Guaranteed Control of Switched Control Systems Using Model Order Reduction and Bisection

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April 11, 2015

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Introduction

Framework

- Goal: control the evolution of an operating system with the help of actuators
- Framework of the switched control systems: one selects the working modes of the system over time, every mode is described by differential equations (ODEs or PDEs)
- Application to medium/high dimensional systems:
  - Model Order Reduction
  - Error bounding
  - State space bisection

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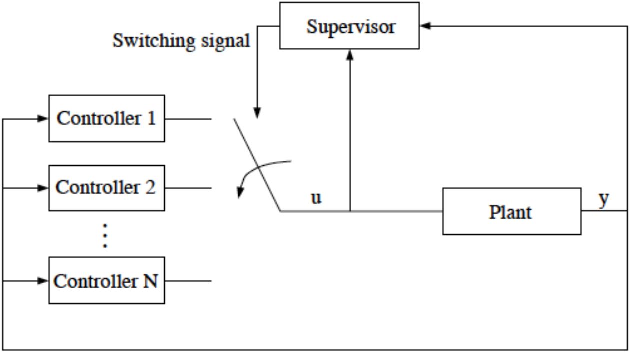
- Instead of considering all the continuous evolution, one observes the system only at periodic switching instants at times: τ, 2τ, $\ldots$

Controlled Switched Systems: Schematic View



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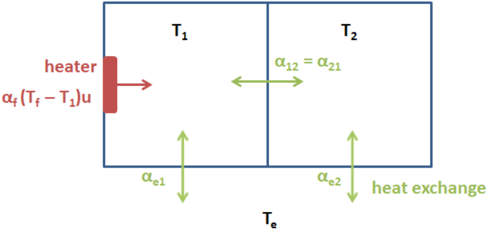
NB: classic stabilization impossible here (no common equilibrium pt)
$\leadsto$ practical stability

Example: Two-room apartment



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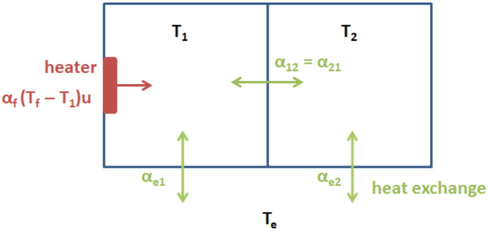

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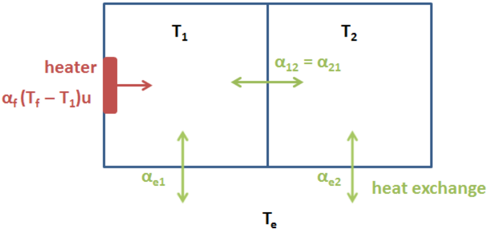

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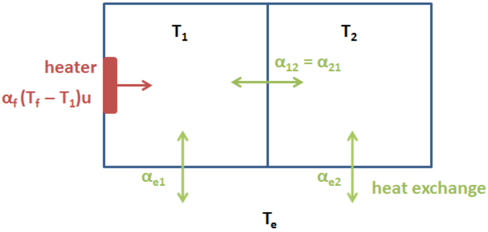

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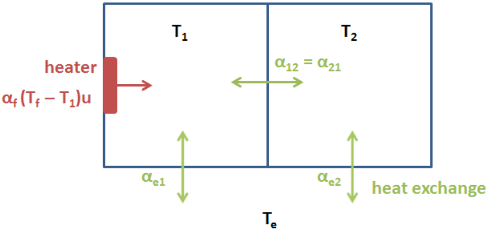

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NB: Each mode has its basic proper equilibrium point; by appropriate switching, one can drive the system to a specific stability zone

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- Example of stability property to be checked: temperature regulation

$$|T_i(t) - T_{reference}| \leq \varepsilon \text{ as } t \to \infty$$

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- A sampled switched system can thus be viewed as a piecewise affine discrete-time system.

