Chapitre 7

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The pioneering work of Vickrey (1939, 1947) on the income-tax frequency has not inspired a lot of further studies. We propose an investigation of this issue both from a theoretical and empirical viewpoint within the year. We first show that increasing the income-tax frequency (from an annual tax basis to a monthly tax basis) and refunding the extra-revenue for the taxpayer is Pareto-improving for any convex tax scheme (increasing marginal tax) and any risk-adverter. Welfare gains are all the larger as the infra-annual volatility of income is large and the possibility to borrow is low, implying that the benefits should be larger for the bottom part of the income distribution. We submit an empirical illustration using French administrative tax data and simulations of the current tax-benefit system. Despite that this system is not convex over the whole income domain, we show that increasing the tax frequency can lead to substantial social welfare gains. However the average welfare gain is lower with the proposed system than with the Vickrey's one even though there are less losers.

7.1 Introduction

Time and risk are difficult matters and the design of tax system does not make an exception when it deals with incomes which are not steady. Usually, the practionners are applying some methods which are both understandable by the layman and easy to implement. For instance, in withholding income tax systems, tax collection is based on monthly income while tax schedules are designed over yearly income, and filing at the end of the year allows using tax allowances to obtain rebates. Benefits also have specific timing of income assessment (for instance in France, earnings of the three past months form the basis of means-tests for social assistance and in-work benefits). Effective taxation then possibly treats various time periods and, hence, different streams of income differently. In this paper, we question the welfare implications of varying the period of observation of the tax base within a year.

Arguably, our question would be irrelevant if people had constant income flows over time. Yet it is clear that many households do face important income variation during the year, either expected (pay rise, seasonal work) or unexpected (income shocks due to business failure or unemployment), voluntary (chosen retirement) or involuntary (low-skill workers forced into temporary work). This is a major motivation for studying the question of within-year tax frequency and its impact on tax-payer utility. Note also that our question would be meaningless if effective taxation was flat. However, this never happens. Effective taxation, i.e. the impact of taxes and benefits, is usually nonlinear. Even in flat-tax countries, the presence of many other instruments, e.g. means-tested benefits, impose some regressivity or progressivity upon the effective tax schedule.

Interestingly, and potentially important for our welfare analysis, the question of tax frequency may be more relevant at the bottom of the distribution. Indeed, for high income households, large income variations are usually required to change tax brackets (especially given the global trend that has consisted in reducing the number of brackets, Peter et al. (2010, Fig. 3, p.468)). For them, this is as if they faced a marginal flat tax around the pre-shock income level. In the lower part of the distribution, however, effective brackets are narrower because of the numerous (and often overlapping) redistributive schemes at low and middle income levels (e.g. social assistance, tax credits, means-tested family benefits, etc.). Thus, even small variations of income may lead to change in effective marginal tax rates (MTR). There is also empirical evidence that this population experience more income fluctuations, and that income variation is more often involuntary, e.g. income shocks due to unemployment (Hills et al., 2006). These aspects are likely to contribute much to the welfare implications of tax frequency that we shall study. [1]

Strangely enough, the question of tax temporality has received little attention in the economic literature. Very few studies make the link between tax-payer utility and tax temporality. The seminal work of Vickrey (1939) focuses on yearly versus lifetime taxation. Vickrey develops the horizontal equity argument that over the lifecycle, people earning the same income should pay the same tax. This leads William Vickrey to propose the celebrated "Cumulative Averaging" tax principle. Initially, the idea was formulated for taxing capital gains and then extended to other types of income. The idea was that an investor should have no incentive to realize her capital gain at one time rather than another on account of the tax. The upshot is that the tax should be completly neutral with respect to the time at which the gain is realized [2].

The present research is devoted to the temporality of the tax within a year. Most developed countries operate a wage withholding tax system [3] which represents the benchmark system in our reasoning. As already mentioned, the amount withheld and paid by the employer to the government each month is applied as a prepayment of income taxes and is refundable if it exceeds the income tax liability. The tax liability is determined by filing the tax return at the end of the year and applying the tax schedule including tax deductions. The current system is far remote from applying a cumulative averaging system à la Vickrey. The first idea which comes out is then to apply the cumulative average proposition.

Three ingredients are needed to compute the tax liabilities according to the cumulative averaging formula : a starting date, a termination date and a tax schedule appropriate to the period covered. The idea of a termination date raises some difficulty for applying the principle over the life cycle because the time of death is uncertain. This difficulty vanishes when applying the mechanism within a year. Each month the taxpayer is paying a withhold tax. This tax is computed as the difference between the theoretical tax liability

1. Note also that poorer people are less able to smooth income because lower resources limit their saving possibilities (Dynan et al. (2004) finds a median saving rate of 1 percent for households in the bottom quintile for US households aged 30-59). In addition, they are often more credit constrained and possibly more myopic (cf. Hastings and Washington, 2010).

2. Vickrey's contribution has triggered important advances in capital taxation (Auerbach, CITE) and dynamic optimal taxation (CITE). An interesting extension of Vickrey's setting is Liebman (2002). He studies a distributionally-neutral Vickrey tax system that would equalize MTRs both over-time for the same individual and across individuals with the same lifetime earnings level, incorporating various motives for efficiency and equity gains (income shocks, myopic agents). Finally, note that the temporality of taxation has also received some attention in law studies following Vickrey, for instance Batchelder (2003) and Fennell and Stark (2005).

3. For capital incomes and earnings of self-employed, the tax payment usually remains on a yearly basis.

of the subperiod running until that month minus the cumulated withheld taxes of all previous months. Two mathematical operations are performed to compute the former. First, the taxbase is computed on a annual basis by assuming that the monthly average income since the beginning of the year will prevail until the end of year. Second, the tax liability is computed on a pro rata temporis basis. That is, the theoretical tax liability for the five first months in May is only the 5/12 of the annual tax liability.

The extension of Vickrey's idea on earnings is not without dispute. Two arguments can be stressed. From a more philosophical view point, we can remind Derek Parfit's provocative stance : *The difference between different persons at the same time is more like the difference between the same person at different times*. That difference in income over time should be taken as a difference in ability to contribute over each period of time. A sport superstar who is able to earn a lot of money over a period of 10 years is a different man from what he will become years after. It is very in line with the static Mirrlees optimal tax model with the efficiency equity tradeoff. Vickrey's tax scheme looks at equity over the life cycle, but if Derek Parfit view is embodied, equity is sacrified for efficiency.

Second it can be emphasized that in many ways human capital is different from financial capital. If the capital market is liquid, you can sell your asset when you want. The labor market is less liquid at least in a given year than the capital market. In many places and many times, for different reasons that we do not develop, the wage does not clear. It means that from the view point of an individual the variability of earnings within a year reflect not only preferences but also features on the local labor market demand as aforementioned. We name the latter the unvoluntary variability. In that case, neutrality with respect to time is not the only property we may want to see respected by the income tax payment. Because of risk aversion, tax smoothing is arguably a property that will come high on the list. The cumulative averaging tax schedule is smoothing the payment of the tax within a year. However it is not the only one and we are here investigating other mechanisms.

In particular, there is a main difference between the framework that we deal with and Vickrey's one. The big strength of the cumulative average formula is each month payment to only depend on the available information. Each month, there is a kind of bayesian revision of what the taxpayer owes, by incorporating the earning information that is disclosed that month. We here adopt a perfect information framework for tax authorities. Namely, the tax authority knows the within-year income stream of the taxpayer from the beginning of the year. This is of course unrealistic, even if there are (econometric) means for the tax authority to forecast the variability of income that prevails for different types of taxpayer. We will comment on that in the conclusion. However we claim that at least from a theoretical perspective and maybe more than that, it is always interesting to compare what we could do in a context of perfect information and what we can do in a context of available information. The difference represents the cost of uncertainty.

In contrast, instead of averaging the income basis of each month, we consider the opposite direction which is to increase the frequency of the tax within the year,empirically illustrated with a move from annual to monthly taxation. The tax liability will be monthly based. If the tax scheme is convex (marginally progressive), the taxpayer will pay more with a monthly tax liability than with an annual tax liability. We show that it is possible to refund each taxpayer the extra amount of taxes and any risk-adverter tax payer will be happier with this system termed the *Monthly tax frequency with compensation,* in short, *Monthly compensated*. We should immediatly stress that our proposal remains neutral with respect to the timing of income within the year. Besides neutrality, there is the issue of welfare gains which depend on the convexity of the tax schedule and the concavity of the utility function, two features that are absent from Vickrey's paradigm. We are more specific but we do not deviate from usual assumptions considered in economic theory.

To the best of our knowledge, we are the first to study the welfare impact of tax frequency within the year, in particular the implications of a shift from annual to monthly taxation. We first suggest a simple theoretical framework in which Jensen's inequality implies that a taxpayer with irregular income streams will pay more with a monthly tax scheme. We derive compensation schemes for the increase in tax liability due to income variation, i.e. compensations equating the amount of tax paid in both annual and monthly systems. We show that the reform with compensation is Pareto improving if utility is increasing, concave and intertemporally additive. [4] We characterize those most concerned by welfare gains, namely those whose incomes vary much over time and who face borrowing constraints.

We then submit an empirical illustration using French administrative data (tax filing data combined with work sequences from the French Labor Force surveys (*Enquête Revenus Fiscaux et sociaux ERFS*). Using simulations of the current income tax, we implement the higher-frequency tax reform and characterize winners and losers. We extend this empirical exercise to effective taxation, i.e. we additionally account for the main redistributive

4. At the center of our theoretical results, Varian (1980) emphasizes the insurance property of an income tax and shows in a two-period model that this insurance component leads to a convex optimal tax scheme.

scheme in France. While our Pareto improving results require the income tax to be convex, real-world effective taxation is rarely so. Moreover, non-convex parts of the effective schedule are typically located in the first half of the income distribution, with the cumulative effect of different instruments (tax credit and benefit withdrawals, social contributions and income tax, etc). Despite this feature, which is particularly present in the case of French taxation, we show that increased tax frequency with compensation leads to substantial welfare gains.

The article is organized as follows. Section 2 presents our theoretical framework and results. Section 3 shows descriptive statistics on France taxpayers earnings. Section 4 submits simulation of a tax frequency reform on French data while section 5 concludes.

7.2 Theory

The assessment of the tax determines the amount an individual is entitled to pay based on its situation (income streams, family situation, etc). The determined amount known as tax liability leads to a payment of the tax either directly by the taxpayer, or by a third party institution (employer, banks, etc). Appart from the frequency of the assessment of the tax, it can leads to different schedule of payments. In practice in countries with Pay-as-you-earn (PAYE) systems, an amount is withholded at each payroll period which can be viewed as sub-periods of taxation, even though in most countries final tax liability is computed over whole year incomes. Meaning that tax liability is based on yearly income, but tax payments are made before the full realisation of incomes that constitutes the tax base.

Hence, there exists a *reference period* for the tax base in order to determine th teax liability. In cases the government wish to recover taxes before full realisation of income, it may want to apply some rules on sub-periods to compute *sub-period tax liability* based on income, concomitant with the curent sub-period, or based on past income streams.

Those cumulative payments may lead to some differences with respect to tax liability and may imply some implicit lending or borrowing between the taxpayer and the state : in the US, fiscal household has to fill a tax return in April for last year income to pay (or get refunded) the difference between the withholded amount and the tax liability. France is moving slowly to a withholding system which is due to be implemented in 2019.

We first define the framework, then expose five tax frequency regimes before presenting some theoretical results about the comparison of the well-beings they generate. We end the section by discussing the external validity of the results when we embody behavioral reactions.

7.2.1 Framework

Let us consider a reference period T and sub-periods t from 1 to T. If we take the year length as the reference period and months as the single periods, then T=12, and each month is labeled from 1 to 12. There are twelve sub-periods corresponding to the first month, the two first months, the first quarter and so on. We denote y_t the gross income the taxpayer at period $t = 1, ..., T$ with $y_t \geq 0$. We call $G(.)$ the reference tax schedule, that is the tax schedule for the reference period which is supposed to be increasing and convex. G can be positive (taxes) or negative (benefits). We first study the case of positive taxes and then indicates how the analysis can be extended to the case of a negative income tax. z_t is the disposable income or the net-of-tax income.

We define G_t the *pro rata temporis* single period tax schedule, or in short, the sub-period equivalent tax schedule where $G_t = \frac{1}{T}G(Ty_t)$. The tax liability of period t is denoted g_t. In our proposal, sub-period equivalent tax function only depends of the sub-period income. We should note that for equal income at each sub-periods the reference income tax is just equal to the sum of the sub-period income tax, formally :

$$\sum_{1}^{T} \frac{1}{T} G(Ty) = G(\sum_{1}^{T} y), \forall y \in \mathbb{R}^{+} \tag{7.1}$$

We consider that tax-payers have additive/separable utility functions defined on disposable income :

$$U = \sum_{t=1}^{T} u(z_t)\,, \tag{7.2}$$

where U is the reference period global utility function and u_t is the increasing and concave sub-period utility function of disposable income. $u(.)$ can be viewed as the indirect utility function at constant wage and price.

