Distributed synthesis of state-dependent switching control

RP'16, Aalborg, Denmark

Adrien Le Coënt [1], Laurent Fribourg [2], Nicolas Markey [2],
Florian De Vuyst [1], Ludovic Chamoin [3]

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[1] CMLA Centre de Mathématiques et de Leurs Applications
[3] LMT-Cachan Laboratoire de Mécanique et Technologie
[2] LSV Laboratoire de Spécification et Vérification

Introduction

Framework

- Framework of the switched control systems: one selects the working modes of the system over time, every mode is described by ordinary differential equations (ODEs)
- The tools we have:
  - Tiling (space discretization)
  - Zonotopes (symbolic representation of sets)
  - Reachability procedure
  - Compositionality

Implementation in

- Octave (linear version as an extension of MINIMATOR https://bitbucket.org/ukuehne/minimator/)
- C++ (non linear prototype)

Outline

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Switched Systems

A discrete-time switched system

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is a family of discrete-time dynamical systems with a rule σ that determines at each time which one is active

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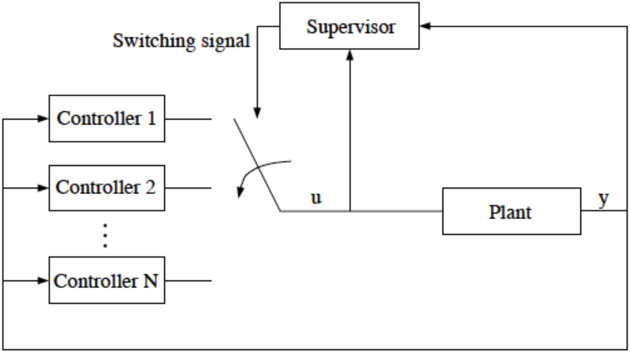
$$x(t+1) = A_u x(t) + B_u$$

Controlled Switched Systems: Schematic View



Switched Systems

We suppose that the system can be written:

$$x_1(t+1) = f_1(x_1(t), x_2(t), u_1)$$
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$$U = U_1 \times U_2$$

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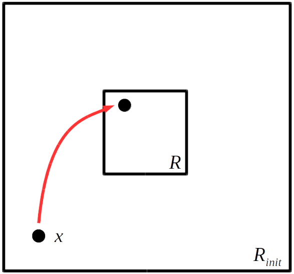

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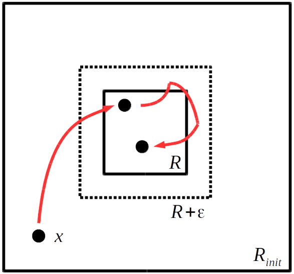

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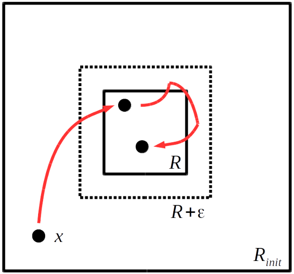
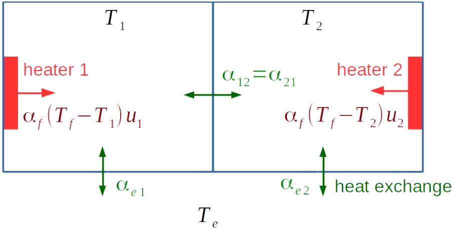


NB: classic stabilization impossible here (no common equilibrium pt) $\rightsquigarrow$ practical stability

Example: Two-room apartment



$$\dot{\begin{pmatrix}T_1\\T_2\end{pmatrix}}=\begin{pmatrix}-\alpha_{21}-\alpha_{e1}-\alpha_f u_1 & \alpha_{21}\\ \alpha_{12} & -\alpha_{12}-\alpha_{e2}-\alpha_f u_2\end{pmatrix}\begin{pmatrix}T_1\\T_2\end{pmatrix}+\begin{pmatrix}\alpha_{e1}T_e+\alpha_f T_f u_1\\ \alpha_{e2}T_e+\alpha_f T_f u_2\end{pmatrix}.$$

