- Firms with quantity $O = 1$ to sell (supply inelastically because they are importers), choose a share v (volume) to move to G (global/major market)
- Firms differ in transportation costs (distance) per unit of transported volume from major markets θ with density f
- Choose local price $p_l(\theta)$ but global price determined by equilibrium.

$$\int v(\theta) df(\theta) = Q^G(p) \tag{1}$$

- Firms maximize profit by choosing share volume v they move to global market

$$\Pi = \max_{v,p_l} [p_l(1-v) + v(p-\theta)] \tag{2}$$

- Local demand $Q^l(p_l) = Q_l p_l^{-\varepsilon_{Dl}}$ so $Q^l(p_l)/Q_l = p_l^{-\varepsilon_{Dl}}$
- That is the equilibrium local price $p_l = (\frac{Q_l}{1-v})^{1/\varepsilon_{Dl}}$
- So firms maximize

$$\max_v \left[Q_l^{1/\varepsilon_{Dl}} (1-v)^{1-1/\varepsilon_{Dl}} + v(p-\theta) \right] \tag{3}$$

- We get assuming $\varepsilon_{Dl} > 1$: If $\theta > p$ then $v = 0$ and firms don't want to send anything and profit is [Think of the role of local elasticity]

$$\Pi = Q_l^{1/\varepsilon_{Dl}} \tag{4}$$

- If $\theta \leq p$, then v according to,

$$(1 - 1/\varepsilon_{Dl}) Q_l^{1/\varepsilon_{Dl}} (1-v)^{-1/\varepsilon_{Dl}} = p - \theta \tag{5}$$

- In summary firms with high transportation cost won't want to send oil to the major market. So if the government puts important weight on this major market, they might want to introduce oil subsidies.
- Notice that if firms could set different prices in the hub, they could just repercute the transportation cost to their price, and send some.
- Now let's set a uniform oil tax τ and subsidy per unit of moved volume $t(\theta)$
- Firms maximize profit

$$\max_{v,p_l} [p_l(1-\tau)(1-v) + v(p(1-\tau) + t(\theta) - \theta)] \tag{6}$$

- After the local price is set:

$$\max_{v} \left[Q_l^{1/\varepsilon_{Dl}} (1-v)^{1-1/\varepsilon_{Dl}} (1-\tau) + v(p(1-\tau) + t(\theta) - \theta) \right] \tag{7}$$

- Assuming $\varepsilon > 1$: If $\theta > p(1-\tau_g) + t(\theta)$, then $v = 0$, and firms do not send anything. The profit is:

$$\Pi = Q_l^{1/\varepsilon_{Dl}} (1-\tau) \tag{8}$$

- If $\theta \leq p(1-\tau_g) + t(\theta)$, then v is determined by:

$$(1 - 1/\varepsilon_{Dl}) Q_l^{1/\varepsilon_{Dl}} (1 - v(\theta))^{-1/\varepsilon_{Dl}} (1-\tau) = p(1-\tau) + t(\theta) - \theta \tag{9}$$

- Government's problem maximize weighted consumer surplus

$$\max_{t(\theta),\tau_g} \left[\mu^G \int_p^\infty Q^G(p')\, dp' + \mu^L \int \int_{p_l(\theta)} Q^L(p')\, dp'\, df(\theta) + \int \left[\tau \cdot (p_l(\theta)(1-v(\theta)) + pv(\theta)) - t(\theta)v(\theta) \right] df(\theta) \right] \tag{10}$$

subject to:

$$\int v(\theta) df(\theta) = Q^G(p) = Q_G p^{-\varepsilon_{D_G}} \quad \text{(global market clearing)} \tag{11}$$

$$Q^L(p_l(\theta)) = Q_L p_l(\theta)^{-\varepsilon_{Dl}} = 1 - v(\theta) \quad \text{(local demand)} \tag{12}$$

$$(1-1/\varepsilon_{Dl}) Q_l^{1/\varepsilon_{Dl}} (1-v(\theta))^{-1/\varepsilon_{Dl}} (1-\tau) = p(1-\tau) + t(\theta) - \theta \quad \text{for } \theta \leq p(1-\tau_g) + t(\theta) \quad \text{(price equality condition)} \tag{13}$$

$$\int \tau \left[p_l(\theta)(1-v(\theta)) + pv(\theta) \right] df(\theta) = \int t(\theta)v(\theta) df(\theta) \quad \text{(government budget constraint)} \tag{14}$$

