8

ACTIVE FILTERS

The operational amplifier, especially the integrated-circuit operational amplifier, proves to be an extremely useful active device in the realization of active RC networks. Operational amplifiers have high input impedance, low output impedance, large open-loop gain, and low cost. These qualities are used to advantage in the circuits to be discussed in this chapter. Enough has been discussed about operational amplifiers in the first chapters of this book so that we will not dwell on their properties here.

We will begin our discussion of active filters in Sec. 8.1 by making statements about active filters in general. Then in Sec. 8.2 we will discuss transfer functions and their parameters. Useful formulas are presented to help evaluate the effects of tolerances and temperature coefficients of resistors and capacitors. In Sec. 8.3 we will then describe several realizations and provide design equations and sensitivity equations. The basic sensitivity relations are derived and discussed in Appendix C. After we have become familiar with the circuit realizations, we will discuss tuning (Sec. 8.4), how operational amplifier characteristics affect filter performance (Sec. 8.5), and, briefly, the characteristics of resistors and capacitors (Sec. 8.6). The chapter concludes with a set of filter design and tuning tables (Sec. 8.7).

8.1 Active Filter Characteristics

The active element, in this case the operational amplifier, in active networks is necessary to permit the realization of complex left hand plane poles using only resistors and capacitors for the passive elements. The operational amplifier permits the use of reasonable-valued resistors and capacitors even at frequencies as low as 10^{-3} Hz. An added bonus is the isolation afforded by the low output impedances of individual stages so that network stages can be designed and tuned independently with minimal interaction. Other active elements, the negative immittance converter and the gyrator, can be implemented with operational amplifiers but for practical reasons are not widely used.

Active filters have some characteristics of their own that make them sufficiently different from passive filters that one who uses them must be aware of these differences. For example, active filters usually have single-ended inputs and outputs and thus do not "float" with respect to the system power supply or common as a passive RLC network can. Amplifiers used for active elements have a limited input and output voltage range (± 10 V for most operational amplifier circuits) and an output current capability of a few milliamperes.

The outputs of active filters built with operational amplifiers have a dc voltage offset which drifts with ambient temperature changes. The voltage offset might range from a few microvolts up to several hundred millivolts. Drifts may range from 1 to 100 μV/°C or even more from a multiple-pole low-pass filter built up of many pole-pair stages. The inputs of active filters may have a bias current; this would be true for low-pass and band-reject filters and may be true for bandpass and high-pass filters, depending upon the particular circuit realization. The bias current may range from a few picoamperes for field-effect transistor operational amplifiers to a few microamperes for bipolar transistor and integrated-circuit amplifiers.

Active filters can provide excellent isolation capabilities, that is, a high input impedance ranging from a few kilohms to several thousand megohms if input buffer amplifiers are used, and a low output impedance ranging from a few hundred ohms down to less than 1 Ω. Unity-gain bandwidths as high as 100 MHz are available in operational amplifiers and permit filter designs in the vicinity of 1 Mc. Slewing rate, which is related to full-power response, is the limiting factor for large-signal characteristics. Frequencies as low as 10^{-3} Hz are possible, but filters at these frequencies can become rather bulky because of capacitor sizes. Active filters can have voltage gain, as much as 40 dB in low-frequency low-Q filters.

The primary advantage of active filters is their small size and weight for low-frequency applications and their ruggedness.

All types of responses are possible: the old standards, Butterworth, Chebyshev, and Bessel (Thompson), single-tuned and stagger-tuned bandpass as well as other responses that meet special needs.

The range of Q's possible for active filters extends up to Q's of a few hundred. However, high-Q networks capable of maintaining stability of their characteristics, in the face of element changes with time, temperature, voltage, and frequency, require more expensive (and usually larger size) resistors and capacitors and generally more operational amplifiers than low-Q (less than 10) filters. These facts will become apparent as individual circuits are discussed.

8.2 Pole Pairs, Network Functions, and Parameters

The circuits to be described realize a single-pole or a single complex pole pair. More complicated filters are then built up from these individual building blocks. This approach permits ease of design and tuning of a complex filter, an important practical matter, by reducing interactions between elements. This approach also permits a single systematic approach to answering the question: What happens to the response of a filter if the network element values are not accurate and if they drift with time and temperature?

The filter network functions that are of most interest are magnitude, phase, and group delay. The network parameters that are important are some characteristic frequency, Q, and passband gain. In this section these functions and parameters will be briefly examined for single-pole and complex-pole-pair networks for low-pass, high-pass, and bandpass networks. These relations will all be useful in the next section for deriving the sensitivity functions of these network realizations.

8.2.1 Low-pass network functions

Single Pole. The single-pole low-pass transfer function in the complex frequency variables is

$$H(s) = \frac{H_o\omega_o}{s + \omega_o}$$

The magnitude of the transfer function for the response to sinusoidal steady-state excitation is

$$|H(j\omega)| = G(\omega) = \left(\frac{H_o^2\omega_o^2}{\omega^2 + \omega_o^2}\right)^{1/2}$$

The phase is

$$\phi(\omega) = -\arctan\frac{\omega}{\omega_o}$$

and the group delay is

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} = \frac{\cos^2\phi}{\omega_o}$$

Complex Conjugate Pole Pair. The complex-conjugate-pole-pair low-pass transfer function and the sinusoidal steady-state magnitude and phase functions are

$$H(s) = \frac{H_o\omega_o^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$|H(j\omega)| = G(\omega) = \left[\frac{H_o^2\omega_o^4}{\omega^4 + \omega^2\omega_o^2(\alpha^2 - 2) + \omega_o^4}\right]^{1/2}$$

$$\phi(\omega) = -\arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} + \sqrt{4 - \alpha^2}\right)\right] - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} - \sqrt{4 - \alpha^2}\right)\right]$$

The relation for phase given above is expressed in a form suitable for general computer use since, on many computers, the arctan function can be determined only for the principal angle. Note that α^2 is usually never greater than 4. If it is, the poles will no longer be complex. The Q of a complex pole pair equals $1/\alpha$.

The group delay for a complex conjugate pole pair is

$$\tau(\omega) = \frac{2\sin^2\phi}{\alpha\omega_o} - \frac{\sin 2\phi}{2\omega}$$

8.2.2 High-pass network functions

Single Pole. The single-pole high-pass transfer function and the sinusoidal steady-state magnitude, phase, and delay functions are

$$H(s) = \frac{H_o s}{s + \omega_o}$$

$$G(\omega) = \left(\frac{H_o^2\omega^2}{\omega^2 + \omega_o^2}\right)^{1/2}$$

$$\phi(\omega) = \frac{\pi}{2} - \arctan\frac{\omega}{\omega_o}$$

$$\tau(\omega) = \frac{\sin^2\phi}{\omega_o}$$

Complex Conjugate Pole Pair. The complex-conjugate-pole-pair high-pass transfer function and the sinusoidal steady-state magnitude, phase, and delay functions are

$$H(s) = \frac{H_o s^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$G(\omega) = \left[\frac{H_o^2\omega^4}{\omega^4 + \omega^2\omega_o^2(\alpha^2 - 2) + \omega_o^4}\right]^{1/2}$$

$$\phi(\omega) = \pi - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} + \sqrt{4 - \alpha^2}\right)\right] - \arctan\left[\frac{1}{\alpha}\left(2\frac{\omega}{\omega_o} - \sqrt{4 - \alpha^2}\right)\right]$$

$$\tau(\omega) = \frac{2\sin^2\phi}{\alpha\omega_o} - \frac{\sin 2\phi}{2\omega}$$