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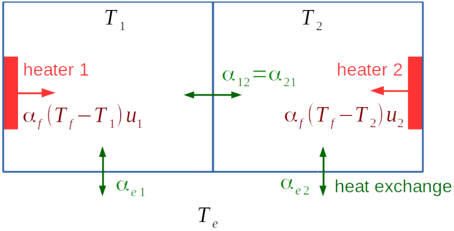

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- Modes: $\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}0\\ 1\end{pmatrix}, \begin{pmatrix}1\\ 0\end{pmatrix}, \begin{pmatrix}1\\ 1\end{pmatrix}$; sampling period τ

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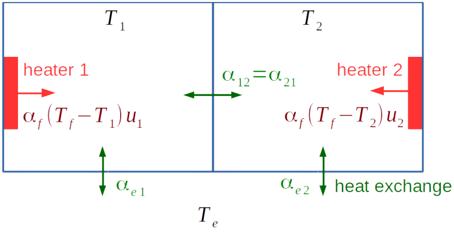

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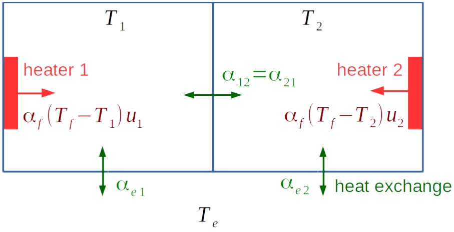

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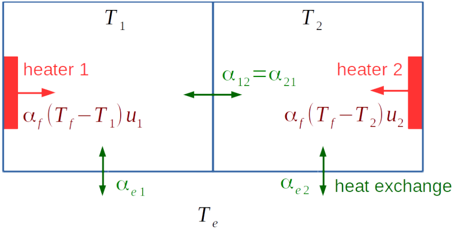

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- A pattern π is a finite sequence of modes, e.g. $\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$
- A state dependent control consists in selecting at each τ a mode (or a pattern) according to the current value of the state.

Reachability and Stability Properties for the two-room apartment

Input: R, ε

Output: a, controlled tiling of $R + a$

Guaranteed properties: reachability from $R + a$ to R, stability in $R + \varepsilon$, safety in $R + a + \varepsilon$

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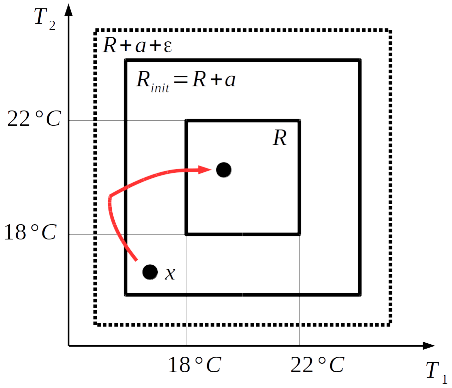
- Reachability: after some steps of time, the temperature reaches R with safety in $R + a + \varepsilon$



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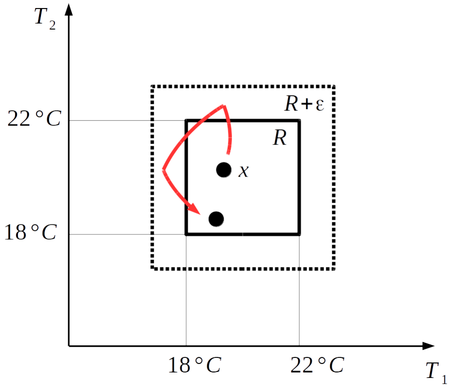
- Stability: special case of reachability, with $a = 0$.

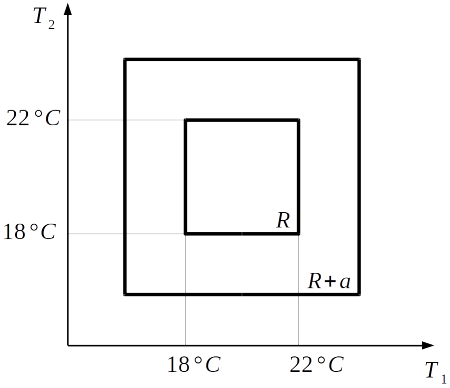


Outline

Centralized control synthesis

$$x(t+1) = f(x(t), u)$$

Goal: from any $x \in R + a$, reach the target zone R.



