

# Guaranteed simulation and synthesis of Cyber-Physical Systems

INRIA Rennes

Audition Concours CRCN

Adrien Le Coënt

<https://adrienlecoent.github.io>

Aalborg University

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## Education

### ■ Postdoctoral fellow (Aalborg University, 2017 - Present)

Guaranteed synthesis of hybrid systems using timed automata abstractions

Kim G. Larsen

### ■ ■ PhD (École Normale Supérieure de Cachan, 2014 - 2017)

Guaranteed control synthesis for switched space-time dynamical systems  
Florian De Vuyst, Laurent Fribourg, Ludovic Chamoin

### ■ ■ Master (École Normale Supérieure de Cachan, 2013 - 2014)

Advanced Techniques in Structural Computations

### ■ ■ Agrégation de Sciences Industrielles (2013)

### ■ Research internship (Technical University of Denmark, 2012)

Passive shunt damping of vibrating beams using piezoelectric devices

## Context: control systems



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### Issues

guaranteed control synthesis for safety/stability/reachability  
nonlinear systems, PDEs

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nonlinear systems, PDEs

### Applications

safety critical systems  
guaranteed performance

# Switched Systems



# Switched Systems



A continuous-time **switched system**

$$\dot{x}(t) = f_{u(t)}(x(t), w(t))$$

is a family of continuous-time dynamical systems with a **control input**  $u(t) \in U = \{1, \dots, N\}$  that determines at each time which one is active

## Control Synthesis Problem

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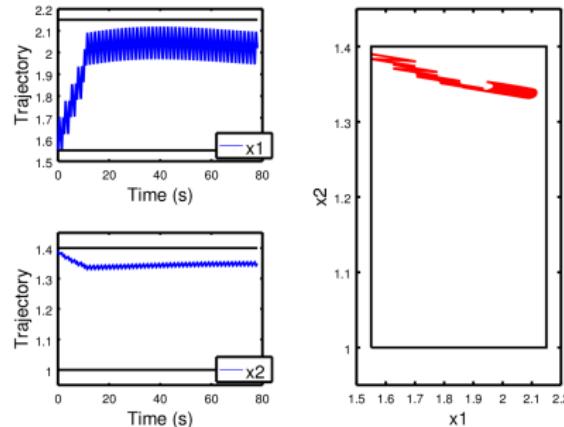
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DC-DC converter:  
stabilize voltage and current

$$x = \begin{pmatrix} V \\ I \end{pmatrix}$$



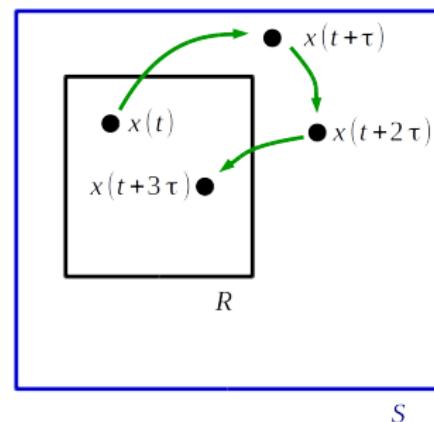
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Given two sets  $R, S$ :

- $(R, S)$ -stability:  $x(t)$  returns in  $R$  infinitely often, at some multiples of sampling period  $\tau$ , and always stays in  $S$



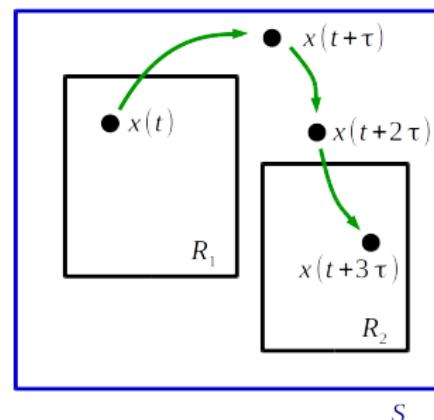
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Given three sets  $R_1, R_2, S$ :

