Efficient Sovereign Debt Management in Emerging Economies[*]

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Abstract

This paper assesses the efficiency of the sovereign debt management in emerging economies. I consider a market economy in which a sovereign borrower trades non-contingent bonds of different maturities with two foreign lenders. The borrower is impatient and lacks commitment. I show that the market economy cannot implement the Planner's constrained efficient allocation through defaults but instead by costly debt buybacks. Moreover, as the lenders must enforce those buybacks, the implementation generally requires history-dependent strategies. Nevertheless, interpreting the borrower's impatience as a form of bounded memory, small perturbations in the payoff of the market participants rule out any other strategies than Markov ones. In this case, the Planner's allocation can only be approximated by Markov strategies. I show that emerging economies such as Argentina and Brazil present evidence of such approximation albeit with different outcomes. The multiplicity in outcome comes from the strategic interaction between the two lenders. I find that Brazil has a more efficient sovereign debt management than Argentina.

Keywords: sovereign debt, default, maturity, buyback, Markov, multiplicity, emerging economies, constrained Pareto efficiency

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1 Introduction

Argentina and Brazil experienced an economic crisis in the late 1990s. Maturity shortened in both countries during that period. Argentina defaulted in 2001 and got excluded from the international financial market until 2006. In opposition, Brazil did not default and started a sovereign debt buyback program in the early 2000s. Buybacks had the peculiarity that they entailed a premium and were therefore costly for the Brazilian government.[1] In light of this historical episode, the question that arises is twofold. What can explain these opposite debt management? And which of these two debt management is more efficient? I analyze the role of the lenders in the multiplicity of equilibria and the role of maturity, buyback and default in implementing or approximating the constrained (Pareto) efficient allocation.

The literature on sovereign debt focuses on the borrower's decision to default. Differences between countries come from differences in terms of preferences, default costs or productivity shocks. My view is that Argentina and Brazil were not dissimilar in terms of economic fundamentals at the end of the 1990s. What truly differentiates those two countries is the strategic interaction between the lenders.

In terms of maturity, the literature on fiscal policy with commitment suggests to trade non-contingent bonds of different maturities to replicate the return of Arrow securities.[2] The portfolio of bonds emanating from this approach is however empirically implausible.[3] To reconcile the model's prediction with the data, the literature has introduced different frictions such as limited commitment and trade constraints.[4] I provide an alternative explanation: market participants often lack the strategical sophistication required to implement the aforementioned maturity management. Focusing on emerging economies, one ought to consider Markov strategies under which market participants can usually only approximate the return of Arrow securities. Such an approximation is consistent with the data, though.

In terms of default and buyback, the literature on sovereign debt argues that it might be optimal to conduct the former as this provides a source of state contingency, while the latter is suboptimal as it only benefits the lenders.[5] I argue the opposite. A default generates deadweight losses which impact both the borrower and the lenders. Hence, it is Pareto

[1] Argentina conducted buybacks at a discount (i.e. below par) which usually correspond to a default in the form of distressed debt exchange.

[2] See Kreps (1982), Angeletos (2002) and Barro (2003).

[3] Buera and Nicolini (2004) and Faraglia et al. (2010) show that the borrower ought to *sell* long-term bonds and *buy* short-term bonds in the magnitude of several multiples of GDP.

[4] Debortoli et al. (2017) introduce limited commitment in fiscal policy, Faraglia et al. (2019) limit the extent of debt repurchase and reissuance and Kiiashko (2022) adds limited commitment in repayment.

[5] See notably Grossman and Van Huyck (1988), Adam and Grill (2017), Roettger (2019) and Hatchondo et al. (2020a) for defaults as a source of risk sharing and Bulow and Rogoff (1988, 1991) and Cohen and Verdier (1995) for buybacks as suboptimal policy.

improving to avoid such an event. In opposition, buybacks at a premium can generate state contingency without causing the aforementioned deadweight losses.[6] As the bond price incorporates such premium, it is possible to generate state-contingent capital losses and gains with the appropriate buyback policy. As a result, the optimal sovereign debt management consists of no default and occasional costly buybacks.

I consider a small open economy populated by two foreign lenders and one sovereign borrower. The lenders supply the capital input and buy bonds issued by the borrower. Conversely, the borrower takes decisions on behalf of the small open economy, runs the production technology and issues non-contingent defaultable bonds of different maturities. Domestic production is subject to persistent productivity shocks and the borrower is impatient. There is one friction: the borrower cannot commit to repay the lenders.

Relying on the concept of sustainable equilibria, I first show that the Second Welfare Theorem holds in the market economy. For this, I derive an optimal contract in which a Planner allocates resources between the borrower and the lenders. This defines a constrained efficient allocation which features state-contingent debt relief and production distortions. Particularly, the borrower records capital gains in low productivity states and capital losses otherwise. In addition, when the borrower receives sufficiently high utility in the contract, the contract can sustain the productivity-maximizing level of capital. Otherwise, the threat of autarky fades and the Planner reduces the level of capital to relax the participation constraint. The Planner never finds optimal to set capital to zero, though.

I then implement the optimal contract in the market economy. Given that the Planner never distorts capital to zero, defaults – which imply markets exclusion – cannot implement the constrained efficient allocation. Instead, the borrower conducts buybacks at a premium. Such buybacks implicitly introduce state contingency in the bond contract as the bond price incorporates the premium paid. They occur in high productivity states implying that the price of long-term bonds increases after the realization of high productivity shocks. This in turn increases the value of outstanding long-term debt resulting in capital losses for the borrower. The opposite happens after the realization of low productivity shocks. Thus, costly buybacks can generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract.

The Second Welfare Theorem holds but the First Welfare Theorem generally fails even when I restrict the analysis to Markov equilibria. I first link the study of emerging economies with the concept of Markov equilibrium. Given that such economies suffer from important political instability, I interpret the borrower's impatience as a form of bounded memory. In addition, as emerging economies' fundamentals are difficult to assess for foreign creditors, I

[6] I consider an exogenous premium on buybacks and provide foundations for such cost in Appendix B.

introduce small and independent perturbations in payoffs.[7] Under these two assumptions, I show that all sustainable equilibria are Markov. Hence, the study of emerging economies ought to be limited to Markov equilibria.

I then show that there are multiple Markov equilibria in the market economy. The multiplicity comes from strategic interactions between the lender holding legacy debt and the other lender. The legacy lender is willing to avoid default and supports costly buybacks while the other lender is indifferent. I find two Markov equilibria. The first one is a Markov equilibrium without default in which premium-bearing buybacks can occur on equilibrium path. The second one is a Markov equilibrium with default in which premium-bearing buybacks never occur as in Arellano and Ramanarayanan (2012). I relate the second Markov equilibrium to the experience of Argentina and the first Markov equilibrium to the experience of Brazil. Hence, the difference between the two countries can be solely attributed to the lenders' interaction.

Beside suffering from multiplicity, Markov strategies generally fail to implement the constrained efficient allocation. The reason is that costly buybacks need to be enforced by the lenders. In a Markov equilibrium, such enforcement is possible only if the borrower does not issue assets and buybacks are not too costly. However, to replicate the Planner's allocation, the borrower needs to hold short-term assets unless the buyback premium is sufficiently large. Particularly, I find that Markov strategies fail to implement the Planner's allocation under empirically plausible buyback premia. Thus, being restricted to Markov strategies, emerging economies can only approximate the constrained efficient allocation.

To gauge the goodness of the Markov approximation, I calibrate the Markov equilibrium with default to match moments of the Argentine economy over the period 1995-2019. The calibrated model fits the data well and features default episodes in which indebtedness increases with respect to output and maturity shortens. Conversely, during restructurings, the level of debt remains substantial and the maturity lengthens. In addition, using the calibration for Argentina, I find that the Markov equilibrium without default is quantitatively close to Brazil. I therefore interpret Brazil as the counterfactual of Argentina with costly buybacks and without defaults. This supports the fact that the difference between Argentina and Brazil can be solely attributed to the lenders. Finally, in line with the literature on fiscal policy with commitment, I find that the implementation of the constrained efficient allocation generates unrealistic debt portfolios unlike Markov equilibria.

I then compare the Markov equilibria with the implementation of the optimal contract through various simulation exercises. Relying on costly buybacks instead of defaults im-

[7] See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019) for the political instability and Tsyrennikov (2013) and Morelli and Moretti (2021) for the lack of financial transparency.

plies important welfare gains for both the borrower and the lenders. In that logic, the Markov equilibrium without default is quantitatively the closest to the constrained efficient allocation. Nonetheless, it is far from the Pareto frontier indicating that the Markov approximation remains crude. Thus, the limitation to Markov strategies in emerging economies is very costly in terms of welfare.

The paper is organized as follows. I review the literature in Section 2. The economic environment is in Section 3 and the market economy in Section 4. I present the constrained efficient and the Markov debt management in Sections 5 and 6, respectively. The calibration and quantitative analyses are in Section 7. Finally, I conclude in Section 8.

2 Literature Review

The paper combines elements of the literature on sovereign defaults and buybacks with elements of the literature about optimal contracts and their implementation.

The literature on sovereign defaults assumes that markets are incomplete and agents follow Markov strategies (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008).[8] There, the borrower has access to only non-contingent claims and can obtain limited state contingency through defaults. My study is the closest to Arellano and Ramanarayanan (2012) and Niepelt (2014) given that I adopt two bonds with different maturities and to Mendoza and Yue (2012) given that the default cost is endogenous. Also, similar to Kovrijnykh and Szentes (2007), I consider strategic lenders. I contribute to this literature in two ways. First, I show that the reliance on defaults to obtain state contingency is inefficient. Second, I provide a foundation for the use of Markov strategies interpreting the assumption of impatience as evidence of bounded memory and then applying the result of Bhaskar et al. (2012) and Angeletos and Lian (2021). This second contribution relates to Krusell and Smith (1996) and Krusell et al. (2002) as it connects the equilibrium outcome with the sophistication of agents' strategies.[9]

On a similar note, this paper relies on costly buybacks as a way to implement the constrained efficient allocation. It therefore relates to the seminal contribution of Bulow and Rogoff (1988, 1991) who document that buybacks are suboptimal as they increase the recovery value per unit of bond and therefore fail to reduce indebtedness. In light of this, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Similarly, Aguiar et al. (2019) find that buybacks reduce welfare as they shift the maturity structure and therefore affect the default risk. In opposition, Rotemberg (1991) shows that buybacks

[8]See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

[9]See also the concept of robust prediction in Passadore and Xandri (2021).

can be advantageous to all parties as they lower the bargaining costs. Moreover, Acharya and Diwan (1993) find that buybacks provide a positive signal about the borrower's willingness to repay. Finally, Kovrijnykh and Szentes (2007) find that buybacks are socially efficient despite the fact that only lenders benefit from it *ex post*. My analysis goes in this direction as it emphasizes the efficiency of buybacks as a source of risk sharing between the borrower and the lenders.

Showing multiplicity in the Markov equilibrium, this study also relates to the work of Calvo (1988), Cole and Kehoe (2000), Conesa and Kehoe (2017), Lorenzoni and Werning (2019) and Aguiar et al. (2022).[10] Unlike Cordella and Powell (2021), I relate the enforcement of a no-default borrowing limit to strategic decisions of the lenders rather than commitment. This generates multiple equilibria different than in Alvarez and Jermann (2000) and Kirpalani (2017). Especially, multiplicity comes from the attitude of the lenders towards defaults and costly buybacks.

The paper derives an optimal contract between foreign lenders and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). My study accounts for limited commitment similar to Aguiar et al. (2009) and is close to Kehoe and Perri (2002) and Restrepo-Echavarria (2019) as it relies on the approach of Marcet and Marimon (2019) with the difference that I implement the contract in a market economy.[11]

The paper therefore addresses the literature on optimal contract's implementation. Note that I discuss the following studies in more details in Appendix A. Unlike Aguiar et al. (2019) and Müller et al. (2019), my implementation is not generally Markov. On the one hand, Aguiar et al. (2019) account for multiple maturities but consider a Planner's problem which has no participation constraint, unlike my Planner problem. On the other hand, Müller et al. (2019) use preemptive restructurings and GDP-linked bonds, whereas I rely on the maturity structure. An alternative to this approach is Dovis (2019) who develop an implementation through partial defaults and an active debt maturity management. He builds on Angeletos (2002) and Buera and Nicolini (2004) who show that one can replicate the state-contingency of Arrow securities using non contingent bonds of different maturities.[12] My implementation is the closest to Dovis (2019) with the difference that I rely on debt buybacks without defaults and haircuts. Moreover, similar to Hatchondo et al. (2020a), I connect my implementation to the Markov allocation. Especially, as buybacks need to be enforced, I explain why and when history dependence matters.

[10]See also Stangebye (2020), Galli (2021), Corsetti and Maeng (2021) and Bloise and Vailakis (2022).

[11]The other difference is that I adopt a capital depreciation rate of 1 which simplifies the equilibrium computation and the proofs.

[12]See also the references in footnotes 2, 3 and 4.

3 Environment

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The small open economy is populated by a benevolent government and a large number of homogenous households which own domestic firms, while two foreign lenders invest in the small open economy.[13]

Foreign lenders are risk neutral, strategic and break even in expectations. They discount the future at rate $\frac{1}{1+r}$ with r being the exogenous risk-free rate. The representative domestic household discounts the future at rate $\beta \leq \frac{1}{1+r}$. Preference over consumption is represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t corresponds to the consumption at time t. The instantaneous utility function $u(\cdot)$ is continuous, increasing and concave.

Domestic households have access to a production technology, $F(k_t, l_t)$, where l_t is the labor input and k_t is the capital input at time t. They are endowed with one unit of labor in every t. The lenders supply the capital input, k_t, at price p_t in every t. I denote $f(k_t) \equiv F(k_t, 1)$ and assume it is continuous, increasing, concave, satisfies the Inada condition, $\lim_{k \to 0} f_k(k) = \infty$, and $f(0) > 0$. For tractability, capital depreciates at rate 1.

Domestic production is subject to a shock g_t which takes value on the discrete set $G \equiv \{g_L, g_H\}$ with $0 < g_L < g_H$ and follows a Markov chain of order one with $\pi(g_{t+1}|g_t)$ corresponding to the probability of drawing g_{t+1} at date $t+1$ conditional on drawing g_t at t. I further assume that shocks are persistent meaning that $\pi(g|g) > 0.5$ for all $g \in G$. The fact that $f(\cdot)$ is concave implies that there exists a level $k^*(g_t)$ which maximizes the net production such that $g f_k(k^*(g_t)) = p_t$.

The government is benevolent and takes the decision on behalf of the small open economy.[14] There is a tax on the import of capital made by domestic firms at rate $\tau_t = 1 - \frac{1}{p_t}$ meaning that $p_t(1 - \tau_t)k_t = k_t$.[15] Thus, the household's after-tax income is given by

$$y(g_t, k_t) \equiv g_t F(k_t, l_t) - k_t.$$

The government has access to bond contracts with two different maturities: short-term and long-term. The short-term bond $b_{st,t+1}$ is a one-period bond with unit price $q_{st,t}$ which pays a coupon of one next period. The long-term bond $b_{lt,t+1} \leq 0$ is a consol with unit price $q_{lt,t}$ which pays a coupon of one every period. I denote debt as a negative asset meaning that $b_j < 0$ is a debt, while $b_j > 0$ is an asset for all $j \in \{st, lt\}$. The government can hold

[13]The present environment is similar to the one of Quadrini (2004), Aguiar et al. (2009) and Dovis (2019).

[14]As in Aguiar et al. (2009) and Dovis (2019), I combine the income of households and government together. Households provide labor inelastically and receive lump sum transfers from the government.

[15]This technical assumption is necessary for the implementation of the constrained efficient allocation in the market economy. Having τ_t a choice variable would make taxation time inconsistent in Markov equilibria.

short-term assets but not long-term assets.

A bond auction determines the new issuance of bond contracts. Similar to Kovrijnykh and Szentes (2007), the two lenders simultaneously offer a couple $(b_{st,t+1}, b_{lt,t+1})$ during the auction in t and the borrower chooses among the two offers. For the long-term bond, all outstanding bonds have to be repurchased before issuing new ones. This implies that only one of the two lenders is holder of legacy claim each period. I call this lender the *legacy* lender and the other the *new* lender.

The bond contract specifies the conditions for default and buyback. A default corresponds to a missed coupon payment which triggers exclusion from both the capital and the bond markets. Nevertheless, the borrower can regain access to those markets with probability $\lambda \geq 0$. There is no recovery value. Conversely, a buyback is defined as any new long-term debt issuance below the initial amount outstanding, i.e. $b_{lt,t+1} \geq b_{lt,t}$. There are two types of buybacks: *official* and *unofficial*. In the latter, the government repurchases the outstanding debt at $q_{lt,t}$.[16] In the former, it repurchase the outstanding debt at $q^{bb}_{lt,t} = q_{lt,t} + \chi$ where $\chi > 0$ is the buyback premium. Only the *legacy* lender can make an offer entailing χ. Note that I provide foundations for $\chi > 0$ in Appendix B.

The government cannot commit to pay the lenders. In particular, it cannot commit to pay the coupon due every period and to conduct *official* instead of *unofficial* buybacks.

If necessary, the lenders can coordinate their beliefs with a sunspot $\upsilon_t \in \{0, 1\}$ which follows a Markov chain of order one with $\pi_\upsilon(\upsilon_{t+1}|\upsilon_t)$. The exogenous shock space is therefore given by $s_t \equiv (g_t, \upsilon_t)$.

The timing of actions is the following. At the beginning of each period $t \geq 0$, g_t and υ_t realize and the lenders jointly supply k_t. Subsequently, domestic production takes place, capital depreciates and is taxed. The government decides whether to default. Conditional on no default, the bond auction determines $b_{st,t+1}$ and $b_{lt,t+1}$.

4 The Market Economy

In this section, I define the set of sustainable equilibrium outcomes in the market economy following the approach of Abreu (1988) and Chari and Kehoe (1990). Keeping track of the entire history of play, it is possible to sustain multiple equilibrium outcomes.

[16] *Unofficial* buybacks are a version of what Cohen and Verdier (1995) call a "secret buyback".

4.1 The Government's problem

Define $D_t \in \{0,1\}$ as the default policy at time t. If $D_t = 0$, the government repays, while if $D_t = 1$, it defaults. Correspondingly, as market re-access is stochastic, let the default status $\mathbb{I}_{D,t}$ be 1 if the government is in default at the beginning of t and 0 otherwise. Further define $M_t \in \{0,1\}$ as the *official* buyback policy at time t. If $M_t = 1$, the government buys its debt back paying the premium χ, while if $M_t = 0$, it does not.

In addition, define the government's choice set as $\mathcal{C}_{b,t} = \{D_t, M_t, b_{st,t+1}, b_{lt,t+1}\}$ and the government's strategy as σ_b. Furthermore, let $h^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t, k_t, \mathcal{C}_{b,t})$ be the history up to time t taking the initial debt $\{b_{j,0}\}_{j\in\{st,lt\}}$ as given. Due to the specific timing of actions, further define the history of the two lenders and the government as $h_l^t = (h^{t-1}, s_t, \mathbb{I}_{D,t})$ and $h_b^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t, k_t)$, respectively. I also define the history for the choice of capital as $h_k^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t)$. Finally, denote the value of the two lenders and the government after any specific history by $W^l(h_l^t)$ and $W^b(h_b^t)$, respectively.

In the case in which the government decides to repay (i.e. $D_t = 0$), it determines its consumption and prospective borrowing given the realization of the history h_b^t. In the case of no *official* buyback (i.e. $M_t = 0$), the budget constraint reads

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})(b_{lt,t+1} - b_{lt,t}) = y(g_t, k_t) + b_{st,t} + b_{lt,t}.$$

There is no restriction on the issue of long-term debt meaning that the government can potentially conduct *unofficial* buybacks. Conversely, in the case of an *official* buyback (i.e. $M_t = 1$), budget constraint is given by

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t) + b_{st,t} + b_{lt,t}(1 + q_{lt}^{bb}) \ \wedge \ b_{lt,t+1} \geq b_{lt,t}.$$

The government pays the premium χ and issues new long-term debt such that $b_{lt,t+1} \geq b_{lt,t}$. Conversely, if the government decides to default (i.e. $D_t = 1$), it gets excluded from the markets and consumes

$$c_t = g_t f(k_t).$$

Neither capital nor debt are repaid. Due to the specific timing of capital, the government enjoys $k_t \geq 0$ in the first period of autarky and then $k_t = 0$. The capital market exclusion therefore forms an endogenous cost of default. Upon markets re-access,

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t).$$

There is no recovery value of debt. Thus, after any history h_b^t, the optimal strategy of the government, σ_b, is the solution of

$$W^b(h_b^t) = \max_{\{\mathcal{C}_{b,t}\}_{t=0}^{\infty}} \; u(c_t) + \beta \mathbb{E}\Big[W^b(h_b^{t+1}) \Big| h_b^t, \mathcal{C}_{b,t} \Big], \tag{1}$$

subject to the budget constraint.

4.2 Sustainable equilibria

Having derived the government's problem, I can define and characterize the set of sustainable equilibria. The lenders break even meaning that in expectations they make zero profit. The price of one unit of bond is given by,

$$\begin{aligned} q_{lt}(h^t) &= \frac{1}{1+r}\mathbb{E}\Big[(1-D(h^{t+1}))\Big\{1+(1-M(h^{t+1}))q_{lt}(h^{t+1})+M(h^{t+1})q_{lt}^{bb}\Big\}\Big|h^t\Big], \\ q_{st}(h^t) &= \frac{1}{1+r}\mathbb{E}\Big[(1-D(h^{t+1}))\Big|h^t\Big]. \end{aligned} \tag{2}$$

As I rely on the history of play, it is sufficient to characterize the problem from the borrower's perspective. In Section 6, I develop the lenders' perspective.