- Will optimize this to find the optimal subsidy schedule.
- Let's have linear subsidy $t(\theta) = t\theta$
- Send cutoff is then $\theta \leq \frac{p(1-\tau_g)}{1-t}$, denote $\tilde{\theta}$ the minimum cut-off resulting.
- The choice of shipped volume below $\tilde{\theta}$ is

$$(1 - 1/\varepsilon_{Dl}) Q_l^{1/\varepsilon_{Dl}} (1 - v(\theta))^{-1/\varepsilon_{Dl}} \frac{\tilde{\theta}}{p} = \tilde{\theta} - \theta \quad \text{for } \theta \leq \tilde{\theta} \tag{15}$$

By rearranging the equality we obtain:

$$(1-v(\theta)) = \left(\frac{p(\tilde{\theta}-\theta)}{\left(1-\frac{1}{\varepsilon_{Dl}}\right)Q_l^{1/\varepsilon_{Dl}}\tilde{\theta}}\right)^{-\varepsilon_{Dl}} = \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)Q_l^{1/\varepsilon_{Dl}}\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}} = Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}} \tag{16}$$

and then:

$$(1-v(\theta))^{\frac{\varepsilon_{Dl}-1}{\varepsilon_{Dl}}} = \left(\frac{p(\tilde{\theta}-\theta)}{\left(1-\frac{1}{\varepsilon_{Dl}}\right)Q_l^{1/\varepsilon_{Dl}}\tilde{\theta}}\right)^{-(\varepsilon_{Dl}-1)} = \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)Q_l^{1/\varepsilon_{Dl}}\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{(\varepsilon_{Dl}-1)} \tag{17}$$

$$(1-v(\theta))^{\frac{\varepsilon_{Dl}-1}{\varepsilon_{Dl}}} = Q_l^{\frac{\varepsilon_{Dl}-1}{\varepsilon_{Dl}}} \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{(\varepsilon_{Dl}-1)} \tag{18}$$

- The objective:

$$CS_L(\theta) = \int_{p_l(\theta)}^{\infty} Q^L(p)\,dp = \int_{p_l(\theta)}^{\infty} Q_L \cdot p^{-\varepsilon_{Dl}}\,dp = \frac{Q_L}{\varepsilon_{Dl}-1} \cdot p_l(\theta)^{-(\varepsilon_{Dl}-1)} = \frac{Q_L}{\varepsilon_{Dl}-1} \cdot p_l(\theta)^{1-\varepsilon_{Dl}} \tag{19}$$

- Developing it further:

$$CS_L(\theta) = \frac{Q_L}{\varepsilon_{Dl}-1} \cdot \left(\frac{1-v}{Q_L}\right)^{\frac{\varepsilon_{Dl}-1}{\varepsilon_{Dl}}} = \frac{Q_L^{\frac{1}{\varepsilon_{Dl}}}}{\varepsilon_{Dl}-1} \cdot (1-v)^{\frac{\varepsilon_{Dl}-1}{\varepsilon_{Dl}}} \tag{20}$$

$$CS_L(\theta) = \frac{Q_L^{\frac{1}{\varepsilon_{Dl}}}}{\varepsilon_{Dl}-1} \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)Q_l^{1/\varepsilon_{Dl}}\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{(\varepsilon_{Dl}-1)} = \frac{Q_L}{\varepsilon_{Dl}-1} \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{(\varepsilon_{Dl}-1)} \tag{21}$$

$$CS_G = \int_{p}^{\infty} Q^G(p)\,dp = \int_{p}^{\infty} Q_G \cdot p^{-\varepsilon_{DG}}\,dp = \frac{Q_G}{\varepsilon_{DG}-1} \cdot p^{-(\varepsilon_{DG}-1)} = \frac{Q_G}{\varepsilon_{DG}-1} \cdot p^{1-\varepsilon_{DG}} \tag{22}$$

- Budget constraint

$$\int \tau\left[p_l(\theta)(1-v(\theta)) + pv(\theta)\right] df(\theta) = t\int \theta v(\theta) df(\theta) \tag{23}$$

$$\int \tau p_l(\theta)(1-v(\theta))df(\theta) + \tau Q_G p^{1-\varepsilon_{D_G}} = t \int \theta v(\theta) df(\theta) \tag{24}$$

$$\tau p_l(1-v) = \tau Q_L \left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} \left(\frac{\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}-1} \tag{25}$$

$$tv(\theta)\theta = t\theta \left[1 - \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right) Q_l^{1/\varepsilon_{Dl}} \tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] = t\theta \left[1 - Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] \tag{26}$$

$$\tau Q_L \left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} \int \left(\frac{\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}-1} df(\theta) + \tau Q_G p^{1-\varepsilon_{D_G}} = t \int \theta \left[1 - Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] df(\theta) \tag{27}$$