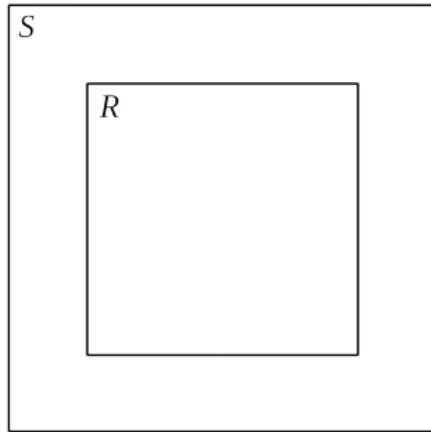
- $(R_1, R_2, S)$ -reachability:  $x(t)$  starting in  $R_1$  reaches  $R_2$  after some multiples of sampling period  $\tau$ , and always stays in  $S$



## Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

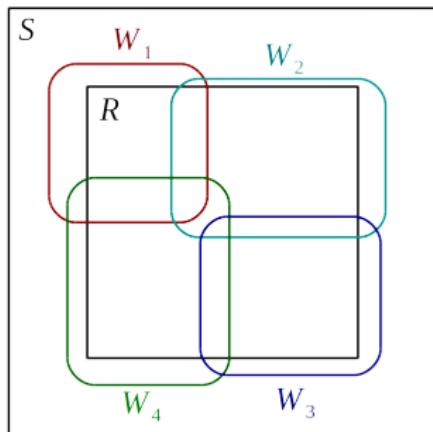
Goal: from any  $x \in R$ , return in  $R$  while always staying in  $S$ .



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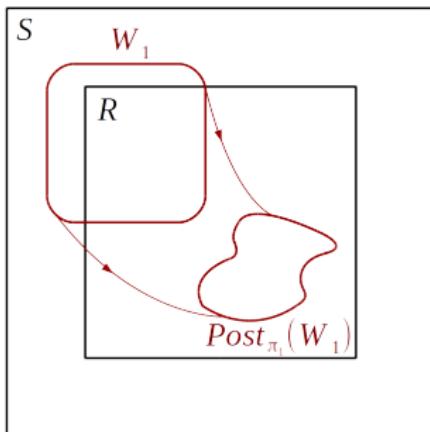
Basic idea:

- Generate a **covering** of  $R$

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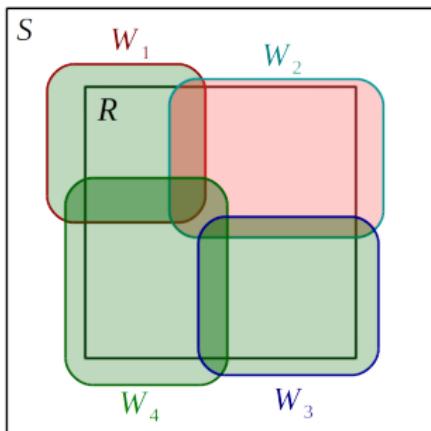
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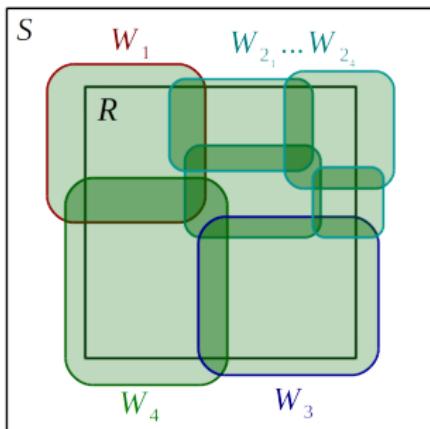
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- Look for **input sequences** mapping (set propagation) the tiles into  $R$  while always staying in  $S$
- If it fails, generate another covering.

## Limits

- Requires the computation of the reachable set
  - unknown in general for nonlinear systems
  - can be approximated using numerical schemes and/or strong hypotheses (restricted class)

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 $\Rightarrow O(N^k)$
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## Contributions

- 💡 Guaranteed reachable set computation for a large class of nonlinear systems with guaranteed numerical schemes
- 💡 Handling higher dimensions using compositionality
- 💡 Synthesizing controllers for PDEs using Model Order Reduction

# Renewing the Euler scheme with the OSL property

[Goubault, Putot, Forward inner-approximated reachability of non-linear continuous systems]

One-sided Lipschitz: for all  $j \in U$ , there exists a constant  $\lambda_j \in \mathbb{R}$  s.t.

$$\langle f_j(y) - f_j(x), y - x \rangle \leq \lambda_j \|y - x\|^2 \quad \forall x, y \in S,$$