Definition 1 (Sustainable Equilibrium). *Given $\{b_{j,0}\}_{j\in\{st,lt\}}$, a sustainable equilibrium in this environment consists of strategy for the government, σ_b, policy for the firm's capital, k as well as price schedule for capital, p, and for bonds, q_{st} and q_{lt} such that*

1. *Taking p, q_{st} and q_{lt} as given, σ_b is the solution to (1).*

2. *Taking p as given, the choice of capital by domestic firms is such that*

$$\mathbb{E}\Big[u_c(c(h_b^t))(gf_k(k(h^t))-p(h^t))\Big|h_k^t\Big] = 0. \tag{3}$$

3. *The price of capital is consistent with*

$$\max_{k_t} \; \mathbb{E}\Big[p(1-\tau)k_t - k_t\Big|h_l^t\Big]. \tag{4}$$

4. *The bond prices satisfy (2).*

Following the approach of Abreu (1988) and Chari and Kehoe (1990), I characterize the set of outcomes that can be sustained in equilibrium using reversion to the worst equilibrium. The following lemma shows that permanent autarky is the worst equilibrium outcome. All proofs are in Appendix K.

Lemma 1 (Worst Equilibrium Outcome). *In this environment, the worst possible outcome is permanent autarky which can be supported as an equilibrium.*

Keeping track of the entire history of play and relying on trigger strategies, I can sustain multiple equilibrium outcomes. The only two requirements are: the allocation and price satisfy the optimality conditions for all market participants and the borrower's value cannot be lower than the value of the worst equilibrium.

Lemma 2 (Sustainable Outcomes). *Given $\{b_{j,0}\}_{j\in st,lt}$, an allocation $\{\mathcal{C}_{b,t}, k_t\}_{t=0}^{\infty}$ with prices $\{q_{st,t}, q_{lt,t}, p_t\}_{t=0}^{\infty}$ is the outcome of a sustainable equilibrium if and only if it satisfies (1), (2), (3), (4) and for every t, h_b^t, $W^b(h_b^t) \geq \sum_{j=t}^{\infty}\beta^{j-t}\sum_{g^j}\pi(g^j|g_t)u(g_j f(0))$.*

In what follows, I show that the Second Welfare Theorem holds in the market economy – Section 5 – but the First Welfare Theorem generally fails even when I restrict the analysis to Markov equilibria – Section 6.

5 Constrained Efficient Debt Management

This section presents the constrained efficient debt management. I first derive an optimal contract between the borrower and the two lenders and subsequently characterize the underlying constrained efficient allocation before implementing it the market economy.

5.1 The optimal contract

The optimal contract is the outcome of a problem in which a Planner allocates capital and consumption to maximize the lenders' and the borrower's weighted utility subject to a participation constraint.[17] The participation constraint accounts for limited commitment in repayment (Thomas and Worrall, 1994). Denoting g^t as the history of realized value of g at time t, it must hold that for all t and g^t

$$\sum_{j=t}^{\infty}\beta^{j-t}\sum_{g^j}\pi(g^j|g_t)u(c(g^j)) \geq V^D(g_t, 0, k_t). \tag{5}$$

If the borrower breaks the contract, it is sent to autarky for some time but can regain access to the market with probability λ and resumes the Markov equilibrium specified in Section 6 where the *new* lender always satiates the borrower's demand for bonds.[18] I write

[17] As I consider a Planner's perspective, I disregard the sunspots in this section.

[18] As we will see in Section 6, in this specific Markov equilibrium, the sunspot is irrelevant. I can therefore write g_t instead of s_t in the outside option.

$V^D(g_t, \mathbb{I}_{D,t}, k_t)$ to make explicit the dependence on k_t. As a result, the participation constraint ensures that the borrower's value of remaining in the contract is at least as large as the value of opting out.

Given the above constraint, the optimal contract between the borrower and the lenders in sequential form is the result of the following Planner's maximization problem

$$\max_{\{k(g^t),c(g^t)\}_{t=0}^{\infty}} \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) u(c(g^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) T(g^t) \tag{6}$$
$$\text{s.t.} \quad (5),\ T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t) \text{ for all } g^t, t \text{ with } (\mu_{b,0}, \mu_{l,0}) \geq 0 \text{ given.}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial non-negative Pareto weights assigned by the Planner to the borrower and the lenders, respectively. The above maximization problem combines the utility function $u(\cdot)$ with the production function $f(\cdot)$ and therefore might not be convex.

Assumption 1 (Convexity)**.** *Define the optimal level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ and $h := gf(k) - k$ for $k \in [0, k^*(g)]$ with $h^*(g) = gf(k^*(g)) - k^*(g)$. Let $K(h)$ denote the inverse mapping from $[0, h^*(g)]$ to $[0, k^*(g)]$ such that $k = K(h)$. For all $g \in G$, $u(gf(k(h)))$ is convex in h for $h \in [0, h^*(g)]$.*

Following, Aguiar et al. (2009), Assumption 1 ensures that there is no need for randomization. I now derive the recursive formulation of the above maximization problem. Following Marcet and Marimon (2019), I reformulate (6) as a saddle-point Lagrangian problem,

$$\begin{aligned}
\mathcal{SP} \min_{\{\gamma(g^t)\}_{t=0}^{\infty}} \max_{\{k(g^t),c(g^t)\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) \mu_{b,t}(g^t) u(c(g^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) \mu_{l,t}(g^t) T(g^t) \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) \gamma(g^t) \Big[\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(c(g^j)) - V^D(g_t, 0, k_t) \Big] \\
\text{s.t.} \quad & T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t), \\
& \mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t) \text{ and } \mu_{l,t+1}(g^t) = \mu_{l,t}(g^t) \text{ for all } g^t, t \\
& \text{with } \mu_{b,0}(g_0) \equiv \mu_{b,0} \text{ and } \mu_{l,0}(g_0) \equiv \mu_{l,0} \text{ given.}
\end{aligned}$$

In this formulation, $\gamma(g^t)$ denotes the Lagrange multiplier attached to the participation constraint at time t. As the value of the borrower appears in both the Planner's objective function and the participation constraint, there is a direct link between $\mu_{b,t}(g^t)$ and $\gamma(g^t)$. More precisely, the borrower's Pareto weight evolves according to $\mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t)$, while the lenders' Pareto weight, $\mu_{l,t}(g^t)$, remains constant.

Following Marcet and Marimon (2019), the saddle-point Lagrangian problem is homogenous of degree one in $(\mu_{b,t}(g^t), \mu_{l,t}(g^t))$. I can therefore redefine the contracting problem over $(x_t(g^t), 1)$ where $x_t(g^t) = \frac{\mu_{b,t}(g^t)}{\mu_{l,t}(g^t)}$ corresponds to the relative Pareto weight – i.e. the Pareto weight attributed to the borrower relative to the lenders. Given that $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} \leq \infty$. Moreover,

$$x_{t+1}(g^t) = (1 + \nu(g^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}}, \tag{7}$$

where $\eta \equiv \beta(1+r) \leq 1$ corresponds to the borrower's impatience relative to the lenders and $\nu(g^t) \equiv \frac{\gamma(g^t)}{\mu_{b,t}(g^t)}$ represents the normalized multiplier attached to the participation constraint. Following Marcet and Marimon (2019), the state vector for the problem reduces to (g, x) and the Saddle-Point Functional Equation is given by

$$\begin{aligned} FV(g,x) = \mathcal{SP} \min_{\nu(g)} \max_{k(g),c(g)} \; & x\Big[(1+\nu(g))u(c(g)) - \nu(g)V^D(g,0,k)\Big] \\ & + T(g) + \frac{1}{1+r}\sum_{g'} \pi(g'|g) FV(g', x') \\ \text{s.t.} \quad & x'(g) = (1+\nu(g))\eta x, \; T(g) = gf(k(g)) - c(g) - k \; \forall g. \end{aligned} \tag{8}$$

The value function takes the form of $FV(g,x) = xV^b(g,x) + V^l(g,x)$ with $V^b(g,x) = u\left(c(g)\right) + \beta \mathbb{E}_{g'|g}\left[V^b\left(g', x'\right)\right]$ being the value of the borrower and $V^l(g,x) = T(g) + \frac{1}{1+r}\mathbb{E}_{g'|g}\left[V^l\left(g', x'\right)\right]$ being the value of the lenders. I obtain the optimal consumption and capital policies by taking the first-order conditions in (8)

$$u_c(c(g)) = \frac{1}{x(1+\nu(g))} \quad \text{and} \quad gf_k(k(g)) - 1 = \nu(g)u_c(gf(k(g)))gf_k(k(g))x.$$

In terms of consumption, the binding participation constraint of the borrower (i.e. $\nu(g) > 0$) induces an increase in consumption. Regarding capital, the economy does not reach the production-maximiging level of capital $k^*(g)$ as long as the participation constraint binds in g. In the next subsection, I formalize this argument in Proposition 2.

5.2 Contract properties

I characterize the main properties of the contract in terms of Pareto frontier and risk sharing. Additional characterization can be found in Appendix C.

I start with the definition of two threshold values for the relative Pareto weight: the one for which the borrower's participation constraint binds with $k = 0$ and with $k = k^*(g)$.

Definition 2 (Threshold)**.** *Define $x_a(g)$ such that $V^b(g, x_a(g)) = V^D(g, 0, 0)$ and $x^*(g)$ such that $V^b(g, x^*(g)) = V^D(g, 0, k^*(g))$.*

In words, $x^*(g)$ is the lowest relative Pareto weight that can sustain $k^*(g)$. Conversely, $x_a(g)$ is the weight associated with the autarkic allocation (i.e. $k = 0$) and is therefore the lowest possible weight in the contract. I show next that $x_a(g)$ is never attained.

Proposition 1 (Efficiency)**.** *Under Assumption 1, the autarkic allocation is not optimal meaning that $x \in \tilde{X} \equiv [\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. Moreover, $V^l(g, x)$ is strictly decreasing, while $V^b(g, x)$ is strictly increasing in $x \in \tilde{X}$ for all $g \in G$.*

The proposition is made of two parts. First, autarky (i.e. $k = 0$) is not optimal.[19] Due to the Inada condition on the production function, there are always strictly positive gains from trade between the borrower and the lenders when k is close to zero. This already means that defaults – which imply markets exclusion – cannot implement the Planner's constrained efficient allocation. Second, the proposition states that the optimal contract is constrained efficient which makes it the best achievable outcome in this environment.

The following proposition highlights the main properties of the constrained efficient allocation. The contract features production distortions, risk sharing across states and state-contingent debt relief.

Proposition 2 (Constrained Efficient Allocation)**.** *Under Assumption 1,*

I. (Production)*. $k(g, x) = k^*(g)$ for $x \geq x^*(g)$ and $x^*(g_H) > x^*(g_L)$. Conversely, for all $x, \ddot{x} \in \tilde{X}$ with $x^*(g) > x > \ddot{x}$, $0 < k(g, \ddot{x}) < k(g, x) < k^*(g)$.*

II. (Risk-Sharing)*. $c(g_L, x) < c(g_H, x)$ and $x'(g_L, x) < x'(g_H, x)$ for all $x < x^*(g_H)$ and $c(g_L, x) = c(g_H, x)$ and $x'(g_L, x) = x'(g_H, x)$ otherwise.*

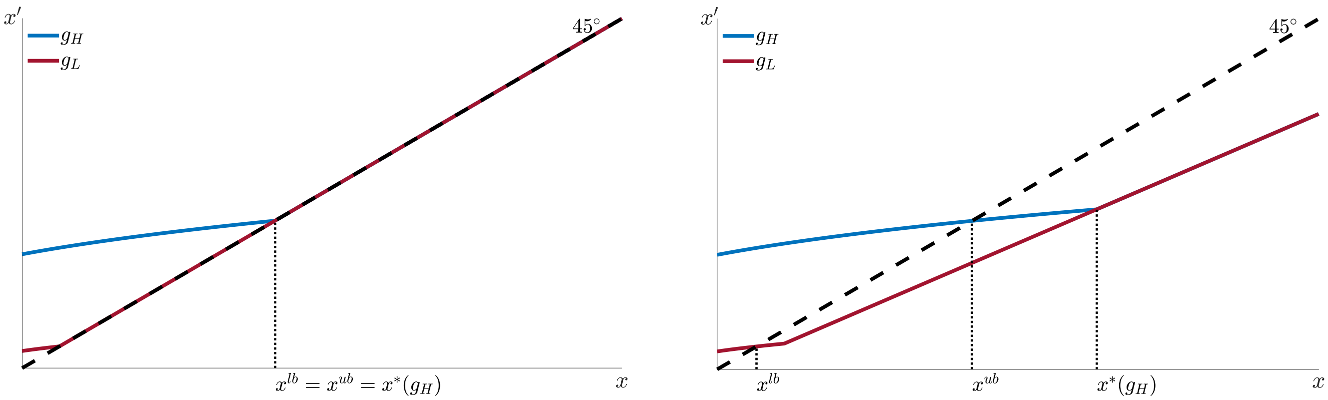
III. (Liabilities)*. $V^l(g_L, x) < V^l(g_H, x)$ for all $x \in \tilde{X}$.*

Part I of the above proposition states that the production-maximizing level of capital $k^*(g)$ such that $g f_k(k^*(g)) = 1$ is attained only if the relative Pareto weight, x, is above a certain threshold. Capital distortion is a consequence of the binding participation constraint (5). As the autarky value depends on the level of capital in the economy, the Planner finds optimal to reduce k to relax the constraint. It continues to decrease k as long as x decreases but never finds optimal to set $k = 0$ following Proposition 1.

[19] In Appendix D, I show that if one introduces domestic capital, this result continues to hold. Note further that the depreciation rate of capital is irrelevant here.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind in all productivity states. Otherwise, the Planner provides more consumption and a greater continuation value when the high productivity state realizes.

Part III relates to the liabilities of the borrower. In this environment, $T(g)$ corresponds to the borrower's current account balance. Hence, the value of the two lenders represents the net foreign asset position in the contract. A positive value of $V^l(g,x)$ indicates the extent to which the borrower is indebted. The proposition states that the liabilities increase when g is high. This implies that the Planner adopts a state-contingent policy as it provides debt relief in low productivity states. This state contingency will be replicated through *official* buybacks in the market economy.



(a) Patience ($\eta = 1$) (b) Impatience ($\eta < 1$)

Note: The figure depicts the law of motion of the relative Pareto weight in the case of a patient (i.e. $\eta = 1$) and impatient (i.e. $\eta < 1$) borrower. The blue line corresponds to the law of motion in g_H and the red line to the law of motion in g_L. The black dotted line represents the 45° line. x^{lb} and x^{ub} correspond to the lower and upper bounds of the ergodic set, respectively. $x^*(g)$ corresponds to the weight at which the participation constraint ceases to bind in g.

Figure 1: Steady State Dynamic

Having determined the constrained efficient allocation, I now show that the long-term contract is characterized by an ergodic set of relative Pareto weights.[20]

Proposition 3 (Steady State)**.** *A steady state is defined by an ergodic set of relative Pareto weights $x \in [x^{lb}, x^{ub}] \subset \tilde{X}$. Under Assumption 1, it holds that $x'(g_H, x^{ub}) = x^{ub}$ and $x'(g_L, x^{lb}) = x^{lb}$ and*

I. If $\eta = 1$, then $x^{lb} = x^{ub} = x^*(g_H)$.

II. If $\eta < 1$, then $x^{lb} < x^{ub} < x^*(g_H)$.

[20]The term ergodic refers to the fact that the relative Pareto weights in this set are aperiodic and recurrent with non-zero probability.

The proposition states that whenever the borrower is patient (i.e. $\eta = 1$), the steady state does not display any dynamic. Conversely, whenever the borrower is relatively impatient (i.e. $\eta < 1$), the steady state is dynamic. This dynamic is however bounded below by x^{lb} and above by x^{ub}. For instance, after a sufficiently long series of g_L (g_H), the contract hits x^{lb} (x^{ub}). It then stays there until g_H (g_L) realizes and that irrespective of the past realizations of the shock. The bounds of the ergodic set represent therefore regions of amnesia in the contract. Figure 1 illustrates each of the two types of steady states.

5.3 Contract implementation

Having derived and characterized the constrained efficient allocation, I construct a sustainable equilibrium that implements the constrained efficient allocation in the market economy. That is, I show that the Second Welfare Theorem applies in the market economy.

Proposition 4 (Implementation)**.** *Under Assumption 1, given a constrained efficient allocation, a sustainable equilibrium exists that implements it.*

The implementation works as follows. The government conducts *official* buybacks when the economy hits the upper bound of the ergodic set (i.e. $x = x^{ub}$). As this bound is reached after a sufficiently long series of high productivity shocks, this buyback policy generates a specific term structure in which high productivity shocks are related to relatively larger long-term bond prices than low productivity shocks, while the short-term bond price remains unchanged. Given this, I can equalize the value of debt in the contract, $V^l(g, x)$, with the value of the debt in the market economy, $b_{st} + b_{lt}[1 + q_{lt}]$, for each (g, x). As I have two productivity states and two bonds, this gives a system of two equations with two unknowns which has a unique solution for each x given the specified term structure.

Lemma 3 (Official Buyback and No Default)**.** *Under Assumption 1, the implementation features official buybacks on equilibrium path and no default. However, official buybacks cannot occur when g_L realizes.*

Lemma 3 is made of two parts. First, the implementation does not rely on defaults. As shown in Proposition 1, the Planner never finds it optimal to distort capital to zero. This means that there is no proper markets exclusion in any region of the contract. It is therefore not possible to interpret the borrower's binding constraint as a default in my environment.[21] In Appendix D, I show that ignoring domestic capital is without loss of generality here.

[21] Müller et al. (2019) and Restrepo-Echavarria (2019) interpret the borrower's binding constraint as a form of preemptive restructuring which does not trigger markets exclusion. Nonetheless, Asonuma and Trebesch (2016) show that even preemptive restructurings are followed by markets exclusion in the data.

Second, *official* buybacks generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract. In particular, as *official* buybacks involve a premium χ, they arise in the high productivity state. Unlike defaults, they are an efficient source of risk sharing. Defaults entail costs for both the lenders and the borrower, while *official* buybacks are solely costly for the latter. A default is therefore not renegotiation proof as both contracting parties would be strictly better off avoiding this event *ex post*.

A corollary of Lemma 3 is that the long-term bond spread is negative.[22] On the one hand, in the absence of default, there is no positive spread. On the other hand, *official* buybacks entail a premium χ implying that the long-term bond price exceeds the risk-free price.

Another corollary is that whether maturity shortens in the low productivity state depends on the exact parameters of the model. On the one hand, there is a substitution effect which pushes the maturity towards the long end in g_L. In particular, every successive realization of g_L makes q_{lt} less sensitive to changes in g. This is because the lenders anticipate that *official* buybacks are less likely to occur. More long-term debt is therefore required to replicate the state contingency in the contract. On the other hand, there is an income effect which increases the total indebtedness in g_L. In steady state $x'(g_L) \leq x$ as shown in Proposition 3. By Proposition 1, this implies that the value of the lenders increases as g_L realizes. Thus, more short-term and long-term debt are needed to replicate the liabilities of the contract.

In Appendix E, I explore alternatives to *official* buybacks. Empirically, such alternatives do not exist or remain underdeveloped. Moreover, they raise similar enforcement issues as *official* buybacks. That is why I do not consider them in the main analysis.

6 Emerging Economies Debt Management

The previous section showed that the Second Welfare Theorem holds in the market economy. This section analyzes whether First Welfare Theorem holds. In particular, it restricts the analysis to Markov equilibria and shows that the First Welfare Theorem generally fails.

6.1 Foundation for Markov equilibrium

As highlighted in Section 4, the market economy features multiple equilibria. I therefore restrict my attention to Markov equilibria. In particular, I present two assumptions under

[22]This is a feature that one finds in other implementations such as the ones of Ábrahám et al. (2019) and Liu et al. (2020). The mechanism is different though. The negative spread restricts the trade of state-contingent securities in a two-sided limited commitment problem when the participation constraint of the risk-neutral lenders binds.

which all sustainable equilibria are Markov and relate these assumptions to the characteristics of emerging economies.

Following Maskin and Tirole (2001), Markov equilibria rely on strategies conditioned the payoff-relevant state which corresponds to $\Omega \equiv (s, \mathbb{I}_D, b_{st}, b_{lt})$ in my environment. Formally,

Definition 3 (Markov Equilibrium)**.** *A sustainable equilibrium is Markov if for any* $(h_b^t, h_l^t) \neq (\tilde{h}_b^t, \tilde{h}_l^t)$ *ending with the same* $\Omega_t \equiv (s_t, \mathbb{I}_{D,t}, b_{st,t}, b_{lt,t})$*, strategies are the same such that* $W^b(h_b^t) = W^b(\tilde{h}_b^t) \ \wedge \ W^l(h_l^t) = W^l(\tilde{h}_l^t)$*.*

The definition relates to a strong-Markov equilibrium as it requires that strategies – and not only payoffs – be the same (Chari and Kehoe, 1993).[23] All Markov equilibria are sustainable equilibria as they restrict the information set to $\Omega_t \subset h^t$ for any $t > 0$.

I now present two assumptions under which all sustainable equilibria are Markov. The first one is that the government's memory is bounded. In other words, the government's memory goes back to a certain number of periods $\mathcal{T} = \frac{1-\psi}{\psi}$ with $\psi \in [0, 1]$.[24] This can be related to the government's relative impatience, $\beta(1 + r) < 1$. In the political economy literature, it is reduced form for the fact that governments are subject to re-elections and may lose office (Alesina and Tabellini, 1990).