8.2.3 Bandpass network function The complex-conjugate-pole-pair bandpass transfer function is

$$H(s) = \frac{H_o\alpha\omega_o s}{s^2 + \alpha\omega_o s + \omega_o^2}$$

where

$$\alpha = \frac{1}{Q} \qquad \text{and} \qquad Q = \frac{\omega_o}{\omega_2 - \omega_1} = \frac{f_o}{f_2 - f_1}$$

and where f_2 and f_1 are the frequencies where the magnitude response is -3 dB from H_o, the passband gain which occurs at $\omega_o = 2\pi f_o$. The sinusoidal steady-state transfer function may be written in the form

$$H(j\omega) = \frac{H_o}{1 + jQ(\omega/\omega_o - \omega_o/\omega)}$$

Thus, the magnitude, phase, and delay functions are

$$G(\omega) = \left[\frac{H_o^2}{1 + Q^2(\omega/\omega_o - \omega_o/\omega)^2}\right]^{1/2} = \left[\frac{H_o^2\alpha^2\omega_o^2\omega^2}{\omega^4 + \omega^2\omega_o^2(\alpha^2 - 2) + \omega_o^4}\right]^{1/2}$$

$$\phi(\omega) = \frac{\pi}{2} - \arctan\left(\frac{2Q\omega}{\omega_o} + \sqrt{4Q^2 - 1}\right) - \arctan\left(2Q\frac{\omega}{\omega_o} - \sqrt{4Q^2 - 1}\right)$$

$$\tau(\omega) = \frac{2Q\cos^2\phi}{\omega_o} + \frac{\sin 2\phi}{2\omega}$$

8.2.4 Band-reject network function A band-reject filter can be realized by performing the operation $1 - H_{BP}(s)$, where $H_{BP}(s)$ is a bandpass

transfer function (Fig. 8.1). If $R' = RH_o$ (bandpass)

$$H(s) = -\frac{s^2 + \omega_o^2}{s^2 + \alpha\omega_o s + \omega_o^2}\frac{R_F}{R}$$

Since this filter is very closely related to the bandpass filter, its properties will not be discussed.

8.3 Filter Realizations[1,2]

In this section we shall present some realizations for active filters. The operational amplifier filter circuits to be analyzed, and for which design procedures and sensitivity equations are given, are the infinite-gain multiple feedback, controlled source, infinite-gain state-variable feedback, and negative immittance converter realizations. Another realization sometimes used is the infinite-gain single-feedback type. This type involves the use of bridged-T and twin-T networks as well as requiring the cancellation of unwanted zeros and poles. Thus this type requires many elements to realize a transfer function with complex poles and is therefore uneconomical. Trimming and adjustment of bridged-T or twin-T networks is difficult since the passive elements interact to a high degree in such networks. For these reasons this circuit will not be discussed. Single real pole realizations will not be shown since these are rather trivial and easy to design. In addition, single-operational-amplifier single-pole circuits are rather uneconomical.

Design procedures given in this section are only suggested procedures. Other choices are possible, and as one gains experience with these circuits, it becomes desirable to design procedures for minimizing sensitivity in certain network parameters or to ensure a convenient spread of element values. In the design procedures given, the capacitors are always made equal. In addition, one usually starts the design process by selecting the capacitor value because there are fewer standard values of capacitors than there are resistors. Resistors are less expensive than capacitors and are more easily used in trimming schemes. In some cases the passband gain

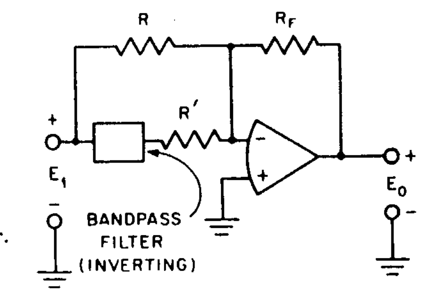

Fig. 8.1 Band-reject filter.

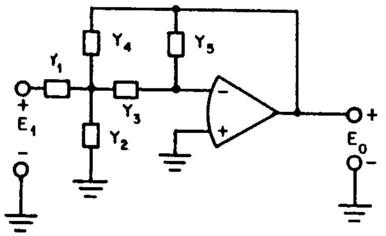

Fig. 8.2 Infinite-gain multiple-feedback circuit.

H_o is a free parameter. It is often convenient to let H_o be a variable which can be used as a parameter for determining optimum, or at least small, sensitivities of certain parameters. An example of this is given in the controlled-source circuit designs. Also, letting H_o be a free parameter simplifies complicated design equations.

8.3.1 Infinite-gain multiple-feedback circuits Figure 8.2 illustrates the infinite-gain multiple-feedback connection for a pair of complex conjugate s-plane poles with zeros restricted to the origin or infinity. The amplifier is used in its inverting configuration, with the + input grounded. Each element Y_i represents a single resistor or capacitor. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-Y_1Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3Y_4 + (1/A_{OL})[(Y_3 + Y_5)(Y_1 + Y_2 + Y_4) + Y_3Y_5]}$$

In the limiting case as A_{OL} approaches infinity we obtain

$$\frac{E_o}{E_1}(s) = \frac{-Y_1Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3Y_4}$$

Examples that follow show how these five elements may be chosen so as to realize low-pass, high-pass and bandpass network functions.

Low Pass. The infinite-gain multiple-feedback circuit for a low-pass network function is shown in Fig. 8.3. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-1/R_1R_3C_2C_5}{s^2 + (s/C_2)(1/R_1 + 1/R_3 + 1/R_4) + 1/R_3R_4C_2C_5}$$

Note that this circuit produces a signal inversion, as will all circuits realized by this technique.

For this circuit, following the notation of the low-pass network function,

$$H_o = \frac{R_4}{R_1}$$

$$\omega_o = \left(\frac{1}{R_3R_4C_2C_5}\right)^{1/2}$$

$$\alpha = \sqrt{\frac{C_5}{C_2}}\left(\sqrt{\frac{R_3}{R_4}} + \sqrt{\frac{R_4}{R_3}} + \frac{\sqrt{R_3R_4}}{R_1}\right)$$

$$\phi = \pi + \phi_{LP}$$

$$\tau = \tau_{LP}$$

Note that the phase inversion has been incorporated into the phase function. A tuning procedure for this circuit would be first to adjust ω_o with R_3 at a frequency of 10ω, as outlined in the section on tuning. Then adjust α with R_1 at the α peaking frequency.

The sensitivities of the network parameters to circuit element changes follow. Remember that the open-loop gain of the operational amplifier is assumed to be infinite (at least very large), and so sensitivity functions for open-loop gain changes are not considered.

$$S_{R_3}^{\omega_o} = S_{R_4}^{\omega_o} = S_{C_2}^{\omega_o} = S_{C_5}^{\omega_o} = -\frac{1}{2}$$

$$S_{C_5}^{\alpha} = -S_{C_2}^{\alpha} = \frac{1}{2}$$

$$S_{R_1}^{\alpha} = \frac{1}{\alpha\omega_o R_1C_2}$$

$$S_{R_3}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_3C_2}$$

$$S_{R_4}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_4C_2}$$

$$S_{R_4}^{H_o} = -S_{R_1}^{H_o} = 1$$

Note that $S_{C_2}^{\alpha}$ and $S_{C_5}^{\alpha}$ are constant and opposite in sign and so are $S_{R_1}^{H_o}$ and $S_{R_4}^{H_o}$.

DESIGN PROCEDURE

Given: H_o, α, $\omega_o = 2\pi f_o$

Choose: $C_2 = C$, a convenient value

$C_5 = KG$

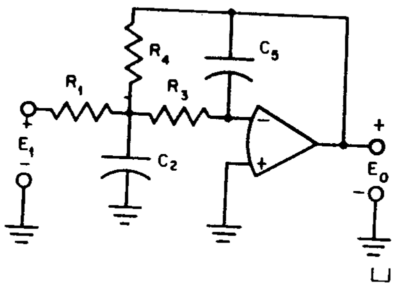

Fig. 8.3 Multiple-feedback low-pass filter.

$$\text{Calculate: } R_4 = \frac{\alpha}{2\omega_o C}\left[1 \pm \sqrt{1 - \frac{4(H_o + 1)}{K\alpha^2}}\right]$$

$$R_1 = \frac{R_4}{H_o}$$

$$R_3 = \frac{1}{\omega_o^2 C^2 R_4 K}$$

For best results H_o should be less than 10 for circuits with an α of about 0.1 ($Q = 10$) and can be as high as 100 for an α of about 1 ($Q = 1$) or less. These extreme limits assume that the operational amplifier has an open-loop gain of at least 80 dB at the frequency of interest. The effects of finite open-loop gain for multiple feedback circuits will be discussed later.