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  if π is a pattern of the form $(u_1 \cdots u_m)$
- The unfolding of $Post_\pi(X)$ is the union of X, $Post_\pi(X)$ and the intermediate sets:

  $X \cup Post_{u_1}(X) \cup Post_{u_1 \cdot u_2}(X) \cup \cdots \cup Post_{u_1 \cdots u_{m-1}}(X) \cup Post_\pi(X)$

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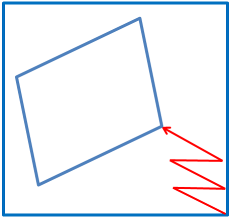

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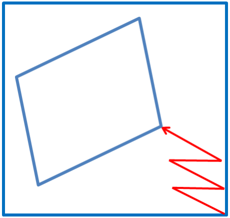

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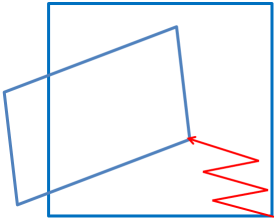

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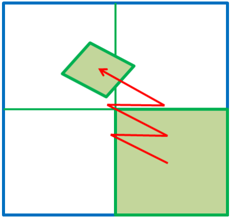

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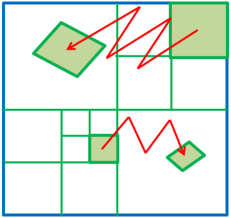

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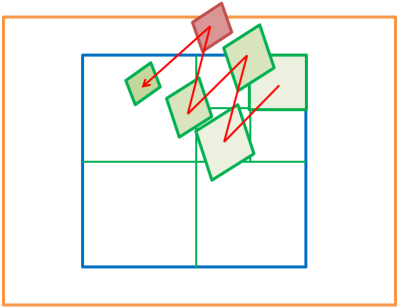
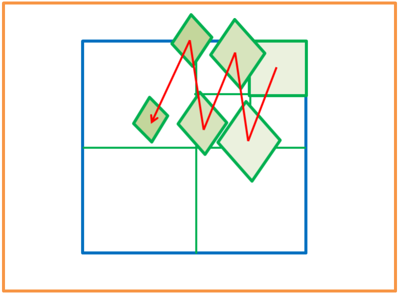
- Extension for safety: the unfolding must stay in the safety set S.





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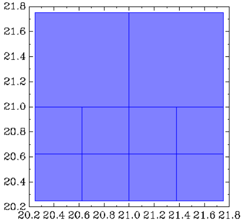

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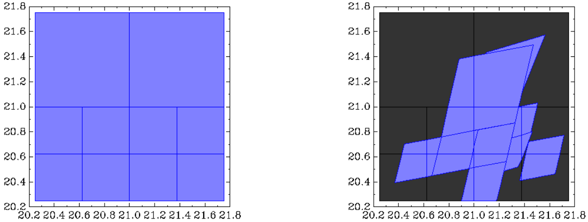

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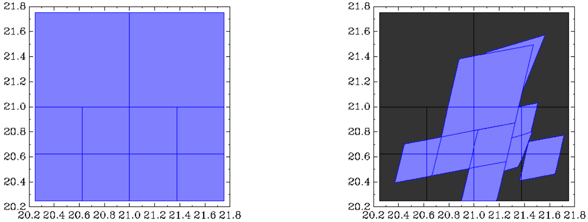

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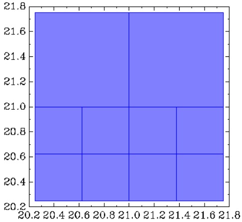

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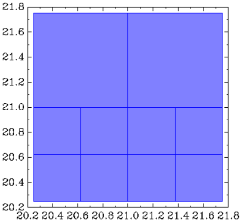

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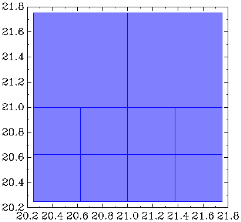