At this stage, we do not include time discounting or interest rate in this framework. These extensions are discussed in the discussion subsection as well as behavioral responses to the change in taxation frequency.

We recall the well-known mathematical result about partial sums and dominance for a convex or concave function which will be useful in deriving results.

Theorem 1. *Let x and y two income streams $x = (x_1, ..., x_n)$ and $y = (y_1, ..., y_n)$, ranked in decreasing order, i.e., $x_1 \geq x_2 \geq \cdots \geq x_n$ and $y_1 \geq y_2 \geq \cdots \geq y_n$ such that $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$ and $\sum_{i=1}^k x_i \geq \sum_{i=1}^k y_i$ for $k = 1, ...n-1$. Then $\sum_{i=1}^n f(x_t) \geq \sum_{i=1}^n f(y_t)$ for any f convex function.*

The version of the theorem for a concave function holds for x and y two income streams ranked in increasing orders. The obtained ranking of income distributions corresponds then to the Lorenz ranking of income distributions.

7.2.2 Five within-year tax payments

Current payment In the US [5] and in many countries, income tax is withholded by the employer based solely on monthly income and monthly situation (family structure). At the end of the year a tax adjustment is proceeded to make the total annual tax liability exactly given by the reference period tax schedule.

$$g_t^C = \begin{cases} \frac{G(Ty_t)}{T}, & \text{if } t \in [1, 11] \\ G(\sum_1^{12} y_t) - \sum_1^{11} \frac{G(Ty_t)}{T}, & \text{if } x = 12. \end{cases}$$

Cumulated Averaging Tax Vickrey's proposal was that each sub-period payment should be equal to the amount one would have paid under the sub-period tax schedule if the total income had been earned in equal amounts in each single period composing the sub-period. Without interest rate it leads to :

$$g_t^V = \frac{t}{T} G\left(T \frac{\sum_{p=0}^t y_p}{t}\right) - \sum_{p=0}^{t-1} g_p^V \tag{7.3}$$

Where g_t^V is the tax paid for a given single period t with Vickrey's tax scheme. On a year span, the taxpayer will have paid exactly the same amount as in the current system $G(\sum_1^{12} y_t)$. We can thus implement a Vickrey's tax scheme on a yearly span – instead of a lifetime span. This is indeed the solutions that seems to be chosen by the Irish PAYE system. [6]

5. https://www.irs.gov/publications/p505/index.html
https://www.irs.gov/pub/irs-pdf/p505.pdf
In fact, the adjustement is made in April after the taxpayer has filled his tax return.
6. http://www.revenue.ie/en/business/paye/guide/employers-guide-paye-calculation.html

Equal Payment Regime In this system, the amount of the tax is based over income of the reference period, $G(\sum_1^T y_t)$. The tax is split equally between all the single periods. If we denote g_t^E the equal monthly payment made at period t, then :

$$g_t^E = \frac{G(\sum_1^T y_t)}{T}$$

As a matter of fact, the Equal Payment Tax is not implementable since the government has to know future income before their realisation. It can still be used as a benchmark. Another limitation is that the monthly income should be higher than the average tax liability, a constraint that can be viewed as severe.

Monthly Tax The monthly tax is the current tax without adjustment at the end of the year

$$g_t^M = \frac{G(Ty_t)}{T}.$$

With a strictly convex tax function those with varying income will pay more tax over the reference period with the montly tax than with the tax on the reference period.

$$\sum_1^T \frac{G(Ty_t)}{T} \geq G(\sum_1^T y_t))$$

Concerning the utility of the tax-payer, its single period utility might be lower or higher with the monthly tax than with the equal payment tax. Two opposite effects are in play : the first one is that more tax is paid with the sub-period tax, which reduces overall income, and thus overall utility. But on the other hand, the fact that the tax is directly linked to the single period income is smoothing the disposable income with a convex tax scheme. The taxpayer pays a smaller tax when earning little and a bigger tax when earning a lot. A risk adverter will like the reduced risk. On the whole, the effect is ambiguous with a mean loss and a reduced variance. The effect depends thus on the shapes of the tax and of the utility function, but also on the different income streams and their dispersion around the annual mean.

Monthly Compensated Tax It is easy to equalize total tax assessement between the equal payment tax and the sub-period tax. We just need to define the parameter λ which will decrease the single period tax schedule and wipe out the extra-tax collected with a monthly taxation scheme.

$$\sum_{1}^{T} \frac{1}{T+\lambda} G(Ty_t) = G\left(\sum_{1}^{T} y_t\right). \tag{7.4}$$

λ is a function of both the convexity of the tax function and the variability of income over the year. It is positive or null.

$$\lambda(y, G) = \frac{\sum_1^T G(Ty_t)}{G\left(\sum_1^T y_t\right)} - T.$$

The uncertainty about the income stream should be resolved to compute the parameter λ. Observe also that the value of λ is specific to the income realization of a given taxpayer. We will come back to that point on the discussion sub-section. The tax paid for the monthly compensated tax (MC) is then :

$$g_t^{MC} = \frac{1}{T+\lambda} G(Ty_t)$$

Figure 1 illustates that the compensated income tax represented by (yellow) stars leads to small absolute reduction for the low income sub-period, and a big absolute reduction for the high income sub-period.
We must observe that except the monthly equal tax, the total amount paid at the end of the year to the treasury is the same for the other four tax payments. The neutrality property essential to Vickrey is then satisfied by the four taxation schemes. We must find other criteria to decide between them. The well-being associated to them seems at least an important criterion.

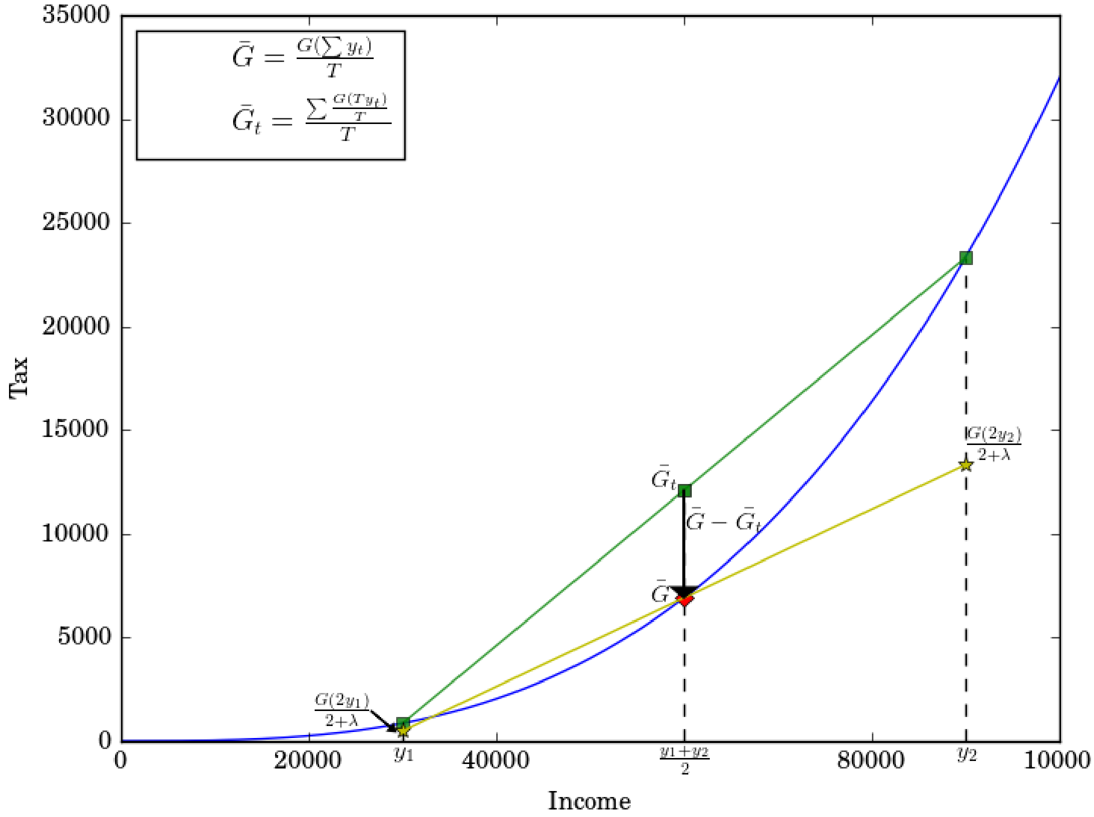
7.2.3 Comparison of well-being for the tax regimes

We first show that the annual well-being is higher with the monthly compensated scheme than with the equal monthly scheme. The current system is also dominated by the monthly compensated scheme. We end up with a comparison with the Vickrey's proposal by means of simple examples.

Monthly compensated versus equal payment

Theorem 1. *If the subutility is concave and the underlying annual tax schedule is convex, the intertemporal utility of the taxpayer over the yearly span is higher with a monthly*

Figure 7.1 – Monthly Compensated tax versus Monthly tax



compensated scheme than with an equal split scheme.

$$\sum_{1}^{T} u_t \left(y_t - \frac{G\left(\sum y_t\right)}{T} \right) \leq \sum_{1}^{T} u_t \left(y_t - \frac{G(Ty_t)}{T+\lambda} \right) \tag{7.5}$$

Démonstration. We check that the conditions of the dominance theorem are satisfied. Clearly because both systems are neutral to the earning timing within the year, the sum of the monthly disposal incomes is the same with the two systems. Now we have to check that the partial sums of monthly tax amounts are lower with the monthly compensated scheme than with the equal payment regime, implying that we have the reverse relation for disposable incomes. It amounts to compare for the first k months which are reranked according to increasing gross income order that :

$$\sum_{t=1}^{k} \frac{1}{T+\lambda} G(Ty_t) \leq \frac{kG(T\bar{y})}{T}$$

with $\bar{y}$ the monthly mean. By definition

$$T + \lambda = \frac{\sum_1^T G(Ty_t)}{G(T\bar{y})}$$

Substituting the value of $T + \lambda$ into the above expression we obtain

$$\frac{\sum_{t=1}^{k} G(Ty_t)}{\sum_1^T G(Ty_t)} G(T\bar{y}) \leq \frac{kG(T\bar{y})}{T}$$

which simplifies in

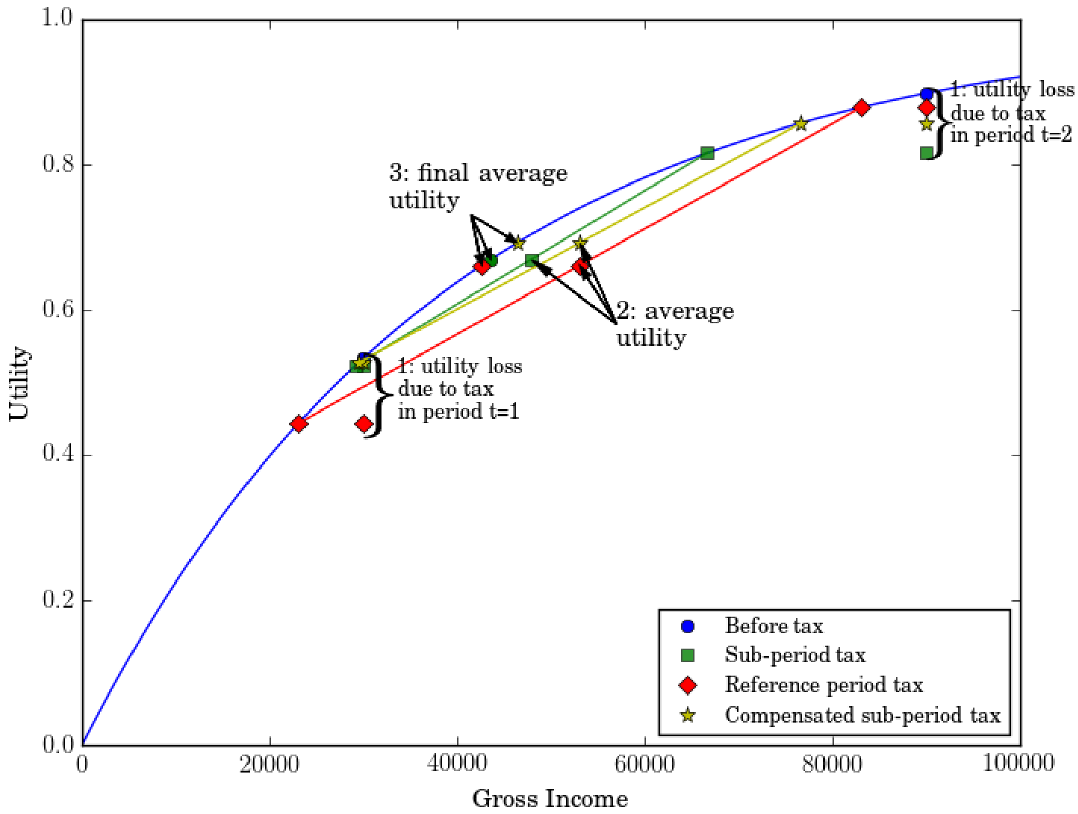
$$\frac{1}{k} \sum_{t=1}^{k} G(Ty_t) \leq \frac{1}{T} \sum_{1}^{T} G(Ty_t)$$

which must be true because the partial average of k increasing numbers is always lower than the total average of the same T numbers with $T > k$. □

The proof shows that the result is indeed stronger than the statement of the theorem suggests. It indicates for instance that if the sequence of income is increasing over the year, then all partial intertemporal utility streams covering any sub-period starting from January are higher for the monthly compensated tax scheme than for the equal payment regime. The taxpayer is happier for any subperiod considered, the first k months from $k = 1, .., 12$.

