figure 2: Main Policy Functions of the CEA

Besides this, capital is distorted for low values of x. However, as soon as x is sufficiently high the productivity-maximizing level, $k^*(g)$, is reached. Nevertheless, such values of x are outside the steady state and therefore capital remains distorted in the long-run when $g = g_H$. Regarding borrowing, when x is low, the government accumulates a substantial amount of long-term debt. However, it holds short-term assets – especially when *official* buybacks occur. This violates Part II of Lemma 2 and explains the reliance on trigger strategies. To obtain short-term debt holdings at the buyback, χ should be at least 8 times larger than the calibrated one. As I calibrated χ as the maximum enforceable *official* buyback premium, Markov strategies fail to implement the Planner's allocation under empirically plausible *official* buyback premia.

I can now compare the bond policy of the CEA with the ones related to the Markov equilibria. Figure 3 depicts the bond policy functions for the MA and the MAND. Starting with the MA, the government increases its indebtedness in the low productivity state as in the CEA. However, it increases both its holdings of short-term and long-term debt. One can even say that the former increases more than the latter consistent with the maturity structure of Argentina. In the high productivity state, the debt level is lower compared to the low state, while the share of long-term debt increases.

Turing to the MAND, one notices the borrowing limit which takes the form of an horizontal line. This indicates that the government cannot borrow above a certain level. When productivity is low, the government does not change its holdings of debt in a significant



(a) MA (b) MAND

Figure 3: Bond Policies of the Markov Equilibria

manner. However, when productivity is high, the *official* buyback program forces the government to maintain its holdings of long-term debt unchanged. As a result, the government adapts its short-term debt position. Note that the level of long-term debt is lower in the MAND than in the MA due to the costly buyback program.

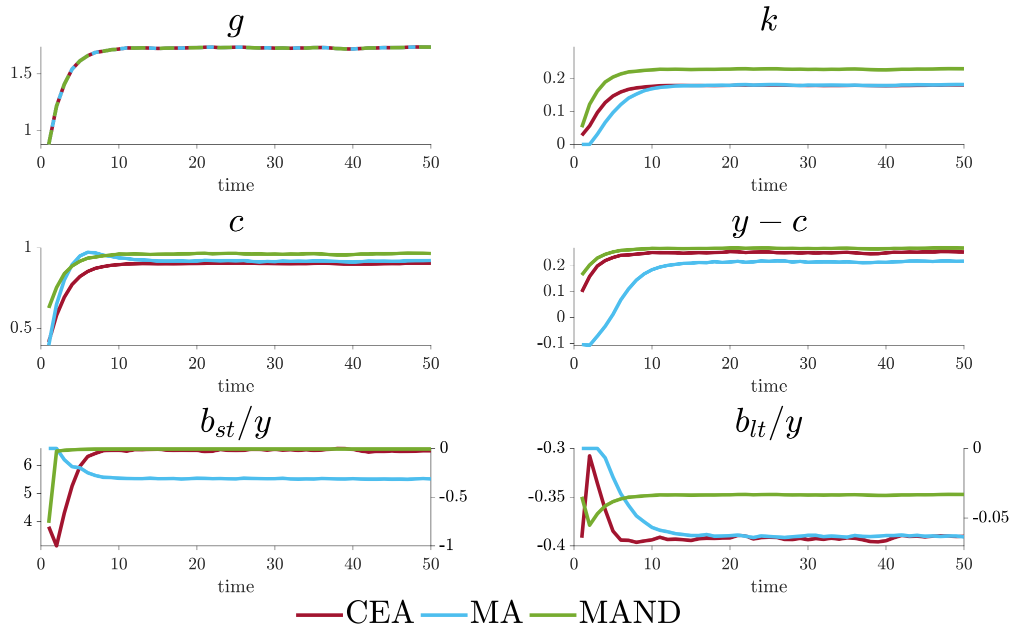
7.4 Equilibria comparison

In this subsection, I explore in more details the differences between the Markov equilibria and the CEA. For this purpose, I conduct three main exercises. First, I construct impulse response functions following a stark negative shock in the economy. Second, I look at the dynamic of a specific shock path in both the Markov equilibria and the CEA. Finally, I compute welfare gains with respect to the MA.

Figure 4 depicts the impulse response functions resulting from a stark negative shock on selected key variables. The responses are computed as the mean of 1,000 independent shock histories starting with the lowest shock as well as initial debt holdings and relative Pareto weight drawn from the ergodic set. The blue line represents the MA, the green line the MAND and the red line the CEA. Notice that the relevant axis for the debt figures are on the right-hand sides for the Markov equilibria and the left-hand side for the CEA.[31]

We see that at the outbreak of the shock's realization, capital is distorted in both the MA and the CEA – albeit to a lesser extent in the latter – but not in the MAND. Consumption drops as economies in the MA fall into default. Capital and debt therefore go to zero. The

[31]This is because the implementation of the CEA relies on short-term assets.



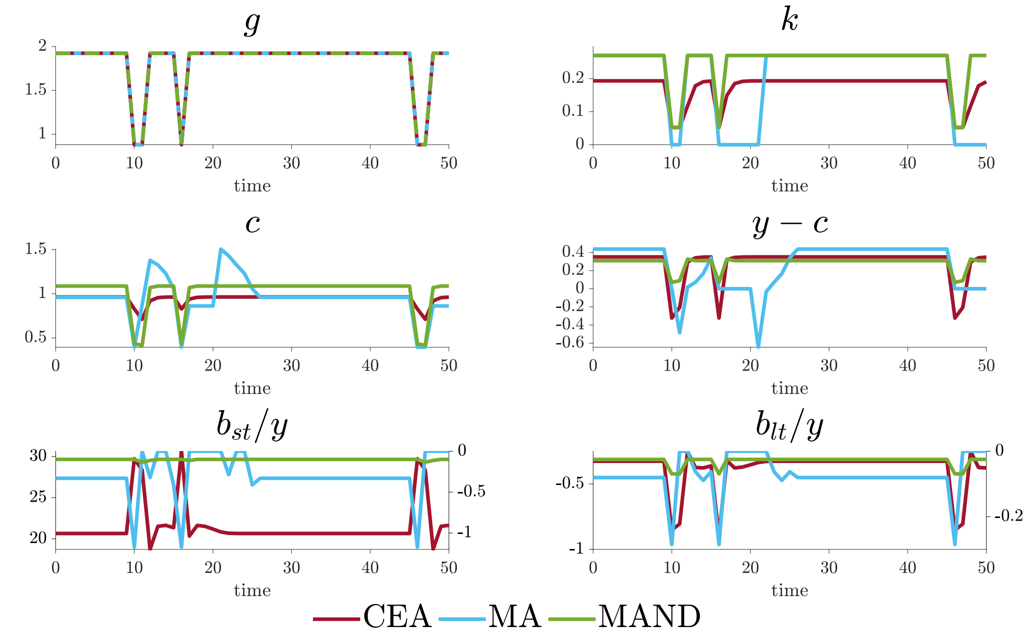
Note: For the debt figures, the relevant axis are on the right-hand sides for the MA and the MAND and the left-hand side for the CEA.

Figure 4: Impulse Response Functions to a Negative g Shock

hump-shaped pattern of consumption observed in the MA is due to the debt restructuring and the underlying regain of access to the market. In opposition, defaults do not arise in the CEA and the MAND which can both increase the indebtedness on impact. Maturity shortens at the outbreak of the bad shock's realization in the MAND and lengthens in the CEA. The Markov equilibria rely mostly on short-term debt, while the CEA use both in opposite directions.

The impulse response functions give an idea of the long-run dynamic of the economy. However, it does not tell how the economy reacts in the short run especially when there is a transition between two values of g. Thus, I simulate the economy and generate one history of shocks for 400 periods. To avoid that the initial conditions blur the results, the first 350 periods are discarded. Again, the blue line represents the MA, the green line the MAND and the red line the CEA. Moreover, the relevant axis for the debt figures are on the right-hand sides for the Markov equilibria and the left-hand side for the CEA.

Figure 5 depicts the simulation results. One observes that, in the MA, the economy defaults in the transition from g_H to g_L. This causes market exclusion and therefore $k = 0$. Consumption largely drops and jumps once the government can re-access the markets. The volatility of consumption is therefore very high. In opposition, there are no defaults in the CEA and the MAND. In the transition from g_H to g_L, the government adapts the maturity of the debt and increases its indebtedness. Especially, one sees that the level of short-term bonds have opposite movements in the MAND and the CEA. The magnitude of the changes is nonetheless different. Consistent with the findings of Buera and Nicolini (2004), the movements in debt holdings are the most pronounced for the CEA and are substantially different than the movements implied by the MA.



Note: For the debt figures, the relevant axis are on the right-hand sides for the MA and the MAND and the left-hand side for the CEA.

Figure 5: Simulation of a Typical Path

Having identified the main difference between the MA, the MAND and the CEA, I can now conduct a welfare analysis. Table 4 depicts the welfare gains of the CEA and the MAND in consumption equivalent terms with respect to the MA. Welfare gains are computed through the simulation of 10,000 independent shock histories starting with the lowest shock as well as initial debt holdings and relative Pareto weights drawn from the ergodic set. The details of the welfare computations are presented in Appendix E.

Table 4: Welfare Analysis

Allocation	Welfare gains (percent)	Capital distortion (percent)	$\mathbb{E}\mathcal{F}(g)$ (percent)
MA	-	28.9	29.6
MAND	1.04	0.0	32.4
CEA	1.10	26.8	100.0

As one can see, the CEA and the MAND imply substantial welfare gains compared to the MA. The gains come mainly from the absence of defaults. Default implies markets exclusion and therefore $k = 0$ which is extremely costly for both the lender and the borrower given the Inada conditions on the production function. In that logic, capital distortions are weaker in the CEA and inexistent in the MAND.

Besides the welfare gains, I can compute the distance with respect to the Pareto frontier. For this purpose, I derive a metric measuring the distance between the constrained efficient allocation and any alternative allocation. Consider the value of the lender in any given allocation as a function of the shock and the value of the borrower as $\ddot{V}^l : G \times \ddot{V}^b \to \mathbb{R}$. I then define

$$\mathcal{I}(g) = \frac{\int \ddot{V}^l(g, \ddot{V}^b) d\ddot{V}^b}{\int_{\underline{x}}^{\overline{x}} V^l(g, x) dx}.$$

The metric $\mathcal{I}$ is bounded between 0 and 1 given Proposition 4. A value of $\mathcal{I}$ near 1 indicates that an allocation is close to the constrained efficient benchmark, whereas a value close to 0 indicates the opposite.[32] I compute $\mathcal{I}$ for $b_{st} \leq 0$ and $b_{lt} \leq 0$ in the Markov equilibrium without default risk to stay consistent with Lemma 2.