High Pass. A high-pass realization is illustrated in Fig. 8.4. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-(C_1/C_4)s^2}{s^2 + s(1/R_5)(C_1/C_3C_4 + 1/C_4 + 1/C_3) + 1/R_2R_5C_3C_4}$$

In terms of our high-pass network function

$$H_o = \frac{C_1}{C_4}$$

$$\omega_o = \left(\frac{1}{R_2R_5C_3C_4}\right)^{1/2}$$

$$\alpha = \sqrt{\frac{R_2}{R_5}}\left(\frac{C_1}{\sqrt{C_3C_4}} + \sqrt{\frac{C_3}{C_4}} + \sqrt{\frac{C_4}{C_3}}\right)$$

$$\phi = \pi + \phi_{HP}$$

$$\tau = \tau_{HP}$$

Tuning this high-pass filter will have to be done in the reverse order to that of the low-pass filter. First, adjust α with R_2 or R_5 at the frequency where the α peak occurs (the ω_α frequency is not known

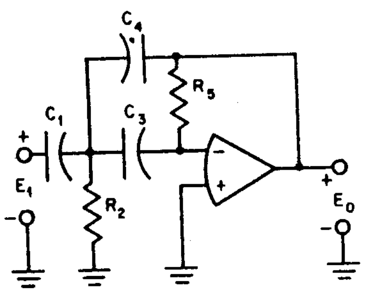

Fig. 8.4 Multiple-feedback high-pass filter.

because ω_o has not been set yet). Then adjust ω_o by adjusting R_2 and R_5 simultaneously by the *same* percentage: α will remain constant. A trimming scheme involving C_1 would be simpler. The sensitivities to element value changes are

$$S_{R_2}^{\omega_o} = S_{R_5}^{\omega_o} = S_{C_3}^{\omega_o} = S_{C_4}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_2}^{\alpha} = -S_{R_5}^{\alpha} = \frac{1}{2}$$

$$S_{C_3}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_5 C_3}\left(\frac{C_1}{C_3} + 1\right)$$

$$S_{C_4}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_5 C_4}\left(\frac{C_1}{C_3} + 1\right)$$

$$S_{C_1}^{\alpha} = \frac{1}{\alpha\omega_o R_5}\frac{C_1}{C_3C_4}$$

$$S_{C_1}^{H_o} = -S_{C_4}^{H_o} = 1$$

DESIGN PROCEDURE

Given: H_o, α, $\omega_o = 2\pi f_o$

Choose: $C = C_1 = C_3$, a convenient value

$$\text{Calculate: } R_5 = \frac{1}{\alpha\omega_o C}(2H_o + 1)$$

$$R_2 = \frac{\alpha}{\omega_o C(2H_o + 1)}$$

$$C_4 = \frac{C_1}{H_o}$$

Again, restrictions on H_o are the same as those for the low-pass case. Note that this realization requires three capacitors, a feature which might make it undesirable when compared with other circuits.

Bandpass 1. There are several configurations of the five elements which may be used to realize a bandpass function. One of the more practical configurations is the one shown in Fig. 8.5. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-s(1/R_1C_4)}{s^2 + s(1/R_5)(1/C_3 + 1/C_4) + (1/R_5C_3C_4)(1/R_1 + 1/R_2)}$$

In terms of our bandpass network function

$$H_o = \frac{1}{(R_1/R_5)(1 + C_4/C_3)}$$

$$\omega_o = \left[\frac{1}{R_5C_3C_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]^{1/2}$$

$$\frac{1}{Q} = \alpha = \sqrt{\frac{1}{R_5(1/R_1 + 1/R_2)}}\left[\sqrt{\frac{C_3}{C_4}} + \sqrt{\frac{C_4}{C_3}}\right]$$

$$\phi = \pi + \phi_{BP}$$

$$\tau = \tau_{BP}$$

Tuning this filter appears rather formidable. In practice $R_1 \gg R_2$ and so R_2 can be used to trim the Q. Then, to adjust the center frequency, R_2 and R_5 can be simultaneously adjusted by the same percentage with negligible effect on the Q.

The sensitivities of the network parameters with respect to the elements are

$$S_{R_5}^{\omega_o} = S_{C_3}^{\omega_o} = S_{C_4}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_1}^{\omega_o} = \frac{-1}{2\omega_o^2R_1R_5C_3C_4}$$

$$S_{R_2}^{\omega_o} = \frac{-1}{2\omega_o^2R_2R_5C_3C_4}$$

$$S_{R_1}^{Q} = \frac{R_1}{2(R_1 + R_2)} - \frac{1}{2}$$

$$S_{R_2}^{Q} = \frac{R_2}{2(R_1 + R_2)} - \frac{1}{2}$$

$$S_{R_5}^{Q} = \frac{1}{2}$$

$$S_{C_3}^{Q} = \frac{Q}{\omega_oR_5C_3} - \frac{1}{2}$$

$$S_{C_4}^{Q} = \frac{Q}{\omega_oR_5C_4} - \frac{1}{2}$$

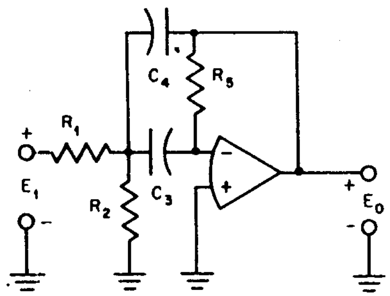

Fig. 8.5 Multiple-feedback bandpass filter.

DESIGN PROCEDURE

Given: H_o, $Q = \frac{1}{\alpha}$, $\omega_o = 2\pi f_o$

Choose: $C = C_3 = C_4$

Calculate: $Q = \frac{1}{\alpha}$

$$R_1 = \frac{Q}{H_o\omega_oC}$$

$$R_2 = \frac{Q}{(2Q^2 - H_o)\omega_oC}$$

$$R_5 = \frac{2Q}{\omega_oC}$$

Again, restrictions on H_o apply to guarantee that the design equations give fairly accurate results.

Bandpass 2. Another multiple-feedback circuit uses an additional active element to overcome some of the disadvantages of the single-amplifier circuit, especially the bandpass realization for Q's roughly between 10 and 50. High Q's realized with bandpass 1 have large spreads of element values and high Q sensitivities to element value changes. The multiple-feedback circuit with positive feedback is shown in Fig. 8.6. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{s(K/R_1C_4)}{s^2 + (s/R_5C_4)(1 + C_4/C_3 - KR_5/R_6) + (1/C_3C_4R_5)(1/R_1 + 1/R_2 + 1/R_6)}$$

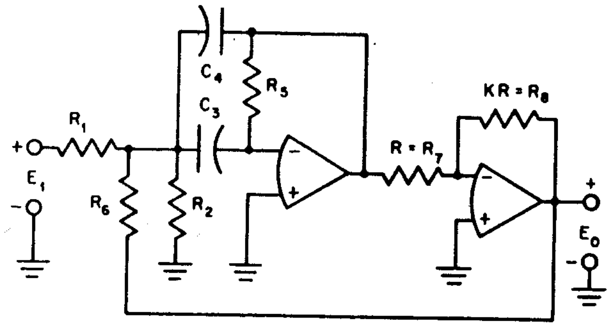

Fig. 8.6 Multiple-feedback bandpass circuit with positive feedback.