The second assumption pertains to small utility shock perturbations. Following Bhaskar et al. (2012) and Angeletos and Lian (2021), in each period t, a utility shock $\epsilon\varrho_{b,t}$ and $\epsilon\varrho_{l,t}$ with $\epsilon \geq 0$ is drawn for the government and the lenders, respectively. It has compact support $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ with absolutely continuous density $\varsigma_{P_i} > 0$ where $|\mathcal{C}_i|$ is the cardinality of the choice set of market participant $i \in \{b, l\}$. Moreover, it is independently distributed across market participants, histories and other shocks. If the market participant $i \in \{b, l\}$ chooses a particular action, say $a \in \mathcal{C}_i$, its utility is augmented by $\epsilon\varrho_{i,t}^a$.[25] Finally, the utility shock $\epsilon\varrho_{i,t}$ is privately observed by market participant $i \in \{b, l\}$.

Assumption 2 (Perturbation)**.** *The government's memory goes back to* $\mathcal{T} = \frac{1-\psi}{\psi}$ *periods in the past with* $\psi \in [0, 1]$*. In addition, in each* t*, a utility shock* $\epsilon\varrho_{i,t}$ *with* $\epsilon \geq 0$ *is drawn from the compact support* $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ *with absolutely continuous and i.i.d. density* $\varsigma_{P_i} > 0$ *for each* $i \in \{b, l\}$*. The utility shock is additive and privately observed.*

Under Assumption 2, the market economy considered in Section 4 corresponds to $\psi = \epsilon = 0$. My equilibrium selection boils down to what happens when $(\psi, \epsilon) > 0$ but arbitrarily small. It means that (a) the government eventually forgets the history of play in the very

[23] See also Definition K.5 in Appendix K.

[24] I consider here that the lenders are long-run players. However this is without loss of generality. The result holds as long as at most one market participant has unbounded memory.

[25] For instance, the instantaneous utility of the government taking action a is given by $u(c_t) + \epsilon\varrho_{b,t}^a$.

distant past and (b) market participants have imperfect knowledge of the other participant's fundamentals. These two assumptions particularly fit the case of emerging economies. On the one hand, such economies suffer from important political instability which can explain (a).[26] On the other hand, the fundamentals of these economies are difficult to assess from the perspective of foreign creditors. Particularly, governments of emerging economies often release data of poor quality or even distort some statistics which can explain (b).[27]

The presence of the privately observed shocks – albeit small and independent – coupled with the bounded memory of the government prevent both participants to rely on past history. This causes all non-Markov equilibria to unravel.

Proposition 5 (Foundation of Markov equilibria). *Under Assumption 2, with* $(\psi, \epsilon) > 0$, *every sustainable equilibrium is a Markov equilibrium.*

The rationale behind that result follows Bhaskar (1998) and Bhaskar et al. (2012). Suppose the lenders condition their action at time t on a payoff-irrelevant past event, then the government must also condition on this past event given that moves are sequential. Nevertheless, as long as $\psi \neq 0$, the government eventually forgets everything that happened in an arbitrarily distant point in the past. This means that, asymptotically, the government – and consequently the lenders – cannot condition on past history. In addition, the utility shocks with $\epsilon \neq 0$ ensure that each decision node is a singleton and that the equilibrium outcome is strong instead of weak-Markov.

Both parts of Assumption 2 are necessary for Proposition 5 to hold. On the one hand, with $\psi = 0$ and $\epsilon \neq 0$, the markets participants could condition their actions on the entire history of play relying on the law of large numbers for the distribution of utility shocks. On the other hand, with $\psi \neq 0$ and $\epsilon = 0$, payoffs are independent of history but not necessarily strategies making the equilibrium weak-Markov as in Chari and Kehoe (1993).

6.2 Markov equilibrium

The Markov equilibrium I obtain in this environment is a version of Arellano and Ramanarayanan (2012) with strategic lenders and additional provisions on buybacks.

Let $\Omega_P \equiv (s, 0, b_{st,t}, b_{lt,t})$ be the payoff-relevant space of the borrower in repayment. Given this, the government's overall beginning of the period value is

$$W^b(\Omega_P) = \max_{D \in \{0,1\}} \Big\{ (1 - D)V^P(\Omega_P) + DV^D(s, 0, k) \Big\}, \tag{9}$$

[26] See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019).

[27] See notably Tsyrennikov (2013) and Morelli and Moretti (2021).

where V^P and V^D correspond to the value of repayment and default, respectively. Under repayment, the government chooses whether to conduct *official* buybacks. Thus

$$V^P(\Omega_P) = \max_{M\in\{0,1\}} \Big\{(1-M)V^{NB}(\Omega_P) + MV^B(\Omega_P)\Big\}, \tag{10}$$

where V^B and V^{NB} are the values under *official* buyback and no *official* buyback, respectively. If the government decides to officially repurchase its long-term debt,

$$\begin{aligned} V^B(\Omega_P) = &\max_{b'_{st},b'_{lt}} u(c) + \beta\mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\ &\text{s.t.} \quad c + q_{st}(s,b'_{st},b'_{lt})b'_{st} + q_{lt}(s,b'_{st},b'_{lt})b'_{lt} = y(g,k) + b_{st} + b_{lt}(1+q^{bb}_{lt}), \\ &\qquad b'_{lt} \geq b_{lt}. \end{aligned}$$

Conversely, under no *official* buyback,

$$\begin{aligned} V^{NB}(\Omega_P) = &\max_{b'_{st},b'_{lt}} u(c) + \beta\mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\ &\text{s.t.} \quad c + q_{st}(s,b'_{st},b'_{lt})b'_{st} + q_{lt}(s,b'_{st},b'_{lt})(b'_{lt} - b_{lt}) = y(g,k) + b_{st} + b_{lt}. \end{aligned}$$

The value under default is given by

$$V^D(s,0,k) = u(gf(k)) + \beta\mathbb{E}_{s'|s}\Big[(1-\lambda)V^D(s',1,0) + \lambda W^b(s',0,0,0)\Big]. \tag{11}$$

This represents the optimal contract's outside option used in Section 5. After the choice of capital,[28] the value of the *legacy* lender is given by

$$W^l_{\text{legacy}}(\Omega) = \max_{b'_{st},b'_{lt}} -\Big[b_{st} + b_{lt}(1 + q_{lt}(s,b'_{st},b'_{lt}) + M(\Omega)\chi)\Big](1 - D(\Omega)). \tag{12}$$

The continuation value is equal to zero given the break-even assumption. The value of the *new* lender is then $W^l_{\text{new}}(\Omega) = 0$. To avoid redundancy with Section 4, the pricing equations are presented in Appendix F.

6.3 Equilibrium multiplicity

I now characterize the Markov equilibrium and show that there are two equilibria: one without default where *official* buybacks can occur and one with default where *official* buybacks never occur on equilibrium path.

[28] The value before the capital choice should include $p(1-\tau)k - k$ with $p = \infty$ if $D = 1$ implying $k = 0$.

The multiplicity of equilibria originates from tensions between the *legacy* and the *new* lender. On the one hand, the former lender is unwilling to let the borrower dilute legacy debt as this directly reduces its payoff. Similarly, it is supportive of *official* buybacks as the payment of the premium χ increases its payoff. On the other hand, the *new* lender is indifferent to both dilution and *official* buybacks owing to the break-even assumption.[29]

Proposition 6 (Legacy Debt)**.** *The legacy lender is unwilling to dilute legacy long-term debt claim and is willing to have official buybacks. The new lender is always indifferent.*

Being indifferent, the *new* lender can offer debt contracts that either satisfy the borrower's problem in (10) or that satisfy the *legacy* lender's problem in (12). This has implications on whether borrowing is risky and on whether buybacks entail a premium.

Regarding risky borrowing, define the endogenous borrowing limit as $\mathcal{B} \geq \min_{s'}\{(b'_{st}, b'_{lt}) : V^P(\Omega'_P) = V^D(s', 0, k')\}$.[30] The borrower has no incentive to default if it does not accumulate debt beyond $\mathcal{B}$. Moreover, the *legacy* lender never offers $(b_{st,t+1}, b_{lt,t+1}) < \mathcal{B}$ given Proposition 6. Thus, if the *new* lender decides to satisfy the *legacy* lender's problem, the borrower never enters the risky borrowing region.[31] In opposition, if it decides to satisfy the borrower's problem, the borrower can enter the risky borrowing region if willing to do so. As a result, whether an equilibrium entails defaults on equilibrium path depends on the behavior of the *new* lender.

Regarding *official* buybacks, the behavior of the *new* lender is also key. An *official* buyback is a reverse default as it corresponds to an overpayment, whereas a default is an underpayment of liabilities. While underpayments are sanctioned by markets exclusion, there is no direct reward after overpayments. Especially, the government can avoid the payment of χ through *unofficial* buybacks. As a result,

Proposition 7 (Official Buyback Aversion)**.** *In any state Ω_P, the borrower cannot commit to conduct official buybacks as it always strictly prefers unofficial buybacks for the same (b'_{st}, b'_{lt}).*

This means that for two debt contract offering the same (b'_{st}, b'_{lt}), the borrower would never choose the one in which it has to pay χ. Thus, if the *new* lender decides to satisfy the borrower's problem, *official* buybacks never occur on equilibrium path.

In opposition, if the *new* lender decides to satisfy the *legacy* lender's problem, *official* buybacks may occur on equilibrium path. The key point is that only the *legacy* lender can

[29]Note that the break even assumption with zero recovery value rules out any monopoly power of the *legacy* lender close to the default decision. My environment therefore does not feature default cycles as in Kovrijnykh and Szentes (2007).

[30]This is what Zhang (1997) defines as a no-default borrowing constraint.

[31]This corresponds to the "saving equilibrium" defined by Aguiar and Amador (2020).

offer debt contracts entailing the payment χ. This means that an *official* buyback occurs only if the borrower chooses the offer of the *legacy* lender. As there are many debt contract satisfying the *legacy* lender's problem, I consider that the *new* lender offers either $\mathcal{B}$ or a sudden stop debt contract consisting of $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$. In particular, if $\upsilon = 0$, it offers $\mathcal{B}$ to the borrower. This does not mean that an *official* buyback occurs whenever $\upsilon = 1$.

Lemma 4 (Official Buyback Enforcement)**.** *When the new lender offers $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$, the borrower accepts the legacy lender's official buyback offer if it does not want to save (i.e. $b'_{st} < 0$) and either $-b_{st} > 0$ is sufficiently large or $-b_{lt}q^{bb}_{lt}$ is not too large.*

The lemma states that *legacy* lender's offer is preferable when the borrower does not possess any assets and *official* buybacks are not too costly. I further develop this argument in Appendix B when I endogenize χ. A direct corollary of Lemma 4 is that the state space Ω_P can be separated in two zones: the enforcement and the impunity zone.

1. The enforcement zone:

   In this zone, the borrower is worse off with a sudden stop rather than paying the premium χ. Hence, if $\upsilon = 1$, an *official* buyback happens as of Lemma 4. Otherwise, it does not happen as of Proposition 7.

2. The impunity zone:

   In this zone, the borrower is always worse off paying the premium χ. *Official* buybacks therefore do not happen as of Proposition 7.

In the impunity zone, $V^{NB}(\Omega_P) > V^{B}(\Omega_P)$ even if new bond issuance is restricted to $b'_{lt} \geq b_{lt}$ and $b'_{st} \geq 0$ when $M(\Omega_P) = 0$. Hence, the enforcement zone is the only zone in which an *official* buyback can occur.

Given the above multiplicities, there are two Markov equilibria. The first one is a Markov equilibrium without default in which *official* buybacks can occur on equilibrium path. It happens when the *new* lender offers debt contracts that satisfy the *legacy* lender's problem in (12). The second one is a Markov equilibrium with default in which *official* buybacks never occur. It happens when the *new* lender offers debt contracts that satisfy borrower's problem in (10).

The Markov equilibrium with default is the one characterized by Arellano and Ramanarayanan (2012). On the one hand, defaults arise on equilibrium path and especially when productivity is low. On the other hand, maturity shortens during debt crises. The repayment of long-term debt is laddered through multiple periods which implies that the claim of legacy creditors can be diluted. The long-term bond therefore admits a greater default premium than the short-term bond. As a result, close to default, the long-term debt price drastically drops which encourages shorter maturity.[32] Finally, *official* buybacks never occur. In terms of sovereign debt management, this equilibrium is the closest to what is observed in Argentina as one will see in the next section.

Furthermore, the Markov equilibrium without default predicts the opposite of what the previous Markov equilibrium does. Defaults do not arise on equilibrium path, while *official* buybacks can. Moreover whether maturity shortens in the low productivity state depends on the exact parameters of the model and the specification of υ. The predictions of the model are therefore the closest to what is observed in Brazil as one will see in the next section.

The Markov equilibrium without default is very close to the implementation presented in Section 5. However, under Markov strategies, *official* buybacks can only arise in the enforcement zone. The following lemma shows that *official* buybacks that implement the constrained efficient allocation do not generally arise in this zone.

Lemma 5 (Non-Markov Implementation)**.** *Under Assumption 1, for a given implementation of the constrained efficient allocation, the point of official buyback is not necessarily located in the enforcement zone.*

Lemma 4 states that *official* buybacks are enforceable in a Markov equilibrium when there is no short-term assets and *official* buybacks are not too costly. However, Lemma 5 shows that, in the implementation of the constrained efficient allocation, at the point of *official* buyback, the borrower needs to hold short-term assets unless the buyback premium is sufficiently large. Hence, *official* buybacks are not automatically enforceable through Markov strategies. Especially in the next section, I show quantitatively that that Markov strategies fail to implement the constrained efficient allocation under empirically plausible buyback premia. Trigger strategies are therefore more often than not necessary, which puts the implementation at the mercy of Proposition 5.

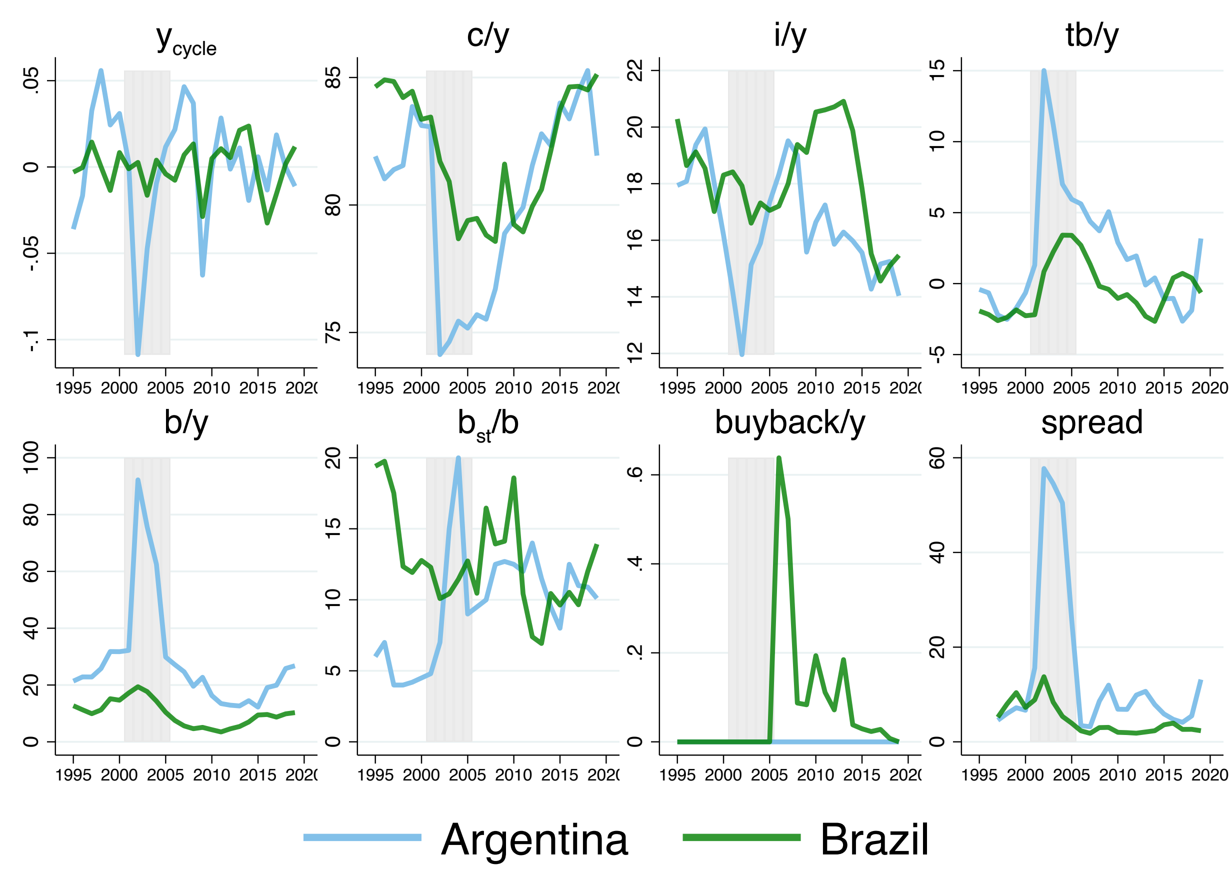
7 Quantitative Analysis

This section starts with a comparison of Argentina and Brazil since 1995. I then calibrate the Markov equilibrium with default to Argentina and assess the fit of the model to the data. I show that the Markov equilibrium without default is quantitatively close to Brazil and contrast the two Markov equilibria with the constrained efficient allocation.

[32] As shown by Niepelt (2014), this result is also a consequence of the fact that a default implicates the entire long-term and short-term debt. It would not arise if the government would only default on the maturing portion of the long-term debt. See also Perez (2017).

7.1 Argentina vs. Brazil

I compare the experience of Argentina and Brazil starting in 1995 as Brazil defaulted last in the 1980s and regained access to the international market after the implementation of the Brady Plan in 1994.[33] Additional results can be found in Appendix H.



Note: $\mathrm{y}_{\text{cycle}}$ corresponds to output detrended using the Hodrick–Prescott filter with a smoothing parameter of 6.25, spread is the EMBI spread, i is the investment, tb/y is the trade balance over output and b is the public sector external debt stock with $b = b_{st} + b_{lt}$. Series for Argentina are depicted in blue and in green for Brazil. The grey area represents the periods in which Argentina is in default.

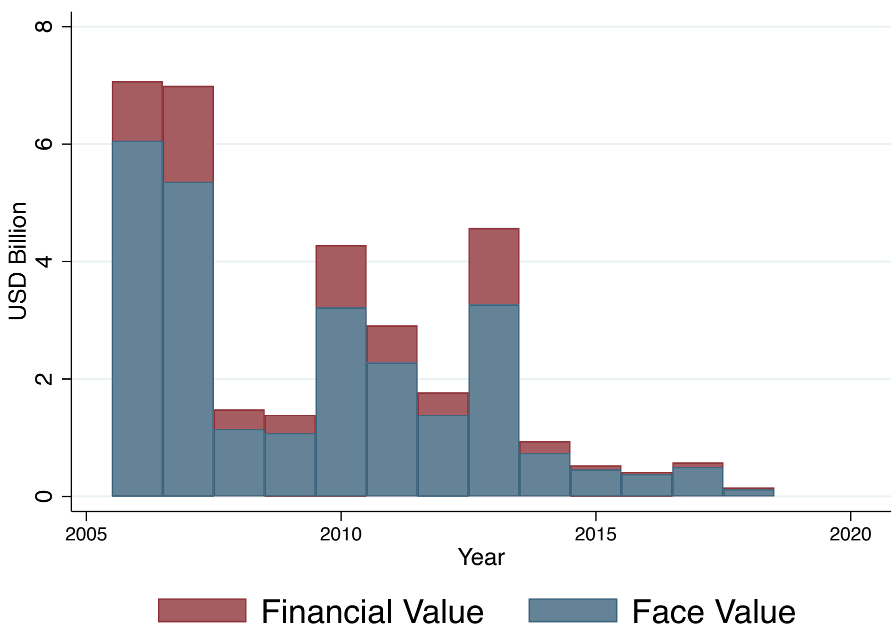
Source: Author's calculation, Buera and Nicolini (2021), Tesouro Nacional, Global Financial Data and World Bank.

Figure 2: Argentina vs. Brazil

Figure 2 presents the main statistics of interests for both countries. The blue (green) line represents Argentina (Brazil). As one can see, the two countries recorded a sudden drop in output at the end of the 1990s. Brazil experienced a major currency crisis following a

[33]The Brady Plan is an extensive debt restructuring program aimed at resolving the numerous of sovereign debt defaults in 1980s especially in Latin America. In general, see Buera and Nicolini (2021) for for the economic history of Argentina and Ayres et al. (2021) for Brazil.

speculative attack on the real, while Argentina suffered from a banking crisis. Consumption and investment drastically reduced, while indebtedness and the spread largely increased. In addition, the average maturity shortened for both countries in the years preceding the crisis.[34] Most importantly, Argentina eventually defaulted in 2001 (represented by the grey area), whereas Brazil did not. This triggered markets exclusion for Argentina which could re-access the market in 2006 (Cruces and Trebesch, 2013).



Note: The figure depicts the amount of external debt bought back by the Brazilian government in USD billion. The Financial value in red corresponds to the amount required for payment of the securities redeemed. The face value in blue corresponds to the value of debt in the national statistics.

Source: Author's calculation and Tesouro Nacional.

Figure 3: Official Buyback in Brazil

Moreover, Brazil conducted *official* buybacks while Argentina did not under the period considered.[35] Figure 3 depicts the *official* buybacks conducted by Brazil. In 2006, the country started the Early Redemption Program which aimed at correcting the average maturity of the debt and reducing the potential refinancing risk.[36] Repurchases were conducted by the Brazilian National Treasury either directly on the secondary market or indirectly through call options and special auctions. I identify two features in the Brazilian *official* buybacks.

[34] See also the maturity regression analysis in Appendix H.

[35] Note that Argentina repurchased external debt at a discount (i.e. below par) on the secondary market in various occasions. However, this usually corresponds to a default in the form of distressed debt exchange.