The general idea illustrated by Figure 7.2 is that the compensation takes out the potential negative effect of paying more taxes with a monthly tax base and the only remaining effect

FIGURE 7.2 – Utility changes with the Monthly compensating scheme



is the income smoothing.

Monthly compensated versus current system For reasons that will appear in a transparent way in the next subsection, we will compare both tax payments when the sequence of gross incomes is increasing within the year. It should be the case for many full-time employees with year-end bonuses.

So let us consider the case where $y_1 \leq y_2 \leq, ..., \leq y_{12}$.

Claim 2. *The monthly compensated system brings out higher welfare than the current system for every subperiod starting from the first month, including the full year. It brings also higher welfare for every month except may be December.*

The statement follows from the fact that $\lambda > 0$ and the mathematical theorem.

Vickrey versus Monthly compensated The Vickrey mechanism is backward looking. The monthly compensated is taking advantage of knowing the full income stream. It is a bit unfair in some sense to compare them since we compare a perfect information mechanism with a mechanism which relies upon the updated information at each month. Nevertheless, it is still interesting to understand in which situations the Vickrey mechanism performs badly. We are considering simple cases which allow us to develop the intuition for more complex situations. Comparing the two mechanisms for more general income streams is not that simple since Vickrey involves substractions and the monthly compensated scheme divisions. This makes the comparision less trackable. There are cases where we cannot conclude from a theoretical view point.

Two sub-periods We suppose that there exists two subperiods, 1 and $T = 2$. Either (first case) $y_1 > y_2$ or (second case) $y_1 < y_2$. In both cases, $g_1^{MC} < g_1^V$ implying that $g_2^{MC} > g_2^V$ because the cumulated taxes are the same. It suffices then to build on the mathematical result to state that :

Claim 3. *In case of an increasing income stream within the year, there will be a welfare gain at each subperiod starting from the first month including the full year to switch from the Vickrey mechanism to the monthly compensated one. In case of a decreasing income stream, the opposite statement holds.*

Three sub-periods Involving a third period in the framework introduces some intricacies. We are able to compare both mechanisms by building upon the two-period case and considering a variable income in the third and last period $T = 3$. We begin with generalizing the previous claim about monotone income streams.

Monotone income streams

Case 1 Increasing income streams $y_1 = y_2 < y_3$

We can think this situation as featuring a year-end bonus following a steady income over the previous months.

The idea is always the same : to check that the conditions of the mathematical theorem holds.

It is still obvious that $g_1^{MC} < g_1^V$.

We now have to look at the second partial sum and we prove that $g_1^{MC} + g_2^{MC} < g_1^V + g_2^V$

By definition $g_1^{MC} + g_2^{MC} = \frac{1}{3+\lambda}[G(3y_1) + G(3y_2)] = \frac{[G(3y_1)+G(3y_2)]}{[G(3y_1)+G(3y_2)+G(3y_3)]} G(y_1 + y_2 + y_3)$

Using now the fact that $y_1 = y_2 = y$ we obtain

$g_1^{MC} + g_2^{MC} = \frac{2G(3y)}{[2G(3y)+G(3y_3)]} G(2y + y_3) = \frac{\frac{2}{3}G(3y)}{[\frac{2}{3}G(3y)+\frac{1}{3}G(3y_3)]} G(2y + y_3)$

On the other hand, $g_1^V + g_2^V = \frac{2}{3}G(\frac{3}{2}(y_1 + y_2)) = \frac{2}{3}G(3y)$

Therefore $g_1^{MC} + g_2^{MC} = \frac{g_1^V + g_2^V}{[g_1^V + g_2^V + \frac{1}{3}G(3y_3)]} G(2y + y_3)$

Then the issue of $g_1^{MC}+g_2^{MC} < g_1^V+g_2^V$ is equivalent to $\frac{g_1^V+g_2^V}{[g_1^V+g_2^V+\frac{1}{3}G(3y_3)]}G(2y+ y_3) < g_1^V+g_2^V$
Factorizing gives the following condition $(g_1^V+g_2^V)^2+(g_1^V+g_2^V)(\frac{1}{3}G(3y_3)-G(2y+ y_3) > 0$
A sufficient condition is : $\frac{1}{3}G(3y_3) > G(2y+ y_3)$.
For a given y, it is clear that due to the convexity of y_3, there exists a cutoff point y_3^* such that $G(3y_3) > 3G(2y+ y_3)$
The following claim makes stock of the above reasoning

Claim 4. *In case of an increasing income stream within the year, $y_1 = y_2 < y_3$, for y_3 big enough, there will be a welfare gain at each subperiod starting from the first month including the full year to switch from the Vickrey mechanism to the monthly compensated one. By continuity it is true for y_2 slightly larger than y_1.*

We shall keep in mind that we have the intuition that the result is more general than that but now on we do not have a more general proof.

Case 2 Decreasing income streams $y_1 > y_2 > y_3 = 0$
This situation features the case of an individual which is employed in the first part of the year, then become unemployed and receives the employment benefits to end the year with no income. The increasing ranking of income denoted $y_{[i]}$ is the opposite of the calendar ranking. That is $y_{[1]} = y_3, y_{[2]} = y_2$ and $y_{[3]} = y_1$.
It follows from the definitions that $g_{[3]}^{MC} < g_{[3]}^V$. We immediatly deduce that it must be true that $g_{[1]}^{MC} + g_{[2]}^{MC} > g_{[1]}^V + g_{[2]}^V$. To apply the mathematical theorem, we must also have that $g_{[1]}^{MC} < g_{[1]}^V$ that is, $g_3^{MC} > g_3^V$.
Be definition we know that $g_3^{MC} = 0$. We must prove that $g_3^V < 0$.
It goes back to the computation of $G(y_1+y_2)-\frac{2}{3}G(\frac{3}{2}(y_1+y_2))$ which amounts to compare $\frac{3}{2}$ and $\frac{G(\frac{3}{2}(y_1+y_2))}{G(y_1+y_2)}$
If we assume in addition that $G(0) = 0$, then convexity implies that $G(\frac{3}{2}(y_1 + y_2)) > \frac{3}{2}G(y_1 + y_2)$. We can then state

Claim 5. *Assume that $G(0) = 0$. In case of a decreasing income stream within the year, $y_1 > y_2 > y_3 = 0$ there will be a welfare gain at each subperiod starting from the first month including the full year to switch from the monthly compensated mechanism to the Vickrey one. By continuity it is true for y_3 close to 0.*

We have dealt with the case where y_3 is small or very large. It remains to deal with the case of y_3 is between y_1 and y_2. We will consider the case where $y_3 = \frac{y_1+y_2}{2}$. Indeed, it is an interesting case because it corresponds to the minimal value of λ as a function of y_3 for a given y_1 and y_2. It is when the correction brought by the monthly compensated mechanism is minimal with respect to the monthly mechanism.

Non-monotone profiles

We are going to deal with two profiles, case 1 where period 2 is the peak, $y_2 = Max(y_1, y_2)$, and case 2 where period 2 is the dip, $y_2 = Min\ (y_1, y_2)$.

Case 1 : Period 2 is the peak

Period 1 is then the worst-off period for the taxpayer. By definition, $g_1^{MC} < g_1^V$. The next poor period is period 3. Let us compare the tax payment for this period. $g_3^{MC} = \frac{1}{3+\lambda}[G(3y_3)] = \frac{1}{3+\lambda}[G(\frac{3}{2}(y_1 + y_2))]$

Now let us compute g_3^V. $g_3^V = G(y_1+y_2+y_3) - \frac{2}{3}G(\frac{3}{2}(y_1+y_2)) = G(\frac{3}{2}(y_1+y_2) - \frac{2}{3}G(\frac{3}{2}(y_1 + y_2)) = \frac{1}{3}[G(\frac{3}{2}(y_1 + y_2))]$

We conclude that $g_3^{MC} < g_3^V$ and therefore $g_1^{MC} + g_3^{MC} < g_1^V + g_3^V$. The second partial sum of taxes for monthly income ranked in increasing order is lower for Monthly compensated scheme than for Vickrey. We can then state

Claim 6. *When $y_1 < y_3 = \frac{y_1+y_2}{2} < y_2$, the taxpayer is better of with the Monthly compensated scheme than with the Vickrey mechanism and this is true for any subperiod starting from the first month including the full year.*

Case 2 : Period 2 is the worst-off period.

By definition, period 1 is the best-off period and it turns out that $g_1^{MC} < g_1^V$. We derive that $g_2^{MC} + g_3^{MC} > g_2^V + g_3^V$.

From the previous derivation of the above claim, we know that $g_3^{MC} < g_3^V$. Therefore, it must be that $g_2^{MC} > g_2^V$. Then the conditions of the theorem applies to the benefit of the Vickrey mechanism.

Claim 7. *When $y_2 < y_3 = \frac{y_1+y_2}{2} < y_1$, the taxpayer is better of with the Vickrey scheme than with the Monthly compensated mechanism and this is true for any subperiod starting from the first month including the full year.*

It appears that there is no global winner nor looser when comparing Vickrey and the Monthly compensated mechanism. The discussion below will pave the way to hint that the income configuration when the Monthly compensated mechanism dominates are those for which the taxpayer appears as the most constrained by the imperfect financial market.

7.2.4 Discussion

We will discuss five points. The first two are related to a less crude representation of preferences of individuals. They care about consumption and leisure and they adapt their behavior to the tax changes. We introduce them in the analysis in turn. We then discuss the negative income tax, the convexity assumption of the tax schedule and the implementation of the monthly compensated tax scheme.

Borrowing and saving If an individual can fully borrow or save within a year, then it is clear that what matters is the total disposable income over the year. The four systems are then equivalent in terms of consumer welfare and the discussion vanishes, except that we can say that with the monthly compensated schedule, the need of precautionary saving weakens since the profile of disposable income displays less variability than with the current system. Our results are worth it for individuals with no or very small saving. It is then a result that can be useful for developing countries and that matters much for poor households in rich and developed country. It is common for poor household that the beginning of the month is easier than the end because they have spent most of their wage or their social welfare benefits during the month (see Hastings and Washington (2010)). We can extend the validity of our results in a framework when the household can save but not borrow (or at a much higher rate). We will prove below that we are back to the framework that we have considered above. We will prove it on the basis of a simple example with two periods and a zero interest rate.

Let us call c_1 and c_2 the consumption levels for these two periods. What matters for the household is the intertemporal utility of consumption streams $U = u(c_1) + u(c_2)$. We will look at the expected utility denoted $E(U)$ at the beginning of the year when the taxpayer does not know what would be the income stream during the year.

There are two cases. In the first case where $y_1 > y_2$, the individual can save in the first period and then the optimal equal consumption plan between the two periods can be achieved. In that regime, the consumption in each period is given by the average income over the year, that is, for the full year we get $2u(\frac{y_1+y_2}{2})$ as the annual utility. In the second case where $y_1 < y_2$, the individual cannot borrow and then the consumption of her first period is constrained to be $c_1 = y_1$ and the consumption of the second period is then $c_2 = y_2$. Therefore, the intertemporal utility is given by $u(y_1)+ u(y_2)$.

Combining both elements and introducing the probability of the two events, the expected utility reads :

$$E(U) = \Pr(y_1 < y_2)[u(y_1) + u(y_2)] + \Pr(y_1 > y_2)[2u(\frac{y_1 + y_2}{2})]$$

The above theorems are related to the first part of the expected utility, $[u(y_1) + u(y_2)]$. The second part of the expected utility is constant for the four tax schemes in discussion. We conclude that the results remain valid when the taxpayer is not allowed to borrow at the same conditions as she saves.

Provided that the consumer starts the new year with almost no saving – which occurs in the US at least for the average people in the bottom quintile – an increasing income sequence within a year is a sequence where the absence of a perfect capital market bites the most. It is the case where the individual will benefit the most of consumer credit since by assumption it cannot draw on her precautionary savings. It is here when the monthly compensated system will help and we recall that, in these circumstances, it dominates the Vickrey mechanism. Of course, the reasoning depends on the assumption that the accounts are cleared at the start of the year which can be viewed as an hoc assumption. Here, we are in some sense trapped by the annual fiscal framework.

