Expt. # 04: Reversible pendulum (Kater's pendulum)

Aim: Precise determination of local gravitational acceleration (g) from the study of small angle oscillation of a reversible pendulum (Kater's pendulum)

Why bother about gravitational acceleration? Any motion of an object on earth is influenced by earth's gravity. For calculation of such motion under earth's gravity one needs to use the gravitational acceleration (g). Therefore it is important to determine g precisely.

Why not use a simple pendulum for this? We know that time period of a simple pendulum of length l is given by (**under small oscillation approximation!**)(*Do you know what it is?*) $T = 2\pi\sqrt{l/g}$ $\quad (1)$. Why not use a simple pendulum and such a simple relation as in Eq. (1) to determine g? Look at the idealized approximations used in deriving Eq. (1)! A simple pendulum is ***point mass m*** hanging from a ***massless string*** of length l! Can you realize in practice a point mass and a massless string? No. So in practice you use a spherical heavy bob and long and light string (see Fig. 1) such that the mass of the string is negligible compared to that of the bob and the diameter of the bob is negligible compared to the length of the string. Such approximation limits the precision of estimation of g using a simple pendulum.

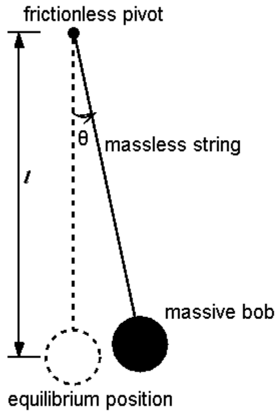

Figure 1: Simple pendulum.

Why not use a physical pendulum (compound pendulum)? A physical (compound) pendulum is a rigid body (with regular or irregular shape) swinging in vertical plane about any horizontal axis passing through the rigid body (see Fig. 2). The time period for such a pendulum is given by $T = 2\pi\sqrt{\frac{I}{MgL}} = 2\pi\sqrt{\frac{L^2+K^2}{gL}} = 2\pi\sqrt{\frac{L^*}{g}}$ $\quad (2)$, where, I is the moment of inertia about the pivot, K is the radius of gyration about center of mass, L is the distance of pivot from the center of mass, and $L^* = L\left(1+\frac{K^2}{L^2}\right)$ is the e*quivalent length* of a simple of pendulum which would have the same time period as the compound pendulum. A point situated below the center of mass of the compound pendulum at a distance L^* from the pivot point is known as the *center of oscillation*. Its position can be estimated theoretically for a regular shaped pendulum with uniform mass distribution. But for an arbitrary shaped pendulum and with uncertainty in the actual mass distribution, it is practically very difficult to accurately locate the center of oscillation. Same problem arises for precise determination of center of mass and in calculation of I, K, and L. **** What are center of mass, moment of inertia, and radius of gyration? ****

What is a reversible pendulum? Dutch scientist Christian Huygens had shown mathematically that the pivot point and center of oscillation for a compound pendulum are interchangeable! That is, if a compound pendulum is suspended upside down with center of oscillation as the new pivot it will have the same time period and old pivot point will be the new center of oscillation. The distance between these two conjugate points will then be the equivalent length L^*.

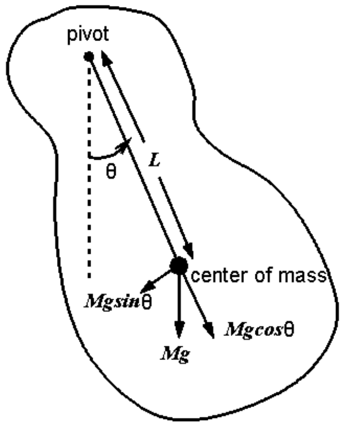

Figure 2: Compound pendulum.

British physicist Henry Kater used this idea to build a reversible pendulum which would practically allow to determine center of oscillation accurately. It is known as Kater's pendulum. It consists of a long bar and two mass blocks whose position can be adjusted along the bar (Fig. 3). In practice, one of the mass block is kept fixed and position of the other is adjusted which allows to vary the mass distribution and thereby the position of center of mass and center of oscillation. This results in changing the time period of oscillation of the pendulum. The pendulum has two knife-edge suspension points (pivot points). It can be set to oscillation from either of the two pivot points. Time period of oscillation is measured for small oscillation from both the pivot points, in turn, for different position of the movable mass block till one gets same time period from both the pivot points. This ensures that for this mass configuration we have got the two conjugate points. If one of them is used as the pivot point, the other will be the point of oscillation, and separation between these points will the equivalent length L^*.