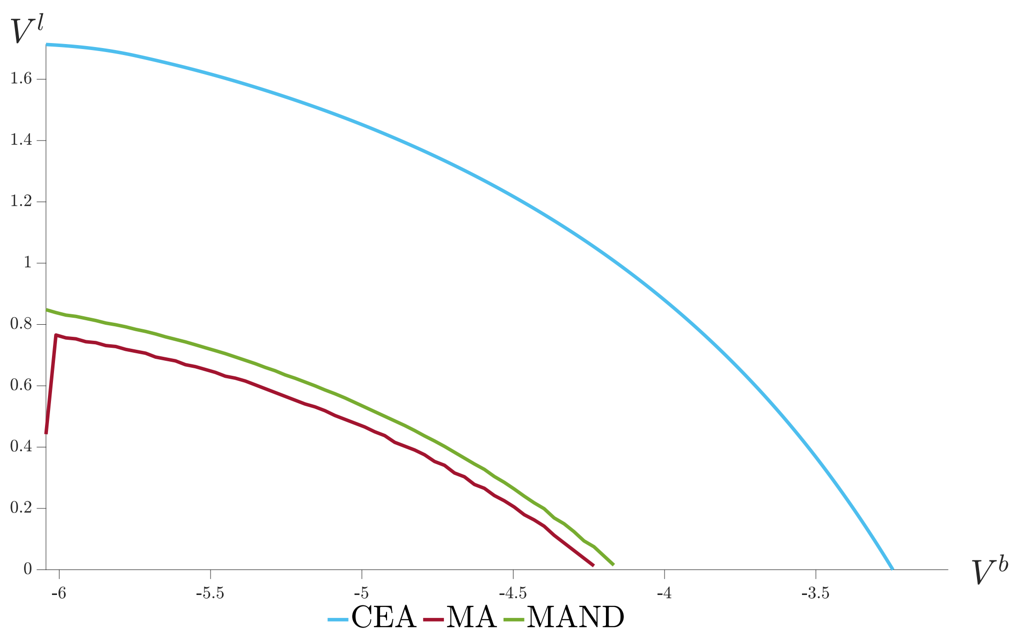


Figure 6: Distance to Pareto Frontier

Figure 6 depicts the different frontiers: in blue the Pareto frontier and in red and green

[32]The metric $\mathcal{I}$ is based on the same concept as the Gini coefficient which measures the distance between the Lorenz curve and the equity line.

the utility possibility frontier related to the MA and the MAND, respectively. Defaults in the MA produce a upward sloping part of the frontier in which both the borrower and the lender can be made better off. This shows the inefficiency of default.

Looking at the metric $\mathcal{F}$, the MAND is superior to the MA but not to the CEA. More precisely, the MA is relatively far from the CEA and the MAND can get the economy closer to it. The MAND therefore provides a better approximation of the CEA, whereas the MA performs poorly. Nevertheless, the MAND remains far from the CEA. The metric $\mathcal{F}$ is important as it relates to the entire value of the debt contract and not only on the steady state unlike the welfare gains computed above.

7.5 Alternative implementation

In this subsection, I present an alternative way of implementing the optimal contract. Most notably, I follow, the approach of Alvarez and Jermann (2000) which relies on state-contingent securities and an endogenous borrowing limit. This limit ensures that the participation constraint is satisfied in equilibrium.

Table 5: Alternative Implementation

	Benchmark	Alternative
b/y	-614.40	8.99
Spread	4.14	4.34
$\sigma(b/y)/\sigma(y)$	9.79	0.09
$\sigma(spread)/\sigma(y)$	0.00	0.00
$\rho(-b/y, y)$	-0.83	-0.98
$\rho(spread, y)$	-0.99	0.00

The main difference between the two implementations is that the benchmark case of Section 5 relies on the term structure, while the alternative case relies on changes in security holdings provided that securities are state-contingent. See Appendix A for a more detailed discussion on this.

Appendix F presents the details of the alternative implementation. I only highlight here the asset structure of the economy. At the start of a period, the government holds a perpetual security a. The government can trade G state contingent securities $a'(g')$ with a unit price of $q(g', a'(g')|g)$. The portfolio $a'(g')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q(g, a') \equiv \sum_{g'|g} q(g', a'(g')|g)$, and an insurance portfolio of G Arrow securities $\hat{a}'(g')$. Thus, $a'(g') = \bar{a}' + \hat{a}(g')$.

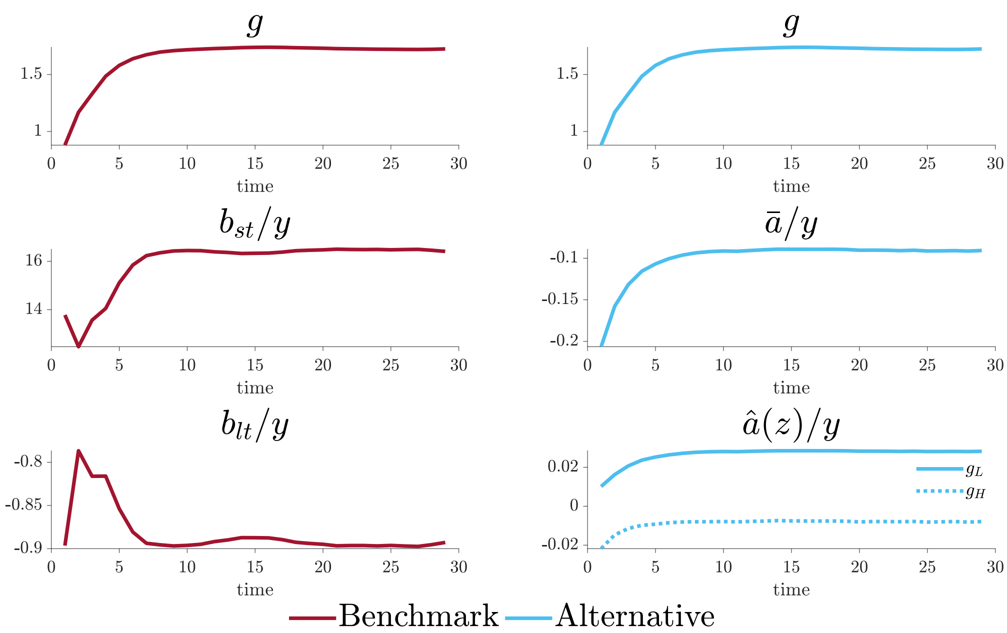


Figure 7: Impulse Response Functions to a Negative g Shock

Table 5 presents the main difference between the two implementations. The benchmark case is related to a lower level of indebtedness and a larger volatility of the debt ratio. This comes from the fact that bonds are non-contingent and the borrower alternates between short-term assets and long-term debt. Thus, large movements in debt holdings are necessary to replicate the state contingency in the contract (Buera and Nicolini, 2004). Besides this, the benchmark implementation displays a lower spread owing to *official* buybacks. As explained before, the reason behind this is that the alternative implementation does not rely on changes in prices to mimic the state-contingency of the contract given that securities are state-contingent by definition.

Similar to the previous section, I construct impulse responses to see how the two implementations work. Figure 7 depicts the responses in red for the benchmark implementation and in blue for the alternative one. The Arrow securities do most of the job in the alternative case, while the benchmark implementation needs to adapt both the long-term and the short-term bonds at the same time.

Turning to the simulation in Figure 8, one can see that the level of long-term bond in the benchmark case closely follows the pattern of bonds in the alternative case. The magnitude of change in the former is nonetheless larger than in the latter. In terms of Arrow securities, $\hat{a}(g_L)$ closely follows the evolution of $\bar{a}$, while $\hat{a}(g_H)$ has the opposite sign. The evolution of $\hat{a}(g_H)$ is therefore closely mimicking the evolution of b_{st}. Given that $\hat{a}(g)$ is state contingent, the alternative implementation needs to change the debt portfolio with lower magnitude.

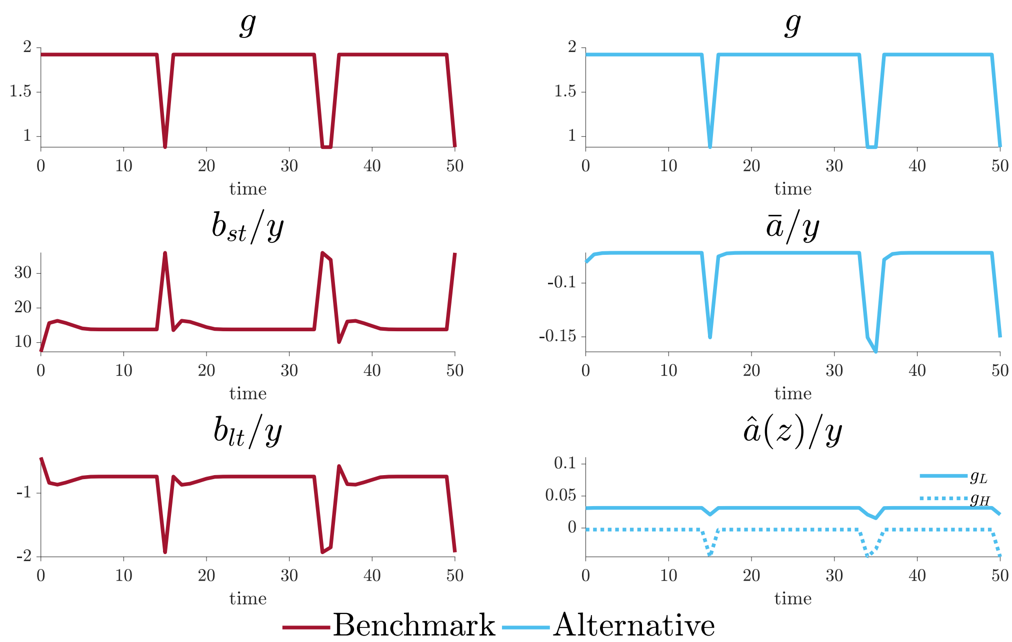


Figure 8: Simulation of a Typical Path

8 Conclusion

This paper derives the constrained efficient allocation emanating from an optimal contract to deduce the optimal sovereign debt management policy. The bottom line is that the reliance on defaults on equilibrium path is inefficient. Instead, changes in maturity and costly debt buyback can implement the constrained efficient allocation. Nevertheless, the implementation often requires highly sophisticated agents capable of building on past history. I show that less sophisticated agents would in fact rely on Markov strategies. Given this, I derive history-invariant debt management policies inspired by the optimal contract and assess their efficiency. I show that a Markov equilibrium with a no-default constraint and an *official* buyback program provides a better approximation of the constrained efficient allocation than a Markov equilibrium with defaults. The comparison of Argentina and Brazil after the 1990s supports this evidence.