- Maximization of weighted surplus:

$$\max_{t,\tau,p} \left[\mu^G \frac{Q_G}{\varepsilon_{DG}-1} \cdot p^{1-\varepsilon_{DG}} + \mu^L \int_0^{\tilde{\theta}} \frac{Q_L}{\varepsilon_{Dl}-1} \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{(\varepsilon_{Dl}-1)} df(\theta)\right] \tag{28}$$

- subject to the budget constraint:

$$\tau Q_L \left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} \int \left(\frac{\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}-1} df(\theta) + \tau Q_G p^{1-\varepsilon_{D_G}} = t \int \theta \left[1 - Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] df(\theta) \tag{29}$$

- global market clearing

$$\int_0^{\tilde{\theta}} \left[1 - Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] df(\theta) = Q_G p^{-\varepsilon_{D_G}} \tag{30}$$

- Cutoff:

$$\tilde{\theta} = \frac{p(1-\tau_g)}{1-t} \tag{31}$$

Assuming $f(\theta) = 1$, we can rewrite the objective function and the constraints. The integral on the left hand side of the market clearing condition can then be easily computed:

$$\int_0^{\tilde{\theta}} \left[1 - Q_l \cdot \left(\frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta} - \theta)} \right)^{\varepsilon_{Dl}} \right] d\theta$$

The first term becomes:

$$\int_0^{\tilde{\theta}} 1 \, d\theta = \tilde{\theta}$$

For the second term, we have:

$$\int_0^{\tilde{\theta}} Q_l \cdot \left(\frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta} - \theta)} \right)^{\varepsilon_{Dl}} d\theta = Q_l \cdot \left(\frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p} \right)^{\varepsilon_{Dl}} \int_0^{\tilde{\theta}} \frac{1}{(\tilde{\theta} - \theta)^{\varepsilon_{Dl}}} d\theta$$

Using the substitution $u = \tilde{\theta} - \theta$ we get:

$$\int_0^{\tilde{\theta}} \frac{1}{(\tilde{\theta} - \theta)^{\varepsilon_{Dl}}} d\theta = \int_{\tilde{\theta}}^{0} u^{-\varepsilon_{Dl}} (-du) = \int_0^{\tilde{\theta}} u^{-\varepsilon_{Dl}} \, du = \frac{u^{1-\varepsilon_{Dl}}}{1 - \varepsilon_{Dl}} \Big|_0^{\tilde{\theta}} = \frac{\tilde{\theta}^{1-\varepsilon_{Dl}}}{1 - \varepsilon_{Dl}} \tag{32}$$

The second term becomes:

$$Q_l \cdot \left(\frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p} \right)^{\varepsilon_{Dl}} \cdot \frac{\tilde{\theta}^{1-\varepsilon_{Dl}}}{1 - \varepsilon_{Dl}}.$$

Simplifying further, we get:

$$Q_l \cdot \frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} \cdot \tilde{\theta}}{p^{\varepsilon_{Dl}}(1 - \varepsilon_{Dl})}$$

Combining the first and second terms, we get:

$$\tilde{\theta} - Q_l \cdot \frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} \cdot \tilde{\theta}}{p^{\varepsilon_{Dl}}(1 - \varepsilon_{Dl})} = \tilde{\theta} + Q_l \cdot \frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} \cdot \tilde{\theta}}{p^{\varepsilon_{Dl}}(\varepsilon_{Dl} - 1)}$$

The market clearing condition can then be rewritten as:

$$\tilde{\theta} + Q_l \cdot \frac{\left(1 - \frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} \cdot \tilde{\theta}}{p^{\varepsilon_{Dl}}(\varepsilon_{Dl} - 1)} = Q_G p^{-\varepsilon_{D_G}} \tag{33}$$