[36] See https://www.gov.br/tesouronacional/en/federal-public-debt/external-market/buyback-program.

First, such buybacks are costly. The financial value (i.e. the red bar) is systematically above the face value (i.e. the blue bar) meaning that the Brazilian government always paid a premium to extract its debt out of the market.[37] This premium is primarily explained by the fact that the repurchased Brazilian bonds entailed high coupon rates relative to the market interest rate. On average, the financial value is 24.5% above the face value. This figure provides the basis of calibration of $\chi > 0$ in the next subsection. Second, those buybacks were the largest when the output of the Brazilian economy was on or above trend consistent with the predictions in Section 5. I provide more details on that in Appendix H.

As a result, the experience of Argentina and Brazil qualitatively relate to the predictions of the Markov equilibrium with and without default, respectively. In what follows, I gauge the quantitative fit.

7.2 Calibration

I calibrate the Markov equilibrium with default as it corresponds to the workhorse model in the literature on sovereign defaults. The calibration aims at matching some specific moments of the Argentine economy over the period 1995-2019. Table 1 summarizes each parameter.

Table 1: Calibration

Parameter	Value	Description	Targeted Moment
A. Based on Literature			
ϑ	2.00	Relative risk aversion	
r^f	0.01	Risk-free rate	
B. Direct Measure from the Data			
$\pi(g_H\|g_H)$	0.93	Probability staying high state	
$\pi(g_L\|g_L)$	0.68	Probability staying low state	Real total factor productivity
g_L	0.44	Productivity in low state	
$1-\alpha$	0.70	Labor share	Labor income share
χ	4.59	*Official* buyback premium	Financial over face value of debt
r^e	0.04	Excess return	US excess return on debt
C. Based on Model solution			
β	0.80	Discount factor	Debt-to-GDP ratio
g_H	1.12	Productivity in high state	Correlation consumption and output
ϕ	1.50	CES production	Investment-to-GDP ratio
λ	0.281	Probability re-accessing market	Average spread

The instantaneous utility function takes the CRRA form with a coefficient of relative risk aversion of ϑ, i.e. $u(c) = \frac{c^{1-\vartheta}}{1-\vartheta}$. I adopt $\vartheta = 2$ as it is standard in the real business cycle literature and set the discount factor to $\beta = 0.8$ to match the average public sector external debt-to-GDP ratio of 28.71%. This corresponds to a quarterly discounting of 0.945 which

[37]The Financial value corresponds to the amount required for payment of the securities redeemed, while the face value in blue corresponds to the value of debt in the national statistics.

is standard in studies on emerging economies. In addition, the production function has the CES form

$$F(k,l) = \left[\alpha k^{\frac{\phi-1}{\phi}} + (1-\alpha) l^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where α represents the capital share and ϕ the CES parameter. The value of $1-\alpha$ is set to the standard labor share in GDP adopted in the literature on emerging economies (Mendoza and Yue, 2012). The CES parameter is $\phi = 1.5$ to match the share of investment in GDP and is within the range of admissible values in the business cycle literature. In addition, I estimate the Markov transition matrix by means of a Markov-switching AR(1) process with two states. For this, I use data on the real total factor productivity of Argentina from 1990 to 2019 from the Penn World Table 10.0 (Feenstra et al., 2015). I then select g_H to match the correlation between consumption and output and accordingly set g_L to obtain the average real TFP of Argentina given the estimated transition matrix.

Regarding the exogenous rate r, I relax the assumption of break-even lending and set $r = r^f + r^e$ where r^f represents the risk-free rate and r^e corresponds to the lenders' excess return. This means that the lenders borrow at r^f and lend at $r > r^f$. This has two purposes. First, it better captures the potential risk premium US investors demand on emerging market bonds. I therefore set $r^e = 0.0434$ consistent with the US excess return on debt instruments estimated by Gourinchas et al. (2017) and $r^f = 0.01$ as it is standard in the literature. Second, as the spread is calculated with respect to the risk-free rate, modelling an excess return enables to correct the negative spread which has little empirical support for the countries under study.

I choose $\lambda = 0.281$ to match the average (EMBI) spread of 14.17%. The value selected implies an expected default length of roughly 3.5 years. This is below the value of 5.1 years Cruces and Trebesch (2013) find in the data. Finally, for the *official* buyback premium, I set $\chi = 4.59$ to match the wedge between the financial and the face value recorded on the Brazilian Early Redemption program highlighted in the previous subsection.

In light of Section 6, the difference between Argentina and Brazil lies in the lenders' strategic interaction. I therefore consider that the two countries are identical in terms of economic fundamentals and use the same calibration to solve the two Markov equilibria and the constrained efficient allocation. The only difference lies in the specification the *new* lender's offer. In particular, for the Markov equilibrium without default, to stay consistent with the data, $\upsilon = 1$ if $g = g_H$ and $\upsilon = 0$ otherwise.

7.3 Numerical results

This subsection presents the result of the calibration. It gauges the fit of the model with respect to the data for both targeted and non targeted moments. It also compares the outcome of the Markov allocation with default (MA), without default (MAND) and the constrained efficient allocation (CEA) together.

Table 2: Targeted and Non-Targeted Moments

A. Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
i/y	14.26	14.22	17.98	16.22	14.61
$-b/y$	28.71	28.15	10.12	7.18	-353.20
Spread	14.17	12.88	4.97	3.85	3.95
corr(c, y)	0.96	0.94	0.88	0.95	0.68
B. Non-Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
c/y	78.90	85.80	81.43	82.90	82.52
$\sigma(c)/\sigma(y)$	1.04	0.84	0.83	0.71	0.21
$\sigma(i)/\sigma(y)$	3.31	0.08	3.21	0.08	1.25
$\sigma(tb/y)/\sigma(y)$	1.22	0.40	1.38	0.40	0.96
$\sigma(spread)/\sigma(y)$	4.53	0.77	2.47	0.00	0.01
corr(i, y)	0.97	0.91	0.93	1.00	0.99
corr$(tb/y, y)$	-0.60	0.53	-0.34	0.79	0.97
corr$(spread, y)$	-0.63	-0.24	-0.20	-0.71	-0.67

Note: The variable $\sigma(\cdot)$ denotes the volatility, tb/y denotes the trade balance over output and i the investment which corresponds to k in the model given the depreciation of 1. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

The upper part of Table 2 presents the fit of the MA with respect to the Argentine economy in terms of targeted moments. It also reports the result of the CEA and the MAND. As one can see, the MA replicates relatively well the main features of the Argentine economy in terms of consumption, investment, spreads and indebtedness.

The lower part of Table 2 presents the fit of the MA in terms of non-targeted business-cycle moments. In general, the fit is poor. This is because I only consider 2 productivity states meaning that I rule out tail events. The MA generates too low volatilities for most variables. Moreover, the trade balance is pro-cyclical unlike the data. The model however produces empirically plausible correlations for the spread and investment relative to output.

Having said that, the MA generates a realistic debt dynamic. Table 3 depicts the under-

Table 3: Debt Structure

	Mean $-b/y$ (percent)	Mean $-b/y$ in g_H (percent)	Mean $-b/y$ in default (percent)	Mean $-b/y$ in restructuring (percent)
Argentina	28.7	22.0	65.7	29.9
MA	28.2	24.1	216.4	17.5
Brazil	10.1	9.0	-	-
MAND	7.2	2.3	-	-
CEA	-353.2	-353.3	-	-
	Mean b_{st}/b (percent)	Mean b_{st}/b in g_H (percent)	Mean b_{st}/b in default (percent)	Mean b_{st}/b in restructuring (percent)
Argentina	9.7	8.3	11.7	9.0
MA	44.0	43.6	84.1	64.5
Brazil	12.6	12.4	-	-
MAND	21.7	11.3	-	-
CEA	112.5	112.0	-	-

Note: In the CEA, $b_{st}/b > 1$ as $b_{st} > 0$ in some states while $b_{lt} \leq 0$. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

lying debt structure of the Markov equilibria and the CEA. Two points deserve to be noted. First, the MA replicates well the data as maturity shortens during debt crises, while indebtedness relative to GDP increases. Second, during a restructuring, the maturity lengthens and the level of debt remains substantial.[38]

Turning to the MAND, Table 2 presents the similarities with Brazil. As discussed at the beginning of the section, Brazil has not defaulted since the end of the 1980s, whereas Argentina defaulted 3 times since 1995 with the most recent episode being in 2023. Second, Brazil conducted an official buyback program from 2006 to 2018. Third, maturity shortens in the low productivity state. Fourth, in terms of economic fundamentals, Brazil records a lower average debt ratio, a greater average investment ratio and a lower average spread than Argentina for the period 1995 to 2019. The MAND is capable of matching most of the main moments of the Brazilian economy despite the fact that none of them where directly targeted.[39] This suggests that Brazil can be interpreted as the counterfactual of Argentina with buybacks and without default in the period 1995-2019.

Looking at the CEA in the last column of Table 2, one directly observes that it predicts an empirically implausible average indebtedness. In fact, the borrower holds a net asset position. Such prediction is well known in the literature on fiscal policy under commitment as highlighted by notably Buera and Nicolini (2004) and Faraglia et al. (2010). Even though I consider an alternative environment without commitment, the bond portfolio implementing

[38] See Benjamin and Wright (2013), Mihalache (2020) and Dvorkin et al. (2021) for related results.

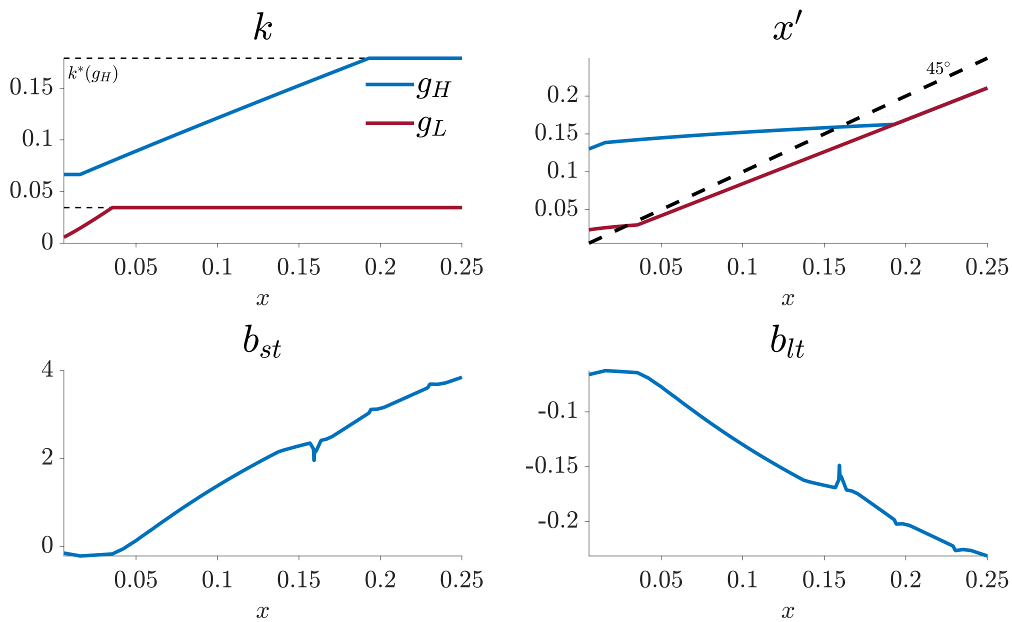
[39] Note that the excess return corrects the negative spread in the model.

the CEA remains at odds with the data.

The MAND and the CEA achieve better risk sharing than the MA. In the CEA, consumption corresponds to a lower share of output, correlates less with output and is less volatile. Investment corresponds to a larger share of output, correlates more with output and is more volatile. Finally, the bond spread is lower than in the MA given that defaults do not arise on equilibrium path and *official* buybacks exceed the risk-free price. The same holds true for the MAND with the exception of a slightly larger consumption correlation than in the MA.

7.4 Implementation and buyback cost

In this subsection, I discuss the implementation of the CEA in the market economy. Under empirically plausible *official* buyback premia, the point of *official* buyback is located outside the enforcement zone.

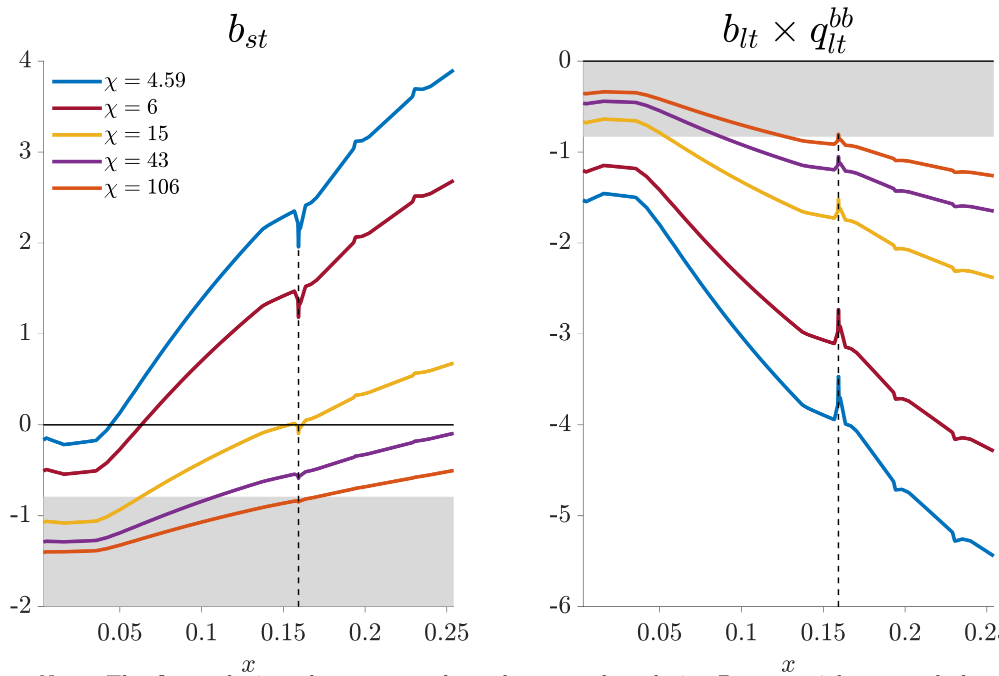


Note: The figure depicts the policy functions for capital, k, prospective relative Pareto weight, x', short-term debt, b_{st}, and long-term debt, b_{lt}.

Figure 4: Main Policy Functions of the CEA

Figure 4 depicts the main policy functions related to the optimal contract. The law of motion of the relative Pareto weight is consistent with the fact that the borrower is impatient (i.e. $\eta < 1$). Similarly, capital remains distorted in steady state in line with Propositions 2 and 3. Regarding borrowing, when x is low, the government accumulates more short-term debt and less long-term debt. In opposition, when x gets larger, the opposite is true.

Furthermore, the borrower holds short-term assets – especially when *official* buybacks occur. This means that the point of *official* buyback is in the impunity zone which explains the reliance on trigger strategies.



Note: The figure depicts the correspondence between the relative Pareto weight, x, and the portfolio of bonds, (b_{st}, b_{lt}), for different *official* buyback premia, χ. The grey area depicts the enforcement zone in the Markov equilibrium.

Figure 5: Implementation and χ

To obtain short-term debt holdings when *official* buybacks occur, the premium χ should be larger than the calibrated one. The rationale behind this is that with a larger χ, the long-term bond price is more sensitive to the realization of g. As a result, more short-term debt and less long-term debt are required to replicate the state-contingent liabilities of the optimal contract. Figure 5 depicts the portfolio of bonds necessary to implement the CEA for different values of χ. The black dashed line represents the relative Pareto weight at which the *official* buyback occurs – i.e. $x = x^{ub}$. As one can see, it is possible that the borrower holds short-term debt – and not asset – by more than tripling χ relative to the calibration benchmark. This means that Markov strategies fail to implement the Planner's allocation under empirically plausible *official* buyback premia in emerging economies.

Thus, with respect to the literature on fiscal policy under commitment and the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), I reconcile the model's prediction with the data by arguing that the borrower lacks the strategical sophistication to implement the CEA. Under an empirically plausible cost of *official* buyback, the borrower can only approximate – as opposed to replicate – the returns of Arrow securities with non-contingent bonds of multiple maturities. Nevertheless, such approximation is consistent with the sovereign debt management of emerging economies as shown previously.

7.5 Equilibria comparison

In this subsection, I explore in more details the differences between the Markov equilibria and the CEA. For this purpose, I conduct two main exercises. First, I compute welfare gains with respect to the MA. Second, I measure the distance of each equilibria from the Pareto frontier. Additional results can be found in Appendix I.

Table 4: Welfare Analysis

State	Borrower welfare gains (percent)		Lenders welfare gains (percent)		$\mathcal{F}(g)$ (percent)		
	MAND	CEA	MAND	CEA	MA	MAND	CEA
g_H	0.01	0.07	0.5	1.7	23.6	26.3	100.0
g_L	0.84	0.85	1.9	3.4	18.7	21.2	100.0
average	0.17	0.22	0.8	2.0	22.6	25.3	100.0

Note: The table presents the welfare gains in consumption equivalent relative to the MA. See Appendix J for details on the computation. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

Table 4 depicts the welfare gains of the CEA and the MAND in consumption equivalent terms with respect to the MA for both the borrower and the lenders. Welfare gains are computed through the simulation of 5,000 independent shock histories starting with initial debt holdings and relative Pareto weights drawn from the ergodic set. The details of the welfare computation are presented in Appendix J.

As one can see, the CEA and the MAND imply substantial welfare gains compared to the MA, on average for both the lenders and the borrower. The CEA leads to the largest welfare gains in all states for all market participants. Those are more pronounced when g_L realizes. Hatchondo et al. (2020a) find similar results when comparing the MA with a Ramsey plan. Note that in the MAND, the borrower's welfare gains become negligible compared to the MA when g_H realizes. This is due to the fact that *official* buybacks occur whenever g_H realizes in the enforcement zone, unlike the implementation of the CEA in which such buybacks occur conditional on a certain portfolio holding in addition to the realization of g_H. It therefore seems that the borrower in the MAND conducts *official* buybacks too frequently.

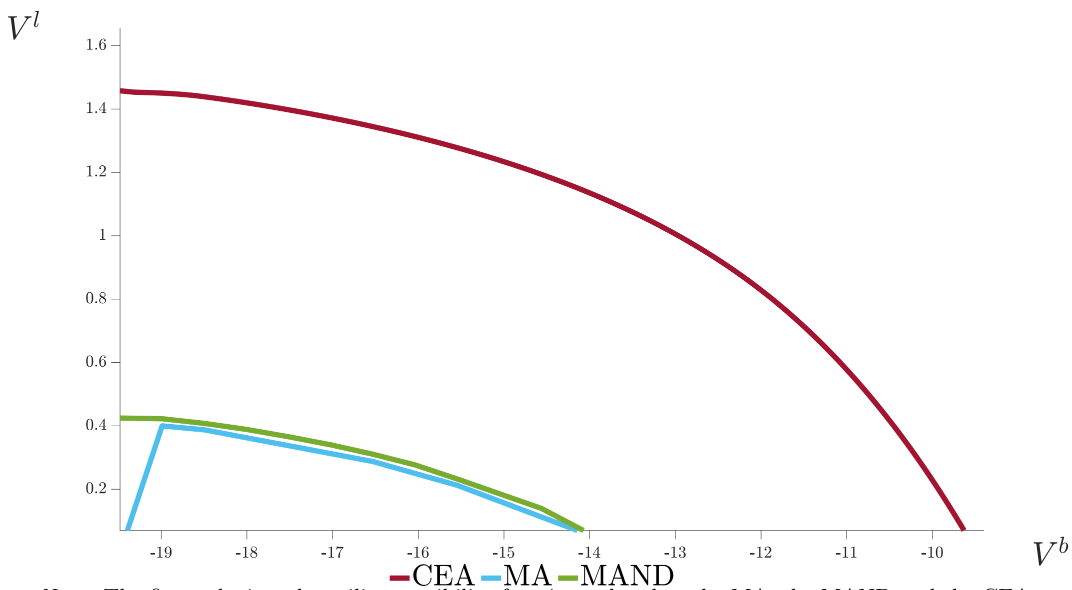
In Table 5, I decompose the borrower's welfare gains of the MAND and the CEA by calculating the percentage of gains that can be attributed to the following two factors: cost

Table 5: Borrower Welfare Decomposition

State	MAND		CEA	
	State contingency (percent)	Cost of default (percent)	State contingency (percent)	Cost of default (percent)
g_H	1.90	98.10	15.67	84.33
g_L	99.13	0.87	98.70	1.30
average	20.38	79.62	31.45	68.55

Note: The table presents the decomposition of the borrower's welfare gains in two factors: cost of default and state-contingency. The gains related to state contingency come from the computation of the MAND without *official* buybacks (i.e. $\upsilon_t = 0$ for all t). The residual welfare gains is attributed to the cost of default. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

of default and state-contingency. I isolate those two factors in the following way. To compute the gains related to state contingency, I compute the MAND without *official* buybacks (i.e. $\upsilon_t = 0$ for all t). The residual welfare gains can then be attributed to the cost of default. Doing so I find that, in the MAND, 20% of the welfare gains come from the state contingency on average and the remaining part can be attributed to the cost of default. In the CEA, we find that 31% of the gains come from state contingency and the rest comes from the cost of default. In both cases, the share of gains related to state contingency is the highest in g_L.