This paper stresses the fact that incomplete markets might not be the reason why a market economy fails to attain constrained efficiency. Rather it can be linked to the incapacity of market participants to build on past history. The literature on sovereign debt and defaults has shown that Markov equilibria are an empirically relevant equilibrium concept. I show that Markov equilibria as (time-invariant) approximation of the constrained efficient allocation are also the policy-relevant equilibrium concept.

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Appendix

A Discussion on Alternative Implementations

This section discusses the relationship between the implementation presented in Section 5 and the principal alternatives that exist in the literature.

Dovis (2019) considers an environment similar to the one presented in Section 2 with the only difference that g is privately observed by the borrower. He derives an optimal contract subject to a participation constraint and an incentive compatibility constraint to account for limited commitment and adverse selection, respectively. He subsequently decentralizes the aforementioned contract through partial defaults and an active debt maturity management. The main difference with my study is that he explicitly uses defaults – instead of buybacks – to implement the constrained efficient allocation. This is because the combination of limited enforcements and adverse selection generates a region of *ex post* inefficiencies in which the Planner sets $k = 0$.[33] As I only consider limited enforcement, this region does not exist in my analysis – as shown in Proposition 4. Nevertheless, my implementation works in the environment of Dovis (2019), while the opposite is not true. In general, the implementation of Dovis (2019) does not apply to renegotiation-proof contracts, while mine applies to contracts with or without *ex post* inefficiencies.

Besides this, Alvarez and Jermann (2000) propose a way to implement the allocation derived in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. I apply their approach in my environment in Appendix F. The main difference with my analysis is that the two authors assume a greater financial sophistication as securities are state contingent, while I generally need higher sophistication in the strategy of the market participants – unless the implementation works under Markov strategies.

The study of Müller et al. (2019) considers a small open economy with a stochastic default cost and two productivity states: recession and normal time. The authors assume a financial market formed by two securities: a one-period non-contingent defaultable bond and a state-contingent bond which pays out only in normal time. The authors additionally assume that (1) the borrower lacks commitment only in recession and (2) renegotiation upon default is endogenous. This coupled with the aforementioned market structure, enables the two bonds to act as Arrow securities. In other words, the defaultable bond is recession contingent and spans the different stochastic default costs through renegotiation, while the contingent bond spans the good state which is free from default risk. Hence, as the bonds act as proper Arrow securities, there is no need to rely on past history as I do.

[33] Using different environments, Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007) and Yared (2010) also characterize a region of *ex-post* inefficiencies in optimal contracts.

The last study that I would like to discuss is the one of Aguiar et al. (2019) who consider a small open economy with a stochastic default cost and two productivity states as in Müller et al. (2019). The authors assume a continuum of maturities. They show the equivalence between the Markov equilibrium and the constrained efficient equilibrium. The Planner's problem is nonetheless peculiar as it does not take into consideration the legacy creditors in the objective function. In other words, the Planner problem is sequential and only accounts for the current creditors, taking as given the inherited debt level. Furthermore, there is no participation constraint of the borrower. That is, the Planner cannot prevent the occurrence of defaults on equilibrium path. Hence, in the absence of a participation constraint, the Planner needs not build on past history. This combined with the disregard of legacy creditors directly leads to the Markov equilibrium in Eaton and Gersovitz (1981).

B Price in Markov Equilibrium

The price of one unit of bond of maturity $j \in \{st, lt\}$ is given by

$$q_j(g, b'_{st}, b'_{lt}) = \mathbb{E}_{g'|g}\Big[(1 - D(\Omega'))q_j^P(g', b'_{st}, b'_{lt}) + D(\Omega')q_j^D(g', b'_{st}, b'_{lt})\Big], \tag{B.1}$$

where recovery value given by

$$q_j^D(g', b'_{st}, b'_{lt}) = \frac{1}{1+r}\left[(1-\lambda)q_j^D(g', b'_{st}, b'_{lt}) + \lambda\frac{W_j\frac{1+r}{1+r-\mathbb{I}_{j=lt}}}{b'_j}\right],$$

where $\mathbb{I}_{j=lt}$ is an indicator function taking value one if $j = lt$ and zero otherwise. In case of repayment, the price depends on the maturity, the repayment productivity and the buyback decision.

$$\begin{aligned}
q_{st}^P(g', b'_{st}, b'_{lt}) &= \frac{1}{1+r}, \\
q_{lt}^P(g', b'_{st}, b'_{lt}) &= \frac{1}{1+r}\Big[1 + (1 - M(\Omega'))q_{lt}(g', b''_{st}, b''_{lt}) + M(\Omega')q_{lt}^{bb}\Big],
\end{aligned}$$

where $b''_j = B_j(g', b'_{st}, b'_{lt})$ for $j \in \{st, lt\}$. Given this, a Markov equilibrium can be defined as

Definition B.2 (Markov Equilibrium)**.** *In this environment, a Markov equilibrium consists of a set of prices, $\{p(\Omega), q_{st}(g, b'_{st}, b'_{lt}), q_{lt}(g, b'_{st}, b'_{lt})\}$, a set of policy functions $\mathcal{G}(\Omega) = \{D(\Omega), M(\Omega), B_{st}(\Omega), B_{lt}(\Omega), \tau(\Omega)\}$ such that, at every possible state Ω,*

1. *Taking p, q_{st} and q_{st} as given, $\mathcal{G}(\Omega)$ solves the government's problem (5)-(7).*

2. Taking p as given, the choice of capital by domestic firms is such that

$$g f_k(k) = p.$$

3. Taking $\mathcal{G}(\Omega)$ as given, the price of capital is consistent with

$$\max_k \; p(\Omega)(1-\tau(\Omega))k - k.$$

4. The prices of bond satisfy (B.1)

C Nash Bargaining in Markov Equilibrium

In this section, I introduce a Nash bargaining game in the Markov equilibrium. This first shows how to endogenize the *official* buyback premium. It also reinforces the argument made in Lemma 2 about the enforcement of *official* buybacks in Markov equilibria.

The threat point of the game is that the borrower is not able to roll over its debt in the current period if the *official* buyback does not take place. In such circumstance, the borrower's value is given by

$$\begin{aligned}
\bar{V}^{NB}(\Omega) = &\max_{\tau, b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{g'|g}\Big[V(\Omega')\Big] \\
\text{s.t.} \quad & c + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g, p, k, \tau) + b_{st} + b_{lt}, \\
& b'_{lt} \geq b_{lt}, \\
& b'_{st} \geq 0.
\end{aligned}$$

Notice that the borrower can issue short-term assets. For any *official* buyback premium χ, I define the surplus of the borrower as

$$\Delta^b(\Omega; \chi) = V^B(\Omega; \chi) - \bar{V}^{NB}(\Omega).$$

The borrower's surplus corresponds to the difference between the value of conducting the *official* buyback and the value of rejecting it and suffering the underlying sudden stop.

To define the surplus of the lender, I first need to derive the lender's value under *official* buyback, under no *official* buyback and under default. The former reads

$$\begin{aligned}
V_l^B(\Omega) = &\max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r} \mathbb{E}_{g'|g}\Big[V_l(\Omega')\Big] \\
\text{s.t.} \quad & c_l + b_{st} + b_{lt}(1 + q_{lt}^{bb}) = q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},
\end{aligned}$$

while under no *official* buyback

$$
\begin{aligned}
\bar{V}_l^{NB}(\Omega) = & \max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[V_l(\Omega')\Big] \\
& \text{s.t.} \quad c_l + b_{st} + b_{lt} = q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}), \\
& \qquad\quad b'_{lt} \geq b_{lt}, \\
& \qquad\quad b'_{st} \geq 0.
\end{aligned}
$$

and finally, under default

$$
V_l^D(g) = \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[(1-\lambda)V_l^D(g') + \lambda V_l(g', W_{st}, W_{lt}\frac{1+r}{r})\Big]
$$

The continuation value under repayment is then $V_l(\Omega) = (1 - D(\Omega) - M(\Omega))\bar{V}_l^{NB}(\Omega) + M(\Omega)V_l^B(\Omega) + D(\Omega)V_l^D(g)$. The surplus of the lender corresponds to the difference in the value under *official* buyback and no *official* buyback

$$
\Delta^l(\Omega; \chi) = V_l^B(\Omega; \chi) - \bar{V}_l^{NB}(\Omega).
$$

If the lender has all the bargaining power, then it could extract a large *official* buyback premium (i.e. $\chi \to 1$). In opposition, if the borrower has all the bargaining power, it can conduct *official* buybacks at no additional cost (i.e. $\chi \to 0$). To consider the case in between those two extremes, I assume that the lender has a bargaining power of $\zeta \in [0, 1]$ and the borrower $1 - \zeta$. Given a specific state Ω, the *official* buyback premium $\chi(\Omega)$ is the solution to

$$
\begin{aligned}
\chi(\Omega) = & \arg\max_{\chi \in [0,1]} \Big[\Delta^l(\Omega; \chi)^{\zeta} + \Delta^l(\Omega; \chi)^{1-\zeta}\Big] \\
& \text{s.t.} \quad \Delta^l(\Omega) \geq 0, \\
& \qquad\quad \Delta^b(\Omega) \geq 0.
\end{aligned}
$$

In light of Lemma 2, the above bargaining problem has a solution only if the threat of the sudden stop is credible. If the threat is not credible, $\Delta^b(\Omega; \chi) < 0$ for all $\chi \in [0, 1]$ meaning that there is no $\chi > 0$ for which the borrower is willing to conduct *official* buybacks instead of being punished.

D Further Characterization of the Optimal Contract

The following lemma derives the inverse Euler Equation which gives the consumption dynamic in the contract.