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[Le Coënt, Sandretto, Chapoutot, Fribourg, FMSD journal, 2018]

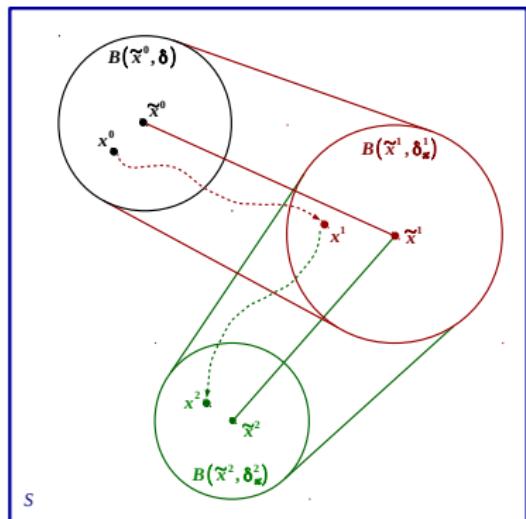
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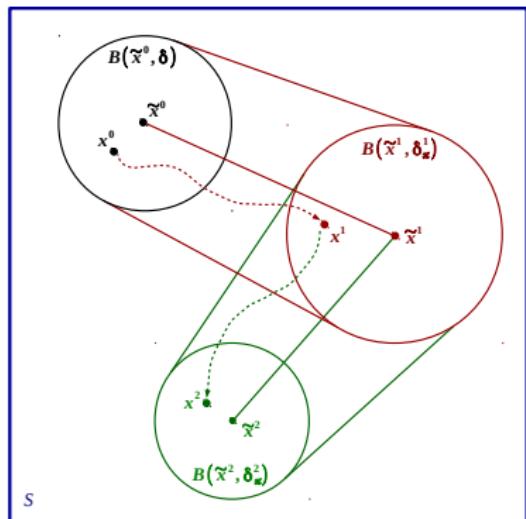
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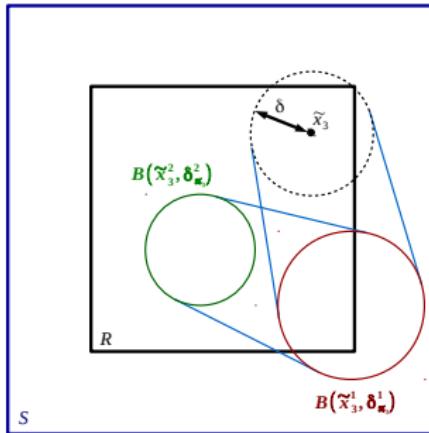
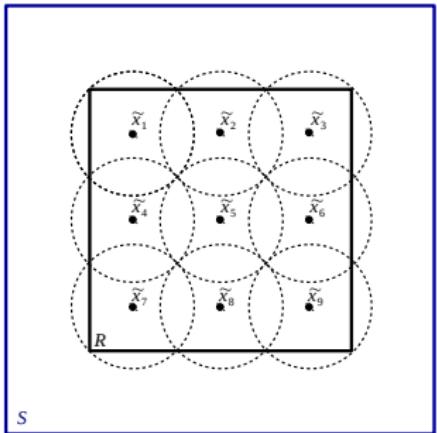


- Extremely fast computations
- Applicable to a large class of systems
- Arbitrary sampling time
- Simple implementation (no interval arithmetics)
- Lack of accuracy when  $\lambda_j > 0$

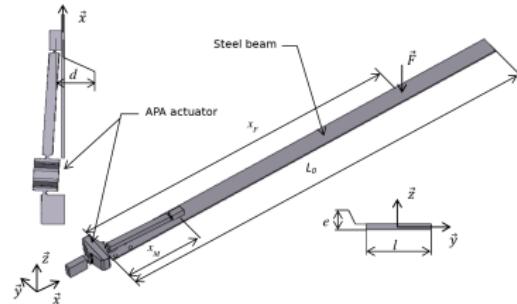
[Le Coënt, Sandretto, Chapoutot, Fribourg, FMSD journal, 2018]

[Le Coënt, De Vuyst, Chamoin, Fribourg, SNR'17]

# Control synthesis



# Dimensional limits



# Distributed control synthesis

Splitting of the system

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2, u_1) \\ \dot{x}_2 &= f_2(x_1, x_2, u_2)\end{aligned}$$