Note: The figure depicts the utility possibility frontiers related to the MA, the MAND and the CEA. Those frontiers express the value of the lenders V^l as a function of the value of the borrower V^b. The CEA is in red, the MA in blue and the MAND in green. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

Figure 6: Distance to Pareto Frontier in g_L

Besides the welfare gains, I can compute the distance with respect to the Pareto frontier. For this purpose, I derive a metric measuring the distance between the constrained efficient allocation and any alternative allocation. From Proposition 4, I obtain a direct correspondence between x and (b_{st}, b_{lt}). I can therefore express the value of the lenders in any Markov equilibrium as a function of x instead of (b_{st}, b_{lt}), i.e. $\ddot{V}^l : G \times X \to \mathbb{R}$. I then define

$$\mathcal{F}(g) = \frac{\int_{\underline{x}}^{\overline{x}} \ddot{V}^l(g, x)dx}{\int_{\underline{x}}^{\overline{x}} V^l(g, x)dx}.$$

The metric $\mathcal{F}(g)$ measures the distance between the Markov allocation and the CEA. Given Proposition 1, it is bounded between 0 and 1. A value of $\mathcal{F}$ near 1 indicates that an allocation is close to the constrained efficient benchmark, whereas a value close to 0 indicates the opposite.[40] I compute $\mathcal{F}(g)$ for $b_{st} \leq 0$ and $b_{lt} \leq 0$ in the Markov equilibrium without default risk to stay consistent with Lemma 4.

Figure 6 depicts the different frontiers: in red the Pareto frontier and in blue and green the utility possibility frontier related to the MA and the MAND, respectively. Defaults in the MA produce an upward sloping part of the frontier in which both the borrower and the lenders can be made better off. Neither the CEA nor the MAND display such upward slope. This shows the inefficiency of default (see Fudenberg et al. 1990).

Looking at the metric $\mathcal{F}(g)$ in the last column of Table 4, the MAND is superior to the MA but not to the CEA. More precisely, the MA is relatively far from the CEA and the MAND can get the economy closer to it. The MAND therefore provides a better approximation of the CEA than the MA. Nevertheless, the MAND remains far from the CEA meaning that the Pareto improvement is small relative to what can be achieved with trigger startegies. The metric $\mathcal{F}(g)$ is important as it relates to the entire value of the debt contract (i.e. the combined value for the borrower and the lenders) and not only on the steady state unlike the welfare gains computed above.

8 Conclusion

This paper derives the constrained efficient allocation emanating from an optimal contract to deduce optimal sovereign debt management policies. The bottom line is that the reliance on defaults on equilibrium path is inefficient. Instead, changes in maturity and costly debt buybacks can implement the constrained efficient allocation. Nevertheless, the implemen-

[40] The metric $\mathcal{F}(g)$ is based on the same concept as the Gini coefficient which measures the distance between the Lorenz curve and the equity line.

tation often requires highly sophisticated agents capable of building on past history. Less sophisticated agents – in the spirit of emerging economies – would in fact rely on Markov strategies. Given this, I derive history-invariant debt management policies inspired by the optimal contract and assess their efficiency. I show that there are multiple Markov equilibria depending on the behavior of the lender which does not hold legacy claims. These equilibria can be Pareto ranked and can rationalize the experience of Argentina and Brazil since 1995.

This paper stresses two points. The first one is that incomplete markets might not be the reason why a market economy fails to attain constrained efficiency. Rather it can be linked to the incapacity of market participants to build on past history. I show that this restriction in the strategies followed by the market participants makes sense in the context of emerging economies. In that logic, Markov equilibria as (time-invariant) approximation of the constrained efficient allocation are not only the empirically-relevant but also the policy-relevant equilibrium concept for such economies.

The second point this paper highlights is that the strategic interaction of lenders is key. The literature on sovereign debt and default has focused on the borrower's side. However, it is possible to explain a variety of alternative dynamics in equilibrium by looking at the lenders and the way they interact.

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Appendix

A Discussion on Alternative Implementations

This section discusses the relationship between the implementation presented in Section 5 and the main alternatives that exist in the literature.

Dovis (2019) considers an environment similar to the one presented in Section 3 with the only difference that g is privately observed by the borrower. He derives an optimal contract subject to a participation and an incentive compatibility constraint to account for limited commitment and adverse selection, respectively. He subsequently decentralizes the aforementioned contract through partial defaults and an active debt maturity management. The main difference with my study is that he explicitly uses defaults – instead of costly buy-backs – to implement the constrained efficient allocation. This is because the combination of limited commitment and adverse selection generates a region of *ex post* inefficiencies in which the Planner sets $k = 0$.[41] As I only consider limited commitment, this region does not exist in my analysis – as shown in Proposition 1. Nevertheless, my implementation works in the environment of Dovis (2019), while the opposite is not true. In general, his implementation does not apply to renegotiation-proof contracts, while mine applies to contracts with or without *ex post* inefficiencies.

Besides this, Alvarez and Jermann (2000) propose a way to implement the allocation derived in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. I apply their approach in my environment in Appendix G. The main difference with my analysis is that the two authors assume a greater financial sophistication as securities are state contingent, while I generally need higher sophistication in the strategy of the market participants – unless the implementation works under Markov strategies.

The study of Müller et al. (2019) considers a small open economy with a stochastic default cost and two productivity states: recession and normal time. The authors assume a financial market formed by two securities: a one-period non-contingent defaultable bond and a state-contingent bond which pays out only in normal time (i.e. GDP-linked bond). The authors additionally assume that the borrower lacks commitment only in recession and renegotiation upon default is endogenous. This coupled with the aforementioned market structure, enables the two bonds to act as Arrow securities. In other words, the defaultable bond is recession contingent and spans the different stochastic default costs through renegotiation, while the contingent bond spans the good state which is free from default risk. Hence, as the bonds act as proper Arrow securities, there is no need to rely on past history.

[41] Using different environments, Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007) and Yared (2010) also characterize a region of *ex-post* inefficiencies in optimal contracts.

The last study that I would like to discuss is the one of Aguiar et al. (2019) who consider a small open economy with a stochastic default cost and two productivity states as in Müller et al. (2019). The authors assume a continuum of maturities. They show the equivalence between the Markov equilibrium and the constrained efficient equilibrium. The Planner's problem is nonetheless peculiar as it does not take into consideration the legacy creditors in the surplus maximization. In other words, the Planner problem is sequential and only accounts for the current creditors, taking as given the inherited debt level. Furthermore, there is no participation constraint of the borrower. That is, the Planner cannot prevent the occurrence of defaults on equilibrium path. Hence, in the absence of a participation constraint – i.e. a forward-looking constraint – the Planner needs not build on past history. This combined with the disregard of legacy creditors directly leads to the Markov equilibrium in the spirit of Eaton and Gersovitz (1981).

B Foundations for Costly Debt Buybacks

In what follows, I endogenize the cost of *official* buybacks in two ways. First, I develop a standard Nash bargaining in the Markov equilibrium. Second, I present a signalling game in which costly *official* buybacks enable the borrower to signal its productivity.

Before that, I present the mechanism of Bulow and Rogoff (1988, 1991) and highlight why this does not suit my framework. The two authors show that a buyback increases the value of debt as the recovery value is divided among fewer creditors.

To see this, consider a Markov equilibrium in which $\mathbb{E}_{s'|s}D(s', 0, b'_{st}, b'_{lt}) > 0$ for all (s, b'_{st}, b'_{lt}). In addition, suppose that there is a fixed recovery value of w after default. The bond price can therefore be separated into two parts: the return when the government decides to repay and the recovery value when the government defaults.

$$q_{lt}(s, b'_{st}, b'_{lt}) = \mathbb{E}_{s'|s}\bigg[(1 - D(\Omega'_P))q^P_j(s', b'_{st}, b'_{lt}) + D(\Omega'_P)q^D_j(g', b'_{st}, b'_{lt})\bigg],$$

where the recovery value is given by

$$q^D_{lt}(s', b'_{st}, b'_{lt}) = \frac{1}{1+r}\bigg[(1-\lambda)q^D_{lt}(s', b'_{st}, b'_{lt}) + \lambda\frac{w}{b'_{lt}}\bigg],$$

and the repayment value reads

$$q^P_{lt}(s', b'_{st}, b'_{lt}) = \frac{1}{1+r}\bigg[1 + (1 - M(\Omega'_P))q_{lt}(s', b''_{st}, b''_{lt}) + M(\Omega'_P)q^{bb}_{lt}\bigg].$$

The buyback premium naturally emerges from the bond price as w is constant. When $-b'_{lt}$ increases, q_{lt}^D decreases which implies that q_{lt} decreases given the strictly positive default probability. This is what the literature calls dilution. With a buyback the opposite happens as $-b'_{lt}$ decreases. There is a *reverse* dilution which increases q_{lt}^D and therefore q_{lt}.

This mechanism however works as long as there is a strictly positive default probability. If defaults never arise on equilibrium path, the long-term bond price remains constant. Moreover, this mechanism can only rationalize buybacks at a discount (i.e. below par) on the secondary market. For instance, it cannot explain the case of Brazil which bought back its debt at a premium (i.e. when the financial value is above the face value) as shown in Section 7.

B.1 Nash Bargaining in Markov Equilibrium

In this subsection, I introduce a Nash bargaining game in the Markov equilibrium. This first shows how to endogenize the *official* buyback premium. It also reinforces the argument made in Lemma 4 about the enforcement of *official* buybacks in Markov equilibria.

The threat point of the game is that the borrower is not able to roll over its debt in the current period if the *official* buyback does not take place. In such circumstance, the borrower's value is given by

$$\begin{aligned}
\bar{V}^{NB}(\Omega_P) = &\max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\
&\text{s.t.} \quad c + q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g,k) + b_{st} + b_{lt}, \\
&\qquad\quad b'_{lt} \geq b_{lt}, \\
&\qquad\quad b'_{st} \geq 0.
\end{aligned}$$

Notice that the borrower can issue short-term assets. For any *official* buyback premium χ, I define the surplus of the borrower as

$$\Delta^b(\Omega_P; \chi) = V^B(\Omega_P; \chi) - \max\Big\{\bar{V}^{NB}(\Omega_P), V^D(s, 0, k)\Big\}.$$

The borrower's surplus corresponds to the difference between the value of conducting the *official* buyback and the value of rejecting it and suffering the underlying sudden stop.

To define the surplus of the lenders, I first need to derive the lenders' value under *official* buyback, under no *official* buyback and under default. The former reads

$$V_l^B(\Omega_P) = \max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r}\mathbb{E}_{s'|s}\Big[W^l(\Omega'_P)\Big]$$

$$\text{s.t.} \quad c_l + b_{st} + b_{lt}(1 + q_{lt}^{bb}) = q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})b'_{lt},$$

while under no *official* buyback

$$\begin{aligned}
\bar{V}_l^{NB}(\Omega_P) = & \max_{b'_{st}, b'_{lt}} c_l + \frac{1}{1+r}\mathbb{E}_{s'|s}\Big[W^l(\Omega'_P)\Big] \\
& \text{s.t.} \quad c_l + b_{st} + b_{lt} = q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}), \\
& \qquad\quad b'_{lt} \geq b_{lt}, \\
& \qquad\quad b'_{st} \geq 0.
\end{aligned}$$

and finally, under default

$$V_l^D(s, 0, k) = -k + \frac{1}{1+r}\mathbb{E}_{s'|s}\Big[(1-\lambda)V_l^D(s', 1, 0) + \lambda W^l(s', 0, 0, 0)\Big]$$

The continuation value under repayment is then $W^l(\Omega_P) = (1 - D(\Omega_P) - M(\Omega_P))\bar{V}_l^{NB}(\Omega_P) + M(\Omega_P)V_l^B(\Omega_P) + D(\Omega_P)V_l^D(s, 0, k)$. The surplus of the lenders corresponds to the difference in the value under *official* buyback and no *official* buyback

$$\Delta^l(\Omega_P; \chi) = V_l^B(\Omega_P; \chi) - \Big[(1 - D(\Omega_P))\bar{V}_l^{NB}(\Omega_P) + D(\Omega_P)V_l^D(s, k)\Big].$$

If the lenders have all the bargaining power, then it could extract a large *official* buyback premium (i.e. $\chi \to \infty$). In opposition, if the borrower has all the bargaining power, it can conduct *official* buybacks at low cost (i.e. $\chi \to 0$). To consider the case in between those two extremes, I assume that the lenders have a bargaining power of $\zeta \in [0, 1]$ and the borrower of $1 - \zeta$. In Ω_P, the *official* buyback premium $\chi(\Omega_P)$ is the solution to

$$\begin{aligned}
\chi(\Omega_P) = & \arg\max_{\tilde{\chi} \in (0,1)} \Big[\Delta^l(\Omega_P; \tilde{\chi})^{\zeta} + \Delta^b(\Omega_P; \tilde{\chi})^{1-\zeta}\Big] \\
& \text{s.t.} \quad \Delta^l(\Omega_P; \tilde{\chi}) \geq 0, \\
& \qquad\quad \Delta^b(\Omega_P; \tilde{\chi}) \geq 0.
\end{aligned}$$

In light of Lemma 4, the above bargaining problem has a solution only if the threat of the sudden stop is credible. If the threat is not credible in a given state Ω_P, $\Delta^b(\Omega_P; \chi) < 0$ for all $\chi \in (0, 1)$ meaning that there is no $\chi > 0$ for which the borrower is willing to conduct *official* buybacks instead of being punished. In other words, there is no solution to the Nash bargaining program meaning that *official* buybacks are not enforceable.

B.2 Signalling in Markov Equilibrium

Besides Nash Bargaining, I can rationalize costly *official* buybacks with a signalling game. For this purpose, consider that g is privately observed by the borrower. The lenders must therefore form beliefs on g – i.e. the borrower's type.

To be an equilibrium, beliefs have to be consistent with the market participants' strategies and, given the beliefs, each market participant's strategy must be optimal. A belief system for the lenders, $\Gamma(b_{st}, b_{lt})$, specifies the a probability distribution over G,

$$\Gamma(b_{st}, b_{lt}) = Pr(g = g_H | b_{st}, b_{lt}).$$

The lenders rely on the debt repayment, say S, as signal for the borrower's type. I therefore construct a separating equilibrium in which the borrower signals its type through debt repayment as in Cole et al. (1995) and Phan (2017a,b). The timing of actions is the following. First, g realizes and is privately observed by the borrower which then decides how much debt to repay, S. Conditional on the repayment, the lenders offer capital $k(S)$ and a bond price schedule $q_{st}(S, b'_{st}, b'_{lt,H})$ and $q_{lt}(S, b'_{st}, b'_{lt})$ for the short-term bond, b'_{st}, and the long-term bond, b'_{lt}, respectively.[42]

The repayment signal works as follows. If the repayment is sufficiently large, then the lenders believe that g_H realized. In opposition, a low repayment signals that g_L realized. The signal therefore fully reveals the shock. However, to be an equilibrium, the low type should not be willing to choose a high repayment and *vice versa*.

I assume the following. If the borrower draws g_H, it chooses to conduct an *official* buyback. In opposition, if it draws g_L, there is neither *official* buyback nor default. Hence, the repayment of the high type for a given (b_{st}, b_{lt}) is

$$S_H(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q^{bb}_{lt}),$$

and for the low type,

$$S_L(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q_{lt}).$$

With $\chi > 0$, it directly follows that $S_L(b_{st}, b_{lt}) > S_H(b_{st}, b_{lt})$ for all (b_{st}, b_{lt}). Thus, costly *official* buybacks are necessary to signal types in the absence of defaults. Whenever the lenders receive a repayment lower than $-S_H(b_{st}, b_{lt})$, it believes that g_L realized. Obviously,

[42] Note that similar to the timing in Eaton and Gersovitz (1981), it is implicitly assumed that the borrower can commit to repayment decision made before the debt auction. See Cole and Kehoe (2000) and Ayres et al. (2018) for more details.

those beliefs are consistent only if the high type has no incentive to repay according to the low type and *vice versa*. Thus, it must hold that for any (b_{st}, b_{lt}),

$$\begin{aligned} &u(y(g_H, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) = \\ &u(y(g_H, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}), \end{aligned} \tag{B.1}$$

where $b'_{j,i}$ denotes the bond choice of maturity $j \in \{st, lt\}$ of reported borrower's type $i \in \{L, H\}$. Equation (B.1) makes the high type indifferent between paying S_H or S_L. Moreover, given that $y(g_H, k(S_H)) > y(g_H, k(S_L))$ as $g_H > g_L$, (B.1) implies by concavity of the utility function that

$$\begin{aligned} &u(y(g_L, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) \leq \\ &u(y(g_L, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}). \end{aligned}$$

As a result, if the the high type indifferent between paying S_H or S_L, the low type has no incentive to pay S_H instead of S_L. Thus, if (B.1) holds, the beliefs are updated according to

$$\Gamma(b_{st}, b_{lt}) = \begin{cases} 1 & \text{if} \quad S \leq S_H(b_{st}, b_{lt}) \\ 0 & \text{else} \end{cases}$$

By Proposition 3 in Phan (2017b), the set of strategies and beliefs presented in this subsection constitutes a separating Markov equilibrium.

We see that from the definition of S_H and S_L, the *official* buyback premium χ ought to be strictly larger than zero for the signal to be informative.[43] On the other hand, as $\chi \to \infty$, $S_H(b_{st}, b_{lt}) \to \infty$. By (B.1) this would imply that the low type has to accumulate an infinite amount of assets. There is therefore a cap on how large χ can be and – similar to Lemma 4 – the lower is the *official* buyback premium, the easier is (B.1) satisfied in a given state Ω.

C Further Theory Developments

I start this section with the existence and uniqueness of the optimal contract. For this, following Marcet and Marimon (2019), I need the following interiority assumption.

Assumption C.1 (Interiority)**.** *There is an $\epsilon > 0$, such that, for all $g^t, t \geq 0$, there is a*

[43]Otherwise, instead of *official* buybacks, the borrower would need to rely on defaults to signal its type as in Cole et al. (1995) and Phan (2017a,b).

sequence $\{\tilde{c}(g^t), \tilde{k}(g^t)\}$ *satisfying,*

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(\tilde{c}(g^j)) \geq V^D(g_t, 0, \tilde{k}_t) + \epsilon.$$

Assumption C.1 ensures the uniform boundedness of the Lagrange multipliers. It states that there are strictly positive rents to be shared among the contracting parties. In my environment, this assumption is satisfied given the Inada condition on the production function.[44]

Proposition C.1 (Existence and Uniqueness)**.** *Under Assumptions 1 and C.1, there exists a unique contract allocation with initial condition* (g_0, x_0).

Having shown existence and uniqueness of the contract allocation, the following lemma derives the inverse Euler Equation which gives the consumption dynamic in the contract.

Lemma C.1 (Inverse Euler Equation)**.** *Under Assumption 1, the inverse Euler equation for a given* $g \in G$ *reads*

$$\mathbb{E}_{s'|s}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

If the participation constraint never binds, I obtain that for all (g, x),

$$\frac{1}{u_c(c(g))} \geq \mathbb{E}_{s'|s}\left[\frac{1}{u_c(c(g'))}\right]$$

with strict inequality when $\eta < 1$. In this case, the inverse Euler Equation is a positive super-martingale. Immiseration is a consequence of Doob's theorem stating that such super-martingales converge almost surely. With $\eta < 1$, the inverse of the marginal utility of consumption converges to 0. Under limited commitment of the borrower (i.e. $\nu(g) \geq 0$), one obtains a left bounded positive submartingale. The borrower's participation constraints therefore sets an upper bound on the supermartingale and prevents immiseration. Alternatively, when $\eta = 1$ consumption remains constant.

D Domestic and Foreign Capital

In this subsection I show that the absence of domestic capital in my environment is without loss of generality for the implementation of the constrained efficient allocation.

[44] See Proposition 1.

Suppose that k is made of the aggregation of domestic capital, say k_d, and foreign capital, say k_f. More precisely, I consider a CES aggregator such that

$$k = \left[\alpha_k k_d^{\frac{\phi_k - 1}{\phi_k}} + (1 - \alpha_k) k_f^{\frac{\phi_k - 1}{\phi_k}} \right]^{\frac{\phi_k}{\phi_k - 1}},$$

where $\alpha_k \in (0, 1)$ is the share of domestic capital and ϕ_k is the elasticity of substitution between domestic and foreign capital. During a default, the borrower loses access to the international capital market (i.e. $k_f = 0$) but now can rely on the domestic capital market (i.e. $k_d > 0$). This dampens the cost of default for the borrower. Given this, I consider two cases: $\frac{\phi_k - 1}{\phi_k} = 1$ and $\frac{\phi_k - 1}{\phi_k} \neq 1$.

When $\frac{\phi_k - 1}{\phi_k} = 1$, aggregation becomes linear. Particularly, with $\alpha_k = 0.5$ domestic and foreign capital are perfect substitutes. This means that during a default, the borrower can substitute k_f by k_d one to one. In other words, the borrower completely averts the output cost of default.

However, this does not mean that the economy can rely on defaults to implement the constrained efficient allocation. A default generates (international) markets exclusion with a fixed re-access probability λ. This prevents the implementation of the constrained efficient allocation. If the default arises at the lower bound of the ergodic set (i.e. $x = x^{lb}$), then markets re-access should occur as soon as g_H realizes. This is not guaranteed in my environment as market re-access follows a poisson process independent of g.[45] Besides this, markets exclusion means that $b_{lt} = b_{st} = 0$ which is not necessarily what the Planner would choose.

When $\frac{\phi_k - 1}{\phi_k} \neq 1$, domestic and foreign capital are not perfect substitutes. This means that it is not possible for the borrower to completely avert the output cost of default. In other words, production without foreign capital is not as efficient in terms of output maximization as production with both types of capital together. This means that, by averting default, a strict Pareto improvement is possible. It is therefore not possible to implement the constrained efficient allocation through defaults.

E Alternatives to Official Buybacks

In this section, I provide alternatives to *official* buybacks: "excusable" defaults, variable-coupon bonds and variable-maturity bonds.

[45]Endogenizing the post-default renegotiation would not necessarily solve the problem. With a Nash bargaining, there is no delay meaning that a default would be settled on the spot (Yue, 2010). With a Rubinstein bargaining, there is the possibility to generate delays (Bi, 2008; Benjamin and Wright, 2013; Dvorkin et al., 2021). Depending on the parameters, the model can predict re-access when g_H realizes.

