

# Reconstruct Orbits from Symbols

This notebook is intended to produce the figures found on our arXiv article, found here:  
<https://arxiv.org/pdf/1912.02940.pdf>

There is another notebook which includes additional symbolic and phase representation of the trajectories, but that were not selected for the final version of the paper.

## Configuration A: Two different sequences, sharing a common core

In this configuration, the idea is to find how two symbolic dynamics with a shared rectangular-shaped sequence of symbols (blue symbols) generate phase space trajectories that overlap one another exponentially toward the core of their common blocks. To that end:

- we generate the sequence of symbols and highlight in blue the shared symbols
- we generate the trajectories in phase space  $(q, p)$  and only retain the Lagrangian formulation ( $q$ 's only)
- we plot the difference between the 2 trajectories of all  $(q_{n,t})$
- we repeat the experiment twice with symbols ranging between  $[0, 3]$  ( $s = 7$ , slow convergence) and  $[0, 9]$  ( $s = 11$ , fast convergence).

### ■ with value $s = 7$ (slow exponential convergence)

```
In[ ]:= sA = 7;  
intA = sA - 4;  
mA1 = RandomInteger[{0, intA}, {27, 28}];  
mA2 = RandomInteger[{0, intA}, {27, 28}];  
{T, Np} = Dimensions[mA1];
```

The two sequences are forced to share a common rectangular-shaped core (blue symbols).

```

In[ ]:= pos = {4, 5};
tout = 20;
nout = 19;
tfac = 0;
nfac = 0;
posbis = {tfac, nfac};

{oA1, oA2} = diffoutside[intA, mA1, mA2, pos, tout, nout];
oA1C = coloring1blockwithfont[oA1, pos, posbis,
  tout, nout, tout - 2 * tfac, nout - 2 * nfac, Blue, 17.5, 17];
oA2C = coloring1blockwithfont[oA2, pos, posbis, tout, nout,
  tout - 2 * tfac, nout - 2 * nfac, Blue, 17.5, 17];

posbig1 = {pos[[2]] - 1, T - pos[[1]] - tout + 1};
posbig2 = {pos[[2]] + nout - 1, T - pos[[1]] + 1};
possmall1 = {pos[[2]] + nfac - 1, T - pos[[1]] - tout + tfac + 1};
possmall2 = {pos[[2]] + nout - nfac - 1, T - pos[[1]] - tfac + 1};

GA1 = oA1C // MatrixForm
GA2 = oA2C // MatrixForm

```

Out[ ]:=MatrixForm=

1	0	0	2	0	2	1	3	3	0	2	0	0	0	3	0	0	2	1	2	3	3	2	2	1	2	1	2
3	2	3	0	2	0	0	3	2	1	2	3	3	2	1	2	2	1	1	1	3	0	0	0	2	1	0	0
2	2	3	1	3	2	1	2	0	0	1	3	2	1	2	2	2	0	2	3	3	2	2	3	1	1	2	
3	3	2	0	<b>2</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	0	3	1	0	2	
0	0	1	0	<b>1</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>3</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>3</b>	0	0	3	1	3
0	1	1	1	<b>2</b>	<b>0</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>0</b>	2	3	3	3	
0	2	3	0	<b>3</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>2</b>	3	0	1	0	
2	3	0	3	<b>0</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>3</b>	0	1	2	3	
3	0	0	1	<b>3</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>3</b>	1	0	2	2	
3	0	0	3	<b>3</b>	<b>0</b>	<b>0</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>0</b>	<b>0</b>	0	3	1	3	
3	1	0	1	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>2</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>0</b>	2	1	1	2	
0	0	1	2	<b>1</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>3</b>	<b>3</b>	0	1	0	3	
2	0	0	1	<b>2</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>1</b>	<b>3</b>	<b>3</b>	0	1	1	3	
1	0	0	2	<b>3</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>3</b>	1	3	0	2	
0	1	1	1	<b>1</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>3</b>	<b>1</b>	3	0	0	1	
1	2	2	1	<b>1</b>	<b>0</b>	<b>3</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>3</b>	1	0	3	1	
3	2	3	0	<b>1</b>	<b>0</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>2</b>	0	3	2	3	
2	1	2	0	<b>0</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>3</b>	<b>1</b>	<b>3</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>3</b>	<b>0</b>	3	2	2	0	
0	0	0	1	<b>0</b>	<b>3</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	0	3	0	1	
3	3	2	1	<b>2</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>3</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>3</b>	<b>3</b>	<b>3</b>	3	1	2	2	
1	0	3	2	<b>1</b>	<b>2</b>	<b>2</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>0</b>	<b>3</b>	<b>0</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>0</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>0</b>	<b>2</b>	2	2	2	0	
3	1	0	0	<b>3</b>	<b>0</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>3</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>3</b>	2	1	2	1	
2	1	0	1	<b>2</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>3</b>	<b>0</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>1</b>	0	1	1	3	
1	2	0	2	0	3	3	1	1	3	3	3	1	2	0	1	3	0	0	2	2	2	2	2	2	1	0	3
3	0	3	3	1	1	2	0	1	0	2	3	2	0	0	3	1	0	2	1	2	3	1	3	3	2	1	
2	0	3	0	2	1	0	2	3	0	1	2	1	1	2	2	2	2	2	1	2	1	0	3	3	1	2	
1	2	2	1	3	3	1	1	3	3	1	0	0	1	1	2	0	0	1	3	1	3	2	0	1	2	1	

