$$= \lim_{n \to \infty} \Delta x \cdot (f(a) + f(a + \Delta x) + f(a + 2 \cdot \Delta x) + f(a + 3 \cdot \Delta x) + \dots + f(a + (n - 1) \cdot \Delta x))$$

$$= \lim_{n \to \infty} \Delta x \left( a^2 + (a + \Delta x)^2 + (a + 2 \cdot \Delta x)^2 + (a + 3 \cdot \Delta x)^2 + \dots + (a + (n - 1) \cdot \Delta x)^2 \right)$$

 $= \lim_{n \to \infty} \left( \Delta x \cdot f(a) + \Delta x \cdot f(a + \Delta x) + \Delta x \cdot f(a + 2 \cdot \Delta x) + \Delta x \cdot f(a + 3 \cdot \Delta x) + \ldots + \Delta x \cdot f(a + (n - 1) \cdot \Delta x) \right)$ 

 $A = \lim_{n \to \infty} U = \lim_{n \to \infty} \sum_{i=0}^{n-1} (\Delta x \cdot f(a+i \cdot \Delta x))$ 

$$= \lim_{n \to \infty} \Delta x \left( a^2 + \left( a^2 + 2a\Delta x + (\Delta x)^2 \right) + \left( a^2 + 2 \cdot 2a\Delta x + 2^2 (\Delta x)^2 \right) + \left( a^2 + 2 \cdot 3a\Delta x + 3^2 (\Delta x)^2 \right) + \dots + \left( a^2 + 2 \cdot (n-1)a\Delta x + (n-1)^2 (\Delta x)^2 \right) \right)$$

$$-\frac{1}{4}\left(a^{2}+2\cdot(n-1)a\Delta x+(n-1)^{2}\left(\Delta x\right)^{2}
ight)$$

$$2\cdot (n-1)a\Delta x + (n-1)^2 \left(\Delta x
ight)^2 \Big) \Big)$$

$$2 \cdot (n-1)a\Delta x + (n-1)^2 (\Delta x)^2 \bigg) \bigg)$$

$$\left((n-1)a\Delta x + (n-1)^2(\Delta x)^2\right)\right)$$

$$-1)a\Delta x + (n-1)^2 (\Delta x)^2$$
 )

$$\Delta x + (n-1)^2 (\Delta x)^2$$
  $\Big)$ 

$$+ (n-1)^{2} (\Delta x)^{2}$$
  
 $+ 3 + ... + (n-1)) + (\Delta x)^{2} (1^{2} + 2^{2} + 3^{2} + ... + (n-1)^{2})$ 

$$(x)^{2} (\Delta x)^{2}$$
  $)$   $(x)^{2} (x^{2} + 2^{2} + 3^{2} + ... + (n-1)^{2})$ 

$$(2^{2}(\Delta x)^{2})$$
  
  $(2^{2} + (n-1)) + (\Delta x)^{2} (1^{2} + 2^{2} + 3^{2} + ... + (n-1)^{2})$ 

$$^{2}(\Delta x)^{2}$$
)  
+  $(n-1)$ ) +  $(\Delta x)^{2}(1^{2} + 2^{2} + 3^{2} + ... + (n-1)^{2})$ )

- $= \lim_{n \to \infty} \Delta x \left( na^2 + 2a\Delta x \left( 1 + 2 + 3 + \dots + (n-1) \right) + (\Delta x)^2 \left( 1^2 + 2^2 + 3^2 + \dots + (n-1)^2 \right) \right)$
- $= \lim_{n \to \infty} \frac{b a}{n} \left( na^2 + 2a \frac{b a}{n} \frac{n(n-1)}{2} + \left( \frac{b a}{n} \right)^2 \frac{n(2n-1)(n-1)}{6} \right)$  $= \lim_{n \to \infty} \Delta x \left( na^2 + 2a\Delta x \frac{n(n-1)}{2} + (\Delta x)^2 \frac{n(2n-1)(n-1)}{6} \right)$ 
  - $= \lim_{n \to \infty} \frac{b-a}{n} \left( na^2 + a(b-a)(n-1) + \frac{(b-a)^2}{n} \frac{(2n-1)(n-1)}{6} \right)$

- $= \lim_{n \to \infty} (b a) \left( a^2 + a(b a) \frac{n 1}{n} + \frac{(b a)^2}{n} \frac{(2n 1)(n 1)}{n} \right)$ 
  - - $= \lim_{n \to \infty} (b a) \left( a^2 + a(b a) \left( 1 \frac{1}{n} \right) + (b a)^2 \frac{1}{6} \left( 2 \frac{3}{n} + \frac{1}{n^2} \right) \right)$

jeweils Nullfolgen für  $n \to \infty$