Control objective:  $(R, S)$ -stability with  $R = R_1 \times R_2$ ,  $S = S_1 \times S_2$

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[Le Coënt, Fribourg, Markey, De Vuyst, Chamoin, TCS journal, 2018]

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- Basic idea:  $(R_1, S_1)$ -stability synthesis for sub-system 1 by considering sub-system 2 as a bounded perturbation (in  $S_2$ ) and vice-versa

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- Basic idea:  $(R_1, S_1)$ -stability synthesis for sub-system 1 by considering sub-system 2 as a bounded perturbation (in  $S_2$ ) and vice-versa
- Requirements:
  - Separated control (often possible)
  - Handling of bounded perturbations
  - Few interactions between sub-systems
  - Both syntheses successful

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[Le Coënt, Fribourg, Markey, De Vuyst, Chamoin, TCS journal, 2018]

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## Seluxit case study



## Seluxit case study, guaranteed reachability and stability

Decomposition in 5 + 6 rooms

### Input:

$$R = [18, 22]^{11}$$

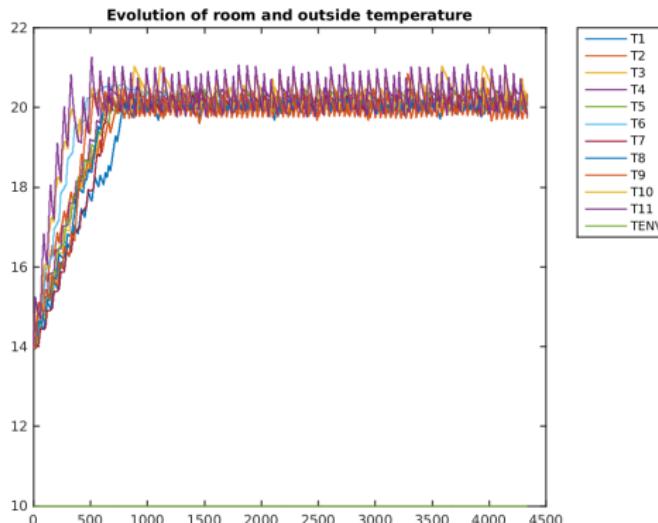
$$S = [17.5, 22.5]^{11}$$

$$T_{env} = 10$$

### Results:

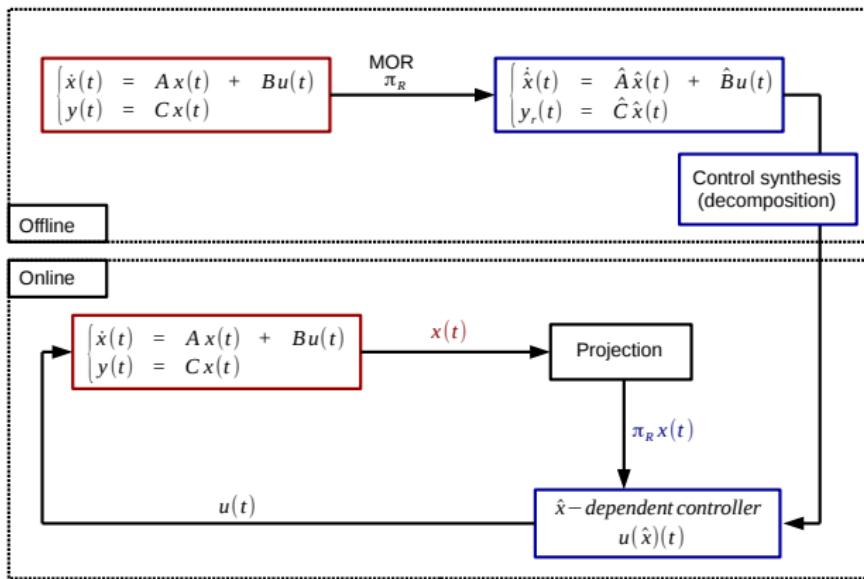
Iterated reachability  
in 15 steps

Synthesis time: 6h



Simulation of the Seluxit case study plotted with time (in min) for  
 $T_{env} = 10^\circ C.$