We do not introduce a monthly discount rate in the intertemporal utility. The monthly discount rate will be very low if any and it can be argued that many people have a spending behavior as if they have a higher marginal utility in let say December, July and August than in March or November. A low discount rate will not be enough to invalidate the above results. It will just make the results more difficult to prove.

Labor supply We already argue in the introduction that caring about the variability of earnings within a year does make sense if this variability is not the result of a choice. We can easily think of situations where the individual prefers to some extent to gather her hours of work rather than spreading them out. The summer vacations are an example of corner solutions for leisure. Many professors prefer to concentrate their teaching duties on one semester. Many pilots and stewarts also prefer to travel with little rest for a while followed by a long period of leisure. The same goes for seasonal workers in seaside or ski resorts. Even if the change of the tax base does not stem from the fact we wish to improve the situation of these individuals, it is important to check that the new tax configuration will not have adverse effect on their behavior and welfare.

The utility will be of the form $U(c + v(l))$ with v can have some convex parts, in such

a way that the individual is choosing a corner solution. For instance take the case of US university professors with a 9 month contract. Does the monthly compensated system will make them more inclined to accept a work during the summer ?
The answer is not obvious and depends on their knowledge of the tax system. We will say that people are myopic if they only look at the monthly marginal tax. The monthly marginal tax rate will be lower with the monthly compensated system than with the current system. TConsequently, the substitution effect will pushed toward working more each month ! As illustrated in Figure 1, the decrease of marginal tax rates will be higher for very active months than in rather inactive months. By definition, the income effect would be zero. So we conclude that in the myopic case, people will work more each month and even more the loaded months. Now, it is also possible that the rational university professor will only contemplate the annual tax schedule which has not changed by definition and then decide not to change her behavior. In all cases, we expect that the magnitude of the behavioral reaction will be small if the actual choice is a corner solution.

Negative income tax The results were obtained for an income tax which does not include transfers and benefits. The case of a negative income tax can be analyzed along the same lines and all the results hold. The main difference is that λ is now negative. Indeed we still have

$$\sum_1^T \frac{G(Ty_t)}{T} \geq G(\sum_1^T y_t))$$

but assuming that $G(Ty_t)$ and $G(\sum_1^T y_t))$ are negative, the value of λ which will make both sides of the inequation equal is negative (or alternatively we will have $T - \lambda$ with $\lambda > 0$).

$$\sum_1^T \frac{1}{T-\lambda} G(Ty_t) = G\left(\sum_1^T y_t\right)$$

In the similar figure as Figure 1, the chord will be always above the function since the tax schedule remains convex. However the amount of benefits transferred to poor people are now lower with the monthly compensated tax than with the equal payment system. We then have to adjust with a negative λ and it turns out than the poor months will be more heavily subsidized than the months when the incomes are higher because the gap

$$\frac{1}{T-\lambda} G(Ty_t) - \frac{1}{T} G(Ty_t) = G(Ty_t)(\frac{\lambda}{(T-\lambda)\,T})$$

will be the higher the tax amount $G(Ty_t)$ is far from 0, meaning that t is a bad month. The smoothing effect of the monthly compensated tax will still operate and the results remain valid in the benefit side as well as in the tax side.

Convexity We assume from the outset that the tax schedule is convex. Many applied works have shown that the effective marginal tax is in fact decreasing on the bottom part of the income distribution in many countries because of the phasing out of the benefit system. In some cases, Diamond (1998) has shown that the static optimal marginal income tax rate should follow a U-shape, a feature also observed in many countries (see Diamond and Saez (2011) and Mankiw et al. (2009).) It adds an additional layer of complexity and it is difficult from a theoretical view point to figure out whether adopting the monthly compensating scheme will be good or harmful for the taxpayer welfare. It militates for empirical studies and it is what we are doing in the next section. The only thing that we can add is that on the income range for which the marginal tax rate will be decreasing, switching to a monthly compensating scheme will be harmful for benefit recipients. The proof of this statement works exactly on the same way as the proofs or our results. By the way, it is not fully sure that in a dynamic context with poverty traps and risk the subsidization of earnings should phased out at a relatively high rate

Implementation On those 4 tax schemes, only the current and the Vickrey system are implementable on the spot taking account the monthly available information. The equal payment and the monthly compensated tax needs for the government to foresee incomes before their realisation. If it is indeed impossible to do it without error, it is possible for the tax authorities to forecast the current variability of income for each taxpayer or for each taxpayer type. Indeed we think that a local analysis can be performed to compute the adjustement parameter, λ, as a function of the degree of convexity of the taxation scheme (the relative Arrow-Pratt degree of risk loving, $\frac{G''(y)}{G'(y)}y$) and a measure of risk, the standard deviation or the coefficient of variation. Once the formulae will be established, it remains to estimate the within-year SD for different types of taxpayers. The available information will be the past information that the tax authorities have, the characteristics of the taxpayer (education, type of job, sector, gender, age etc.) and why not the information that the taxpayer will be able to provide about the variability of income she foresees for the coming year. An individual forecasting error will be made for sure, but the good thing will be that the (positive or negative) adjustment at the end of the fiscal year will likely be much less big than it is actually. It means that in average the tax payment will be more accurate each month than now. An exploration on actual data is then needed to evaluate the heterogeneity dimension of taxpayers in terms of the adjusment parameter λ. This is the purpose of the next sections.

7.3 Data & French tax system

We first give some information about the French tax system and compare it to some other systems. We then move to the data and show an estimate of the between-month variabilility.

7.3.1 Tax Schedules : From theory to practice

In the theoretical section, the general shape of G was very general and stated to be convex for the welfare gains results. Obviously the precise shape of the tax function would be of primary importance for the matter discussed, weather in terms of gain or loss, and even in a more significant way for the size of the gains. G could be seen as a specific tax such as income tax or as the effective tax (i.e the sum of all taxes and benefit one is entitled). Even though effective taxation is the salient point for public economist to work on, it should be clear that since all taxes are not based on the same temporality, a static view point can overwrite some information and can lead to misleading conclusions. We see as as first step to focus on a simulation which only involves the income tax schedule.

Piecewise linear tax scheme Most countries (to the notable exception of Germany) have piecewise linear income tax schemes.
A piecewize linear tax scheme is defined as a sequence of (usually) increasing marginal tax rate,[7] :

$$0 < m_0 < m_1 < ... < m_p$$

and a sequence of thresholds specifying the bands of taxable income to which the respective rates apply.

$$0 = \beta_0 < \beta_1 < ... < \beta_p$$

If $y \in [\beta_k, \beta_{k+1}]$, then the tax liability is :

7. as defined in P.J. Lambert *The distribution and redistribution of income, third edition* P.181

Table 7.1 – France 2009 Income Tax Schedule

from 0 to 6011 euros	0%
from 6011 to 11991 euros	5,5%
from 11991 to 26631 euros	14%
from 26631 to 71397 euros	30%
from 71397 to 151200 euros	41%
beyond 151200 euros	45%

$$s(y) = \sum_{j \leq k-1} m_j(\beta_{j+1} - \beta_j) + m_k(y - \beta_j)$$

It is clear that if one's income stays between two tax brackets, the taxpayer is comparable to one facing a flat tax, and all the characteristics concerned by the above paragraph still holds. So with a piecewise linear tax scheme, one would need to have large enough variation in income to be concerned by a change in tax frequency concerning the amount paid.

France tax system France has an annual income tax that most taxpayers pays on a monthly basis (one tenth of the tax based on last year income for most fiscal household).[8] It consists of a tax applied to a fiscal household. A fiscal household is an entity based on family situation : married couples or under civil union[9] will constitute one unit, single constitutes one fiscal household and couples without civil union will form two fiscal households.

France Income tax consists of a piecewise linear tax scheme composed in 2009 of 5 tax brackets.

Thus in the formalised version of the piecewise linear tax scheme

- $\vec{m} = [6011, 11991, 26631, 71397, 151200]$, and,
- $\vec{\beta} = [0, 5.5, 14, 30, 41, 45]$

To compute tax liability, for a single without kid we just have to apply taxable income to the tax scheme.

8. In fact, there is several systems among which the taxpayer can choose. A three-time payment of the tax (in February, May, and September), a monthly payment of the tax, or a monthly payment of the tax consisting of 10 payments from January to October that correspond to one tenth of last year due tax. If the due income tax is greater for one year than the preceding year, additional payment are required in November and December. More than two-thirds of households choose that later option.

9. *Pacte civil de solidarité* (Pacs)

$$\text{Final tax} = G(\text{taxable income}) = \sum_{j \le k-1} m_j(\beta_{j+1} - \beta_j) + m_k(y - \beta_j)$$

Joint taxation consist of a system called *fiscal shares*[10]. A certain number of fiscal shares are attributed to each fiscal household based on its composition : 1 share for a single, two shares for a couple under civil union. Then each dependent child gives additional shares : 0.5 for each two first childs, 1 share for each child after the second one. [11] We then divide taxable income by the number of fiscal shares and we refer to it later on as normalized tax base. The piecewise linear scheme is then applied to the normalized tax base and finally the obtained amount is multiplied by the number of fiscal shares to obtain the tax liability.

$$\begin{aligned}
\text{Final tax} &= G\left(\frac{\text{taxable income}}{\text{fiscal shares}}\right) \times \text{fiscal shares} \\
&= \sum_{j \le k-1} m_j(\beta_{j+1} - \beta_j) + m_k\left(\frac{y}{\text{fiscal shares}} - \beta_j\right) \times \text{fiscal shares}
\end{aligned}$$

There exists also a mechanism called *la décote* which is a tax break for household paying a small amount of tax. This mechanism doubles the marginal tax rates in the two first brackets. It does not take family situation into account appart from being in couple or not. This mechanism introduces a non-convexity in the tax schedule. Another source of local non-convexity is that for a tax due lower than €62 the taxpayer does not have to send a check to the tax authorities.

France has also many benefits that mainly concerns households exempted to the income tax, and that do not have a smooth or convex shape of the effective MTR.

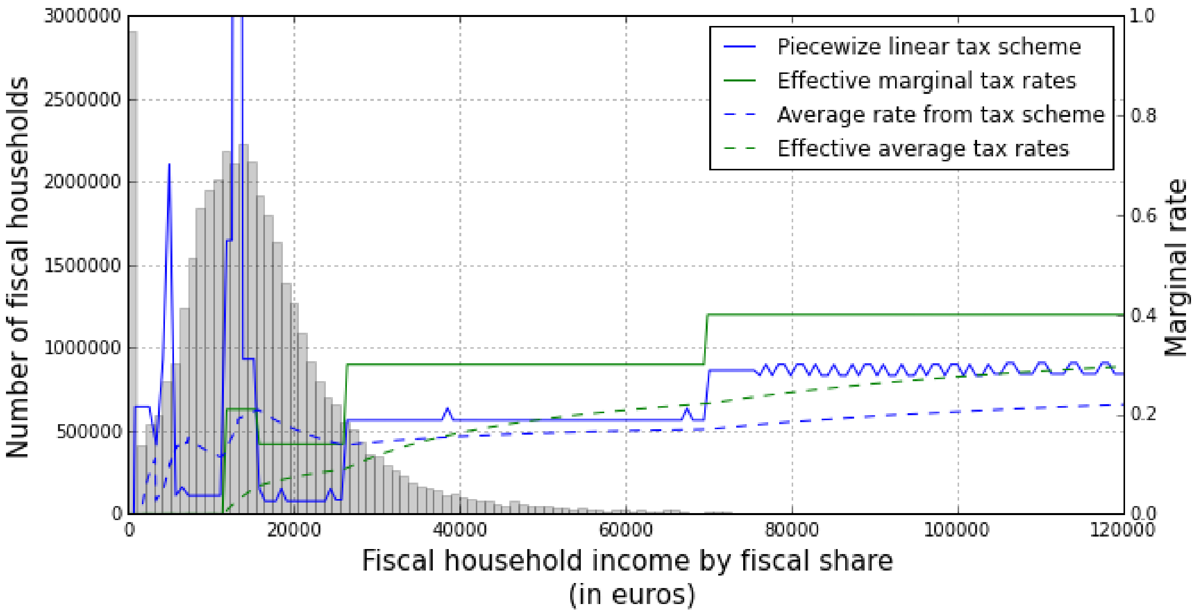
As we can see on Figure 7.3, on the range €10,000 to €30, 000, there are 4 tax brackets where there is only 2 from €30, 000 to €120, 000. With the introduction of benefits to this analysis and their high number into the analysis, the number of tax-brackets soar at the very bottom of the distribution as we can see on Figure 7.3 effective MTRs (which is for single individuals). The inclusion of family support benefits, local taxation, "social tariffs" for telephone and electricity utility services, Universal Health Care Cover, etc, would even make the picture more complicated. [12]

10. *Parts fiscales* in French

11. There exists a certain number of exceptions and special case, single parents benefit from an additional half-fiscal share, there exist specific rules for disable people, etc.