Lemma D.1 (Inverse Euler Equation). *Under Assumptions 1 and 3, the inverse Euler equation for a given $g \in G$ reads*

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

Proof. See Appendix G $\square$

If the participation constraint of the borrower never binds, I obtain that for all (g, x),

$$\frac{1}{u_c(c(g))} \geq \mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))}\right]$$

with strict inequality when $\eta < 1$. In this case, the inverse Euler Equation is a positive supermartingale. Immiseration is a consequence of the theorem stating that supermartingales converge almost surely to $-\infty$. Alternatively, when $\eta = 1$, consumption remains constant. Under limited commitment of the borrower (i.e. $\nu(g) \geq 0$), one obtains a left bounded positive submartingale. The borrower's participation constraints therefore sets an upper bound on the supermartingale and prevents immiseration.

E Welfare Analysis

To compute the borrower's welfare, first define the borrower's value for a sequence of consumption $\{c(g^t)\}$ starting from an initial state at $t = 0$ as

$$V(\{c(g^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c(g^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c(g^t)^{1-\sigma}}{1-\sigma},$$

I denote the borrower's consumption allocation in the benchmark model by $\{c^b(g^t)\}$ and the consumption allocation in the alternative model by $\{c^a(g^t)\}$. In addition, I define the consumption-equivalent welfare gain of the alternative model with respect to the benchmark model by ι such that

$$V(\{(1+\iota)c^b(g^t)\}) = V(\{c^a(g^t)\}).$$

Given the functional form of the instantaneous utility one obtains

$$(1+\iota)^{1-\sigma}\Big[V(c^b(g^t)\})\Big] = V(\{c^a(g^t)\}).$$

The welfare gain therefore boils down to

$$\iota = \left[\frac{V(\{c^a(g^t)\})}{V(\{c^b(g^t)\})}\right]^{\frac{1}{1-\sigma}} - 1.$$

F Alternative Implementation

In what follows, I propose an alternative implementation as the one derived in Section 5. More precisely, I rely on the approach of Alvarez and Jermann (2000) using trade in state-contingent securities and an endogenous borrowing limit.

The structure of the financial market is the following. At the start of a period, the government holds a perpetual security a.[34] The government can trade G state contingent securities $a'(g')$ with a unit price of $q(g', a'(g')|g)$. The portfolio $a'(g')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q(g, a') \equiv \sum_{g'|g} q(g', a'(g')|g)$, and an insurance portfolio of G Arrow securities $\hat{a}'(g')$. Thus we have that $a'(g') = \bar{a}' + \hat{a}(g')$ with

$$\bar{a}' = \frac{\sum_{g'|g} q(g', a'(g')|g)a'(g')}{q(g, a')} \quad \text{and} \quad \sum_{g'|g} q(g', a'(g')|g)\hat{a}'(g') = 0.$$

The last equation represents the market clearing condition of the Arrow securities.

The capital market is the same as in the main text. The lender provides k at price p and the government can decide to tax the repayment of capital to the lender at rate τ. The government's problem therefore reads

$$\begin{aligned} W^b(g, a) = & \max_{c, \tau, \{a'(g')\}_{g' \in g}} u(c) + \beta \mathbb{E}_{g'|g}\big[W^b(g', a'(g'))\big] && \text{(F.1)} \\ & \text{s.t. } c + \sum_{g'|g} q(g', a'(g')|g)(a'(g') - a) \leq g f(k) - p(1-\tau)k + a \\ & \qquad \bar{a}' + \hat{a}(g') \geq \mathcal{A}(g', k'), && \text{(F.2)} \end{aligned}$$

[34] The maturity is unimportant in this implementation. The security a could also be a one-period security.

where $\mathcal{A}(g', k')$ represents the endogenous borrowing limit and is defined such that

$$W(g', \mathcal{A}(g', k')) = V^D(g', k'). \tag{F.3}$$

One can see here the similarity with the borrowing limit defined in Section 4. The lender's problem is static. I nonetheless express it in recursive form.

$$\begin{aligned} W^l(g, a_l) = &\max_{c_l, k_l, \{a_l'(g')\}_{g' \in g}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\left[W^l(g', a_l'(g'))\right] \\ &\text{s.t. } c_l + \sum_{g'|g} q(g', a_l'(g')|g)(a_l'(g') - a_l) \leq p(1-\tau)k_l - k_l + a_l. \end{aligned} \tag{F.4}$$

Given this environment, I can determine a recursive competitive equilibrium in the following terms.

Definition F.3 (Recursive Competitive Equilibrium (RCE))**.** *A recursive competitive equilibrium (RCE) is a sequence of prices $q(g', a'(g')|g)$ and $p(g, a)$, value functions, $W^b(g, a)$ and $W^l(g, a)$, an endogenous borrowing limit, $\mathcal{A}(g', k')$, as well as policy functions for (i) consumption, $c(g, a)$ and $c_l(g, a)$, (ii) capital, $k = k(g, a)$ and $k_l = k_l(g, a)$ as well as (iii) asset holdings $a'(g') = A(g', g, a)$ and $a_l'(g') = A_l(g', g, a)$ such that,*

1. *Given the value function for the outside option of the government, $V^D(g', k')$ as well as the asset price $q(g', a'(g')|g)$,*
   - *(a) the policy functions $c(g, a)$ and $A(g', g, a)$, together with the value function $W^b(g, a)$, solve the government problem (F.1) with the endogenous limit, $\mathcal{A}(g', k')$.*
   - *(b) the policy functions $c_l(g, a_l)$, $k_l(g, a)$, and $A_l(g', g, a_l)$, together with the value function $W^l(g, a_l)$, solve the lender's problem (F.4) and*
2. *Taking p as given, the choice of capital by domestic firms is such that*

$$u_c(c)(g f_k(k) - p) = 0.$$

3. *The asset market clears, $a'(g') + a_l'(g') = 0$ for all $g' \in G$.*
4. *The product and capital markets clear, $c(g, a) + c_l(g, a_l) = gf(k)$ with $k = k_l$.*

For the government's problem, taking the first-order conditions with respect to consumption and assets, one obtains

$$u_c(c) = \mu_{BC}(g, a),$$

$$q(g', a(g')|g) = \beta\pi(g'|g)\frac{u_c(c')}{u_c(c)}[1 + \sum_{g''|g} q(g'', a''(g'')|g')] + \frac{\mu_{EBL}(g', a'(g'))}{u_c(c)},$$

where μ_{BC} and μ_{EBL} are the Lagrange multipliers attached to the budget constraint and the endogenous borrowing limit, respectively. Especially, $\mu_{EBL}(g', a'(g')) \geq 0$ with $\mu_{EBL}(g', a'(g')) = 0$ if $a'(g') > \mathcal{A}(g', k')$.

Conversely, taking the first-order conditions with respect to consumption, capital and assets of the lender's problem

$$\begin{aligned}
1 &= \mu^l_{BC}(g, a_l), \\
1 &= p(1 - \tau), \\
q(g', a(g')|g) &= \frac{1}{1+r}\pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')).
\end{aligned}$$

Following Krueger et al. (2008), the price is determined by the agent whose constraint is not binding. Therefore the price is determined by

$$q(g', a(g')|g) = \pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')) \max\left\{\beta\frac{u_c(c(g', a'(g'))}{u_c(c(g, a))}, \frac{1}{1+r}\right\}. \quad \text{(F.5)}$$

The following lemma states that the constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.

Proposition F.1 (Alternative Implementation)**.** *Given initial conditions $\{g_0, x_0\}$, a constrained efficient allocation can be implemented as a competitive equilibrium with state-contingent securities and an endogenous borrowing limit.*

Proof. See Appendix G ☐

The benchmark implementation presented in Section 5 relies on changes in the term premium to mimic the state-contingency in the optimal contract, while this alternative implementation relies on changes in security holdings provided that securities are state-contingent. More importantly, given that securities are state contingent, the assumption that the borrower and the lender keep track of the entire history of play is not anymore necessary. The implementation of the constrained efficient allocation now lies on the assumption of a greater financial sophistication.

G Proofs

Proof of Lemma 1.

The value of permanent autarky is given by

$$v_a(g_t) = \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0)),$$

as the lender sets $k = 0$ in case of default. Permanent autarky is the worst equilibrium outcome as the government could always be better off with $k = \epsilon$ for small $\epsilon > 0$ given the Inada conditions on the production function. I show this in Proposition 4.

Permanent autarky is an equilibrium of the market economy. Suppose that the lender believes that $\tau_t = 1$ as well as that $D_t = 1$ for all t. Then, it sets $k_t = 0$ and $q_{j,t} = 0$ for all $j \in \{st, lt\}$. Given this, the government finds optimal to choose $\tau_t = 1$ and $D_t = 1$ for all t confirming the lender's beliefs. $\square$

Proof of Proposition 1.

Assume by contradiction that in a given state Ω, the borrower wants to conduct an *official* buyback. That is, the borrower picks a pair (b'_{st}, b'_{lt}) such that

$$V^{NB}(\Omega) < V^B(\Omega).$$

The consumption under *official* buyback is given by

$$c^B(\Omega) = y(g, p, k, \tau) + b_{st} + b_{lt}(1 + q^{bb}_{lt}) - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},$$

and the expected continuation value by

$$\mathbb{E}_{g'|g}\Big[V(g', b'_{st}, b'_{lt})\Big].$$

Now consider the alternative strategy of picking the same pair (b'_{st}, b'_{lt}) but conducting an *unofficial* buyback. In such circumstance, consumption is given by

$$c^{NB}(\Omega) = y(g, p, k, \tau) + b_{st} + b_{lt} - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}).$$

It is clear from the *official* buyback premium that $c^{NB}(\Omega) > c^B(\Omega)$. Moreover, as the borrower repays the debt in both types of buybacks, the continuation value is the same. Hence,

$$V^{NB}(\Omega) = u(c^{NB}(\Omega)) + \beta\mathbb{E}_{g'|g}\Big[V(g', b'_{st}, b'_{lt})\Big] > u(c^B(\Omega)) + \beta\mathbb{E}_{g'|g}\Big[V(g', b'_{st}, b'_{lt})\Big] = V^B(\Omega),$$

which contradicts the fact that an *official* buyback is ever optimal. □

Proof of Lemma 2.