Out[ ]//MatrixForm=

1	1	0	0	1	2	1	1	2	0	0	1	1	2	0	0	0	3	3	3	2	3	0	1	2	0	2	1
3	2	3	0	2	0	1	3	2	2	3	2	0	3	3	3	3	3	1	2	3	2	1	2	2	1	1	1
2	0	2	3	3	3	1	0	3	1	3	1	0	0	2	1	3	1	0	2	2	2	3	0	3	3	2	0
0	2	3	0	2	1	2	2	1	0	0	1	2	3	2	2	2	0	0	0	0	1	2	1	2	2	1	2
0	3	2	1	1	3	2	2	1	0	3	1	0	0	3	3	0	2	1	2	2	3	3	2	0	3	3	2
0	0	2	3	2	0	3	1	2	2	1	2	2	1	1	1	2	3	0	2	1	0	0	3	0	2	1	3
0	3	0	3	3	2	1	2	0	1	0	3	0	1	3	0	1	3	0	2	1	0	2	3	0	1	2	1
2	2	0	2	0	3	0	2	1	3	0	3	0	1	3	3	3	2	0	2	0	1	3	2	3	2	1	0
3	0	1	1	3	0	1	2	2	0	2	0	0	3	0	1	0	3	0	3	0	3	3	1	0	3	0	2
0	2	1	0	3	0	0	2	2	3	0	3	2	3	3	0	0	1	2	3	1	0	0	1	1	3	0	3
2	2	3	2	3	0	1	3	2	3	0	1	2	3	0	2	0	2	0	2	1	0	0	2	1	2	1	2
1	3	2	2	1	1	0	1	2	3	1	1	2	1	3	0	2	1	3	0	2	3	3	0	3	1	2	0
2	0	0	0	2	2	1	1	1	3	3	3	1	2	0	2	1	1	0	3	1	3	3	2	2	0	2	2
1	2	0	0	3	2	3	1	2	3	3	2	2	0	1	1	1	1	3	0	1	0	3	0	0	2	2	3
2	0	1	3	1	3	0	2	0	2	2	1	3	0	1	3	2	0	0	0	0	3	1	1	0	0	0	3
0	3	3	1	1	0	3	2	1	1	2	3	2	3	3	1	2	3	2	0	1	1	3	0	3	1	3	0
3	3	0	3	1	0	2	2	0	0	1	0	3	1	1	1	0	1	3	2	0	1	2	1	1	0	1	1
3	2	3	0	0	2	2	2	0	1	2	3	3	1	3	1	3	1	3	0	2	3	0	2	1	3	1	0
0	3	3	1	0	3	1	1	3	1	2	1	0	1	0	1	2	0	0	1	0	1	0	2	3	3	0	0
2	2	3	2	2	2	3	1	2	1	0	3	0	3	0	3	3	3	0	2	3	3	3	2	2	1	0	3
3	3	3	0	1	2	2	0	3	0	2	0	3	0	2	2	3	0	1	3	0	0	2	1	0	0	0	2
2	1	1	1	3	0	2	1	1	0	3	2	0	0	0	2	2	1	3	2	3	1	3	1	2	3	2	2
2	1	1	1	2	0	1	1	2	3	1	2	3	1	3	0	3	3	2	0	1	2	1	0	1	2	1	2
0	2	1	1	2	1	2	1	2	2	3	3	2	1	0	2	0	1	2	1	2	1	2	2	2	3	3	2
0	2	0	3	2	1	3	3	1	2	3	1	1	1	3	1	0	0	1	0	1	1	0	0	2	3	3	3
0	1	0	0	0	3	2	3	2	2	1	0	3	2	2	3	0	3	3	0	1	3	2	1	0	3	2	2
0	3	3	1	3	1	1	2	0	1	3	3	0	0	0	1	3	2	2	0	2	0	2	0	1	0	1	1

In[ ]:= **Evaluating and Plotting;**

{QA1, PA1} = Orbits[sA, oA1];

{QA2, PA2} = Orbits[sA, oA2];

Our focus here is on the distance in between  $q_i$ 's (or  $x_i$ 's) only, per the Lagrangian formulation of our equation (which ignores the value of the particles's momentum  $p_i$ 's)

## ■ with value s = 13 (faster exponential convergence)

In[ ]:= sB = 13;

intB = sB - 4;

mB1 = RandomInteger[{0, intB}, {27, 28}];

mB2 = RandomInteger[{0, intB}, {27, 28}];

{T, Np} = Dimensions[mB1];

```

In[ ]:= pos = {4, 5};
tout = 20;
nout = 19;
{oB1, oB2} = diffoutside[intB, mB1, mB2, pos, tout, nout];

oB1C = coloring1blockwithfont[oB1, pos, posbis,
  tout, nout, tout - 2 * tfac, nout - 2 * nfac, Blue, 17.5, 17];
oB2C = coloring1blockwithfont[oB2, pos, posbis, tout, nout,
  tout - 2 * tfac, nout - 2 * nfac, Blue, 17.5, 17];
posbig1 = {pos[[2]] - 1, T - pos[[1]] - tout + 1};
posbig2 = {pos[[2]] + nout - 1, T - pos[[1]] + 1};

GB1 = oB1C // MatrixForm
GB2 = oB2C // MatrixForm

