Numerical Methods Notes

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Why numerical methods?

- Give numerical solution to a mathematical problem
- When problem is too complex (no closed form solution)
  - Solution doesnt exist
  - Too dumb to figure out solution
- Numerical solutions only produce *approximations* of the actual solution
  - We should be able to determine how good such approximations are
  - With an upper boundary

What are numerical methods?

1. Algorithms

- Precise set of rules(instructions) to follow
- Generate approximation of mathematical problem

2. Method to estimate error

- Without this approximation is useless
- Will differ depending on type of mathematical model

Algorithms

- Most are iterative
  - An initial guess that is gruadually improved
  - It us up to you to find the initial guess
  - And up to you to decide when to stop the iteration
- Can have quality control step (Check how good a job you did)
  - This step is to decide when to stop the iteration
  - Used to identify the difference in precision between approximation and real answer.
  - We typically have a target *tolerance* or *precision*

How do we decide target precision?

- Will depend on specific problem and why you are solving it
- Usually given lmao

Error

- Difference between actual mathematical solution r and the numerical approximation

$$x_r$$

- Can be measured as an *abosulute error* or *relative error*
  - Usually we use relative error (%).

True Error vs Estimated Erro

- The True Error is the actual difference between solution and estimate. Which is never really known unless you know the real solution.
- The Estimated Error is a conservative estiamtion of the true error.

Estimated absolute error

$$E_a \geq |x_r - r|$$

Estimated relative error

$$e_a \geq \frac{|x_r - r|}{r}$$

Source of errors

- Error in modelling
  - Model is not 100% accurate
  - Bugs
  - Errors in inputs
  - *Round off errors*
  - *Truncation errors*