The main element that has to be understood is that an *official* buyback represents a reverse-default as it corresponds to an overpayment – in opposition to an underpayment – of liabilities. Not conducting *official* buybacks does not lead to market exclusion as the borrower repaid its debt. The only punishment the lender can impose in case of no *official* buyback is to not roll over the debt, meaning that $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$. This punishment is clearly Markov as it only pertains to the current period, unlike trigger strategies that rely on all future paths of play.

In addition, when the borrower issues short-term assets, there is no threat available to the lender as the borrower is in fact the lender of short-term debt. In other words, when $b'_{st} > 0$, it is the lender who is auctioning and it cannot exclude the borrower from the auction – as the borrower repaid the debt and has market access. In opposition, when $b'_{st} < 0$, the borrower is auctioning – as it seeks to raise resources – and the lender can decide not to participate to this auction.

Part I

The proof follows the same logic as the one of Proposition 1. Suppose by contradiction that the lender can enforce *official* buybacks in a state Ω such that $B_{st}(\Omega) \geq 0$. Formally, in the case of an *official* buyback, the borrower chooses $b'_{st} = B_{st}(\Omega) \geq 0$ and $b'_{lt} = B_{lt}(\Omega) \geq b_{lt}$ such as to maximize its utility. The borrower's value in this situation is

$$\begin{aligned} V^B(\Omega) =& u(c^B) + \beta\mathbb{E}_{g'|g}\Big[V(g', b'_{st}, b'_{lt})\Big] \\ &\text{s.t.} \quad c^B + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})b'_{lt} = y(g, p, k, \tau) + b_{st} + b_{lt}(1 + q^{bb}_{lt}). \end{aligned}$$

Now consider the case in which the borrower does not conduct the *official* buyback but mimics the debt choice in the case of *official* buyback. As explained before, this is possible as the lender cannot prevent the borrower to issue assets. The borrower's value value is given by

$$\begin{aligned}\bar{V}^{NB}(\Omega) =& u(c^{NB}) + \beta \mathbb{E}_{g'|g}\Big[V(g', b'_{st}, b'_{lt})\Big] \\ & \text{s.t.} \quad c^{NB} + q_{st}(g, b'_{st}, b'_{lt})b'_{st} + q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g, p, k, \tau) + b_{st} + b_{lt},\end{aligned}$$

The contradiction is immediate as the continuation value is the same in the two cases and $c^{NB} < c^B$. Thus, *official* buybacks are not enforceable in case of short-term asset issuance.

Part II

I consider a state Ω in which, $B_{st}(\Omega) < 0$ and $B_{lt}(\Omega) \geq b_{lt}$. Moreover, I assume without loss of generality that the choice of private debt is the same in the case with and without *official* buyback. Given this, we have that for all $g' \in G$, $V(g', B_{st}(\Omega), B_{lt}(\Omega)) \leq V(g', 0, B_{lt}(\Omega))$. In words, the continuation value under the no-roll-over punishment is weakly larger than the continuation value under an *official* buyback.

I consider two cases. First, if $b_{st} < 0$ is such that $\bar{V}^{NB}(\Omega) = V^D(g)$, the *official* buyback is enforceable given the definition of the endogenous borrowing limit.

Second, given that the continuation value under punishment is weakly larger, to obtain that $c^{NB} > c^B$, it must be that

$$q_{st}(g, b'_{st}, b'_{lt})b'_{st} < b_{lt}(q_{lt}^{bb} - q_{lt}(g, b'_{st}, b'_{lt})).$$

Hence, provided that $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$, if χ is sufficiently close to zero and $-b_{lt}$ is relatively small compared to b'_{st}, it is possible to have $V^B(\Omega) > \bar{V}^{NB}(\Omega)$ ensuring the enforcement of *official* buybacks.

□

Proof of Proposition 2.

I prove the proposition by construction. Denote the objects related to the Markov equilibrium with default incentives with "MA" and without default incentives with "MAND". Given equation (10), for all Ω,

$$V^P_{MAND}(\Omega) \geq V^D(g).$$

Hence, there is no default on equilibrium path. This combined with the state contingent buyback program implies that

$$
\begin{aligned}
q_{lt}^{MAND}(g, b'_{st}, b'_{lt}) = q_{lt}^{MAND}(g) \in \left(\frac{1}{r}, \frac{1}{(1-\chi)r}\right), \\
q_{st}^{MAND}(g, b'_{st}, b'_{lt}) = q_{st}^{MAND} = \frac{1}{1+r}.
\end{aligned}
$$

As a result the long-term (short-term) bond price in the Markov equilibrium without default incentives is always strictly (weakly) greater the in the Markov equilibrium with default incentives. Formally for all (g, b'_{st}, b'_{lt}),

$$
\begin{aligned}
q_{lt}^{MAND}(g) > q_{lt}^{MA}(g, b'_{st}, b'_{lt}), \\
q_{st}^{MAND} \geq q_{st}^{MA}(g, b'_{st}, b'_{lt}).
\end{aligned}
$$

First, consider that there is no default in the MA for any Ω meaning that

$$
\begin{aligned}
q_{lt}^{MAND}(g) > q_{lt}^{MA} \equiv q_{lt}^{rf} = \frac{1}{r}, \\
q_{st}^{rf} \equiv q_{st}^{MAND} = q_{st}^{MA} = \frac{1}{1+r}.
\end{aligned}
$$

Define the consumption in the MA as

$$
c_{MA}(\Omega) = y(g, p, k, \tau) + b_{st} + b_{lt} - q_{st}^{rf} B_{st}^{MAND}(\Omega) - q_{lt}^{rf}(B_{lt}^{MAND}(\Omega) - b_{lt}).
$$

Given the absence of default, the maturity structure is indeterminate in the MA. Conversely, the consumption in the MAND under (9) when $g = g_L$ is

$$
\begin{aligned}
c_{MAND}(g_L, b_{st}, b_{lt}) =& y(g_L, p, k, \tau) + b_{st} + b_{lt} - q_{st}^{rf} B_{st}^{MAND}(g_L, b_{st}, b_{lt}) - \\
& q_{lt}^{MAND}(g_L)(B_{lt}^{MAND}(g_L, b_{st}, b_{lt}) - b_{lt}),
\end{aligned}
$$

and when $g = g_H$

$$
\begin{aligned}
c_{MAND}(g_H, b_{st}, b_{lt}) = & y(g_H, p, k, \tau) + b_{st} + b_{lt}(1 + q_{lt}^{bb}) - q_{st}^{rf} B_{st}^{MAND}(g_H, b_{st}, b_{lt}) - \\
& q_{lt}^{MAND}(g_H) B_{lt}^{MAND}(g_H, b_{st}, b_{lt}), \\
B_{lt}^{MAND}(g_H, b_{st}, b_{lt}) \geq b_{lt}. &
\end{aligned}
$$

Under the assumption that $B_{lt}^{i}(\Omega) \geq (1-\chi) b_{lt}$ for all $i \in \{MA, MAND\}$ when $g = g_H$, it is always possible for the borrower in the MAND to mimic the choice of debt given in the MA.

Consider that in the MAND, the borrower adopts the following policy: $\tilde{B}_{lt}^{MAND}(\Omega) \leq B_{lt}^{MA}(\Omega)$ and $\tilde{B}_{st}^{MAND}(\Omega) = B_{st}^{MA}(\Omega)$ for all Ω such that

$$c_{MA}(g_H, b_{st}, b_{lt}) > \tilde{c}_{MAND}(g_H, b_{st}, b_{lt}),$$
$$c_{MA}(g_L, b_{st}, b_{lt}) < \tilde{c}_{MAND}(g_L, b_{st}, b_{lt}),$$

and such that for any $g_- \in G$

$$\mathbb{E}_{g|g_-} c_{MA}(\Omega) = \mathbb{E}_{g|g_-} \tilde{c}_{MAND}(\Omega).$$

This is feasible given the assumption that $B_{lt}^{i}(\Omega) \geq (1-\chi)b_{lt}$ for all $i \in \{MA, MAND\}$ and the fact the the borrowings terms are strictly better for the long-term debt in the MAND compared to the MA.

The above construction implies that the consumption in the MA is a mean-preserving spread of the consumption in the MAND. Given the properties of the borrower's utility, this implies that for all Ω and $g_- \in G$

$$\mathbb{E}_{g|g_-} \tilde{V}_{MAND}^{P}(g, b_{st}, b_{lt}) > \mathbb{E}_{g|g_-} V_{MA}^{P}(g, b_{st}, b_{lt}),$$

where $\tilde{V}_{MAND}^{P}$ is the value in the MAND when the borrower follows $\tilde{B}_{lt}^{MAND}(\Omega) \leq B_{lt}^{MA}(\Omega)$ and $\tilde{B}_{st}^{MAND}(\Omega) = B_{st}^{MA}(\Omega)$. Such behavior might not be optimal and therefore

$$\mathbb{E}_{g|g_-} V_{MAND}^{P}(g, b_{st}, b_{lt}) \geq \mathbb{E}_{g|g_-} \tilde{V}_{MAND}^{P}(g, b_{st}, b_{lt}) > \mathbb{E}_{g|g_-} V_{MA}^{P}(g, b_{st}, b_{lt}).$$

Provided that $c_{MA}(g_L, b_{st}, b_{lt}) < \tilde{c}_{MAND}(g_L, b_{st}, b_{lt})$, this directly implies that

$$V_{MAND}^{P}(g_L, b_{st}, b_{lt}) \geq \tilde{V}_{MAND}^{P}(g_L, b_{st}, b_{lt}) > V_{MA}^{P}(g_L, b_{st}, b_{lt}).$$

The case when $g = g_H$ is more delicate as

$$c_{MA}(g_H, b_{st}, b_{lt}) > \tilde{c}_{MAND}(g_H, b_{st}, b_{lt}).$$

Assume by contradiction that there is a state for which

$$V_{MAND}^{P}(g_H, b_{st}, b_{lt}) < V_{MA}^{P}(g_H, b_{st}, b_{lt}).$$

Now consider the strategy in the MAND which consists of having $\ddot{B}_{st}^{MAND}(\Omega)$ and $\ddot{B}_{lt}^{MAND}(\Omega)$ such that $\ddot{c}_{MAND}(\Omega) = c_{MA}(\Omega)$ for all Ω. Under such strategy, the borrower completely replicates the consumption path of the MA in the MAND. This is feasible as $B^i_{lt}(\Omega) \geq b_{lt}(1-\chi)$ for all $i \in \{MA, MAND\}$ and the borrowing terms for long-term debt in the MAND are always strictly better than in the MA. This then implies that for all Ω

$$\ddot{V}^P_{MAND}(\Omega) = V^P_{MA}(\Omega).$$

Since this strategy might not be optimal, it must be that

$$V^P_{MAND}(g_H, b_{st}, b_{lt}) \geq \ddot{V}^P_{MAND}(g_H, b_{st}, b_{lt}) = V^P_{MA}(g_H, b_{st}, b_{lt}).$$

which contradicts the fact that $V^P_{MAND}(g_H, b_{st}, b_{lt}) < V^P_{MA}(g_H, b_{st}, b_{lt})$. Hence, the borrower is always at least weakly better off in the MAND compared to the MA.