```

Out[ ]//MatrixForm=

1	3	2	4	3	1	9	9	0	6	7	2	3	9	2	1	7	4	5	6	1	2	0	0	6	5	6	3
2	7	7	4	6	7	6	4	8	3	2	3	7	5	8	7	2	7	6	1	3	8	6	3	9	2	8	5
5	5	9	7	6	4	1	0	8	1	5	8	9	7	2	9	7	3	2	3	7	4	3	0	1	0	7	1
8	4	2	2	6	4	4	5	1	8	1	2	6	9	9	8	2	3	2	6	3	8	7	2	1	0	4	4
7	1	0	6	5	1	5	5	7	3	4	9	7	1	9	0	1	0	4	0	1	5	7	5	7	3	5	9
3	3	8	6	1	4	5	5	9	0	0	0	2	0	4	5	0	3	5	2	6	0	0	3	0	6	8	1
9	2	8	4	4	1	2	4	8	8	1	1	6	4	0	3	8	4	3	2	6	0	8	3	9	8	7	7
8	5	8	4	3	7	5	7	9	8	7	6	0	6	4	7	6	6	8	9	4	8	8	4	8	6	7	4
9	5	4	7	6	1	6	0	3	6	0	8	0	4	9	1	6	8	3	3	0	5	5	8	4	4	8	9
9	0	4	0	5	8	6	9	6	9	4	1	5	3	5	4	9	0	3	8	4	9	7	2	6	1	7	8
9	5	2	8	6	8	7	1	5	7	2	7	7	9	3	1	4	6	2	2	1	7	4	5	1	3	7	5
3	8	8	2	8	4	4	2	4	4	2	0	3	3	9	4	6	6	4	4	0	3	9	6	6	4	5	1
8	4	8	4	2	2	3	1	2	4	3	8	4	8	4	9	4	6	5	7	9	2	6	9	8	1	1	2
9	4	9	1	0	6	0	6	6	8	9	6	9	3	2	7	1	4	2	7	8	3	8	3	2	7	5	5
0	8	3	1	2	0	1	8	4	1	2	1	1	3	9	1	3	3	8	0	6	1	6	4	4	4	3	0
9	4	5	7	8	3	9	1	6	6	5	9	1	2	6	4	6	9	1	0	7	7	5	4	4	5	9	2
1	2	4	6	6	3	8	9	5	5	8	8	0	4	8	3	5	8	8	1	3	2	8	2	1	7	4	0
4	8	1	2	2	3	1	2	2	5	3	8	8	9	2	9	0	6	3	1	4	0	6	6	8	1	6	0
3	0	2	9	9	0	6	2	3	9	6	8	4	2	9	8	1	2	4	5	6	7	7	6	1	3	2	8
5	2	7	5	5	4	4	3	9	4	9	1	8	6	5	6	5	0	4	5	3	3	7	7	4	8	2	4
9	0	1	6	6	1	8	5	4	7	2	9	0	1	9	6	9	5	2	2	5	4	7	5	7	6	0	9
7	0	9	3	2	8	9	1	0	2	7	0	6	5	0	6	7	3	9	8	3	4	1	0	2	8	7	7
7	8	4	9	0	0	5	5	6	3	7	7	0	5	2	1	3	2	3	6	8	8	0	4	1	3	7	2
7	8	4	1	2	0	6	7	8	3	9	5	6	4	7	6	8	4	7	3	0	2	6	5	1	2	1	1
6	2	6	9	0	0	5	0	4	5	5	7	0	8	7	3	9	2	8	1	8	3	5	4	5	4	2	9
7	1	5	8	1	2	2	1	2	3	1	6	9	5	5	7	6	5	6	4	1	6	5	2	9	8	1	7
4	9	7	5	1	2	6	5	7	5	3	6	8	8	6	2	4	1	5	2	6	8	7	8	9	1	4	9

Out[ ]:=MatrixForm=

8	8	0	3	0	6	3	5	6	9	2	9	9	8	9	9	8	9	0	2	6	4	1	6	3	5	4	6
7	1	2	2	0	7	8	8	7	6	0	1	2	0	7	5	9	6	2	6	1	5	1	0	0	7	0	9
0	0	6	4	9	6	8	7	1	1	0	3	3	9	1	0	2	6	5	9	8	1	0	6	8	9	2	3
8	3	4	1	6	4	4	5	1	8	1	2	6	9	9	8	2	3	2	6	3	8	7	1	2	1	3	3
4	8	5	4	5	1	5	5	7	3	4	9	7	1	9	0	1	0	4	0	1	5	7	3	8	5	0	7
8	4	8	7	1	4	5	5	9	0	0	0	2	0	4	5	0	3	5	2	6	0	0	8	9	0	7	1
9	0	4	2	4	1	2	4	8	8	1	1	6	4	0	3	8	4	3	2	6	0	8	4	3	9	1	0
5	9	9	9	3	7	5	7	9	8	7	6	0	6	4	7	6	6	8	9	4	8	8	2	5	2	7	7
7	3	1	7	6	1	6	0	3	6	0	8	0	4	9	1	6	8	3	3	0	5	5	3	9	6	8	4
2	8	0	6	5	8	6	9	6	9	4	1	5	3	5	4	9	0	3	8	4	9	7	8	8	4	4	2
7	0	6	5	6	8	7	1	5	7	2	7	7	9	3	1	4	6	2	2	1	7	4	6	8	5	7	1
7	5	5	1	8	4	4	2	4	4	2	0	3	3	9	4	6	6	4	4	0	3	9	9	2	8	9	5
5	2	1	6	2	2	3	1	2	4	3	8	4	8	4	9	4	6	5	7	9	2	6	4	4	4	5	2
0	6	7	8	0	6	0	6	6	8	9	6	9	3	2	7	1	4	2	7	8	3	8	7	1	2	7	4
5	6	3	1	2	0	1	8	4	1	2	1	1	3	9	1	3	3	8	0	6	1	6	4	1	1	3	4
1	7	8	7	8	3	9	1	6	6	5	9	1	2	6	4	6	9	1	0	7	7	5	7	3	3	6	9
8	6	3	2	6	3	8	9	5	5	8	8	0	4	8	3	5	8	8	1	3	2	8	2	3	3	8	9
1	6	2	0	2	3	1	2	2	5	3	8	8	9	2	9	0	6	3	1	4	0	6	1	8	0	4	1
1	6	4	6	9	0	6	2	3	9	6	8	4	2	9	8	1	2	4	5	6	7	7	2	5	7	0	3
9	5	5	1	5	4	4	3	9	4	9	1	8	6	5	6	5	0	4	5	3	3	7	4	3	6	0	9
1	9	3	5	6	1	8	5	4	7	2	9	0	1	9	6	9	5	2	2	5	4	7	0	1	6	0	7
2	9	0	7	2	8	9	1	0	2	7	0	6	5	0	6	7	3	9	8	3	4	1	0	4	3	3	5
1	8	9	1	0	0	5	5	6	3	7	7	0	5	2	1	3	2	3	6	8	8	0	5	2	8	1	3
8	1	9	0	9	7	9	1	4	8	9	6	1	1	4	1	9	6	3	0	2	9	6	0	3	7	7	8
6	6	0	9	5	4	7	0	5	3	0	4	9	0	8	8	4	8	6	8	0	0	6	5	3	6	9	3
5	5	2	0	8	0	6	1	3	9	8	1	3	9	3	5	0	9	5	9	4	7	3	6	8	3	3	6
4	4	2	7	6	0	4	2	4	3	7	8	8	9	9	9	5	6	2	1	3	4	7	9	9	8	3	7