Now we will consider the budget constraint (29). By using the same substitution technique as in 32, the integral on the left hand side of the equation can be written as::

$$\int_0^{\tilde{\theta}} \frac{1}{(\tilde{\theta}-\theta)^{\varepsilon_{Dl}-1}}\, d\theta = \int_{\tilde{\theta}}^{0} u^{-(\varepsilon_{Dl}-1)}(-du) = \int_0^{\tilde{\theta}} u^{-(\varepsilon_{Dl}-1)}\, du = \frac{u^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}}\Big|_0^{\tilde{\theta}} = \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}} \tag{34}$$

The left-hand side of the equation of the budget constraint can then be simplified as:

$$\tau Q_L \left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} \cdot \frac{\tilde{\theta}^{\varepsilon_{Dl}-1}}{p^{\varepsilon_{Dl}-1}} \cdot \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}} + \tau Q_G p^{1-\varepsilon_{D_G}} = \frac{\tau Q_L \cdot \theta}{p^{\varepsilon_{Dl}-1}\cdot(2-\varepsilon_{Dl})}\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} + \tau Q_G p^{1-\varepsilon_{D_G}}$$

Now we will focus on the right hand side:

We are still assuming $f(\theta) = 1$ (uniform distribution). The term on the right-hand side of the budget constraint can then be computed:

$$t\int \theta \left[1 - Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}}\right] d\theta.$$

The first term becomes:

$$t\int_0^{\tilde{\theta}} \theta\, d\theta = t\cdot\frac{\theta^2}{2}\Big|_0^{\tilde{\theta}} = t\cdot\frac{\tilde{\theta}^2}{2}.$$

For the second term, we have:

$$t\int_0^{\tilde{\theta}} \theta\cdot Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p(\tilde{\theta}-\theta)}\right)^{\varepsilon_{Dl}} d\theta = t\cdot Q_l\cdot\left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p}\right)^{\varepsilon_{Dl}} \cdot \int_0^{\tilde{\theta}} \frac{\theta}{(\tilde{\theta}-\theta)^{\varepsilon_{Dl}}}\, d\theta.$$

Using the substitution $u = \tilde{\theta} - \theta$, we have:

$$\int_0^{\tilde{\theta}} \frac{\theta}{(\tilde{\theta}-\theta)^{\varepsilon_{Dl}}}\, d\theta = \int_{\tilde{\theta}}^{0} \frac{\tilde{\theta}-u}{u^{\varepsilon_{Dl}}}(-du) = \int_0^{\tilde{\theta}} \frac{\tilde{\theta}}{u^{\varepsilon_{Dl}}}\, du - \int_0^{\tilde{\theta}} u^{-(\varepsilon_{Dl}-1)}\, du. \tag{35}$$

Each term is computed as follows: For the first term:

$$\int_0^{\tilde{\theta}} \frac{\tilde{\theta}}{u^{\varepsilon_{Dl}}}\, du = \tilde{\theta}\int_0^{\tilde{\theta}} u^{-\varepsilon_{Dl}}\, du = \tilde{\theta}\cdot\frac{u^{1-\varepsilon_{Dl}}}{1-\varepsilon_{Dl}}\Big|_0^{\tilde{\theta}} = \frac{\tilde{\theta}\cdot\tilde{\theta}^{1-\varepsilon_{Dl}}}{1-\varepsilon_{Dl}} = \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{1-\varepsilon_{Dl}}.$$

For the second term:

$$\int_0^{\tilde{\theta}} u^{-(\varepsilon_{Dl}-1)}\, du = \frac{u^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}}\Big|_0^{\tilde{\theta}} = \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}}.$$

Combining the two terms, we get:

$$\int_0^{\tilde{\theta}} \frac{\theta}{(\tilde{\theta}-\theta)^{\varepsilon_{Dl}}}\, d\theta = \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{1-\varepsilon_{Dl}} - \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{2-\varepsilon_{Dl}} = \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}$$

The second term becomes:

$$t \cdot Q_l \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)\tilde{\theta}}{p} \right)^{\varepsilon_{Dl}} \cdot \frac{\tilde{\theta}^{2-\varepsilon_{Dl}}}{(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}$$

Simplifying further, we get:

$$t \cdot \tilde{\theta}^2 \cdot \frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} Q_L}{p^{\varepsilon_{Dl}}(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}$$

Combining the first and second terms, we have:

$$t \cdot \tilde{\theta}^2 \cdot \left[\frac{1}{2} - \frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} Q_L}{p^{\varepsilon_{Dl}}(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}) \right]$$

