$$
\begin{aligned}
y &= x^2 + bx + c \\
&= x^2 + 2 \cdot \frac{b}{2}x + c \\
&= \underbrace{x^2 + 2 \cdot \frac{b}{2}x + \left(\frac{b}{2}\right)^2}_{\left(x + \frac{b}{2}\right)^2} - \left(\frac{b}{2}\right)^2 + c \\
&= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \qquad\qquad \left| \; + \left(\frac{b}{2}\right)^2 - c \right. \\
y + \left(\frac{b}{2}\right)^2 - c &= \left(x + \frac{b}{2}\right)^2 \qquad\qquad\qquad\quad \left| \; \text{(Scheitelpunktform)} \right. \\
y - y_S &= (x - x_S)^2 \\
S(x_S; y_S) \quad &\text{bzw.} \quad S\left(-\frac{b}{2}; \left(\frac{b}{2}\right)^2 - c\right)
\end{aligned}
$$