Expt. # 20: Study of resonance phenomena in series LCR AC circuit

Aim: To draw the resonance curve and to obtain quality factor from resonance curve for two different values of R.

Description of apparatus and principle of experiment:

This manual assumes that you have basic knowledge of electrical circuit elements like inductance (L), capacitance (C), resistance (R) and their responses to applied AC or DC voltages. If required, read it from a book.

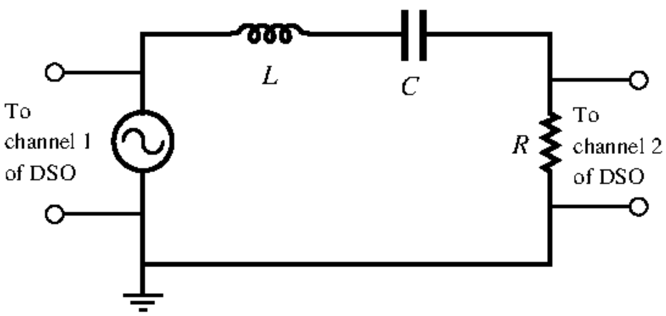

Fig.1: The circuit diagram

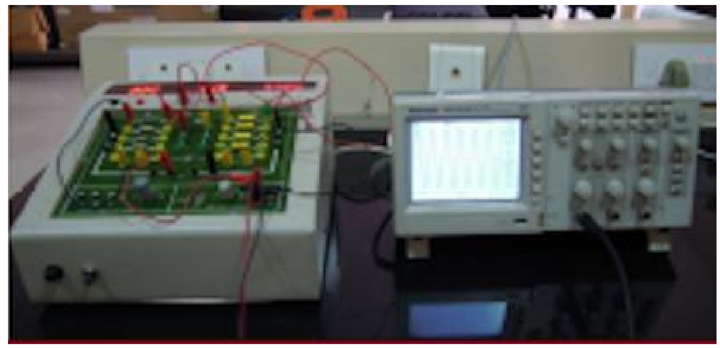

Fig.2: A photograph of the set-up

Schematic of a series LCR circuit is shown in Fig. 1 and a photograph of the actual setup in our laboratory is shown in Fig. 2. The symbols have their usual meaning. Experimental setup consists of a LCR panel board and a digital storage oscilloscope (DSO). The LCR panel board has several resistors, capacitors, and inductors. It also has a voltage source whose frequency and amplitude can be varied. The frequency is displayed in a frequency meter on board. Besides these, the panel has a voltmeter and an ammeter. These components can be appropriately connected to make the series LCR circuit shown in Fig. 1.

The voltmeter and the ammeter of the LCR panel are not very precise and do not work well over the entire frequency range of the voltage source. So, we shall not use these. Instead, we shall use an oscilloscope to measure (peak to peak) input voltage V_I. The current passing through the system will be obtained by measuring the voltage across the resistor, as $i = V_R / R$.

The current i flowing through the circuit depends on the amplitude and frequency of the sinusoidal input voltage V_I. At low frequency, the current i is small because the reactance of the capacitor is high at low frequency. As the frequency of V_I is increased, keeping its amplitude fixed, the current i increases. It reaches a maximum at an appropriate frequency and then decreases with further increase in frequency and becomes small at high frequency when the reactance due to the inductor becomes large. This behaviour of the current is known as resonance and the frequency at which the current reaches maximum is known as the resonance frequency. **We want to emphasize that the resonance curve is not at all symmetric with respect to the resonance frequency. We shall show it theoretically in the Appendix.**

Look at Fig. 1. Impedance of the circuit: [neglecting resistances associated with a voltage source, inductor and capacitor]

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)\ \left(j = \sqrt{-1}\right) \tag{1}$$

$$\therefore |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2}$$

We can write $V_I = Zi$ or $i = \frac{V_I}{Z}$.

$$\therefore |i| = \frac{|V_I|}{|Z|} = \frac{|V_I|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{3}$$

We see from Eq. (3) that the magnitude of current ($|i|$) depends on frequency through the bracketed squared term in the denominator. The magnitude of current ($|i|$) will be maximum (resonance occurs) when the denominator of Eq. (3) will be minimum. This will happen if for some frequency ω_0(the resonance frequency), the bracketed squared term in the denominators of Eq. (3) becomes zero, i.e., $\omega_0 L - \frac{1}{\omega_0 C} = 0$.