The budget constraint can then be expressed as:

$$\frac{\tau Q_L \cdot \theta}{p^{\varepsilon_{Dl}-1} \cdot (2-\varepsilon_{Dl})} \left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} + \tau Q_G p^{1-\varepsilon_{D_G}} = t \cdot \tilde{\theta}^2 \cdot \left[\frac{1}{2} - \frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} Q_L}{p^{\varepsilon_{Dl}}(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}) \right]$$

Replacing $\tilde{\theta}$, we get:

$$\frac{p(1-\tau_g)}{1-t} \cdot \frac{\tau Q_L}{p^{\varepsilon_{Dl}-1}(2-\varepsilon_{Dl})} \left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1} + \tau Q_G p^{1-\varepsilon_{D_G}} = t \cdot \left[\frac{p(1-\tau_g)}{1-t} \right]^2 \cdot \left[\frac{1}{2} - \frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}} Q_L}{p^{\varepsilon_{Dl}}(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})}) \right] \tag{36}$$

Now, doing the same substitution as 34, we can simplify the objective function (28):

$$\max_{t,\tau,p} \left[\mu^G \frac{Q_G}{\varepsilon_{DG}-1} \cdot p^{-(\varepsilon_{DG}-1)} + \mu^L \frac{Q_L}{\varepsilon_{DL}-1} \cdot \left(\frac{\left(1-\frac{1}{\varepsilon_{DL}}\right)\tilde{\theta}}{p} \right)^{\varepsilon_{DL}-1} \cdot \frac{\tilde{\theta}^{2-\varepsilon_{DL}}}{2-\varepsilon_{DL}} \right]$$

By further simplifying we get:

$$\max_{t,\tau,p} \left[\mu^G \frac{Q_G}{(\varepsilon_{DG}-1)} \cdot p^{-(\varepsilon_{DG}-1)} + \mu^L \frac{Q_L \cdot \theta}{(\varepsilon_{DL}-1)\cdot(2-\varepsilon_{DL})} \cdot \left(\frac{1-\frac{1}{\varepsilon_{DL}}}{p} \right)^{\varepsilon_{DL}-1} \right]$$

Replacing $\tilde{\theta}$, we get:

$$\max_{t,\tau,p}\left[\mu^G\frac{Q_G}{(\varepsilon_{DG}-1)}\cdot p^{-(\varepsilon_{DG}-1)}+\mu^L\frac{Q_L\cdot\frac{p(1-\tau_g)}{1-t}}{(\varepsilon_{DL}-1)\cdot(2-\varepsilon_{DL})}\cdot\left(\frac{1-\frac{1}{\varepsilon_{DL}}}{p}\right)^{\varepsilon_{DL}-1}\right] \tag{37}$$

Finally by combining everything toghether, we get the following simplified version of the problem:

- Maximization of weighted surplus:

$$\max_{t,\tau,p}\left[\mu^G\frac{Q_G}{(\varepsilon_{DG}-1)}\cdot p^{-(\varepsilon_{DG}-1)}+\mu^L\frac{Q_L\cdot\frac{p(1-\tau_g)}{1-t}}{(\varepsilon_{DL}-1)\cdot(2-\varepsilon_{DL})}\cdot\left(\frac{1-\frac{1}{\varepsilon_{DL}}}{p}\right)^{\varepsilon_{DL}-1}\right] \tag{38}$$

- Subject to the budget constraint:

$$\frac{p(1-\tau_g)}{1-t}\cdot\frac{\tau Q_L}{p^{\varepsilon_{Dl}-1}(2-\varepsilon_{Dl})}\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}-1}+\tau Q_G p^{1-\varepsilon_{D_G}}=t\cdot\left[\frac{p(1-\tau_g)}{1-t}\right]^2\cdot\left[\frac{1}{2}-\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}}Q_L}{p^{\varepsilon_{Dl}}(1-\varepsilon_{Dl})(2-\varepsilon_{Dl})})\right] \tag{39}$$

- Global market clearing:

$$\frac{p(1-\tau_g)}{1-t}\cdot\left[1+\frac{\left(1-\frac{1}{\varepsilon_{Dl}}\right)^{\varepsilon_{Dl}}Q_L}{p^{\varepsilon_{Dl}}(\varepsilon_{Dl}-1)}\right]=Q_G p^{-\varepsilon_{D_G}} \tag{40}$$

Now we want to solve the problem