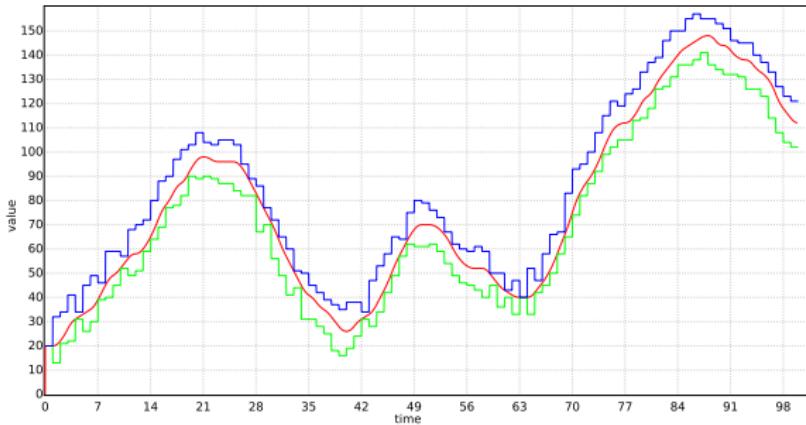
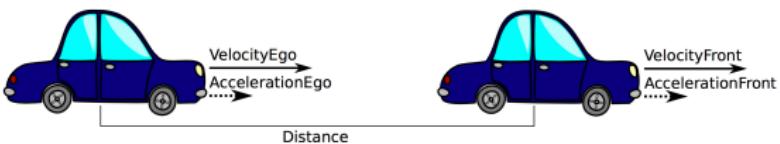
# Dealing with high dimensionality : Model Order Reduction



[Le Coënt, De Vuyst, Chamoin, Rey, Fribourg, IJDC journal, 2016]

[Le Coënt, De Vuyst, Chamoin, Rey, Fribourg, SynCoP'15]

# Timed game abstraction of a cruise control application in UPPAAL TIGA



[Larsen, Le Coënt, Mikučionis, Taankvist, CyPhy'18]

# Cyber-Physical Systems

- Specification, modeling and analysis
  - Hybrid and heterogeneous models [CyPhy'18, QEST'19 (subm.)]
  - Networking
  - Interoperability
  - Time synchronization
- Scalability and complexity management
  - Modularity and composability [TCS, RP'17, RP'16, RP'16]
  - Synthesis [IJDC, SynCoP'15, JMES]
  - Interfacing with legacy systems
- Validation and verification
  - Assurance (guaranteed computations) [FMSD, SNR'17, SNR'16, CyPhy'18]
  - Certification
  - Simulation
  - Stochastic models [ADHS'18, CDC'19 (subm.), NAHS (subm.)]

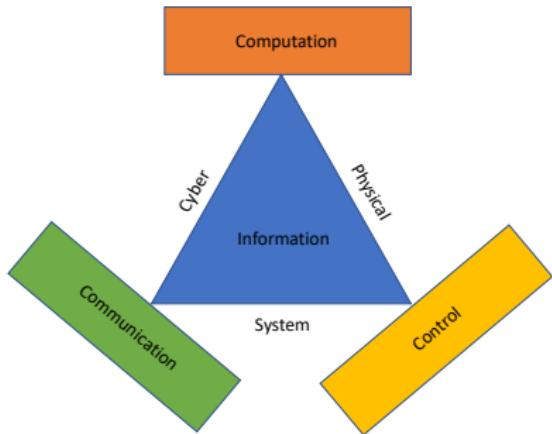
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# Research program : guaranteed simulation and synthesis of Cyber-Physical Systems

## Why ?

Safety critical applications: autonomous vehicles, smart grids, medical monitoring, robotics...