In[ ]:= **Evaluating and Plotting;**  
 {QB1, PB1} = Orbits[sB, oB1];  
 {QB2, PB2} = Orbits[sB, oB2];

## ■ The two configuration in Lagrangian Space

In[ ]:= QA21abs = Abs[QA1 - QA2];  
 {T, L} = QA21abs // Dimensions;  
 lminA = Min[QA21abs] // N // Log10  
 lmaxA = Max[QA21abs] // N // Log10  
 flminA = lminA // Floor  
 clmaxA = lmaxA // Ceiling

Out[ ]:= -8.68319

Out[ ]:= -0.218326

Out[ ]:= -9

Out[ ]:= 0

We store each of the logmin / logmax entries for both data sets. The floor and ceiling of those are actually what matter since we look to range colors from an integer to another.

```
In[ ]:= QB21abs = Abs[QB1 - QB2];
      lminB = Min[QB21abs] // N // Log10
      lmaxB = Max[QB21abs] // N // Log10
      flminB = lminB // Floor
      clmaxB = lmaxB // Ceiling
```

```
Out[ ]:= -11.7726
```

```
Out[ ]:= -0.0902288
```

```
Out[ ]:= -12
```

```
Out[ ]:= 0
```

```
In[ ]:= fmin = Min[flminA, flminB];
      cmax = Max[clmaxA, clmaxB];
```

The minimum and the maximum of the log scales of the two diff-plots.

```
In[ ]:= cs = ColorData["DeepSeaColors"];
      myCF[x_] := colorRange[1 / (cmax - fmin) * x - (cmax - fmin) * cmax] /.
        colorRange -> ColorData[{"DeepSeaColors", {-1, 0}}]
      break[y_] := Table[y[[i]], {i, 3}]
```

We derive a color function code that spreads over the DeepSeaColors gradient in reverse {-1, 0}.

The min value represents a threshold in darkness, while the max value is the ceiling in brightness.

The idea is that fmin maps to -1 and cmax maps to 0. Any change in the direction below the threshold or above the ceiling won't affect the color coding.

```
In[ ]:= myCF[cmax]
      Table[myCF[cmax][[i]], {i, 3}]
      Table[myCF[cmax + 0.03][[i]], {i, 3}]

      myCF[fmin]
      Table[myCF[fmin][[i]], {i, 3}]
      Table[myCF[fmin - 0.03][[i]], {i, 3}]

      Table[myCF[fmin + i], {i, 0, cmax - fmin}]
```

```
Out[ ]:= 
```

```
Out[ ]:= {0.772061, 0.92462, 0.998703}
```

```
Out[ ]:= {0.772061, 0.92462, 0.998703}
```

```
Out[ ]:= 
```

```
Out[ ]:= {0.16791, 0., 0.301671}
```

```
Out[ ]:= {0.16791, 0., 0.301671}
```

```
Out[ ]:= {
```

We then map from the data's value to a gradient scheme spread between a (0, 1) bar.

Technically even min and max of each data set won't reach the ends of the bar because those corresponds to floor of min and ceiling of max:

fmin -> 0 and cmax -> 1

nA represents the total number of ticks

The list represents the position and the string display at each tick, given that the position must be between 0 and 1.

However, for listA we look to shift and compress the color spread so that:

fmin\_overall -> 0 and cmax\_overall -> 1

fmin and cmax will land in between 0 and 1.

The reason we do so is for the bar-legend function: the range of values chosen (bar\_min, bar\_max) (which must be within 0 and 1)

must correspond to the range of colors we would like to choose for both pairs of system considered.

```
In[ ]:= myTFA[x_] := (x - flminA) / (clmaxA - flminA);
myTFAS[x_] := x * flminA / fmin + 1 - flminA / fmin;
nA = clmaxA - flminA
listticksA = Transpose[{Table[myTFA[flminA + i], {i, 0, nA}],
  Join[{"1"}, Table[Superscript["10", -i], {i, nA}]] // Reverse}]
listticksAS = Transpose[{Table[myTFAS[myTFA[flminA + i]], {i, 0, nA}],
  Join[{"1"}, Table[Superscript["10", -i], {i, nA}]] // Reverse}]
```

Out[ ]:= 9

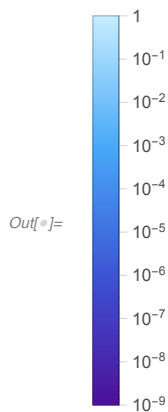
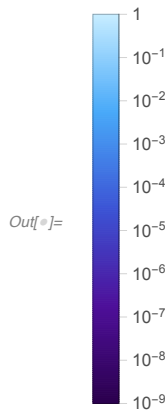
```
Out[ ]:= {{0, 10-9}, {1/9, 10-8}, {2/9, 10-7}, {1/3, 10-6},
  {4/9, 10-5}, {5/9, 10-4}, {2/3, 10-3}, {7/9, 10-2}, {8/9, 10-1}, {1, 1}}
```

```
Out[ ]:= {{1/4, 10-9}, {1/3, 10-8}, {5/12, 10-7}, {1/2, 10-6},
  {7/12, 10-5}, {2/3, 10-4}, {3/4, 10-3}, {5/6, 10-2}, {11/12, 10-1}, {1, 1}}
```

```

In[ ]:= barA = BarLegend[{cs, {0, 1}}, Ticks → listticksA]
barA = BarLegend[{cs, {1 - flminA / fmin, 1}}, Ticks → listticksAS]