12. See Anne and L'Horty (2012) for more information about the diffent benefits (monetary or in kind) depending of the administrative level.

Figure 7.3 – 2009 French Income Tax, and Individual Income Distribution.



External Validity France has thus many benefits that imply non-convex effective MTR, and income tax that embodied a tax break increasing the MTR at the beginning of the income tax. It is the case for most OECD countries. For instance US has EITC that increase marginal tax rate at the beginning of the piecewise tax scheme. UK has also a working tax credit and income support that gives decreasing marginal tax rate at the beginning of the income distribution. Both UK, and US also have means tested benefits with phasing out implying high effective MTR, and diminishing MTR. Thus as in France, households in UK and US are likely to face both locally concave or convex effective tax schedule. Even if the labor market and tax institutions of every country are specific, we believe that some of the conclusions of our simulations for France can be extended to other OECD countries.

FIGURE 7.4 – UK scheme from Brewer et al. (2010) .

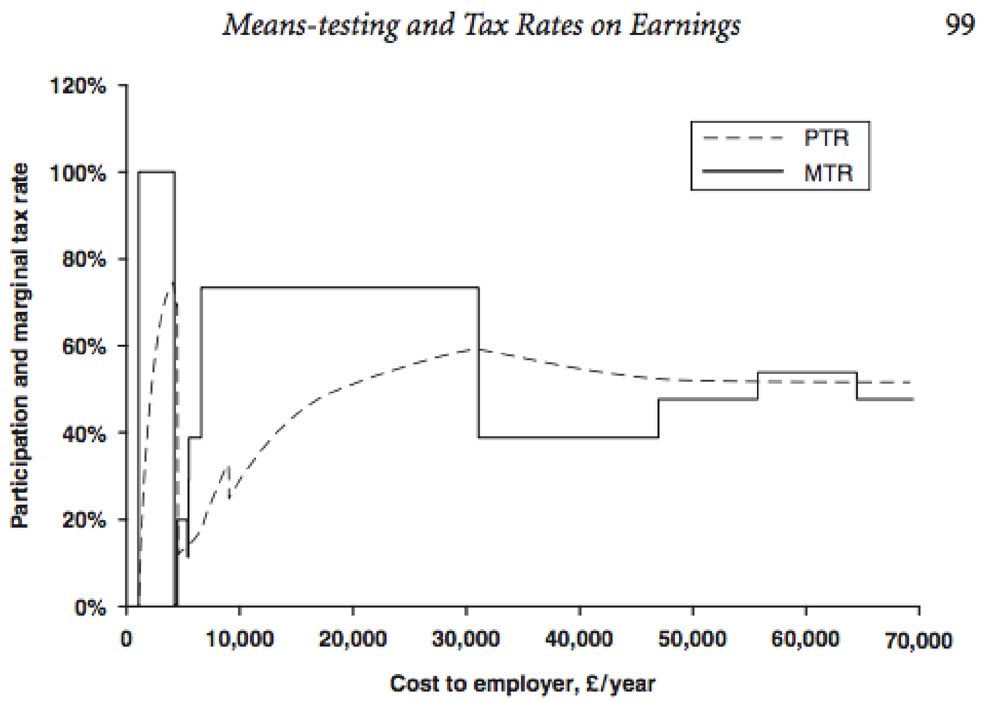
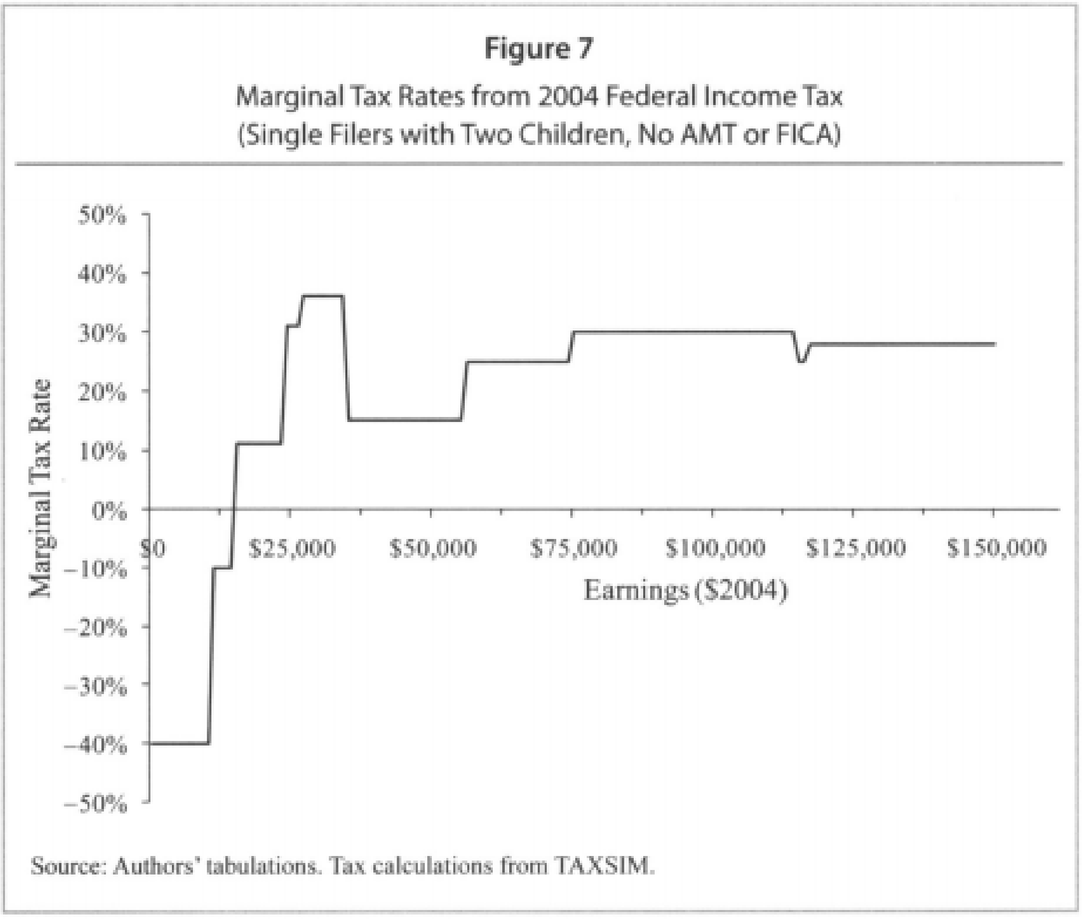


FIGURE 7.5 – 2004 US tax scheme from Eissa and Hoynes (2008)



7.3.2 Descriptive statistics

To our knowledge, France does not have yet available database that allows for the monitoring of the income tax at a monthly scale. We dodge that difficulty by resorting to an administrative database which coupled detailed information on the labor force and tax files. [13] We here explain how we infer monthly income from French labor survey matched with tax returns. Our attempt to deal with the earning time scale can to some extent be compared to the practices in the British Household Panel Survey (see (Jenkins et al., 2000, p. 4-5)).

From the labor force survey, we have individual declared income by categories over the year. Work related categories are labor income, unemployment income, and retirement income. We have for each individual its monthly situation (employed, unemployed, retired, student, inactive).

In order to infer a monthly income for each individual, we equally split income-tax returns into the months for which an individual was in the corresponding category. For example, employment income is allocated to months where the individual was declared employed, unemployment income to months where the individual was unemployed, and retirement income to the months where the individual was retired. If an individual declares for a given month that she was a student or inactive, the inferred income is zero. Other incomes (capital incomes, etc) are equally split over the year. .

Without saying, our method is far from perfect. For those reasons, descriptive statistics and results of the simulations should be taken with a piece of salt and just as a rough estimation of the impact of a change of the tax system toward a monthly compensated tax. In following sections, except when specified, the observations are weighted such that it represents French population in 2009. Technical details concerning the method are in the appendix.

Income Distribution and Variations Over Months Figure 7.6 shows aggregate income per month, that is of a bit more than €60 billions. The sources of income are steady, to the exception of November due to a higher number of declared retired persons in November 2009 in the French labor survey [14].Figure 7.7 represents a simplified version of the possible transitions, such that taking a job that we define as not having a work

13. The official name of the survey is *enquête sur les revenus fiscaux et sociaux* (ERFS)

14. Usually, retirement takes place in September and they can be a delay between the retirement date and the month when the person receives her first pension benefit.

FIGURE 7.6 – Aggregate Disposable Income by Months.