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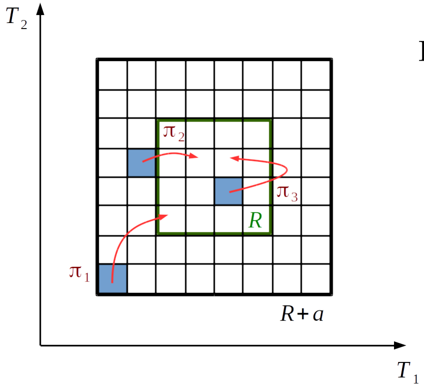

Basic idea:

- Generate a tiling of $R + a$
- Look for patterns (input sequences) mapping the tiles into R
- If it fails, generate another tiling.

Centralized control synthesis

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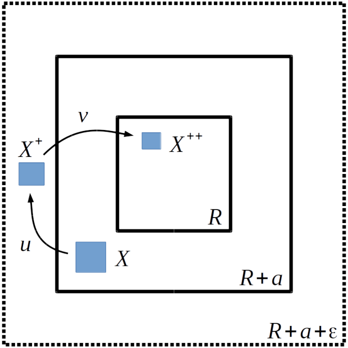
Example of a validated pattern of length 2 mapping the tile X into R with a tolerance in $R + a + \varepsilon$:

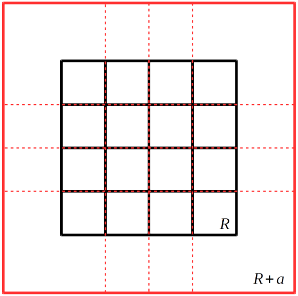


- $X \subset R + a$
- $X^+ = f(X, u) \subset R + a + \varepsilon$
- $X^{++} = f(X^+, v) \subset R$

- Pattern $u \cdot v$ depends only on X

Reachability

Parametric extension of a tiling:



Problem to solve: Find (the maximum value of) $a \geq 0$ such that $R + a$ can be mapped into R.

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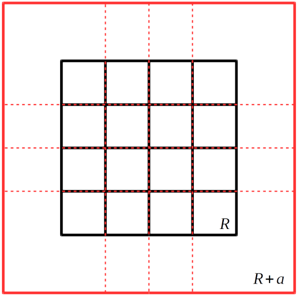
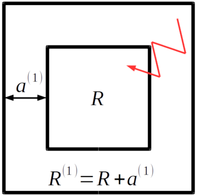


Problem to solve: Find (the maximum value of) $a \geq 0$ such that $R + a$ can be mapped into R.

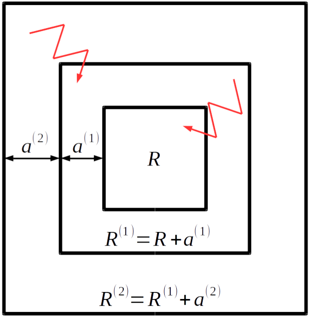
$\Rightarrow$ Can be solved by constrained optimization algorithms

Reachability: backward iteration of the procedure



Iterated control of $R^{(1)} = R + a^{(1)}$ towards R,

Reachability: backward iteration of the procedure



Iterated control of $R^{(1)} = R + a^{(1)}$ towards R, and $R^{(2)} = R^{(1)} + a^{(2)}$ towards $R^{(1)}$.

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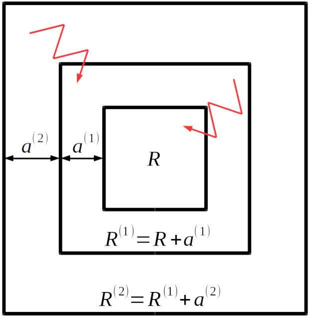

Iterated control of $R^{(1)} = R + a^{(1)}$ towards R, and $R^{(2)} = R^{(1)} + a^{(2)}$ towards $R^{(1)}$.