```



This is the bar-legend of system A (slow convergence) so that the plotting never reaches the darker regions (below  $10^{-9}$ ) since this range is reserved to system B

```

In[ ]:= myTFB[x_] := (x - flminB) / (clmaxB - flminB);
nB = clmaxB - flminB;
listticksB = Transpose[{Table[myTFB[flminB + i], {i, 0, nB}],
  Join[{"1"}, Table[Superscript["10", -i], {i, nB}]] // Reverse}]

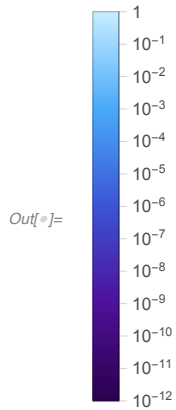
```

Out[ ]:=  $\left\{ \left\{ 0, 10^{-12} \right\}, \left\{ \frac{1}{12}, 10^{-11} \right\}, \left\{ \frac{1}{6}, 10^{-10} \right\}, \left\{ \frac{1}{4}, 10^{-9} \right\}, \left\{ \frac{1}{3}, 10^{-8} \right\}, \left\{ \frac{5}{12}, 10^{-7} \right\}, \right.$

$\left. \left\{ \frac{1}{2}, 10^{-6} \right\}, \left\{ \frac{7}{12}, 10^{-5} \right\}, \left\{ \frac{2}{3}, 10^{-4} \right\}, \left\{ \frac{3}{4}, 10^{-3} \right\}, \left\{ \frac{5}{6}, 10^{-2} \right\}, \left\{ \frac{11}{12}, 10^{-1} \right\}, \{1, 1\} \right\}$



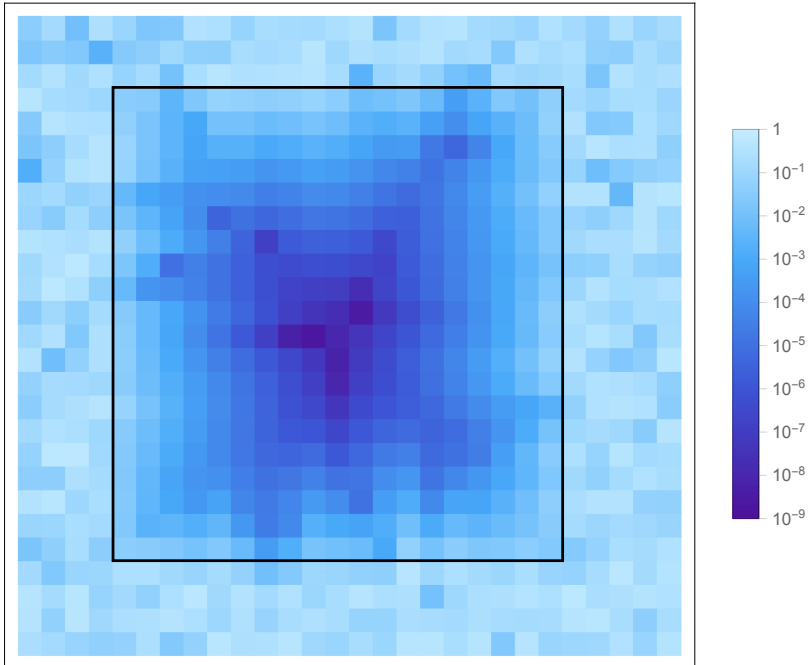
```
In[ ]:= barB = BarLegend[{cs, {0, 1}}, Ticks → listticksB]
```



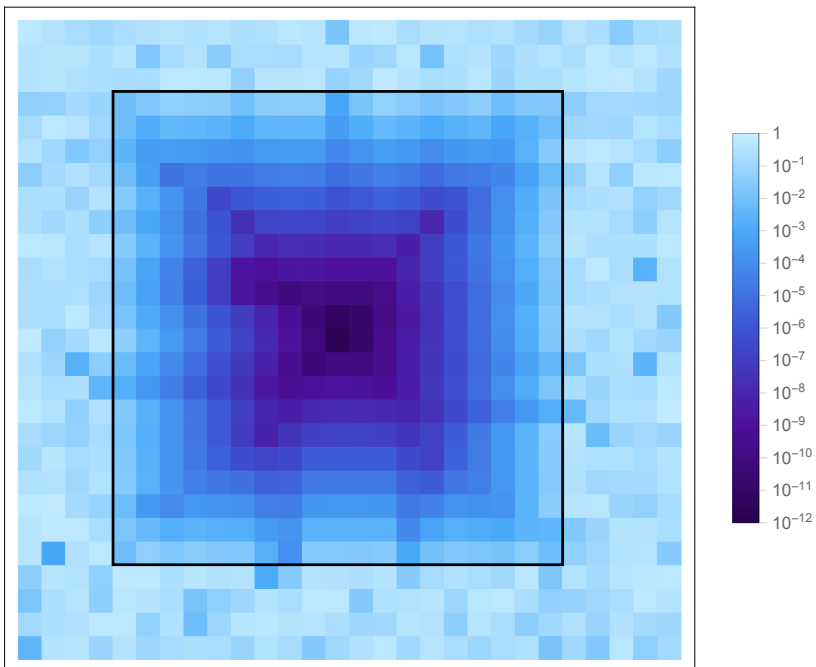
```
In[ ]:= G1 = ArrayPlot[QA21abs // Log10,
  FrameTicks → {None, None},
  ColorFunction → (myCF[#1] &),
  ColorFunctionScaling → False,
  PlotLegends → barA,
  Epilog → {EdgeForm[Thickness[0.004]], Opacity[0], Rectangle[posbig1, posbig2]}]
```

```
G2 = ArrayPlot[QB21abs // Log10,
  FrameTicks → {None, None},
  ColorFunction → (myCF[#1] &),
  ColorFunctionScaling → False,
  PlotLegends → barB,
  Epilog → {EdgeForm[Thickness[0.004]], Opacity[0], Rectangle[posbig1, posbig2]}]
```

