Gestion de Portefeuille

Ex 7: Risk Parity and Risk Budgeting

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1 Données

On utilisera les données de l'article de Litterman et He.

```
spl <- function (
  s,      # input string
  delim = ',' # delimiter
) {
  unlist(strsplit(s,delim))
}


 data =
'1,0.4880,0.4780,0.5150,0.4390,0.5120,0.4910
 0.4880,1,0.6640,0.6550,0.3100,0.6080,0.7790
 0.4780,0.6640,1,0.8610,0.3550,0.7830,0.6680
 0.5150,0.6550,0.8610,1,0.3540,0.7770,0.6530
 0.4390,0.3100,0.3550,0.3540,1,0.4050,0.3060
 0.5120,0.6080,0.7830,0.7770,0.4050,1,0.6520
 0.4910,0.7790,0.6680,0.6530,0.3060,0.6520,1'

  Corrmat = matrix( as.double(spl( gsub('\n', ',', data), ',')),
                    nrow = length(spl(data, '\n')), byrow=TRUE)

  stdevs = c(16.0, 20.3, 24.8, 27.1, 21.0,  20.0, 18.7)/100
  w.eq = c(1.6, 2.2, 5.2, 5.5, 11.6, 12.4, 61.5)/100
  # Prior covariance of returns
  Sigma = Corrmat * (stdevs %*% t(stdevs))
```

Rendements d'équilibre

```
# risk aversion parameter
delta = 2.5
Pi = delta * Sigma %*% w.eq
```

Assets	Std Dev	Weq	PI
Australia	16	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9
Japan	21	11.6	4.3
UK	20	12.4	6.8
USA	18.7	61.5	7.6

2 Questions

2.1 Calculer une allocation telle que les contributions au risque du portefeuille sont idfentiques pour tous les titres (optimisation non-linéaire).

On doit avoir:

$$\min_w \sum_i (CR_i - CR_{i-1})^2$$

Définissez la fonction objectif et la matrice de contraintes, puis utilisez solnl pour obtenir la solution.

```
f.obj <- function(w) {
  CR <- w * Sigma %*% w
  sum(diff(CR) ** 2)*1000
}

n <- length(asset.names)
Aeq <- matrix(rep(1,n), nrow=1)
Beq <- 1
lb <- rep(0, n)
ub <- rep(1,n)
w.0 <- rep(1/n, n)

res.optim <- solnl(X=w.0, objfun=f.obj, Aeq=Aeq, Beq=Beq, lb=lb,
                   ub=ub)
```

Solution:

Table 1: Portefeuille Risk Parity

	weight
Australia	19.61
Canada	13.90
France	10.72
Germany	9.79
Japan	17.61
UK	13.42
USA	14.96

2.2 Solution analytique (méthode de Newton).

Risk parity condition:

$$w_i \frac{\partial \sigma_P}{\partial w_i} = w_j \frac{\partial \sigma_P}{\partial w_j} = \lambda$$

Let $1/w = [1/w_1, \ldots, 1/w_n]$, the above expression in matrix form is, dropping the denominator $w^T \Sigma w$ which is common to both sides of the equalities:

$$\Sigma w = \lambda \times 1/w$$

Define the vector-valued function

$$F(w, \lambda) = \begin{bmatrix} \Sigma w - \lambda \times 1/w \\ 1^T w - 1 \end{bmatrix}$$

We look for w^*, λ^* such that $F(w^*, \lambda^*) = 0$.

```
f.obj <- function(x) {
  n <- length(x)
  w <- matrix(x[1:(n-1)], ncol=1)
  lambda <- x[n]
  res.1 <- Sigma %*% w - lambda * 1/w
  res.2 <- sum(w) - 1
  as.vector(rbind(res.1, res.2))
}

x.0 <- c(rep(1/n, n), .1)
res.newton = nleqslv(x.0, f.obj)
```

The solution is:

Table 2: Portefeuille Risk Parity (Newton)

	weight
Australia	19.61
Canada	13.90
France	10.72
Germany	9.79
Japan	17.61
UK	13.42
USA	14.96

“‘