Provable Controlled Invariance for hybrid systems using symbolic reachability: Application to mechanical systems with impacts

CyPhy 2018

Adrien Le Coënt[1], Laurent Fribourg [2]

October 4, 2018

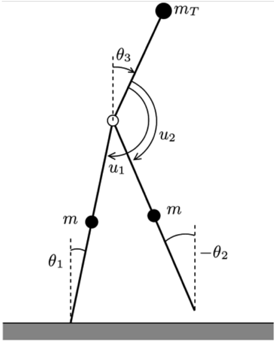
[1]Aalborg University, Department of Computer Science
[2]LSV, ENS Paris-Saclay, CNRS, INRIA

Outline

1. Biped robot: model and control synthesis objective
2. State-of-the art method: Poincaré plan and Hybrid Zero Dynamics
3. Our method for control synthesis of sampled switched systems: controlled recurrence, tool MINIMATOR
4. Adaptation to the biped robot model (hybrid model with impact)
5. Results and future work

Outline

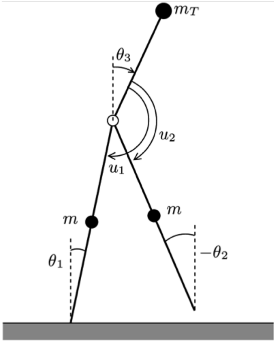
Biped robot: model



- During a step, the dynamics of the robot is given by the nonlinear equation

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + G(\theta) = Bu$$

Biped robot: model



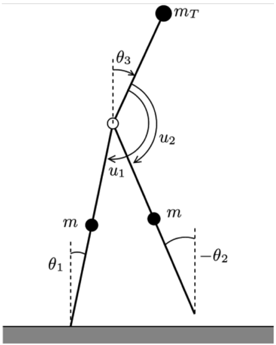
- During a step, the dynamics of the robot is given by the nonlinear equation

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + G(\theta) = Bu$$

- A collision happens when both feet touch the ground. Collision happens when:

$$\theta_1 + \theta_2 = 0$$

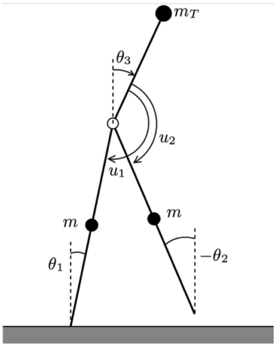
Biped robot: collision model



- A collision happens when both feet touch the ground. Collision happens when:

$$\theta_1 + \theta_2 = 0$$

Biped robot: collision model



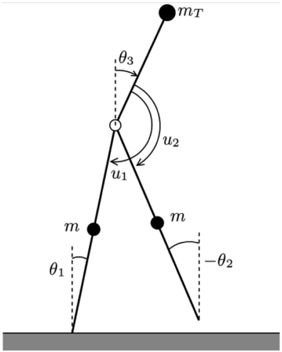
- A collision happens when both feet touch the ground. Collision happens when:

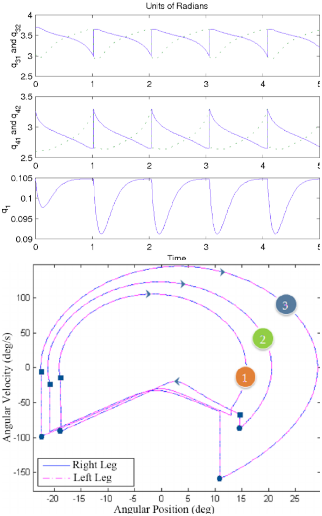
$$\theta_1 + \theta_2 = 0$$

- Once the collision happens, conservation of the momentum and considering of symmetries in the system leads to a *reset* to apply. The equations of the reset are:

$$\begin{pmatrix} I & 0 \\ 0 & H^n(\theta^+) \end{pmatrix} \begin{pmatrix} \theta^+ \\ \dot{\theta}^+ \end{pmatrix} = \begin{pmatrix} Q & 0 \\ 0 & H^o(\theta^-) \end{pmatrix} \begin{pmatrix} \theta^- \\ \dot{\theta}^- \end{pmatrix}$$