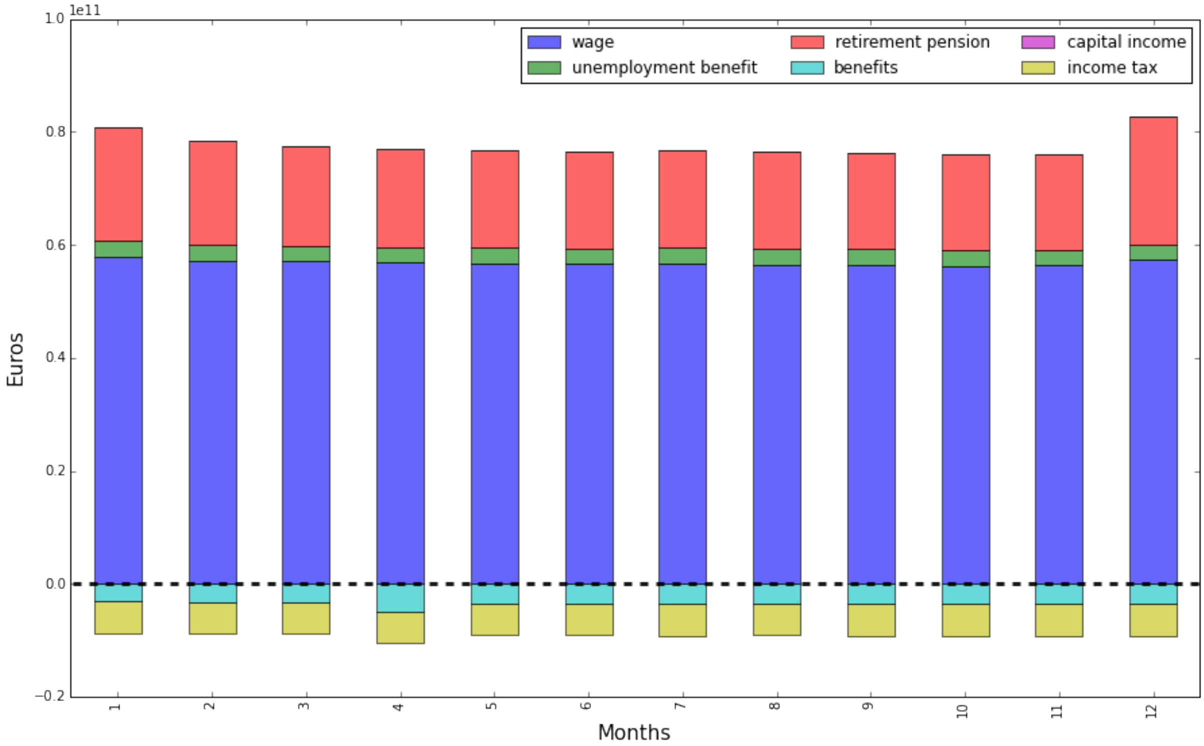


TABLE 7.2 – Share of Type of Transitions

Job loss	Job taking	Retirement
44%	48%	8%

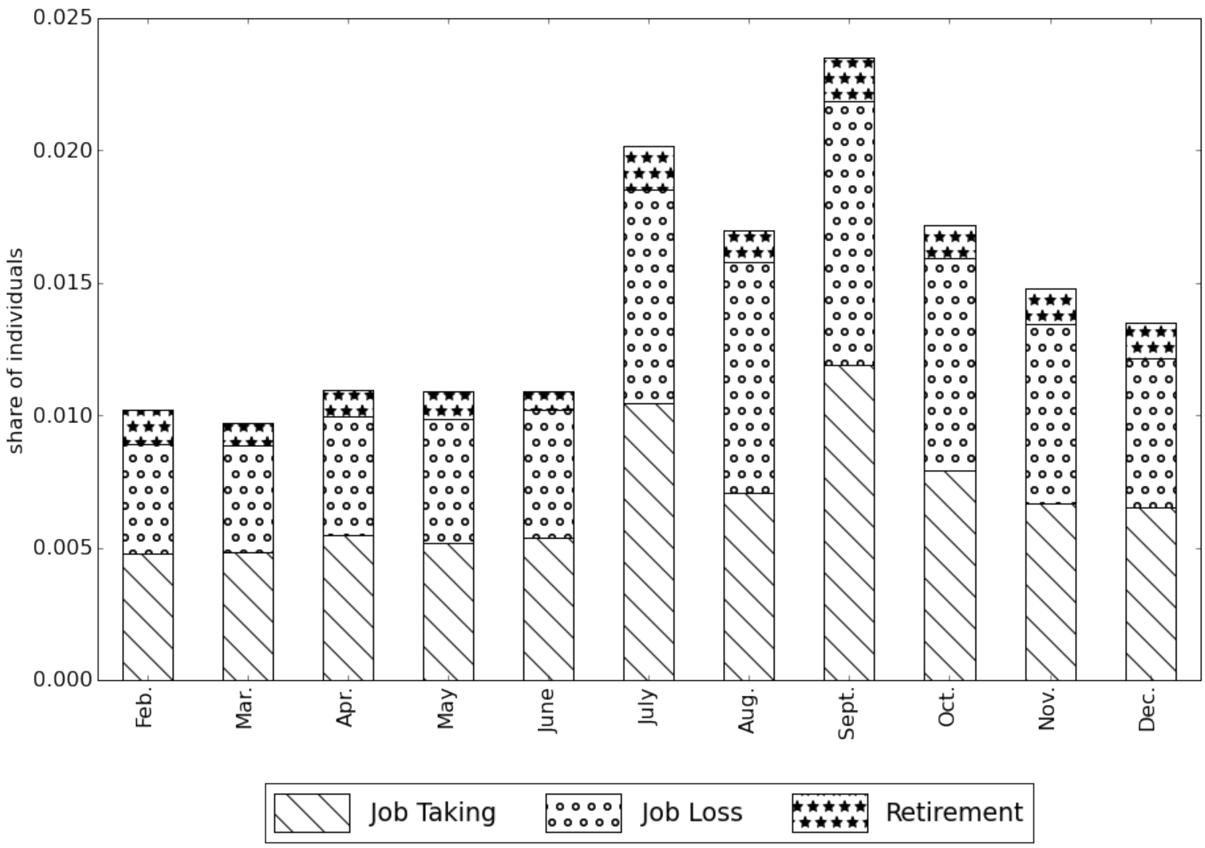
activity last month to having a work activity on the concerned month. Job loss is defined as switching from being employed to unemployed, student or inactive. Retirement is defined as switching from employed or inactive to retired. [15]

We see that on average 1.4 % of the individuals has a change of situation each month. From those 1.4%, 44% are job losses, 48% are job taking, and 8 % are people that get retired.

Figure 7.8 represents the CDF of individual maximum variation of income within the year (the difference between lowest and highest income). With the assumptions we made, 86 % have a constant income over the year, the remaining share having a variable income. 5 % has a variation that is higher than 2000 euros. After some data cleaning to remove most obvious cases of bad correspondence between income and status, we obtain less impressive variations as in Figure 7.9, where only 2 % has an income variation over € 1000.

15. Since we have 5 types of situation (employed, unemployed, inactive, retired, student), we can thus have $5 \times (5 - 1) = 20$ different types of transition.We simplify here those transitions into 3 categories to have a clearer view of the transitions. A more complete description of the transitions are provided in the appendix.

Figure 7.7 – Aggregate Monthly Transitions



	No Variation	Job Taking	Retirement	Job Loss	Sum of Variations
Feb.	79609	381	102	328	811
Mar.	79646	384	66	324	774
Apr.	79547	434	80	359	873
May	79553	411	84	372	867
June	79552	427	57	384	868
July	78832	823	128	637	1588
Aug.	79076	561	95	688	1344
Sept.	78575	937	127	781	1845
Oct.	79062	626	100	632	1358
Nov.	79248	530	106	536	1172
Dec.	79350	516	104	450	1070
Total	872050	6030	1049	5491	12570

Figure 7.8 – Infra-annual Variation of Income.

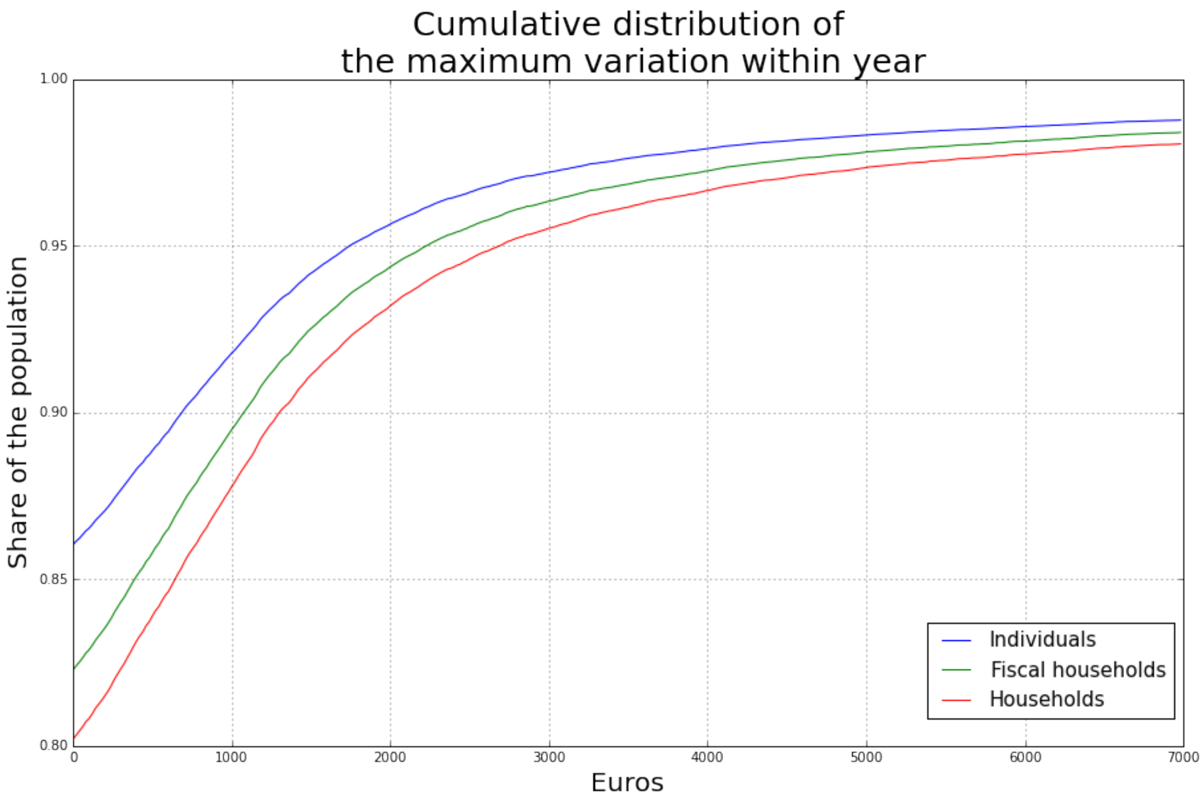


Figure 7.9 – Infra-annual Variation of Income Corrected.

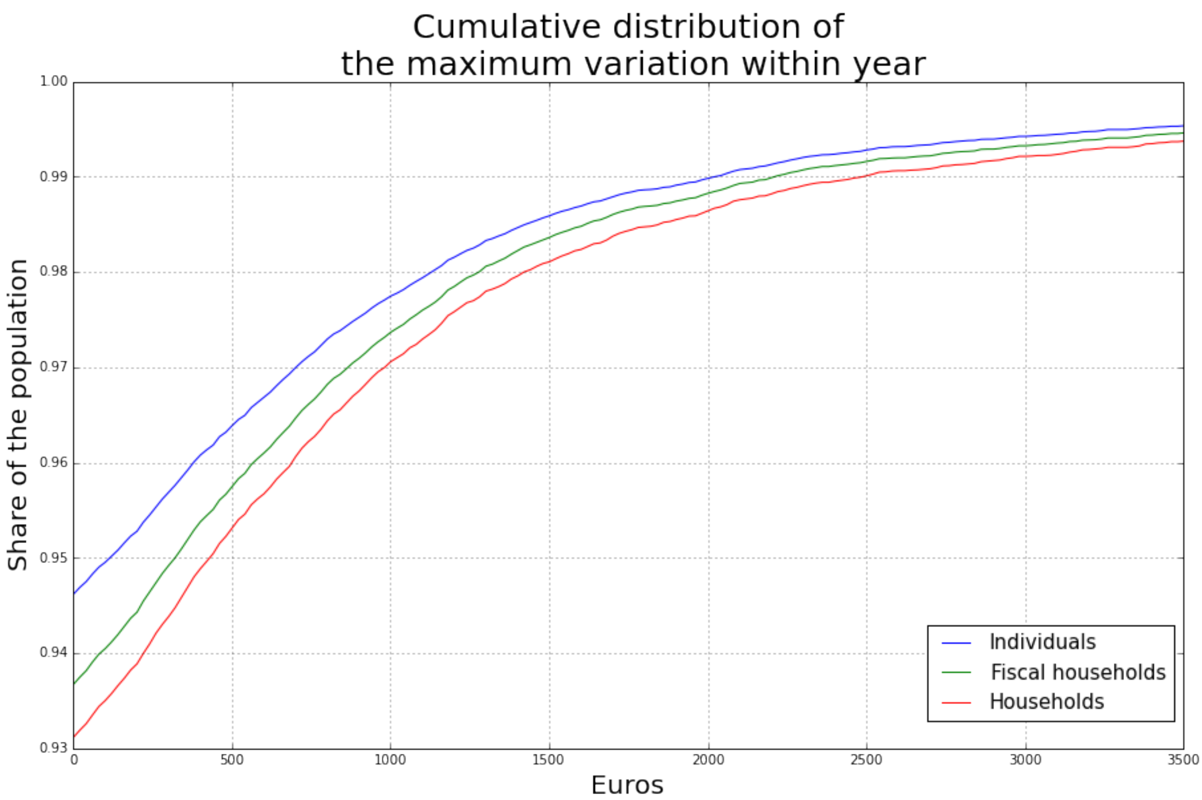
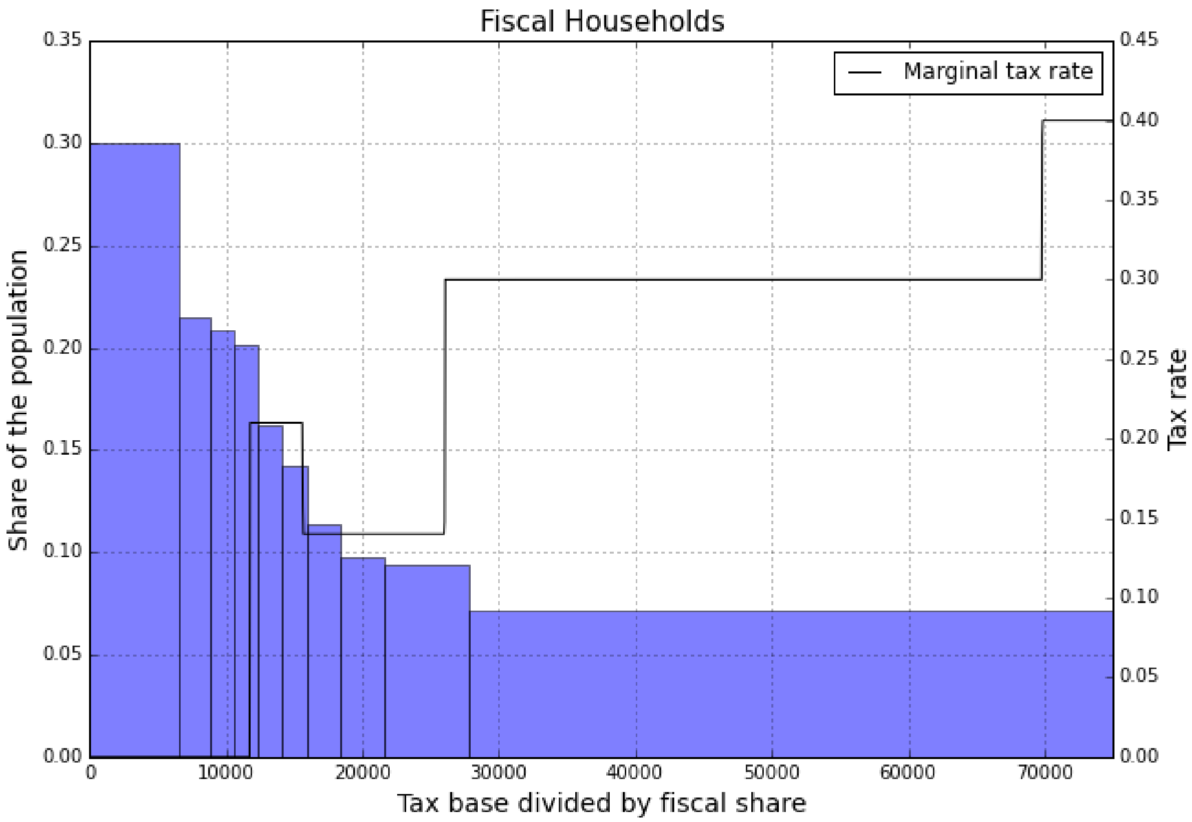


Figure 7.10 – Share per decile of individuals having more than a 20% gap between the worst month and the best month. .



