Downside Beta

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1 Introduction

Our aim is to utilize downside beta as a signal to assess the strength of selected companies stocks. The idea is inspired by the paper written by Andrew Ang, Joseph Chen and Yuhang Xing from 2006 titled ,"Downside risk[J]. The review of financial studies". Our motivation for selecting downside beta as topic is that we want to evaluate the situation where the economy might place greater emphasize on downside risk than upside gains[1], in an easy and simple manner to implement.

We compute downside betas over periods when the excess market return is below its mean, which is shown below.

$$\beta^- = \frac{\text{Cov}(r_i, r_m \mid r_m < \mu_m)}{\text{Var}(r_m \mid r_m < \mu_m)}$$

By establishing the signal, which is the difference between the downside beta and the normal market beta, we want to show that stocks with high downside betas have, on average, low unconditional average returns. For decile creation, we determine the cutoffs, re-balance the selected stocks annually each January, and use the signal from the previous December to sort stocks into 10 deciles. The portfolio is constructed by taking a long position in decile 10, which has downside beta smaller than the market beta and a short position in decile 1, which has an average downside beta larger than the market beta. The detailed procedures will be explained further in part 2 and part 3.

The reason of investors follow our strategy is that we hope this downside beta implementation help us better measure the systematic risk of individual stocks. In addition, hopefully the strategy allows an obvious downside risk premium.

2 Data Preparation

- CRSP dataset, combined with the de-listing return, the same as we used from all the problem sets.

- Compute lagged individual capitalization and lagged market capitalization for each month.
- Fama French dataset, the benchmark data, from problem set 1.
- Compute lagged annual moving average of the market return for each month.
- Merge the dataset into 1 on date.

3 Portfolio Construction

3.1 Signal Creation

The signal is created by investing in stocks with lower market beta when the market is underperforming its moving average, and it aims to capitalize on the relative stability of these stocks during periods of market downturns.

- Create function to compute annual beta condition on where monthly market return is smaller than the average return, denoted as downside beta
- Compute normal market beta for each year
- Create the signal as the difference between the downside beta and market beta
- $$\text{Signal} = \beta_{\text{rolling, lagged}}^{(12 \text{ months})} - \beta_{\text{downside}}$$

(1)

3.2 Decile Creation

- Use only NYSE stocks to compute the cutoffs for each decile.
- Re-balance annually, on each January, use the signal from last December to sort the stocks into 10 deciles.
- Where 10 denotes the rolling beta is most smaller than normal market beta and 1 denotes the rolling beta is close or larger to normal market beta.
- Construct the portfolio as long the 10 decile and short 1 decile.

4 Results

After acquiring a signal for all stocks and all months, the first 10 decile portfolios of equal weighting were created. Decile 1 containing the lowest signal stocks and decile 10 the highest signal stocks, i.e. the stocks that have a lower beta during market underperformance. The following summary statistics from 2000 were calculated for these 10 deciles and presented below in table 1. As well as a final column for the net neutral portfolio of decile 10-1 (Which is the factor column).

4.1 Decile Statistics

Decile	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**10-1**
Excess Return	5.86	7.24	7.62	7.21	7.74	8.17	6.98	7.28	7.93	7.35	1.49
Volatility	26.80	22.94	19.16	16.45	16.39	15.40	15.50	16.06	16.11	16.75	18.39
Sharpe Ratio	0.22	0.32	0.40	0.44	0.47	0.53	0.45	0.45	0.49	0.44	0.08
Skewness	-0.60	-0.45	-0.32	-0.56	-0.65	-0.57	-0.46	-0.27	-0.41	-0.45	0.56

Table 1: Decile Summary Statistics

The first interesting point to make is that apart from the first column for Decile 1, all the deciles see pretty similar excess returns. This suggests that this factor only affects stock returns negatively in the bottom 10%.

Another observation is that the volatility decreases with increasing decile until about decile 4 and then remains flat. This gives decile 4-10 similar sharpe ratio results with decile 6 performing best.

A hypothesis for this is that markets are most volatile during stress periods which normally lead to overall market losses. As you increase in decile and have an average greater signal, or a portfolio of stocks which are lower beta in underperforming market periods and higher beta during good market periods, you are essentially reducing market beta during volatile periods which in turn leads to lower portfolio volatility which is a good thing.