Biped robot: limit cycles

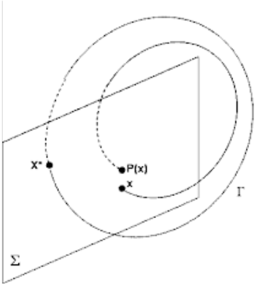




Outline

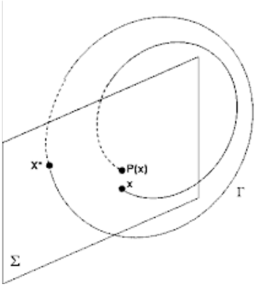
Dynamical systems: state-of-the-art method for proving convergence towards a limit cycle

- Intersection of trajectories with Poincaré plan converge to a fixed-point $P(x)$. Here the Poincaré plan corresponds to the contact with the floor: $\rightarrow$ zero dynamics



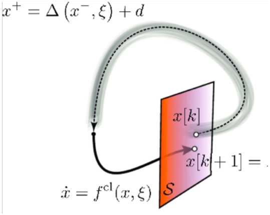
Dynamical systems: state-of-the-art method for proving convergence towards a limit cycle

- Intersection of trajectories with Poincaré plan converge to a fixed-point $P(x)$. Here the Poincaré plan corresponds to the contact with the floor: $\rightarrow$ zero dynamics
- In the biped model: complications due to the reset on impact



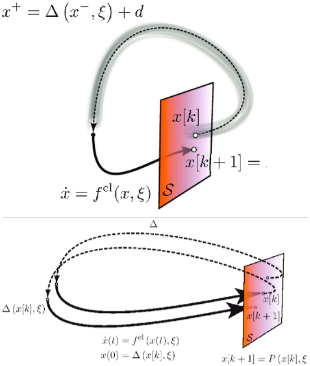
Dynamical systems: state-of-the-art method for proving convergence towards a limit cycle

- In the biped model: complications due to the reset on impact
  $\rightarrow$ Extension needed: Hybrid Zero Dynamics (HZD)



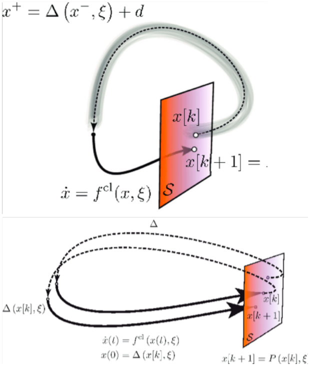
Dynamical systems: state-of-the-art method for proving convergence towards a limit cycle

- In the biped model: complications due to the reset on impact
  $\rightarrow$ Extension needed: Hybrid Zero Dynamics (HZD)
  $\rightarrow$ Requires additional actuators for nullifying movements transverse to the Poincaré plan



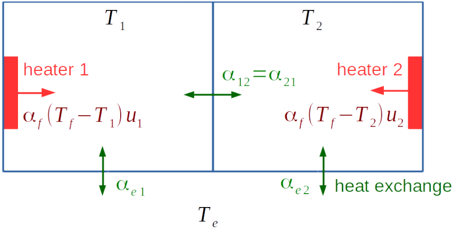
Dynamical systems: state-of-the-art method for proving convergence towards a limit cycle

- In the biped model: complications due to the reset on impact
  → Extension needed: Hybrid Zero Dynamics (HZD)
  → Requires additional actuators for nullifying movements transverse to the Poincaré plan
- NB: HZD proves only limit convergence for sufficiently "high gain" of actuators; no explicit bounds for the bassin of attraction.



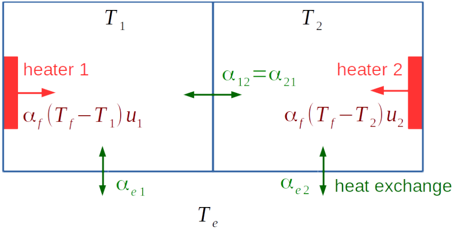
Outline

Example: Two-room apartment



$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \end{pmatrix} = \begin{pmatrix} -\alpha_{21} - \alpha_{e1} - \alpha_f u_1 & \alpha_{21} \\ \alpha_{12} & -\alpha_{12} - \alpha_{e2} - \alpha_f u_2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + \begin{pmatrix} \alpha_{e1} T_e + \alpha_f T_f u_1 \\ \alpha_{e2} T_e + \alpha_f T_f u_2 \end{pmatrix}.$$