Note that the output is taken at the second amplifier. The overall signal transfer is noninverting. The circuit parameters are

$$H_o = \frac{1}{R_1}\,\frac{1}{(1/KR_5)(1 + C_4/C_3) - 1/R_6}$$

$$\omega_o = \left[\frac{1}{R_5C_3C_4}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_6}\right)\right]^{1/2}$$

$$\frac{1}{Q} = \alpha = \sqrt{\frac{1}{R_5(1/R_1 + 1/R_2 + 1/R_6)}}\,\sqrt{\frac{C_3}{C_4}}\left(1 + \frac{C_4}{C_3} - \frac{KR_5}{R_6}\right)$$

$$\phi = \phi_{BP}$$

$$\tau = \tau_{BP}$$

Since R_1 and R_6 are larger than R_2, R_2 is used to trim the center frequency. Note that in this circuit Q can be adjusted with K without influencing ω_o. The sensitivity of the network parameters to element value changes are

$$S_{R_1}^{H_o} = -1$$

$$S_{C_3}^{H_o} = -S_{C_4}^{H_o} = \frac{H_o}{K}\,\frac{R_1}{R_5}\,\frac{C_4}{C_3}$$

$$S_K^{H_o} = S_{R_5}^{H_o} = \frac{H_o}{K}\,\frac{R_1}{R_5}\left(1 + \frac{C_4}{C_3}\right)$$

$$S_{R_6}^{H_o} = -H_o\,\frac{R_1}{R_6}$$

$$S_{C_3}^{\omega_o} = S_{C_4}^{\omega_o} = S_{R_5}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_1}^{\omega_o} = \frac{-1}{2\omega_o^2R_1R_5C_3C_4}$$

$$S_{R_2}^{\omega_o} = \frac{-1}{2\omega_o^2R_2R_5C_3C_4}$$

$$S_{R_6}^{\omega_o} = \frac{-1}{2\omega_o^2R_5R_6C_3C_4}$$

$$S_{R_1}^{Q} = \frac{-1}{2\omega_o^2R_1R_5C_3C_4}$$

$$S_{R_2}^{Q} = \frac{-1}{2\omega_o^2R_2R_5C_3C_4}$$

$$S_{R_6}^{Q} = -\frac{1}{2}\,\frac{1}{(1 + R_6/R_1 + R_6/R_2)} - \frac{1}{(R_6/KR_5)(1 + C_4/C_3) - 1}$$

$$S_{C_3}^{Q} = \frac{Q}{\omega_oR_5C_3} - \frac{1}{2}$$

$$S_{C_4}^{Q} = \frac{Q}{\omega_oR_5C_4}\left(1 - \frac{KR_5}{R_6}\right) - \frac{1}{2}$$

$$S_{R_5}^{Q} = \frac{Q}{\omega_oR_5C_4}\left(1 + \frac{C_4}{C_3}\right) - \frac{1}{2}$$

$$S_K^{Q} = \frac{-KQ}{\omega_oR_6C_4}$$

$$S_{R_6}^{K} = -S_{R_5}^{K} = 1$$

DESIGN PROCEDURE

Given: $Q = 1/\alpha$, $\omega_o = 2\pi f_o$
H_o must be a free parameter.
Choose: $C = C_3 = C_4$, $R = R_1 = R_5$
K is chosen to reduce the spread of element values or to optimize sensitivity. It might typically be between 1 and 10.

Calculate: $R = \dfrac{Q}{\omega_oC}$

$$R_6 = R\,\frac{KQ}{2Q - 1}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{R}\left(Q - 1 - \frac{2}{K} + \frac{1}{KQ}\right)$$

For this procedure, $H_o = \sqrt{Q}\,K$.

This completes the section on infinite-gain multiple-feedback realizations. A few general comments are in order. An advantage of this realization is that the output impedance is low; thus networks may be cascaded with negligible interaction. A disadvantage is that it is not possible to obtain high Q without resorting to large spreads of element values and also incurring large Q sensitivities. The multiple-feedback realization with positive feedback can overcome this and allow reasonable sensitivities up to a Q of 50.

8.3.2 Controlled-source circuits A noninverting voltage-controlled voltage source (VCVS) implemented with an operational amplifier is illustrated in Fig. 8.7. The input impedance is very large, tens to hundreds of thousands of megohms, depending upon the type of operational amplifier, and the output impedance is very low, usually less than 1 Ω for K between 1 and 10. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = 1 + \frac{R_b}{R_a} = K$$

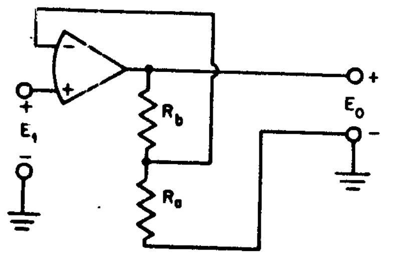

Fig. 8.7 Noninverting operational amplifier VCVS.

The sensitivities of K to the two resistors are

$$S_{R_b}^{K} = 1$$
$$S_{R_a}^{K} = -1$$

Figure 8.8 shows the controlled-source connection for a circuit which may be used to realize voltage transfer functions with a single pair of complex conjugate s-plane poles with zeros restricted to the origin or infinity. The Y_i are restricted to be single elements, R's and C's. These five elements may be chosen so as to realize low-pass, high-pass, and bandpass network functions. Realizations are possible with $K < 0$; but, since this operational amplifier circuit always has K greater than +1, these will not be discussed. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{KY_1Y_4}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + [Y_1 + Y_2(1 - K) + Y_3]}$$

Low Pass. A VCVS circuit for a low-pass network function is shown in Fig. 8.9. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{K/R_1R_2C_1C_2}{s^2 + s[1/R_1C_1 + 1/R_2C_1 + (1 - K)/R_2C_2] + 1/R_1R_2C_1C_2}$$

The network parameters are

$$H_o = K$$
$$\omega_o = \left(\frac{1}{R_1R_2C_1C_2}\right)^{1/2}$$

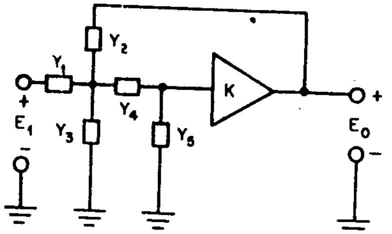

Fig. 8.8 VCVS configuration for a second-degree voltage transfer function.

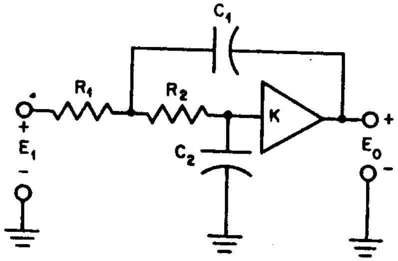

Fig. 8.9 VCVS low-pass network.

$$\alpha = \left(\frac{R_2C_2}{R_1C_1}\right)^{1/2} + \left(\frac{R_1C_2}{R_2C_1}\right)^{1/2} + \left(\frac{R_1C_1}{R_2C_2}\right)^{1/2} - K\left(\frac{R_1C_1}{R_2C_2}\right)^{1/2}$$
$$\phi = \phi_{LP}$$
$$\tau = \tau_{LP}$$

Controlled-source circuits are easier to tune than other circuit realizations. In fact, they can be adjusted over wide ranges without interaction of the network parameters. ω_o is tuned by adjusting R_1 and R_2 by equal percentages: α will not be affected. Capacitance C_1 and C_2 can be adjusted in the same way for the same result. α is trimmed by adjusting K. The sensitivities of the network parameters to element value changes are

$$S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2}$$
$$S_K^{H_o} = 1$$
$$S_{R_1}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_1C_1}$$
$$S_{R_2}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o R_2}\left(\frac{1}{C_1} + \frac{1 - K}{C_2}\right)$$
$$S_{C_1}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$S_{C_2}^{\alpha} = \frac{1}{2} - \frac{1 - K}{\alpha\omega_o R_2C_2}$$
$$S_K^{\alpha} = \frac{-K}{\alpha\omega_o R_2C_2}$$

DESIGN PROCEDURE

Given: H_o, α, $\omega_o = 2\pi f_o$
Choose: $C_1 = C_2 = C$, a convenient value
Calculate: $K = H_o > 2$

$$R_2 = \frac{\alpha}{2\omega_o C}\left[1 + \sqrt{1 + \frac{4(H_o - 2)}{\alpha^2}}\right]$$

$$R_1 = \frac{1}{\omega_o^2 C^2 R_2}$$

If H_o is large, say greater than 10, there will be large spreads in element values and high sensitivities. An interesting design procedure is to use K to vary the sensitivities of circuit parameters.