Finally, whilst the factor portfolio of decile 10 minus decile 1 has a relatively low excess return and Sharpe ratio, it is still positive whcih for a 0 cost portfolio is promising and warrants further analysis.

4.2 Strategy alpha and beta to other factors

To quantify the success of the strategy, the following regressions were carried out as seen in equation 2 and 3 for the factor net neutral portfolio, against the Fama-French 3 factor model. Due to the high performance of Decile 6, one factor portfolio was decided as Decile 6 minus Decile 1, as well as comparing the

more standard Decile 10 minus Decile 1. Further analysis as to which portfolio is optimum could be taken including the potential for the average of Decile 5 - 10 minus Decile 1 and analyzing in different time periods.

$$\text{DownsideBeta}_{it} = \alpha + \beta_{\text{MKT-RF}}(\text{MKT})_t + \beta_{\text{SMB}}(\text{SMB})_t + \beta_{\text{HML}}(\text{HML})_t + \epsilon_{it} \quad (2)$$

Decile 6-1: $\alpha = 0.0059$ (T-stat = 2.47), Market $\beta = -0.47$ (T-stat = -8.75), SMB $\beta = -0.59$ (T-stat = -6.21), HML $\beta = 0.29$ (T-stat = 3.93)
Decile 10-1: $\alpha = 0.0051$ (T-stat = 1.87), Market $\beta = -0.50$ (T-stat = -8.19), SMB $\beta = -0.42$ (T-stat = -3.88), HML $\beta = 0.29$ (T-stat = 3.47)

The regression analysis of the downside beta using the Fama-French three-factor model yields insightful results for the 6-1 and 10-1 decile portfolios. For the 6-1 decile, the intercept (α) of 0.0059 with a T-statistic of 2.47 indicates statistically significant abnormal returns. The market beta ($\beta_{\text{MKT-RF}}$) is -0.47 with a T-statistic of -8.75, suggesting a strong negative sensitivity to market movements. The size beta (β_{SMB}) is -0.59 (T-stat = -6.21), indicating a preference for larger stocks, while the value beta (β_{HML}) is 0.29 (T-stat = 3.93), highlighting a tilt towards value stocks.

Similarly, the 10-1 decile portfolio shows an intercept (α) of 0.0051 (T-stat = 1.87), which is also significant though slightly less so than the 6-1 decile. The market beta ($\beta_{\text{MKT-RF}}$) is -0.50 (T-stat = -8.19), the size beta (β_{SMB}) is -0.42 (T-stat = -3.88), and the value beta (β_{HML}) is 0.29 (T-stat = 3.47), all indicating similar trends in factor loadings but with slightly varying magnitudes.

These findings underscore the effectiveness of the strategy in capturing significant abnormal returns through exposure to market, size, and value factors. The negative market and size betas, coupled with positive value betas, suggest that these portfolios are positioned defensively with a preference for larger, value-oriented stocks. This aligns with the strategy's objective to invest in lower beta stocks during market downturns, potentially offering investors a way to achieve favorable risk-adjusted returns.

$$\text{DownsideBeta}_{it} = \alpha + \beta_{\text{MKT-RF}}(\text{MKT-RF})_t + \epsilon_{it} \quad (3)$$

Decile 6-1: $\alpha = 0.0055$ (T-stat = 2.13), Market beta = -0.57 (T-stat = -10.14)
Decile 10-1: $\alpha = 0.0048$ (T-stat = 1.72), Market beta = -0.57 (T-stat = -9.32)

This further regression shows that just running a traditional alpha calculation of against the market returns the 6-1 decile portfolio again produces statistically significant alpha, whilst the 10-1 decile is still positive it is slightly less significant.

4.3 Cumulative performance of portfolios

The best way to really see how this strategy performs over time is to plot the cumulative returns as seen in Figure 1 below. Here plotted are the decile 1, decile 6, decile 10, risk-free and decile 6-1 excess log returns since 2000.