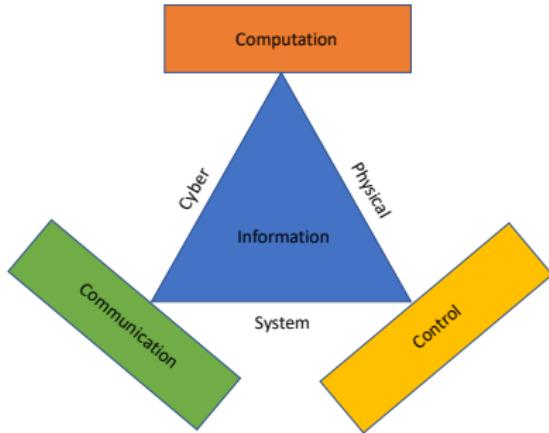


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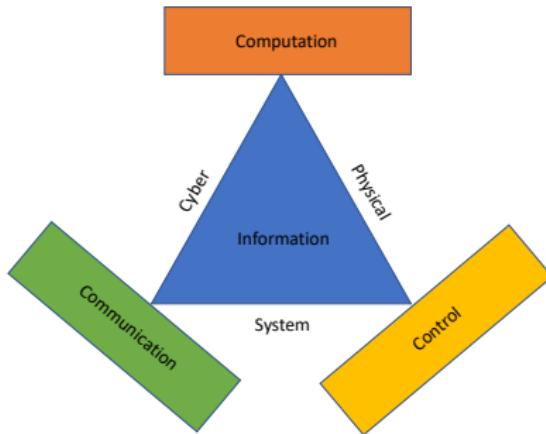
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## Lines of research :

- Guaranteed simulation
- Compositional synthesis
- General framework for PDEs



## Guaranteed simulation

- Guaranteed simulation with numerical schemes and OSL property

Extends previous work for ODEs using new numerical schemes

**Issues:** Accuracy, understanding of the OSL property, implicit schemes needed

**Approach:** implicit Euler, Runge-Kutta, link with Grönwall's inequality and incremental stability

[Le Coënt et al., Control Synthesis of Nonlinear Sampled Switched Systems using Euler's Method, 2017]

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- Guaranteed simulation for DAEs and hybrid systems

**Issues:** algebraic condition, state guards, topology for sound approximation

**Approach:** implicit schemes [Petzold], incremental difficulty

$$\dot{x} = f(y, x)$$

$$0 = g(y, x)$$

[Caillaud, Ghorbal, Benveniste et al., Structural Analysis of Multi-Mode DAE, 2017]

## Compositional synthesis

- Contract-based design for synchronous systems

Motivated by the 11-room case study

Goals: better performances, application to smart grids

Approach: energy distribution could be solved with

Assume-Guarantee contracts

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implies

$$\forall t, S_1(t) \wedge S_2(t)$$

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### ■ Asynchronous inter-connected systems

Compositional synthesis works for synchronous switched systems

But we need asynchronous switchings !

Goals: realistic case studies (e.g. cruise-control)

Approach: guaranteed simulation with arbitrary switching times & timed game abstraction

[Larsen, Le Coënt et al., Guaranteed control synthesis for continuous systems in Uppaal Tiga, 2018]

# Partial Differential Equations

## ■ A general framework for PDEs

Basic idea :

$$\text{PDE} + \text{discretization} = \text{HD-ODE}$$

$$\text{HD-ODE} + \text{MOR} = \text{ODE}$$

$$\text{ODE} + \text{tiling based synthesis} + \text{error bounding} = \text{guaranteed control}$$

### ■ First: linear 1D

[Le Coënt, Guaranteed control synthesis for switched space-time dynamical systems, 2017]

### ■ Then: nonlinear 1D

[De Vuyst, Toumi, Empirical Interpolation Decomposition, 2018]

### ■ Last: multi-D

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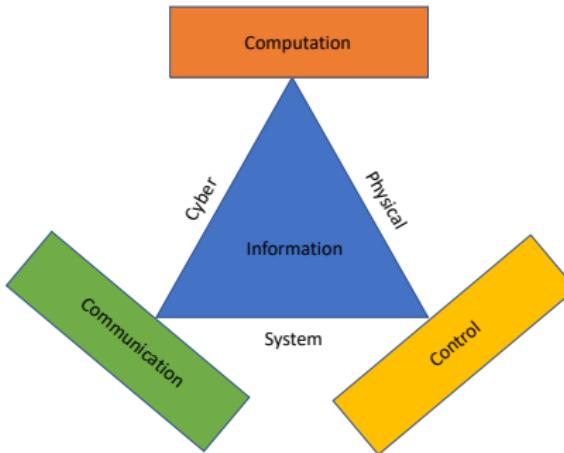
## ■ Domain decomposition methods and compositional analysis

Long term idea for overcoming any dimensional barrier

## Research program

Équipe-Projet HYCOMES (Benoît Caillaud, Khalil Ghorbal, Albert Benveniste)