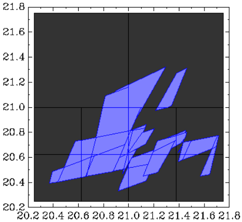

Decomposition for the two-room apartment

For: $\alpha_{12} = 5 \times 10^{-2}, \alpha_{21} = 5 \times 10^{-2}, \alpha_{e1} = 5 \times 10^{-3}, \alpha_{e2} = 3.3 \times 10^{-3}, \alpha_f = 8.3 \times 10^{-3}, T_e = 10, T_f = 50$ and $\tau = 5$.
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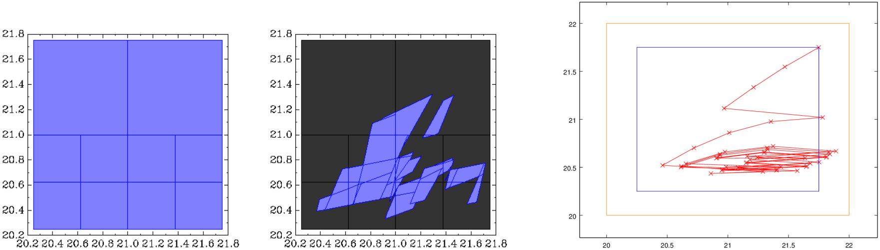

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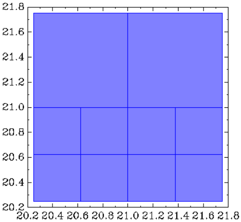

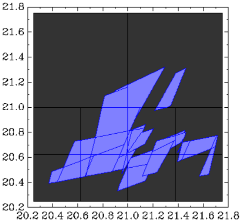

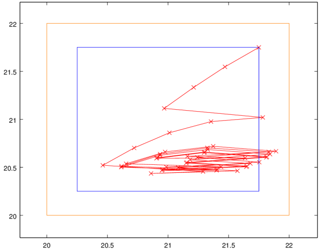

Figure : Decomposition (left) ; unfolding (middle) ; unfolded trajectory (right) in plane (T_1, T_2)

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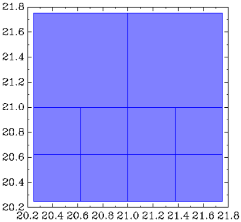

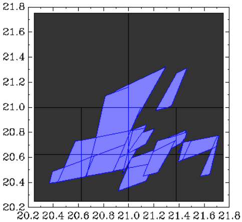

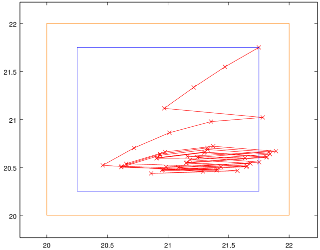

Figure : Decomposition (left) ; unfolding (middle) ; unfolded trajectory (right) in plane (T_1, T_2)

Decomposition found for $k = 4, d = 3$.

Outline

A Sampled Switched System with Output

- Described by the differential equation:

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bu(t) \\ y(t) & = Cx(t) \end{cases}$$

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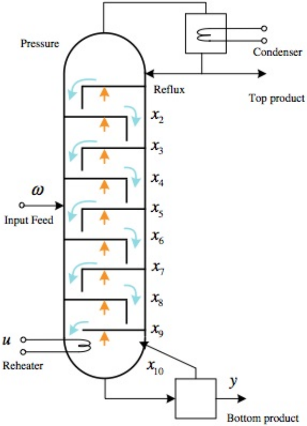
A Sampled Switched System with Output

A distillation column



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definition

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Computational cost of decomposition: at most in $O(2^{nd} N^k)$.

Outline

Model Order Reduction by Projection

Construction of a reduced order system $\hat{\Sigma}$ of order n_r:

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Reduced Order Control

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Questions:

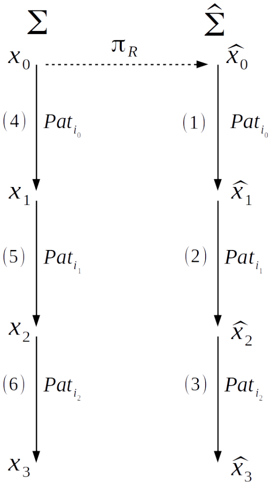
- How is it applied?
- Is the reduced-order control effective at the full-order level?