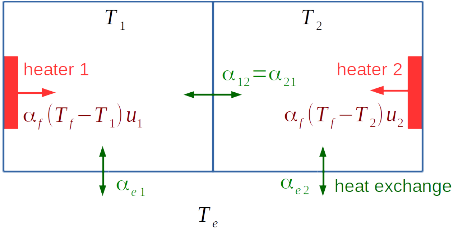
Example: Two-room apartment



$$\begin{pmatrix}\dot{T}_1\\ \dot{T}_2\end{pmatrix} = \begin{pmatrix}-\alpha_{21}-\alpha_{e1}-\alpha_f u_1 & \alpha_{21}\\ \alpha_{12} & -\alpha_{12}-\alpha_{e2}-\alpha_f u_2\end{pmatrix}\begin{pmatrix}T_1\\ T_2\end{pmatrix}+\begin{pmatrix}\alpha_{e1}T_e+\alpha_f T_f u_1\\ \alpha_{e2}T_e+\alpha_f T_f u_2\end{pmatrix}.$$

- Modes: $\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}$; sampling period τ

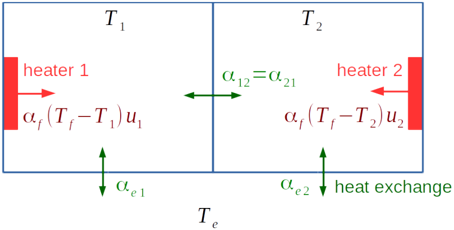
Example: Two-room apartment



$$\begin{pmatrix}\dot{T}_1\\ \dot{T}_2\end{pmatrix}=\begin{pmatrix}-\alpha_{21}-\alpha_{e1}-\alpha_f u_1 & \alpha_{21}\\ \alpha_{12} & -\alpha_{12}-\alpha_{e2}-\alpha_f u_2\end{pmatrix}\begin{pmatrix}T_1\\ T_2\end{pmatrix}+\begin{pmatrix}\alpha_{e1}T_e+\alpha_f T_f u_1\\ \alpha_{e2}T_e+\alpha_f T_f u_2\end{pmatrix}.$$

- Modes: $\begin{pmatrix}u_1\\ u_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix},\begin{pmatrix}0\\ 1\end{pmatrix},\begin{pmatrix}1\\ 0\end{pmatrix},\begin{pmatrix}1\\ 1\end{pmatrix}$; sampling period τ
- A pattern π is a finite sequence of modes, e.g. $\left(\begin{pmatrix}0\\ 1\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\end{pmatrix}\cdot\begin{pmatrix}1\\ 1\end{pmatrix}\right)$

Example: Two-room apartment



$$\begin{pmatrix}\dot{T}_1\\ T_2\end{pmatrix} = \begin{pmatrix}-\alpha_{21}-\alpha_{e1}-\alpha_f u_1 & \alpha_{21}\\ \alpha_{12} & -\alpha_{12}-\alpha_{e2}-\alpha_f u_2\end{pmatrix}\begin{pmatrix}T_1\\ T_2\end{pmatrix} + \begin{pmatrix}\alpha_{e1}T_e+\alpha_f T_f u_1\\ \alpha_{e2}T_e+\alpha_f T_f u_2\end{pmatrix}.$$

- Modes: $\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, \begin{pmatrix}0\\ 1\end{pmatrix}, \begin{pmatrix}1\\ 0\end{pmatrix}, \begin{pmatrix}1\\ 1\end{pmatrix}$; sampling period τ
- A pattern π is a finite sequence of modes, e.g. $\left(\begin{pmatrix}0\\ 1\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\end{pmatrix}\cdot\begin{pmatrix}1\\ 1\end{pmatrix}\right)$
- A state dependent control consists in selecting at each τ a mode (or a pattern) according to the current value of the state.

(R, S)-stability property for the two-room apartment

Input:

- R, S
- an integer K (maximal length of patterns)

Output: controlled covering of R (each covering set is coupled with a pattern)

Guaranteed properties: (R, S)-stability

(R, S)-stability property for the two-room apartment

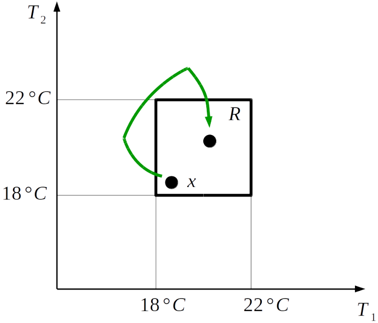
Input:

- R, S
- an integer K (maximal length of patterns)

Output: controlled covering of R (each covering set is coupled with a pattern)

Guaranteed properties: (R, S)-stability

- Recurrence in R: after some ($\leq K$) steps of time, the temperature returns in R



(R, S)-stability property for the two-room apartment

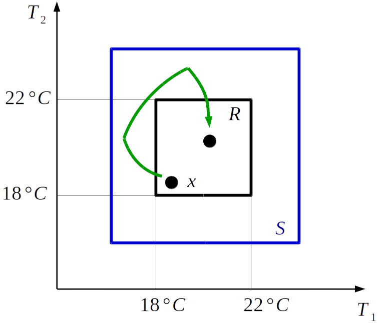
Input:

- R, S
- an integer K (maximal length of patterns)

Output: controlled covering of R (each covering set is coupled with a pattern)

Guaranteed properties: (R, S)-stability

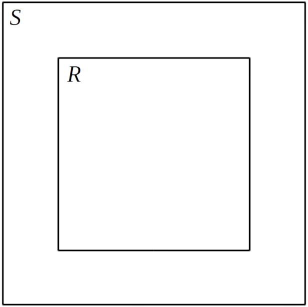
- Recurrence in R: after some ($\leq K$) steps of time, the temperature returns in R
- Safety in S: $x(t)$ always stays in S.



Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

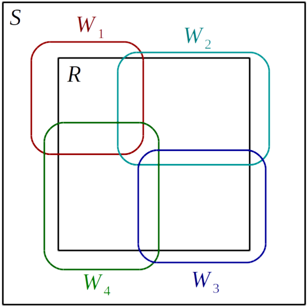
Goal: from any $x \in R$, return in R while always staying in S.



Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

Goal: from any $x \in R$, return in R while always staying in S.



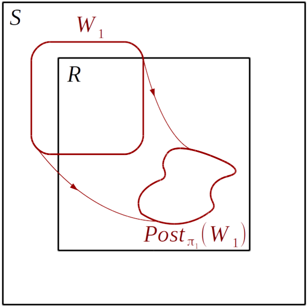
Basic idea:

- Generate a covering of R

Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

Goal: from any $x \in R$, return in R while always staying in S.



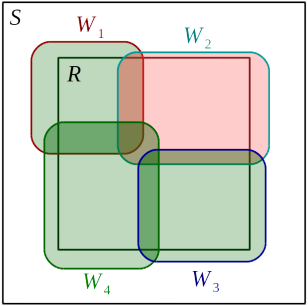
Basic idea:

- Generate a covering of R
- Look for patterns (input sequences) mapping the tiles into R while always staying in S

Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

Goal: from any $x \in R$, return in R while always staying in S.



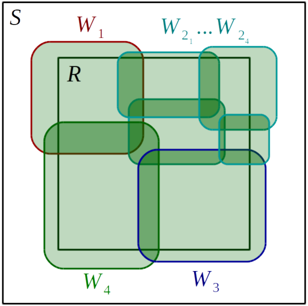
Basic idea:

- Generate a covering of R
- Look for patterns (input sequences) mapping the tiles into R while always staying in S
- If it fails,

Control tiling procedure

$$\dot{x}(t) = f_{\sigma(t)}(x(t), d(t))$$

Goal: from any $x \in R$, return in R while always staying in S.

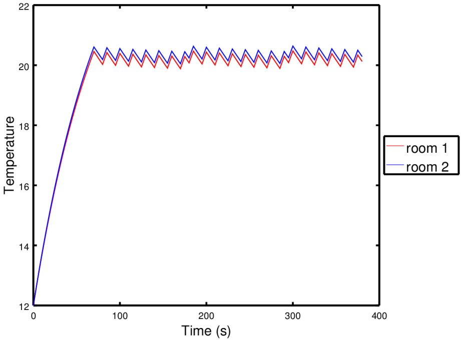


Basic idea:

- Generate a covering of R
- Look for patterns (input sequences) mapping the tiles into R while always staying in S
- If it fails, generate another covering.