Out[ ]:=



Out[ ]:=



```

In[ ]:= SetDirectory["/Users/adriensaremi/Downloads/Figs/"];
Export["AKSs7BlockBorderM1.pdf", GA1];
Export["AKSs7BlockBorderM2.pdf", GA2];
Export["AKSs7distM1M2_2.pdf", GA21C];

In[ ]:= Export["AKSs13BlockBorderM1.pdf", GB1];
Export["AKSs13BlockBorderM2.pdf", GB2];
Export["AKSs13distM1M2.pdf", GB21C];

```

```
Export["AKSLPS12.pdf", G1];
Export["AKSLPS12_1.pdf", G2];
```

```
In[ ]:= minbA = 1;
maxbA = intA;
colorschemeA =
  Table[ColorData[{"DeepSeaColors", {minbA, maxbA}}][i], {i, 0, intA}] // Reverse;
```

```
minbB = 1;
maxbB = intB;
colorschemeB =
  Table[ColorData[{"DeepSeaColors", {minbB, maxbB}}][i], {i, 0, intB}] // Reverse;
```

The colorscheme used here is simply obtained by “fractionating” the DeepSea Gradient, where each color corresponds to a symbol (ranging from light to dark using the Reverse command).

```
In[ ]:= colorschemeA // Reverse
colorschemeB // Reverse
Table[myCF[fmin + i], {i, 0, cmax - fmin}]
```

```
Out[ ]:= {■, ■, ■, ■}
```

```
Out[ ]:= {■, ■, ■, ■, ■, ■, ■, ■, ■, ■}
```

```
Out[ ]:= {■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■, ■}
```

## Configuration B : Swapping 2 cores between 2 sequences with shared surrounding

```
In[ ]:= s = 13;
int = s - 4;
m = RandomInteger[{0, int}, {25, 21}];
```

From the m matrix defined above, we generate two matrices m1 and m2, identical, except to small submatrices B and C that are interchanged between m1 and m2. The blocks B (green) and C (red) on both matrices are always surrounded by a bigger “frame” A (blue).

```

In[ ]:= pos1 = {3, 2};
pos2 = {14, 11};
tout = 9;
nout = 8;
tin = 3;
nin = 2;
posin = {3, 3};
{m1, m2} = swapping2blocks[int, m, pos1, pos2, posin, tout, nout, tin, nin] ;

```

```

m1C = coloring2blocks[m1, pos1, posin, tout, nout, tin, nin, Blue, Green];
m1C = coloring2blocks[m1C, pos2, posin, tout, nout, tin, nin, Blue, Red];
m2C = coloring2blocks[m2, pos1, posin, tout, nout, tin, nin, Blue, Red];
m2C = coloring2blocks[m2C, pos2, posin, tout, nout, tin, nin, Blue, Green];

```

```
m1C // MatrixForm
```

```
m2C // MatrixForm
```

Out[ ]//MatrixForm=

```

3 1 9 0 1 7 1 0 2 3 4 1 8 1 7 3 5 0 3 0 1
4 8 1 7 2 6 0 7 5 7 8 8 7 7 6 5 9 3 9 5 1
3 9 8 1 5 8 2 4 5 0 7 0 4 6 9 2 1 0 0 0 4
1 1 9 6 0 9 5 5 8 1 1 0 5 6 9 4 3 8 5 5 5
4 7 0 4 8 6 1 7 7 3 7 4 5 9 4 8 4 9 3 6 5
3 3 5 8 7 0 6 0 1 8 2 1 0 7 2 2 9 2 0 3
5 9 4 1 4 4 2 1 9 7 3 2 1 8 8 7 2 0 9 0 3
3 4 5 4 6 2 8 5 2 4 7 3 5 0 3 7 7 2 2 9 9
5 4 8 6 9 6 0 7 9 9 4 2 4 9 0 0 7 6 8 0 2
9 6 6 9 6 0 6 7 7 9 6 5 5 4 7 0 7 1 3 7 2
7 7 8 4 2 6 7 3 2 1 9 8 0 6 0 7 9 4 1 8 5
8 5 7 5 4 3 6 8 6 7 4 7 7 1 2 1 0 7 3 3 7
4 3 7 0 3 9 9 8 9 2 0 8 7 7 6 0 4 3 3 5 6
8 8 0 3 0 5 2 5 8 7 9 8 1 5 8 2 4 5 8 9 0
1 6 9 2 8 6 3 7 1 7 1 9 6 0 9 5 5 8 7 1 0
8 1 2 8 8 7 9 9 8 4 7 0 4 8 6 1 7 7 4 3 1
4 6 5 3 1 7 6 2 8 7 3 5 8 8 0 0 6 0 1 5 7
9 9 1 8 9 0 2 1 4 4 9 4 1 9 1 2 1 9 5 7 2
6 9 0 6 2 1 0 2 7 3 4 5 4 4 4 8 5 2 9 4 9
8 9 0 4 1 5 5 1 5 9 4 8 6 9 6 0 7 9 7 4 0
3 7 9 2 7 5 8 7 3 0 6 6 9 6 0 6 7 7 4 2 0
9 0 8 1 1 5 5 1 8 9 7 8 4 2 6 7 3 2 3 7 9
6 6 2 2 7 1 8 6 0 5 9 6 8 6 9 1 6 5 1 2 2
5 3 4 0 6 0 3 8 1 6 7 1 6 2 2 9 7 1 5 7 7
1 8 0 4 3 1 8 6 1 7 9 8 0 0 3 6 7 1 1 0 8