Outline

Offline Procedure

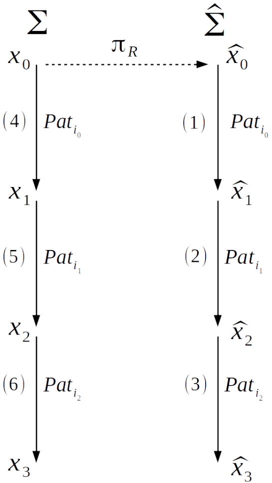
$$
\begin{tikzcd}[row sep=large, column sep=huge]
\Sigma & \widehat{\Sigma} \\
x_0 \arrow[r, dashed, "\pi_R"] \arrow[d, "(4)"', "Pat_{i_0}"] & \widehat{x}_0 \arrow[d, "(1)"', "Pat_{i_0}"] \\
x_1 \arrow[d, "(5)"', "Pat_{i_1}"] & \widehat{x}_1 \arrow[d, "(2)"', "Pat_{i_1}"] \\
x_2 \arrow[d, "(6)"', "Pat_{i_2}"] & \widehat{x}_2 \arrow[d, "(3)"', "Pat_{i_2}"] \\
x_3 & \widehat{x}_3
\end{tikzcd}
$$

Offline Procedure



1. Projection of the initial state x_0

Offline Procedure



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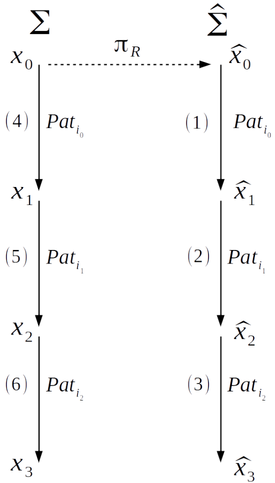

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3. Application of the pattern sequence at the full-order level (steps (4),(5),(6)).

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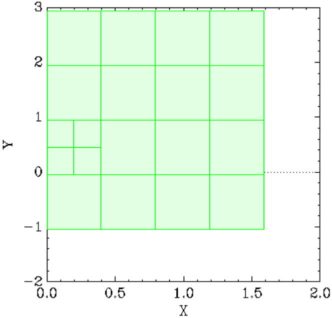
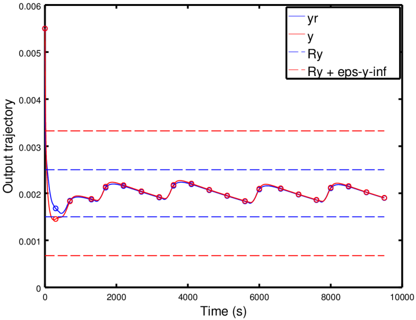
Consequence: the output of the full order system is sent in $R_y + \varepsilon_y^\infty$.

Guaranteed Offline Control

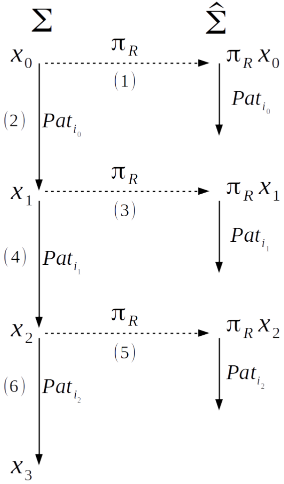
Simulation on a linearized model of a distillation column [5]: $n = 11$ and $n_r = 2$:



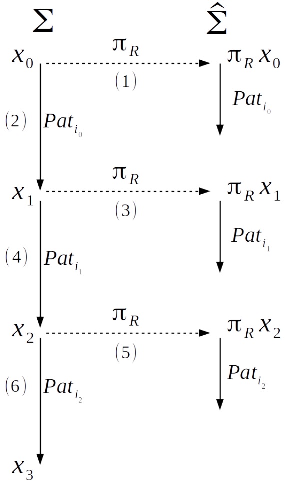


Outline

Online Procedure

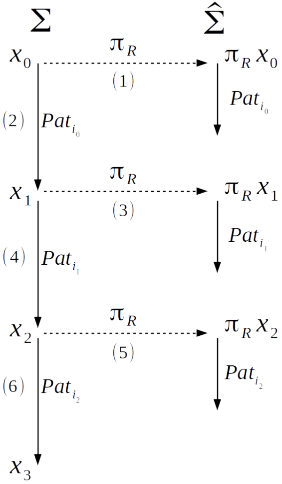


Online Procedure



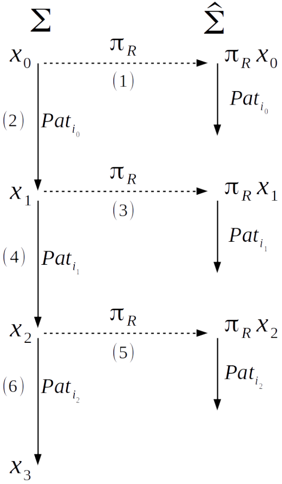
1. Projection of the initial state x_0 (step (1))

Online Procedure



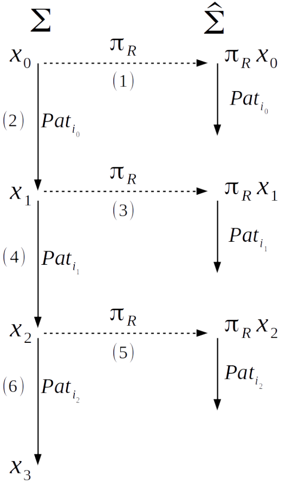
1. Projection of the initial state x_0 (step (1))
2. Computation of the pattern Pat_{i_0} at the reduced-order level

Online Procedure



1. Projection of the initial state x_0 (step (1))
2. Computation of the pattern Pat_{i_0} at the reduced-order level
3. Application of the pattern Pat_{i_0} at the full-order level, Σ is sent to a state x_1 (step (2))

Online Procedure



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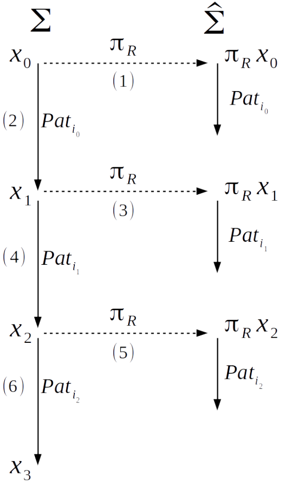
4. Projection of the (new initial) state x_1 (step (3))

Online Procedure



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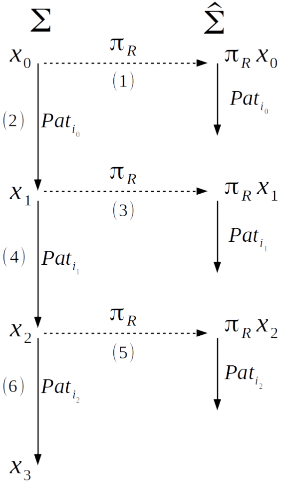
4. Projection of the (new initial) state x_1 (step (3))
5. Computation of the pattern Pat_{i_1} at the reduced-order level

Online Procedure



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5. Computation of the pattern Pat_{i_1} at the reduced-order level
6. Application of the pattern Pat_{i_1} at the full-order level, Σ is sent to a state x_2 (step (4))...

Guaranteed Online Control

Requirement to apply the online procedure:

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- Ensure that $\pi_R Post_{Pat_i}(x) \in \hat{R}_x$ at every step.