Capacitors are often the components which have the largest temperature coefficients. It is possible to set K such that the overall α sensitivity is minimum, assuming that the capacitors drift equally. In this case we set $S_{C_1}{}^{\alpha} = -S_{C_2}{}^{\alpha}$.

Choose $C = C_1 = C_2$ and let $R_1 = R_2 = R$; then $K = 3 - \alpha$ and $R = 1/\omega_o C$.

High Pass. A VCVS circuit realization of a high-pass network function is shown in Fig. 8.10. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{Ks^2}{s^2 + s[1/R_2C_1 + 1/R_2C_2 + (1 - K)/R_1C_1] + 1/R_1R_2C_1C_2}$$

The network parameters are

$$H_o = K$$

$$\omega_o = \left(\frac{1}{R_1R_2C_1C_2}\right)^{1/2}$$

$$\alpha = \left(\frac{R_1C_1}{R_2C_2}\right)^{1/2} + \left(\frac{R_1C_2}{R_2C_1}\right)^{1/2} + \left(\frac{R_2C_2}{R_1C_1}\right)^{1/2} - K\left(\frac{R_2C_2}{R_1C_1}\right)^{1/2}$$

The same comments about frequency adjustment and tuning that we mentioned in the low-pass case apply for the high-pass case also. The network parameter sensitivities with respect to element value changes are

$$S_{R_1}{}^{\omega_o} = S_{R_2}{}^{\omega_o} = S_{C_1}{}^{\omega_o} = S_{C_2}{}^{\omega_o} = -\frac{1}{2}$$

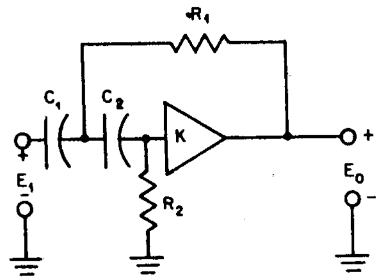

Fig. 8.10 VCVS high-pass network.

$$S_{R_1}{}^{\alpha} = \frac{1}{2} - \frac{1 - K}{R_1C_1\alpha\omega_o}$$

$$S_{R_2}{}^{\alpha} = \frac{1}{2} - \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{\alpha\omega_o}$$

$$S_{C_1}{}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o C_1}\left(\frac{1 - K}{R_1} + \frac{1}{R_2}\right)$$

$$S_{C_2}{}^{\alpha} = \frac{1}{2} - \frac{1}{\alpha\omega_o C_2 R_2}$$

$$S_K{}^{\alpha} = \frac{-K}{\alpha\omega_o R_1 C_1}$$

$$S_K{}^{H_o} = 1$$

DESIGN PROCEDURE

Given H_o, α, $\omega_o = 2\pi f_o$

Choose $C_1 = C_2 = C$

Calculate: $R_1 = \dfrac{\alpha + \sqrt{\alpha^2 + 8(H_o - 1)}}{4\omega_o C}$

$$R_2 = \frac{4}{\omega_o C}\frac{1}{\sqrt{\alpha^2 + 8(H_o - 1)}}$$

Naturally, $H_o = K$ must be such that R_1 and R_2 are positive-valued resistors. Again, a large H_o will result in a large spread of element values and high sensitivities. We can use the same scheme for making $S_{C_1}{}^{\alpha} = -S_{C_2}{}^{\alpha}$ as in the low-pass case.

Choose $C_1 = C_2 = C$; let $R_1 = R_2 = R$. Then $K = 3 - \alpha$ and $R = 1/\omega_o C$.

Bandpass 1. A VCVS realization for the bandpass network function is shown in Fig. 8.11. The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{Ks/R_1C_2}{s^2 + s\left[\frac{1}{R_3C_2} + \frac{1}{R_1C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1 - K}{R_2C_2}\right] + \frac{1}{R_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1C_2}}$$

The network parameters are

$$H_o = \frac{K}{1 + R_1/R_3 + C_2/C_1(1 + R_1/R_2) + (1 - K)(R_1/R_2)}$$

$$\omega_o = \left[\frac{1}{R_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1C_2}\right]^{1/2}$$

$$\frac{1}{Q} = \alpha = \sqrt{\frac{R_3}{(1/R_1 + 1/R_2)}}\left[\sqrt{\frac{C_1}{C_2}}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1 - K}{R_2}\right) + \sqrt{\frac{C_2}{C_1}}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]$$

The sensitivities of the network parameters to element changes are

$$S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = S_{R_3}^{\omega_o} - \frac{1}{2}$$

$$S_{R_1}^{\omega_o} = \frac{-1}{2\omega_o^2 R_1C_1R_3C_2} \quad S_{R_2}^{\omega_o} = \frac{-1}{2\omega_o^2 R_3C_1R_2C_2}$$

$$S_K^Q = \frac{+KQ}{\omega_o R_2C_2} \quad S_{R_1}^Q = \frac{-1}{2(1 + R_1/R_2)} + \frac{Q_1}{\omega_o R_1}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$S_{R_2}^Q = \frac{-1}{2(1 + R_2/R_1)} + \frac{Q}{\omega_o R_2}\left(\frac{1}{C_1} + \frac{1 - K}{C_2}\right)$$

$$S_{R_3}^Q = \frac{-1}{2} + \frac{Q}{\omega_o R_3C_2} \quad S_{C_1}^Q = -\frac{1}{2} + \frac{1}{\alpha\omega_o C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$S_{C_2}^Q = \frac{-1}{2} + \frac{Q}{\alpha\omega_o C_2}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1 - K}{R_2}\right)$$

$$S_K^{H_o} = 1 + \frac{Q}{\omega_o R_2C_2}$$

$$S_{R_1}^{H_o} = \frac{Q}{\omega_o R_1}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) - 1$$

$$S_{R_2}^{H_o} = \frac{Q}{\omega_o R_2}\left(\frac{1}{C_1} + \frac{1 - K}{C_2}\right)$$

$$S_{R_3}^{H_o} = \frac{Q}{\omega_o R_3C_2}$$

$$S_{C_1}^{H_o} = \frac{Q}{\omega_o C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$S_{C_2}^{H_o} = \frac{Q}{\omega_o C_2}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1 - K}{R_2}\right) - 1$$

DESIGN PROCEDURE

The *general* design formulas obtained by solving the network parameter equations for the circuit elements are very complicated. The following design procedure, however, has been found to be useful. It gives a fairly good spread of element values.

Given: Q, $\omega_o = 2\pi f_o$

H_o will be a free parameter,

Choose: $C = C_1 = C_2$, a convenient value

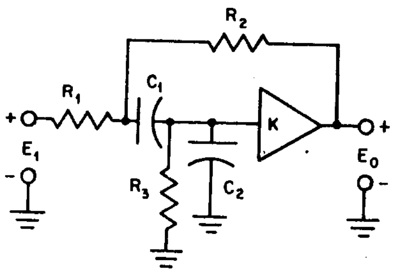

Fig. 8.11 VCVS bandpass network.