First, Grossman and Van Huyck (1988) develop the concept of "excusable" defaults.[46] The idea is that defaults which are on the path of play agreed by all market participants are not punished. In other words, the debt contract specifies *ex ante* the circumstances in which the borrower is allowed to repudiate its debt without suffering from markets exclusion. Given this, if defaults were "excusable", then the borrower's binding constraint – i.e. $x = x^{lb}$ – could be interpreted as a default. The issue is that the borrower might be willing to repudiate debt more often than what the debt contract specifies. To deal with this, one can either use trigger strategies or introduce an endogenous borrowing limit. Nevertheless, the concept of "excusable" defaults has little empirical relevance. The closest policy that has been implemented to this date is a sovereign debt standstill analyzed by Hatchondo et al. (2020b) with the only difference that there is no arrears accumulation in "excusable" defaults. In addition, Mateos-Planas et al. (2023) show that if the borrower were to choose the conditions for "excusable" defaults, such events would be extremely rare if not inexistent.

Second, the long-term debt can have variable coupon as in Faraglia et al. (2019) and Aguiar et al. (2021). Particularly, assume that the coupon payment is a choice variable, say $\kappa \in [0, 1]$, for the borrower. Obviously, the variability of the coupon is a covenant in the debt contract. In other words, changes in coupon are agreed by the contracting parties *ex ante* and do not pertain to a contract renegotiation – e.g. an outright default in case of reduced coupon payment. With such debt contract, it is possible to implement the constrained efficient allocation in two ways: the borrower sets a standard coupon payment $\tilde{\kappa}$ and either increases it to $\bar{\kappa} > \tilde{\kappa}$ when $x = x^{ub}$ or decreases it to $\underline{\kappa} < \tilde{\kappa}$ when $x = x^{lb}$. In the former case, a variant of Proposition 7 applies as the borrower is not willing to pay a larger coupon payment. Hence, the same enforcement issue arises as with *official* buybacks and trigger strategies remain necessary in general. In opposition, in the case of reduced coupon payment, the borrower might be tempted to reduce the coupon payment more frequently than the Planner would. Thus, the lenders would also need to supervise the coupon policy.

Lastly, bonds can have variable maturities. That is, the maturity of outstanding short-term (long-term) debt can be lengthened (shortened). Similar to variable-coupon bonds, this is a feature which should be explicitly mentioned in the debt contract. To implement the constrained efficient allocation, the borrower ought to either lengthen the maturity of short-term debt when $x = x^{ub}$ or shorten the maturity of long-term debt when $x = x^{lb}$. Implicitly, by shortening the maturity, the borrower pays less coupons than it initially promised. In other words, the claim of legacy creditors is reduced. The opposite happens in the case of maturity lengthening. Thus, similar to variable-coupon bonds, maturity lengthening would need to be enforced, while maturity shortening should be closely supervised to avoid lowering

[46] See also the recent proposal on contingent convertible bonds by Hatchondo et al. (2022).

legacy creditors' claim too frequently.

F Price in Markov Equilibrium

The definition of price and equilibrium directly follow from Definition 3 stating that Markov equilibria are sustainable equilibria restricted to the payoff-relevant space. Thus, the price of one unit of bond is given by

$$
\begin{aligned}
q_{lt}(s, b'_{st}, b'_{lt}) &= \frac{1}{1+r}\mathbb{E}_{s'|s}\Big[(1 - D(\Omega'_P))\Big\{1 + (1 - M(\Omega'_P))q_{lt}(s', b''_{st}, b''_{lt}) + M(\Omega'_P)q^{bb}_{lt}\Big\}\Big], \\
q_{st}(s, b'_{st}, b'_{lt}) &= \frac{1}{1+r}\mathbb{E}_{s'|s}\Big[(1 - D(\Omega'_P))\Big],
\end{aligned}
\tag{F.1}
$$

The equilibrium definition follows directly from Definitions 1 and 3.

G Alternative Implementation

In what follows, I propose an alternative implementation as the one derived in Section 5. More precisely, I rely on the approach of Alvarez and Jermann (2000) using trade in state-contingent securities and an endogenous borrowing limit.

The structure of the financial market is the following. At the start of a period, the government holds a perpetual security a.[47] The government can trade $|G|$ state contingent securities $a'(g')$ with a unit price of $q(g', a'(g')|g)$. The portfolio $a'(g')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q(g, a') \equiv \sum_{g'|g} q(g', a'(g')|g)$, and an insurance portfolio of $|G|$ Arrow securities $\hat{a}'(g')$. Thus we have that $a'(g') = \bar{a}' + \hat{a}(g')$ with

$$
\bar{a}' = \frac{\sum_{g'|g} q(g', a'(g')|g)a'(g')}{q(g, a')} \quad \text{and} \quad \sum_{g'|g} q(g', a'(g')|g)\hat{a}'(g') = 0.
$$

The last equation represents the market clearing condition of the Arrow securities. Given that in equilibrium, there is going to be no default, I omit $\mathbb{I}_D$ when not necessary.

As I am only interested in applying the Second Welfare Theorem, I disregard the sunspot variables in the state space. Nevertheless, it is true that such variables are important as the First Welfare Theorem does not hold in the environment of Alvarez and Jermann (2000).

The capital market is the same as in the main text: the lenders supply k at price p which

[47] The maturity is unimportant in this implementation. The security a could also be a one-period security.

is taxed at rate $\tau = 1 - \frac{1}{p}$. The government's problem therefore reads

$$
\begin{aligned}
W^b(g,a) = & \max_{c,\{a'(g')\}_{g'\in g}} u(c) + \beta \mathbb{E}_{g'|g}\big[W^b(g', a'(g'))\big] && \text{(G.1)}\\
& \text{s.t. } c + \sum_{g'|g} q(g', a'(g')|g)(a'(g') - a) \le y(g,k) + a \\
& \bar{a}' + \hat{a}(g') \ge \mathcal{A}(g', k'), && \text{(G.2)}
\end{aligned}
$$

where $\mathcal{A}(g', k')$ represents the endogenous borrowing limit and is defined such that

$$W^b(g', \mathcal{A}(g', k')) = V^D(g', 0, k'). \tag{G.3}$$

One can see here the similarity with the borrowing limit defined in Section 6. The lenders' problem is static. I nonetheless express it in recursive form.

$$
\begin{aligned}
W^l(g, a_l) = & \max_{c_l, k_l, \{a_l'(g')\}_{g'\in g}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\big[W^l(g', a_l'(g'))\big] && \text{(G.4)}\\
& \text{s.t. } c_l + \sum_{g'|g} q(g', a_l'(g')|g)(a_l'(g') - a_l) \le p(1-\tau)k_l - k_l + a_l.
\end{aligned}
$$

Given this environment, I can determine a recursive competitive equilibrium in the following terms.

Definition G.4 (Recursive Competitive Equilibrium (RCE))**.** *A* recursive competitive equilibrium *is a sequence of prices $q(g', a'(g')|g)$ and $p(g,a)$, value functions, $W^b(g,a)$ and $W^l(g,a)$, an endogenous borrowing limit, $\mathcal{A}(g', k')$, as well as policy functions for (i) consumption, $c(g,a)$ and $c_l(g,a)$, (ii) capital, $k = k(g,a)$ and $k_l = k_l(g,a)$, (iii) asset holdings $a'(g') = A(g', g, a)$ and $a_l'(g') = A_l(g', g, a)$ such that,*

1. *Given the value function for the outside option of the government, $V^D(g', 0, k')$ as well as the prices $q(g', a'(g')|g)$ and $p(g,a)$,*
   - *(a) the policy functions $c(g,a)$ and $A(g', g, a)$, together with the value function $W^b(g,a)$, solve the government problem (G.1) with the endogenous limit, $\mathcal{A}(g', k')$.*
   - *(b) the policy functions $c_l(g, a_l)$, $k_l(g,a)$, and $A_l(g', g, a_l)$, together with the value function $W^l(g, a_l)$, solve the lenders' problem (G.4) and*
2. *Taking p as given, $k(g,a)$ is such that $u_c(c)(gf_k(k) - p) = 0$.*
3. *The price of capital is consistent with $\max_k\{p(1-\tau)k - k\}$.*

4. *The asset market clears,* $a'(g') + a_l'(g') = 0$ *for all* $g' \in G$.

5. *The product and capital markets clear,* $c(g,a) + c_l(g,a_l) = gf(k(g,a))$ *and* $k(g,a) = k_l(g,a)$.

For the government's problem, taking the first-order conditions with respect to consumption and assets, one obtains

$$
\begin{aligned}
u_c(c) &= \mu_{BC}(g,a), \\
q(g',a(g')|g) &= \beta\pi(g'|g)\frac{u_c(c')}{u_c(c)}[1 + \sum_{g''|g} q(g'',a''(g'')|g')] + \frac{\mu_{EBL}(g',a'(g'))}{u_c(c)},
\end{aligned}
$$

where μ_{BC} and μ_{EBL} are the Lagrange multipliers attached to the budget constraint and the endogenous borrowing limit, respectively. Especially, $\mu_{EBL}(g',a'(g')) \geq 0$ with $\mu_{EBL}(g',a'(g')) = 0$ if $a'(g') > \mathcal{A}(g',k')$.

Conversely, taking the first-order conditions with respect to consumption and assets of the lenders' problem

$$
q(g',a(g')|g) = \frac{1}{1+r}\pi(g'|g)(1 + \sum_{g''|g} q(g'',a''(g'')|g')).
$$

Following Krueger et al. (2008), the price is determined by the agent whose constraint is not binding. Therefore the price is determined by

$$
q(g',a(g')|g) = \pi(g'|g)(1 + \sum_{g''|g} q(g'',a''(g'')|g')) \max\left\{\beta\frac{u_c(c(g',a'(g'))}{u_c(c(g,a))}, \frac{1}{1+r}\right\}. \tag{G.5}
$$

The following lemma states that the constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.

Proposition G.2 (Alternative Implementation)**.** *Given initial conditions* (g_0, x_0)*, a constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.*

The benchmark implementation presented in Section 5 relies on changes in the term premium to mimic the state-contingency in the optimal contract, while this alternative implementation relies on changes in security holdings provided that securities are state-contingent. More importantly, given that securities are state contingent, the assumption that the borrower and the lenders keep track of the entire history of play is not anymore necessary. The

Table G.1: Alternative Implementation

	Benchmark	Alternative
$-b/y$	-353.20	15.87
Spread	3.95	4.34
$\sigma(b/y)/\sigma(y)$	8.34	0.31
$\sigma(spread)/\sigma(y)$	0.00	0.00
$\text{corr}(b/y, y)$	-0.72	0.67
$\text{corr}(spread, y)$	-0.67	0.00

Note: The variable $\sigma(\cdot)$ denotes the volatility. In the alternative implementation, $\bar{a} = b$. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25.

implementation of the constrained efficient allocation now lies on the assumption of a greater financial sophistication.

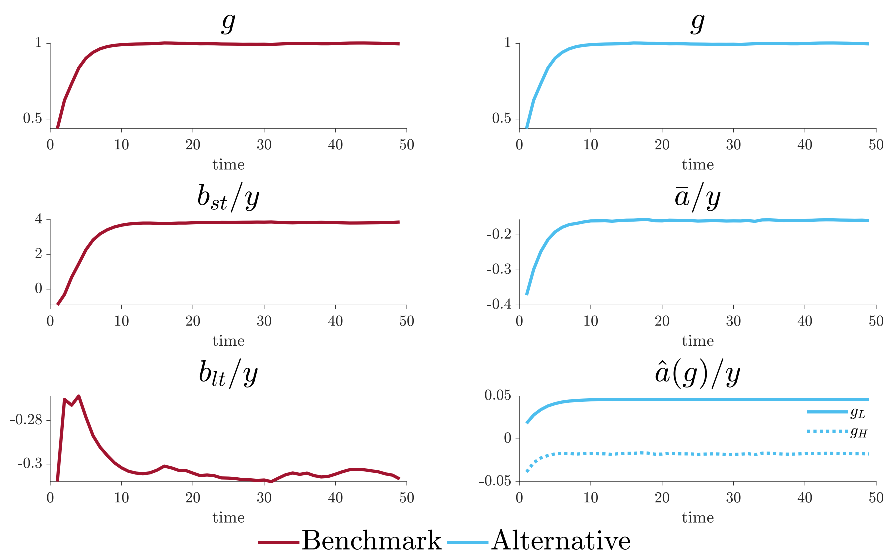

Figure G.1: Impulse Response Functions to a Negative g Shock

Having properly defined the alternative implementation, I now compare it quantitatively to the one presented in Section 5 using the calibration in Section 7.

Table G.1 presents the main difference between the two implementations. The benchmark case is related to a net asset position and a larger volatility of the debt ratio. This comes from the fact that bonds are non-contingent and the borrower alternates between short-term assets and long-term debt. Thus, large movements in debt holdings are necessary to replicate the state contingency in the contract as shown by Buera and Nicolini (2004) and Faraglia et al. (2010). Particularly, we see that the volatility of debt-to-GDP ratio is 26 times larger

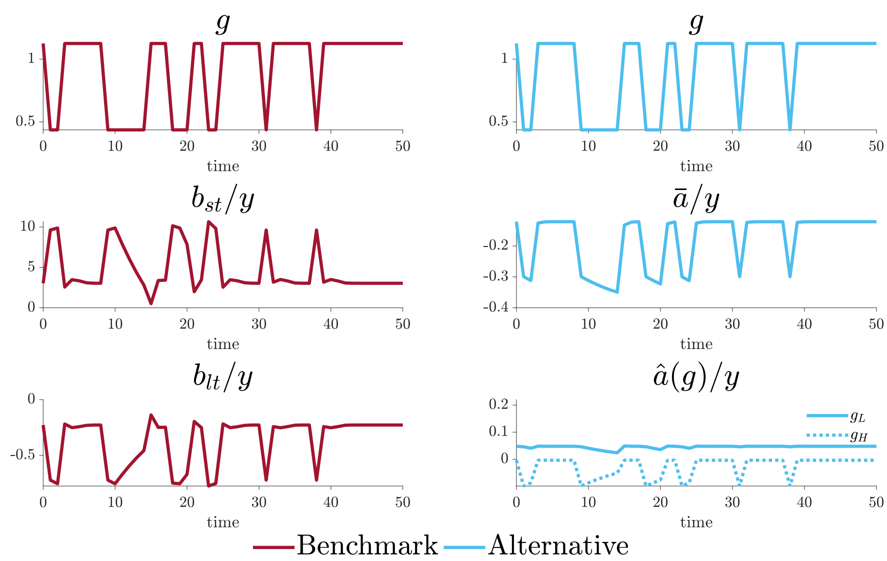

Figure G.2: Simulation of a Steady State Path

in the benchmark than in the alternative implementation. Besides this, the benchmark implementation displays a lower spread owing to *official* buybacks. As explained before, the reason behind this is that the alternative implementation does not rely on changes in prices to mimic the state-contingency of the contract given that securities are state-contingent by definition.

Similar to the previous section, I construct impulse responses to see how the two implementations work. Figure G.1 depicts the responses in red for the benchmark implementation and in blue for the alternative one. The Arrow securities complement the accumulation of bonds in the alternative case, while the benchmark implementation needs to adapt both the long-term and the short-term bonds in opposite directions.

Turning to the simulation in Figure G.2, one can see that the level of long-term bond in the benchmark case closely follows the pattern of bonds in the alternative case. The magnitude of change in the former is nonetheless larger than in the latter. In terms of Arrow securities, $\hat{a}(g_H)$ closely follows the evolution of $\bar{a}$, while $\hat{a}(g_L)$ has the opposite sign. The evolution of $\hat{a}(g_H)$ is therefore closely mimicking the evolution of b_{lt}. Given that $\hat{a}(g)$ is state contingent, the alternative implementation needs to change the debt portfolio with lower magnitude.

H Empirical Analysis on Maturity and Buyback in Brazil

In this section, I establish four main facts related to the prediction of the model for Brazil. The first fact relates to the sovereign debt maturity management. As already noted by

Arellano and Ramanarayanan (2012), Broner et al. (2013), Perez (2017) and Bai et al. (2017), maturity shortens during debt crises and lengthens otherwise.

Fact I. *Average maturity shortens during debt crises and lengthens otherwise.*

Table H.2 presents the result of a typical maturity regression analysis. The dependent variable corresponds to the average maturity (in years) on new external debt retrieved from the World Bank's International Debt Statistics from 1995 to 2019. The explanatory variable is the EMBI spread which I obtain from the Global Financial Data. As one can see, when the spread increases, the average maturity of new issuance shortens.

The second fact relates to *official* buybacks. As already highlighted in Section 7, the Brazilian government always paid a premium to extract its debt out of the market. More precisely, on average, the financial value is 24.5% above the face value.

Fact II. *Official buybacks are costly.*

While the second fact states that *official* buybacks are costly, the third fact explains when *official* buybacks occur. Consistent with the predictions of the model, *official* buybacks tend to arise in good times. Particularly, there is a positive association between the amount repurchased and the economic situation of the country.

Fact III. *Substantial official buybacks are more likely to arise in good economic times.*

Table H.2: Regression Analysis

	(1) Maturity	(2) Buyback	(3) Buyback
EMBI Spread	-0.52**	-0.33***	-1.95***
	[0.20]	[0.10]	[0.59]
Foreign Share of External Debt			9.05*
			[4.55]
Observations	25	25	15
R^2 adjusted	0.17	0.24	0.45

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

Source: Author's calculation, Global Financial Data, Tesouro Nacional and World Bank.

Table H.2 presents the result of the regression analysis on the Brazilian buyback. The dependent variable corresponds to the amount of external debt bought back in USD billion.

I use the same explanatory variables as before. As one can see, when the EMBI spread increases, the amount bought back diminishes.

Using the database of Onen et al. (2023), I include as an additional explanatory variable the share of foreign holdings of long-term external government bonds. This variable only starts in 2005, though. Given this, I come up with the last fact

Fact IV. *Substantial official buybacks are more likely with a low share of domestic holdings of external debt.*

As shown in Table H.2, a larger share of foreign holdings is associated to higher buybacks. However, this fact does not directly relate to the predictions of the model given that I do not consider domestic investors.

I Additional Quantitative Results

This subsection provides additional quantitative results. First, I construct impulse response functions following a stark negative shock in both the Markov equilibria and the CEA. Second, I look at the dynamic of a specific shock path in steady state.

Figure I.3 depicts the impulse response functions resulting from a stark negative shock on selected key variables. The responses are computed as the mean of 2,000 independent shock histories starting with the lowest shock as well as initial debt holdings and relative Pareto weight drawn from the ergodic set. The blue line represents the Markov equilibrium with default (MA), the green line the Markov equilibrium without default (MAND) and the red line the constrained efficient allocation (CEA).

We see that at the outbreak of the shock's realization, capital decreases in the MA, the MAND and the CEA – albeit to a lesser extent in the latter two cases. Capital and debt go to zero as economies in the MA fall into default. In opposition, defaults do not arise in the CEA and the MAND which can both increase the indebtedness on impact. Maturity shortens at the outbreak of the bad shock's realization in both the MAND and the CEA. The Markov equilibria rely mostly on short-term debt, while the CEA uses both in opposite directions. Note further that the CEA switches from short-term debt to asset holdings.

The impulse response functions give an idea of the long-run dynamic of the economy. However, it does not tell how the economy reacts in the short run especially when there is a transition between two values of g. Thus, I simulate the economy and generate one history of shocks for 200 periods. To avoid that the initial conditions blur the results, the first 150 periods are discarded. Again, the blue line represents the MA, the green line the MAND

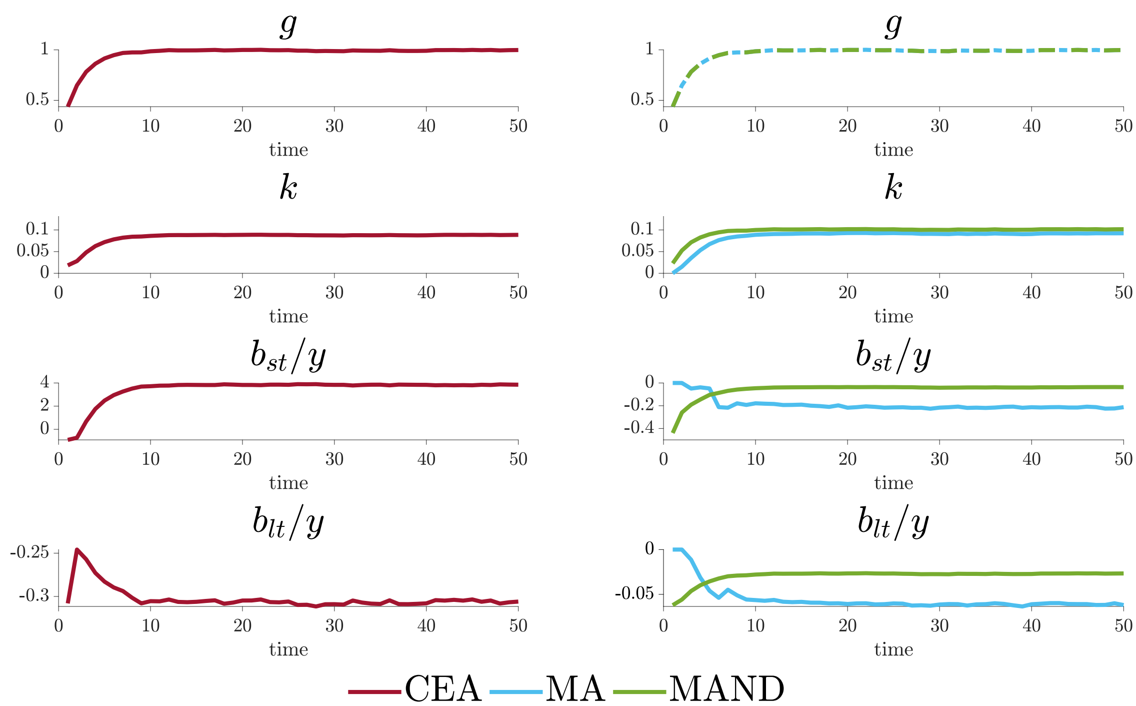

Figure I.3: Impulse Response Functions to a Negative g Shock

and the red line the CEA. In addition, the grey area represents the period in which the MA is in default.