Application to the two-room appartment

Periodic behaviour (limit cycle):



NB: method designed for sampled switched systems
Finite number of modes (ex: heater 1: ON and heater 2: OFF)
The control makes the mode change (switch) every τ seconds (sampling period)

Outline

Extension of the method for the biped model

- No sampling every τ seconds: the change of modes is done when an event occurs: impact with the floor ($\theta_1 + \theta_2 = 0$)
- The mode is now chosen by selecting an appropriate setpoint (constant θ_{SP}) for the expected length of the forthcoming footstep
- The actuator is a classical PD-control (proportional derivative: $u = -Kp(\theta - \theta_{SP}) - K_d \frac{d\theta}{dt}$)
- The reset of the state on impact has to be taken into account

Linearization of the dynamics

Swing phase dynamics:

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + G(\theta) = B\left(-Kp(\theta - \theta_{SP}) - K_d\dot{\theta}\right)$$

Linearization of the dynamics

Swing phase dynamics:

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + G(\theta) = B\left(-Kp(\theta - \theta_{SP}) - K_d\dot{\theta}\right)$$

with

$$M(\theta) = \begin{pmatrix} M_{11}, M_{12}, M_{13} \\ M_{21}, M_{22}, M_{23} \\ M_{31}, M_{32}, M_{33} \end{pmatrix}$$

$$N(\theta, \dot{\theta}) = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

$$G(\theta) = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix}$$

Linearization of the dynamics

Swing phase dynamics:

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + G(\theta) = B\left(-Kp(\theta - \theta_{SP}) - K_d\dot{\theta}\right)$$

and

$M_{11} = (m_u + m_h + m_l)(l_a + l_b)^2 + m_l l_a^2$,
$M_{12} = M_{21} = -m_l(l_a + l_b)l_b cos(\theta_1 - \theta_2)$,
$M_{13} = M_{31} = m_u(l_a + l_b)l_u cos(\theta_1 - \theta_3)$, $M_{22} = m_l l_b^2$, $M_{23} = M_{32} = 0$,
$M_{33} = m_u l_u^2$;
$N_1 = -m_l(l_a + l_b)l_b sin(\theta_1 - \theta_2)\dot{\theta}_2^2 + m_u(l_a + l_b)l_u sin(\theta_1 - \theta_3)\dot{\theta}_3^2$,
$N_2 = m_l(l_a + l_b)l_b sin(\theta_1 - \theta_2)\dot{\theta}_1^2$, $N_3 = -m_u(l_a + l_b)l_u sin(\theta_1 - \theta_3)\dot{\theta}_1^2$;
$G_1 = -((m_h + m_l + m_u)(l_a + l_b) + m_l l_a)g sin(\theta_1)$, $G_2 = m_l l_b g sin(\theta_2)$,
$G_3 = -m_u l_u g sin(\theta_3)$.

Linearization of the dynamics

Let us take $G_1 = -((m_h + m_l + m_u)(l_a + l_b) + m_l l_a) g sin(\theta_1)$.

It can be written as $G_1 = G_1^\star sin(\theta_1)$.

It actually verifies: $G_1 = G_1^\star \theta_1 + d_1^G$ with $|d_1^G| \leq \delta_i^G$ with

$$\delta_i^G := \frac{|G_1^\star|}{6} |\theta_1|^3_{max}$$

G_1 can actually be written as a perturbed linear equation.
Do this for all the nonlinear parameters and...

Linearization of the dynamics

and...

$$M(\theta)\ddot{\theta} + N(\theta,\dot{\theta}) + G(\theta) = B\left(-Kp(\theta-\theta_{SP}) - K_d\dot{\theta}\right)$$

becomes

$$\dot{x} = Ax + \theta_{SP} b + Hd$$

with $x = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \theta_1, \theta_2, \theta_3)^\top$

$$A = \begin{pmatrix} \left(M^{\star-1}B^d\right) & \left(M^{\star-1}(-G^\star + B^p)\right) \\ \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} & \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \end{pmatrix}$$

$$H = \ldots$$

and $d \in D$

Reachability computation

A *zonotope* is a set:

$$Z = < c, G > = \{x \in \mathbb{R}^n : x = c + \sum_{i=1}^{p} \beta^{(i)} g^{(i)},\ -1 \leq \beta^{(i)} \leq 1\}$$

Given the dynamics

$$\dot{x} = Ax + b + Hd$$

with $d \in D$, and an initial zonotope $Z = < c, G >$,
The image of Z at time $t + \tau$ is included in:

$$Z' = < A^d c + b^d, A^d G + A^d H^d >$$

where $A^d = e^{A\tau}$ and $b^d = \int_0^\tau e^{A(t-\tau)} dt$. $H^d = \begin{pmatrix} \epsilon_1 & 0 & \dots & 0 \\ 0 & \epsilon_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \epsilon_n \end{pmatrix}$ and

for all $i = 1, \dots, n$:

$$\epsilon_i = \max_{x \in D} |Hx|_i$$

Reachability computation

Linear system with perturbation :

Antoine Girard. *Reachability of uncertain linear systems using zonotopes*. International Workshop on Hybrid Systems: Computation and Control, 2005.