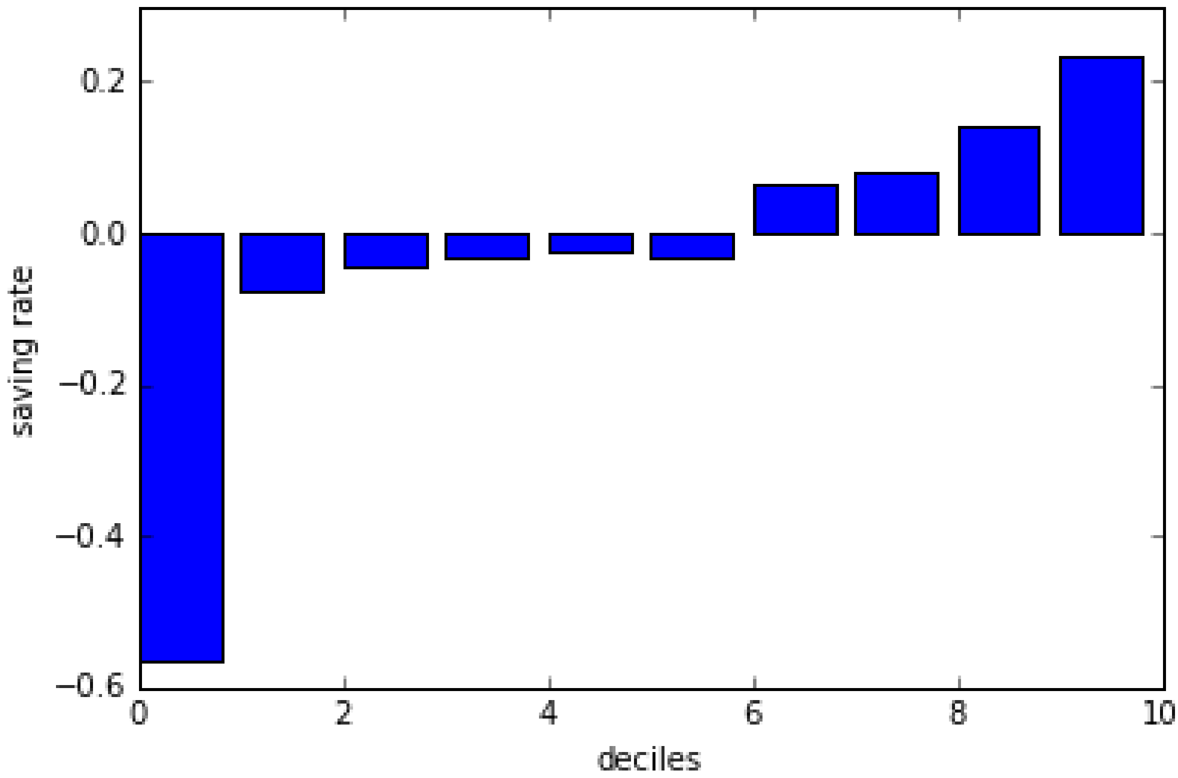
We can see that a significative share of individuals face a variation in income during the year. The French tax system being piecewise linear, a change in income does not necessarily mean a change in the tax amount since a fiscal household would have to change of tax bracket to face a difference in the tax amount. Joint taxation also takes its part on that dimension since it is the average of incomes that are applied to the tax scheme. [16]

Figure 7.10 shows that most population concerned by between-month income variation is at the bottom of the distribution, where the income tax scheme is concave. Figure 7.10 shows the share per decile of people that have more than 20% of income variation between the worst month and the best month. [17] The income limits of deciles are showed on the horizontal axis in terms of normalized tax base. The MTR represents the MTR of a tax-unit composed by one individual. The 22.5 % MTR spike at the beginning of the income range is due to a tax break called *"la décote"* and that –similarly to EITC– creates diminishing MTRs at the beginning of the tax-schedule.

We see that 10 % of the population face the 30% marginal tax bracket or a higher marginal

16. More precision on the joint taxation system in France in appendix
17. Here the graphs do not change much after data cleaning.

Figure 7.11 – Mean saving rate by decile (French household family survey).



tax. If we assume that those tax-units do not move drastically their month-to-month income (from one tax bracket to an other), it means that most tax-payers in the last decile will likely face a flat-tax scheme. However, the other 90% of the population do not face a convex tax scheme : they might pay less tax with a monthly payment ! Moreover those having the smallest income are the one that are the most likely to have variations in income. Thus the poorest are the one facing the most risky labor demand, and those variation are large enough to change the tax bracket they face.

These populations are also on average negative savers as showed by Figure 7.11. Thus marginal propensity to consume is very high for those populations. The focus group faces most variation and poor savings. This population also faces the highest number of potential tax backets, and thus potential effects of disposable-income variation.

7.4 Simulations

We are performing a simulation exercice where the benchmark is the equal payment system who will face three challengers, the Vickrey system, the monthly system and the monthly compensated system.

Simulation Hypothesis We use the microsimulation software *OpenFisca*[18] to simulate a increase in the frequency of the tax from an annual to a monthly one. We did that by changing all the tax formulas by making them working on a monthly period instead of an annual one, multiplying all the inputs by 12, and dividing the output by 12. Details and the code of the reform are available on GitHub.

Switching from the actual system (that has many complexities that will not be described here) to a monthly system without compensation increases the simulated income tax revenues from 48 billion euros for 2009 to 54 billions. To put the income on a monthly basis would generate a gain for the Treasury of 6 billion euros.

We then calculate the compensating term λ for each household. 53% of the sample has a λ equal to 0, 93 % is very close to zero ($|\lambda| < 0.001$). This difference between those two numbers is mainly explained by small rounding differences (usually a cent of euro). We represent the distribution of λ on Figure 7.12, λs equal to zero are taken out for a matter of scale. We see on Figure 7.12 that a small part of the λ are negative. This is due to the non convex part at the beginning of the tax scheme.

One should note that the French income tax being piecewise linear, having income that varies is not sufficient for a change in the income tax. One would need to change tax bracket in order to change the income tax with a monthly basis. French tax schedule having quite large and few brackets, one would need big variation in income in order to observe a significant difference. It is even more true that France has a joint taxation (which double the length of the tax brackets).

To summarize, we could say that a significative part (around 7 %) of the population would face a variation of their income tax if it is put on a monthly basis.

Winner/Losers. Figure 7.13 shows the number of monetary winners and losers among fiscal households according to their normalized tax base.[19] when we implement the monthly tax system instead the equal payment system. We see that losers (in blue – or light grey) represent the vast majority (more than 95%) with respect to winners (in red– or black). As expected, we see that the taxpayers that pay less on a monthly basis are

18. Openfisca registers many intricacies of the French tax system. The drawback is that so far the calibration to macro data is not very accurate.
http://openfisca.org/en/

19. In order to make them comparable we divided the taxable income by the number of fiscal shares (such that they are normalized to face the same tax schedule).

FIGURE 7.12 – λ Distribution

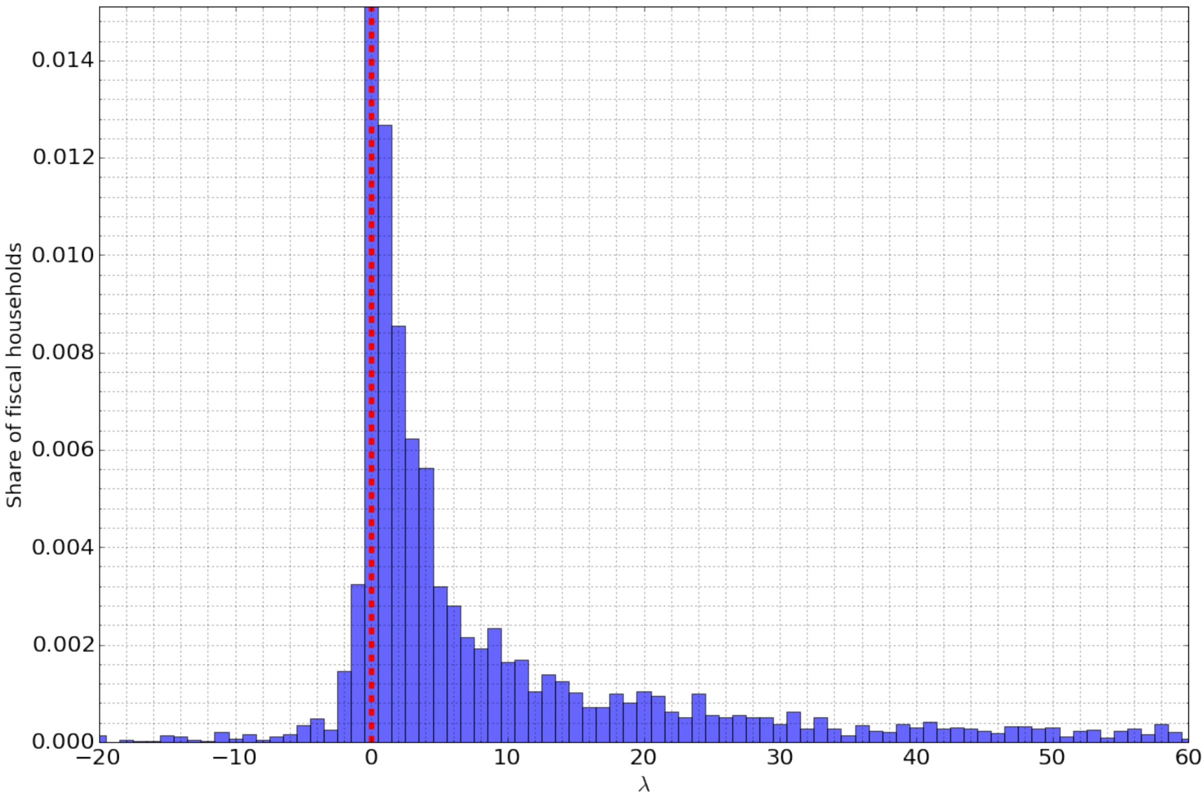
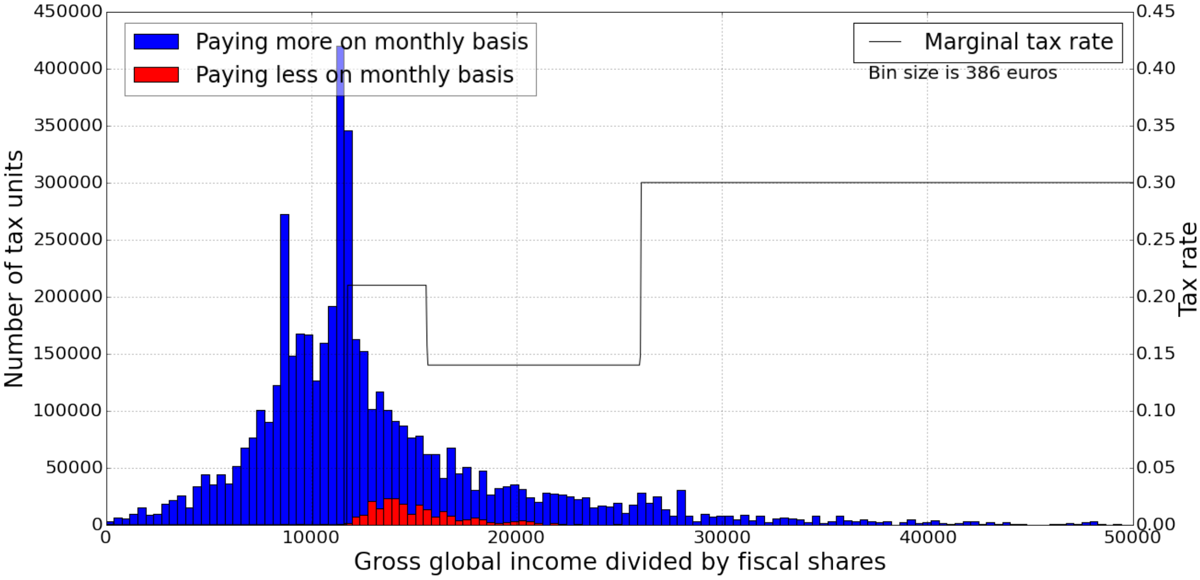


FIGURE 7.13 – Winners and Losers with Monthly Tax.



the one located around the concave part of the tax scheme : those whose some monthly income belongs to the 22.5% tax bracket. There are also winners because of the threshold exemption which also introduces a small non-convexity.

Welfare Analysis. To estimate the welfare gains and losses, we need to estimate a utility function. The theory of equal sacrifice provides a sound theoeretical basis to estimate such a utility function. Young (Young (1990)) provides a method to test the theory of equal sacrifice against data. It shows that the fit is not that bad on various examples, including the US income tax. We did it on French data and we used the estimated utility function to calibrate the welfare changes.