The lender extracts a larger rent than in the MA given that $\chi > 0$. Hence, for the same Ω, the value of the lender is strictly larger in the MAND than in the MA.

Considering no default in the MA gives an upper bound for the price. Hence, the argument derived above generally applies when there is a strictly positive probability of default in the MA. Note that one cannot compare the MAND and the MA for debt portfolio in which (10) would be violated. Hence, the MA and the MAND do not necessarily span each other.

The last part of the proposition follows directly from the fact that *official* buybacks can only be conditioned on g. Otherwise, one could condition *official* buybacks on a specific (g, b_{st}, b_{lt}) which would probably be less recurrent than (9) and therefore less costly for the borrower. $\square$

PROOF OF PROPOSITION 3.

I prove the proposition following Bhaskar et al. (2012). I first show that every equilibrium under Assumption 2 with $\psi, \epsilon > 0$ is essentially sequentially strict. I then prove that every essentially sequentially strict equilibrium is a Markov (perfect) equilibrium.

I start the proof with some definitions. Given the information structure, I split the histories into two categories: public and private. Public histories are the ones defined in Section 3 – that is h^t_b and h^t_l. Private histories of the borrower and the lender at time t are the ones tracking the utility shocks – that is $p^t_b = (p^{t-1}_b, \varrho_{b,t})$ and $p^t_l = (p^{t-1}_l, \varrho_{l,t})$, respectively. Finally, I denote he entire history of the play including the privately observed utility shocks by $\hat{h}^t$.

In addition, I denote σ_b and σ_l as the strategy profile of the borrower and the lender, respectively. Besides this, A_i corresponds to the countable set of actions with typical element

a_i for market participant $i \in \{b, l\}$. For instance, actions taken by the borrower relate to borrowing, defaults, buybacks and taxation. Moreover, $W^b(\sigma_b, \sigma_l | h_b^t, p_b^t)$ and $W^l(\sigma_b, \sigma_l | h_l^t, p_l^t)$ represent respectively the value of the borrower and the lender from the strategy profile (σ_b, σ_l) at the relevant histories.

Given that each market participant has some private information regarding their payoff, they need to form beliefs about the unobserved utility shock of the other participants. Denote the belief of agent $i \in \{b, l\}$ over the entire history $\hat{h}^t$ as $\omega_i^{(h_i^t, p_i^t)}$. I follow Bhaskar et al. (2012) and put the least structure possible on such beliefs. They simply need to be independent of the private payoff shocks and put zero weight to events that history $\hat{h}^t$ is inconsistent with h_i^t. With this, I define a Markov equilibrium as

Definition G.4 (Markov Equilibrium). *A strategy σ_i for $i \in \{b, l\}$ is Markov if for any two histories $(h_i^t, p_i^t) \neq (\tilde{h}_i^t, \tilde{p}_i^t)$ ending with the same state Ω,*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(\tilde{h}_i^t, \tilde{p}_i^t).$$

A strategy profile (σ_b, σ_l) is a Markov equilibrium if (σ_b, σ_l) is Markov and for any alternative strategy $(\tilde{\sigma}_b, \tilde{\sigma}_l)$,

$$W^b(\sigma_b, \sigma_l) \geq W^b(\tilde{\sigma}_b, \tilde{\sigma}_l) \wedge W^l(\sigma_b, \sigma_l) \geq W^l(\tilde{\sigma}_b, \tilde{\sigma}_l).$$

Furthermore, I define a sequential best response as

Definition G.5 (Sequential Best Response). *A strategy σ_i is a sequential best response to (σ_{-i}, ω_i), if for each history (h_i^t, p_i^t) and each alternative strategy $\tilde{\sigma}_i$*

$$\int W^i(\sigma_b, \sigma_l | \hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t) \geq \int W^i(\tilde{\sigma}_b, \sigma_l | h_b^t) d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Strategy σ_i is a sequential best response to σ_{-i} if strategy σ_i is a sequential best response (σ_{-i}, ω_i) for some ω_i.

Given the information structure, there is no general solution concept which can be used here. That is why, Bhaskar et al. (2012) appeal to the very weak concept of sequential optimality. Nonetheless, a profile of mutual sequential best response for the borrower and the lender represents a perfect Bayesian equilibrium.

The other concept defined by the aforementioned authors is the current shock strategy which relies at most on the current value of the private shock. Formally

Definition G.6 (Current Shock Strategy)**.** *A strategy σ_i is a current shock strategy, if for any public history (h_i^t, p_i^t) and for any two histories, p_i^t and $\tilde{p}_i^t$, both finishing with the same ϱ_i, then for almost all ϱ_i*

$$\sigma_i(h_i^t, p_i^t) = \sigma(h_i^t, \tilde{p}_i^t).$$

The next lemma links Definitions G.5 and G.6. In words, any sequential response relies at most on the current value of the private shock. As a result the history of past private shocks becomes irrelevant.

Lemma G.2 (Sequential Strictness and Current Shock Strategy)**.** *If σ_i is a sequential best response to σ_{-i}, then σ_i is a current shock strategy.*

Proof. Consider a market participant i with history (h_i^t, p_i^t). The expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t) = \mathbb{E}_{g'|g} \int \int \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', \hat{h}^t) d\mu_{P_i}(\varrho_i') d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Since σ_{-i} and $\omega_i^{(h_i^t, p_i^t)}$ do not depend on the private history, the value $W^i(a, \sigma_{-i}, \omega_i | h_i^t, p_i^t)$ is also independent of private history. Furthermore, since the density of ϱ_i is absolutely continuous, the market participant i can only be indifferent between two actions on a zero measure of the support. For different values of ϱ_i, the action is unique and independent of the past values of the shock. $\square$

Given that beliefs on the history of past private shock do not matter, I can suppress the dependence on the beliefs and the private shock realization in the value function. Thus, the expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i} | h_i^t) = \int \mathbb{E}_{g'|g} \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', h_i^t) d\mu_{P_i}(\varrho_i').$$

I then arrive to the first step of the proof. Given that beliefs over private histories are irrelevant for optimality, every perfect Bayesian equilibrium (i.e. a profile of mutual sequential best responses) satisfying Assumption 2 with $\psi, \epsilon > 0$ are essentially sequentially strict.

Lemma G.3 (Sequential Best Response and Perfect Bayesian Equilibrium)**.** *Every perfect Bayesian equilibrium satisfying Assumption 2 with $\psi, \epsilon > 0$ is essentially sequentially strict.*

Proof. I need to show that for any period, history and for almost all values of the private shock, the optimal action is unique. I consider the case of the borrower first. The borrower's value from action a_b after the realization of ϱ_b is given by

$$W^b(a_b, \varrho_b, \sigma_b | h_b^t) = u(a_b, g) + \epsilon \varrho_b^a + \beta \mathbb{E}_{g'|g} W^i(\sigma_b | a_b, g', h_b^t).$$

Suppose two actions a_b and $\tilde{a}_b$, the equality $W^b(a_b, \varrho_b, \sigma_b | h_b^t) = W^b(\tilde{a}_b, \varrho_b, \sigma_b | h_b^t)$ implies that

$$\epsilon(\varrho_b^{a_b} - \varrho_b^{\tilde{a}-b}) = u(a_b, g) - u(\tilde{a}_b, g) + \beta \mathbb{E}_{g'|g} \left[W^b(\sigma_b | a_b, g', h_b^t) - W^b(\sigma_b | \tilde{a}_b, g', h_b^t) \right].$$

The set of actions is countable, whereas the set of values of private shocks for which a market participant can be indifferent has measure zero. Hence, for almost all values of ϱ_i, the set of maximizing actions must be a singleton, and the profile is essentially sequentially strict. The proof naturally extends to the case of the lender and is therefore omitted. □

Now that we have that all equilibria satisfying Assumption 2 with $\psi, \epsilon > 0$ are essentially sequentially strict, I simply need to show that sequentially strict equilibria are Markov equilibria.

Lemma G.4 (Sequential Strictness and Markov Equilibrium)**.** *Every essentially sequentially strict perfect Bayesian equilibrium is a Markov perfect equilibrium.*

Proof. Consider a t period history h^t. As shown previously, the private history matters, so the focus is on public history. Define $\mathcal{T}$ as the lowest amount of time s such that $(1-\psi)^s \psi = 0$. Under Assumption 2 with $\psi, \epsilon > 0$, the borrower's behavior will not depend on h^t anymore from $t + \mathcal{T} + 1$ periods onward given that its memory is bounded to $\mathcal{T}$ periods back. This means that the lender's value will not depend on h^t from $t + \mathcal{T} + 1$ periods. As a result, if the lender strategy is sequentially strict, then h^t becomes irrelevant from $t + \mathcal{T} + 1$ periods.

What happens in period $t + \mathcal{T}$? This represents the last period in which strategies could be conditioned on h^t. However, at that time, the borrower's maximization problem is independent of h^t as no conditioning is possible next period. In addition, sequential strictness implies that the maximizing action is a singleton. Applying this argument recursively completes the proof. □

I have therefore shown that, under the assumption of fading memory of the borrower, small perturbations in the payoff of the market participants destroy all equilibria except Markov ones. □

Proof of Proposition 4.

From the first-order conditions on consumption, c is increasing in x'. Hence, so does the value of the borrower. In opposition, with a greater c, the value of the lender decreases.