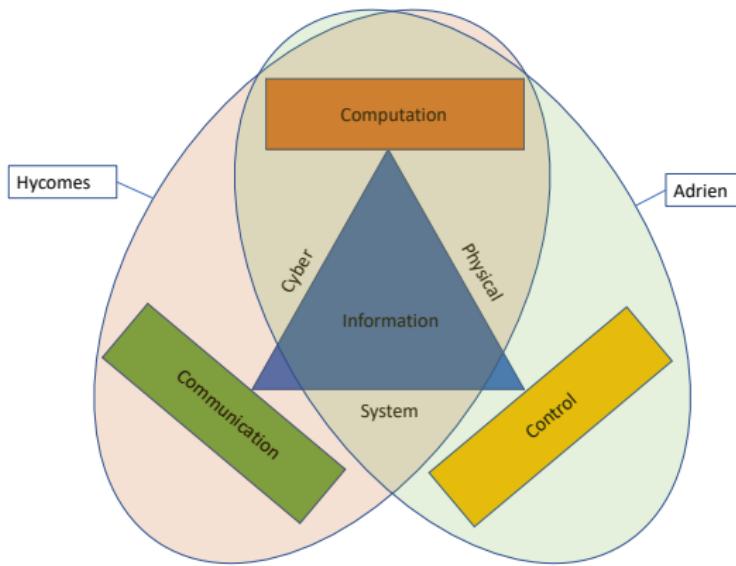
- Faithful simulation, modularity, keeping models close to physics
- Contract-based design and interface theories, with applications to requirements engineering



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# Ecosystem

## Guaranteed simulation

Hycomes            U2IS            LAAS

Khalil Ghorbal    Alexandre Chapoutot    Victor Magron

Benoît Caillaud    Julien Alexandre dit Sandretto

## Compositional synthesis

Hycomes            AAU            LSV            L2S

Albert Benveniste    Kim Larsen    Laurent Fribourg    Antoine Girard

Benoît Caillaud

## DAEs

Hycomes

Khalil Ghorbal

Benoît Caillaud

## PDEs

UTC

LMT

Florian De Vuyst

Ludovic Chamoin

# Industrial collaborations, research supervision, project funding

## ■ Industrial collaborations and case-studies

- Nilfisk (DK) : lifetime and state-of-charge optimization of lead-acid batteries
- Seluxit (DK) : 11 room floor heating case-study

## ■ Research supervision

- Research internship :
  - A numerical-symbolic hybrid method for approx. solutions of parameter-dependent PDEs<sup>1</sup>
  - Analysis and practice of the PARAEXP algorithm<sup>2</sup>
- Bachelor's project : Prediction Methods for Finite Control Set - Model Predictive Control<sup>3</sup>

## ■ Projects supporting this research

ERC Lasso, Innovation Fund Denmark DiCyPS, Institut Farman

**SWITCHDESIGN2** and RAILBOOL, ANR iCODE, ANR Digicosme,  
ERC Cassting

---

<sup>1</sup>A. Guérin, J. Delhom

<sup>2</sup>R. Alison, G. Brunet, P. Jusselin, F. Koechlin

<sup>3</sup>A. G. Weirsøe, C. S. Mathisen, J. L. Haslund, M. Nielsen

# Summary

Project : Guaranteed simulation and synthesis of Cyber-Physical Systems

## Experience

(2017 - 2019) Post doc (Aalborg, 

(2014 - 2017) PhD (Cachan,  

(2012) Research internship (Copenhagen, 

## 13 publications (4 journals, 9 conferences) + 3 submissions

Guaranteed simulation: [FMSD](#), SNR'16, SNR'17, ADHS'18, CyPhy'18, CDC'19 (subm.), [NAHS](#) (subm.)

Compositional synthesis: [TCS](#), RP'17, RP'16, RP'16

Control of PDEs: [IJDC](#), [JMES](#), SynCoP'15

Timed game abstractions: CyPhy'18, QEST'19 (subm.)

## Case of undiscretized PDE problems

Difficulty:

- The problem becomes **infinite-dimensional**;
- Even spatially discretized, the ***curse of dimensionality*** makes the former approaches (bisection, ball overlapping, ...) irrelevant.

⇒ requires **model order reduction (MOR)**