Figure I.4 depicts the simulation results. One observes that, in the MA, the economy defaults in the transition from g_H to g_L. This causes market exclusion and therefore $b_{st} = b_{lt} = k = 0$. In opposition, there are no defaults in the CEA and the MAND. In the transition from g_H to g_L, the government adapts the maturity of the debt and increases its indebtedness. Especially, one sees that the level of short-term bonds have opposite movements in the MAND and the CEA. The magnitude of the changes is also very different. Consistent with the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), the movements in debt holdings are the most pronounced for the CEA. Particularly, holdings of short-term debt oscillate between 2000% and 2500% of output.

J Welfare Analysis

To compute the borrower's welfare, first define the borrower's value for a sequence of consumption $\{c(s^t, \mathbb{I}_D^t)\}$ starting from an initial state at $t = 0$ as

$$W^b(\{c(s^t, \mathbb{I}_D^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c(s^t, \mathbb{I}_D^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c(s^t, \mathbb{I}_D^t)^{1-\vartheta}}{1-\vartheta},$$

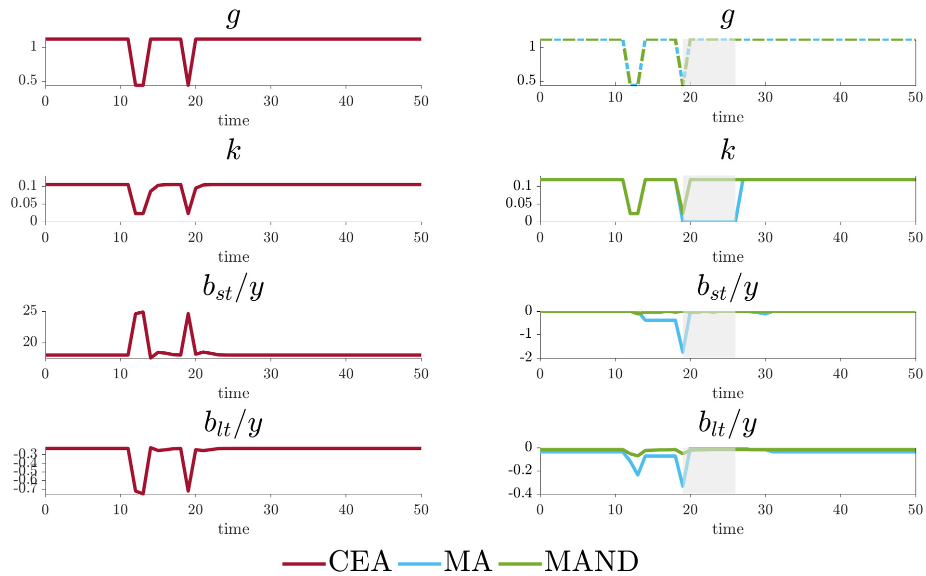

Figure I.4: Simulation of a Typical Path

I denote the borrower's consumption allocation in the benchmark model by $\{c^b(s^t, \mathbb{I}_D^t)\}$ and the consumption allocation in the alternative model by $\{c^a(s^t, \mathbb{I}_D^t)\}$. In addition, I define the consumption-equivalent welfare gain of the alternative model with respect to the benchmark model by ι such that

$$W^b(\{(1+\iota)c^b(s^t, \mathbb{I}_D^t)\}) = W^b(\{c^a(s^t, \mathbb{I}_D^t)\}).$$

Given the functional form of the instantaneous utility one obtains

$$(1+\iota)^{1-\vartheta}\Big[W^b(c^b(s^t, \mathbb{I}_D^t)\})\Big] = W^b(\{c^a(s^t, \mathbb{I}_D^t)\}).$$

The borrower's welfare gain therefore boils down to

$$\iota = \left[\frac{W^b(\{c^a(s^t, \mathbb{I}_D^t)\})}{W^b(\{c^b(s^t, \mathbb{I}_D^t)\})}\right]^{\frac{1}{1-\vartheta}} - 1.$$

The lenders' welfare gains can be computed in the same way by setting $\vartheta = 0$ owing to the risk neutrality.

K Proofs

Proof of Lemma 1

The value of permanent autarky is given by

$$v_a(g_t) = \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0)), \tag{K.1}$$

as the lenders set $k = 0$ in case of default. Permanent autarky is the worst equilibrium outcome as the government could always be better off with $k = \epsilon$ for small $\epsilon > 0$ given the Inada conditions on the production function. I show this in Proposition 1.

Permanent autarky is an equilibrium of the market economy. Suppose that the lenders believe that $D_t = 1$ for all t. Then, they set $p_t = \infty$ and $q_{j,t} = 0$ for all $j \in \{st, lt\}$. Given this, the government finds optimal to choose $D_t = 1$ for all t confirming the lenders' beliefs. □

Proof of Lemma 2

Necessity:

> Conditions (1), (2), (3) and (4) follow directly from the equilibrium's definition. The budget constraints in the repayment (including *official* buybacks) and default states is required by feasibility. Finally, the fact that the value of the borrower is larger than the value under the worst equilibrium – i.e. (K.1) – ensures that the allocation can be sustained by trigger strategies.

Sufficiency:

> Let's rely on trigger strategy (Abreu, 1988). That is, each player is punished by the worst outcome of the game (i.e. permanent autarky which is an equilibrium as shown in Lemma 1) if it decides to deviate. Since the outcome satisfies (1), (2), (3) and (4), it is optimal. Also as it satisfies the different budget constraints it is feasible. Finally, no deviations from play is profitable given that the value of the borrower is larger than the value under the worst equilibrium.

□

Proof of Proposition 1

I first show that the autarkic allocation is not optimal. The proof follows Aguiar et al. (2009). Consider a version of the optimal contract in which the outside option corresponds to the value of permanent autarky is given by (K.1). In autarky, $k = 0$ and from Definition 2 there is an $x_a(g)$ such that $u(c(g, x_a(g))) = u(gf(0))$ at all histories. Using the definition of h in Assumption 1, consider that one increases h by Δh and $u(c(g, x_a(g)))$ by $\theta u_c(gf(0))\Delta h$ where

$$\theta = \frac{u_c(g_L f(0))}{u_c(g_L f(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g_L} u_c(g' f(0))} < 1.$$

I defined θ such that the borrower's participation constraint holds. To see this, note that the increase of h increases the borrower's outside option by $u_c(gf(0))\Delta h$ as it can benefit from the additional level of capital before going to autarky forever. However, if the borrower does not choose autarky, its value increases by $\theta(u_c(gf(0))+\frac{\beta}{1-\beta}\mathbb{E}_{g'|g} u_c(g'f(0)))\Delta h \geq u_c(gf(0))\Delta h$ by definition of θ. Hence the borrower's participation constraint is satisfied. Furthermore, the value of the lenders changes by

$$\Delta h \frac{1}{1+r}\left(1 - \mathbb{E}_{g'|g_L}\left[\frac{u_c(g' f(0))}{u_c(c(g', x_a(g)))}\right]\theta\right) = \Delta h \frac{1}{1+r}(1-\theta) > 0,$$

where the equality comes from the fact that we consider the autarkic allocation (i.e. $c(g, x_a(g)) = gf(0)$) and the inequality from the fact that $\theta < 1$. As a result, the autarkic allocation is not optimal. The contract is therefore restricted to relative Pareto weights $x > \max_{g \in G}\{x_a(g)\} = x_a(g_H)$.

Consider the interval $[\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. From the law of motion of the relative Pareto weight, x' is strictly increasing in x. From the first-order conditions on consumption, c is strictly increasing in x. Hence, so does the value of the borrower. In opposition, with a greater c or x', the value of the lenders decreases. That is the lenders' value is strictly decreasing in x. Moreover, note that the relative Pareto weight, $x_{t+1} = \frac{\mu_{b,t}+\gamma(g_t)}{\mu_{l,t}}$, cannot be negative as $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t. Hence, any continuation of an efficient allocation is itself efficient. □

Proof of Proposition 2

– Part I

The optimal level of capital is given by

$$gf_k(k(g)) - 1 = \nu(g) u_c(gf(k(g))) gf_k(k(g)) x.$$

As one can see, as soon as the participation constraint does not bind (i.e. $\nu(g) = 0$), the contract can attain the production-maximizing level of capital $k^*(g)$ such that

$gf_k(k^*(g)) = 1$. When this condition is not met, $k < k^*(g)$. Thus, define $x^*(g)$ such that

$$V^b(g, x^*(g)) = V^D(g, 0, k^*(g)).$$

By the above definition, if $x < x^*(g)$, capital is distorted in state g, while if $x \geq x^*(g)$, capital is at the production-maximizing level. Moreover as $V^D(g_L, 0, k^*(g_L)) < V^D(g_H, 0, k^*(g_H))$, $x^*(g_H) > x^*(g_L)$.

Assume by contradiction that for $x_1 < x_2 < x^*(g)$ one has that $k(g, x_1) \geq k(g, x_2)$ for all $g \in G$. From Proposition 1, we have that $V^b(g, x_1) < V^b(g, x_2)$. Moreover, as $k^*(g) > k(g, x_1) \geq k(g, x_2)$, the borrower's participation constraint binds which gives

$$\begin{aligned} V^b(g, x_1) &= u(gf(k(g, x_1))) + \beta \mathbb{E}_{g'|g} V^D(g', 0, k') \\ &< V^b(g, x_2) \\ &= u(gf(k(g, x_2))) + \beta \mathbb{E}_{g'|g} V^D(g', 0, k'), \end{aligned}$$

where the two equalities come from the binding borrower's participation constraint and the inequality for the fact that $x_1 < x_2$ for a given g. This is contradiction. Hence, it must be that $k(g, x_1) < k(g, x_2)$ for any $x_1 < x_2$.

The fact that that $k(g, x) > 0$ for all (g, x) follows directly from Proposition 1 which shows that the autarkic allocation is not optimal.$\square$

– Part II

The law of motion of the relative Pareto weight is given by $x'(g) = (1 + \nu(g))\eta x$, while the first-order condition on consumption reads $u_c(c(g)) = \frac{1}{1+\nu(g)}$.

Given the first-order condition, $c(g_L, x) \leq c(g_H, x)$ only when $\nu(g_L) \leq \nu(g_H)$. Assume by contradiction that $\nu(g_L) > \nu(g_H)$. This implies that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Especially, consider that $\nu(g_L) > \nu(g_H) \geq 0$. In this case,

$$\begin{aligned} u(c(g_H, x)) + \beta \mathbb{E}_{g'|g_H} V^b(g', x'(g_H, x)) &\geq u(g_H f(k(g_H))) + \beta \mathbb{E}_{g'|g_H} V^D(g', 0, k'), \\ u(c(g_L, x)) + \beta \mathbb{E}_{g'|g_L} V^b(g', x'(g_L, x)) &= u(g_L f(k(g_L))) + \beta \mathbb{E}_{g'|g_L} V^D(g', 0, k'). \end{aligned}$$

Given that $g_H > g_L$ and $\pi(g|g) > 0.5$ for all $g \in G$, $u(g_H f(k(g_H))) > u(g_L f(k(g_L)))$ and $V^D(g_H, 0, k) > V^D(g_L, 0, k)$. This implies that

$$u(c(g_H, x)) + \beta V^b(g', x'(g_H, x)) > u(c(g_L)) + \beta V^b(g', g_L, x'(g_L, x)).$$

which contradicts the fact that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Hence, $\nu(g_L) \leq \nu(g_H)$ which gives $c(g_L, x) \leq c(g_H, x)$ and $x(g_L, x) \leq x(g_H, x)$ as desired.

Especially, by definition, when $x \geq x^*(g_H)$, then $\nu(g) = 0$ for all g implying that $c(g_L, x) = c(g_H, x)$ and $x(g_L, x) = x(g_H, x)$. Otherwise, $c(g_L, x) < c(g_H, x)$ and $x(g_L, x) < x(g_H, x)$. $\square$

– Part III

This proofs is a modified version of Thomas and Worrall (1990, Lemma 4). The value of liabilities in the optimal contract is given by

$$V^l(g, x) \equiv gf(k(g)) - k(g) - c(g, x) + \frac{1}{1+r}\mathbb{E}_{g'|g}V^l(g', x'(g, x)).$$

Assume by contradiction that for a given x it holds that $V^l(g_H, x) \leq V^l(g_L, x)$. For $x \geq x^*(g_H)$, one directly reaches a contradiction as $c(g_L, x) = c(g_H, x)$ and $x(g_L, x) = x(g_H, x)$ which implies that $V^l(g_H, x) > V^l(g_L, x)$.

For $x < x^*(g_H)$, consider the pooling allocation in which $u(\ddot{c}(g_H, x)) = u(\ddot{c}(g_L, x)) = u(c(g_H, x))$ and $\ddot{V}^b(g_H, x) = \ddot{V}^b(g_L, x) = V^b(g_H, x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to

$$\ddot{V}^l(g_H, x) > \ddot{V}^l(g_L, x)$$

which is a direct contradiction. Hence, $V^l(g_H, x) \geq V^l(g_L, x)$. However, $V^l(g_H, x) = V^l(g_L, x)$ is ruled out by fact that there is no perfect risk sharing when $x < x^*(g_H)$. $\square$

Proof of Proposition 3

Recall the law of motion of the relative Pareto weight

$$x'(g) = (1 + \nu(g))\eta x.$$

The motion of the relative Pareto weight is dictated by the relative impatience, η, and the binding participation constraint, ν. I consider two cases. On the one hand, if $\eta < 1$, the relative Pareto weight increases only if $\nu(g) > 0$ is sufficiently large to overcome impatience. As we know, when $x \geq x^*(g)$, $\nu(g, x) = 0$ meaning that impatience eventually dominates the limited commitment issue. On the other hand, if $\eta = 1$ immiseration due to impatience does not exist and the relative Pareto weight remains constant.

When $\eta = 1$, the upper bound of the ergodic set coincides with the lower bound. As shown in Proposition 2, $x'(g_L, x) \leq x'(g_H, x)$. Moreover, by definition of $x^*(g_H)$ in Proposition 2, $x^{ub} = x^{lb} = x^*(g_H)$. Conversely, when $\eta < 1$, impatience prevents the contract to reach $x^*(g_H)$ as $\nu(g_H, x^*(g_H)) = 0$. Hence, $x^*(g_H) > x^{ub}$. Moreover, $x'(g_L, x) < x'(g_H, x)$ when $x < x^*(g_H)$ implying that $x^{ub} > x^{lb}$. In other words, impatience immiserates the relative Pareto in the low productivity state implying that $x^*(g_H) > x^{ub} > x^{lb}$.

To show the existence of the ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as the midpoint of $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \rightarrow \mathbb{R}$ as

$$Q(x, G) = \sum_{g'|g} \pi(g'|g) \mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of g_L, the relative Pareto weight transit to x^{lb} (either through impatience or because $x^{lb} = x^{ub}$). Hence for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(g_L|g_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M < \infty$ g_H, the relative Pareto weight transit to x^{ub} (either through the binding constraint or because $x^{lb} = x^{ub}$) meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}])^M \geq \pi(g_H|g_H)^M > 0$. Setting $\epsilon = \min\{\pi(g_L)^M, \pi(g_H)^M\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. □

Proof of Proposition 4

The proof of this proposition is by construction. Similar to Dovis (2019), I express the policy functions of the implemented contract as a function of the relative Pareto weight, x, and the productivity state, g. Formally, define

$$\begin{aligned}
&\bar{D}, \bar{M} : G \times X \rightarrow \{0, 1\}, \\
&\bar{k}, \bar{p}, \bar{q}_{st}, \bar{q}_{tl}, \bar{b}_{st}, \bar{b}_{lt} : G \times X \rightarrow \mathbb{R}.
\end{aligned}$$

Given the timing of actions, the price schedules and bond policies depend on the prospective relative Pareto weights after the productivity shock realizes. Those objects can therefore be rewritten as

$$\begin{aligned}
&\bar{b}_j(g, x) = \bar{b}_j(x'(g, x)), \\
&\bar{q}_j(g, x) = \bar{q}_j(x'(g, x)) \quad \text{for all } j \in \{st, lt\}.
\end{aligned}$$

I first determine the default and *official* buyback policies. Subsequently, I compute the underlying prices. I then define the portfolio of bonds to match the total value of debt $V^l(g,x)$ implied by the constrained efficient allocation. Finally, I determine the optimal capital pricing from the optimality conditions of the lenders and the domestic firms.

Given Proposition 1, autarky is never optimal in the contract. Hence, the government never enters into default. That is $\bar{D}(g,x) = 0$ for all (g,x). The government will therefore rely on changes in the maturity structure and *official* buybacks as in the Markov equilibrium without default. I assume that *official* buybacks arise only if the economy hits the upper bound of the ergodic set,

$$\bar{M}(g,x) = \begin{cases} 1 & \text{if} \quad g = g_H \text{ and } x = x^{ub} \\ 0 & \text{else} \end{cases}$$

Given the above policies, the short-term bond price equates the risk-free price,

$$\bar{q}_{st}(x) = \frac{1}{1+r},$$

while the long-term bond price,

$$\bar{q}_{lt}(x) = \frac{1}{1+r}\mathbb{E}_{g'|g}[1 + \bar{q}_{lt}(x')].$$

Note further that, the long-term bond price has the following properties.

Lemma K.2 (Bond Price)**.** *With $q_{lt}^{bb} = q_{lt} + \chi$ and $\chi > 0$, the long-term bond price is the unique fixed point of $\bar{q}_{lt}$, is decreasing and is such that*

$$\frac{1}{r} + \chi > \bar{q}_{lt}(x'(g_H,x)) \geq \bar{q}_{lt}(x'(g_L,x)) > \frac{1}{r},$$

with strict inequality when $\eta < 1$.

Proof. Recall that the long-term bond price is given by

$$\bar{q}_{lt}(g,x) = \frac{1}{1+r}\mathbb{E}_{g'|g}\left[(1-\bar{D}(g',x'))\left\{1 + (1-\bar{M}(g',x'))\bar{q}_{lt}(g',x') + \bar{M}(g',x')q_{lt}^{bb}\right\}\right],$$

I consider that $\bar{D}(g',x') = 0$ for all (g',x') and $\bar{M}(g',x') = 1$ if $g' = g_H$ as well as $x' = x^{ub}$ and $\bar{M}(g',x') = 0$ otherwise. From Proposition 3, g_H and $x = x^{ub}$ arises with strictly positive probability for any (g, x),

$$\frac{1}{r} + \chi > \bar{q}_{lt}(g, x) > \frac{1}{r}.$$

Define Q_{lt} as the space of bounded functions $\bar{q}_{lt} : [\underline{x}, \bar{x}] \to [0, \frac{1}{r} + \chi]$ and $\mathbb{T} : Q_{lt} \to Q_{lt}$ as

$$\mathbb{T}\bar{q}_{lt}(g, x) = \frac{1}{1+r} \sum_{g'} \pi(g'|g)[1 + \bar{q}_{lt}(g', x')].$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$ which is increasing as $\mathbb{T}$ maps increasing functions into increasing functions. This implies that $\bar{q}_{lt}(x'(g_H, x)) \geq \bar{q}_{lt}(x'(g_L, x))$ as $x'(g_H, x) > x'(g_L, x)$ for all x in the above specified domain. Assume now that there exists a x such that $\bar{q}_{lt}(x'(g_H, x)) = \bar{q}_{lt}(x'(g_L, x))$. This requires that $x'(g_H, x)$ and $x'(g_L, x)$ belongs to a subset $[x_t, x_{t+1}]$ where $\bar{q}_{lt}$ stays constant. Hence, for any $\ddot{x} \in [x_t, x_{t+1}]$, it must be that $x'(g_H, \ddot{x}), x'(g_L, \ddot{x}) \in [x_t, x_{t+1}]$ which is a contradiction as $x'(g_H, x_{t+1}) > x_{t+1}$ when $\eta < 1$. Therefore it must be that $\bar{q}_{lt}(x'(g_H, x)) > \bar{q}_{lt}(x'(g_L, x))$ when $\eta < 1$ and $\bar{q}_{lt}(x'(g_H, x)) \geq \bar{q}_{lt}(x'(g_L, x))$ otherwise. $\square$

Having properly determined the different price schedules, I can now determine the bond holdings and the maturity in order to match the total value of the debt implied by the constrained efficient allocation. Particularly, it must hold that when $x = x^{ub}$,

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + q_{lt}^{bb}], \tag{K.2}$$
$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))]. \tag{K.3}$$

Otherwise, the relationship is given by

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_H, x))],$$
$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma K.2 ensures that there exists a unique solution. The maturity structure of the bond portfolio is therefore properly determined.

To complete the proof, I determine the optimal capital price. Given that $\tau = 1 - \frac{1}{\bar{p}(g,x)}$, from the optimality conditions of the domestic firms and the lenders I get

$$g f_k(k) = \bar{p}(g, x). \tag{K.4}$$

Hence, the constrained efficient allocation can be replicated with the above policies for default, *official* buyback, and bond holdings. The optimality conditions of the lenders, the government and the domestic firms are satisfied as well as the price schedules.