$\Rightarrow$ Compute a basin of attraction of R

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$$x_1(t+1) = f_1(x_1(t), x_2(t), u_1)$$
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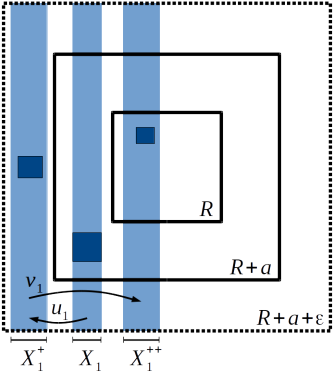
Target zone: $R = R_1 \times R_2$



- $X_1 \subset R_1 + a$
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- $X_1^{++} = f_1(X_1^+, R_2 + a + \varepsilon, v_1) \subset R_1$

- Pattern $u_1 \cdot v_1$ depends only on X_1

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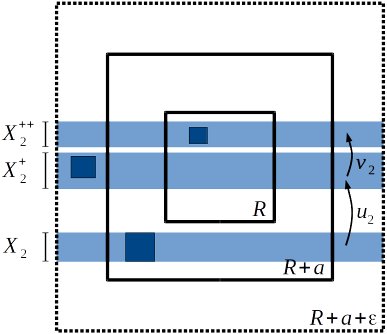
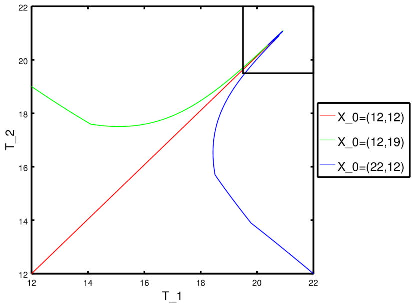


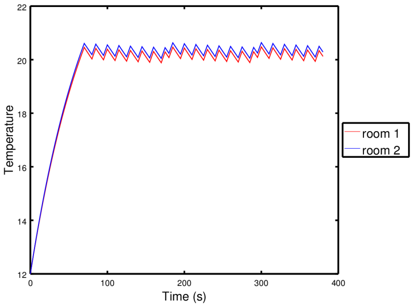
- $X_2 \subset R_2 + a$
- $X_2^+ = f_2(R_1 + a, X_2, u_2) \in R_2 + a + \varepsilon$
- $X_2^{++} = f_2(R_1 + a + \varepsilon, X_2^+, v_2) \in R_2$

- Pattern $u_2 \cdot v_2$ depends only on X_2

Centralized control

Input: $R = [18.5, 22]^2$, $\varepsilon = 1.5$
Output: $a = 6$ in 4 steps, cpu time: ~ 20s





Simulations of the centralized reachability controller for three different initial conditions plotted in the state space plane (left); simulation of the centralized reachability controller for the initial condition $(12, 12)$ plotted within time (right).

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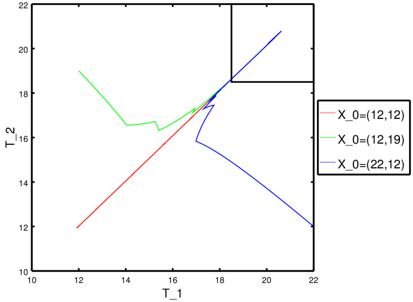

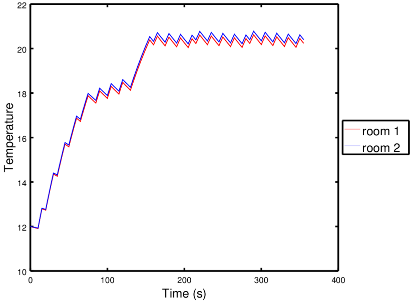

Simulations of the distributed reachability controller for three different initial conditions plotted in the state space plane (left); simulation of the distributed reachability controller for the initial condition $(12, 12)$ plotted within time (right).