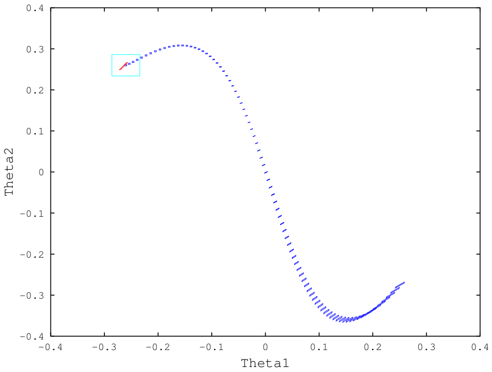
Linearization of the reset similarly as for the swing phase and take the reset/guard into account with:

Antoine Girard, Colas Le Guernic. *Zonotope/hyperplane intersection for hybrid systems reachability analysis*. International Workshop on Hybrid Systems: Computation and Control, 2008.

Outline

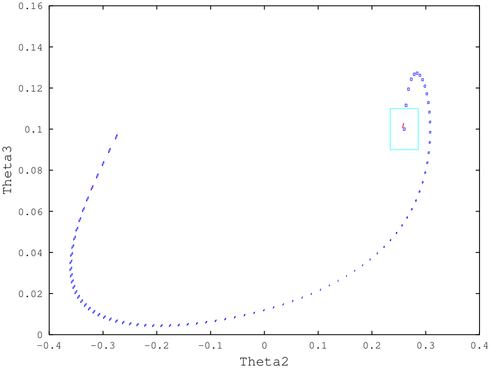
Numerical results

With the parameters: $K_p = 124.675$, $K_d = 19.25$, we get:



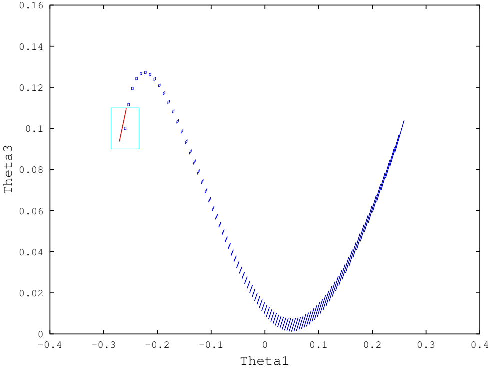
Numerical results

With the parameters: $K_p = 124.675$, $K_d = 19.25$, we get:



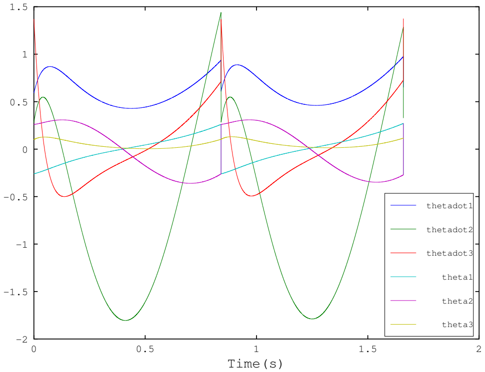
Numerical results

With the parameters: $K_p = 124.675$, $K_d = 19.25$, we get:



Numerical results

With the parameters: $K_p = 124.675$, $K_d = 19.25$, we get:



Conclusions and future work

Advantages of the method w.r.t. HZD:

- Simpler (no need for Poincaré map, additional nullifying actuators, change of coordinates)
- Conventional control (Proportional-derivative)
- Explicit bounds for the bassin of attraction and the zone of recurrence (rectangle R)

Future work:

- Application to more sophisticated biped models (here, only 6-state model vs. 32-state model for classical biped robot)
- Extension to other models with impact (e.g., mechanical arm grasping objects)
- Combination of the method with model reduction and/or compositional analysis