The yearly disposable income z is obviously computed as

$$Z = \sum_{1}^{12} z_t = \sum_{1}^{T} y_t - g_t(y_t)$$

The best fit for an utility function defined on yearly basis we obtain is

$$U(Z) = -(Z)^{-0.89}.$$

We define the utility function to a monthly basis as

$$u_t(z_t) = \frac{-(12z_t)^{-0.89}}{12} \tag{7.6}$$

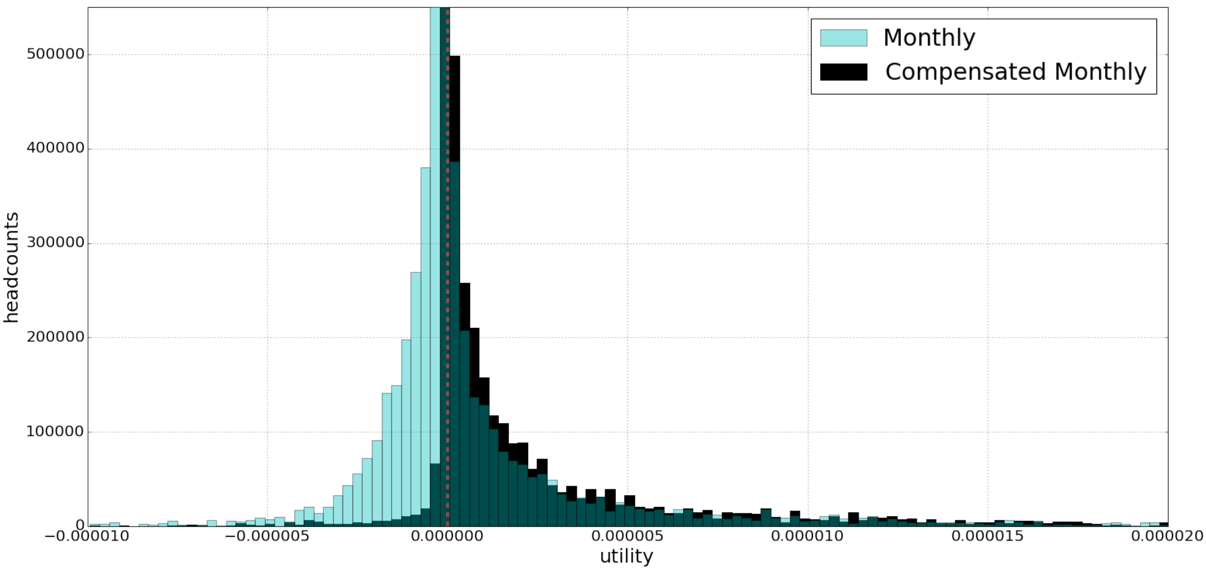
in order the sum over the year gives the yearly utility U.

Figure 7.14 shows all losers and winners by utility points bins gains or loss. Utility gains with the monthly tax scheme are in cyan and dark green while the utility gains with the compensated monthly tax scheme are in black and dark green.

We see as expected that there are losers (10% of the sample) and winners (7.4%) with the monthly tax scheme. The former concerns taxpayers that have higher amount of tax to pay and benefit less from income smoothing over the year. With the monthly compensated tax scheme, almost people are net gainers from the reform. For the few losers (2%), there are two reasons which approximatively are equally important. For 50% of them, their income falls down on the concave part of the tax scheme, and for 50% of them because of the collection threshold of the tax.

Comparison with Vickrey tax scheme We have implemented Vickrey tax scheme as defined in Équation 7.3 with the reference period being year 2009 and the sub-period

FIGURE 7.14 – The distribution of utility gains and losses for the monthly and monthly compensated schemes vs equal split



span being the month. In Figure 7.15 we can see the difference in utility of the monthly compensated tax scheme and the Vickrey tax scheme viz the equal payment scheme. The grey colour shows the overlap between the two histograms ; the black is specific to Vickrey and the cyan to the monthly compensated.

The first message is that the two distributions are quite close and both mechanisms have similar impacts in terms of providing a smoothing income. Both mechanisms make the individuals happier compared to the equal payment scheme but the proportion of winners is higher with Vickrey than with Monthly compensated (13,4% againts 9,2%). In addition on average Vickrey compensation scheme leads to higher utility gains than the monthly compensated tax scheme. However, a higher fraction of the population has a loss in utility with the Vickrey tax scheme (6.2%) compared to equal payment than with the monthly compensated one (2%). These people are taxpayers that earn less at the beginning of the year than at the end. It is an echo of the results found in the theoretical section.

Tableau 7.3 computes the average utility for the different tax systems. Average utility U is of $-1253*10^{-7}$, $-1250*10^{-7}$, $-1249*10^{-7}$, $-12444*10^{-7}$, for respectively the annual basis tax, the monthly tax, the monthly basis compensated tax, and the Vickrey scheme. Mean utilitarian gains of an increase in frequency is of $3*10^{-7}$, and gain from an increase with compensation is of $4*10^{-7}$, while the Vickrey scheme entails a gain of $9*10^{-7}$ utility points.

In order to take into account the effect of joint taxation, we also implement the OECD

FIGURE 7.15 – The distribution of utility gains and losses for the Vickrey and monthly compensated schemes vs equal split

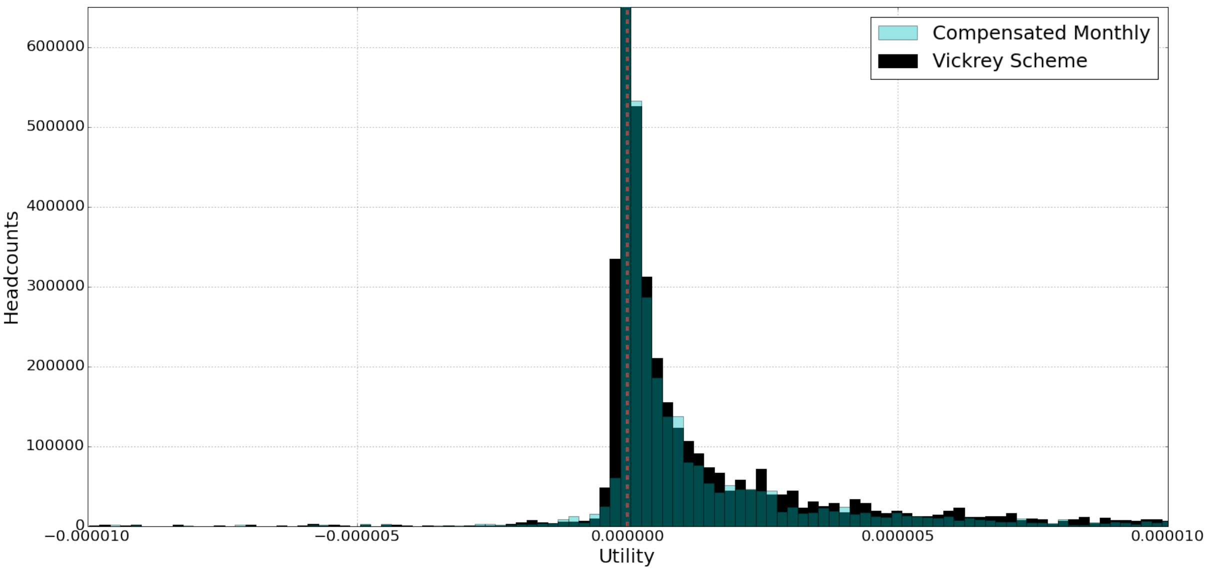


TABLE 7.3 – Utility with the different tax schemes

	Without equivalent scale				With equivalent scale			
Tax scheme	Equal split	Monthly	Compensated	Vickrey	Equal split	Monthly	Compensated	Vickrey
Aggregate	-4206.78	-4195.98	-4194.63	-4178.36	-7599.73	-7582.93	-7579.59	-7552.6
Average	-1.2529 $\times 10^{-4}$	-1.2497 $\times 10^{-4}$	-1.2493 $\times 10^{-4}$	-1.2444 $\times 10^{-4}$	-2.2634 $\times 10^{-4}$	-2.2584 $\times 10^{-4}$	-2.2574 $\times 10^{-4}$	-2.2494$\times 10^{-4}$

equivalent scale to compute utility. The ranking of tax systems is similar.

So the message that comes out of this simulation is the following. The monthly compensated mechanism is more inclusive (less losers) but the gains are more modest than with Vickrey. We conclude this empirical section as well as the theoretical section that this is a closed contest between Vickrey and Monthly compensated payment system. It is too close to call.

7.5 Conclusion

In this article the within-year frequency of the tax and the tax basis for a monthly payment were under scutiny. We have proven given some standard hypothesis that it can be Pareto-improving to adopt a monthly tax base for the monthly payment. We have then tried to confront our theoretical framework to reality by implementing our prescriptive reform to the French income tax.

We have shown that there is a mismatch between our framework where the income tax convex and reality in developed countries such as US, UK and France : taxes on income are not convex and it has some adverse implication in terms of disposable income in a dynamic setting : convex tax schemes promote steady incomes and tax more heavily varying income, while concave tax schemes subsidize varying income. The dataset we used shows that most varying incomes are located at the bottom of the distribution, where precisely the income tax-benefit system is not convex.

We have then run the microsimulation model *OpenFisca* to estimate the impact of a switch from a between-month equal split system to monthly compensated based system. We have quantified monetary gains and loss for taxpayers and identified losers and winners. We have then performed a welfare analysis based on the equal sacrifice principle to determine losers and winners in terms of utility of the switch.

The main results is that in average the Vickrey performs better. The embodied insurance mechanisme works better whatever the convexity of the tax scheme, whereas that of the Monthly compensated supposes the convexity of the tax scheme. In a true sense, this latter mechanism is penalized by the lack of convexity in the bottom part of the income distribution. However, a weakness of the Vickrey systeme has been revealed both by the theory and by the data. It works less well for increasing income stream within a year. This configuration is unfortunately that for which the lack of precautionary saving is the most detrimental.

Those results are driven by within-year income variability. Most income variability studies in developed countries look at year to year incomes, to the notable exception of Hills et al. (2006) who looks at week-by-week effective disposable incomes of 93 poor families over a year in the UK.[20] Hills et al. (2006) emphasize that over nine tenth of those families has monthly income that vary over 10% of their average income[21] and half of the families in the sample has nothing left over for savings, and a quarter for which outgoings exceeded income. Literature on saving behavior –always yearly based– underline that bottom of the distribution has very high marginal propensity to consume.[22] Those facts suggest that a large parts of the bottom of the distribution cope with whatever is their monthly disposable income and gives credit to our "no-saving" framework. Poor savings and income variability can have huge impact on citizen well being, this suggest that the tax-benefit

20. We can also note Jenkins et al. (2000) who studied the difference between snapshot of current income vs. synthesize annual income on the observation of the distribution of income, their findings is that it has nearly no impact.

21.

Only seven of the 93 families had incomes fitting our "highly stable" pattern, that is, varying less than 10 per cent either way from their annual average. Only a third had income in at least eleven periods within 15 per cent of their mean, and within 25 per cent of it in any other periods.

22. Carrolls, etc, choisir les bonnes références.

scheme should be studied in a non purely static models with those effects in mind.
Similarly to Brewer et al. (2010) [23], our public prescription is that bottom of the distribution, tax-benefits should be based on high-frequency in order to smooth disposable income. As we have shown, this is compatible with not increasing the size of transfer payments to the bottom part of the distribution.
Increasing tax frequency could allow for a decrease of the size of the refund which can have some macroeconomic consequences. Such situation has been empirically studied by Shapiro and Slemrod (1993, 2003, 2009) that study the impact on aggregate demand of the US tax-refund.
Those issues open avenues for further research that should be investigated. Optimal taxation is for most contributions developed in a static framework. Studying the incentives and its impact on the optimal tax scheme in a dynamic setting might lead to different public policy prescriptions as the one prescribed by actual models.

23. " The assessment period and timing of taxes and transfers In most countries, individual income taxes are assessed on annual income, and transfers often assessed on a monthly basis (in the UK, weekly). Standard economic models predict that families should budget over long time periods, by borrowing (or using credit) and saving. If families have fluctuating incomes but are able to smooth consumption over time by borrowing and saving, then income assessed over a longer period of time is a better measure of economic welfare or well-being than income assessed over a short period of time. In reality, costs of using financial services and other credit market failures, low levels of literacy, numeracy and financial education, and self-control problems with savings all create significant departures from the standard model. These departures are likely to be more prevalent amongst low-income families, and tend to lead to such families budgeting consumption over short periods of time, such as a month or a fortnight. It therefore seems desirable to operate transfers for low-income families on a **high-frequency basis**, and operate taxes on higher incomes on a lower frequency, such as annual."

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