One further shows that the autarkic allocation is not optimal. The proof follows Aguiar et al. (2009). Consider a version of the optimal contract in which the outside option corresponds to the value of permanent autarky is given by

$$v_a(g_t) = \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0)),$$

In autarky, $k = 0$. Suppose there is an $\underline{\underline{x}}$ such that $u(c(g, \underline{\underline{x}})) = u(gf(0))$ for all g. Consider that one increases h by Δh and $u(c(g, \underline{\underline{x}}))$ by $\theta u_c(gf(0))\Delta h$ where

$$\theta = \frac{u_c(g_L f(0))}{u_c(g_L f(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g_L} u_c(g' f(0))} < 1.$$

I defined θ such that the borrower's participation constraint holds. To see this, note that the increase of h increases the borrower's outside option by $u_c(gf(0))\Delta h$ as it can benefit from the additional level of capital before going to autarky forever. However, if the borrower does not choose autarky, its value increases by $\theta(u_c(gf(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g} u_c(g' f(0)))\Delta h \geq u_c(gf(0))\Delta h$ by definition of θ. Hence the borrower's participation constraint is satisfied. Furthermore, the value of the lender changes by

$$\Delta h \frac{1}{1+r}\left(1 - \mathbb{E}_{g'|g_L}\left[\frac{u_c(g' f(0))}{u_c(c(g', \underline{\underline{x}}))}\right]\theta\right) = \Delta h \frac{1}{1+r}(1-\theta) > 0.$$

As a result, the autarkic allocation is not optimal. Notice as well that the law of motion of the relative Pareto weight is given by

$$x'(g) = (1 + \nu(g))\eta x.$$

As one can see, the only source of immiseration for x' is the borrower's relative impatience. The participation constraint can only increase x' over time. Hence, any continuation of an efficient allocation is itself efficient. In other words, what I showed above is that there exits a region of *ex post* inefficiencies in the vicinity of v_a in which the value of both the lender and the borrower can be increased. However, from the law of motion (13), the binding constraint of the borrower only increases the value of the relative Pareto weight. This together with the fact that the initial value of the contract is efficient ensures that the region of inefficiencies is never attained. □

Proof of Proposition 5.

- Part I

  The optimal level of capital is given by

$$g f_k(k(g)) - 1 = \nu(g) u_c(g f(k(g))) g f_k(k(g)) x.$$

  As one can see, as soon as the participation constraint does not bind (i.e. $\nu(g) = 0$), the contract can attain the production-maximizing level of capital $k^*(g)$ such that $g f_k(k^*(g)) = 1$. As soon as this condition is not met, $k < k^*(g)$. Thus, define $x^*(g)$ such that

$$V^b(g, x^*(g)) = V^D(g, k^*(g)).$$

  By the above definition, if $x < x^*(g)$, capital is distorted in state g, while if $x \geq x^*(g)$, capital is at the production-maximizing level. Moreover as $V^D(g_L, k^*(g_L)) < V^D(g_H, k^*(g_H))$, $x^*(g_H) > x^*(g_L)$.

  Observe that ν is the multiplier attached to the borrower's participation constraint. Hence, when this constraint binds, $\nu > 0$, whereas $\nu = 0$ when it does not. In that logic, the larger is ν the more binding is the constraint.

  Assume by contradiction that for $x_1 < x_2$ one has that $k(g, x_1) \geq k(g, x_2)$ for all $g \in G$. Using the first-order condition with respect to capital, one has

$$x = \frac{g f_k(k) - 1}{\nu(g) u_c(g f(k)) f_k(k)}.$$

  Given that $x_1 < x_2$,

$$\frac{f_k(k(g, x_1)) - 1}{\nu(g) u_c(g f(k(g, x_1))) f_k(k(g, x_1))} < \frac{f_k(k(g, x_2)) - 1}{\nu(g) u_c(g f(k(g, x_2))) f_k(k(g, x_1))}$$

  With the assumption that $k(g, x_1) \geq k(g, x_2)$, the above inequality is satisfied only if $\nu(g, x_1) > \nu(g, x_2)$. This is a contradiction as a lower level of capital should relax the borrower's participation constraint and not the opposite.

  The fact that that $k(g, x) > 0$ for all (g, x) follows directly from Proposition 4 which shows that the autarkic allocation is not optimal.$\square$

- Part II

The law of motion of the relative Pareto weight is given by $x'(g) = (1 + \nu(g))\eta x$, while the first-order condition on consumption reads $u_c(c(g)) = \frac{1}{1+\nu(g)}$.

Given the first-order condition, $c(g_L, x) \leq c(g_H, x)$ only when $\nu(g_L) \leq \nu(g_H)$. Assume by contradiction that $\nu(g_L) > \nu(g_H)$. This implies that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Especially, consider the case in which $\nu(g_L) > \nu(g_H) = 0$. In this case,

$$
\begin{aligned}
u(c(g_H)) + \beta V^b(g', x'(g_H, x)) &> V^D(g_H, k), \\
u(c(g_L)) + \beta V^b(g', x'(g_L, x)) &= V^D(g_L, k).
\end{aligned}
$$

Given that $g_H > g_L$ and $\pi(g|g) > 0.5$ for all $g \in G$, $u(g_H f(k(g_H))) > u(g_L f(k(g_L)))$ and $V^D(g_H, k) > V^D(g_L, k)$. This implies that

$$
u(c(g_H)) + \beta V^b(g', x'(g_H, x)) > u(c(g_L)) + \beta V^b(g', g_L, x'(g_L, x)).
$$

which contradicts the fact that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Hence, $\nu(g_L) \leq \nu(g_H)$ which gives $c(g_L, x) \leq c(g_H, x)$ and $x(g_L, x) \leq x(g_H, x)$ as desired. $\square$

– Part III

This proofs is a modified version of Thomas and Worrall (1990, Lemma 4).The value of liabilities in the optimal contract for all $i \in \{L, H\}$ is given by

$$
V^l(g, x) \equiv g f(k) - k - c(g, x) + \frac{1}{1+r} \mathbb{E}_{g'|g} V^l(g', x'(g, x)).
$$

Assume by contradiction that for a given x it holds that $V^l(g_H, x) \leq V^l(g_L, x)$. For $x \geq x^*(g_H)$, one directly reaches a contradiction as $c(g_L, x) = c(g_H, x)$ and $x(g_L, x) = x(g_H, x)$ which implies that $V^l(g_H, x) > V^l(g_L, x)$.

For $x < x^*(g_H)$, consider the pooling allocation in which $u(\ddot{c}(g_H, x)) = u(\ddot{c}(g_L, x)) = u(c(g_H, x))$ and $\ddot{V}^b(g_H, x) = \ddot{V}^b(g_L, x) = V^b(g_H, x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to

$$
\ddot{V}^l(g_H, x) > \ddot{V}^l(g_L, x)
$$

which is a direct contradiction. Hence, $V^l(g_H, x) \geq V^l(g_L, x)$. However, $V^l(g_H, x) = V^l(g_L, x)$ is ruled out by fact that there is no pooling when $x < x^*(g_H)$. $\square$

Proof of Proposition 6.

Recall the law of motion of the relative Pareto weight

$$x'(g) = (1 + \nu(g))\eta x.$$

The motion of the relative Pareto weight is dictated by the relative impatience, η, and the binding participation constraint, ν. I consider two cases. On the one hand, if $\eta < 1$, the relative Pareto weight increases only if $\nu(g) > 0$ is sufficiently large to overcome impatience. As we know, when $x \geq x^*(g)$, $\nu(g, x) = 0$ meaning that impatience eventually dominates the limited commitment issue. On the other hand, if $\eta = 1$ immiseration due to impatience does not exist and the relative Pareto weight remains constant there.

When $\eta = 1$, the upper bound of the ergodic set coincides with the lower bound. As shown in Proposition 5, $x'(g_L, x) \leq x'(g_H, x)$ meaning that the law of motion of the relative Pareto weight in the low productivity state crosses the 45° line before the one of the high productivity state. This coupled with the fact that $x'(g_L, x)$ lies on the 45° when $\nu(g_L, x) = 0$ leads to $x^{ub} = x^{lb}$. Moreover, by definition of $x^*(g_H)$ in Proposition 5, $x^{ub} = x^{lb} = x^*(g_H)$. Conversely, when $\eta < 1$, impatience immiserates the relative Pareto in the low productivity state implying that $x^*(g_H) > x^{ub} > x^{lb}$.

To show the existence of the ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as the midpoint of $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \to \mathbb{R}$ as

$$Q(x, G) = \sum_{g'|g} \pi(g'|g)\mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of g_L, the relative Pareto weight transit to x^{lb} (either through impatience or because $x^{lb} = x^{ub}$). Hence for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(g_L|g_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M < \infty$ g_H, the relative Pareto weight transit to x^{ub} (either through the binding constraint or because $x^{lb} = x^{ub}$) meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}])^M \geq \pi(g_H|g_H)^M > 0$. Setting $\epsilon = \min\{\pi(g_L)^M, \pi(g_H)^M\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. $\square$

Proof of Lemma 3.

Necessity:

> The pricing equation, (2), as well as the capital choice and price conditions, (3) and (4) follow directly from the equilibrium's definition. The budget constraints in the repayment and default states is required by feasibility. The participation constraint, (12), ensures that neither the lender nor the government has an incentive to break the contract and end up in permanent autarky.

Sufficiency:

> Let's rely on trigger strategy (Abreu, 1988). That is, each player is punished by the worst outcome of the game (i.e. permanent autarky which is an equilibrium as shown above) if he or she decides to deviate. Since the outcome satisfies (2), (3) and (4), it is optimal. Also as it satisfies the different budget constraints it is feasible. Finally, no deviations from play is profitable given that (12) holds.

□

Proof of Proposition 7.

The proof of this proposition is by construction. Similar to Dovis (2019), I express the policy functions of the implemented contract as a function of the relative Pareto weight, x, and the productivity state, g. Formally, define

$$\begin{aligned}
&\bar{\tau}, \bar{p} : X \to \mathbb{R}, \\
&\bar{D}, \bar{H}, \bar{M} : G \times X \to \{0, 1\}, \\
&\bar{q}_{st}, \bar{q}_{tl}, \bar{b}_{st}, \bar{b}_{lt} : G \times X \to \mathbb{R}.
\end{aligned}$$

Notice that the tax policy only depends on x and not g as I want to replicate the constrained efficient allocation through the maturity structure of the debt.

