LINMA 2471 – Optimization models and methods II

Homework assignment I – AMPL modelling

[v1.1]

This first homework assignment asks you to formulate and solve an optimization case study using an AMPL model as the central tool. Work in pairs to formulate the problem, produce AMPL codes, run them, analyze the results and write a PDF report that will describe your modelling process, implementation, results and comments. The AMPL models themselves should be readable, well-structured and easy to maintain (e.g. parameters in a separate file, data-model separation), and can be commented (in which case it is not necessary to repeat what is in the comments in the main report). Provide all command files (scripts) needed to reproduce your results.

Submit your report and AMPL files on Moodle ; deadlines, rules about sharing information and late submission policy can also be found there.

Optimal path at baseball

Read the article located at

`http://sites.williams.edu/Morgan/2009/10/29/baserunners-optimal-path/`

and answer its central question: what is the optimal path at baseball, i.e. the path that minimizes the time it takes to go (exactly) through all four bases (start at rest on first base, 90ft distance between bases) with an acceleration that never exceeds $10\frac{\text{ft}}{s^2}$.

Can you beat the stated $16.7s$ record time ?

Instructions

A central issue in the formulation of this model is how the representation of the runner's path/behaviour is discretized (i.e. how the path is represented with a finite number of variables).

You will develop two successive models. Each model is based on a interval time parameter $h > 0$.

1. A first model will used *discretized physics* in the following (simplest) way: we only consider acceleration a_k, speed v_k and position p_k at times $t = kh$ for $k \in \mathbb{N}$ (all vectors are in $\mathbb{R}^2$).

   Given an initial speed v_0 at time $t = 0$ and initial acceleration a_0, speed at time $t = h$ is equal to $v_1 = v_0 + a_0 h$, speed at time $t = 2h$ is equal to $v_2 = v_1 + a_1 h$, and then $v_{k+1} = v_k + a_k h$ for all k. Similarly, position will be determined according to $p_1 = p_0 + v_0 h, \ldots, p_{k+1} = p_k + v_k h$.

2. A second model will use the actual *continuous physics* from the real world. You will assume that acceleration $a(t)$ is piecewise constant, and that it can only change at times $t = h$, $t = 2h, t = 3h, \ldots, t = kh$ for all $k \in \mathbb{N}$. This means that $a(t) = a_0$ $\forall t$ such that $0 \leq t < h$,

$a(t) = a_1$ $\forall t$ such that $h \leq t < 2h, \ldots,$ and $a(t) = a_k$ $\forall t$ such that $kh \leq t < (k+1)h$ for any integer k.

Hint: you may discover that enforcing that the path goes through all four bases is difficult. One way to proceed is divide the path in four segments, each one connecting a pair of successive bases. Choose the number of discretized time interval on each segment but let the time interval of each segment (hence also its duration) vary.

Your report should discuss the following, for each of the two ways you discretized the problem:

- ◇ Is your model convex ?
- ◇ Does your model computes the exact optimal value for the problem ? If not can does it provide some kind of bound (lower and/or upper) on the optimal value ? Similarly, does your model provide a solution that is exactly feasible for the problem ?
- ◇ What are the results of your model ? Estimate its accuracy (use finer and finer discretizations).
- ◇ How sensitive are your results with respect to the parameters ?