

# 1 Generalization

## 1.1 Introduction

While we computed all the different tensors, one can notice some kind of pattern. Moreover the complexity increase substantially while we add some freedom. A good idea would be to introduce a general metric from which we hope to find some interesting general form for the Christoffel Symbol, the Ricci Tensor and the Einstein tensor.

## 1.2 The metric

Let us introduce the metric :

$$g_{ij} = \eta_{ij} + H k_i k_j \quad (1)$$

with  $k_i$  any null geodesic and  $H$  any function  $H := H(t, x, y, z)$ . The first thing we will need to find is the inverse of this metric. Indeed we will need it to compute the Christoffel symbols. We guess from our previous calculus an inverse of the form :

$$g^{ij} = \eta^{ij} - H k^i k^j \quad (2)$$

We can quickly check it as follow :

$$g^{ij} g_{ij} = \dots\dots$$