This concludes the proof as the market allocation satisfies the necessary and sufficient conditions provided in Lemma 2. Especially, I used the budget constraints to determine the optimal bond holdings given the prices computed according to (2). The capital pricing is set to match the conditions (3) and (4). Finally, the resource constraint and the condition for reversion to the worst equilibrium outcome are satisfied as the constrained efficient allocation meet those requirements. Especially, note that $V^D(g_t, 1, 0) \geq v_a(g_t)$ as defined in (K.1) given that $\lambda \geq 0$. Thus, the participation constraint (5) ensures that the outcome can be sustained by trigger strategies. $\square$

Proof of Lemma 3

The fact that default never occurs is a direct corollary of Proposition 1. Regarding buybacks, I consider that the government conducts *official* buybacks when it hits the upper bound of the ergodic set – i.e. $x = x^{ub}$ and $g = g_H$. I therefore need to consider two alternatives.

First, could *official* buybacks occur in the lower bound of the ergodic set – i.e. $x = x^{lb}$? The answer is negative. To reach the lower point of amnesia, the relative Pareto weight needs to decrease. More precisely, in steady state $x'(g_L) \leq x$ as shown in Proposition 3. This implies that the value of the lenders increases as g_L realizes. This goes against the idea of a debt repurchase which (weakly) reduces indebtedness. Furthermore, from Part III of Proposition 2, it holds that $V^l(g_H, x) > V^l(g_L, x)$ which implies debt relief in the low productivity state. However, the price of long-term debt would increases as g_L realizes (i.e. the reverse of Lemma K.2) which goes against the idea of a debt relief. Hence, it is not possible to have an *official* buyback at any point related to the realization of g_L.

Second, could *official* buybacks occur before the contract hits the upper bound of the ergodic set – i.e. $x < x^{ub}$ and $g = g_H$? The answer is positive as the realization of g_H is associated with a debt decrease. However, one has to be careful that each *official* buyback should be such that $b'_{lt} \geq b_{lt}$. Moreover, note that if *official* buybacks happen at say $\tilde{x} < x^{ub}$ and $g = g_H$, then for all $x \in (\tilde{x}, x^{ub}]$ $\bar{q}_{lt}(x'(g_H, x)) \leq \bar{q}_{lt}(x'(g_L, x))$. $\square$

Proof of Proposition 5

I prove the proposition following Bhaskar et al. (2012). I first show that every equilibrium under Assumption 2 with $(\psi, \epsilon) > 0$ is essentially sequentially strict. I then prove that every essentially sequentially strict equilibrium is a Markov (perfect) equilibrium.

I start the proof with some definitions. Given the information structure, I split the histories into two categories: public and private. Public histories are the ones defined in Section 4 – that is h_b^t and h_l^t. Private histories of the borrower and the lenders at time t are the ones tracking the utility shocks – that is $p_b^t = (p_b^{t-1}, \varrho_{b,t})$ and $p_l^t = (p_l^{t-1}, \varrho_{l,t})$, respectively. Finally, I denote he entire history of the play including the privately observed utility shocks by $\hat{h}^t$.

In addition, I denote σ_b and σ_l as the strategy profile of the borrower and the lenders, respectively. Besides this, $\mathcal{C}_i$ corresponds to the countable set of actions with typical element a_i for market participant $i \in \{b, l\}$. For instance, actions taken by the borrower relate to borrowing, defaults and buybacks, while the lenders choose capital and its price. Moreover, $W^b(\sigma_b, \sigma_l | h_b^t, p_b^t)$ and $W^l(\sigma_b, \sigma_l | h_l^t, p_l^t)$ represent respectively the value of the borrower and the lenders from the strategy profile (σ_b, σ_l) at the relevant histories.

Given that each market participant has some private information regarding their payoff, they need to form beliefs about the unobserved utility shock of the other participants. Denote the belief of agent $i \in \{b, l\}$ over the entire history $\hat{h}^t$ as $\omega_i^{(h_i^t, p_i^t)}$. I follow Bhaskar et al. (2012) and put the least structure possible on such beliefs. They simply need to be independent of the private payoff shocks and put zero weight to events that history $\hat{h}^t$ is inconsistent with h_i^t. With this, I define a Markov equilibrium as

Definition K.5 (Markov Equilibrium)**.** *A strategy σ_i for $i \in \{b, l\}$ is Markov if for any two histories $(h_i^t, p_i^t) \neq (\tilde{h}_i^t, \tilde{p}_i^t)$ ending with the same state Ω_t,*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(\tilde{h}_i^t, \tilde{p}_i^t).$$

A strategy profile (σ_b, σ_l) is a Markov equilibrium if (σ_b, σ_l) is Markov and for any alternative strategy $(\tilde{\sigma}_b, \tilde{\sigma}_l)$,

$$W^b(\sigma_b, \sigma_l) \geq W^b(\tilde{\sigma}_b, \tilde{\sigma}_l) \;\wedge\; W^l(\sigma_b, \sigma_l) \geq W^l(\tilde{\sigma}_b, \tilde{\sigma}_l).$$

Note that this definition is equivalent to Definition 3 in the main text accounting for Assumption 2. Furthermore, I define a sequential best response as

Definition K.6 (Sequential Best Response)**.** *A strategy σ_i is a sequential best response to (σ_{-i}, ω_i), if for each history (h_i^t, p_i^t) and each alternative strategy $\tilde{\sigma}_i$*

$$\int W^i(\sigma_i, \sigma_{-i} | \hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t) \geq \int W^i(\tilde{\sigma}_i, \sigma_{-i} | \hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Strategy σ_i is a sequential best response to σ_{-i} if strategy σ_i is a sequential best response

(σ_{-i}, ω_i) *for some* ω_i.

Given the information structure, there is no general solution concept which can be used here. That is why, Bhaskar et al. (2012) appeal to the very weak concept of sequential optimality. Nonetheless, a profile of mutual sequential best response for the borrower and the lenders represents a perfect Bayesian equilibrium.

The other concept defined by the aforementioned authors is the current shock strategy which relies at most on the current value of the private shock. Formally

Definition K.7 (Current Shock Strategy)**.** *A strategy* σ_i *is a current shock strategy, if for any public history* (h_i^t, p_i^t) *and for any two histories,* p_i^t *and* $\tilde{p}_i^t$*, both finishing with the same* ϱ_i*, then for almost all* ϱ_i

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(h_i^t, \tilde{p}_i^t).$$

The next lemma links Definitions K.6 and K.7. In words, any sequential response relies at most on the current value of the private shock. As a result the history of past private shocks becomes irrelevant.

Lemma K.3 (Sequential Strictness and Current Shock Strategy)**.** *If* σ_i *is a sequential best response to* σ_{-i}*, then* σ_i *is a current shock strategy.*

Proof. Consider a market participant i with history (h_i^t, p_i^t). The expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t) = \mathbb{E}_{g'|g} \int \int \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', \hat{h}^t) d\varsigma_{P_i}(\varrho_i') d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Since σ_{-i} and $\omega_i^{(h_i^t, p_i^t)}$ do not depend on the private history, the value $W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t)$ is also independent of private history. Furthermore, since the density of ϱ_i is absolutely continuous, the market participant i can only be indifferent between two actions on a zero measure of the support. For different values of ϱ_i, the action is unique and independent of the past values of the shock. $\square$

Given that beliefs on the history of past private shock do not matter, I can suppress the dependence on the beliefs and the private shock realization in the value function. Thus, the expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i} | h_i^t) = \int \mathbb{E}_{g'|g} \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', h_i^t) d\varsigma_{P_i}(\varrho_i').$$

I then arrive to the first step of the proof. Given that beliefs over private histories are irrelevant for optimality, every perfect Bayesian equilibrium (i.e. a profile of mutual sequential best responses) satisfying Assumption 2 with $(\psi, \epsilon) > 0$ are essentially sequentially strict.

Lemma K.4 (Sequential Best Response and Perfect Bayesian Equilibrium)**.** *Every perfect Bayesian equilibrium satisfying Assumption 2 with $(\psi, \epsilon) > 0$ is essentially sequentially strict.*

Proof. I need to show that for any period, history and for almost all values of the private shock, the optimal action is unique. I consider the case of the borrower first. The borrower's value from action a_b after the realization of ϱ_b is given by

$$W^b(a_b, \varrho_b, \sigma_b | h_b^t) = u(a_b, g) + \epsilon \varrho_b^a + \beta \mathbb{E}_{g'|g} W^i(\sigma_b | a_b, g', h_b^t).$$

Suppose two actions a_b and $\tilde{a}_b$, the equality $W^b(a_b, \varrho_b, \sigma_b | h_b^t) = W^b(\tilde{a}_b, \varrho_b, \sigma_b | h_b^t)$ implies that

$$\epsilon(\varrho_b^{a_b} - \varrho_b^{\tilde{a}-b}) = u(a_b, g) - u(\tilde{a}_b, g) + \beta \mathbb{E}_{g'|g} \left[W^b(\sigma_b | a_b, g', h_b^t) - W^b(\sigma_b | \tilde{a}_b, g', h_b^t) \right].$$

The set of actions is countable, whereas the set of values of private shocks for which a market participant can be indifferent has measure zero. Hence, for almost all values of ϱ_i, the set of maximizing actions must be a singleton, and the profile is essentially sequentially strict. The proof naturally extends to the case of the lenders and is therefore omitted. □

Now that we have that all equilibria satisfying Assumption 2 with $(\psi, \epsilon) > 0$ are essentially sequentially strict, I simply need to show that sequentially strict equilibria are Markov equilibria.

Lemma K.5 (Sequential Strictness and Markov Equilibrium)**.** *Every essentially sequentially strict perfect Bayesian equilibrium is a Markov perfect equilibrium.*

Proof. Consider a t period history h^t. As shown previously, the private history matters, so the focus is on public history. Under Assumption 2 with $(\psi, \epsilon) > 0$, the borrower's behavior will not depend on h^t anymore from $t + \mathcal{T} + 1$ periods onward given that its memory is bounded to $\mathcal{T}$ periods back. This means that the lenders' value will not depend on h^t from $t + \mathcal{T} + 1$ periods for sure. As a result, if the lenders' strategy is sequentially strict, then h^t becomes irrelevant from $t + \mathcal{T} + 1$ periods.

What happens in period $t + \mathcal{T}$? This represents the last period in which strategies could be conditioned on h^t. However, at that time, the borrower's maximization problem is independent of h^t as no conditioning is possible next period. In addition, sequential strictness implies that the maximizing action is a singleton. Applying this argument recursively completes the proof. □

I have therefore shown that, under the assumption of bounded memory of the borrower, small perturbations in the payoff of the market participants suppress all equilibria except Markov ones. □

Proof of Proposition 7

Assume by contradiction that in a given state Ω_P, the borrower wants to conduct an *official* buyback. That is, the borrower picks a pair (b'_{st}, b'_{lt}) such that

$$V^{NB}(\Omega_P) < V^B(\Omega_P).$$

The consumption under *official* buyback is given by

$$c^B(\Omega_P) = y(g,k) + b_{st} + b_{lt}(1 + q^{bb}_{lt}) - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},$$

and the expected continuation value by

$$\mathbb{E}_{g'|g}\Big[W^b(g', b'_{st}, b'_{lt})\Big].$$

Now consider the alternative strategy of picking the same pair (b'_{st}, b'_{lt}) but conducting an *unofficial* buyback. In such circumstance, consumption is given by

$$c^{NB}(\Omega_P) = y(g,k) + b_{st} + b_{lt} - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}).$$

It is clear from the *official* buyback premium that $c^{NB}(\Omega_P) > c^B(\Omega_P)$. Moreover, as the borrower chooses the same (b'_{st}, b'_{lt}), the continuation value is the same as before. Hence,

$$V^{NB}(\Omega_P) = u(c^{NB}(\Omega_P)) + \beta\mathbb{E}_{g'|g}\Big[W^b(g', b'_{st}, b'_{lt})\Big] > u(c^B(\Omega_P)) + \beta\mathbb{E}_{g'|g}\Big[W^b(g', b'_{st}, b'_{lt})\Big] = V^B(\Omega_P),$$

which contradicts the fact that an *official* buyback is ever optimal. □

Proof of Lemma 4

- If $b'_{st} \geq 0$

  The proof follows the same logic as the one of Proposition 7. Suppose by contradiction that the lenders can enforce *official* buybacks in a state Ω_P such that $B_{st}(\Omega_P) \geq 0$. Formally, in the case of an *official* buyback, the borrower chooses $b'_{st} = B_{st}(\Omega_P) \geq 0$ and $b'_{lt} = B_{lt}(\Omega_P) \geq b_{lt}$ to maximize its utility.

Now consider the case in which the borrower does not conduct the *official* buyback but mimics the debt choice in the case of *official* buyback. This is possible as the *new* lenders offer $b'_{st} \geq 0$.

The contradiction is immediate as the continuation value is the same in the two cases and $\bar{c}^{NB} < c^B$. Thus, *official* buybacks are not enforceable in case of short-term asset issuance.

– If $b'_{st} < 0$

I consider a state Ω_P in which, $B_{st}(\Omega_P) < 0$ and $B_{lt}(\Omega_P) \geq b_{lt}$. Moreover, I assume without loss of generality that the choice of private debt is the same in the case with and without *official* buyback. Given this, we have that for all s', $W^b(s', B_{st}(\Omega_P), B_{lt}(\Omega_P)) \leq W^b(s', 0, B_{lt}(\Omega_P))$. In words, the continuation value under the no-roll-over punishment is weakly larger than the continuation value under an *official* buyback.

I consider two cases. First, for a given g, if the level of short-term liabilities is large, then the incapacity to issue new short-term debt can lead to a default. In such case it is preferable for the borrower to pay the premium χ instead of entering default.

Second, given that the continuation value under punishment is weakly larger, to obtain that $c^B > \bar{c}^{NB}$, it must be that

$$q_{st}(g, b'_{st}, b'_{lt})b'_{st} < b_{lt}(q^{bb}_{lt} - q_{lt}(g, b'_{st}, b'_{lt})).$$

Hence, provided that $q^{bb}_{lt} = \frac{1}{(1-\chi)r}$, if $\chi \to 0$ and $b_{lt} \to 0$, it is possible to have $V^B(\Omega_P) > \bar{V}^{NB}(\Omega_P)$ ensuring the enforcement of *official* buybacks.

□

Proof of Proposition 6

Consider the value of the *legacy* lender when there is no dilution. In that case

$$W^l_{\text{legacy},ND}(\Omega) = -\Big[b_{st} + b_{lt}(1 + \frac{1}{r})\Big].$$

In opposition, when there is dilution

$$W^l_{\text{legacy},D}(\Omega) = -\Big[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega)))\Big] < W^l_{\text{legacy},ND}(\Omega),$$

where the inequality comes from the fact that $q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega)) < \frac{1}{r}$. As a result, the *legacy* lender is never willing to dilute. Similarly, the value of the *legacy* lender when there is an *official* buyback is

$$W^l_{\text{legacy},B}(\Omega) = -\Big[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega))) + \chi)\Big],$$

while when there is no *official* buyback

$$W^l_{\text{legacy},NB}(\Omega) = -\Big[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega))))\Big] < W^l_{\text{legacy},B}(\Omega),$$

As a result, the *legacy* lender is always willing to have an *official* buyback. In opposition, the value of the *new* lender is

$$W^l_{\text{new}}(\Omega) = W^l_{\text{new},ND}(\Omega) = W^l_{\text{new},D}(\Omega) = W^l_{\text{new},B}(\Omega) = W^l_{\text{new},NB}(\Omega) = 0.$$

As a result, the *new* lender is indifferent to dilution and *official* buyback. □

Proof of Lemma 5

From (K.2) and (K.3), the short and long-term holdings at the *official* buyback are respectively

$$\begin{aligned}
\bar{b}_{st}(x) &= \frac{V^l(g_H, x)[1 + \bar{q}_{lt}(x'(g_L, x))] - V^l(g_L, x)[1 + q^{bb}_{lt}]}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} < 0, \\
\bar{b}_{lt}(x) &= -\frac{V^l(g_H, x) - V^l(g_L, x)}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} \lesseqqgtr 0.
\end{aligned}$$

From Part III of Proposition 2, it holds that $V^l(g_H, x) > V^l(g_L, x)$ meaning that $b_{lt} < 0$. However, it is not guaranteed that $b_{st} < 0$. Particularly, b_{st} can be negative only if q^{bb}_{lt} is very large with respect to q_{lt}. Moreover, recall that the *official* buyback takes place in the point of amnesia meaning that $b'_{st} = b_{st}$ and $b'_{lt} = b_{lt}$. As a result, Part **??** of Lemma 4 does not generally apply as b'_{st} is negative when χ is sufficiently large. □

Proof of Lemma C.1

The law of motion of the relative Pareto weight is given by

$$x'(g) = (1 + \nu(g))\eta x.$$

and the level of consumption by

$$u_c(c(g)) = \frac{1}{x(1+\nu(g))}.$$

Isolating x leads to

$$x = \frac{1}{u_c(c(g))(1+\nu(g))}. \tag{K.5}$$

Plugging this back into the law of motion gives

$$x'(g) = (1+\nu(g))\eta \frac{1}{u_c(c(g))(1+\nu(g))}.$$

Replacing $x'(g)$ by with the forward equivalent of (K.5) gives

$$\frac{1}{u_c(c(g'))(1+\nu(g'))} = \eta \frac{1}{u_c(c(g))}.$$

Taking expectations on both sides,

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

which gives the inverse Euler equation. □

Proof of Proposition C.1

Existence and uniqueness follow from Theorem 3 in Marcet and Marimon (2019). The two authors make the following assumptions: A1 a well defined Markov chain process for g, A2 continuity in $\{c, k\}$ and measurability in g, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lenders and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given my environment and Assumption 1. Since feasible c and k are bounded, payoffs functions are bounded as well. This combined with the fact that the outside options are also bounded ensure that A4 is met. Whether A3 is satisfied depends on the initial condition (g_0, x_0). Assumption C.1 ensures feasibility and that the strict interiority condition is satisfied.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding to the original contract problem (6). To obtain the recursive formulation of the contract, I have normalized the co-state variable. I relied on the the homogeneity of degree one in (μ_b, μ_l) to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if I define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}^2_+\}$ and the set of feasible consumption and capital by $A = \{(c, k) \in \mathbb{R}^2_+\}$, the correspondence $SP : A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Marcet and Marimon (2019) additionally show that the the saddle point functional equation (8) is a contraction mapping. Thus, given the concavity assumptions of $u(\cdot)$ and $f(\cdot)$, the allocation is unique. $\square$

Proof of Proposition G.2

Following Alvarez and Jermann (2000) we prove the proposition by construction. First, define the asset price as

$$q(g', x'|g) = \frac{\pi(g'|g)}{1+r}\left[1 + \sum_{g''|g'} q(g'', x''|g')\right] \max\left\{\frac{u'(c(g', x'))}{u'(c(g, x))}\eta, 1\right\}.$$

Second, iterating over the budget constraint of the government and applying the transversality condition gives

$$a(g^t) = \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[c(g^{t+j}, x(g^{t+j})) - Y(g^{t+j}, x(g^{t+j}))], \tag{K.6}$$

where, $Y(g^t, x(g^t)) = g_t f(k(g^t, x(g^t))) - k(g^t, x(g^t))$ for all t and g^t. Similarly, iterating over the budget constraint of the lenders leads to

$$\begin{aligned} a_l(g^t) &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t) c_l(g^{t+j}, x(g^{t+j})) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[Y(g^{t+j}, x(g^{t+j})) - c(g^{t+j}, x(g^{t+j}))] \\ &= -a(g^t). \end{aligned} \tag{K.7}$$

The market clearing condition implies that $a_l(g^t) + a(g^t) = 0$ for all t and g^t.

To ensure that the capital level is the same as in the constrained efficient allocation, I set the capital price according to

$$g f_k(k) = p(g, a).$$

I now need to establish the correspondence between the initial conditions, x_0, in the contract and the initial conditions in the RCE, a_0. Given (K.6) and (K.7) evaluated at $t = 0$, one can determine $\bar{a}'$ using the budget constraint

$$c(g_0, x_0) + q(g_0, a_1)(\bar{a}' - a_0) + \sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) \leq y(g_0, k) + a_0.$$

and the fact that $\sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) = 0$. Once, $\bar{a}'$ is determined, one can find the holdings of Arrow securities $\hat{a}'(g', g_0, a_0)$ for all $g' \in G$. We can then retrieve the entire portfolio recursively for $t > 0$.

Third, define the endogenous borrowing limits such that

$$\mathcal{A}(g, k) = a(g, \tilde{x}(g, k)),$$

where $\tilde{x}(g, k)$ is the relative Pareto weight when the participation constraint binds at (g, k). This definition implies that $a'(g', g, a) \geq \mathcal{A}(g', k')$. Hence, the constructed asset holdings satisfy the competitive equilibrium constraints.

Fourth, to ensure optimality of the policy functions by setting

$$\mu_{BC}(g, a) = \frac{1}{x(1 + \nu(g))}$$

Hence, since $c(g, x)$ satisfies the optimality conditions in the Planner's problem, it is also optimally determined in the RCE. For the lenders, $c_l(g, x)$ is optimal if the asset portfolio is optimally determined. For this observe that

$$\begin{aligned} q(g', a'(g')|g) &= \frac{1}{1+r}\pi(g'|g)\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &> \frac{1}{1+r}\pi(g'|g)\frac{u'(c(g', a'(g')))}{u'(c(g, a))}\eta\left[1 + \sum_{g''|g'} q(g'', a''(g'')|g')\right] \\ &\quad \text{if } a'(g', g, a) = \mathcal{A}(g', k'). \end{aligned}$$

Hence the portfolio is optimally determined. We therefore obtain a one-to-one map between x and a for a given g. More precisely, $c(g, a) = c(g, x)$, $c_l(g, a) = T(g, x)$ and $k(g, a) = k(g, x)$.

Thus, $W^b(g,a) = W^b(g,x)$ and $W^l(g,a) = W^l(g,x)$. Furthermore, the endogenous limits binds uniquely and exclusively when the participation constraints of the government binds. $\square$