Calculate: $K = 5 - \frac{\sqrt{2}}{Q}$

$$R = \frac{\sqrt{2}}{\omega_o C}$$

Then

$$H_o = \frac{5}{\sqrt{2}}Q - 1$$

High-Q circuits will have a large spread of element values and high sensitivities. Q's should be less than 10 for best results.

Four other bandpass circuits are realizable by using the VCVS with $K > 0$. One is obtained by removing C_2 in the circuit of Fig. 8.11 and connecting one terminal to the node formed by $C_1 - R_1 - R_2$ and the other terminal to ground. Two others are generated by interchanging the locations of the resistors and capacitors in the circuit of Fig. 8.11 and the one mentioned above. These two are of less practical interest because they require three capacitors and one of the capacitors is a series capacitor at the input.

Bandpass 2. Still another bandpass realization is illustrated in Fig. 8.12. The voltage transfer function is

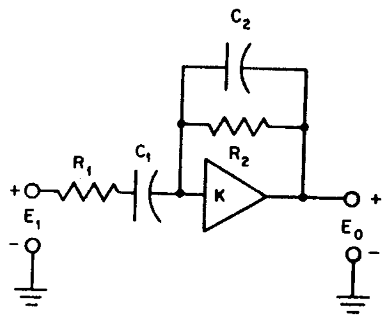

Fig. 8.12 Alternative VCVS bandpass network.

$$\frac{E_o}{E_1}(s) = \frac{s\frac{K}{1-K}\frac{1}{R_1C_2}}{s^2 + s\left[\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_1C_2(1-K)}\right] + \frac{1}{R_1R_2C_1C_2}}$$

$$H_o = \frac{K}{(1-K)(R_1/R_2 + C_2/C_1) + 1}$$

$$\omega_o = \left(\frac{1}{R_1R_2C_1C_2}\right)^{1/2}$$

$$\frac{1}{Q} = \alpha = \sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} - \frac{1}{1-K}\left(\sqrt{\frac{R_2C_1}{R_1C_2}}\right)$$

$$\phi = \pi + \phi_{BP}$$

$$\tau = \tau_{BP}$$

The center frequency can be trimmed by varying R_1 and R_2. If this is done simultaneously so that their ratio remains constant, Q will not change. Q can be trimmed with K. Note that there is a restriction on the *minimum* value K may have for stability. Because of this restriction, the passband gain H_o will be negative. The sensitivities of the network parameters to element value changes are

$$S_{R_1}{}^{\omega_o} = S_{R_2}{}^{\omega_o} = S_{C_1}{}^{\omega_o} = S_{C_2}{}^{\omega_o} = -\frac{1}{2}$$

$$S_K{}^{H_o} = 1 + H_o\left(\frac{R_1}{R_2} + \frac{C_2}{C_1}\right)$$

$$S_{R_2}{}^{H_o} = -S_{R_1}{}^{H_o} = H_o\frac{1-K}{K}\frac{R_1}{R_2}$$

$$S_{C_1}{}^{H_o} = -S_{C_2}{}^{H_o} = H_o\frac{1-K}{K}\frac{C_2}{C_1}$$

$$S_K{}^Q = \frac{-K}{(1-K)^2}\frac{Q}{\omega_oR_1C_2}$$

$$S_{R_1}{}^Q = \frac{1}{2} - \frac{Q}{\omega_oR_1}\left[\frac{1}{C_1} + \frac{1}{C_2(1-K)}\right]$$

$$S_{R_2}{}^Q = \frac{1}{2} - \frac{Q}{\omega_oR_2C_2}$$

$$S_{C_1}{}^Q = \frac{-1}{2} + \frac{1}{\alpha\omega_oR_1C_1}$$

$$S_{C_2}{}^Q = \frac{-1}{2} + \frac{1}{\alpha\omega_oC_2}\left[\frac{1}{R_2} + \frac{1}{R_1(1-K)}\right]$$

DESIGN PROCEDURE

Given: Q, $\omega_o = 2\pi f_o$

H_o is a free parameter,

Choose: $C = C_1 = C_2$, a convenient value

Calculate: $R_1 = R_2 = \frac{1}{\omega_o C}$

$$K = \frac{3Q-1}{2Q-1}$$

Then

$$|H_o| = 3Q - 1$$

Q should be limited to about 10.

A few general comments about controlled-source realizations are in order. The Q (or α) of a circuit may be adjusted independently of ω_o by adjusting K: it is not independent of H_o, however. Networks may be cascaded without interaction occurring between them. The frequency term ω_o can be adjusted independently of α for the low-pass and high-pass cases, as discussed earlier. The characteristics of the network are sensitive to K. The circuit becomes very Q-sensitive to element value changes for high Q's.

8.3.3 Infinite-gain state-variable circuits An infinite-gain state-variable network configuration is illustrated in Fig. 8.13. This configuration makes use of operational amplifiers in the same way they would

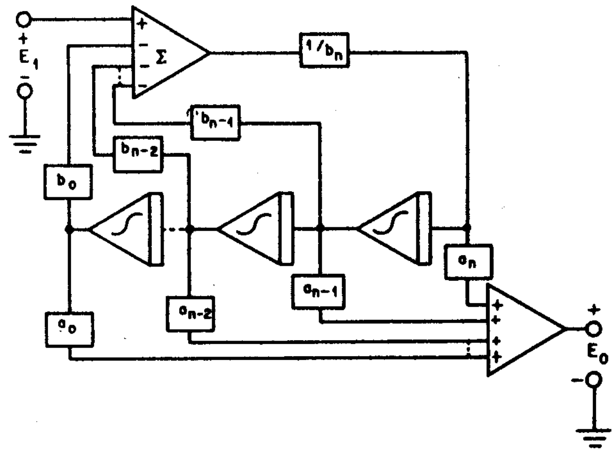

Fig. 8.13 State-variable infinite-gain network configuration.

be used in an analog computer realization of transfer functions (i.e., using integrators and summers). A second-order realization is shown in Fig. 8.14. Here the usual summing amplifier is replaced by a differentially connected operational amplifier to ease the spread in element values.

The voltage transfer function has the form

$$\frac{E_o}{E_1}(s) = \frac{a_o + a_1 s + \cdots + a_{n-1}s^{n-1} + a_n s^n}{b_o + b_1 s + \cdots + b_{n-1}s^{n-1} + b_n s_n}$$

The design procedures used in this section are simplified procedures in that $C_1 = C_2$, $R_1 = R_2$, and $R_5 = R_6$. We set $R_1 = R_2$ and $C_1 = C_2$ in order to scale adequately the output voltages of the operational amplifiers. The condition $R_5 = R_6$ further simplifies design calculations. Note that bandpass, low-pass, and high-pass realizations occur simultaneously. One merely chooses the output at a different point. In addition, one can sum the low-pass and high-pass outputs and form a pair of $j\omega$ axis zeros. The transfer functions are

$$\frac{E_{lp}}{E_1}(s) = \frac{\dfrac{1}{R_1R_2C_1C_2}\dfrac{1 + R_6/R_5}{1 + R_3/R_4}}{s^2 + s\dfrac{1}{R_1C_1}\dfrac{1 + R_6/R_5}{1 + R_4/R_3} + \dfrac{R_6}{R_5}\dfrac{1}{R_1R_2C_1C_2}}$$