```

Out[ ]//MatrixForm=

3	1	9	0	1	7	1	0	2	3	4	1	8	1	7	3	5	0	3	0	1
4	8	1	7	2	6	0	7	5	7	8	8	7	7	6	5	9	3	9	5	1
3	9	8	1	5	8	2	4	5	0	7	0	4	6	9	2	1	0	0	0	4
1	1	9	6	0	9	5	5	8	1	1	0	5	6	9	4	3	8	5	5	5
4	7	0	4	8	6	1	7	7	3	7	4	5	9	4	8	4	9	3	6	5
3	3	5	8	8	0	0	6	0	1	8	2	1	0	7	2	2	9	2	0	3
5	9	4	1	9	1	2	1	9	7	3	2	1	8	8	7	2	0	9	0	3
3	4	5	4	4	4	8	5	2	4	7	3	5	0	3	7	7	2	2	9	9
5	4	8	6	9	6	0	7	9	9	4	2	4	9	0	0	7	6	8	0	2
9	6	6	9	6	0	6	7	7	9	6	5	5	4	7	0	7	1	3	7	2
7	7	8	4	2	6	7	3	2	1	9	8	0	6	0	7	9	4	1	8	5
8	5	7	5	4	3	6	8	6	7	4	7	7	1	2	1	0	7	3	3	7
4	3	7	0	3	9	9	8	9	2	0	8	7	7	6	0	4	3	3	5	6
8	8	0	3	0	5	2	5	8	7	9	8	1	5	8	2	4	5	8	9	0
1	6	9	2	8	6	3	7	1	7	1	9	6	0	9	5	5	8	7	1	0
8	1	2	8	8	7	9	9	8	4	7	0	4	8	6	1	7	7	4	3	1
4	6	5	3	1	7	6	2	8	7	3	5	8	7	6	0	6	0	1	5	7
9	9	1	8	9	0	2	1	4	4	9	4	1	4	4	2	1	9	5	7	2
6	9	0	6	2	1	0	2	7	3	4	5	4	6	2	8	5	2	9	4	9
8	9	0	4	1	5	5	1	5	9	4	8	6	9	6	0	7	9	7	4	0
3	7	9	2	7	5	8	7	3	0	6	6	9	6	0	6	7	7	4	2	0
9	0	8	1	1	5	5	1	8	9	7	8	4	2	6	7	3	2	3	7	9
6	6	2	2	7	1	8	6	0	5	9	6	8	6	9	1	6	5	1	2	2
5	3	4	0	6	0	3	8	1	6	7	1	6	2	2	9	7	1	5	7	7
1	8	0	4	3	1	8	6	1	7	9	8	0	0	3	6	7	1	1	0	8

## Evaluation and Plotting

```
{Q1, P1} = Orbits[s, m1];
```

```
{Q2, P2} = Orbits[s, m2];
```

```
In[ ]:= xplotrangerange = {0.23, 0.35};
```

```
yplotrangerange = {0.83, 0.95};
```

```
size1 = 17;
```

```
size2 = 11;
```

```
dgreen = 0.04;
```

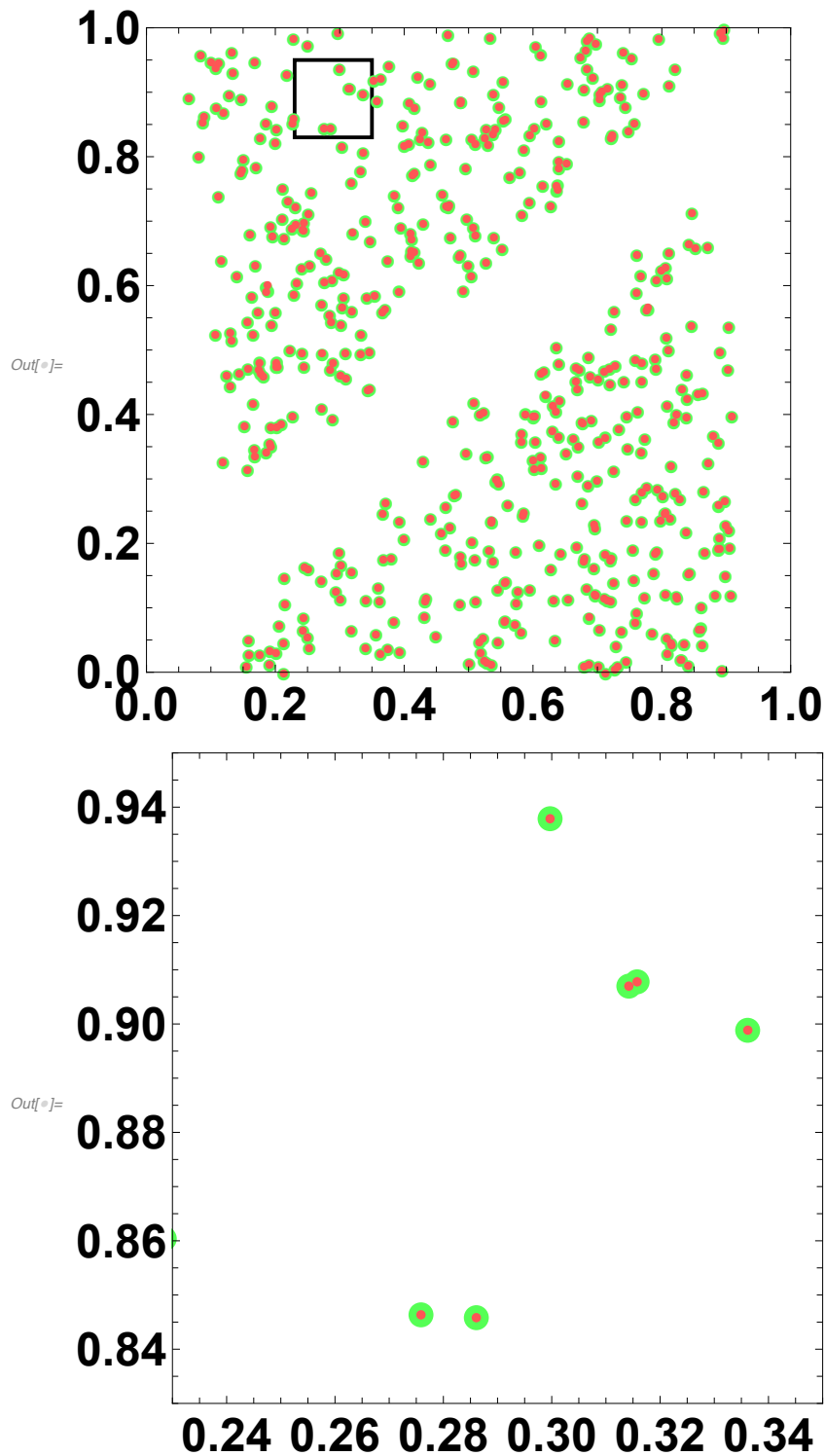
```
dred = 0.015;
```

```
{plot1, plot2} =
```

```
plotting2[Q1, P1, Q2, P2, xplotrangerange, yplotrangerange, dgreen, dred, size1, size2];
```

```
Show[plot1, ImageSize -> 400]
```

```
Show[plot2, ImageSize -> 400]
```