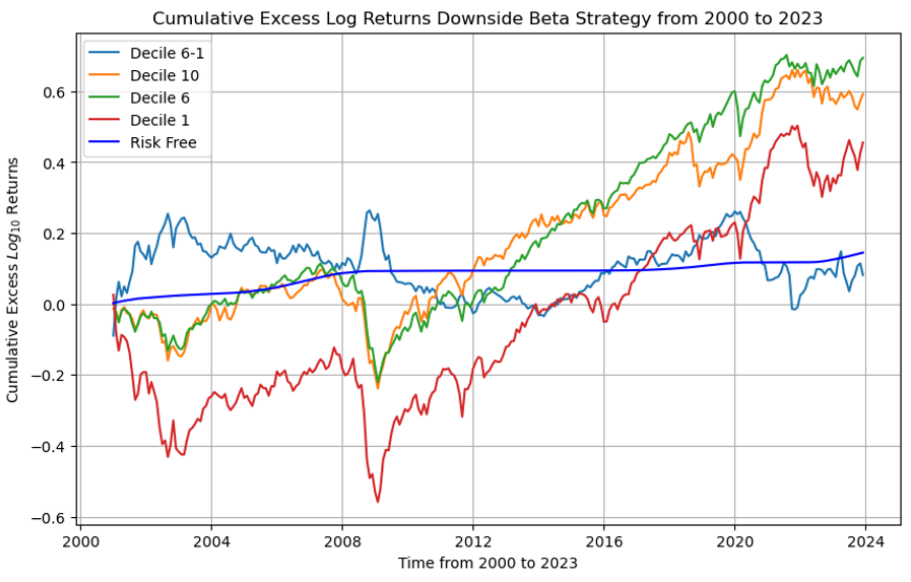

Figure 1: Cumulative Excess Log Returns

The figure firstly shows that by investing not in decile 1 by investing in decile 6 or 10 portfolios, you would have made 20% more in excess returns across the 23 year period which is a significant improvement.

Ontop of this, it is clear that the factor net neutral portfolio has finished positive across the period but does perform differently in different periods. For example, on the onset of market crashes such as the dot come bubble collapse at the beginning of this time period or the Global financial crisis in 2008, it performs very well. That is broadly due to the negative market beta this strategy has, i.e. it performs well when stocks perform badly. However, whilst it does give up some of its gains upon market recovery, due to the positive alpha it does not give up all of them which is positive.

Whilst the concluion from this analysis and the graph is that this factor is not a game changer and does not produce huge amounts of alpha, it is nonetheless a useful factor. Particularly due to its outperformance in market downturns, it is likely a factor you want to tilt slightly towards and more so in periods of market volatility.

5 Main Costs and Risks

5.1 Main Costs

1. **Data and Infrastructure Costs:** Access to high-quality financial data (e.g., from WRDS) and the need for computational resources to handle data processing and analysis incur significant costs.

2. **Transaction costs:** When trading month to month, rebalancing the portfolio will lead to transaction costs being incurred which will lower its returns.

5.2 Main Risks

1. **Changing Betas:** A risk factor in the strategy is the potential for changing betas over time. Betas, can fluctuate due to changes in market conditions, company-specific events, or macroeconomic factors. This variability can impact the reliability of the calculated rolling and downside betas, leading to potential misestimations of the stock's true market sensitivity. Also the 12 month beta estimation period may not be optimum.

2. **Model Risk:** Potential inaccuracies in statistical models used for calculating downside beta can lead to incorrect investment decisions.

3. **Market Risk:** Despite a focus on minimizing downside risk, there is no guarantee that stocks with lower downside betas will not lose value in a downturn.

4. **Overfitting Risk:** The strategy might become too finely tuned to historical data, performing well on past data but poorly in actual trading conditions.

5. **Liquidity Risk:** During market downturns, executing trades at favorable prices might be challenging, especially if the strategy involves less liquid stocks.

6. **Timing Risk:** The strategy's performance depends significantly on timing the market, which is inherently difficult and unpredictable.

5.3 Code wise based Risk

Our script calculates and utilizes rolling betas and makes investment decisions based on these metrics along with market conditions defined by moving averages. This approach assumes that past relationships and trends will persist, exposing it to risks of model inaccuracies and timing errors. The complexity and resource intensity of the strategy imply higher operational and infrastructure costs.

6 Reference

[1]Ang A, Chen J, Xing Y. Downside risk[J]. The review of financial studies, 2006, 19(4): 1191-1239.