Seluxit case study

Kim G. Larsen, Marius Mikučionis, Marco Muniz, Jiri Srba, Jakob H. Taankvist. *Online and Compositional Learning of Controllers with Application to Floor Heating*. Tools and Algorithms for Construction and Analysis of Systems 2016.

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System dynamics:

$$\frac{d}{dt}T_i(t) = \sum_{j=1}^{n} A^d_{i,j}(T_j(t) - T_i(t)) + B_i(T_{env}(t) - T_i(t)) + H_{i,j}.v_j$$

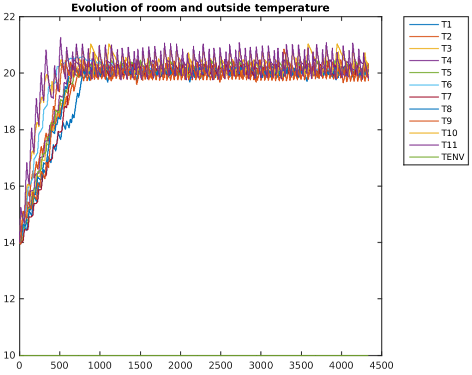
- System of dimension 11
- 2^{11} combinations of v_j (not all admissible, constraint on the number of open valves)
- Pipes heating a room may influence other rooms
- Doors opening and closing (here: average between open and closed)
- Varying external temperature (here: $T_{env} = 10°C$)
- Measures and switching every 15 minutes

Seluxit case study, guaranteed reachability and stability

Decomposition in 5 + 6 rooms (cf. [Larsen et al., TACAS 2016], thanks to the Aalborg team for the simulator)

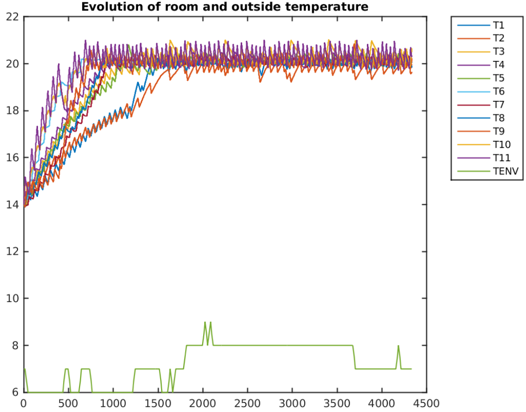
Input:
$R = [18, 22]^{11}$
$\varepsilon = 0.5$
$T_{env} = 10$

Output:
$a = 4$ in 15 steps
cpu time: 6h



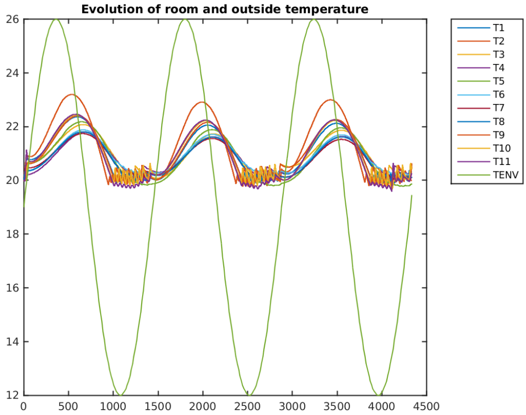
Simulation of the Seluxit case study plotted with time (in min) for $T_{env} = 10°C$.

Seluxit case study, robustness test



Simulation of the Seluxit case study in the soft winter scenario.

Seluxit case study, robustness test (2)



Simulation of the Seluxit case study in the spring scenario.

Conclusion

Recap:

- Guaranteed control
- Compositional principle allowing to handle dimensions up to 11
- Robustness simulations for varying external temperature but no formal proof yet
- Non linear systems

Limitations:

- External temperature constant
- Averaging between open and closed doors
- Parameter ε guessed manually

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Further work:

- Automatic synthesis of the parameter ε (contract-based design)
- Varying external temperature and door openings and closings

Some References

Laurent Fribourg, Ulrich Kühne, and Romain Soulat.
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Thank you ! Questions?