This gives the expression for the **resonance frequency:** $\omega_0 = \frac{1}{\sqrt{LC}}$ (4)

Equation (3) can be rewritten as

$$|i| = \frac{\frac{|V_I|}{R}}{\sqrt{1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{|i_0|}{\sqrt{1 + \frac{L}{R^2 C}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)^2}} = \frac{|i_0|}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} = \frac{|i_0|}{\sqrt{1 + Q^2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2}} \tag{5}$$

Here, $|i_0| = \frac{|V_I|}{R}$ is the magnitude of current at resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonance frequency, and the linear frequency f is related to the angular frequency ω by $\omega = 2\pi f$.

The quality factor Q is defined as:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \tag{6}$$

where, $\Delta\omega$ is the **full width** of the resonance curve **at half-power point**, i.e. when $|i| = \frac{|i_0|}{\sqrt{2}}$. The quality factor Q is a measure of the sharpness of a resonance curve. The larger the Q, the sharper is the resonance curve. It is shown in Appendix that Q as defined in Eq. (6) can also be expressed as:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \tag{7}$$

This shows that for a given L and C (which fixes the resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$) Q is larger for smaller R.

Procedure:

1. The frequency of the voltage source in the LCR panel board can be varied continuously through a frequency knob in 3 different ranges: $10 - 100$ Hz, $100 - 1000$ Hz and $1 - 10$ kHz, accessible through 3 frequency range selector buttons. Use $100 - 1000$ Hz and $1 - 10$ kHz range for your experiment.

2. The LCR panel board has different sets of R, L, and C. Choose the L and C values such that the resonance frequency $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{(2\pi\sqrt{LC})}$ falls well inside the accessible frequency range which is $0.1 - 10$ kHz. Choose the smallest value of R.
3. Make the circuit as shown in Fig. 1. Connect V_I to channel 1 of the oscilloscope using a banana (or crocodile) to BNC cable. Give the voltage across the resistor as the input to channel 2. Note that the neutral lines of the two channels of a CRO are internally connected. So be careful to ensure that only one of them is connected to the circuit.
4. Power on the LCR panel board and the oscilloscope. Set the voltage source frequency to about 100 Hz. Frequency is seen from the frequency counter of the panel. Oscilloscope also gives frequency readings.
5. Use the 10 V peak-to-peak range button for the voltage source. Set the peak-to-peak value of V_I to about 8 V. You will use the oscilloscope to measure V_I.
6. If necessary, take the help of your lab instructor/assistant to set up the oscilloscope. You may start oscilloscope setting by pressing **“Autoset”** button on the oscilloscope.
7. Press Ch1 menu. Check settings of the oscilloscope: Trigger source Ch1, coupling AC. Set appropriate voltage and time range.
8. Press **“Measure”** button of the oscilloscope. This will display the frequency, peak-to-peak value etc. of the Ch1 voltage on the screen.
9. Read the current (peak to peak) from the channel 2 of the DSO. Record the data (current vs frequency) in a tabular form.
10. Change the frequency in steps of $0.2 - 0.4$ kHz and repeat step 9. Make sure that the peak-to-peak value of V_I is the same (say 8 V) for all frequencies. If it changes with frequency, adjust the amplitude button on the LCR panel to set it to 8 V at every frequency, before reading the current i. Do it up to the maximum frequency (10 kHz).
11. **Repeat the experiment for a different value of R, keeping L and C unchanged.**
12. Plot the resonance curve (i against f) for two values of R on the same graph. Determine the resonance frequency. Compare it with the calculated $f_0 = \frac{1}{(2\pi\sqrt{LC})}$.
13. Determine the full widths $\Delta f$of the resonance curves at half-power points, i.e., when $|i| = \frac{|i_0|}{\sqrt{2}}$ (i_0 V is the maximum value of current, i.e. at resonance) and calculate the quality factors $Q = \frac{f_0}{\Delta f}$. Compare it with calculated $Q = \frac{\sqrt{L}}{(R\sqrt{C})}$.
14. Try to fit the resonance curves with the expression given in Eq. (5). Compare values of f_0 and Q obtained from fit to those estimated in steps 11-12.