Given the timing of actions, the price schedules and bond policies depend on the prospective relative Pareto weights after the productivity shock realizes. Those objects can therefore be rewritten as

$$\begin{aligned}
&\bar{b}_j(g, x) = \bar{b}_j(x'(g, x)), \\
&\bar{q}_j(g, x) = \bar{q}_j(x'(g, x)) \quad \text{for all } j \in \{st, lt\}.
\end{aligned}$$

I first determine the default and *official* buyback policies. Subsequently, I compute the underlying prices. I then define the portfolio of bonds to match the total value of debt $V^l(g,x)$ implied by the constrained efficient allocation. Finally, I determine the optimal tax rate from the optimality conditions of the lender and the domestic firms.

Given Proposition 4, autarky is never optimal in the contract. Hence, the government never enters into default. That is $\bar{D}(g,x) = 0$ for all (g,x). The government will therefore rely on changes in the maturity structure and *official* buyback as in the Markov equilibrium without default incentives. I assume that *official* buybacks arise only if the economy hits the upper bound of the ergodic set,

$$\bar{M}(g,x) = \begin{cases} 1 & \text{if} \quad g = g_H \text{ and } x = x^{ub} \\ 0 & \text{else} \end{cases}$$

Given the above policies, the short-term bond price equates the risk-free price,

$$\bar{q}_{st}(x) = \frac{1}{1+r},$$

while the long-term bond price,

$$\bar{q}_{lt}(x) = \frac{1}{1+r}\mathbb{E}_{g'|g}[1 + \bar{q}_{lt}(x')].$$

Note further that, the long-term bond price has the following properties.

Lemma G.5 (Bond Price)**.** *Under Assumption 1, with* $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$ *and* $\chi \in (0,1)$*, the long-term bond price is the unique fixed point of* $\bar{q}_{lt}$*, is decreasing and is such that*

$$\frac{1}{(1-\chi)r} > \bar{q}_{lt}(x'(g_H,x)) > \bar{q}_{lt}(x'(g_L,x)) > \frac{1}{r}.$$

Proof. Recall that the long-term bond price is given by

$$q_{lt}(g,x) = \frac{1}{1+r}\mathbb{E}_{g'|g}\left[(1 - D(g',x'))\left\{1 + (1 - M(g',x'))q_{lt}(g',x') + M(g',x')q_{lt}^{bb}\right\} + D(g',x')q_{lt}^{D}(g',x')\right],$$

I consider that $D(g',x') = 0$ for all (g',x') and $M(g',x') = 1$ if $g' = g_H$ as well as $x' = x^{ub}$ and $M(g',x') = 0$ otherwise. From Proposition 6, g_H and $x = x^{ub}$ arises with strictly positive probability for any (g,x),

$$\frac{1}{(1-\chi)r} > q_{lt}(g,x) > \frac{1}{r}.$$

Define Q_{lt} as the space of bounded functions $q_{lt} : [\underline{x}, \bar{x}] \rightarrow [0, \frac{1}{(1-\chi)r}]$ and $\mathbb{T} : Q_{lt} \rightarrow Q_{lt}$ as

$$\mathbb{T} q_{lt}(g, x) = \frac{1}{1+r} \sum_{i=1}^{N} \pi(g_i)[1 + q_{lt}(g', x')].$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$ which is increasing as $\mathbb{T}$ maps increasing functions into increasing functions. This implies that $q_{lt}(x'(g_H, x)) \geq q_{lt}(x'(g_L, x))$ as $x'(g_H, x) > x'(g_L, x)$ for all x in the above specified domain. Assume now that there exists a x such that $q_{lt}(x'(g_H, x)) = q_{lt}(x'(g_L, x))$. This requires that $x'(g_H, x)$ and $x'(g_L, x)$ belongs to a subset $[x_t, x_{t+1}]$ where q_{lt} stays constant. Hence, for any $\ddot{x} \in [x_t, x_{t+1}]$, it must be that $x'(g_H, \ddot{x}), x'(g_L, \ddot{x}) \in [x_t, x_{t+1}]$ which is a contradiction as $x'(g_H, x_{t+1}) > x_{t+1}$ when $\eta < 1$. Therefore it must be that $q_{lt}(x'(g_H, x)) > q_{lt}(x'(g_L, x))$. □

Having properly determined the different price schedules, I can now determine the bond holdings and the maturity in order to match the total value of the debt implied by the constrained efficient allocation. Particularly, it must hold that when $x = x^{ub}$,

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + q_{lt}^{bb}], \tag{G.1}$$

$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))]. \tag{G.2}$$

Otherwise, the relationship is given by

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_H, x))],$$
$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma G.5 ensures a unique solution. The maturity structure of the bond portfolio is therefore properly determined.

To complete the proof, I determine the optimal tax rate and the level of intermediate price. From the optimality conditions of the domestic firms and the lender I get

$$g f_k(k) = \frac{1}{1 - \bar{\tau}(x)} = \bar{p}(x). \tag{G.3}$$

Hence, the constrained efficient allocation can be replicated with the above policies for default, *official* buyback, and bond holdings. The optimality conditions of the lender and the domestic firms are satisfied as well as the price schedules.

This concludes the proof as the market allocation satisfies the necessary and sufficient conditions provided in Lemma 3. Especially, I used the budget constraints to determine the optimal bond holdings given the prices computed according to (2). The tax level is set to match the conditions (3) and (4). Finally, the resource constraint and (12) are satisfied as the constrained efficient allocation meet those requirements.

For the second part of the proposition, assume that Part II of Lemma 2 applies. If $\beta < \frac{1}{1+r}$, (11) implements the constrained efficient allocation with Markov strategies under $\overline{B}_{st} = B_{st}(g_H, \overline{B}_{st}, \overline{B}_{lt})$ and $\overline{B}_{st} = B_{lt}(g_H, \overline{B}_{st}, \overline{B}_{lt})$. Otherwise, (9) implements the constrained efficient allocation with Markov strategies.

Note that the borrower would also need to implement the appropriate capital taxation consistent with (G.3). This is possible in a Markov environment as the lender sets p before the borrower decides τ. Hence, the lender can indirectly implement the right τ through p. $\square$

Proof of Lemma D.1.

The law of motion of the relative Pareto weight is given by

$$x'(g) = (1 + \nu(g))\eta x.$$

and the level of consumption by

$$u_c(c(g)) = \frac{1}{x(1 + \nu(g))}.$$

Isolating x leads to

$$x = \frac{1}{u_c(c(g))(1 + \nu(g))}. \tag{G.4}$$

Plugging this back into the law of motion gives

$$x'(g) = (1 + \nu(g))\eta \frac{1}{u_c(c(g))(1 + \nu(g))}.$$

Replacing $x'(g)$ by with the forward equivalent of (G.4) gives

$$\frac{1}{u_c(c(g'))(1 + \nu(g'))} = \eta \frac{1}{u_c(c(g))}.$$

Taking expectations on both sides,

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta\frac{1}{u_c(c(g))},$$

which gives the inverse Euler equation. $\square$

Proof of Proposition F.1.

Following Alvarez and Jermann (2000) we prove the proposition by construction. First, define the asset price as

$$q(g', x'|g) = \frac{\pi(g'|g)}{1+r}\left[1 + \sum_{g''|g'} q(g'', x''|g')\right] \max\left\{\frac{u'(c(g', x'))}{u'(c(g, x))}\eta, 1\right\}.$$

Second, iterating over the budget constraint of the government and applying the transversality condition gives

$$a(g^t) = \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[c(g^{t+j}, x(g^{t+j})) - Y(g^{t+j}, x(g^{t+j}))], \tag{G.5}$$

where, $Y(g^t, x(g^t)) = g_t f(k(g^t, x(g^t))) - k(g^t, x(g^t))$ for all t and g^t. Similarly, iterating over the budget constraint of the lender leads to

$$\begin{aligned} a_l(g^t) &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t) c_l(g^{t+j}, x(g^{t+j})) \qquad \text{(G.6)} \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[Y(g^{t+j}, x(g^{t+j})) - c(g^{t+j}, x(g^{t+j}))] \\ &= -a(g^t). \end{aligned}$$

The market clearing condition implies that $a_l(g^t) + a(g^t) = 0$ for all t and g^t.

To ensure that the capital level is the same as in the constrained efficient allocation, I set the capital tax rate and the level of intermediate price according to

$$g f_k(k) = \frac{1}{1-\tau(a)} = p(a).$$

I now need to establish the correspondence between the initial conditions, x_0, in the contract and the initial conditions in the recursive competitive equilibrium, a_0. Given (G.5) and (G.6) evaluated at $t = 0$, one can determine $\bar{a}(g_0, a_0)$ using the budget constraint

$$c(g_0, x_0) + q(g_0, a_1)(\bar{a}' - a_0) + \sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) \leq g_0 f(k) - p(1-\tau)k + a_0.$$

and the fact that $\sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) = 0$. Once, $\bar{a}(g_0, a_0)$ is determined, one can find the holdings of Arrow securities $\hat{a}'(g', g_0, a_0)$ for all $g' \in \Theta$. We can then retrieve the entire portfolio recursively for $t > 0$.

Third, define the endogenous borrowing limits such that

$$\mathcal{A}(g, k) = a(g, \tilde{x}(g, k)),$$

where $\tilde{x}(g, k)$ is the relative Pareto weight when the participation constraint binds at (g, k). This definition implies that $a'(g', g, a) \geq \mathcal{A}(g', k')$. Hence, the constructed asset holdings satisfy the competitive equilibrium constraints.

Fourth, to ensure optimality of the policy functions by setting

$$\mu_{BC}(g, a) = \frac{1}{x(1 + \nu(g))}$$

Hence, since $c(g, x)$ satisfies the optimality conditions in the Planner's problem, it is also optimally determined in the competitive equilibrium. For the lenders, $c_l(g, x)$ is optimal if the asset portfolio is optimally determined. For this observe that

$$\begin{aligned} q(g', a'(g')|g) &= \frac{1}{1+r}\pi(g'|g)\frac{u'(c(g', a'(g')))}{u'(c(g, a))}\eta\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &> \frac{1}{1+r}\pi(g'|g)\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &\quad \text{if } a'(g', g, a) = \mathcal{A}(g', k'). \end{aligned}$$

Hence the portfolio is optimally determined. We therefore obtain a one-to-one map between x and a for a given g. More precisely, $c(g, a) = c(g, x)$, $c_l(g, a) = T(g, x)$ and $k(g, a) = k(g, x)$. Thus, $W^b(g, a) = V^b(g, x)$ and $W^l(g, a) = V^l(g, x)$. Furthermore, the endogenous limits binds uniquely and exclusively when the participation constraints of the government binds. $\square$