Configuration C: Swapping 3 diamond-shaped cores between 2 sequences with shared

# diamond-shaped surrounding

```
In[ ]:= s = 7;
int = s - 4;
m = RandomInteger[{0, int}, {34, 33}];
{T, Np} = Dimensions[m1];
```

Here, we just switch three diamond-shaped blocks between three identical sequences (in a cyclic manner), with identical bordering on the outside of these blocks. From the  $m$  matrix defined above, we generate three matrices  $m_1$ ,  $m_2$  and  $m_3$ , identical, except to small diamond-shaped submatrices  $B$ ,  $C$  and  $D$  that are cyclically permuted between  $m_1$ ,  $m_2$  and  $m_3$ . The blocks  $B$  (green)  $C$  (red) and  $D$  (cyan) on both matrices are always surrounded by a bigger diamond-shaped “frame”  $A$  (blue).

```
In[ ]:= pos1 = {2, 10};
pos2 = {8, 24};
pos3 = {17, 14};
dout = 8;
din = 2;

{o1, o2, o3} = swapping3diamonds[int, m, dout, din, pos1, pos2, pos3];

o1C = coloring2diamonds[o1, dout, din, pos1, Blue, Green];
o1C = coloring2diamonds[o1C, dout, din, pos2, Blue, Red];
o1C = coloring2diamonds[o1C, dout, din, pos3, Blue, Cyan];

o2C = coloring2diamonds[o2, dout, din, pos1, Blue, Red];
o2C = coloring2diamonds[o2C, dout, din, pos2, Blue, Cyan];
o2C = coloring2diamonds[o2C, dout, din, pos3, Blue, Green];

o3C = coloring2diamonds[o3, dout, din, pos1, Blue, Cyan];
o3C = coloring2diamonds[o3C, dout, din, pos2, Blue, Green];
o3C = coloring2diamonds[o3C, dout, din, pos3, Blue, Red];

G1 = o1C // MatrixForm
G2 = o2C // MatrixForm
G3 = o3C // MatrixForm

(*posbig1 = {pos1[[2]], T-pos1[[1]]}+{-1,1};
posbig2 = {pos2[[2]], T-pos2[[1]]}+{-1,1};
posbig3 = {pos3[[2]], T-pos3[[1]]}+{-1,1};
possmall1 = {pos1[[2]], T-pos1[[1]]}-(dout-din)*{0,1}+{-1,1};
possmall2 = {pos2[[2]], T-pos2[[1]]}-(dout-din)*{0,1}+{-1,1};
possmall3 = {pos3[[2]], T-pos3[[1]]}-(dout-din)*{0,1}+{-1,1};*)
```

Out[ ]//MatrixForm=

3	3	3	1	3	1	1	1	2	1	1	0	1	0	2	3	3	3	1	1	0	1	1	1	1	3	0	2	1	1	2	3	
3	1	3	0	0	1	3	2	0	3	0	3	3	3	3	0	1	1	3	3	2	0	0	0	2	0	1	0	1	2	3	1	
2	0	1	3	1	2	3	3	2	2	2	0	3	0	1	3	1	0	1	2	3	0	2	2	1	0	0	1	0	0	0	1	
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2 2 2 0 1 0 1 1 2 0 0 0 3 0 0 3 2 0 0 0 2 1 1 3 2 0 2 0 3 3

```

## Evaluating and Plotting;

```
{Q1, P1} = Orbits[s, o1];
```

```
{Q2, P2} = Orbits[s, o2];
```

```
{Q3, P3} = Orbits[s, o3];
```

```

In[ ]:= (*xplotrange = {0.62,0.74};
        yplotrange = {0.32,0.45};

        size1 = 21;
        size2 = 12;
        size3 = 6;
        dgreen = 0.045;
        dred = 0.022;
        dbblue = 0.011;

        {plot3,plot4} = plotting3[Q1,P1,Q2,P2,Q3,P3,
            xplotrange,yplotrange,dgreen,dred,dbblue,size1,size2,size3];
        plot3
        plot4*)

```