Appendix:

1. Note that the resonance curve is not symmetric about the resonance frequency f_0. We can rewrite Eq. (5) as

$$V_R \propto |i| = \frac{|i_0|}{\sqrt{1+Q^2\left(\frac{f^2-f_0^2}{f_0 f}\right)^2}} = \frac{|i_0|}{\sqrt{1+Q^2\left(\frac{(f-f_0)(f+f_0)}{f_0 f}\right)^2}} = \frac{|i_0|}{\sqrt{1+Q^2(f-f_0)^2\left(\frac{1}{f_0}+\frac{1}{f}\right)^2}} \quad (8)$$

It is clearly seen from Eq. (8) that value of $|i|$at $f = f_0 + \delta$ is larger than that at $f = f_0 - \delta$.

2. We can see from Eq. (5) that $|i|$ at $f_1 = \alpha f_0$ is the same as that at $f_2 = \frac{f_0}{\alpha}$. This means that, if we get a value, say $|i_1|$at $f_1 = 0.5f_0$, then we get the same value $|i_1|$at $f_2 = \frac{f_0}{0.5} = 2f_0$, not at $f_2' = 1.5f_0$. This tells us that if we plot the resonance curve with frequency $f$in logarithmic scale, it will look symmetric about f_0.

3. Resonance curve looks symmetric in a linear scale within a small range of frequency near f_0. Let us say, we get a value, say $|i_1|$at $f = f_0(1 + x)$. Then we shall get the same value $|i_1|$ **at** $f = \frac{f_0}{(1+x)} \simeq f_0(1 - x)$ for $x \ll 1$. For very large Q, i.e., for very sharp resonance when $|i|$ will substancially fall within a short range about f_0 the curve will appear symmetric [see Fig. 3].

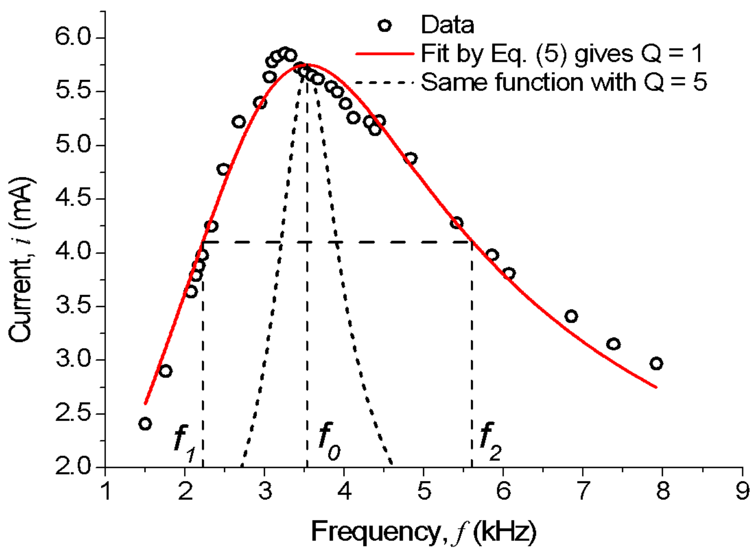

Figure 3: Example of data and fit to data by Eq. (5). For small Q resonance curve is asymmetric. For large Q it appears symmetric.

$$1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 = 2 \quad \Rightarrow \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \pm\frac{1}{Q} \quad \Rightarrow \omega^2 \mp \frac{\omega_0}{Q}\omega - \omega^2 = 0 \qquad (9)$$

Equation (9) has the solutions: ***Here we take only + sign because ω has to be +ve.***

$$\omega_\pm = \frac{\pm\frac{\omega_0}{Q} + \sqrt{\left(\frac{\omega_0}{Q}\right)^2 + 4\omega_0^2}}{2}$$

$$\therefore \Delta\omega = \omega_+ - \omega_- = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{Q}\right)^2 + 4\omega_0^2} + \frac{\omega_0}{2Q} - \sqrt{\left(\frac{\omega_0}{Q}\right)^2 + 4\omega_0^2} = \frac{\omega_0}{Q} \qquad \Rightarrow Q = \frac{\omega_0}{\Delta\omega}$$

4. At half-power points denominator of Eq. (5) should be $\sqrt{2}$, i.e.,

5. For LCR curve fitting, the appropriate function (name: LCRseries) is defined in the computer next to printer. Use it. A good choice of for L and C may be: $L = 21$ mH and $C = 0.1$ µF, R can be chosen to be **100** and **250** Ω.