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Solution: Compute an ε-decomposition

definition

A ε-decomposition Δ of R_x is a set of couples $\{(V_i, Pat_i)\}_{i\in I}$ such that:

- $\bigcup_{i\in I} V_i = R_x$
- $\forall i \in I\ Post_{Pat_i}(V_i) \subseteq R_x - \varepsilon_x^{|Pat_i|}$
- $\forall i \in I\ Post_{Pat_i,C}(V_i) \subseteq R_y$ (y-convergence)

Guaranteed Online Control

An ε-decomposition performed on $\hat{\Sigma}$ permits to iterate the online procedure:

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- we have:

$$\|\pi_R Post_{Pat}(x) - Post_{Pat}(\pi_R x)\| \leq \varepsilon_x^{|Pat_{i_k}|}$$

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- we have:

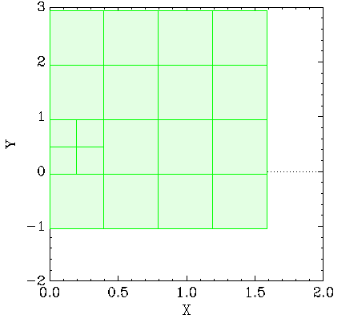
$$\|\pi_R Post_{Pat}(x) - Post_{Pat}(\pi_R x)\| \leq \varepsilon_x^{|Pat_{i_k}|}$$

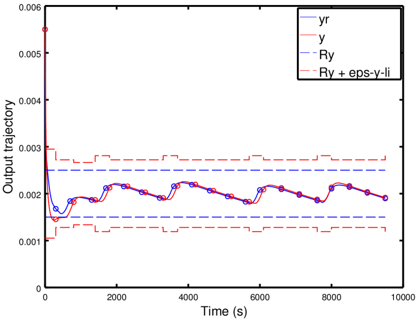
- thus, at every step k:

$$\pi_R Post_{Pat_{i_k}}(x_k) \in \hat{R}_x$$

Guaranteed Online Control

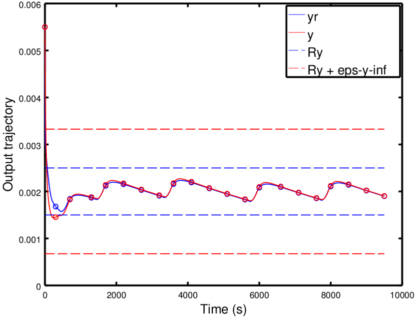
Simulation on a linearized model of a distillation column [5]: $n = 11$ and $n_r = 2$:

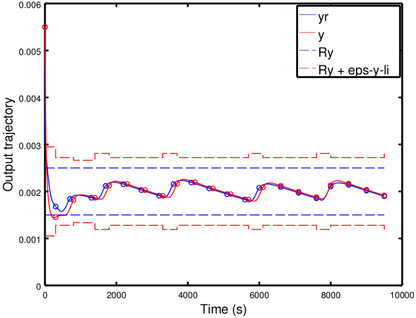




Remark: Output trajectory error depending on the length of the applied pattern: much lower than the infinite bound ε_y^∞

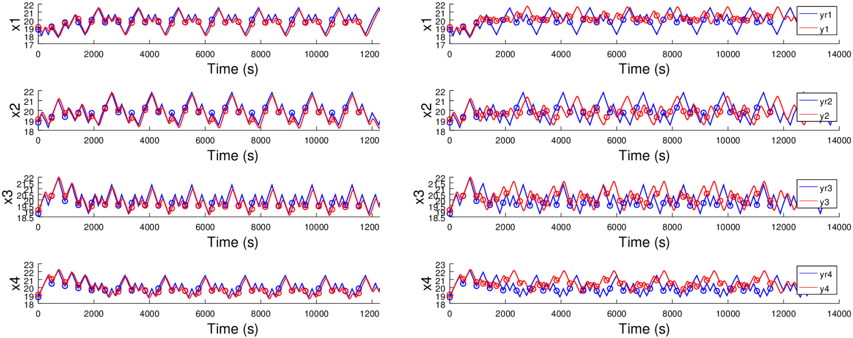
Comparison of the Two Procedures





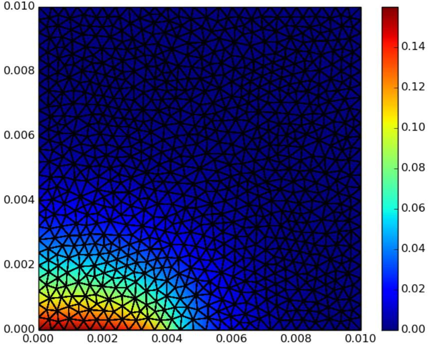
Other Applications

- Control of the temperature of a 4 room appartment: offline and online control
  $n = 8$ and $n_r = 4$



