HW2 2020-2021

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1 Traveling Salesman Problem (TSP)

The Traveling Salesman Problem (TSP) is one of the most famous combinatorial optimization problems. Its objective is to find the tour minimizing total distances. In this homework, we will explore two different formulations of TSP.

Let V be the set of vertices and E be the set of edges in a graph. For a subset $S \subset V$, $\delta(S)$ denote the set of edges whose one of the endpoints lies in S and the other is outside of S.

The following formulation is called Sub-Tour Elimination Formulation because of the constraints (3). Without (3), we might get solutions called sub-tours which makes a tour of only part of the vertices of the graph. The constraints (3) eliminates this issue by imposing every subset $S \subset V$ should be connected to $V \setminus S$.

$$\min \sum_{e \in E} c_e x_e \tag{1}$$

$$s.t. \sum_{e \in \delta(\{i\})} x_e = 2, \qquad \text{for all } i \in V \tag{2}$$

$$\sum_{e \in \delta(S)} x_e \geq 2, \qquad \text{for all } S \subset V, 2 \leq |S| \leq |V| - 2 \tag{3}$$

$$x_e \in \{0, 1\}, \qquad \text{for all } e \in E \tag{4}$$

The issue here is that the number of subsets of V increases exponentially as $|V|$ increases. One of the ways to address this issue is to add only a part of (3). To do that, we need a way to distinguish which constraint is necessary and to know when there are enough of them. Let us denote $\overline{x}$ as the optimal solution for the linear relaxation problem of the above formulation with only subset of (3). The problem of finding a constraint that $\overline{x}$ violates is called the separation problem. If the separation problem cannot find such a constraint, then we can conclude that the subset of (3) we used is enough to represent the whole set of (3).

Q1. Show that the separation problem can be solved either by Minimum Cut problem or Maximum Flow problem.

Q2. The Branch-and-Cut Algorithm or Cutting Plane Method solves an IP problem by adding additional constraints such as (3) on the fly (during the Branch-and-Bound algorithm). We sometimes call a restricted version of this algorithm as Cut-and-Branch when we add constraints only in the root node of the Branch-and-Bound algorithm (before we start the branching). It is similar to the ideas of Branch-and-Cut seen in the lectures but for a different type of relaxation. Implement this Cut-and-Branch algorithm using the TSP 42-Cities. In the root node (before you start solving IP formulation), starting from an empty set of (3) add constraints until the separation problem cannot find one. Report the separation problem you use, the number of cuts (constraints) you add in the end and the GAP between the LP relaxation and the IP optimal solution when the separation problem cannot find extra constraint. (Do not forget to turn off automatic heuristic, presolve, and cut generation.)

Q3. Another formulation of TSP is Miller-Tucker-Zemlin(MTZ) Formulation. This formulation uses additional continuous variables u_i for $i = 2, \ldots, |V|$, which represents somewhat the order of vertices in a tour. Here, instead of x_e we use x_{ij} for the following formulation, where $i, j \in \{1, \ldots, |V|\}$.

$$\min \sum_{i \neq j} c_{ij} x_{ij} \tag{5}$$

$$s.t. \sum_{i: i \neq j} x_{ij} = 1, \qquad \text{for all } j \in \{1, \ldots, |V|\} \tag{6}$$

$$\sum_{j: i \neq j} x_{ij} = 1, \qquad \text{for all } i \in \{1, \ldots, |V|\} \tag{7}$$

$$u_i - u_j + (|V| - 1) x_{ij} \leq |V| - 2, \qquad \text{for all } i, j \in \{2, \ldots, |V|\}, i \neq j \tag{8}$$

$$1 \leq u_i \leq |V| - 1, \qquad \text{for all } i \in \{2, \ldots, |V|\} \tag{9}$$

$$x_{ij} \in \{0, 1\}, \qquad \text{for all } i, j \in \{1, \ldots, |V|\}, i \neq j \tag{10}$$

Implement TSP using the TSP 26-Cities with MTZ formulation (Branch and Bound algorithm: Gurobi automatically does it for you). Compare it with Sub-Tour Elimination Formulation and analyze the result. (Do not forget to turn off automatic heuristic, presolve, and cut generation.)

Submission

Submit both code and report in a zip file until **23:59 30/11/2020**. No late acceptance. This assignment will account for 2 point out of the total 20 points for this course. The assignment will be evaluated as great (2 points), pass (1 point) or fail (0 point).

- Code (.jl file or .ipynb file)
- Report (.pdf file within 3 pages)
- File Name (include "FirstName_LastName" for each group members)

Reference

Miller, Clair E., Albert W. Tucker, and Richard A. Zemlin. "Integer programming formulation of traveling salesman problems." Journal of the ACM (JACM) 7.4 (1960): 326-329.