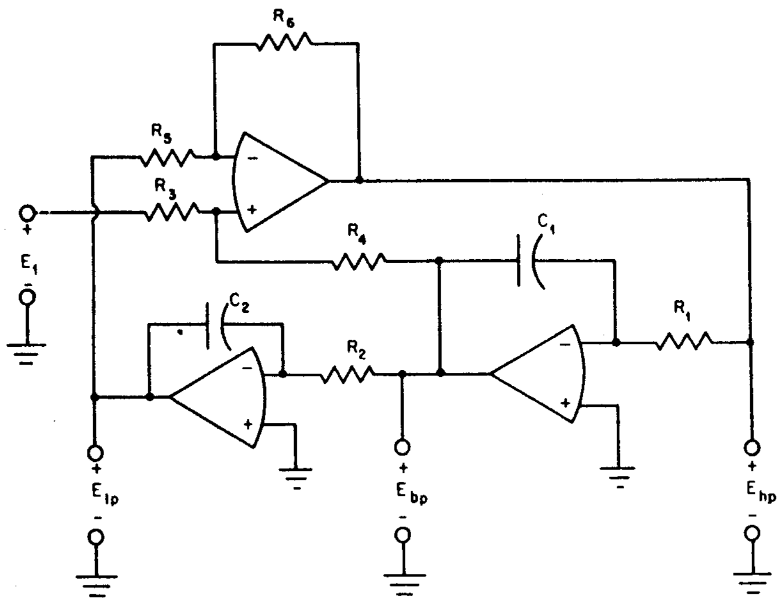

Fig. 8.14 Second-degree state-variable network.

$$\frac{E_{hp}}{E_1}(s) = \frac{s^2\dfrac{1 + R_6/R_5}{1 + R_3/R_4}}{s^2 + s\dfrac{1}{R_1C_1}\dfrac{1 + R_6/R_5}{1 + R_4/R_3} + \dfrac{R_6}{R_5}\dfrac{1}{R_1R_2C_1C_2}}$$

$$\frac{E_{bp}}{E_1}(s) = \frac{-s\dfrac{1}{R_1C_1}\dfrac{1 + R_6/R_5}{1 + R_3/R_4}}{s^2 + s\dfrac{1}{R_1C_1}\dfrac{1 + R_6/R_5}{1 + R_4/R_3} + \dfrac{R_6}{R_5}\dfrac{1}{R_1R_2C_1C_2}}$$

The state-variable realization in general provides less Q sensitivity to element variation than a single-amplifier realization and for this reason is sometimes used for high-Q bandpass applications ($Q > 50$). Of course it requires three amplifiers, which is a disadvantage. For the low-Q low-pass and high-pass applications, it is a rather expensive circuit to use. Since some filter manufacturers use this circuit as a basic building block, the low-pass and high-pass as well as the bandpass outputs are worth some discussion.

Low Pass. The network parameters for the low-pass function are

$$H_o = \frac{1 + R_5/R_6}{1 + R_3/R_4}$$

$$\omega_o = \left(\frac{R_6}{R_5R_1C_1R_2C_2}\right)^{1/2}$$

$$\alpha = \frac{1 + R_6/R_5}{1 + R_4/R_3}\left(\frac{R_5}{R_6}\frac{R_2C_2}{R_1C_1}\right)^{1/2}$$

$$\phi = \phi_{LP}$$

$$\tau = \tau_{LP}$$

The sensitivities of the network parameters to element value changes are

$$S_{R_5}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2} = -S_{R_6}^{\omega_o}$$

$$S_{R_2}^{\alpha} = S_{C_2}^{\alpha} = \frac{1}{2} = -S_{R_1}^{\alpha} = -S_{C_1}^{\alpha}$$

$$S_{R_6}^{\alpha} = -\frac{1}{2} + \frac{R_6/R_5}{R_1C_1\alpha\omega_o(1 + R_4/R_3)} = -S_{R_5}^{\alpha}$$

$$S_{R_3}^{\alpha} = \frac{1}{1 + R_3/R_4} = -S_{R_4}^{\alpha}$$

$$S_{R_3}^{H_o} = -S_{R_4}^{H_o} = \frac{-1}{1 + R_4/R_3}$$

$$S_{R_5}^{H_o} = -S_{R_6}^{H_o} = \frac{1}{H_o}\frac{R_5/R_6}{1 + R_3/R_4}$$

DESIGN PROCEDURE

Given: α, $\omega_o = 2\pi f_o$
H_o is a free parameter.
Choose: $C = C_1 = C_2$, $R_5 = R_6 = R_3$

Calculate: $R_1 = R_2 = \dfrac{1}{\omega_o C}$

$$R_4 = \left(\frac{2}{\alpha} - 1\right) R_3$$

Then

$$H_o = 2 - \alpha$$

High Pass. The network parameters for the high-pass function are

$$H_o = \frac{1 + R_6/R_5}{1 + R_3/R_4}$$

$$\omega_o = \left(\frac{R_4}{R_5 R_1 R_2 C_1 C_2}\right)^{1/2}$$

$$\alpha = \frac{1 + R_6/R_5}{1 + R_4/R_3}\left(\frac{R_5}{R_6}\frac{R_2 C_2}{R_1 C_1}\right)^{1/2}$$

$$\phi = \phi_{HP}$$

$$\tau = \tau_{HP}$$

The sensitivities of the network parameters to element value changes are

$$S_{R_5}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_4}^{\omega_o} = \frac{1}{2}$$

$$S_{R_1}^{\alpha} = S_{C_1}^{\alpha} = -\frac{1}{2}$$

$$S_{R_2}^{\alpha} = S_{C_2}^{\alpha} = \frac{1}{2}$$

$$S_{R_5}^{\alpha} = -S_{R_6}^{\alpha} = \frac{1}{2} - \frac{R_6/R_5}{R_1 C_1 \alpha \omega_o (1 + R_4/R_3)}$$

$$S_{R_3}^{\alpha} = -S_{R_4}^{\alpha} = \frac{1}{1 + R_3/R_4}$$

$$S_{R_3}^{H_o} = -S_{R_4}^{H_o} = \frac{-1}{1 + R_4/R_3}$$

$$S_{R_5}^{H_o} = -S_{R_6}^{H_o} = \frac{1}{H_o}\frac{R_6/R_5}{1 + R_3/R_4}$$

DESIGN PROCEDURE

Given: α, $\omega_o = 2\pi f_o$
H_o is a free parameter.
Again a simplified design procedure is described by setting $R_5 = R_6$.
Choose: $C_1 = C_2 = C$
$R_5 = R_6 = R_3$

Calculate: $R_1 = R_2 = \dfrac{1}{\omega_o C}$

$$R_4 = \left(\frac{2}{\alpha} - 1\right) R_3 \qquad \alpha < 2$$

Bandpass. The network parameters for the bandpass case are

$$H_o = \frac{R_4}{R_3}$$

$$\omega_o = \left(\frac{R_6}{R_5 R_1 C_1 R_2 C_2}\right)^{1/2}$$

$$Q = \frac{1}{\alpha}\frac{1 + R_4/R_3}{1 + R_6/R_5}\left(\frac{R_6}{R_5}\frac{R_1 C_1}{R_2 C_2}\right)^{1/2}$$

$$\phi = \pi + \phi_{BP}$$

$$\tau = \tau_{BP}$$

The sensitivities of the network parameters to element values change are

$$S_{R_5}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_6}^{\omega_o} = \frac{1}{2}$$

$$S_{R_1}^{Q} = S_{C_1}^{Q} = +\frac{1}{2}$$

$$S_{R_2}^{Q} = S_{C_2}^{Q} = \frac{1}{2}$$

$$S_{R_6}^{Q} = S_{R_5}^{Q} = \frac{1}{2} - \frac{R_6/R_5}{R_1 C_1 \alpha_{\omega_o} (1 + R_4/R_3)}$$

$$S_{R_4}^{Q} = -S_{R_3}^{Q} = \frac{1}{1 + R_3/R_4}$$

$$S_{R_3}^{H_o} = -1 = -S_{R_4}^{H_o}$$

DESIGN PROCEDURE

Given: H_o, Q, $\omega_o = 2\pi f_o$
Again the simplified design procedure consists of setting $R_5 = R_6$.
Choose: $C_1 = C_2 = C$ $\qquad R_3 = R_5 = R_6$

$$R_1 = R_2 = \frac{1}{\omega_o C}$$
$$R_4 = R_2(2Q\cancel{K} - 1)$$

Note that all these filters can be tuned by varying R_1 and R_2 or C_1 and C_2 simultaneously. The Q can be independently adjusted by R_4; the gain will change, however.