Refer to Fig. 3. In this figure the center of mass (CM) is a conceptual point (denoted by x). For this experiment, we do not need to know the exact location of the CM and need not measure the distances $\boldsymbol{a}$ and $\boldsymbol{b}$ separately. Using parallel axis theorem, the moment of inertia $\boldsymbol{I_a}$ about the pivot point $\boldsymbol{A}$ can be written in terms of the moment of inertia $\boldsymbol{I_{cm}}$about the center of mass as $\boldsymbol{I_a = I_{cm} + Ma^2 = M(K^2 + a^2)}$, where $\boldsymbol{I_{cm} = MK^2}$ with $\boldsymbol{M}$ being the total mass of the pendulum and $\boldsymbol{K}$ being the radius of gyration about the center of mass. Time period for small oscillation for the pivot point$\boldsymbol{A}$ is then given by

$$\boldsymbol{T_a = 2\pi\sqrt{\frac{I_a}{Mga}} = 2\pi\sqrt{\frac{K^2+a^2}{ga}}} \qquad (3).$$

Similarly, time period for small oscillation for the pivot point $\boldsymbol{B}$is then given by

$$\boldsymbol{T_b = 2\pi\sqrt{\frac{I_b}{Mgb}} = 2\pi\sqrt{\frac{K^2+b^2}{gb}}}. \qquad (4)$$

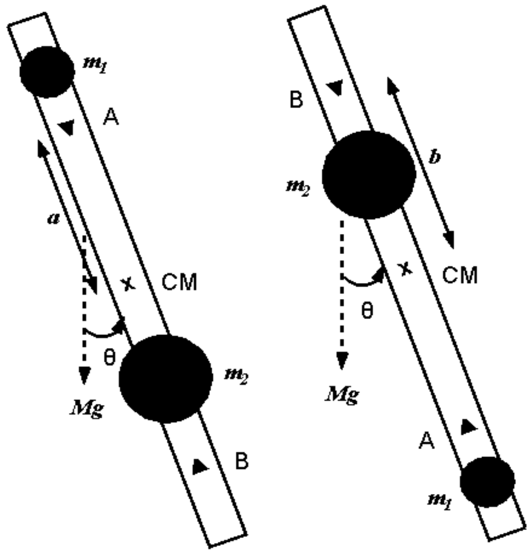

Figure 3: Kater's reversible pendulum.

It can be shown (**do it yourself!**) that for $\boldsymbol{T_a = T_b}$, either $\boldsymbol{a = b}$ or$\boldsymbol{ab = K^2}$. Using second condition it can be shown (**do it yourself!**) that

$$\boldsymbol{T_a = T_b = T = 2\pi\sqrt{(a+b)/g} \quad or \quad g = 4\pi^2(a+b)/T^2} \qquad (5)$$

The quantity $\boldsymbol{a + b}$ is the distance between the two pivot points. Equation (5) can be used to determine $\boldsymbol{g}$.

Experimental procedure:

1. Carefully measure the distance $\boldsymbol{d = a + b}$ between the two knife-edges (pivot points) and tabulate data.
2. Treat the mass block of **1000 g** as fixed mass $\boldsymbol{m_1}$ and mass block of **1400 g** as movable mass $\boldsymbol{m_2}$**.** Treat the pivot point closer to $\boldsymbol{m_1}$as$\boldsymbol{A}$. Set $\boldsymbol{m_1}$ at a distance $\boldsymbol{x_1 = 25}$ cm and $\boldsymbol{m_2}$ at a distance $\boldsymbol{x_2 = 15}$ cm from $\boldsymbol{A}$, taking account of the radius $\boldsymbol{r = 5}$ cm of the mass blocks (Fig. 4).

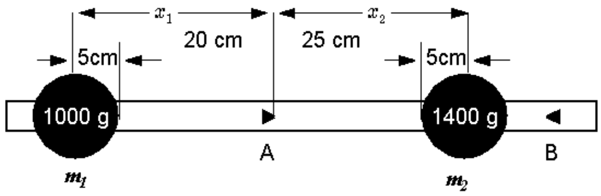

Figure 4: Positioning of mass blocks.

3. Carefully hang the pendulum from pivot point $\boldsymbol{A}$in the wall mount and set it to oscillation (with small amplitude) in a plane parallel to the wall.
4. Wait for 2-3 oscillation (for stabilization) and then use a stop watch to measure carefully the time of **10** oscillations. This is $\boldsymbol{10T_a}$.Repeat the measurement once more.
5. Reverse the pendulum, hang it from pivot point $\boldsymbol{B}$, set it to small oscillation, wait for 2-3 oscillation, and then measure carefully time for **10** oscillations. This is $\boldsymbol{10T_b}$. Repeat the measurement once more.
6. Move $\boldsymbol{m_2}$ by **5** cm to a distance $\boldsymbol{x_2 = 20}$cm and repeat steps 2 – 4.
7. Repeat the procedure until you reach $\boldsymbol{x_2 = 90}$ cm. Record all data in a tabulated form.

Calculation:

1. Plot (on the same graph) $\boldsymbol{T_a^2}$ and $\boldsymbol{T_b^2}$ as a function of $\boldsymbol{x_2}$in origin. Plot data by points only, do not join them by lines. Properly label the graph axes and write units.
2. Fit the data by a second order polynomial (fit both data sets separately). The fitted curves will intersect at two points. At these points$\boldsymbol{T_a = T_b = T}$. Locate the two intersection points precisely using screen reading function of origin and calculate average value of T^2 from these two intersection points.

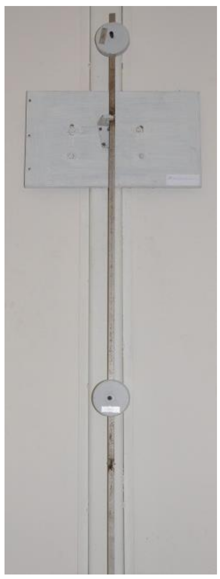
Figure 5: Photograph of experimental setup.

Error analysis: Do the error analysis yourself. Consult the notes on error analysis available at course webpage in WeLearn.