Other Applications

- Control of the temperature of a square plate discretized by finite elements: offline and online control
  $n = 897$ and $n_r = 2$



Other Applications

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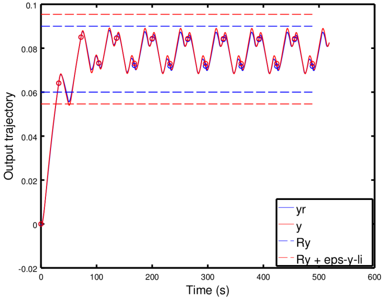
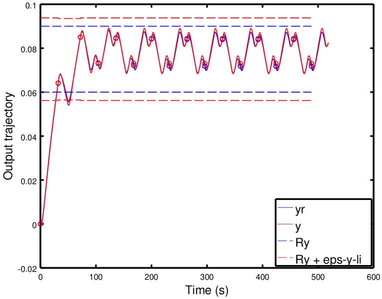
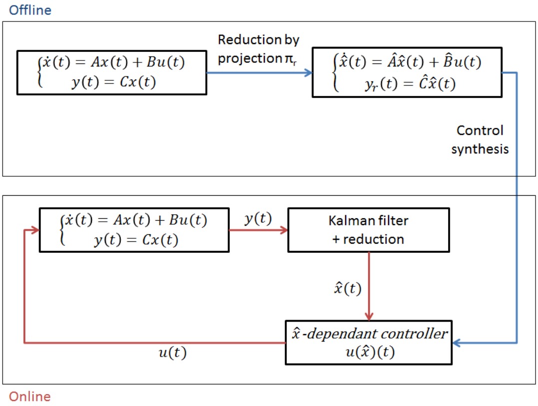




Open Questions and Future Work



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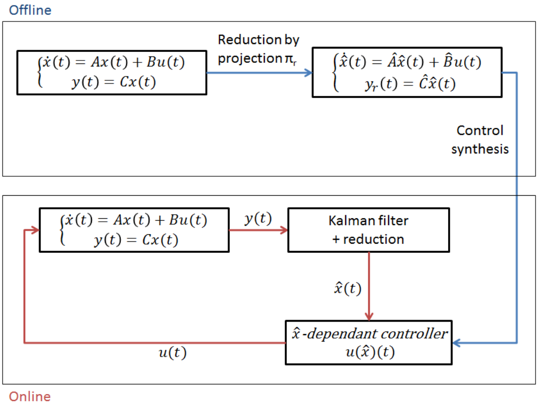

- Online reconstruction of the reduced state
  - $\Rightarrow$ reduced Kalman filter
  - $\Rightarrow$ reconstruction error estimation

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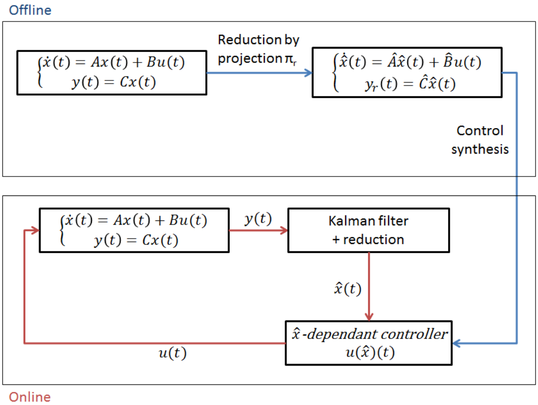

- Online reconstruction of the reduced state
  - $\Rightarrow$ reduced Kalman filter
  - $\Rightarrow$ reconstruction error estimation
- Application to large scale systems

Some References

Peter Benner and André Schneider.
Balanced truncation model order reduction for lti systems with many inputs or outputs.
In *Proceedings of the 19th international symposium on mathematical theory of networks and systems–MTNS*, volume 5, 2010.

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Thank you ! Questions?