8.3.4 Negative immittance converter circuits A realization for an INIC[1,2] (ideal current-inversion negative immittance converter) using a differential input operational amplifier is shown in Fig. 8.15. The voltage and current relationships are

$$E_1 = E_2$$
$$I_1 = \frac{R_2}{R_1} \qquad I_2 = \frac{1}{K} I_2$$

The sensitivity of K to element value changes is $S_{R_1}{}^K = -S_{R_2}{}^K = 1$.

One reason the INIC realization might be used is its low sensitivity to element value changes as compared with other realizations. However, the INIC realization does not have a low output impedance, and isolating stages must be used if stages are to be cascaded. Since low-pass and high-pass filters have low Q's and, hence, low Q sensitivities for filters up to about six poles, we will discuss only the bandpass realization. It is probably not economical to use the INIC for low-pass and high-pass filters.

Bandpass. The INIC realization for a bandpass filter is shown in Fig. 8.16.

The voltage transfer function is

$$\frac{E_o}{E_1}(s) = \frac{-Ks/R_1C_2}{s^2 + s(1/R_1C_1 + 1/R_2C_2 - K/R_1C_2) + 1/R_1C_1R_2C_2}$$

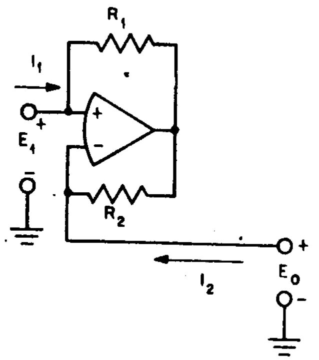

Fig. 8.15 Operational amplifier realization of the INIC.

The network parameters are

$$H_o = \frac{K}{C_2/C_1 + R_1/R_2 - K}$$
$$Q = \frac{1}{\alpha} = \frac{1}{\sqrt{R_1C_1/R_2C_2} + \sqrt{R_2C_2/R_1C_1} - K\sqrt{R_2C_1/R_1C_2}}$$
$$\omega_o = \left(\frac{1}{R_1C_1R_2C_2}\right)^{1/2}$$
$$\phi = \pi + \phi_{BP}$$
$$\tau = \tau_{BP}$$

The sensitivities of the H_o, Q, ω_o network parameters to element value changes are

$$S_K{}^{H_o} = 1 + H_o$$
$$S_{R_1}{}^{H_o} = \frac{-R_1/R_2}{C_2/C_1 + R_1/R_2 - K} = -S_{R_2}{}^{H_o}$$
$$S_{C_1}{}^{H_o} = \frac{C_2/C_1}{C_2/C_1 + R_1/R_2 - K} = -S_{C_2}{}^{H_o}$$
$$S_{R_1}{}^{Q} = \frac{Q}{\omega_o R_1}\left(\frac{1}{C_1} - \frac{K}{C_2}\right) - \frac{1}{2}$$
$$S_{R_2}{}^{Q} = \frac{Q}{\omega_o R_2 C_2} - \frac{1}{2}$$
$$S_{C_1}{}^{Q} = \frac{Q}{\omega_o R_1 C_1} - \frac{1}{2}$$
$$S_{C_2}{}^{Q} = \frac{Q}{\omega_o C_2}\left(\frac{1}{R_2} - \frac{K}{R_1}\right) - \frac{1}{2}$$
$$S_K{}^{Q} = \frac{QK}{\omega_o R_1 C_2}$$
$$S_{R_1}{}^{\omega_o} = S_{R_2}{}^{\omega_o} = S_{C_1}{}^{\omega_o} = S_{C_2}{}^{\omega_o} = -\frac{1}{2}$$

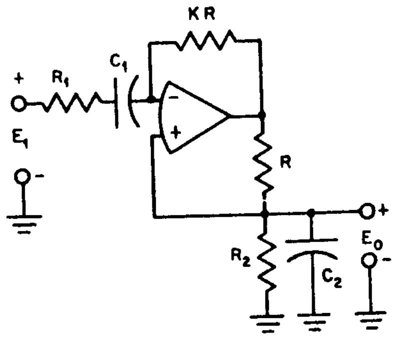

Fig. 8.16 INIC bandpass network.

8.7 Filter Design and Tuning Tables

TABLE 8.2 Butterworth Network Parameters

		Design		Tuning	
Number of poles	Stage	α	ω_o	ω_a or -3 dB* frequency	20 log $G(\omega_a)/G(o)$
2	1	1.414214	1.000000	1.000*	
3	1	a real pole	1.000000	1.000	
	2	1.000000	1.000000	0.707	1.25
4	1	1.847759	1.000000	0.719*	
	2	0.765367	1.000000	0.841	3.01
5	1	a real pole	1.000000	1.000*	
	2	1.618034	1.000000	0.859*	
	3	0.618034	1.000000	0.899	4.62
6	1	1.931852	1.000000	0.676*	
	2	1.414214	1.000000	1.000*	
	3	0.517638	1.000000	0.931	6.02
7	1	a real pole	1.000000	1.000*	
	2	1.801938	1.000000	0.745*	
	3	1.246980	1.000000	0.472	0.22
	4	0.445042	1.000000	0.949	7.25
8	1	1.961571	1.000000	0.661*	
	2	1.662939	1.000000	0.829	
	3	1.111140	1.000000	0.617	0.69
	4	0.390181	1.000000	0.961	8.34
9	1	a real pole	1.000000	1.000*	
	2	1.879385	1.000000	0.703*	
	3	1.532089	1.000000	0.917*	
	4	1.000000	1.000000	0.707	1.25
	5	0.347296	1.000000	0.969	9.32
10	1	1.985377	1.000000	0.655*	
	2	1.782013	1.000000	0.756*	
	3	1.414214	1.000000	1.000*	
	4	0.907981	1.000000	0.767	1.84
	5	0.312869	1.000000	0.975	10.20

* Butterworth filters are frequency-normalized to give -3-dB response at $\omega = 1.0$.

TABLE 8.3 Bessel Network Parameters

		Design		Tuning	
Number of poles	Stage	α	ω_o	ω_a or -3 dB* frequency	20 log $G(\omega_a)/G(o)$
2	1	1.732051	1.732051	1.362*	
3	1	a real pole	2.322185	2.322*	
	2	1.447080	2.541541	2.483*	
4	1	1.915949	3.023265	2.067*	
	2	1.241406	3.389366	1.624	0.23
5	1	a real pole	3.646738	3.647*	
	2	1.774511	3.777893	2.874*	
	3	1.091134	4.261023	2.711	0.78
6	1	1.959563	4.336026	2.872*	
	2	1.636140	4.566490	3.867*	
	3	0.977217	5.149177	3.722	1.38
7	1	a real pole	4.971785	4.972*	
	2	1.878444	5.066204	3.562*	
	3	1.513268	5.379273	5.004*	
	4	0.887896	6.049527	4.709	1.99
8	1	1.976320	5.654832	3.701*	
	2	1.786963	5.825360	4.389*	
	3	1.406761	6.210417	0.637	0.00
	4	0.815881	6.959311	5.680	2.56
9	1	a real pole	6.297005	6.297*	
	2	1.924161	6.370902	4.330*	
	3	1.696625	6.606651	5.339*	
	4	1.314727	7.056082	2.600	0.08
	5	0.756481	7.876636	6.655	3.09
10	1	1.984470	6.976066	4.540*	
	2	1.860312	7.112217	5.069*	
	3	1.611657	7.405447	6.392*	
	4	1.234887	7.913585	3.857	0.25
	5	0.706560	8.800155	7.623	3.60

* Bessel filters are frequency-normalized to unity delay $\tau(\omega) = 1$ sec at $\omega = 0$.