1 Basis of General Relativity

1.1 Introduction

Our overall goal is to find different of the Einstein vacum field equation, we can write this equation as follow :

$$G_{ij} = 0R_{ij} - \frac{1}{2}Rg_{ij} = 0$$

But first we need to fully understand the terms that appears in the equation. We shall start by defining what is a metric. The term g_{ij} is a metric. It is a fundamental part of general relativity.

1.2 Metrics

To describe a metric we will start with the case of a 3D cartesian space and then generalised the definition of a metric. In Cartesian coordinate, we can define an infinitesimal line element as :

$$ds^2 = dx^2 + dy^2 + dz^2$$

But this obviously doesn't work for a more complicated space, if we consider a spherical space for instance. This line element would need some correction. We can also introduce the time dimension to the line element, getting a generalized form as follow :

$$ds^2 = g_{ij}dX^idX^j \tag{1}$$

g_{ij} is a tensor and $dX^i = \{t, x, y, z\}$. The next thing that we'll need to study are the shortest distances between two points. It is for this purpose that we want to study geodesics.

1.3 Geodesics

Considering an arbitrary curve C given by : $X^i = X^i(\lambda)$, in a space with metric g_{ij}, we have :

$$s = \int ds = \int \sqrt{g_{ij}\dot{X}^i\dot{X}^j}d\lambda \tag{2}$$

The s arclength can be use to find the distance between two points. What interest us is to find the shortest distance between two points. [...]

1.4 Christoffel Symbols

The Christoffel Symbols doesn't explicitly appears in the Einstei field equation but it is a central element to compute the Ricci and Rieman tensor. To define a Christoffel symbol, we need to first define what is a covariant derivative :

Definition 1 *(Covariant derivative) A covariant derivative (sometimes derivative operator)* ∇_a *on a manifold* $\mathcal{M}$ *is a mapping which takes a type (p,q) tensor to a tensor (p,q+1) with the follow properties :*

1. *For any smooth function* f *the covariant derivative coincides with the partial derivative*