HW1 2020-2021

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1 Uncapacitated Facility Location (UFL)

The problem consists in minimizing the total cost related to the opening of services (that might be shops, depots, etc.) and related also to the cost of every client using these services (that could come from the distance between a client and a shop).

Let $N = \{1, 2, \ldots, n\}$ be the set of services and $M = \{1, 2, \ldots, m\}$ be the set of clients. A service $j \in N$ can be open or closed, hence we introduce a boolean variable y_j equals respectively to 1 or 0. Each client $i \in M$ satisfies a fraction of its demand x_{ij} thanks to service j. He must receive complete satisfation, thus $\sum_{j \in M} x_{ij} = 1$ and x_{ij} belongs to $[0, 1]$.

By associating a fixed cost f_j to the opening of service j and a proportional cost C_{ij} to the use of service j by client i, we obtain the following IP:

$$z = \min \sum_{(i,j)\in(M\times N)} C_{ij}x_{ij} + \sum_{j\in N} f_j y_j \tag{1}$$

$$s.t. \sum_{j\in N} x_{ij} = 1, \qquad \text{for all } i \in M \tag{2}$$

$$x_{ij} \leq y_j, \qquad \text{for all } (i,j) \in (M \times N) \tag{3}$$

$$x \in [0,1]^{m\times n} \tag{4}$$

$$y \in \{0,1\}^n \tag{5}$$

Q1. Implement UFL with Toy-Store Dataset (Link) [1]. There are 24 potential facility locations for 100 customers. The fixed cost and demand data are annualized value and the unit of the fixed cost is US dollar. Consider only annual transportation cost for C_{ij}. Assume that it costs \$1 for transporting 1 mile (1,61 km) per unit. Calculate the distance using longitude and latitude data by Great-circle Distance formula (Link). Report the optimal cities and the optimal total annual cost.

Q2. In the UFL formulation above, the constraints (3) can be also written as following:

$$\sum_{i\in M} x_{ij} \leq |M| y_j, \qquad \text{for all } j \in N \tag{6}$$

Observe that constraints (6) have much less number than that of (3). However, it was seen in the lecture that formulation with (3) is better. Let's check it numerically in this assignment. Implement the Toy-Store problem with the constraints (6) instead of (3) and report observations. (Do not forget to turn off automatic heuristic, presolve, and cut generation; Detailed explanation in the next page.)

[1] Snyder, Lawrence V., and Zuo-Jun Max Shen. Fundamentals of supply chain theory. John Wiley Sons, Incorporated, 2019.

Julia

In this semester, the students need to use **Julia** for implementation. **Julia** is an open source high-level programming language optimized for numerical analysis and computations. A domain specific language **JuMP** embedded in **Julia** is designed specifically for mathematical programming. **JuMP** is fast, solver independent, and has a syntax structure which mimics natural mathematical expressions. Starting from the USA, the Julia community is growing fast especially for mathematical programming community.

You need to prepare three things before start coding.

- Julia Language JuliaTutorials (Link)
- JuMP JuMP Manual (Link)
- Optimization Solver (Gurobi, CPLEX, etc) Gurobi (Link) / Gurobi JuMP Manual (Link)

For the optimization solver, you can use something else, but we highly recommend Gurobi. You can get a student license for free.

Parameter Settings for Gurobi

For comparing different IP formulations numerically, it is very important to turn off the setting for additional behaviors solvers do. For Gurobi, you need to set following three parameters to zero: **Presolve**, **Heuristics**, and **Cuts**. One way to set the parameters in Gurobi is using *set_optimizer_attribute*. You can check the details in the Gurobi JuMP Manual above.

Submission

Submit both code and report in a zip file until **23:59 18/10/2020**. No late acceptance. This assignment will account for 1 point out of the total 20 points for this course. The assignment will be evaluated as pass (1 point) or fail (0 point).

- Code (.jl file)
- Report (.pdf file within 2 pages)
- File Name (include "FirstName_LastName" for each group members)

Reference

Snyder, Lawrence V., and Zuo-Jun Max Shen. Fundamentals of supply chain theory. John Wiley Sons, Incorporated, 2019.